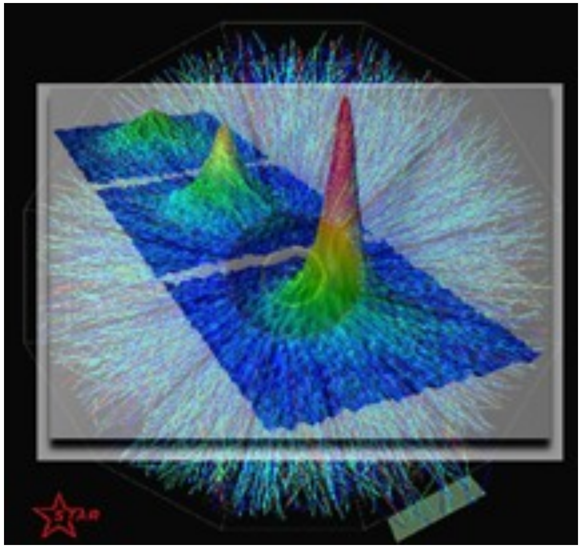
2014 Berkeley Summer School on Collective Dynamics, June 9, 2014

Pre-Equilibrium Physics in Heavy Ion Collisions



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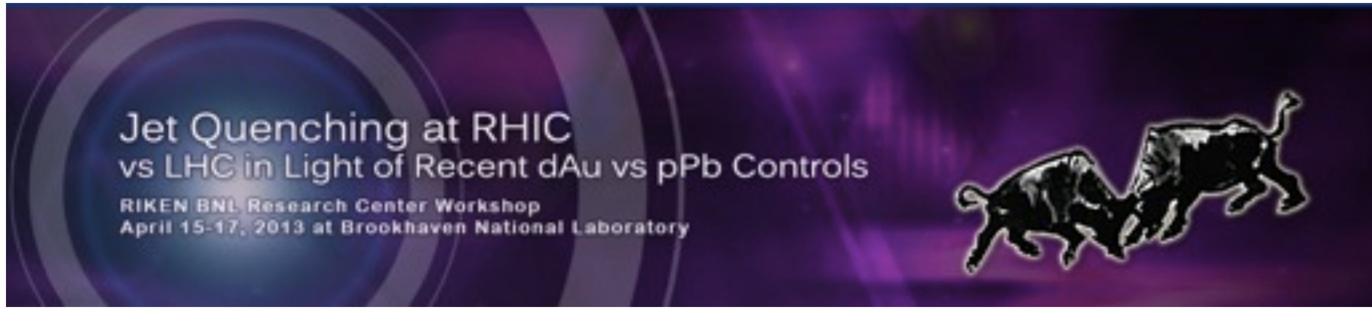
Research Supported by NSF



pA studies 1972-2013

reminiscences

The pA play
as seen through the eyes of one of the actors

Act 1 before the early 1970's	The "A" of "pA" is more of a nuisance than a help!
Act 2 The 1970's	Is there too much or too little cascading?
Act 3 late 1970's, early 1980's	Is there too much or too little quenching in the forward direction?
Act 4 Late 1980's, 1990's & 2000's	Who cares about the details of "pA" ? After all, it's only a reference!
Act 5 To-day	Who is helping whom? pp & pA the understanding of AA or AA the understanding of pp & pA?

Outline

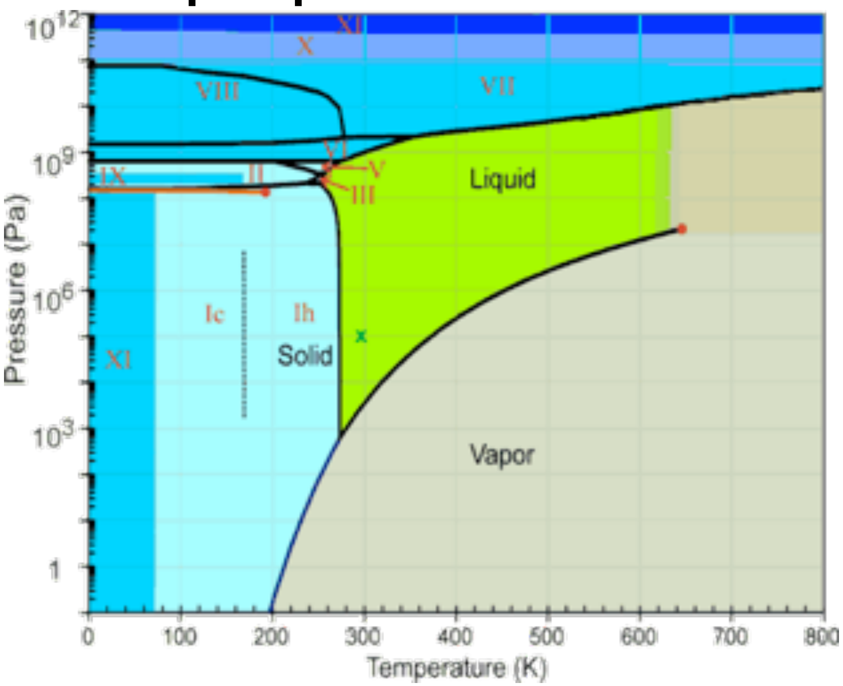
- Pre-Equilibrium: Setting the Stage
- Different Approaches
- A Quick Primer on Kinetic Theory
- Recent Developments: Overpopulated Glasma
- Summary & Outlook

General References (recent review articles):
Huang & JL, arXiv:1402.5578;
Berges, Blaizot, Gelis, arXiv:1203.2042;
Gelis, arXiv:1211.3327;
Strickland, arXiv:1312.2285;
Arnold, arXiv:0708.0812.

Pre-Equilibrium: Setting the Stage

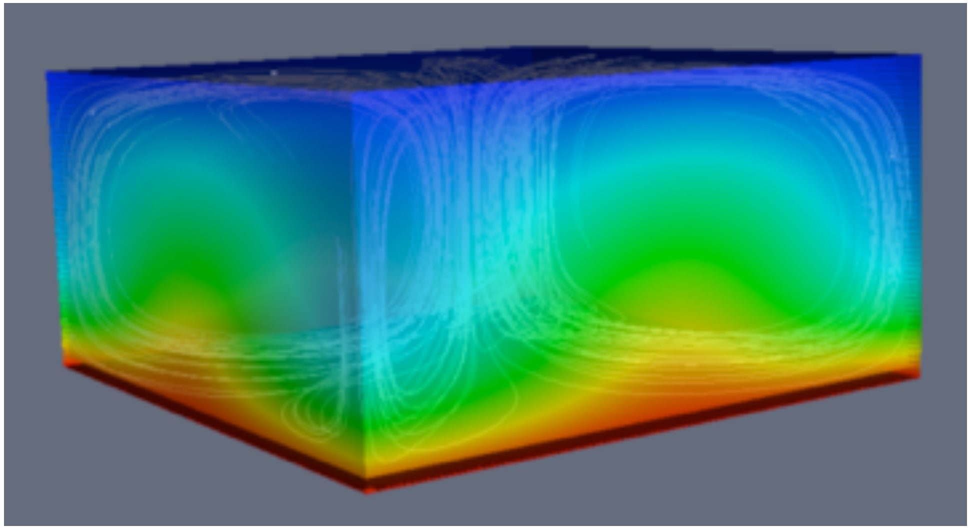
Let's Start with Normal Matter

We study their equilibrium properties in details.



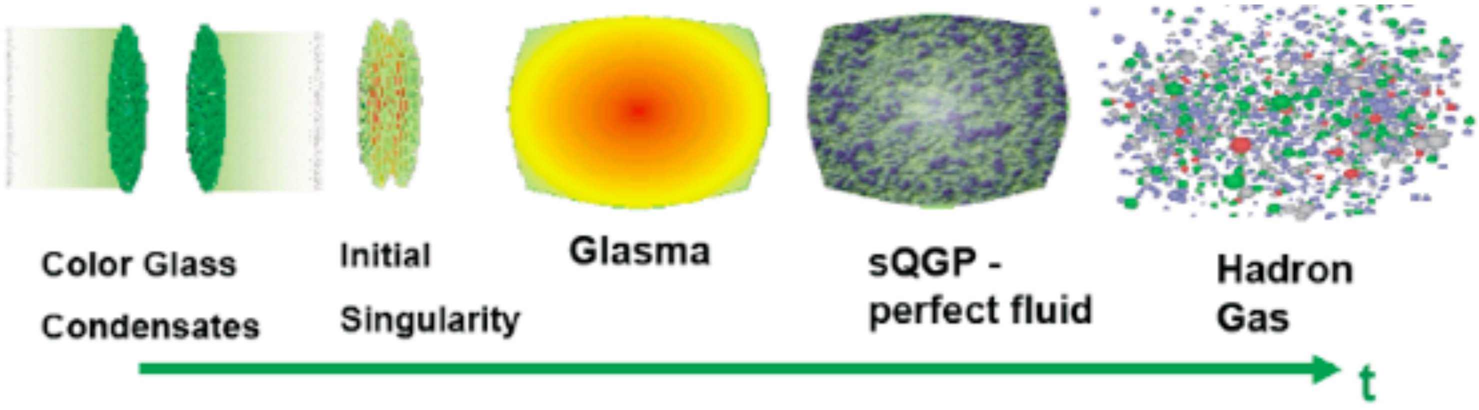
phase diagram of water

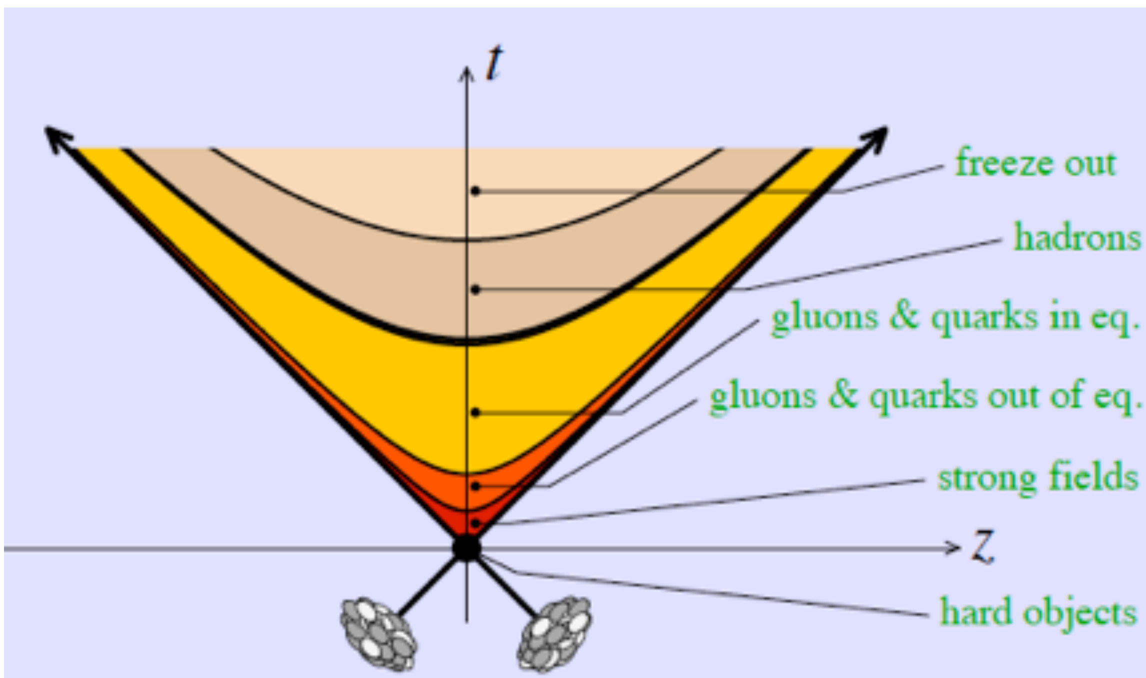
Out-of-equilibrium phenomena are also extremely interesting and rich.



Rayleigh Bernard convection

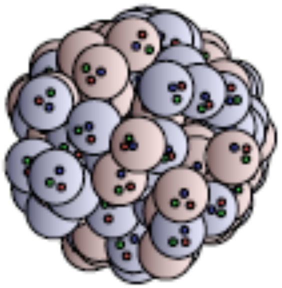
Different Stages of Heavy Ion Collisions

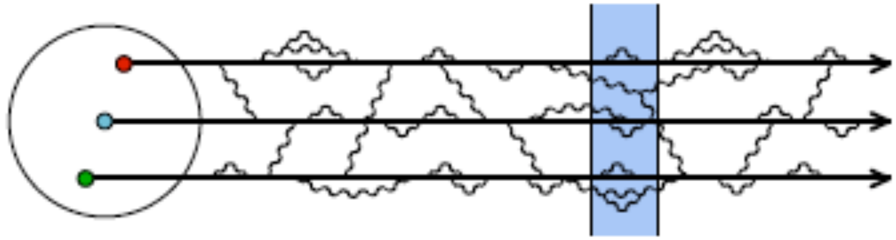




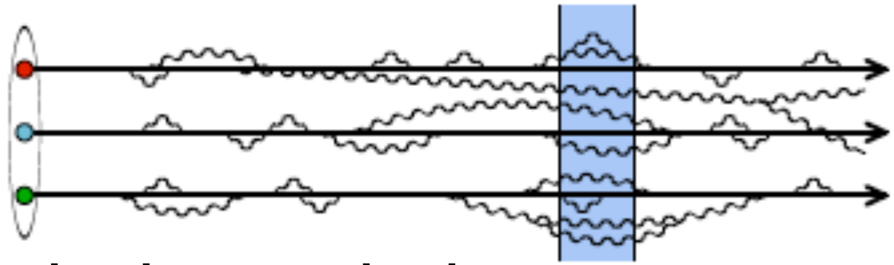
Probing
matter properties:
thermal
&
near thermal
(transport)
&
far-from thermal

What is a Proton/Nucleus?





hadron at low energy



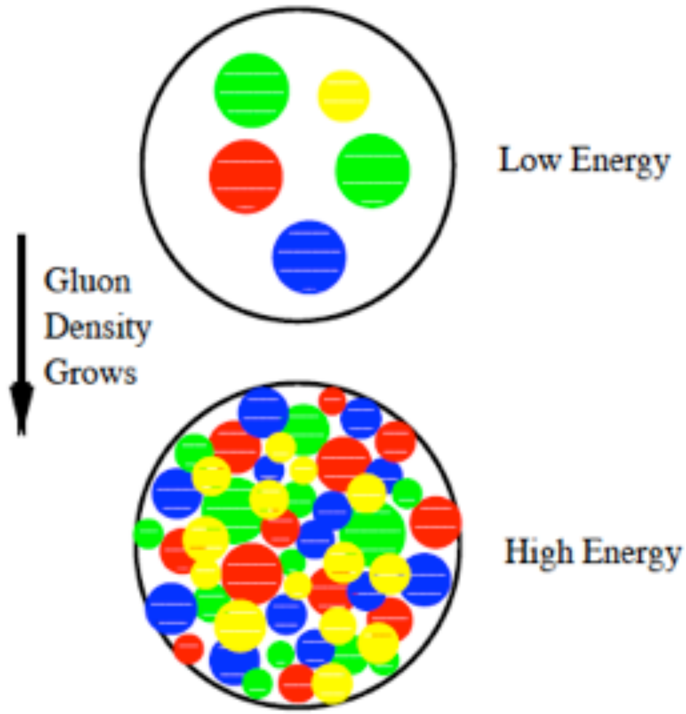
hadron at high energy

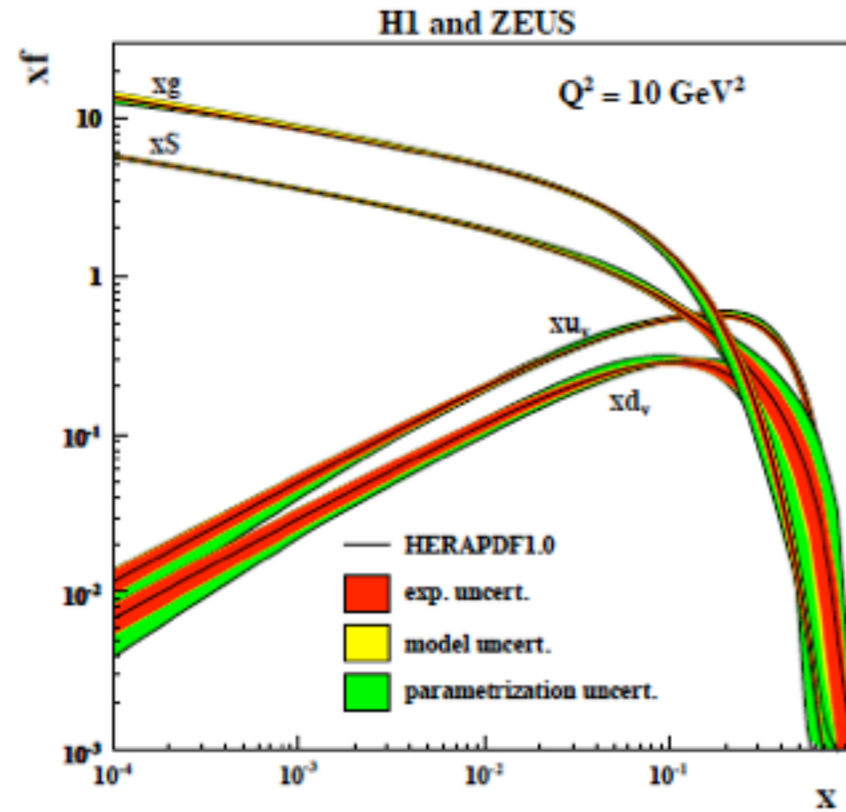
$$\tau \sim 1/\delta E \to \gamma\tau$$

Sending a hadron to high energy --> dilate the quantum fluctuations, and make "snapshot" with high resolution

At the Very Beginning...

Small-x part is important and dominated by gluons



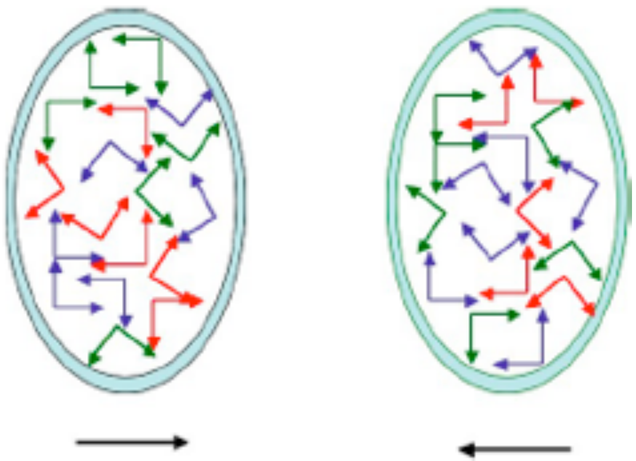


$$Q^2 \le \underbrace{Q_s^2 = \frac{\alpha_s x G(x, Q_s^2)}{A^{2/3}}}_{\text{saturation momentum}} \sim A^{1/3} x^{-0.3}$$

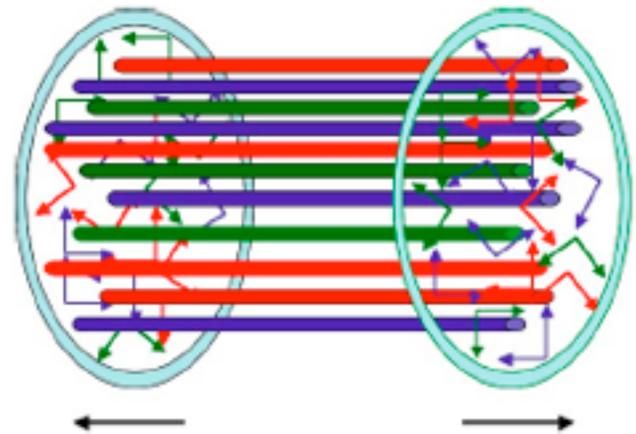
See lectures by Jean-Paul, Ernst about this topic

Immediately After Collision

collision of two sheets
of colored glass



shortly after passage,
random color charge picked up



$$\vec{E} \parallel \vec{B} \parallel \hat{z}$$

Immediately after collision:
Collimated strong, longitudinal E and B color fields;
long range flux tube --> emission of gluons with correlations

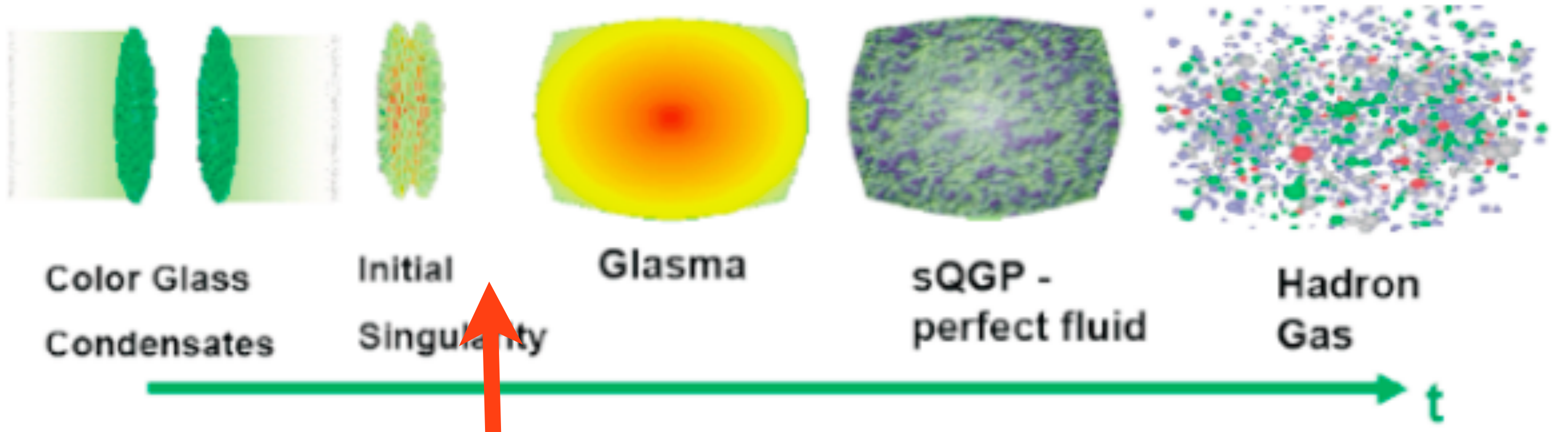
A key feature/issue:
negative longitudinal pressure

$$T_G^{\mu\nu} = Diag(\epsilon, \epsilon, \epsilon, -\epsilon)$$

$$T^{\mu\nu} = -F^{\mu\rho}F^{\nu}_{\rho} + \frac{1}{4}g^{\mu\nu}F^{\rho\lambda}F_{\rho\lambda}$$

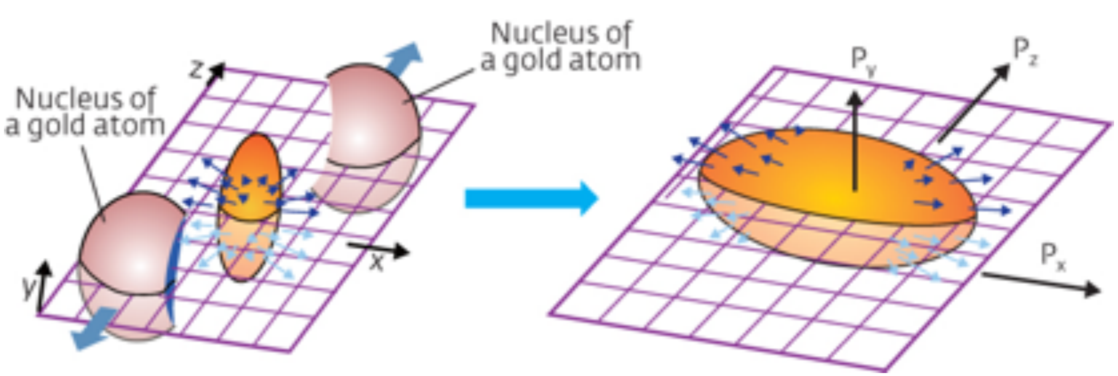
Ex. Verify this energy momentum tensor yourself.

A Little While After Collision: Hydro Triumph



Initial condition is extremely important.

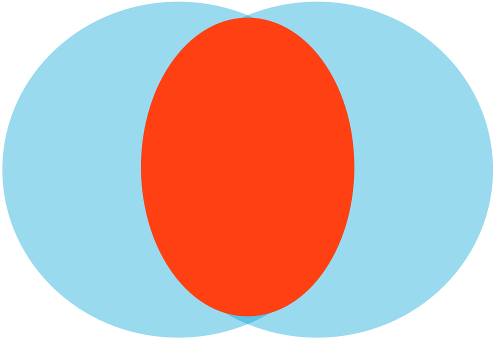
Such info is preserved and transformed into final state observables!



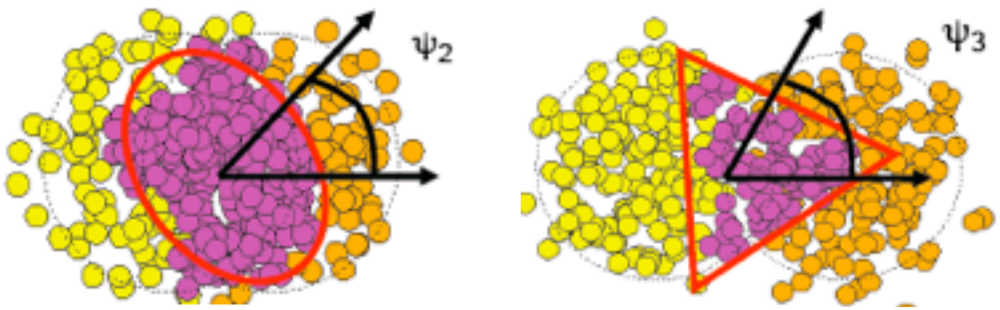
e.g. elliptic flow

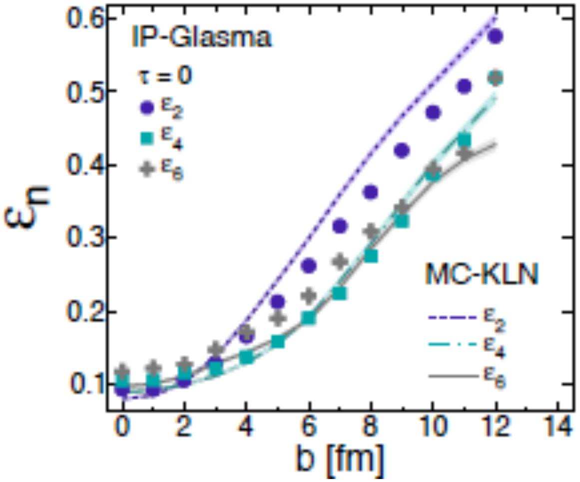
Fluctuating Initial Condition (I.C.)

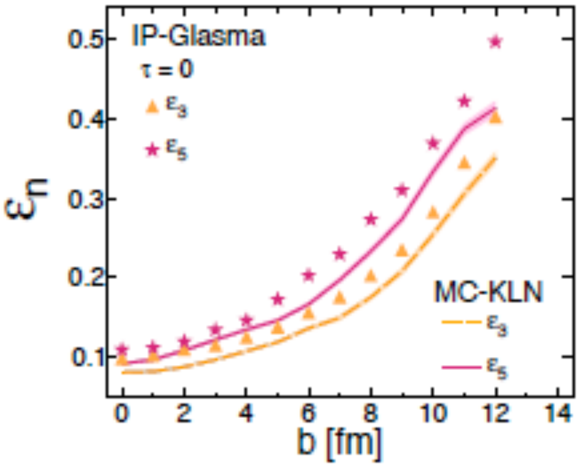
The initial condition used to be like this ...



Thanks to Roland, Alver, and many others, we now know it is actually like this:

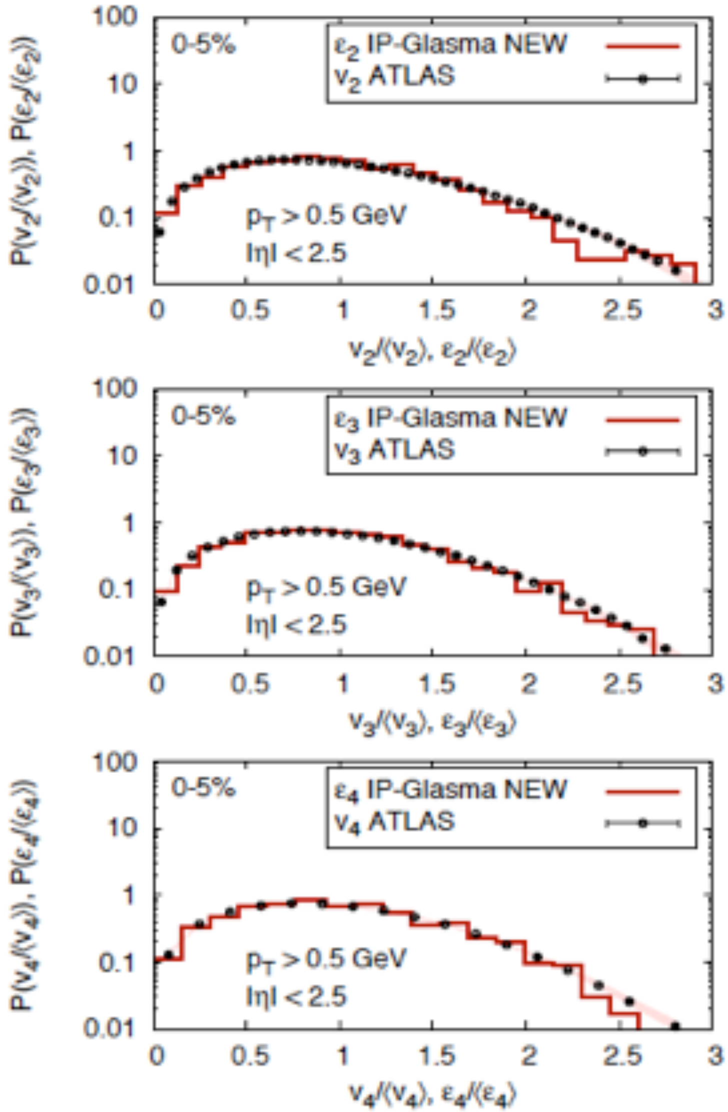






Mapping from I.C. to Final Observables

$$\epsilon_n \xrightarrow[\text{expansion}]{\text{hydrodynamic}} v_n$$



Not only for the mean value, but for the whole distribution, response can be established between I.C. and observables!

--- It would be great to see similar comparison for varied fluctuating quantities at various beam energies.

See lectures by Gunther, Constantin, Fuqiang, Jiangyong, Raju on this topic

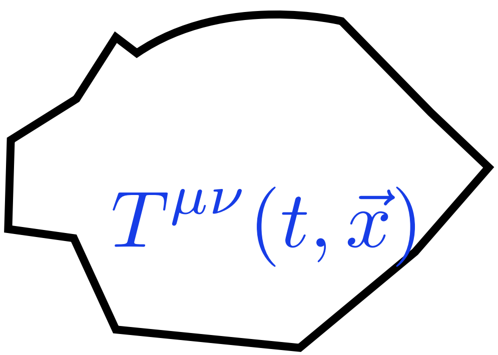
There is however a "little" GAP!

From 0+ time to ~ 1 fermi/c time, what happens?

$$T_G^{\mu\nu} = Diag(\epsilon, \epsilon, \epsilon, -\epsilon)$$

? $\Longrightarrow$ $$T_H^{\mu\nu} = Diag(\epsilon, \frac{\epsilon(1-\delta)}{2}, \frac{\epsilon(1-\delta)}{2}, \delta\,\epsilon)$$

To fully appreciate the challenge, we need ask:
What is hydrodynamics?
What does it take to have hydro?



conservation laws: $\partial_\mu T^{\mu\nu}(t,\vec{x}) = 0$

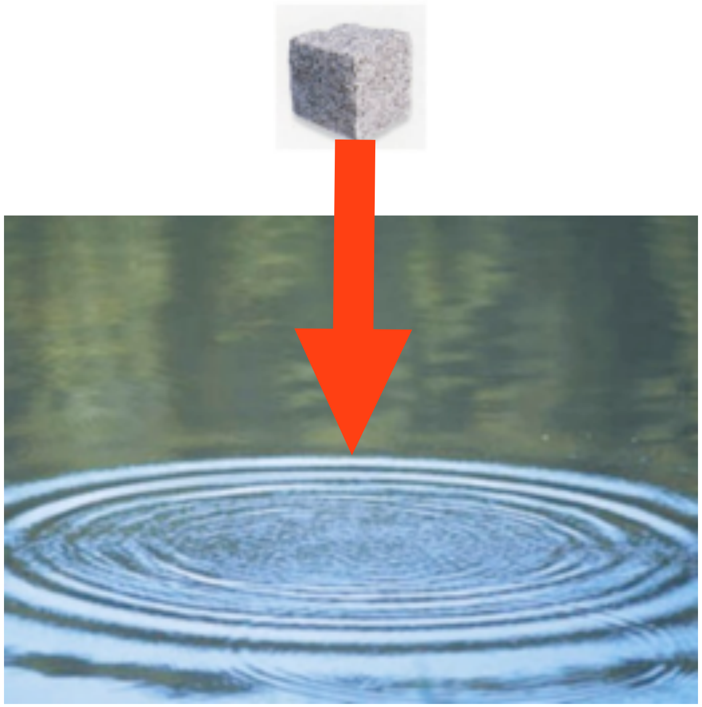
equation of state: $\epsilon = \epsilon(p)$

objects of hydro:
continuum fields
$\epsilon,\ p,\ \vec{v}$

how it works (ideal hydro as example here):

$$\gamma^2(\partial_t + \vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{w/c^2}\left(\vec{\nabla}p + \frac{\vec{v}}{c}\partial_0 p\right)$$

Hydro as Effective Description



After certain **time scale** and watching for patterns over some macroscopic **length scale**, you see the same ripples --> universal hydro emerges.

Hydro describes the long time, large distance behavior of system, which is dictated by conserved quantities like energy & momentum.

Separation of Scales

Important microscopic dissipation scales:
"mean free path", "relaxation time"

$$L_\eta \equiv \frac{\eta}{(w/c^2)c_s} \qquad \tau_\pi = \frac{5}{4}\frac{\eta}{\mathcal{P}}$$

JL & Koch, 2009 $\quad W = \epsilon + P = Ts + \mu n$

Ex. Estimate these scales for waver under normal condition.

Hydrodynamics naturally emerges when scales we are concerned with are well separated, in fact very large, compared with the above:

$$L >> L_\eta \,, \; t >> \tau$$

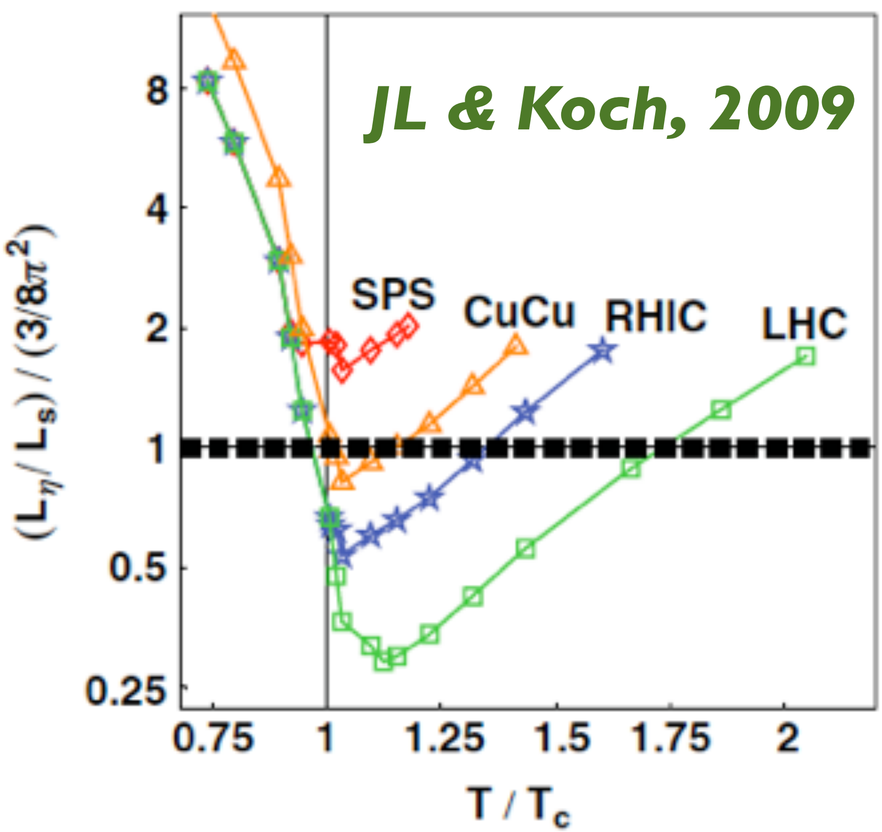
In the QGP case, both requires extremely small dissipative scales: stringent constraint at early time!

$$\boxed{\frac{\eta}{s}\frac{1}{T\tau} << 1}$$

Evolution of Our Conception about Hydro

Hydrodynamic expansion seems emerging quickly in impacts with all different "stones": AuAu, CuCu, CuAu, UU, PbPb, pPb, dAu, pp ?!

Ideal hydro --> viscous hydro --> hydro on its "limit"



* In all cases, we need to understand how such hydro could possibly emerge at such short scales!

* For small system & early time, we need to understand how hydro "dances" well on the edge of cliff.

“Thermalization”: An Outstanding Puzzle

Substantial Evidences of a thermal QGP:

chemical equilibrium freezeout; thermal photons;
hydrodynamical flow from very early time
(elliptic flow; sensitively preserve initial fluctuations)...

Strongly Interacting all along:

very small eta/s; opaque to hard probes;
short equilibration time

“Tension” for understanding thermalization:

early time scale ~Qs is high, coupling NOT large;
weak-coupling-based understanding of initial states

The thermalization problem presents:

* A significant gap in phenomenological description of heavy ion collision experiments;
* A great theoretical challenge to understand the far-from-equilibrium evolution in a non-Abelian gauge theory.

Different Approaches

How to Describe It: Weakly or Strongly Coupled?

pre-equilibrum
glasma

thermal QGP

→ Time

$Q_s \sim 10\Lambda_{QCD}$

$T_{max} \sim 2\Lambda_{QCD}$

it should be amenable to a weakly coupled description

?

it is plausibly a strongly coupled plasma

How to Describe It: Field or Quanta?

just after collision | glasma evolution | close to thermal QGP → Time

$A \sim 1/g$ | $f < 1/g^2$ | $f \sim 1$

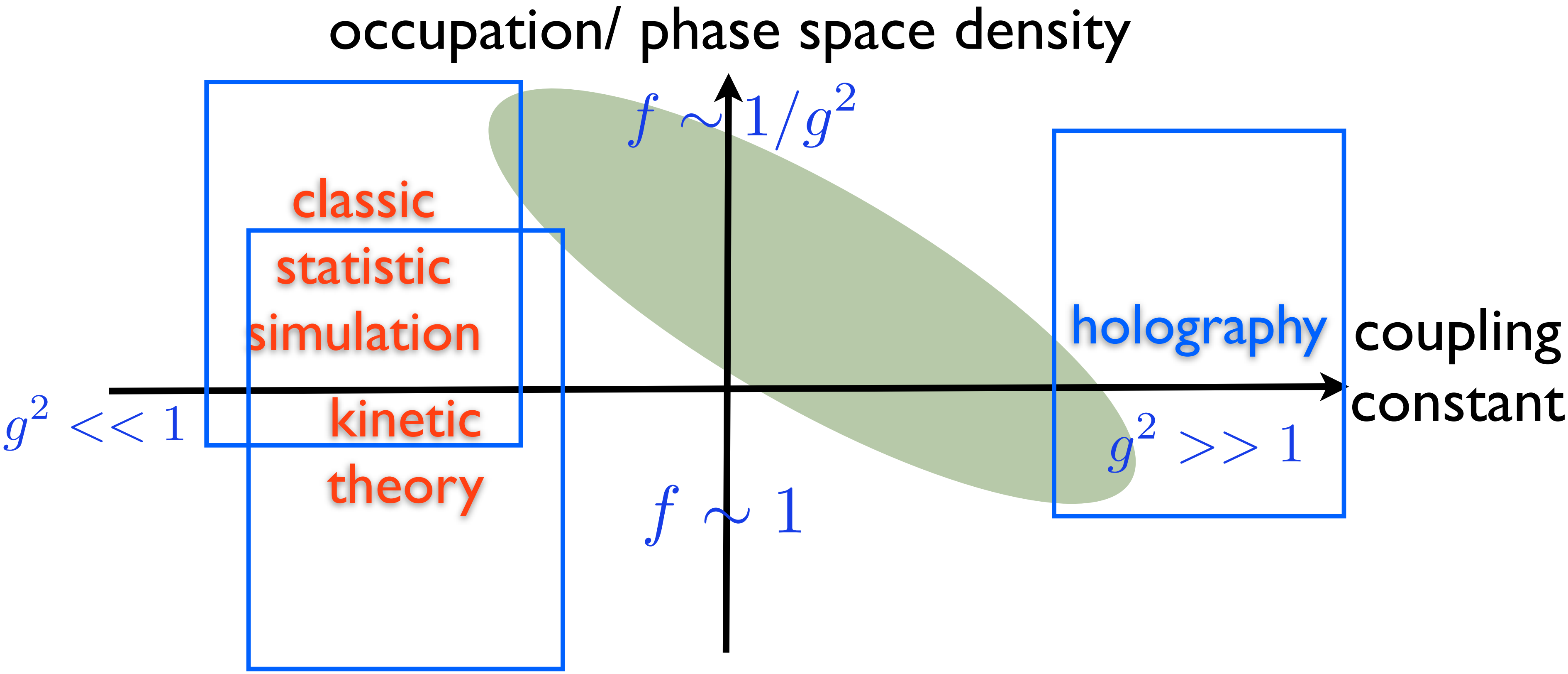
initially dominated by strong classical field

system becomes diluter

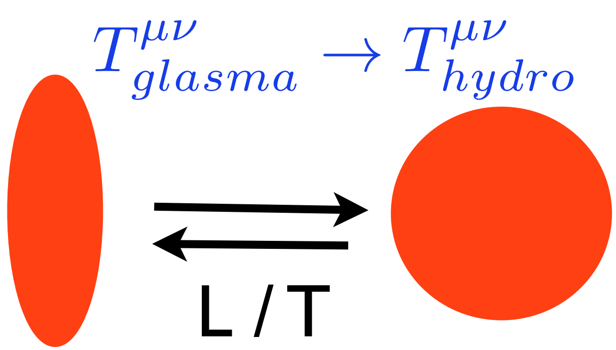
dominated by quanta

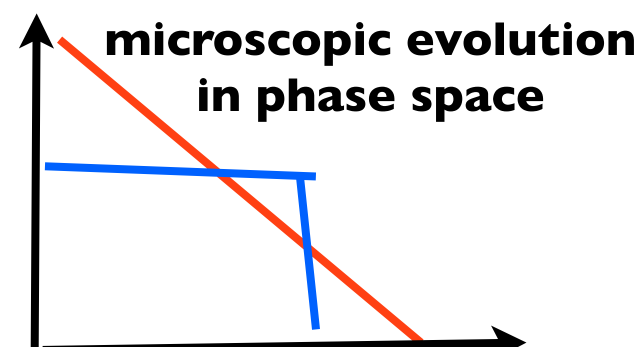
?

Glasma in the Real World



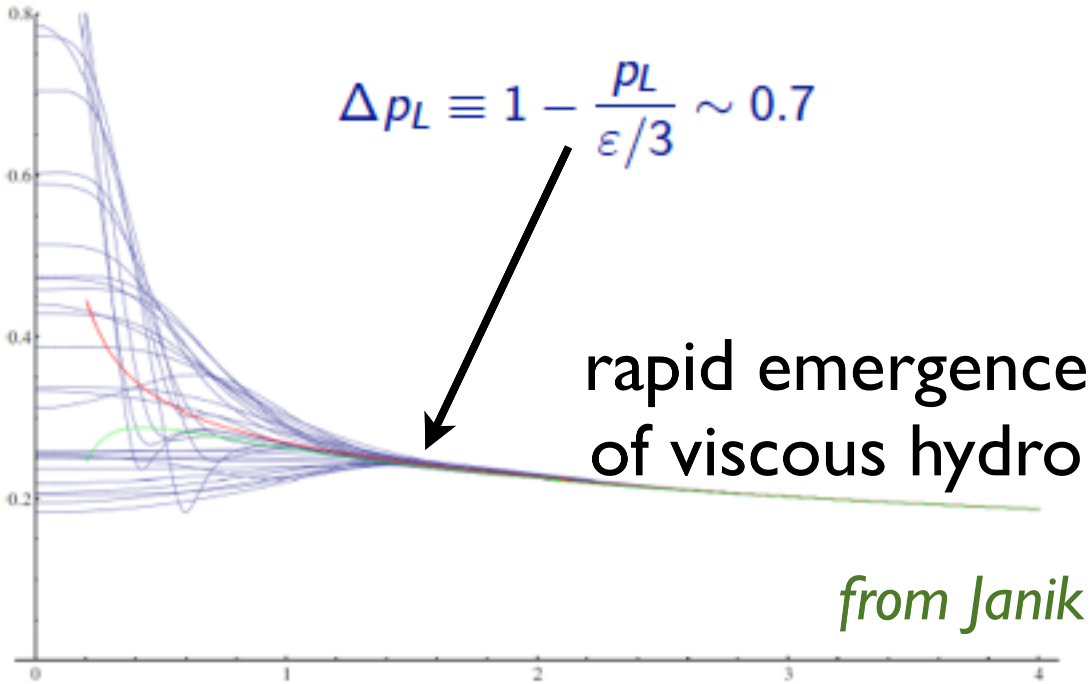
A Few Levels of Questions:





Holography

g_{mn} gravity in 5D bulk $\longrightarrow$ $T_{\mu\nu}$ in 4D boundary



from Janik talk at QM14

Important insights from holography:

** universal hydro emerges*

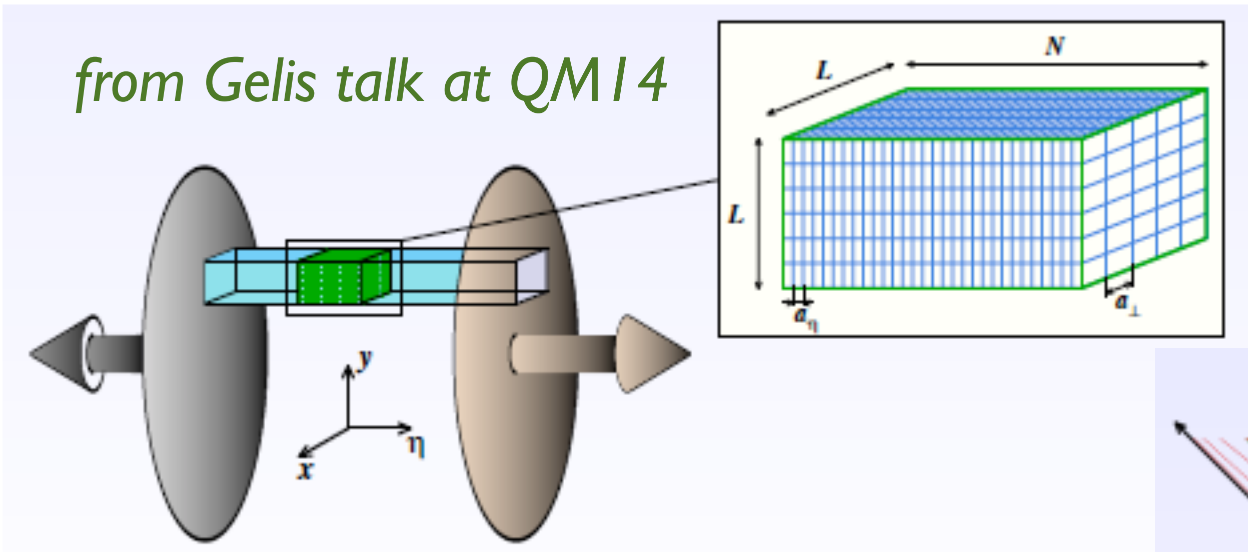
** large anisotropy upon vhydro --> not thermalized yet*

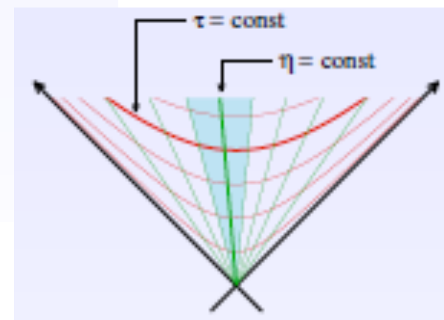
(Anisotropic hydro: Strickland, Heinz, et al)

** hydro works well even with large anisotropy & large gradient*

Classical Field Simulations

$$J^{\mu} = \delta^{\mu +} \underbrace{\rho_1(x^-, x_\perp)}_{\sim\delta(x^-)} + \delta^{\mu -} \underbrace{\rho_2(x^+, x_\perp)}_{\sim\delta(x^+)} \qquad [D_\mu, F^{\mu\nu}] = J^\nu$$

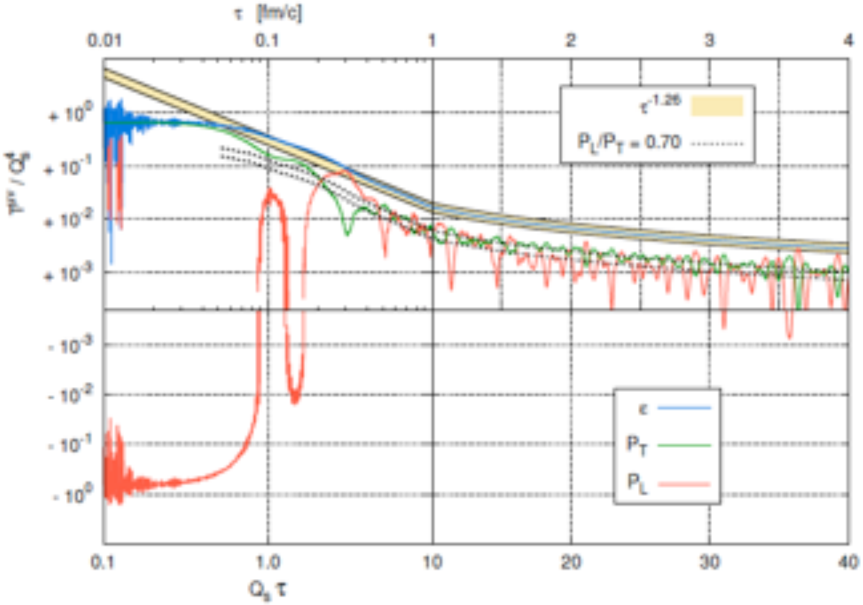




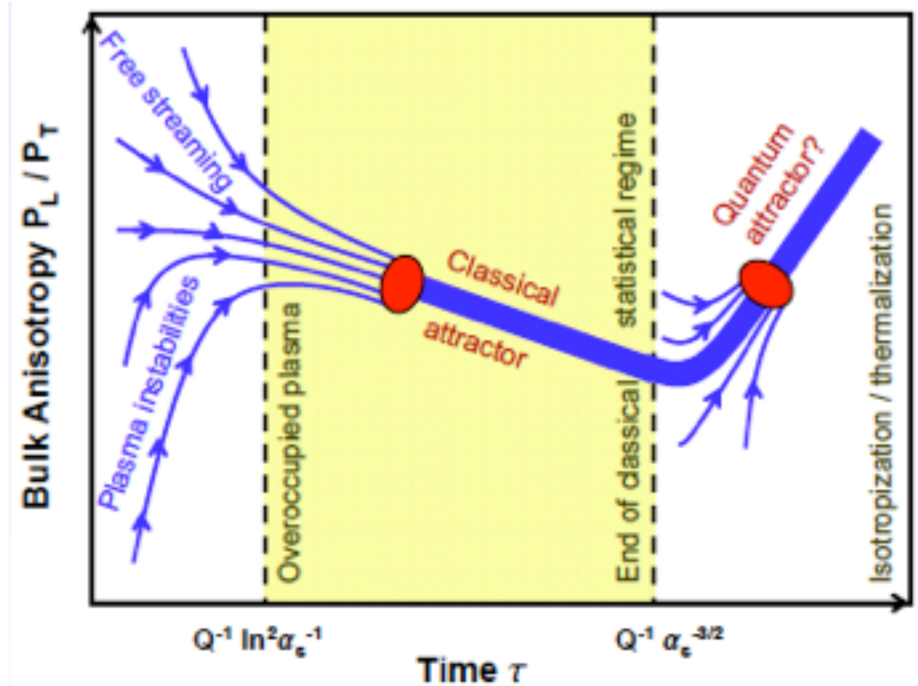
This approach works best
for very small coupling and for large occupation

Please see Raju's lectures for more about this approach.

Classical Field Simulations



Epelbaum&Gelis (2013)

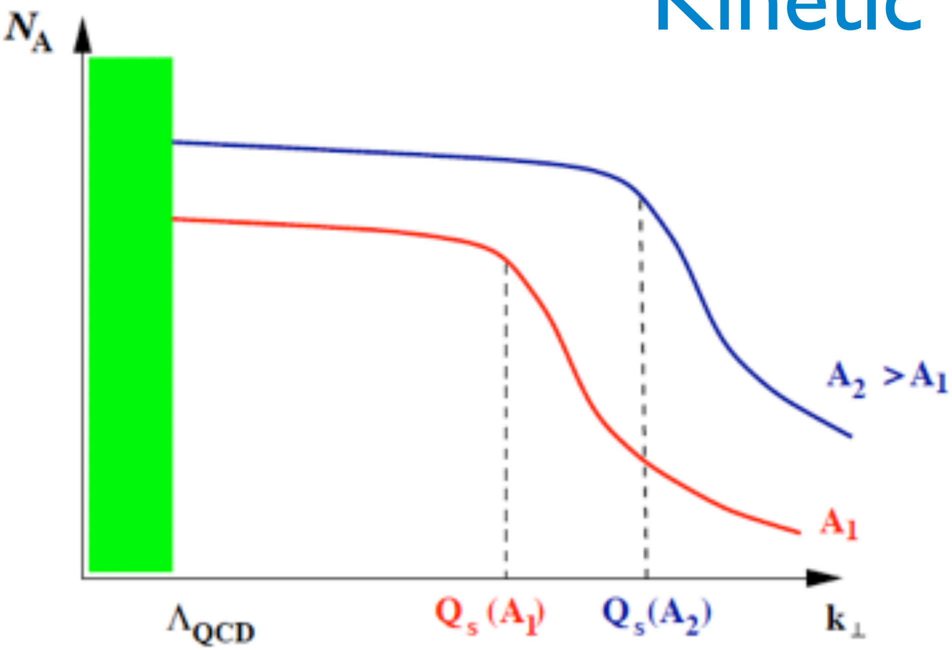


Berges,Boguslavski,Schlitchting,Venugopalan (2013)

A key issue:
could enough longitudinal pressure be built up quickly and matchable to hydro stage?

Please see Raju's lectures for more about this approach.

Kinetic Theory



A theoretically "cleaner" case:
very large nuclei;
very high beam energy.
-->
how the initial distribution
evolves toward thermal case?

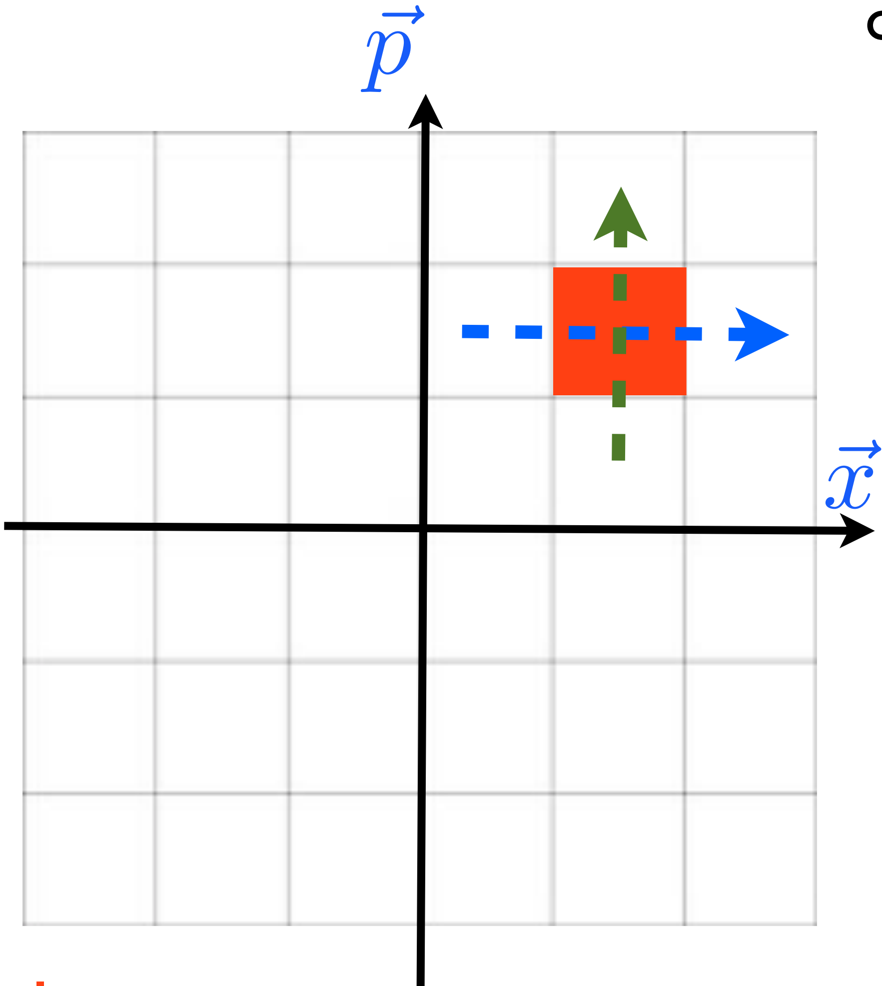
Kinetic theory allows a description from far-from-equilibrium initial condition to nearly thermal point.

* "Bottom-up" (Baier, Muller, Schiff, Son; ...)
* Effective kinetic theory (Arnold, Moore, Yaffe; ...)
* BAMPS (Greiner, Xu, et al)
* Greco group (recently, incorporating full Bose statistics)
* Overpopulated glasma and enhanced elastic scatterings (BGLMV; ...)

The rest of this talk will focus on kinetic approach.

A Quick Primer on Kinetic Theory

Kinetic Theory for Many Body System



Instead of tracing every particle, one focuses on their **phase space density**

$$f(\boldsymbol{x}, \boldsymbol{p}) = \frac{(2\pi)^3}{2(N_c^2 - 1)} \frac{dN}{d^3\boldsymbol{x}\, d^3\boldsymbol{p}}$$

Its evolution is described by the **transport equation**:

$$\mathcal{D}_t f(t, \vec{x}, \vec{p}) = C[f]$$

$$\mathcal{D}_t \equiv \partial_t$$

$$+\vec{v} \cdot \nabla_{\vec{x}}$$

$$+\vec{F} \cdot \nabla_{\vec{p}}$$

Input:

* cross-section * initial condition

Destination:

* fixed point solution

Fixed Point

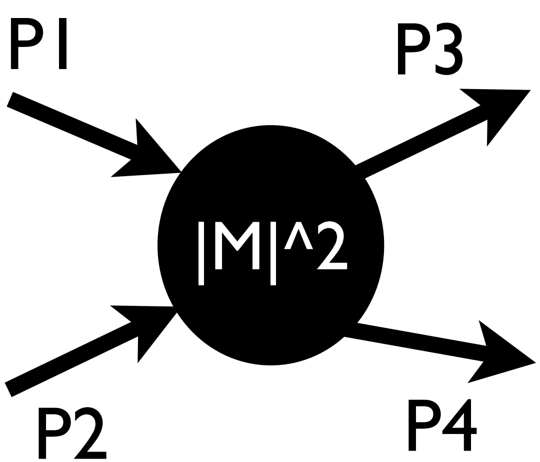
Example of 2--2 scattering: generic form of collision kernel

microscopic conservation

$$\mathcal{C}_{2\to2}[f_1] = \frac{1}{2}\int_{234}\frac{1}{2E_1}|M_{12\to34}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

microscopic detailed balance

$$\times[(1+f_1)(1+f_2)f_3f_4 - f_1f_2(1+f_3)(1+f_4)].$$



* Bose statistics

$$f_{eq} = \frac{1}{e^{(E-\mu)/T}-1}$$

* Fermi statistics:
(1+ f) -> (1-f)

$$f_{eq} = \frac{1}{e^{(E-\mu)/T}+1}$$

* Boltzmann statistics:
(1+ f) -> 1

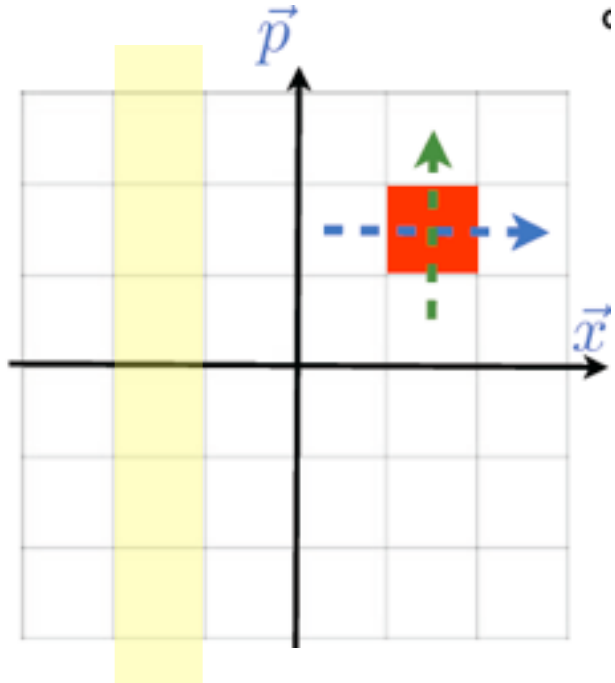
$$f_{eq} = e^{-(E-\mu)/T}$$

$$[f_3f_4 - f_1f_2] \to$$

$$\left[e^{-(E_3+E_4-2\mu)/T} - e^{-(E_1+E_2-2\mu)/T}\right] \to 0$$

Ex. Verify the above fixed points yourself.

Connecting Kinetic Theory with Hydrodynamics



Particle current & density

$$j^\mu = \int_{\vec{p}} \frac{p^\mu}{E} f(p) \qquad n = \int_{\vec{p}} f(p)$$

Energy-momentum tensor

$$T^{\mu\nu} = \int_{\vec{p}} \frac{p^\mu p^\nu}{E_p} f(p)$$

Conservation laws

$$dE/dt = \int_{\vec{p}} E\, C[f(\vec{p})]$$

$$dn/dt = \int_{\vec{p}} C[f(\vec{p})]$$

$$\frac{df}{dt} = C[f]$$

Fixed point solution (equilibrium)
(E.o.S evaluated at this situation)

$$df/dt = C[f = f_{eq}] = 0$$

Conservation Law I.

Let us examine energy momentum conservation generally in the kinetic framework (without external forces)

$$\mathcal{D}_t f(t,\mathbf{x},\mathbf{p}) = \mathcal{C}[f]$$

(consider m to n particle process)

$$\mathcal{D}_t f(t,\mathbf{x},\mathbf{p}) \equiv \frac{p^\mu}{E_p}\partial_\mu f(t,\mathbf{x},\mathbf{p}) = (\partial_t + \mathbf{v}_p \cdot \nabla_{\mathbf{x}}) f(t,\mathbf{x},\mathbf{p})$$

$$\boxed{\partial_\mu T^{\mu\nu} = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} p_1^\nu \mathcal{C}[f_1]}$$

$$\propto \int_{1,\ldots,m}\int_{m+1,\ldots,m+n} p_1^\nu |M_{m\to n}|^2 \delta^{(4)}(\Sigma_{i=1}^m p_i - \Sigma_{j=m+1}^{m+n} p_j)$$

$$\times\{[\Pi_{i=1}^m (1+f_i)][\Pi_{j=m+1}^{m+n} f_j] - [\Pi_{i=1}^m f_i][\Pi_{j=m+1}^{m+n}(1+f_j)]\}$$

$$(\Sigma_{i=1}^m p_i^\nu - \Sigma_{j=m+1}^{m+n} p_j^\nu)$$

Two essential points

* micro. conservation; * micro. cyclic symmetry

Conservation Law II.

Let us examine particle number conservation generally in the kinetic framework (without external forces)

$$\mathcal{D}_t f(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f]$$

(consider m to n particle process)

$$\mathcal{D}_t f(t, \mathbf{x}, \mathbf{p}) \equiv \frac{p^\mu}{E_p} \partial_\mu f(t, \mathbf{x}, \mathbf{p}) = (\partial_t + \mathbf{v}_p \cdot \nabla_{\mathbf{x}}) f(t, \mathbf{x}, \mathbf{p})$$

$$\boxed{\mathcal{D}_t n = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \mathcal{C}[f_1]}$$

$$\propto \int_{1,\ldots,m} \int_{m+1,\ldots,m+n} 1\, |M_{m\to n}|^2 \delta^{(4)}(\Sigma_{i=1}^{m} p_i - \Sigma_{j=m+1}^{m+n} p_j)$$
$$\times \{[\Pi_{i=1}^{m}(1+f_i)][\Pi_{j=m+1}^{m+n} f_j] - [\Pi_{i=1}^{m} f_i][\Pi_{j=m+1}^{m+n}(1+f_j)]\}$$

$$(n-m)$$

Particle # is conserved only for ELASTIC PROCESSES, with n=m .

If n is NOT equal to m, i..e inelastic case, the fixed point must have ZERO chemical potential.

Longitudinal Expansion I.

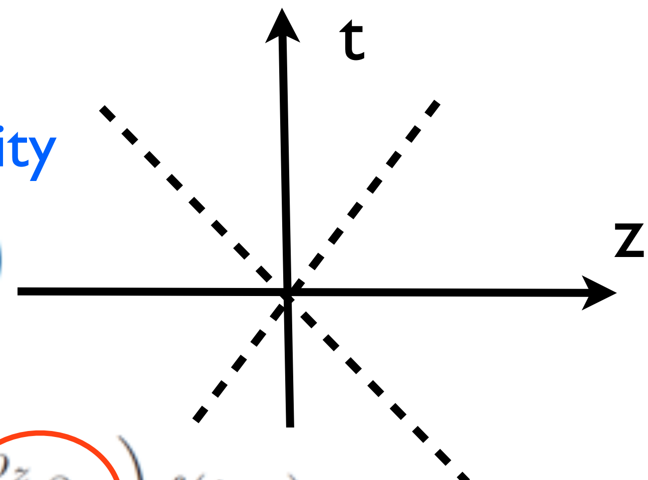
The early stage matter in heavy ion collisions undergoes strong, boost-invariant, longitudinal expansion.

$$\mathcal{D}_t f(t, \mathbf{x}, \mathbf{p}) = (\partial_t + v_z \partial_z) f(t, z, \mathbf{p}) = \mathcal{C}[f]$$

Boost invariant assumption:

y momentum rapidity; \eta: spatial rapidity

$$f(t, z, \mathbf{p}) \rightarrow f(\tau, y - \eta, \mathbf{p}_\perp)$$



At mid-rapidity, \eta --> 0, one gets

$$\mathcal{D}_t f(t, \mathbf{x}, \mathbf{p}) = \left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \mathbf{p}) = \left(\partial_t - \frac{p_z}{t}\partial_{p_z}\right) f(t, \mathbf{p})$$

The drift term implies “leakage” of z-momentum that grows with p_z

$$\left(\partial_t - \frac{p_z}{t}\partial_{p_z}\right) f(t, \mathbf{p}) = \frac{\partial(tf)}{t\partial_t} - \nabla_{\mathbf{p}} \cdot \left[\frac{p_z}{t} f \hat{z}\right]$$

Longitudinal Expansion II.

How particle number evolves? (assuming elastic only)

$$\int_{\vec{p}} \left[\frac{\partial(tf)}{t\partial t} - \nabla_{\vec{p}} \cdot \left(\frac{p_z}{t} f \hat{z} \right) \right] = \frac{1}{t} \partial_t (tn) = 0 \quad \Longrightarrow \quad \boxed{n = \frac{n_0 t_0}{t}}$$

How energy evolves?

$$\int_{\vec{p}} E_p \left[\frac{\partial(tf)}{t\partial t} - \nabla_{\vec{p}} \cdot \left(\frac{p_z}{t} f \hat{z} \right) \right] = \frac{\partial_t (t\epsilon)}{t} + \frac{P_L}{t} = 0$$

$$\Longrightarrow \quad \boxed{\epsilon = \epsilon_0 \left(\frac{t_0}{t} \right)^{1+\delta}}$$

How energy evolves critically depends on the L/T anisotropy!

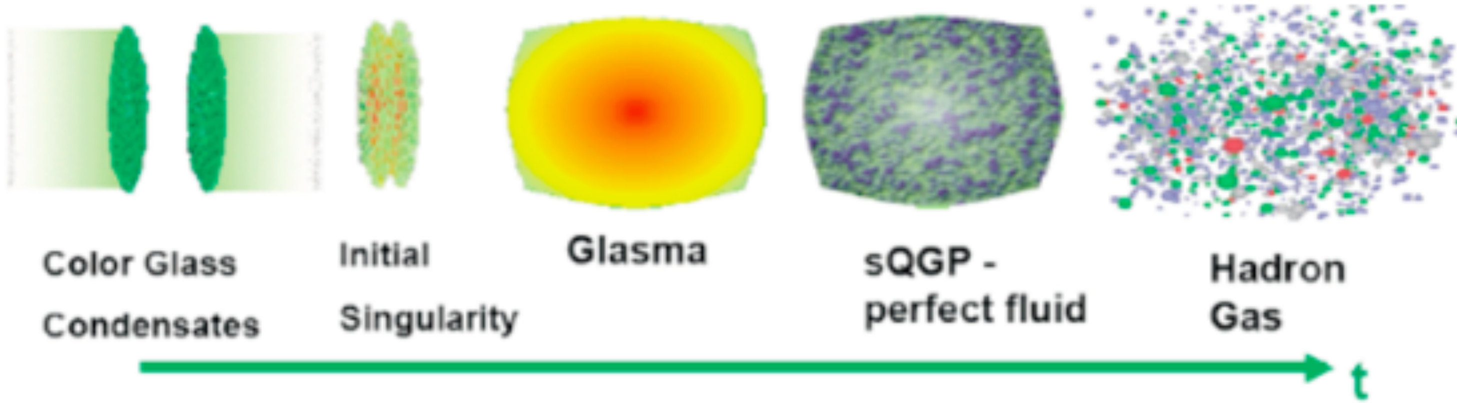
anisotropy parameter $\delta = \frac{P_L}{\epsilon}$

\delta=0, free streaming
\delta=1/3, hydro limit

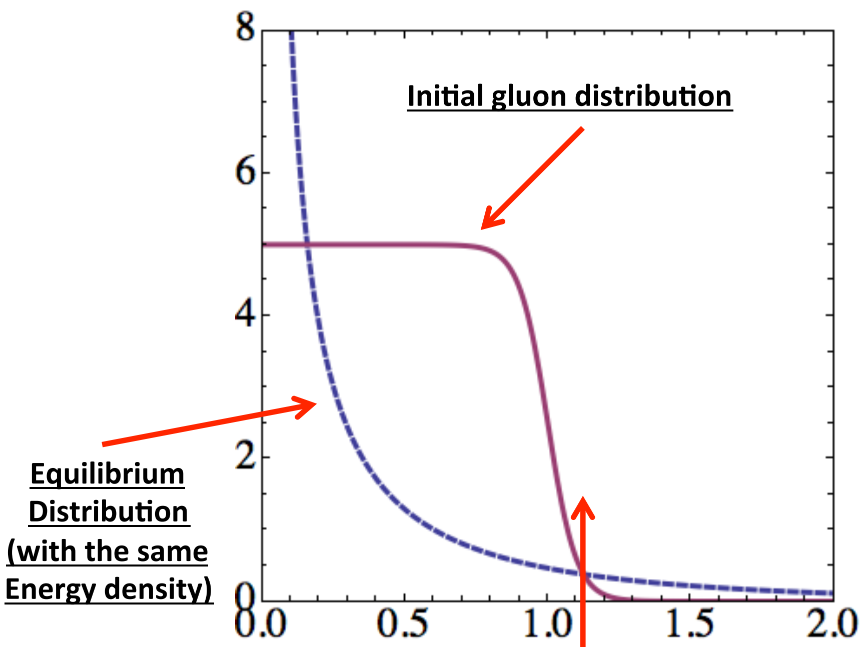
Ex. Verify the above relations yourself.

Recent Developments: Overpopulated Glasma

Overpopulated Glasma



The precursor of a thermal quark-gluon plasma, known as glasma, is born as a gluon matter with **HIGH OVERPOPULATION**:



Saturation Scale Qs ~ 1 GeV or larger, weakly coupled

Very large occupation number $f \sim \dfrac{1}{\alpha_s}$

Saturation fixes initial scale

$$\epsilon_0 \sim \frac{Q_s^4}{\alpha_s}$$

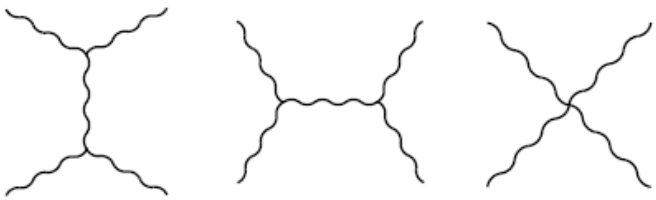
$$n_0 \sim \frac{Q_s^3}{\alpha_s}$$

$$\epsilon_0/n_0 \sim Q_s$$

Kinetic Equation with Elastic Gluon Scatterings

$$\mathcal{C}_{2\to 2}[f_1] = \frac{1}{2}\int_{234}\frac{1}{2E_1}|M_{12\to 34}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$
$$\times\left[(1+f_1)(1+f_2)f_3f_4 - f_1f_2(1+f_3)(1+f_4)\right].$$

$$\boxed{\mid M_{12\to 34}\mid^2 = 72g^4\left[3-\frac{t\,u}{s^2}-\frac{s\,u}{t^2}-\frac{t\,s}{u^2}\right]}$$



$$\mathbf{f} * \mathbf{f} * \alpha_s{}^2 \sim O\ (1)$$

Quantum amplification of scatterings changes the usual power counting in coupling!

A <u>weakly-coupled but strongly interacting</u> regime emerges!

Two approaches possible:

1) directly solve the above (e.g. Greco group; BAMPS)

2) analytically derive approximate equation that captures main physics

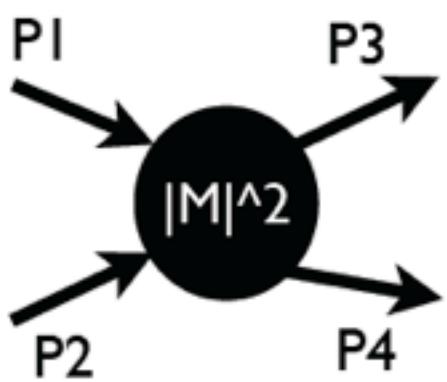
Kinetic Equations with Long Range Interactions

For describing kinetic evolution of a system with long range interactions, the small angle approximation is a very useful approach.

* Landau ~1950 for NR QED plasma (in Boltzmann limit), known as Landau collision integral.

* A. Mueller, 1999, generalized to relativistic gluon plasma with QCD interactions (also in Boltzmann limit).

* Blaizot-Liao-McLerran, 2011, gluon plasma with quantum statistics.



$$|M_{12\to34}|^2 = 72g^4\left[3 - \frac{t\,u}{s^2} - \frac{s\,u}{t^2} - \frac{t\,s}{u^2}\right]$$

Dominant contribution to cross-section comes from small angle scattering:

$$t = (p_1 - p_3)^2 \to 0$$
$$u = (p_1 - p_4)^2 \to 0$$

A particle changes its momentum via a series of small angle scatterings, picking up many "random small kicks" --> diffusion in momentum space!

Kinetic Eq. Under Small Angle Approximation

$$\mathcal{D}_t f(\vec{p}) = \xi \left(\Lambda_s^2 \Lambda\right) \vec{\nabla} \cdot \left[\vec{\nabla} f(\vec{p}) + \frac{\vec{p}}{p} \left(\frac{\alpha_S}{\Lambda_s}\right) f(\vec{p})[1 + f(\vec{p})]\right]$$

Ex. Verify the fixed point and conservation laws of the above equation.

$$\Lambda \left(\frac{\Lambda_s}{\alpha_S}\right)^2 \equiv (2\pi^2) \int \frac{d^3p}{(2\pi)^3} f(\vec{p}) \left[1 + f(\vec{p})\right]$$

$$\Lambda \frac{\Lambda_s}{\alpha_S} \equiv (2\pi^2)\, 2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\vec{p})}{p}$$

Two important scales:

hard scale Lambda

soft scale Lambda_s

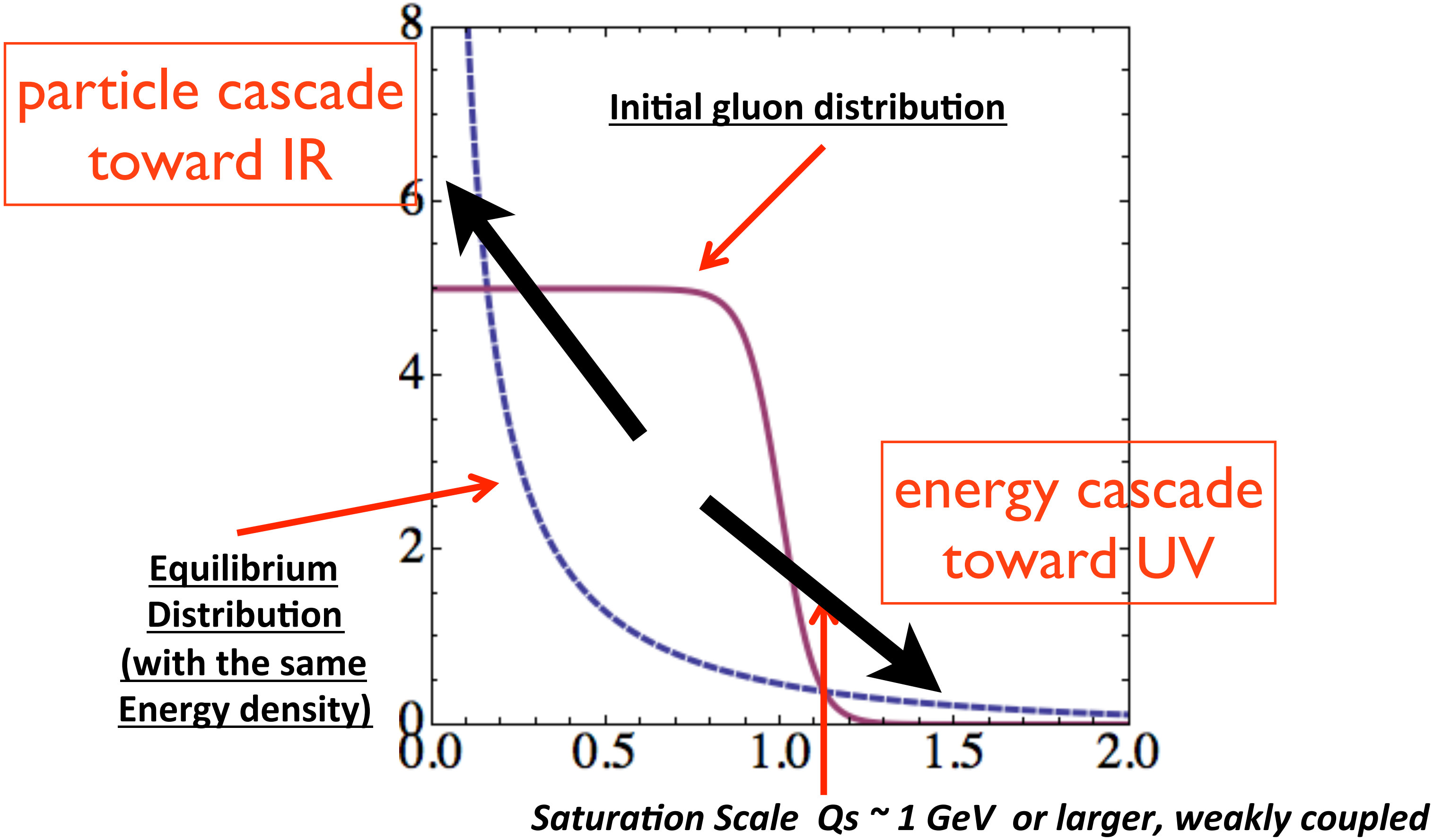
Λ : $f << 1$ for $p > \Lambda$ $\qquad$ Λ_s : $f \sim \dfrac{1}{\alpha_s}$

Initial glasma: $\Lambda \sim \Lambda_s \sim Q_s$

Thermalized weakly-coupled QGP: $\Lambda \sim T$, $\Lambda_s \sim \alpha_s * T$

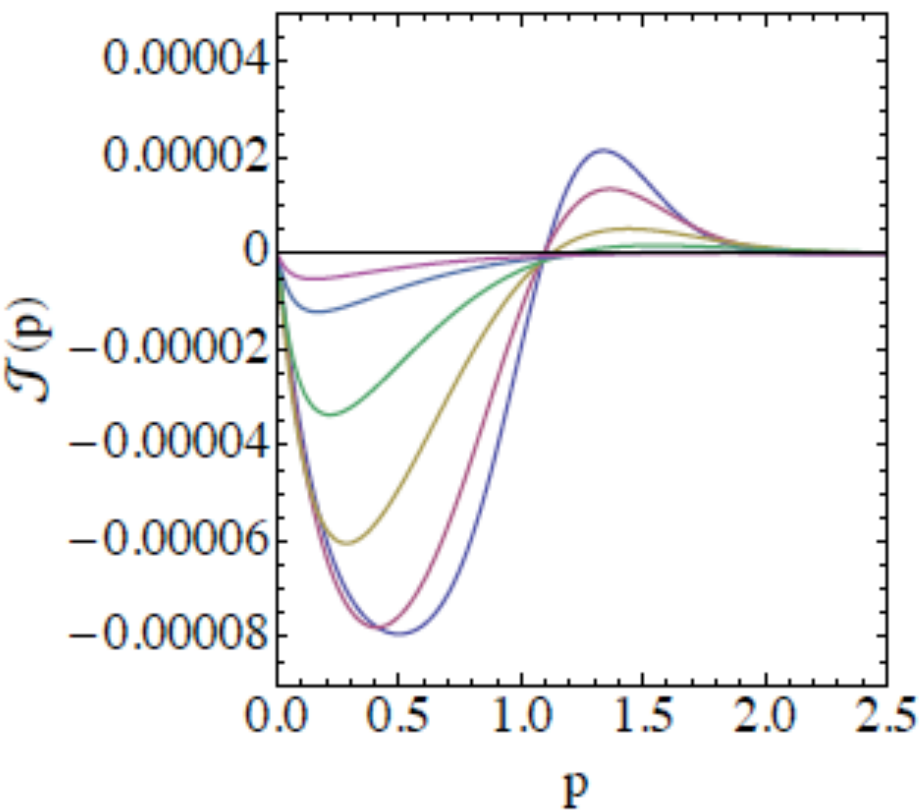
Elastic scattering time scale $\quad t_{scat} \sim \dfrac{\Lambda}{\Lambda_s^2}$

How Thermalization Proceeds

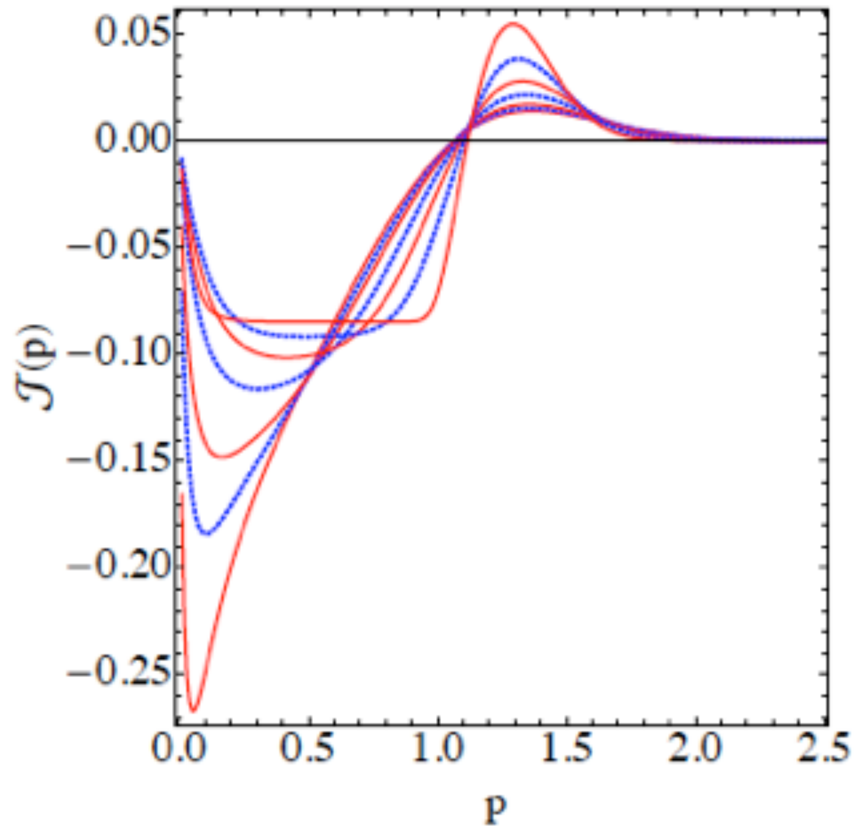


Initial glasma: $\Lambda \sim \Lambda_s \sim Q_s$ → Thermalized weakly-coupled QGP: $\Lambda \sim T$, $\Lambda_s \sim \alpha_s * T$

IR and UV Cascade



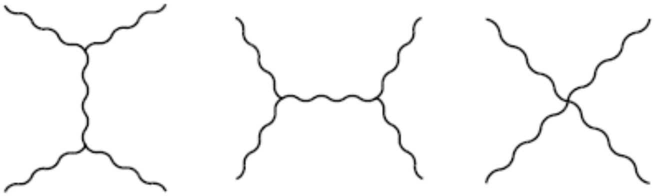
f_0=0.1
(underpopulated)



f_0=1
(overpopulated)

Blaizot, JL, McLerran, 1305.2119, NPA2013

Elastic Driven Thermalization of Overpopulated Glasma



$$f * f * \alpha_s^{\,2} \sim O(1)$$

Thermalization must be accompanied by specific separation of the two scales:

$$s \sim \int_p [(1 + f) * \text{Ln}(1 + f) - f * \text{Ln}(f)]$$

For fixed energy density, the entropy is maximized when f~1 for dominant phase space

Initial glasma: $\Lambda \sim \Lambda_s \sim Q_s$ $\longrightarrow$ Thermalized weakly-coupled QGP: $\Lambda \sim T$, $\Lambda_s \sim \alpha_s * T$

separation of two scales toward thermalization

$$\boxed{\frac{\Lambda_s}{\Lambda} \sim \alpha_s}$$

A Simple Estimate for “Static Box”

A schematic scaling distribution characterized by the two evolving scales:

$$f(p) \sim \frac{1}{\alpha_s} \quad \text{for } p < \Lambda_s, \qquad f(p) \sim \frac{1}{\alpha_s}\frac{\Lambda_s}{\omega_p} \quad \text{for } \Lambda_s < p < \Lambda, \qquad f(p) \sim 0 \quad \text{for } \Lambda < p$$

$$n_g \sim \frac{1}{\alpha_s}\Lambda^2\Lambda_s \qquad \epsilon_g \sim \frac{1}{\alpha_s}\Lambda_s\Lambda^3 \qquad n = n_c + n_g$$

Two conditions fixing the time evolution:

$$\Lambda_s\Lambda^3 \sim \text{constant} \qquad t_{\text{scat}} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

The scaling solution:

$$\Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s\left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

Thermalization time:

$$\Lambda_s \sim \alpha_s \Lambda \quad \Longrightarrow \quad \boxed{t_{\text{th}} \sim \frac{1}{Q_s}\left(\frac{1}{\alpha_s}\right)^{7/4}}$$

Expanding Case

Longitudinal expansion leads to L/T anisotropy:

$$P_L = \delta\epsilon \qquad \epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta}$$

$\delta = 0$: free streaming $\qquad \delta = 1/3$: isotropic

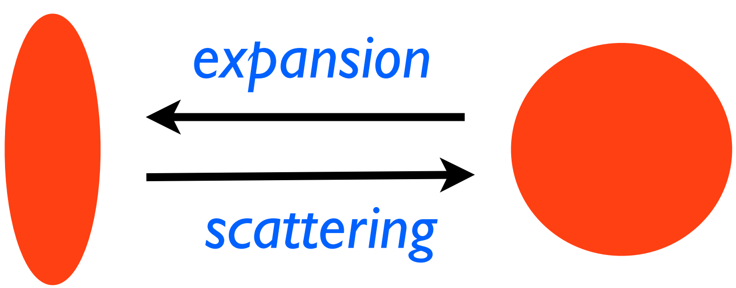
The scaling solution:

$$\Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}.$$

Thermalization time:

$$\Lambda_s \sim \alpha_s \Lambda \quad \Longrightarrow \quad \boxed{\left(\frac{t_{\mathrm{th}}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{7/(3-\delta)}}$$

L/T Anisotropy: expansion versus scattering --> possible balance

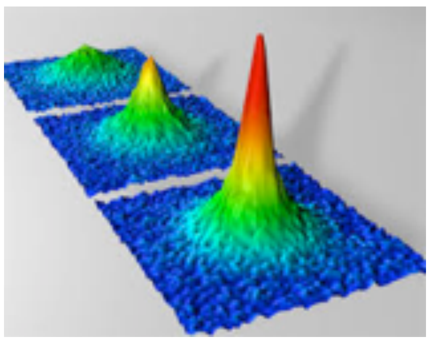


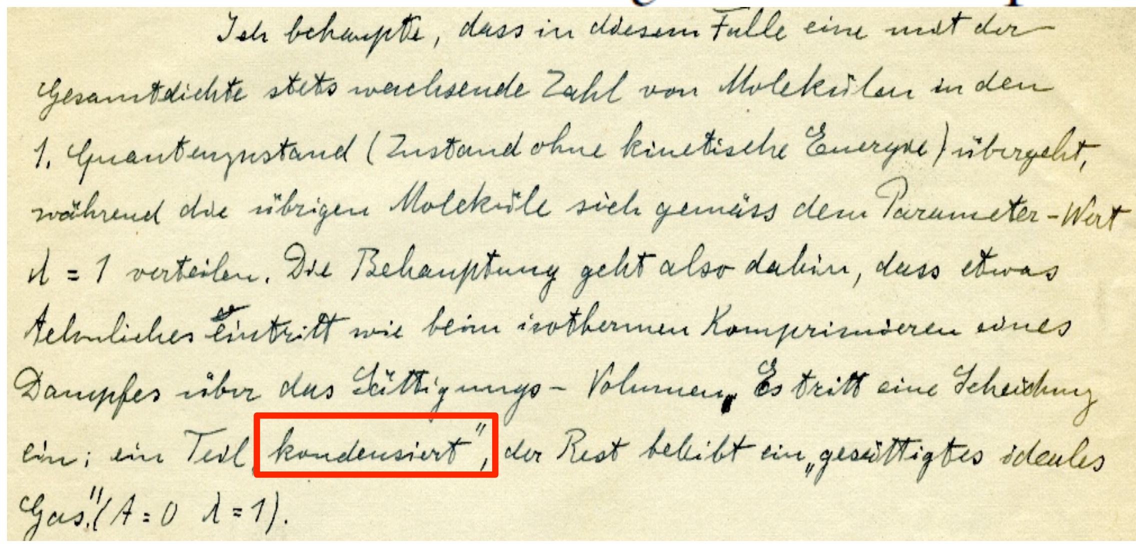
$$\frac{p_z}{t}\partial_{p_z} \sim 1/t$$

$$\Gamma_{scat} \sim \frac{\Lambda_s^2}{\Lambda} \sim \frac{\hat{o}(1)}{t}$$

BEC: Quantum Coherence <=> Overpopulation

$$f_{\rm eq}(\boldsymbol{k}) = n_c \delta(\boldsymbol{k}) + \frac{1}{e^{\beta(\omega_{\boldsymbol{k}} - m_0)} - 1}$$





Ich behaupte, dass in diesem Falle eine mit der Gesamtdichte stets wachsende Zahl von Molekülen in den 1. Quantenzustand (Zustand ohne kinetische Energie) übergeht, während die übrigen Moleküle sich gemäss dem Parameter-Wert $\lambda = 1$ verteilen. Die Behauptung geht also dahin, dass etwas Aehnliches eintritt wie beim isothermen Komprimieren eines Dampfes über das Sättigungs-Volumen. Es tritt eine Scheidung ein; ein Teil „kondensiert", der Rest bleibt ein „gesättigtes ideales Gas" ($A = 0$ $\lambda = 1$).

Einstein: new phase emerges with condensate, when quantum wave scale overlaps with inter-particle scale (--- the 1st application of de Broglie wavelength idea)

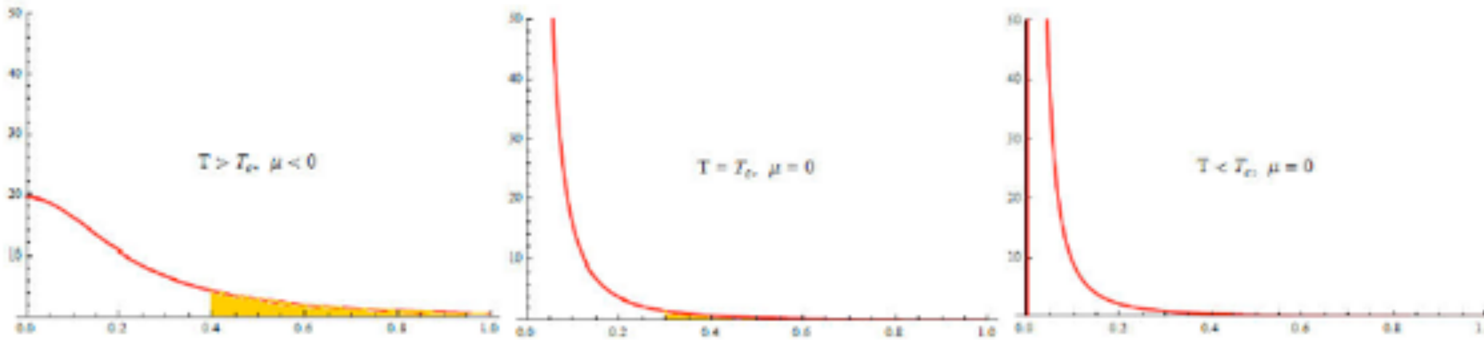
Quantum Coherence implies **OVERPOPULATION**:

$$\boxed{\frac{\lambda_{dB}}{d} \sim \left(n\epsilon^{-3/4}\right)^{\alpha} \sim \hat{O}(1)}$$

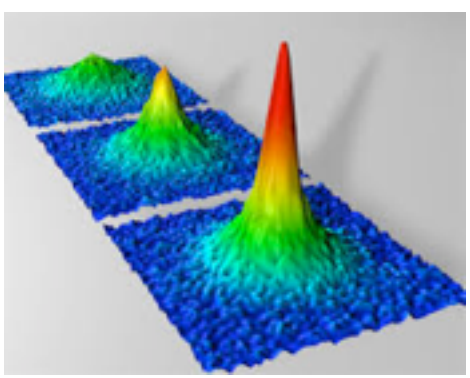
BEC in The Very Cold

Brilliant evaporative cooling: precisely to achieve

OVERPOPULATION



Cooling procedure: kick out fast atoms (truncating UV tail);
then let system relax toward new equilibrium;
relaxation via IR particle cascade & UV energy cascade.



It took ~70 years to achieve

OVERPOPULATION,

thus BEC in ***ultra-cold*** bose gases.

$$n \cdot \epsilon^{-3/4} > \hat{O}(1)\ threshold$$

BEC in the Very Hot!

Temperature

$10^{-8}K$	$10^{0}K$	$10^{1}K$	$10^{2}K$	~~ $10^{12}K$
cold atomic gas	liquid helium; cosmic axion?	magnon	cavity photon; magnon	overpopulated glasma!

Overpopulation: Thermodynamic Consideration

Our initial gluon system is highly OVERPOPULATED:

$$f(p) = f_0 \, \theta(1 - p/Q_s),$$

$$\epsilon_0 = f_0 \, \frac{Q_s^4}{8\pi^2}, \qquad n_0 = f_0 \, \frac{Q_s^3}{6\pi^2}, \qquad n_0 \, \epsilon_0^{-3/4} = f_0^{1/4} \, \frac{2^{5/4}}{3\,\pi^{1/2}},$$

This is to be compared with the thermal BE case:

$$n \, \epsilon^{-3/4}|_{SB} = \frac{30^{3/4} \, \zeta(3)}{\pi^{7/2}} \approx 0.28$$

Overpopulation occurs when: $\boxed{f_0 > f_0^c \approx 0.154}$

Identifying f_0 -> 1/alpha_s, even for alpha_s =0.3,
the system is highly overpopulated!!

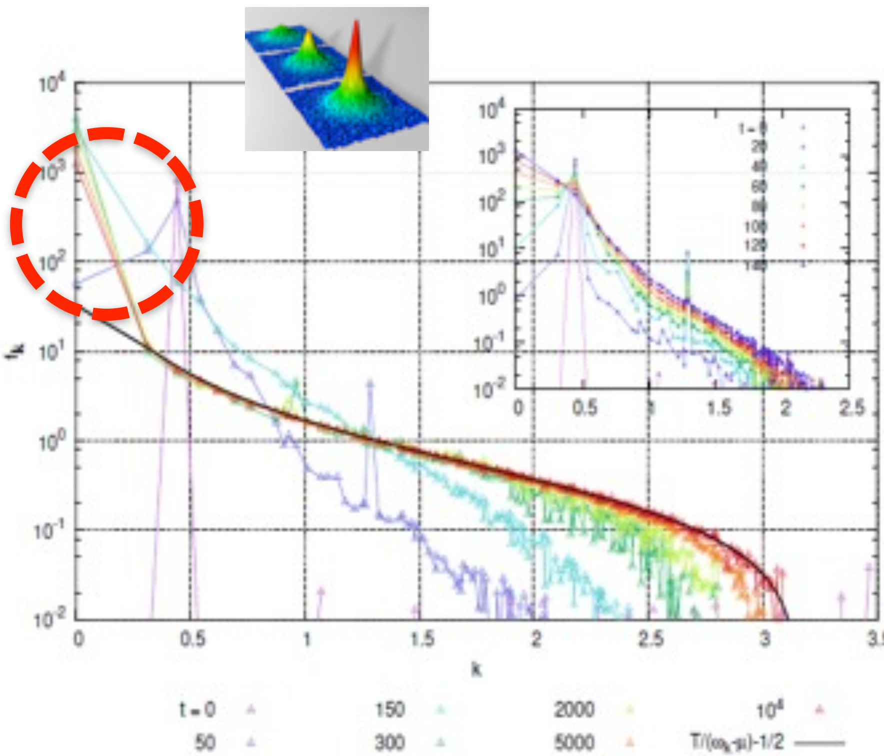
Will the system accommodate the excessive particles by forming a Bose-Einstein Condensate (BEC) ? AND HOW???

Strong Evidence of BEC from Scalar Field Theory Simulations

Bose–Einstein condensation and thermalization of the quark–gluon plasma

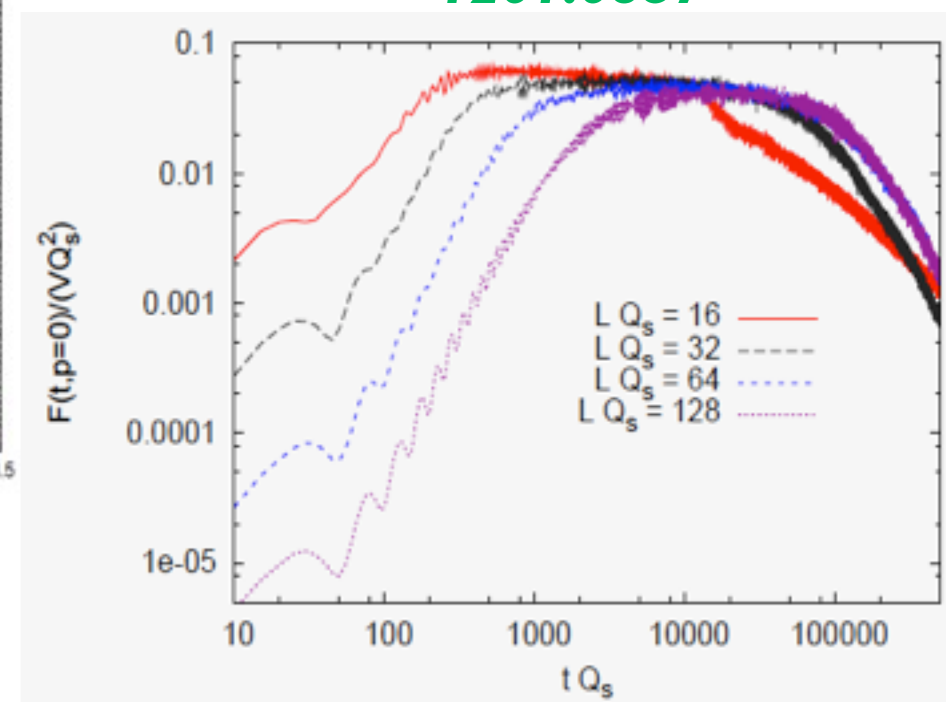
Jean-Paul Blaizot [a], François Gelis [a], Jinfeng Liao [b,*], Larry McLerran [b,c], Raju Venugopalan [b]

Absolutely true for pure elastic scatterings;
True, in transient sense, for systems with inelastic processes



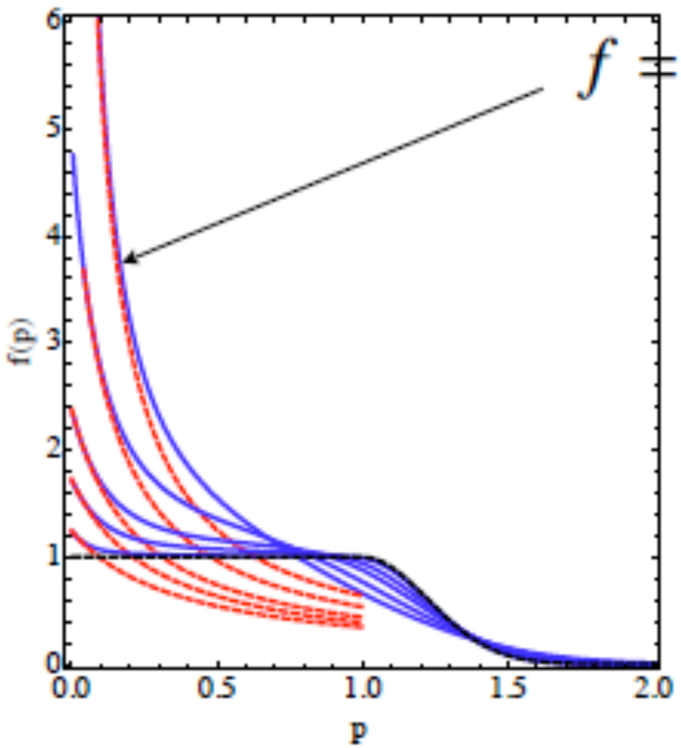
From: Epelbaum & Gelis 1107.0668

From: Berges & Sexty 1201.0687



How BEC Onset Occurs Dynamically?

A crucial step: rapid IR local thermalization



$$f = \frac{T^*}{p-\mu^*} \qquad (\mu^* < 0)$$

Very strong particle flux toward IR,
leading to rapid growth
and almost instantaneous
local thermal distribution
of very soft modes

What happens next depends on INITIAL CONDITION:

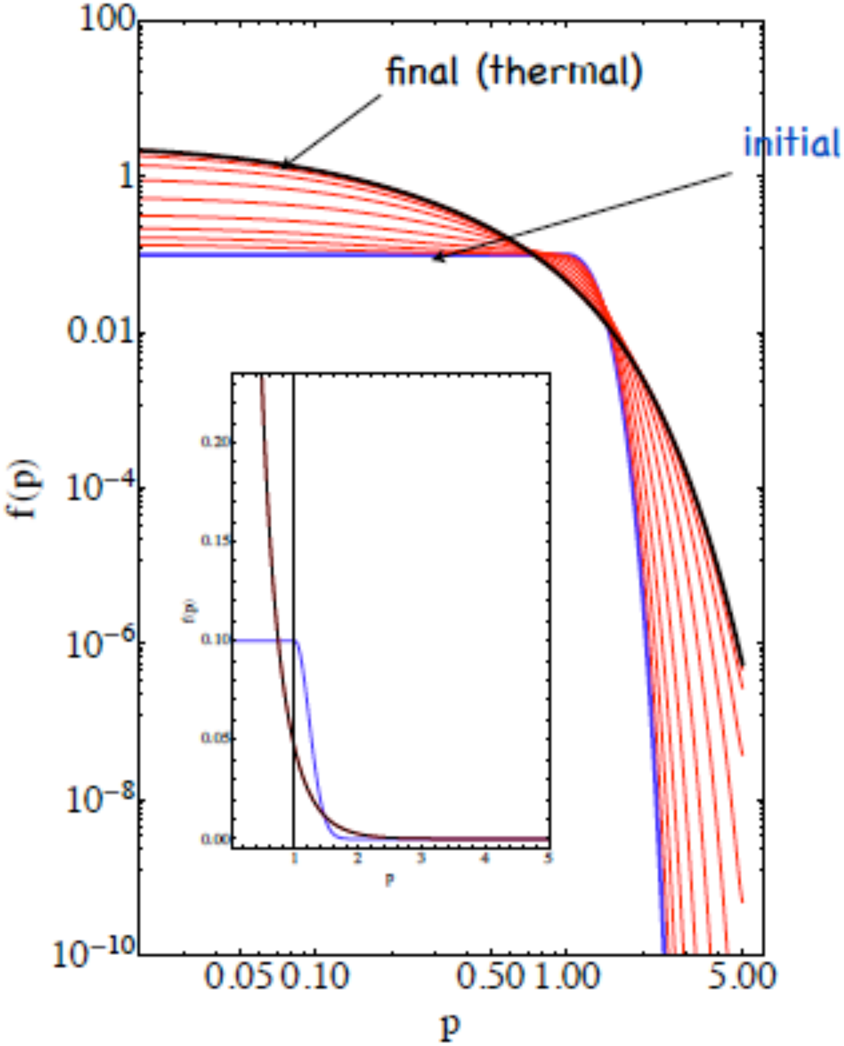
underpopulation v.s. overpopulation

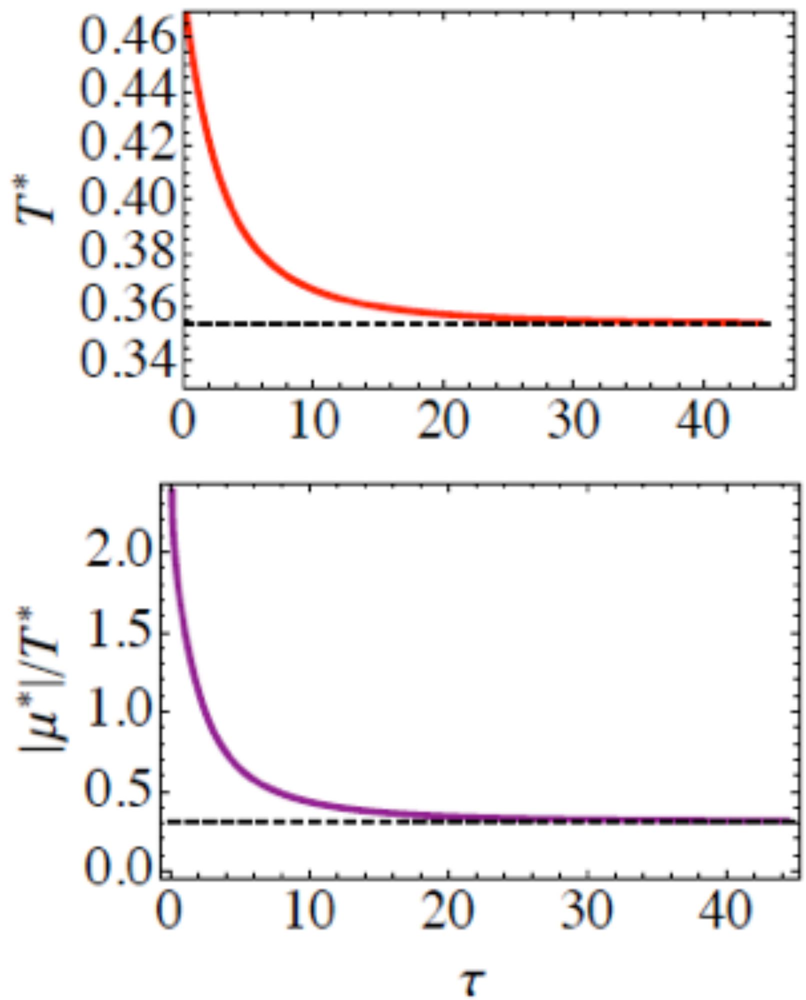
Blaizot, JL, McLerran, 1305.2119, NPA2013

Underpopulated Case

f 0 = 0.1

In underpopulated case, the system thermalizes to thermal BE distribution.



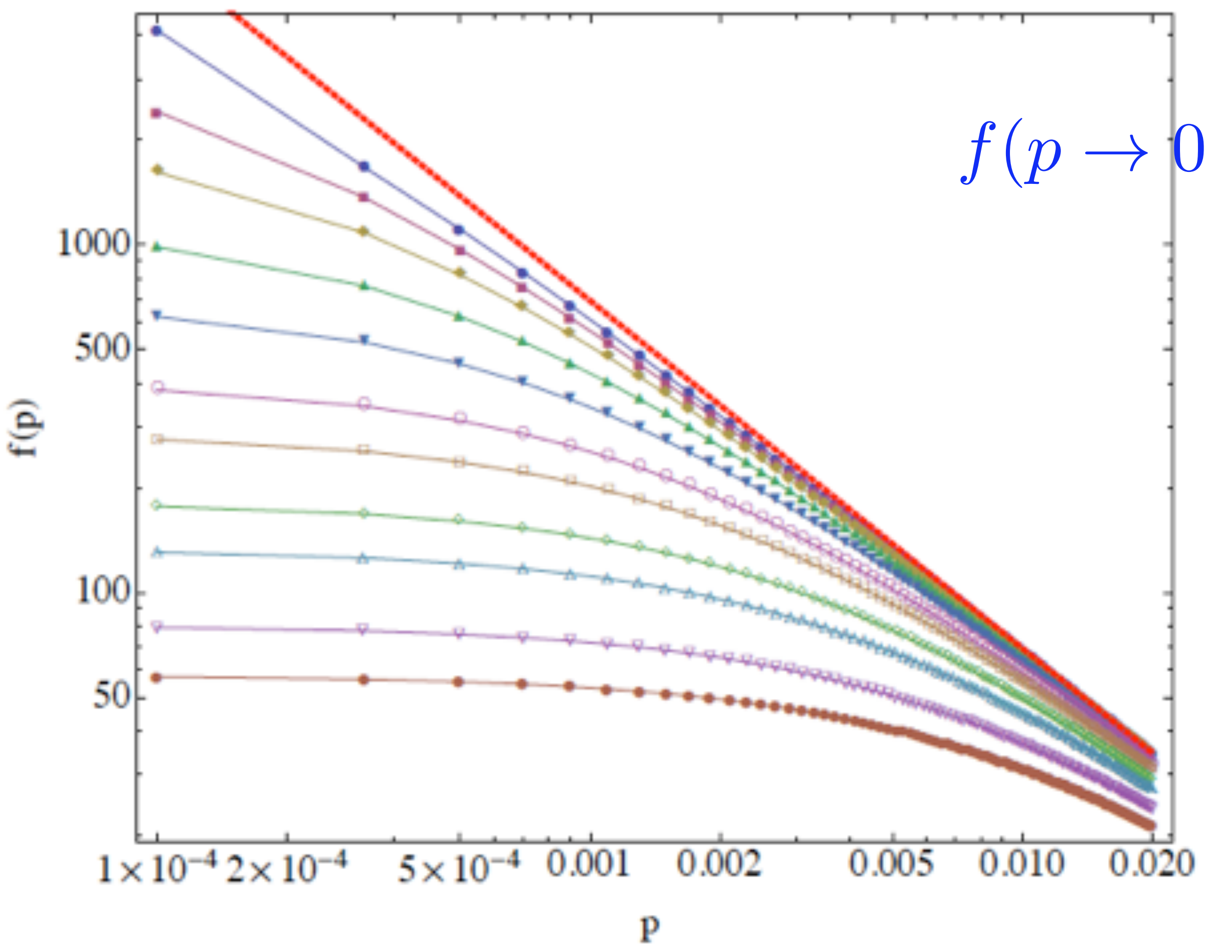


Overpopulated Case: How Onset of BEC Develops?

Before it could reach equilibrium, onset of BEC occurs!

A **critical** IR distribution develops, i.e. Mu* vanishes.

(In thermal BEC: global distribution must be critical.)

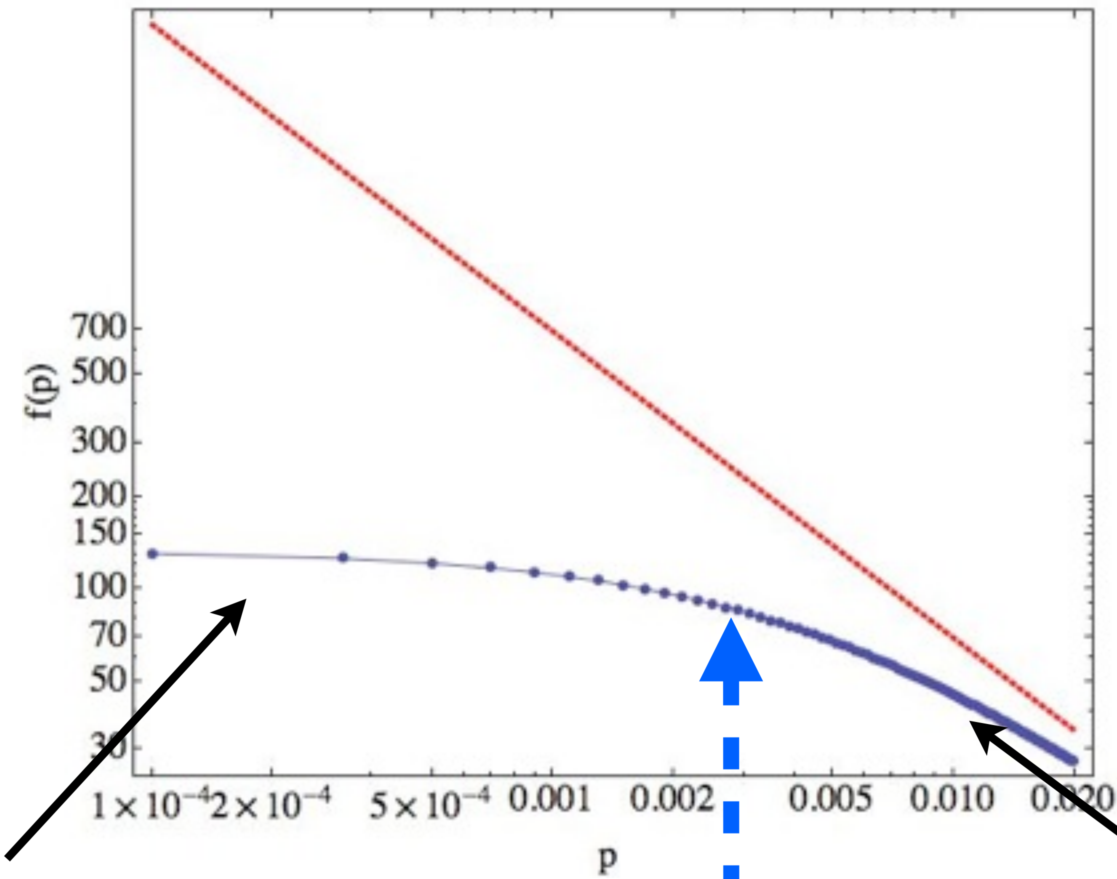


$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

$$\mu^* \to 0$$

f_0=1

Overpopulated Case: How Onset of BEC Develops?



p << *Mu**

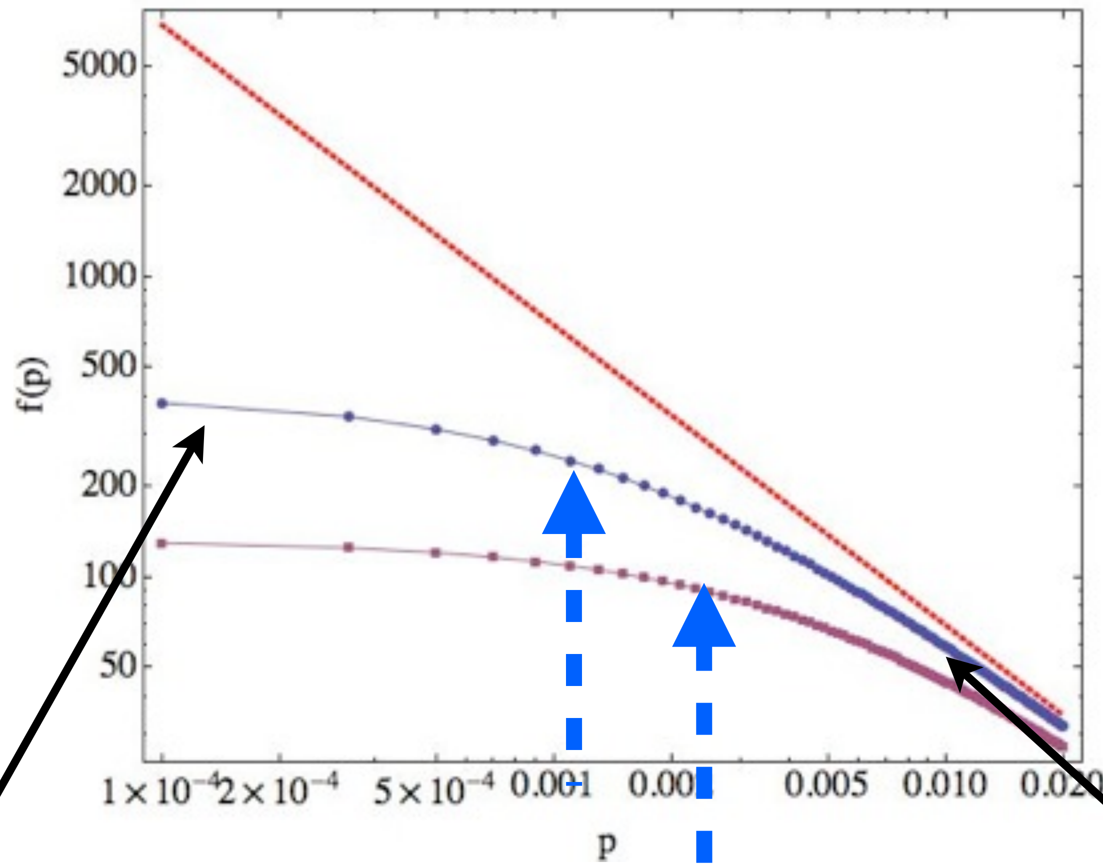
$$f \to T^* / |\mu^*|$$

p >> *Mu**

$$f \to T^* / p$$

$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

Overpopulated Case:
How Onset of BEC Develops?



p << *Mu**

$$f \to T^* / |\mu^*|$$

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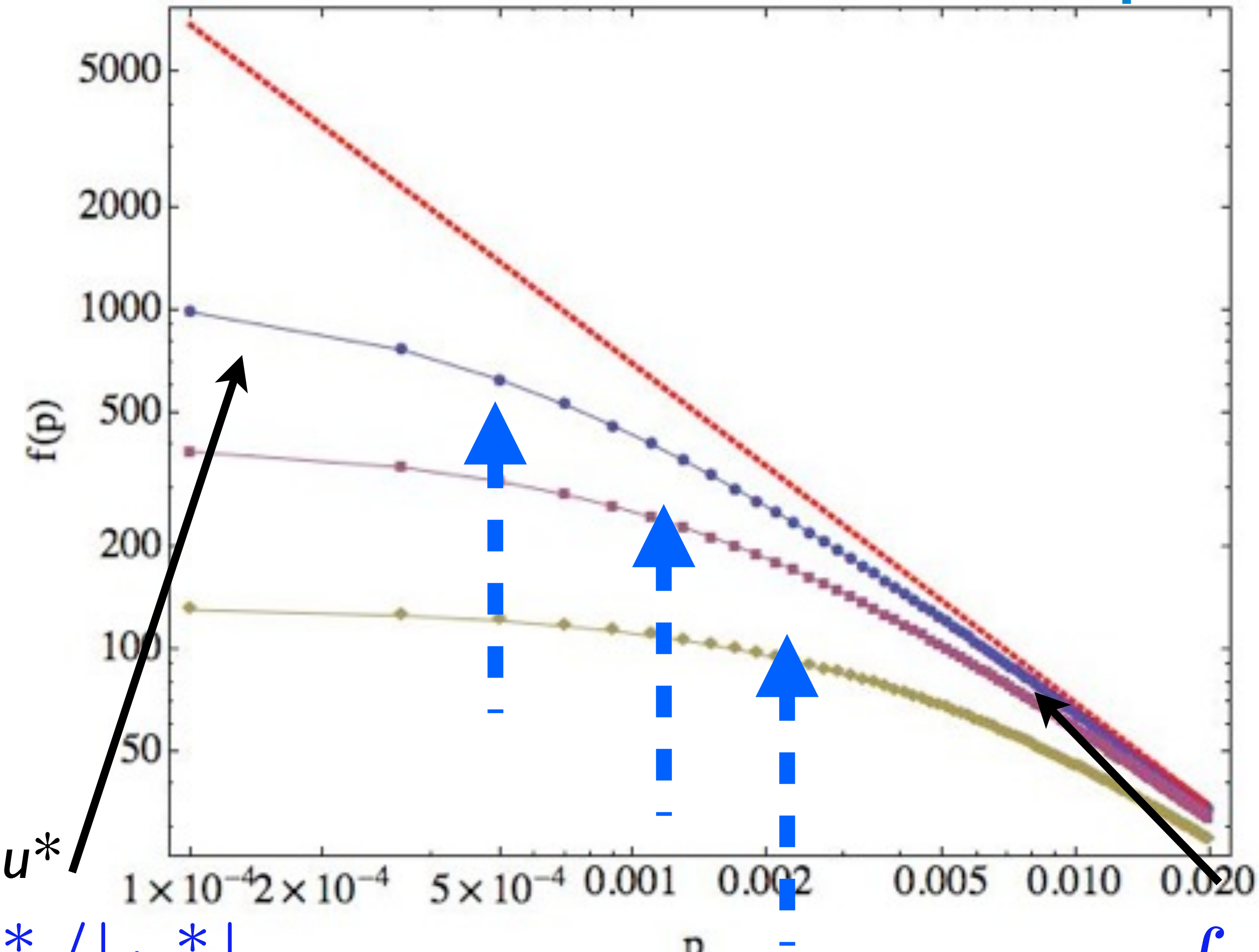
$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

Overpopulated Case: How Onset of BEC Develops?



p << *Mu**

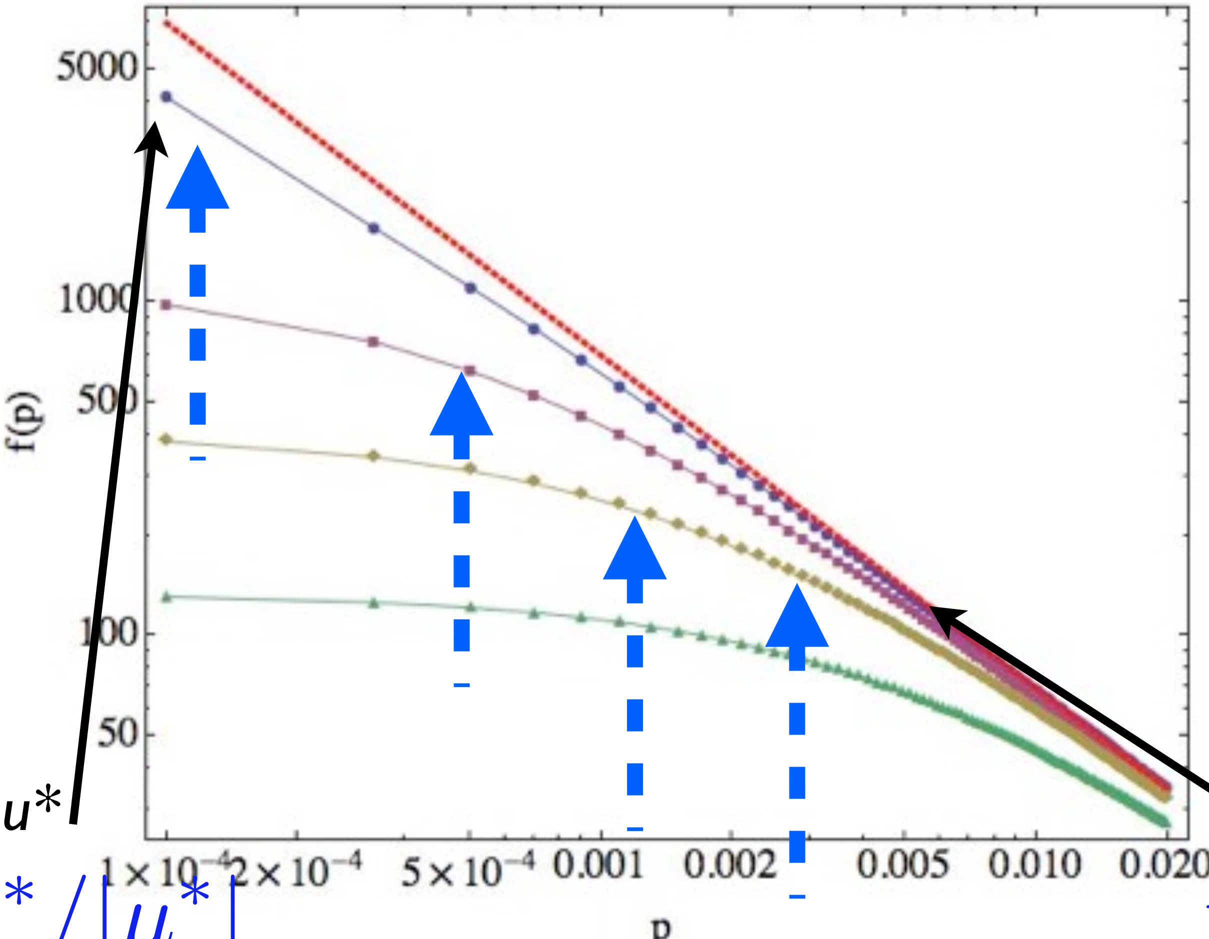
$$f \to T^*/|\mu^*|$$

p >> *Mu**

$$f \to T^*/p$$

$$f(p\to 0) \to \frac{T^*}{p-\mu^*}$$

Overpopulated Case: How Onset of BEC Develops?



$\mu^* \to 0$

proceed in a self-similar scaling way

p << *Mu**

$f \to T^*/|\mu^*|$

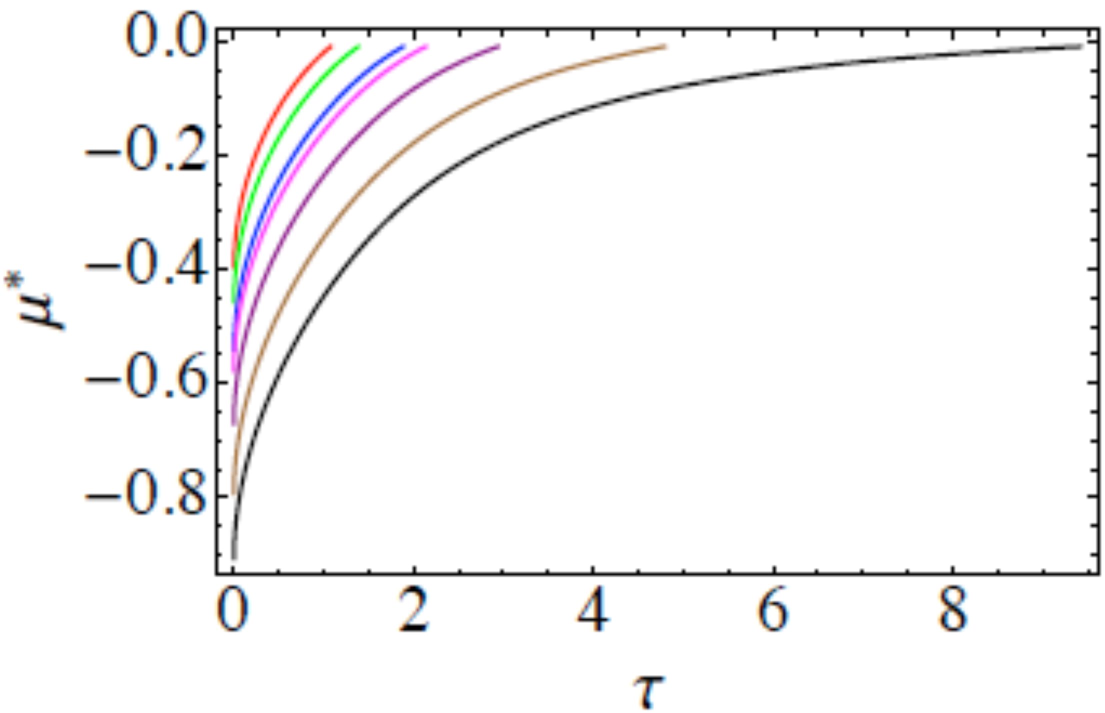
p >> *Mu**

$f \to T^*/p$

$$f(p \to 0) \to \frac{T^*}{p - \mu^*}$$

Onset of Dynamical BEC

Onset of dynamical (out-of-equilibrium) BEC:
* occurring in a finite time
* local Mu* vanishes with a **scaling behavior**
* persistence of particle flux toward zero momentum



$$|\mu^*| = C(\tau_c - \tau)^\eta.$$

$$\eta \simeq 1$$

For different
f_0 = 0.2, 0.3, 0.5, 0.8, 1, 2, 5

Blaizot, JL, McLerran, 1305.2119, NPA2013

How Robust is the BEC Onset Dynamics?

There are a number of important aspects to explore about this dynamical process from initial overpopulation to the onset of BEC:

✦ How does that depend on the initial distribution shape?

--> the same onset dynamics (Blaizot, Liao, McLerran)

✦ How does that depend on a finite mass (e.g. from medium effect)

--> the same onset dynamics (Blaizot, Jiang, Liao)

✦ How is that influenced by the longitudinal expansion?

--> the same onset dynamics (Blaizot, Jiang, Liao, McLerran)

✦ How is that influenced by including quarks?

--> the same onset dynamics (Blaizot, Wu, Yan)

✦ How is that influenced by including inelastic collisions?

--> the same onset dynamics (Huang, Liao)

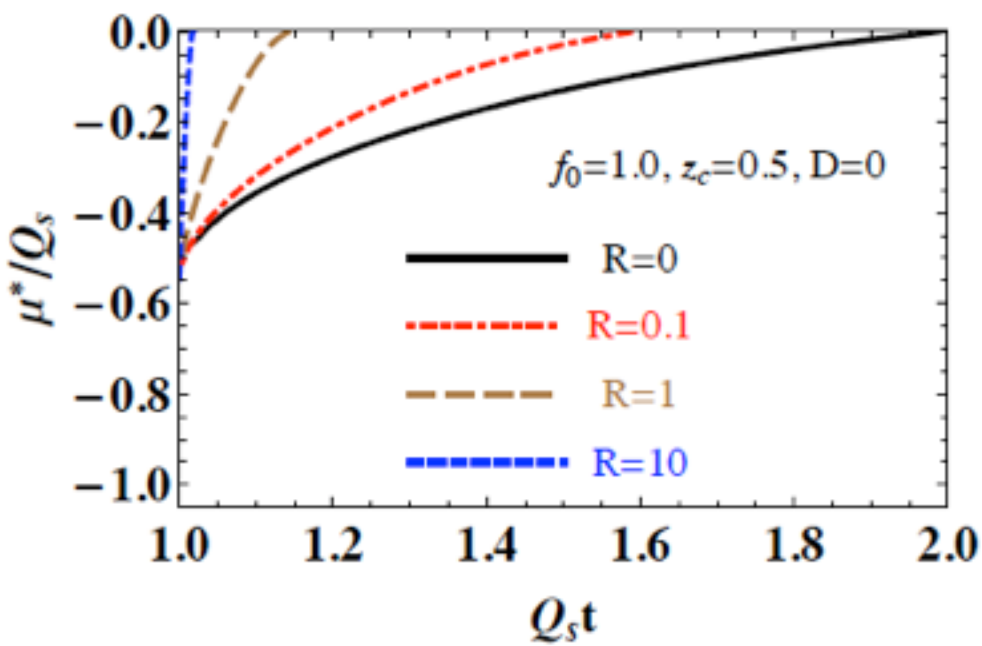
Including the Inelastic

An inelastic kernel including 2<-->3 processes
(Gunion-Bertsch, under collinear and small angle approxation)

$$\mathcal{D}_t f_p = \mathcal{C}^{\rm eff}_{2\leftrightarrow 2}[f_p] + \mathcal{C}^{\rm eff}_{1\leftrightarrow 2}[f_p],$$

Huang & JL, arXiv:1303.7214

$$\begin{aligned}\mathcal{C}^{\rm eff}_{1\leftrightarrow 2} \;=\;& \xi\,\alpha_s^2\, R\,\frac{I_a}{I_b}\left\{\int_0^{z_c}\frac{dz}{z}\left[g_p f_{(1-z)p} f_{zp} - f_p g_{(1-z)p} g_{zp}\right]\right.\\ &\left. + \int_0^{z_c}\frac{dz}{(1-z)^4 z}\left[g_p g_{zp/(1-z)} f_{p/(1-z)} - f_p f_{zp/(1-z)} g_{p/(1-z)})\right]\right\}\end{aligned}$$

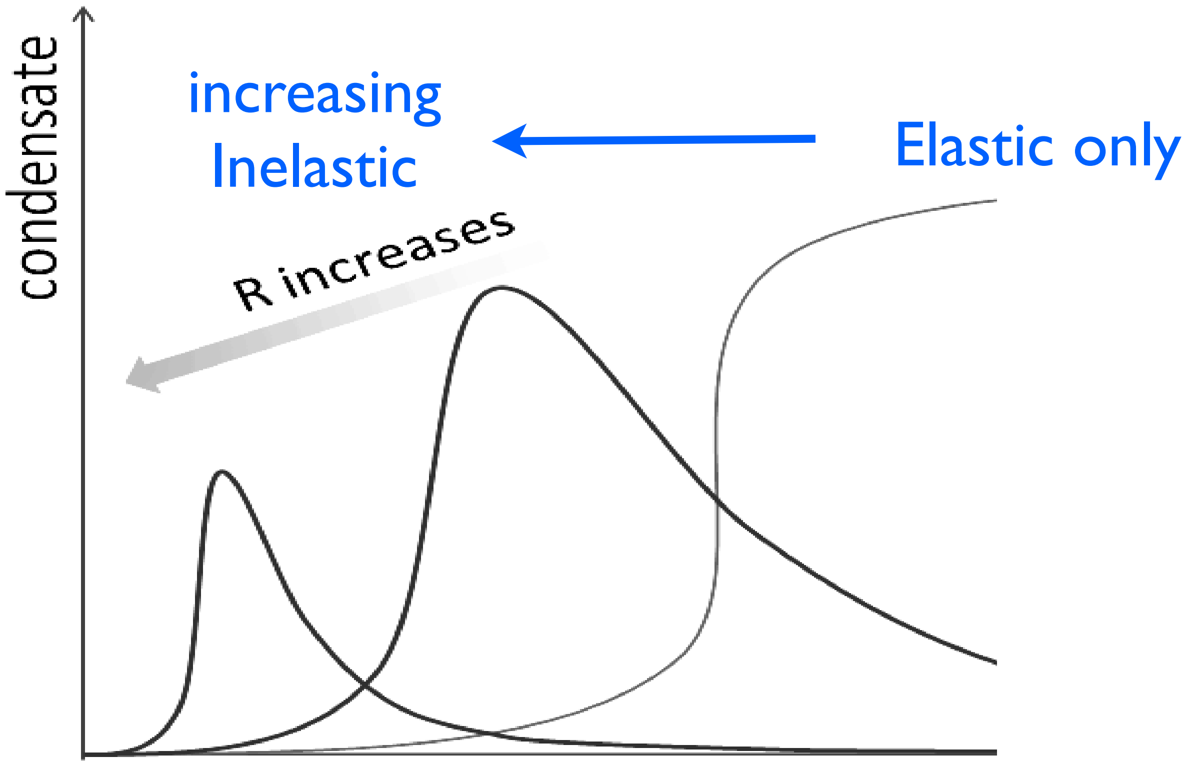


Local effect:
enhance IR growth,
accelerate the onset

Global effect:
reduce number density,
enhance entropy growth

The "Fuller" Picture

What we find: the inelastic process catalyzes
the onset of dynamical (out-of-equilibrium) BEC.
It might sound contradicting with common wisdom ...
but it is NOT.



Summary & Outlook

Summary

* Saturation physics sets the scale and initial conditions before and just after collision, described by color glass condensate.

* A hydrodynamic behavior seems to emerge rather quickly and universally for various colliding systems.

* In between the two, there is the **glasma** stage and understanding its evolution is an outstanding challenge. Various approaches are being developed to provide insights and hopefully solutions in the future.

* At certain point the glasma becomes a dense gluon system characterized by **saturation scale** and **high overpopulation,** describable within a **kinetic theory** framework.

* Elastic process (alone) in highly overpopulated system can induce **very rapid growth of soft modes** and **efficient isotropizating mechanism** in competition against expansion.

* Overpopulation may lead to a transient **Bose-Einstein Condensate**. The dynamical onset of BEC in a scaling way is found to be a very robust feature.

Outlook

* A number of key questions need to be understood:

--- could enough longitudinal pressure emerge quickly, and how?

--- how close is the microscopic picture to the thermal one?

--- could a transient condensate form, with what consequences?

--- how to smoothly account for the running at initial high Qs scale toward later, much lower T_thermal scale?

--- experimental observable with access to pre-equilibrium stage?

--- ...?

* Resolving existing issues within each type of approaches and investigating the "boundary" of their applicability

* Comparing results from varied approaches, and exploring a combination of them that may be ultimately required to describe the real world glasma evolution

* Matching to hydrodynamics & detailed prescription connecting the initial nuclear wave function to initial conditions for hydro