

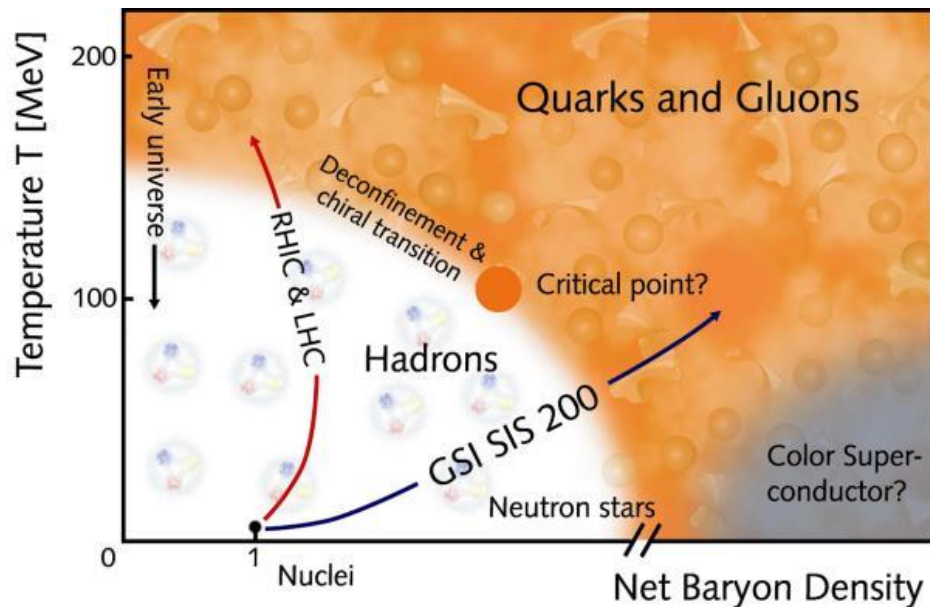
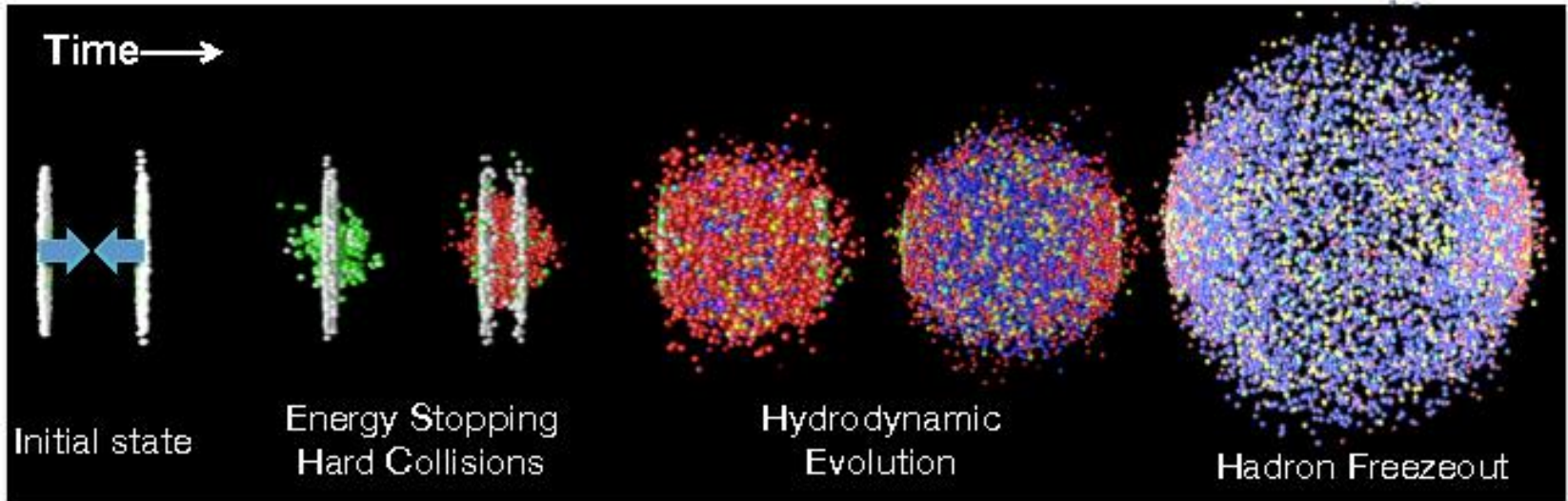
# Particle Correlations

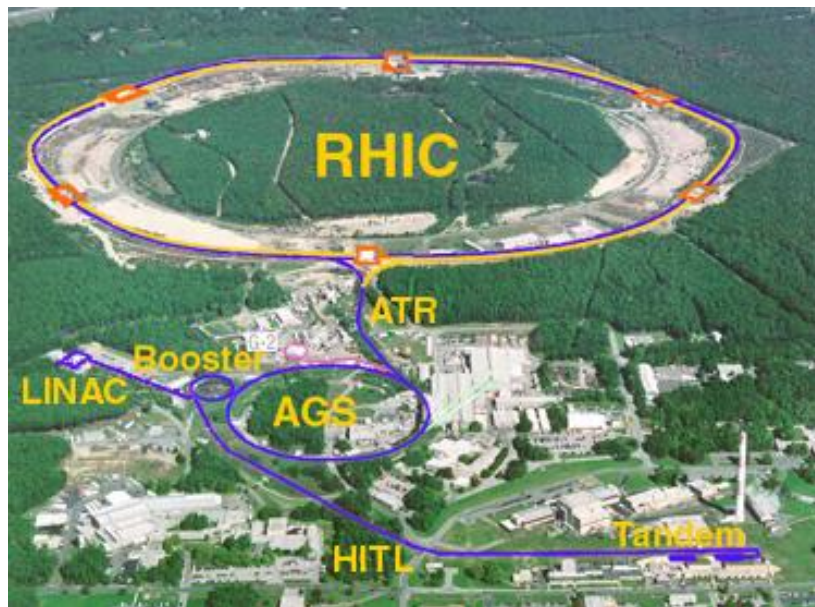
Fuqiang Wang  
Purdue University

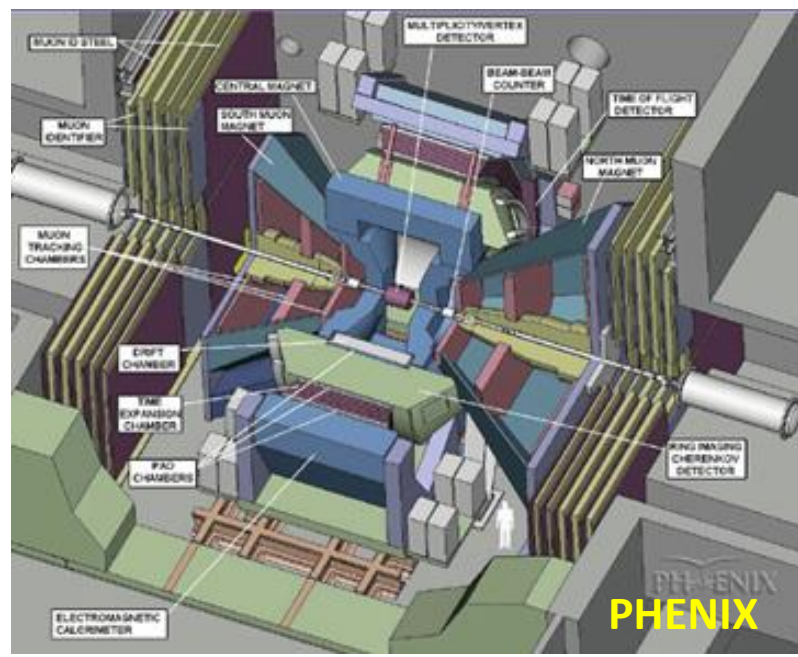
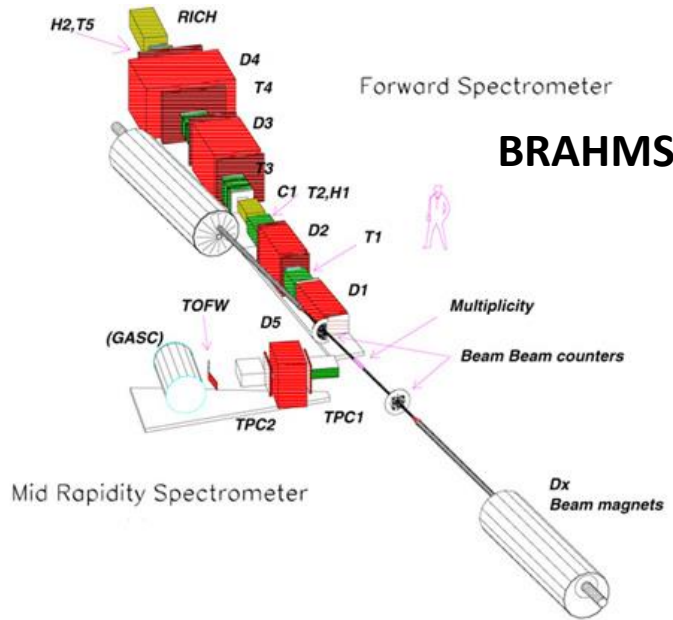
# Outline

- Why particle correlations?
  - Few-body (jet-like) correlations
  - Many-body (flow) correlations
  - Analysis techniques
- Particle correlations in heavy-ion collisions
  - Near-side ridge correlation
  - Away-side double-peak correlation
  - Triangular flow background
- Particle correlations in small systems
  - Revisit two-particle acceptance correction
- Flow correlations
  - Some new idea: initial state anisotropy, quantum mechanics

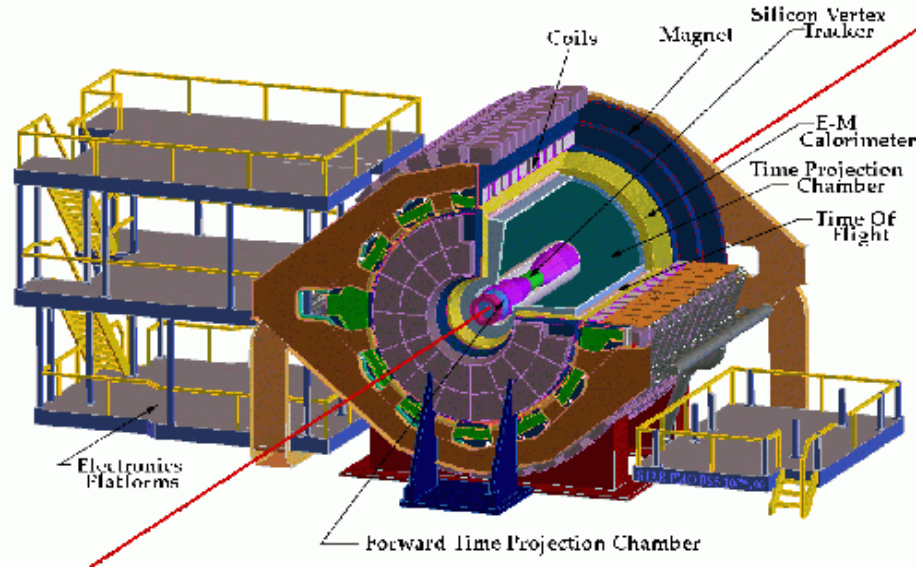
# Artist's view of heavy-ion collisions

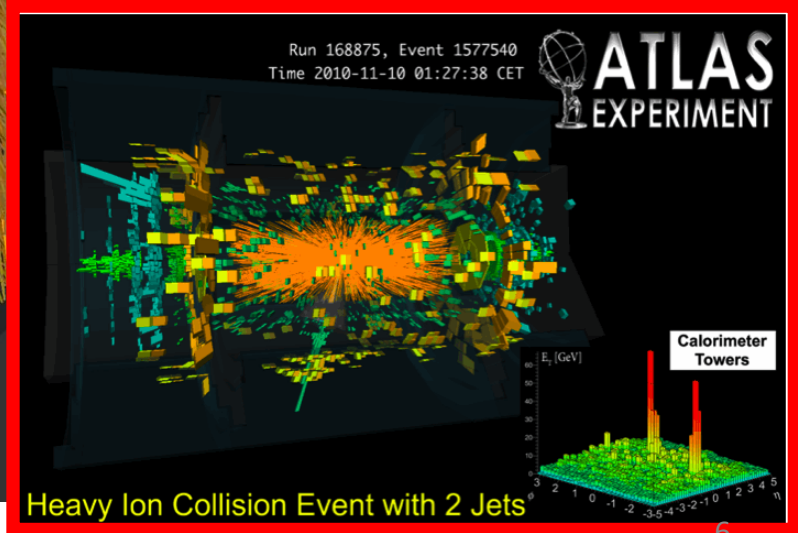
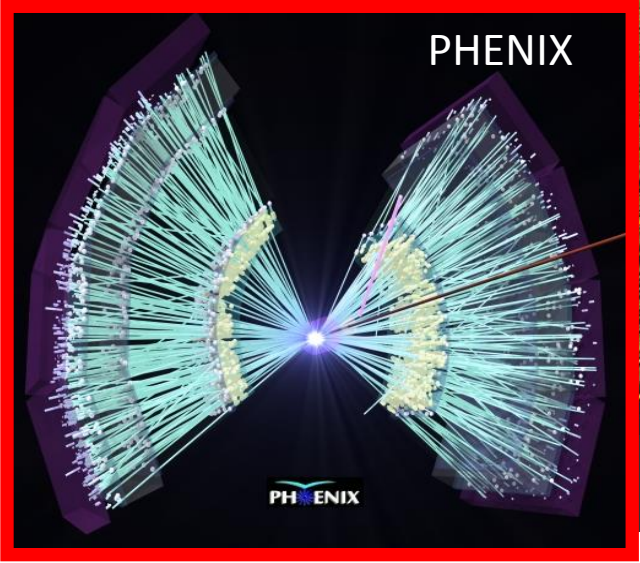
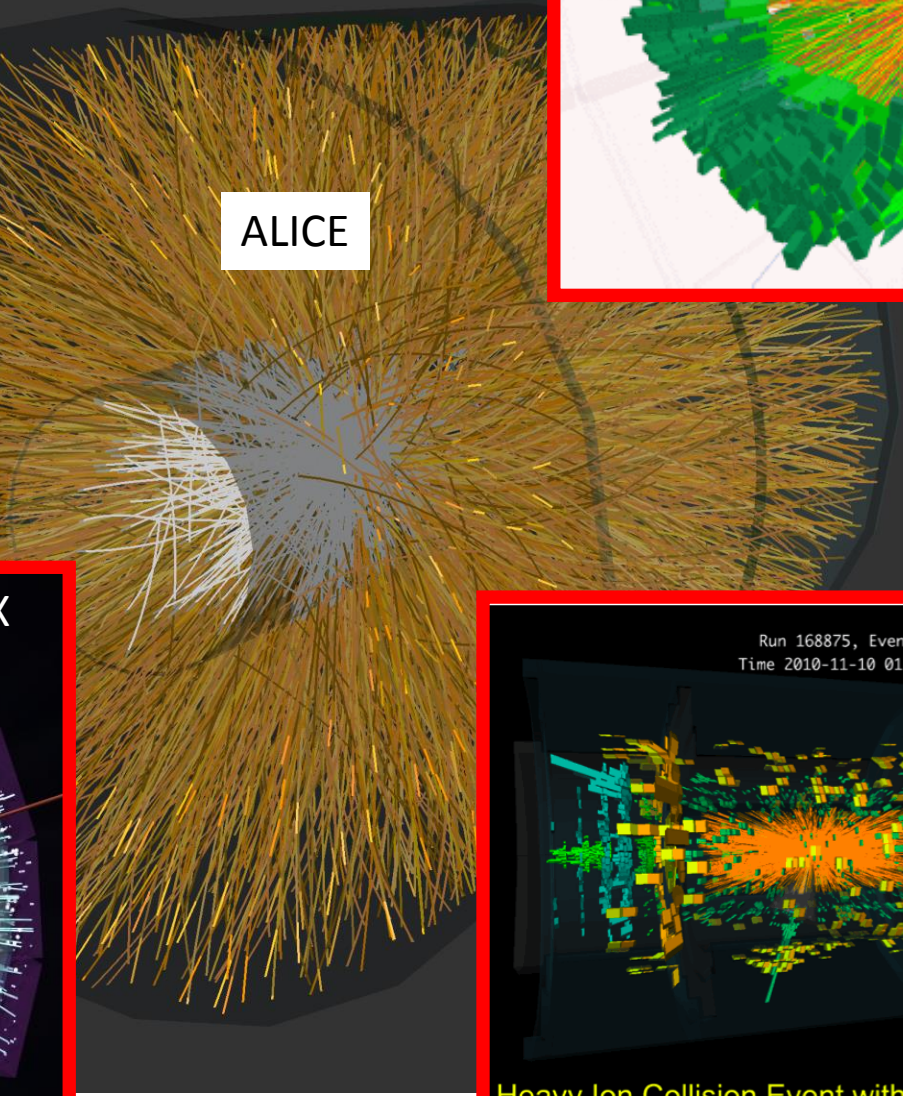
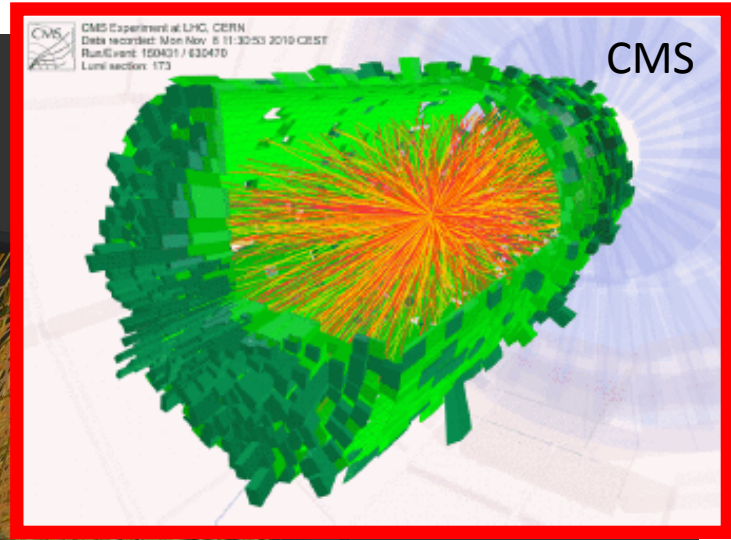
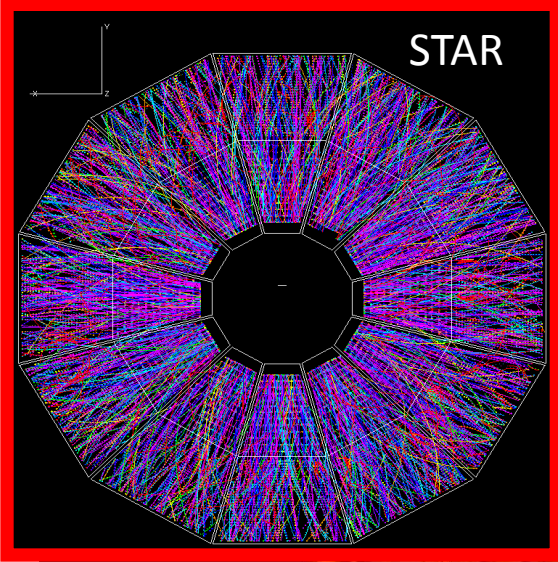






## STAR Detector



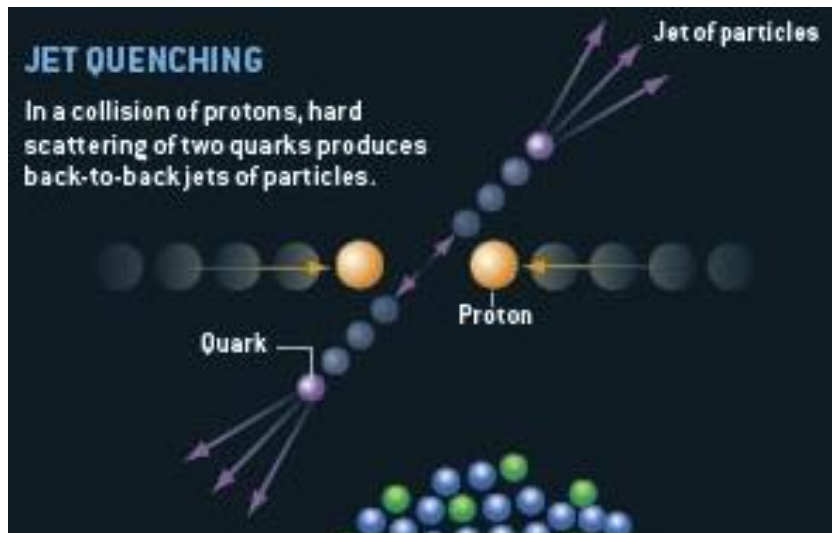


# Why particle correlations?

- Single particles can only measure production rates and kinematic distributions
- High-energy collisions are complex—need particle correlations to measure the complex structure of the collision system
- Particle correlations measure jet-like correlations, flow, etc.
- Majority of measurements in heavy-ion collisions are done by particle correlations

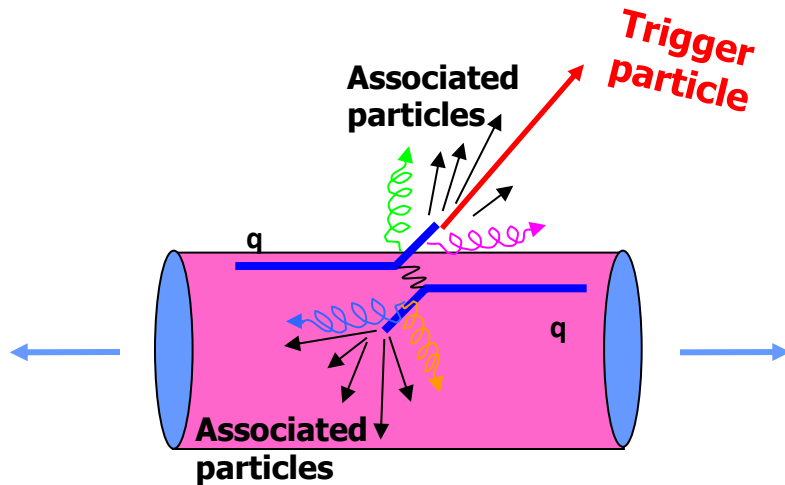
# Two categories of correlations

- Few-body, e.g.
  - Jets
  - Resonance decays
- Many-body, event-wise
  - Collective flow





# Analysis techniques



$$\Delta\phi = \phi_{assoc} - \phi_{trig}, \Delta\eta = \eta_{assoc} - \eta_{trig}$$

$$S(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{same}}{d\Delta\eta d\Delta\phi}$$

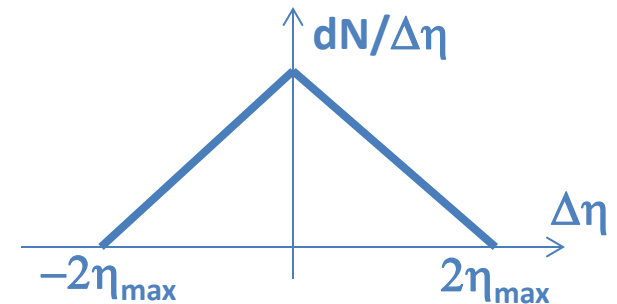
$$B(\Delta\eta, \Delta\phi) = \frac{1}{N_{trig}} \frac{d^2 N^{mix}}{d\Delta\eta d\Delta\phi}$$

- Tracking efficiency is corrected for associated particles.
- Trigger particles are often uncorrected, because correlations are normalized per trigger. Better to have trigger particle correction as well.
- Two-particle acceptance often corrected by mixed-events:  $B(\Delta\eta, \Delta\phi) / B(0, \Delta\phi)$ .

$$dN / d\eta = \text{const.} \quad (-\eta_{\max} < \eta < \eta_{\max})$$

$$dN / d\Delta\eta = \int d\eta_1 \int d\eta_2 (\text{const} \times \text{const}) \delta(\eta_2 - \eta_1 - \Delta\eta)$$

$$\propto 1 - \frac{|\Delta\eta|}{2\eta_{\max}}$$



# Particle correlations in heavy-ion collisions

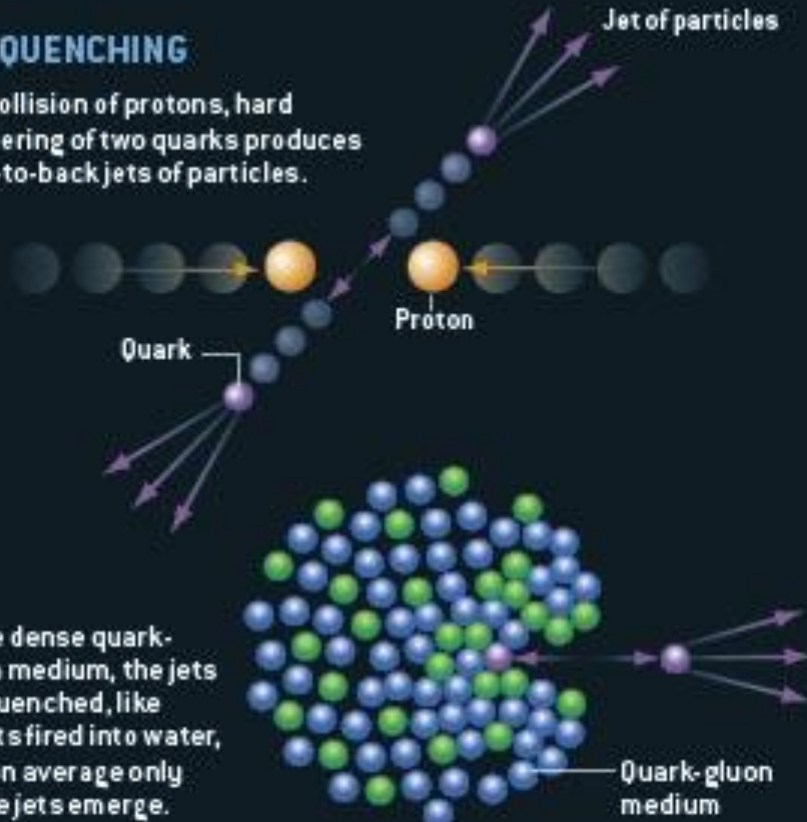
# Jet correlations

**EVIDENCE FOR A DENSE LIQUID**

Two phenomena in particular point to the quark-gluon medium being a dense liquid. Jet quenching implies the quarks and gluons are closely packed, and

**JET QUENCHING**

In a collision of protons, hard scattering of two quarks produces back-to-back jets of particles.

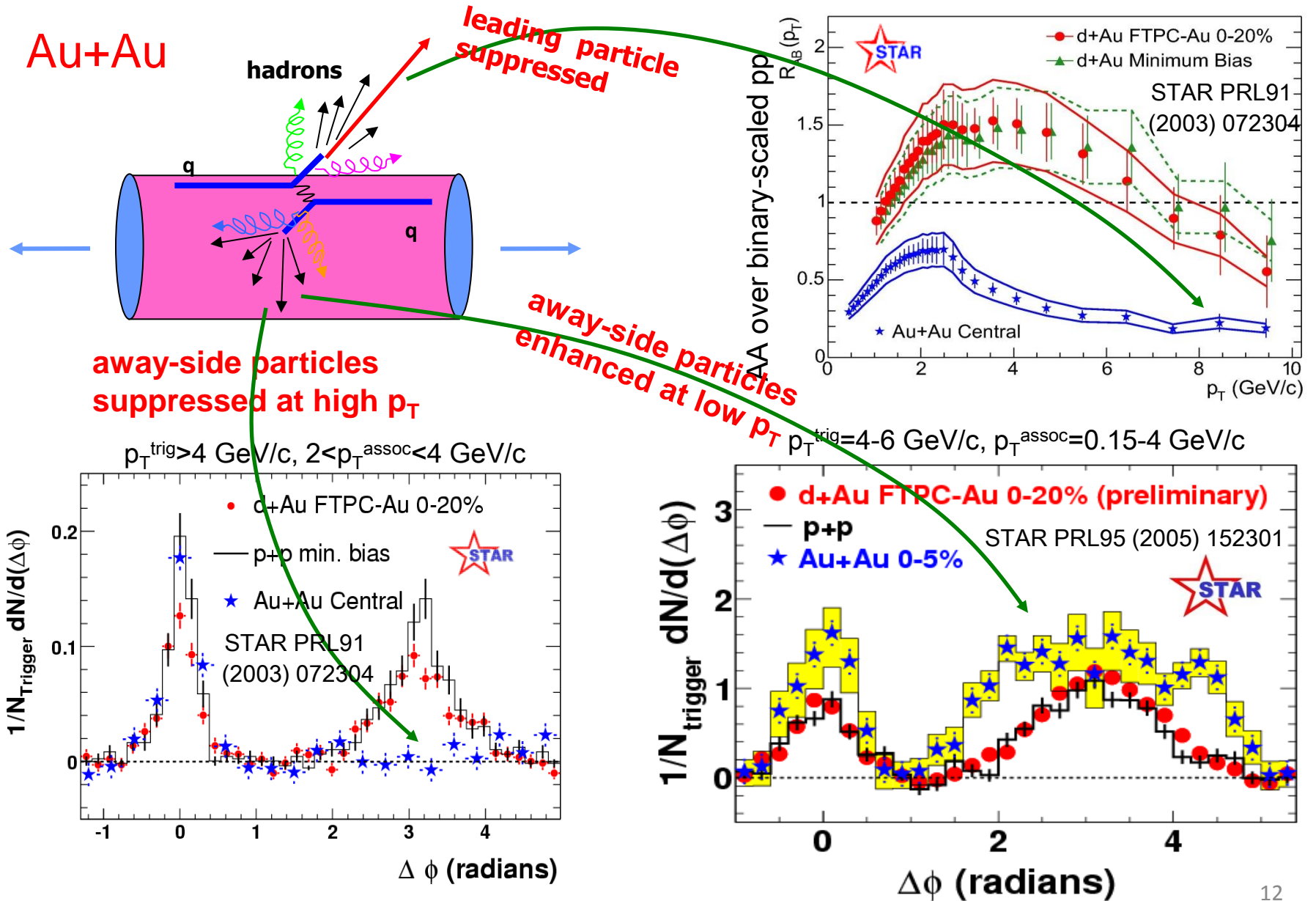


In the dense quark-gluon medium, the jets are quenched, like bullets fired into water, and on average only single jets emerge.

Labels in the diagram: Quark, Proton, Jet of particles, Quark-gluon medium.

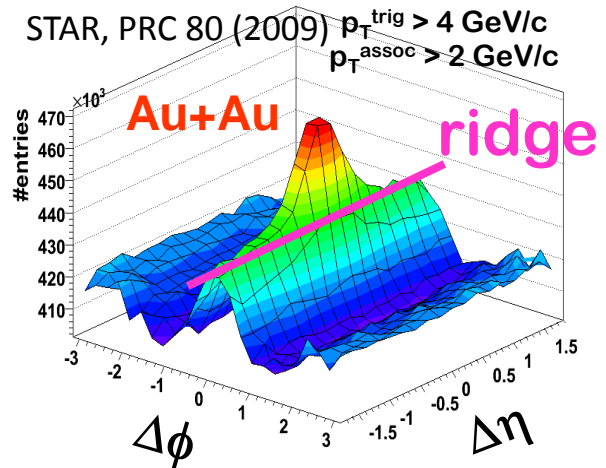
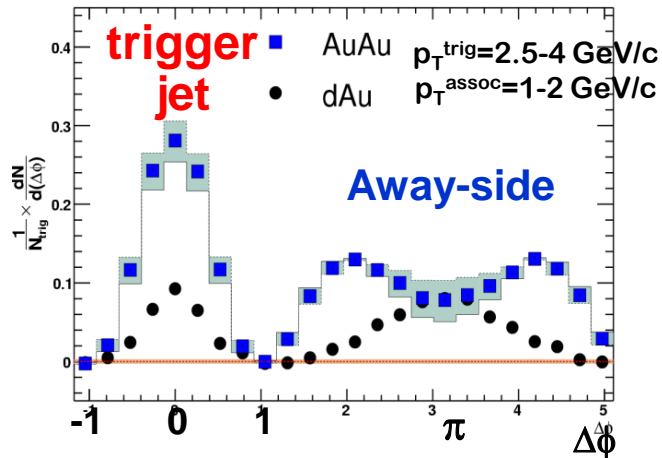
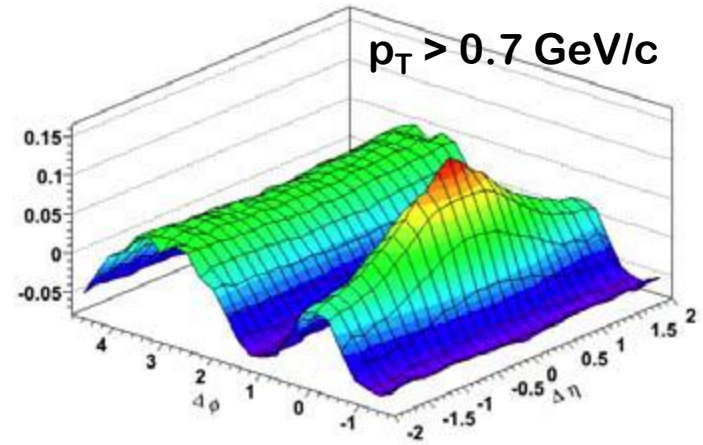
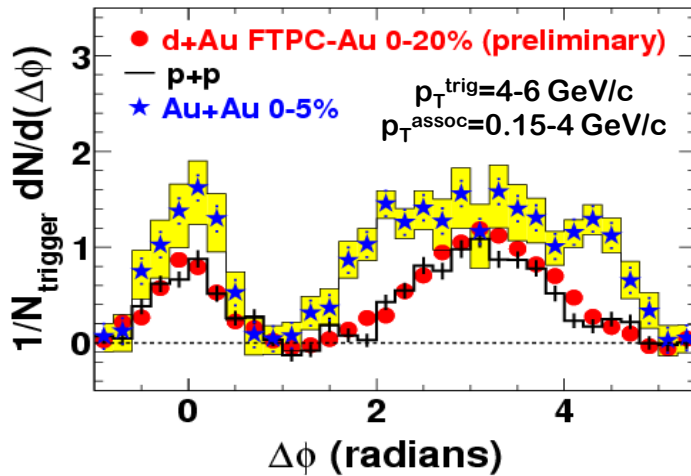
- Hard-scattering between partons in pp.
- Calculable by pQCD
- Fragmentation of partons produces back-to-back jets of hadrons.
- Jets are clustered in angle and rich in high- $p_T$  particles.
- **Jets produced in AA traverse and interact with the medium, lose energy and thus carry information of the medium.**

# Particle correlations: focus on away side

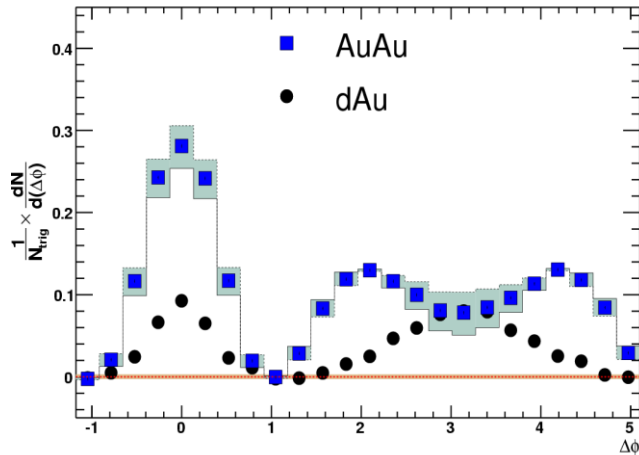


# The near-side is also interesting

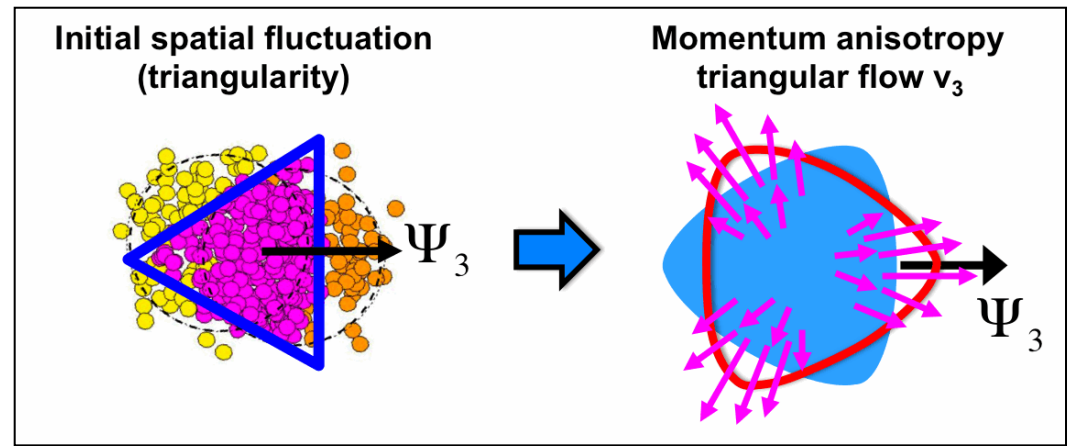
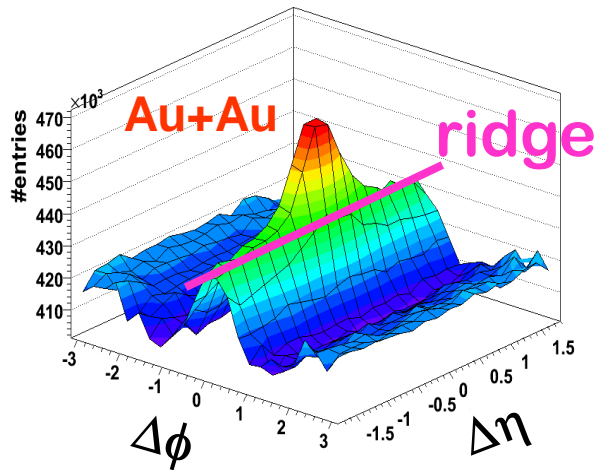
STAR, PRL 95 (2005); PRC82 (2010)



# Triangular flow in heavy-ions

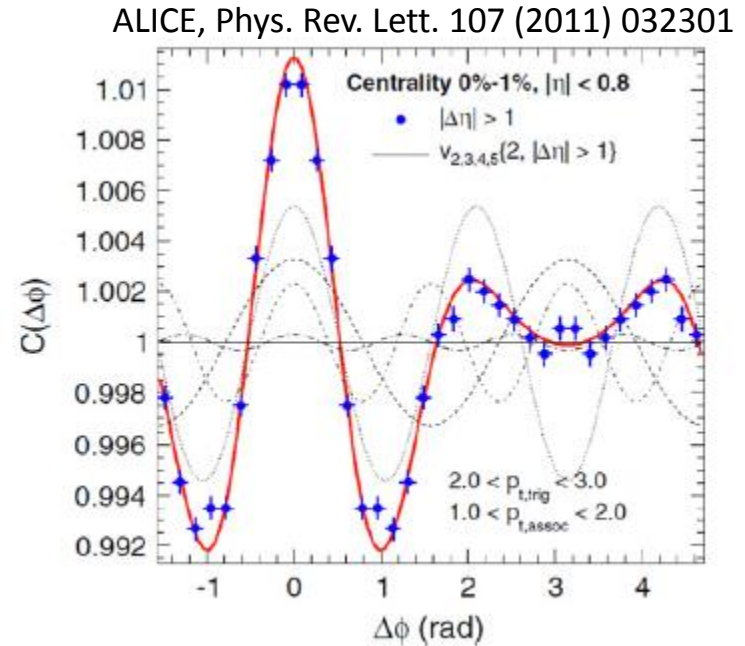


- Double-peak away-side correlations
- Long-range near-side ridge
- Triangular flow,  $v_3$
- Other odd harmonics



# $v_n$ are measured by two-particle correlations

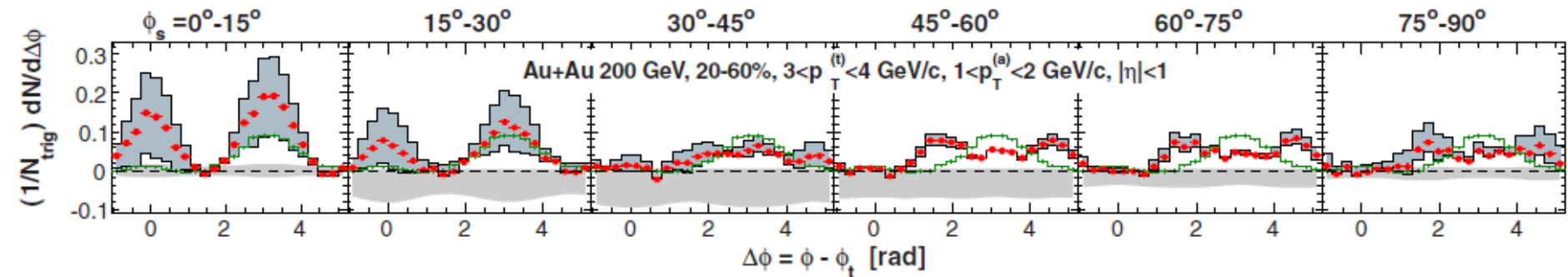
- $V_n$  from two-particle correlation
- Subtract  $v_n$  from two-particle correlation
- Almost a tautology
- Comparison to hydro gives us confidence that  $v_n$  are mostly from flow
- Quantitatively how much is flow and how much is nonflow—still an open question.
- Hydro has some tension to simultaneously describe  $v_2$  and  $v_3$
- Important to reduce/eliminate nonflow contributions to flow; do as best a job as we can.



# EP-dep. correlation with $v_n$ subtraciton

## Strategy:

- Measure  $v_n$  by two-particle correlation with one particle at as low  $p_T$  as feasible, to maximally reduce nonflow contaminations.
- Subtract  $v_n$  measurements from two-particle correlations at high and intermediate  $p_T$ .



## Open questions:

- Effect of jets on event plane reconstruction?
- Are any remaining correlations still coming from hydro flow, i.e. jets are completely gone?



# Particle correlations in small systems

# Ridge in small systems

usual p-p collision

high multiplicity p-p collision

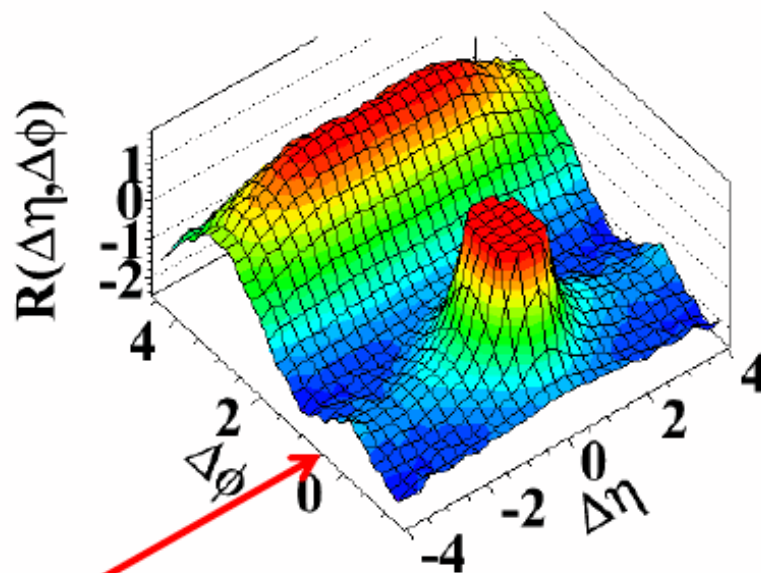
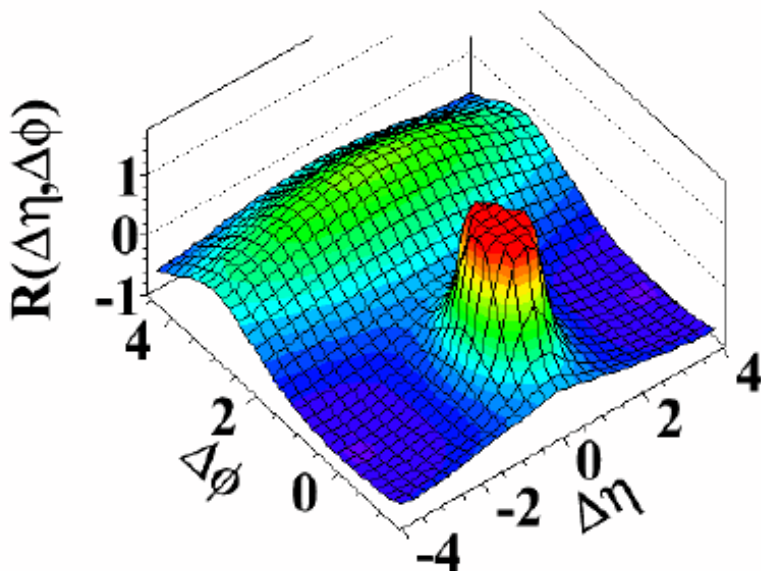
Minimum Bias  
no cut on multiplicity

High multiplicity data set  
and  $N > 110$

(b) MinBias,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$

CMS, JHEP 1009 (2010) 091

(d)  $N > 110$ ,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



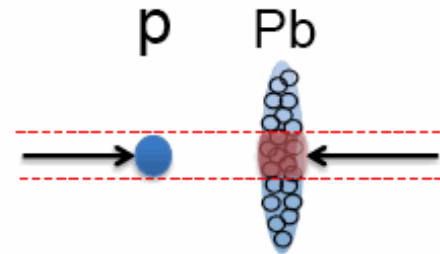
New “ridge-like” structure extending to large  $\Delta\eta$  at  $\Delta\phi \sim 0$

- Why wasn't it discovered long ago by HEP?
- Two types of discoveries:
  - Theoretically predicted, and experimentally verified
  - Surprises
- HEP moved on to more exclusive processes
- There may be still important physics that were missed in last half century

p-p collision (high Mult.)

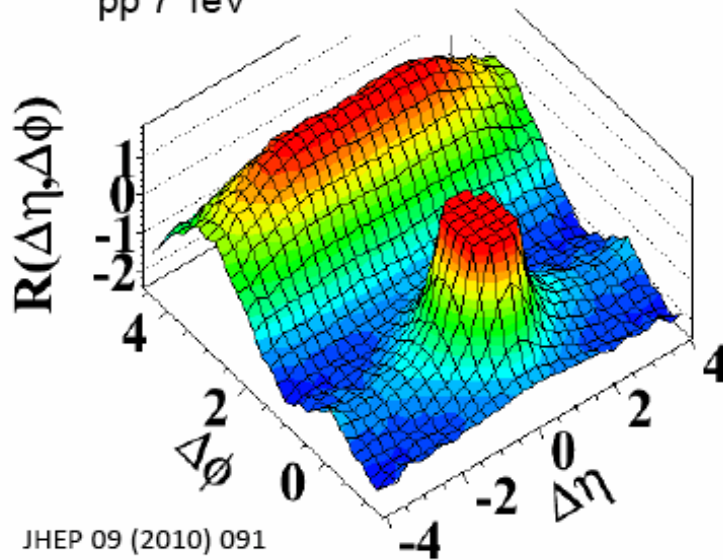
p-Pb collision (high Mult.)

Physical origin unclear



CMS, JHEP 1009 (2010) 091

(d)  $N > 110$ ,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$   
pp 7 TeV

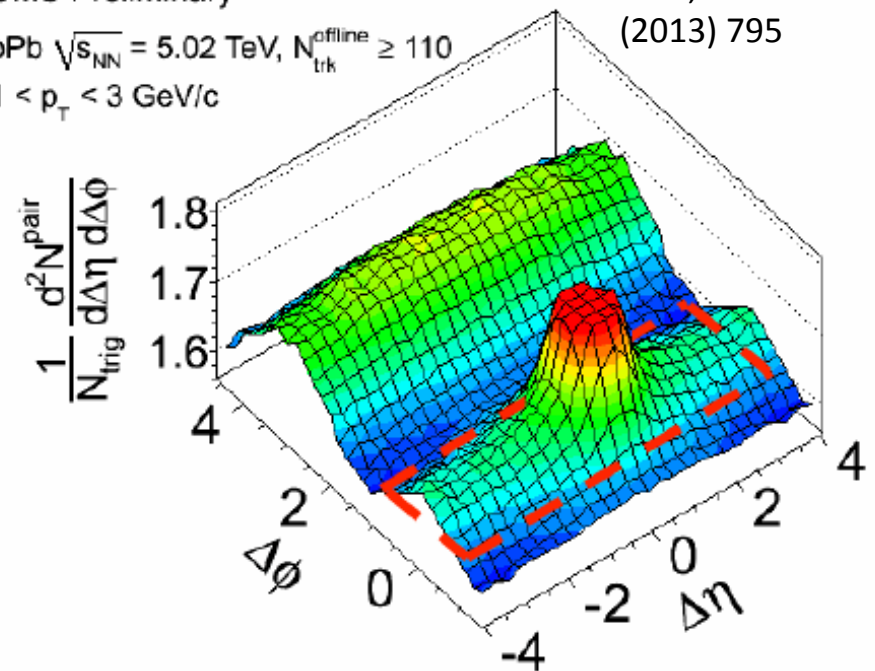


JHEP 09 (2010) 091

CMS Preliminary

pPb  $\sqrt{s_{NN}} = 5.02 \text{ TeV}$ ,  $N_{\text{trk}}^{\text{offline}} \geq 110$   
 $1 < p_T < 3 \text{ GeV}/c$

CMS, PLB718  
(2013) 795

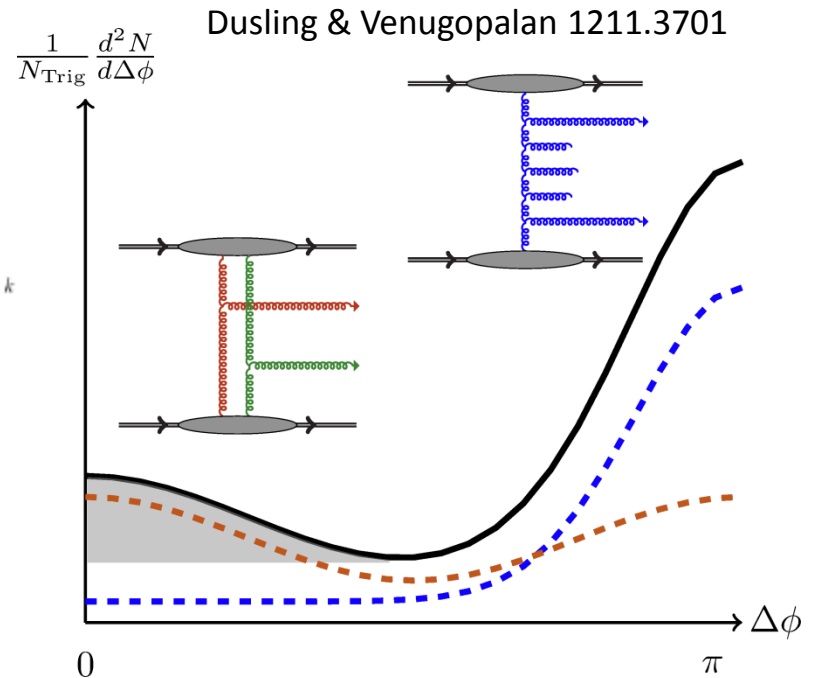
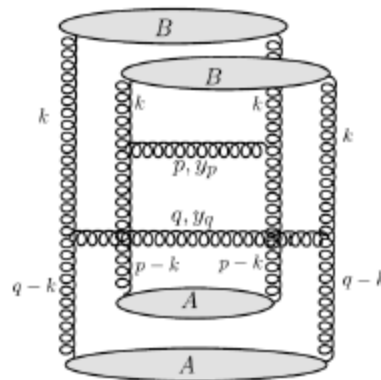
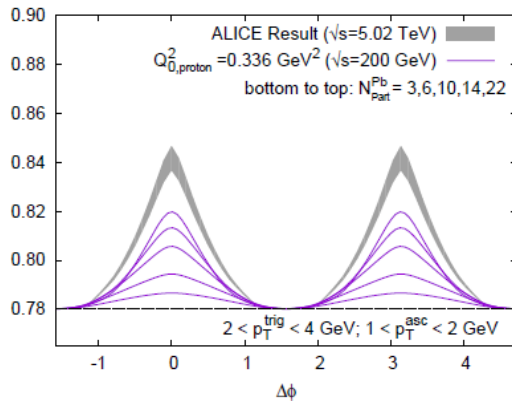


Much bigger than pp

# CGC/Glasma

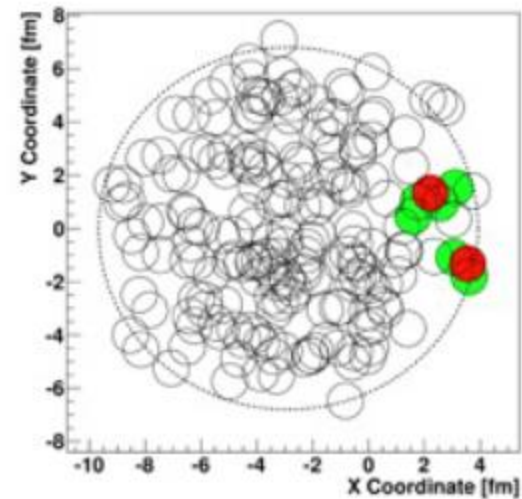
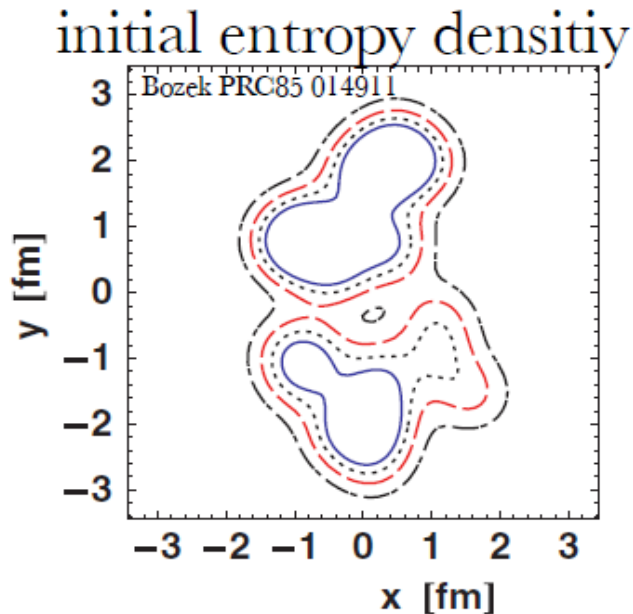
There is an intrinsic correlation in azimuthal angle coming from the two-particle production process, such as the one shown in Fig. 11 [92]. There is only a single loop momentum  $k_T$  in this two-particle production process, causing correlations. Because the single gluon distribution peaks at the saturation scale  $Q_s$ , large probability is found for production of two particles with their momenta  $p_T$  and  $q_T$  parallel to each other such that  $|p_T - k_T| \sim Q_s$  and  $|q_T - k_T| \sim Q_s$ . These processes therefore cause small angle correlations at  $\Delta\phi = 0$ . Because the correlations originate from the very early times of the collision,  $\tau_{\text{init.}}$ , they can persist to large rapidity differences,  $\Delta y = 2 \ln(\tau_{\text{f.o.}}/\tau_{\text{init.}})$  where  $\tau_{\text{f.o.}}$  is the particle freeze-out proper time.

Dusling and Venugopalan, arXiv:1302.7018



# Another explanation: Hydro flow

- In heavy-ions, subtract  $v_2 \rightarrow$  non-zero finite correlation: near-side large  $\Delta\eta$  ridge, away-side double peak  $\rightarrow v_3$
- In pp, pA (and possibly dA) systems, subtract uniform pedestal  $\rightarrow$  non-zero finite correlation: large  $\Delta\eta$  ridge  $\rightarrow v_2$  (and  $v_3$ )



# Acceptance correction *revisited*

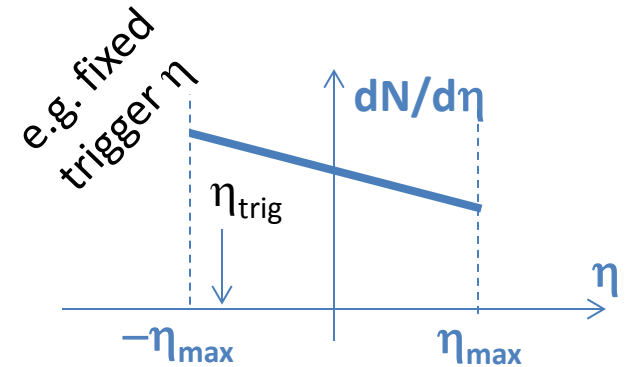
L.Xu, C.H.Chen, FW, PRC88 (2013) 064907

- Two-particle acceptance correction by mixed-events is, in principle, wrong.

$$\frac{1}{N_{trig}} \frac{d^2 N^{same}}{d\Delta\eta d\Delta\phi} \bigg/ \frac{1}{N_{trig}} \frac{d^2 N^{mix}}{d\Delta\eta d\Delta\phi}$$

- Should just be corrected by single particle efficiencies:

$$\frac{1}{N_{trig}} \frac{d^2 N}{d\Delta\eta d\Delta\phi} \bigg/ \mathcal{E}_{trig} \mathcal{E}_{assoc}$$

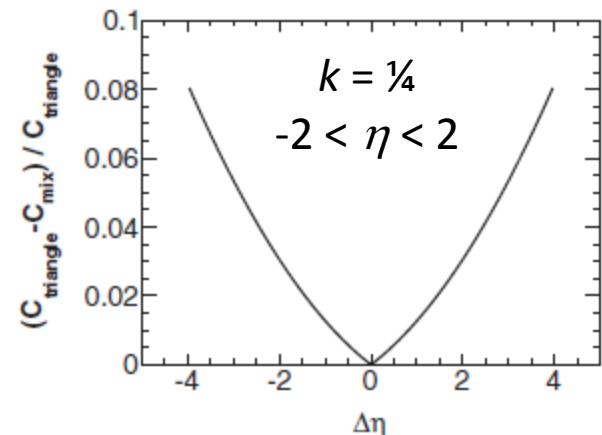


- How much error it makes?

$$\frac{dN}{d\eta} \propto 1 + k \frac{\eta}{\eta_m},$$

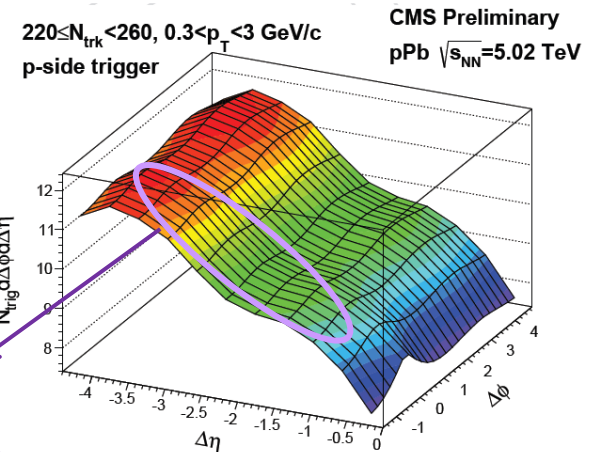
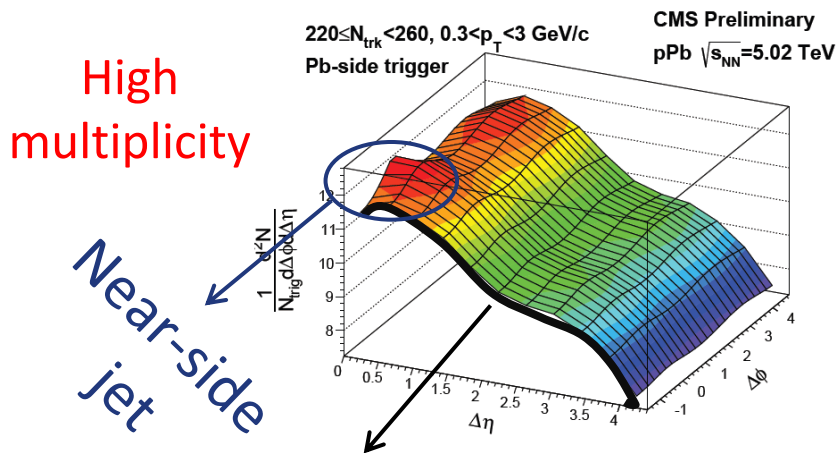
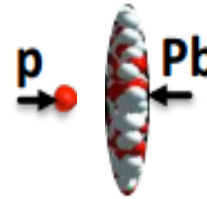
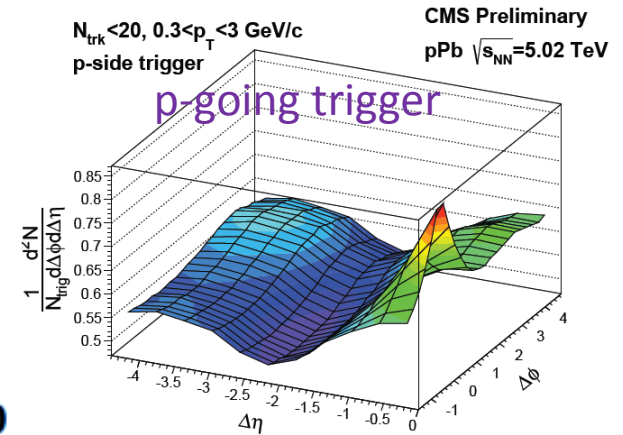
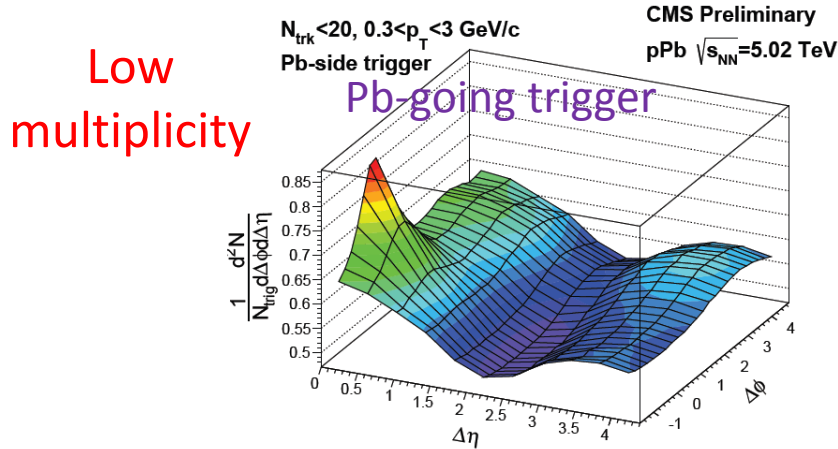
$$\begin{aligned} \frac{dN}{d\Delta\eta} &\propto \int_{\eta_1} \int_{\eta_2} \left(1 + k \frac{\eta_1}{\eta_m}\right) \left(1 + k \frac{\eta_2}{\eta_m}\right) \\ &\quad \times \delta(\eta_2 - \eta_1 - \Delta\eta) d\eta_1 d\eta_2 \\ &= \int_{\max(-\eta_m, -\eta_m - \Delta\eta)}^{\min(\eta_m, \eta_m - \Delta\eta)} \left(1 + k \frac{\eta_1}{\eta_m}\right) \left(1 + k \frac{\eta_1 + \Delta\eta}{\eta_m}\right) d\eta_1 \\ &= (2\eta_m - |\Delta\eta|) \left\{ 1 + \frac{1}{6} k^2 \left[ 2 - 2 \frac{|\Delta\eta|}{\eta_m} - \left( \frac{\Delta\eta}{\eta_m} \right)^2 \right] \right\}. \end{aligned}$$

$$\frac{C_{triangle} - C_{mix}}{C_{triangle}} = \frac{k^2}{6 + 2k^2} \frac{|\Delta\eta|}{\eta_m} \left( 2 + \frac{|\Delta\eta|}{\eta_m} \right)$$



# Dihadron per trigger pair density

L. Xu (CMS) QM 2014

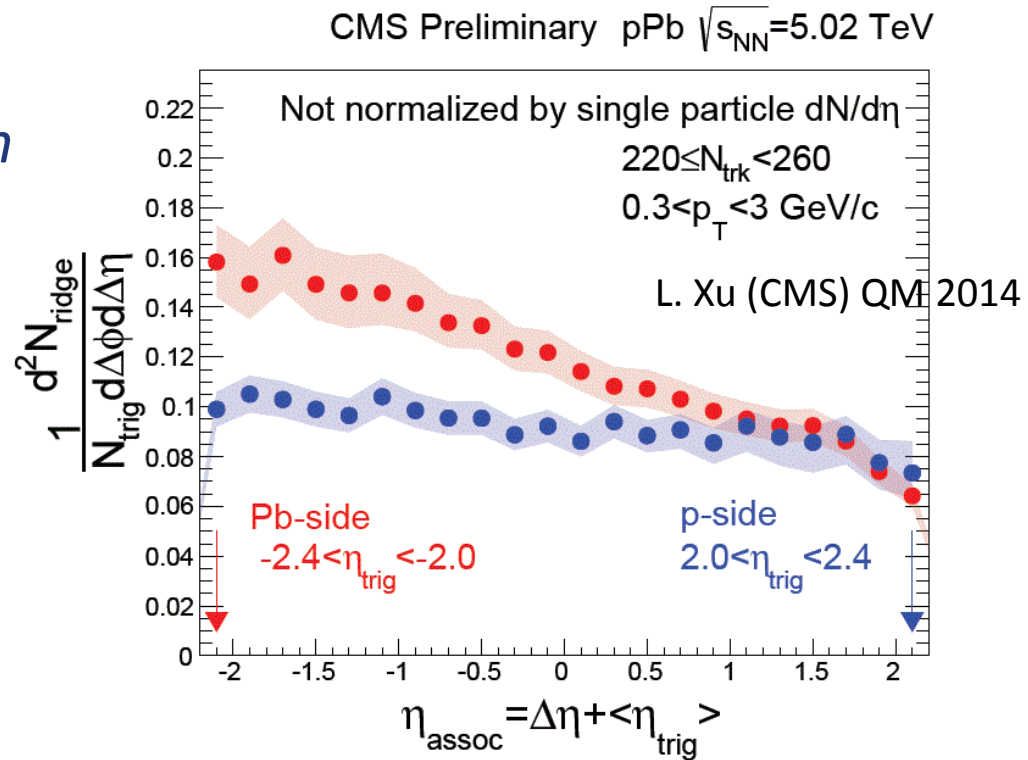
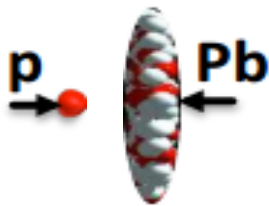


Shape reflects  
single particle  $dN/d\eta$



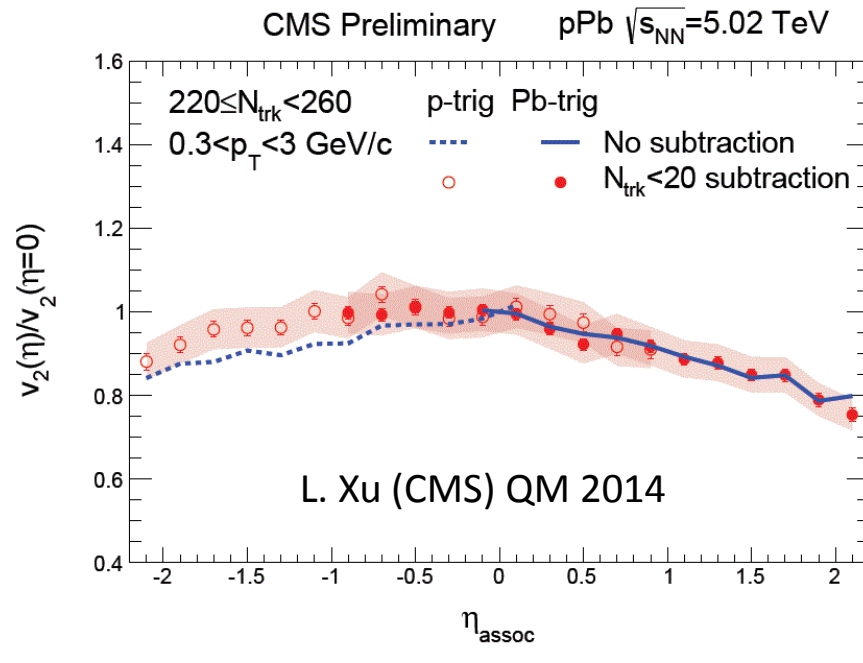
# Ridge yield vs $\eta_{\text{assoc}}$

*Near-side ridge  
after jet subtraction*



- Near-side ridge yield: different  $\eta$  dependences for p-going and Pb-going triggers

# $\eta$ -dependence of $v_2(\eta)/v_2(0)$



- $v_2$  shape is  $\eta$  dependent in p+Pb!
- $v_2$  asymmetric about mid-rapidity

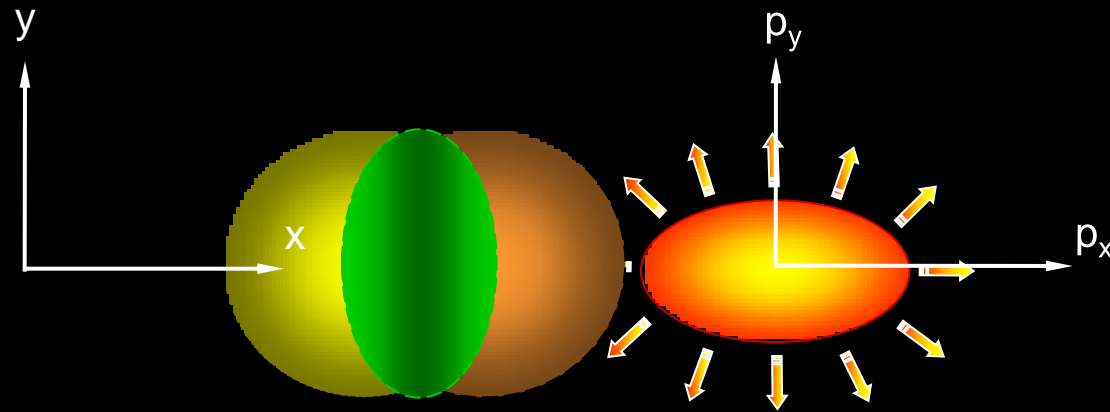
# Flow correlations

# Anisotropy Parameter $v_2$

coordinate-space-anisotropy



momentum-space-anisotropy



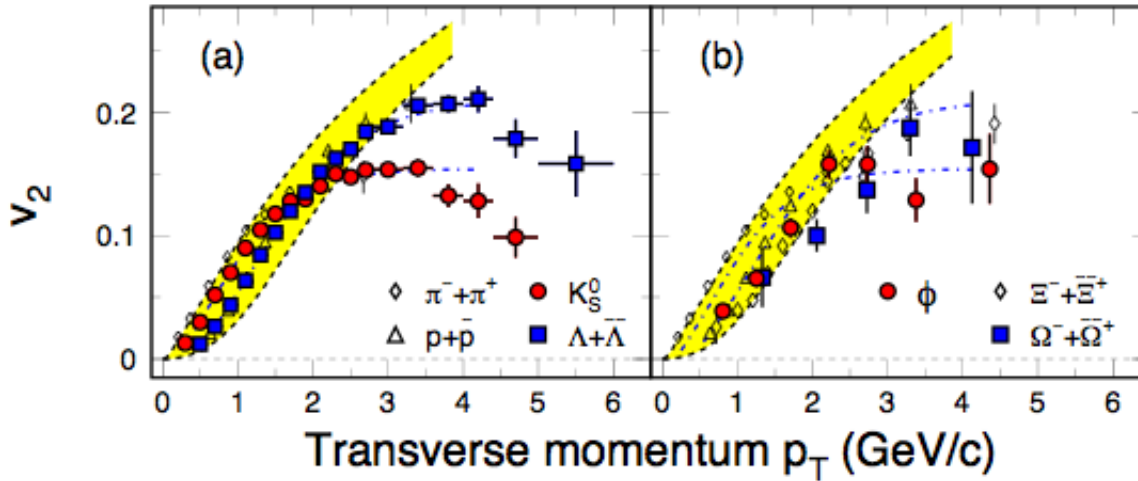
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

$$v_2 = \langle \cos 2\varphi \rangle, \quad \varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

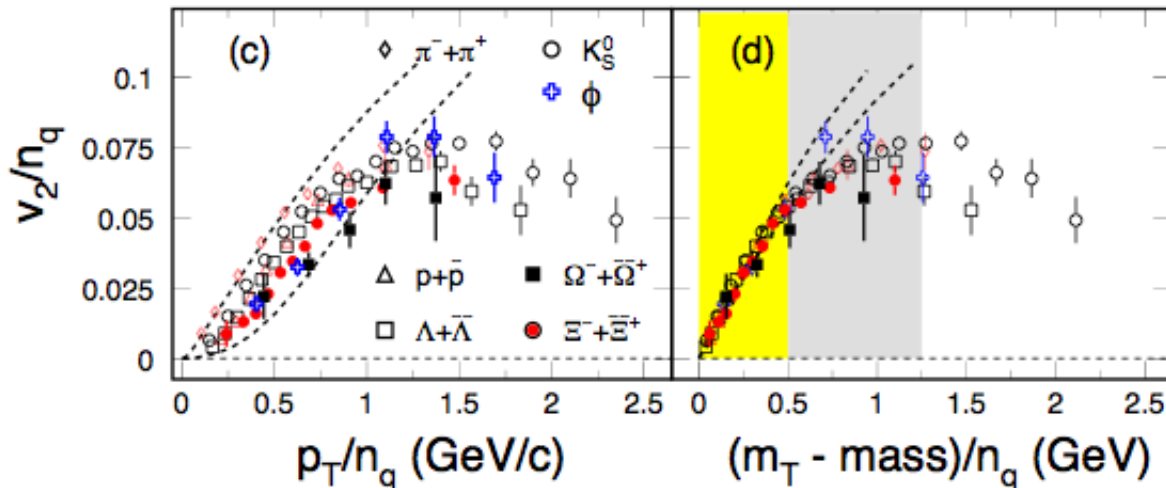
**Initial/final conditions, EoS, degrees of freedom**

# Collectivity, Deconfinement at RHIC

$\sqrt{s_{NN}} = 200 \text{ GeV}$   $^{197}\text{Au} + ^{197}\text{Au}$  Collisions (min. bias)

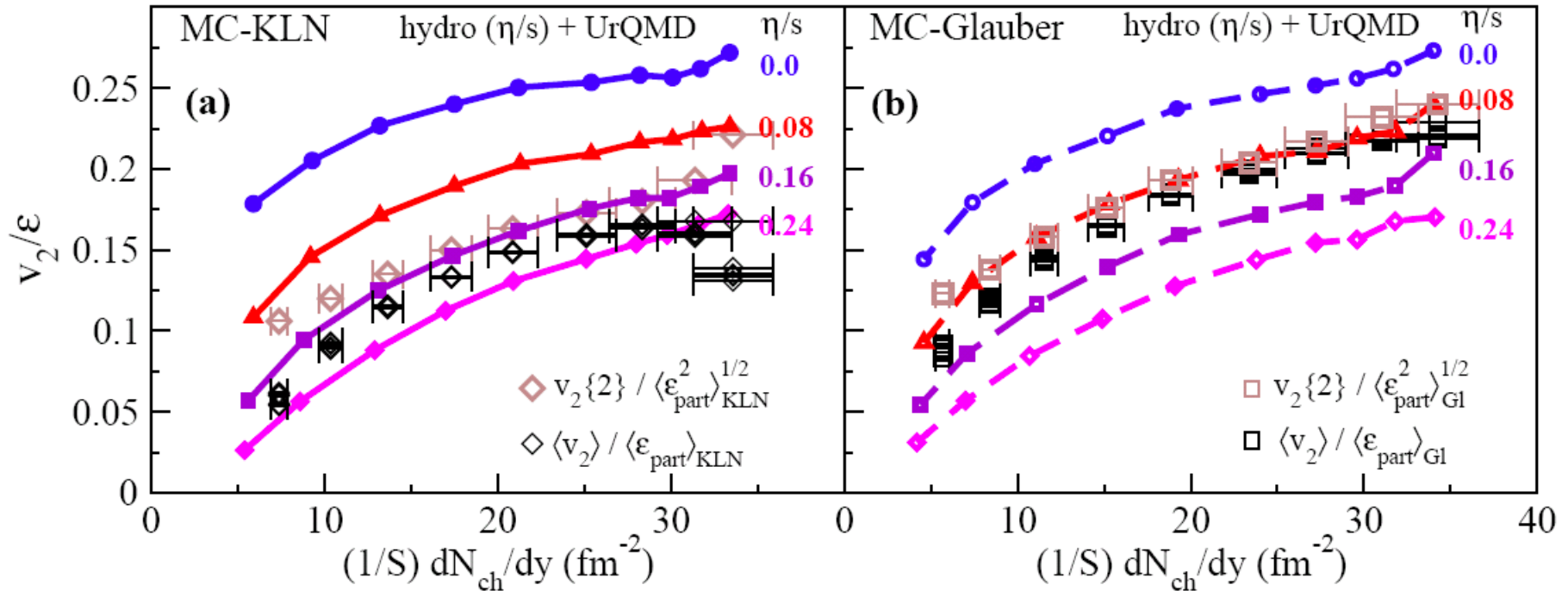


- Low  $p_T$  ( $\leq 2 \text{ GeV/c}$ ): hydrodynamic mass ordering
- High  $p_T$  ( $> 2 \text{ GeV/c}$ ): **number of constituent quarks scaling**



- Quark degrees of freedom, deconfinement, **Partonic Collectivity,**

# Comparison with Hydrodynamics



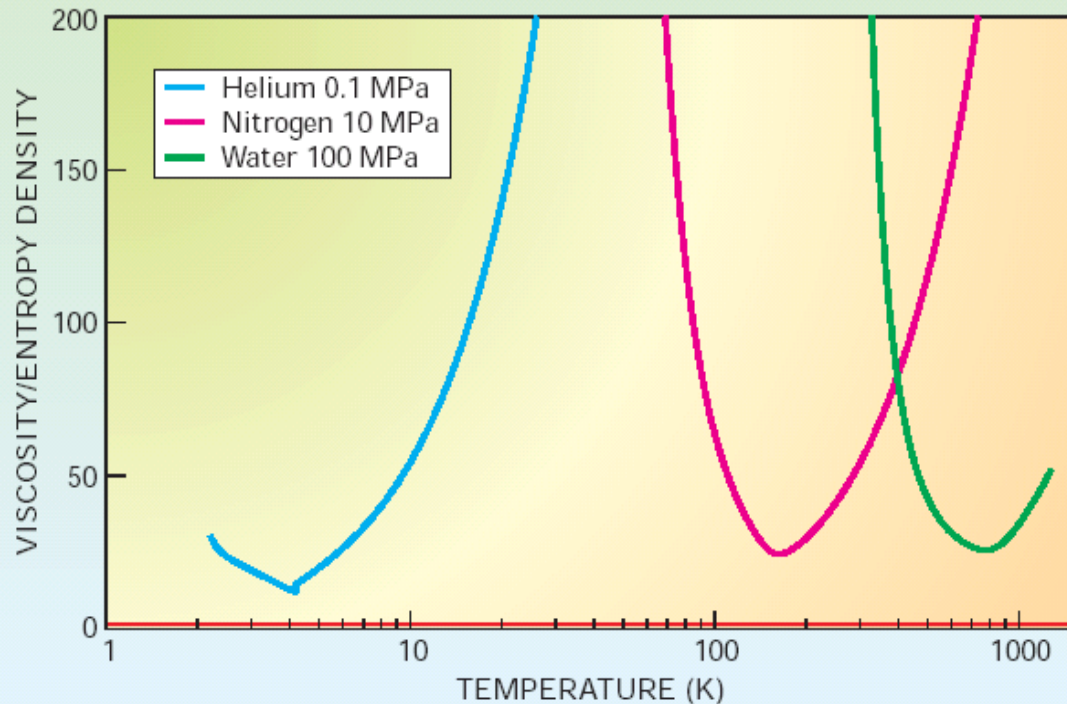
Model: Song *et al.* arXiv:1011.2783

- ➔ **Small value** of viscosity to entropy density ratio  $\eta/s$
- ➔ Model uncertainty dominated by **initial eccentricity  $\epsilon$**

# Low $\eta/s$ for QCD Matter at RHIC

Physics Today, May 2005

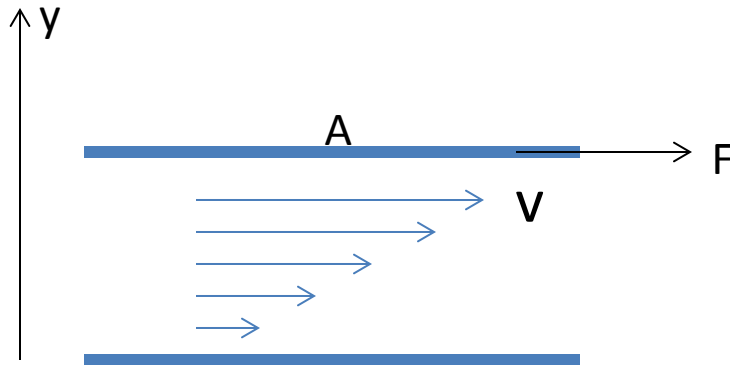
P. K. Kovtun, D. T. Son, A. O. Starinets, Phys. Rev. Lett. 94 111601 (2005).



RHIC results

- $\eta/s \geq 1/4\pi$
- $\eta/s(\text{QCD matter}) < \eta/s(\text{QED matter})$

# Viscosity quantum limit



$$\eta = \frac{1}{3} n p l_{mfp}$$

$$l_{mfp} = 1 / (n \sigma)$$

$$p l_{mfp} \geq \hbar$$

$$s \sim 4 n k_B$$

$$\eta / s > \hbar / 4 \pi k_B$$

$$\eta / s > 1 / 4 \pi$$

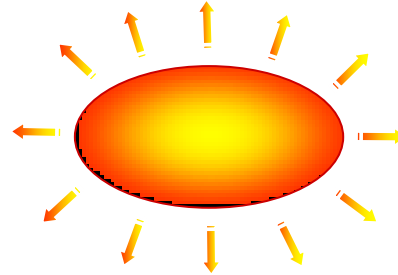
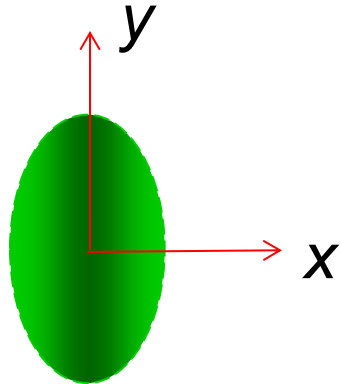
$$\frac{F}{A} = \eta \frac{\partial v}{\partial y}$$

Kovtun, Son, Starinets, PRL 94 (2005) 111601  
 Schafer, arXiv:0912.4236



Does it have to be all  
pressure-driven hydro flow?

# Uncertainty principle



$$\Delta x \cdot \Delta p > \hbar / 2$$

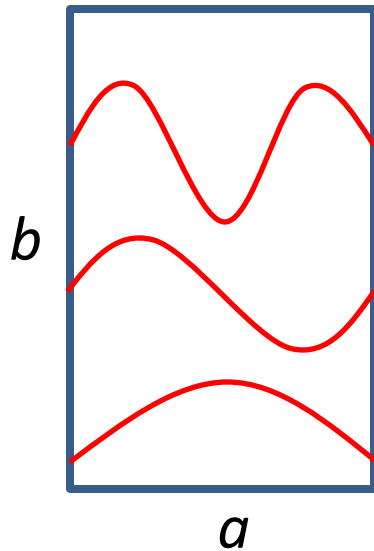
$$p_x > p_y$$

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

$$v_2 = \langle \cos 2\varphi \rangle = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

# Infinite square well



$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \quad \Rightarrow \quad \psi \propto \begin{cases} \cos \frac{n_{\text{odd}}\pi}{a}x \\ \sin \frac{n_{\text{even}}\pi}{a}x \end{cases}$$

Take even mode for example:

$$\langle p_x^2 \rangle = \hbar^2 k^2 ; \quad \langle x^2 \rangle = \frac{a^2}{4} - \frac{2}{k^2} ; \quad k = \frac{n_{\text{odd}}\pi}{a}$$

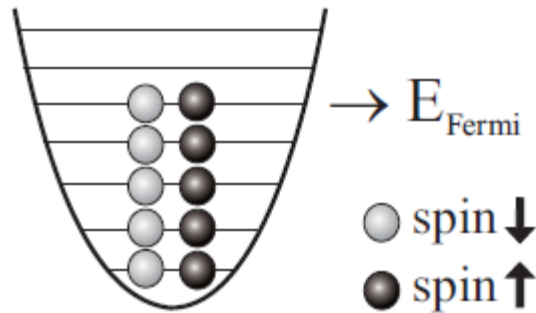
$$\sqrt{\langle p_x^2 \rangle \cdot \langle x^2 \rangle} = \hbar \sqrt{\frac{k^2 a^2}{4} - 2} = \hbar \sqrt{\frac{\pi^2}{4} n_{\text{odd}}^2 - 2} > \hbar / 2$$

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{b^2 - a^2}{b^2 + a^2} = \varepsilon \quad \text{for all } n.$$

# Harmonic oscillator

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega^2 x^2 \right) \psi = E \psi ; \quad E = \left( n + \frac{1}{2} \right) \hbar \omega$$

fermions:  
half-integer spin



$$\left\langle \frac{p_x^2}{2m} \right\rangle = \left\langle \frac{1}{2} m \omega^2 x^2 \right\rangle = \frac{E}{2} = \frac{1}{2} \left( n + \frac{1}{2} \right) \hbar \omega$$

$$\sqrt{\langle p_x^2 \rangle \langle x^2 \rangle} = \left( n + \frac{1}{2} \right) \hbar$$

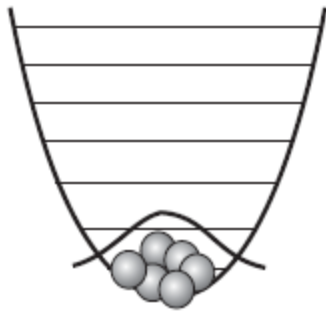
$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

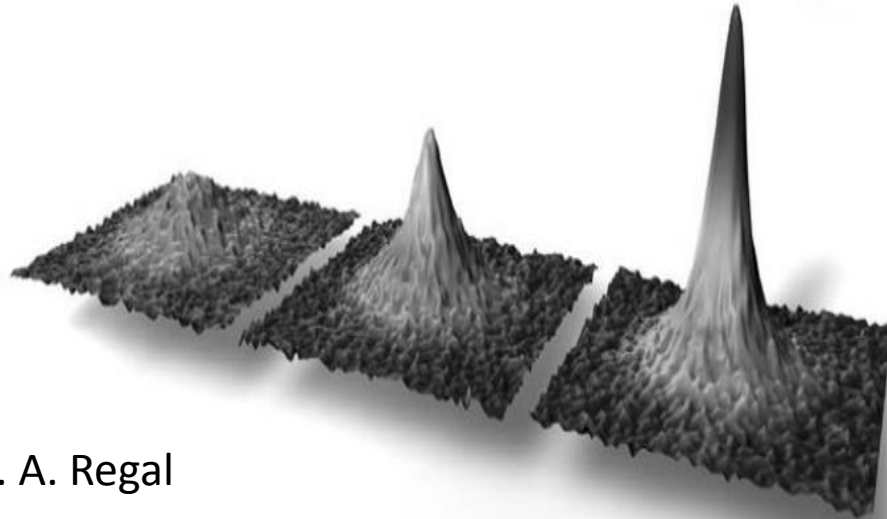
$$v_2 = \varepsilon \quad \text{for each and all } n$$

# Bose-Einstein Condensate

bosons:  
integer spin

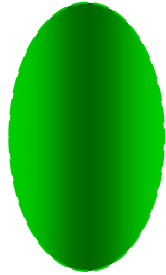


D. S. Jin and C. A. Regal

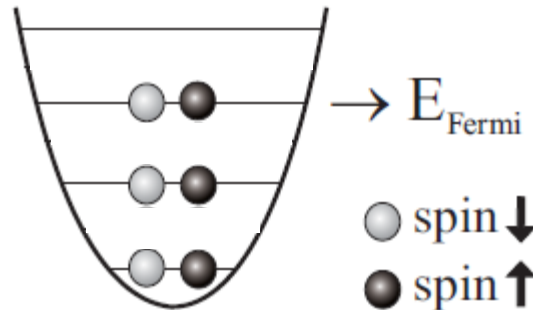


Single ground state in anisotropic trap  $\rightarrow$  large momentum anisotropy

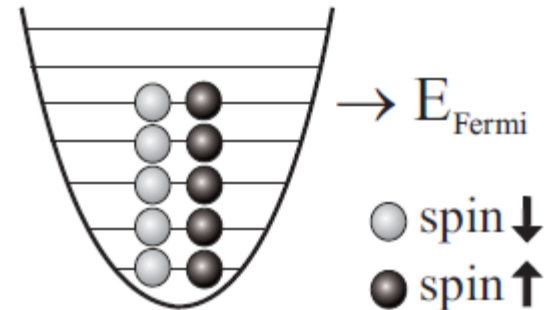
# Thermal probability



fermions:  
half-integer spin



fermions:  
half-integer spin



$x, y$  at same Fermi energy, so different number of filled energy levels.

At high temperature, classical limit, sum is approximated by integral:

$$\frac{dN}{d\mathbf{p}} = N \frac{\int d\mathbf{r} e^{-H_1(\mathbf{p}, \mathbf{r})/T}}{\int d\mathbf{r} d\mathbf{p} e^{-H_1(\mathbf{p}, \mathbf{r})/T}} = N \frac{e^{-K(\mathbf{p})/T}}{\int d\mathbf{p} e^{-K(\mathbf{p})/T}}$$

then it's independent of potential.

It's isotropic at all temperature because  $K=(p_x^2+p_y^2)/2m$  is isotropic.

# Is QGP hot?

Size  $r \sim 1$  fm

Intrinsic momentum/energy scale  $\sim 1/r \sim 200$  MeV

QGP temperature  $T \sim 300$  MeV

Typical momentum/energy  $\sim T \sim 300$  MeV

QGP is **not** hot at all.

Quantum effect must be present.

# Thermal probability weight

$$\rho(\mathbf{r}) \equiv \frac{dN}{d\mathbf{r}} = \frac{1}{Z} \sum_j |\psi_j(\mathbf{r})|^2 e^{-E_j/T}$$

$$f(\mathbf{p}) \equiv \frac{dN}{d\mathbf{p}} = \frac{1}{Z} \sum_j |\psi_j(\mathbf{p})|^2 e^{-E_j/T}$$

$$Z \equiv \sum_j e^{-E_j/T}$$

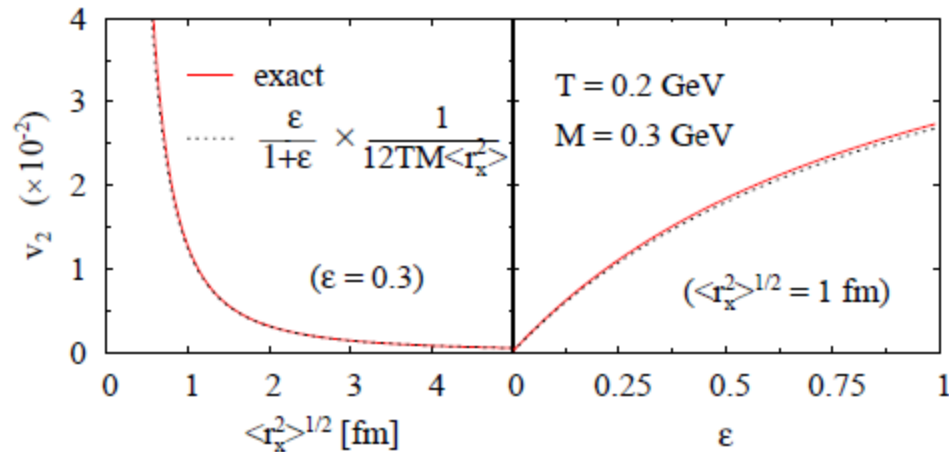
$$\langle p_i^2 \rangle = \frac{M\omega_i}{2} \coth \frac{\omega_i}{2T}$$

$$\langle r_i^2 \rangle = \frac{1}{2M\omega_i} \coth \frac{\omega_i}{2T}.$$



# Initial $v_2$ from QM

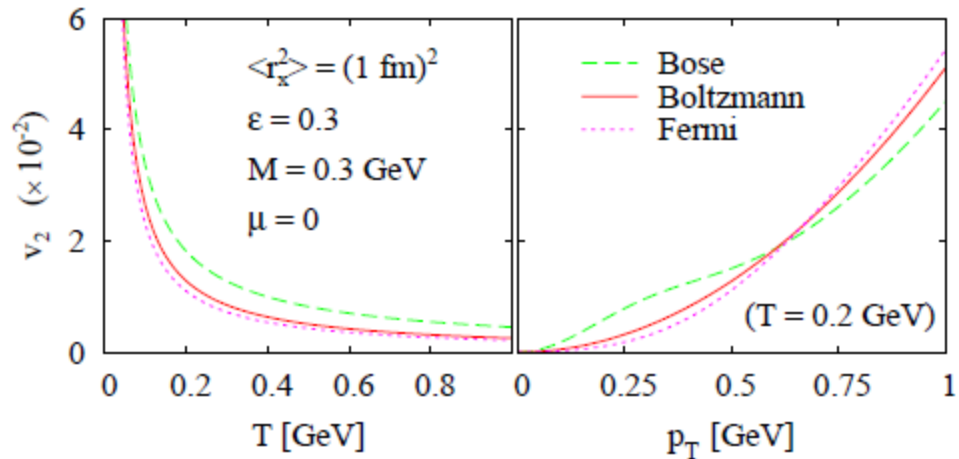
$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\epsilon}{1 + \epsilon}$$



D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1 + \varepsilon}$$

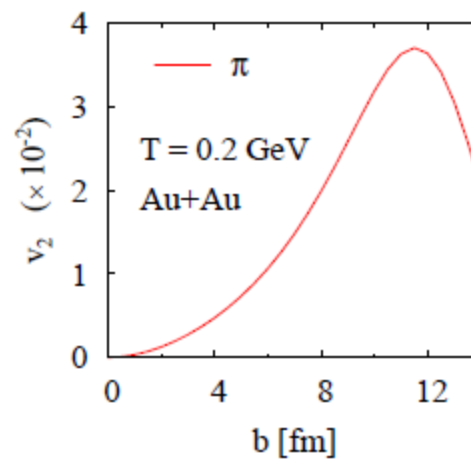
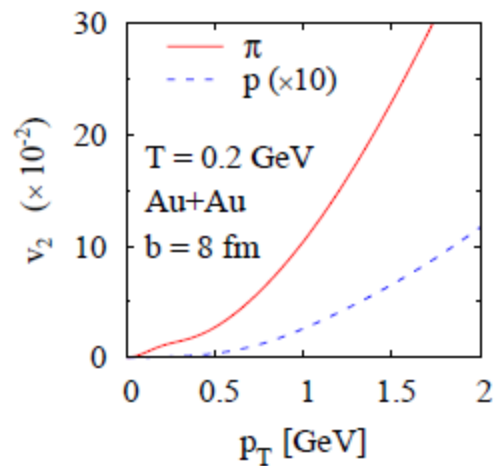
$$v_{2n}(p_T) = h_n \left( \frac{p_T^2}{2MT} (S_y - S_x) \right), \quad S_i \equiv \frac{T}{\omega_i} \tanh \frac{\omega_i}{2T}$$



D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

# Typical Au+Au collisions

$b = 8$  fm:  $\langle r_x^2 \rangle^{1/2} = 1.5$  fm and  $\langle r_y^2 \rangle^{1/2} = 2.2$  fm.



D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

# Transverse profile from SHO

$$\rho(\mathbf{r}) \propto \exp\left(-\sum_i \frac{r_i^2}{2\langle r_i^2 \rangle}\right), \quad f(\mathbf{p}) \propto \exp\left(-\sum_i \frac{p_i^2}{2\langle p_i^2 \rangle}\right)$$

This may not correspond exactly to heavy-ion collision energy density profile, but close.

# Cold atoms

## Strong elliptic anisotropy

K. M. O'Hara *et al.*, Science 298, 2179 (2002).

Lithium atoms  $M \sim 6000 \text{ MeV}$

Temperature  $T \sim 1 \text{ } \mu\text{K} \sim 10^{-16} \text{ MeV}$

Trap size  $x \sim 20 \text{ } \mu\text{m}$ ,  $y \sim 100 \text{ } \mu\text{m}$

Typical momentum  $(TM)^{1/2} \sim 10^{-6} \text{ MeV}$

Intrinsic momentum quantum  $\sim 1/r \sim 10^{-8} \text{ MeV}$ , negligible.

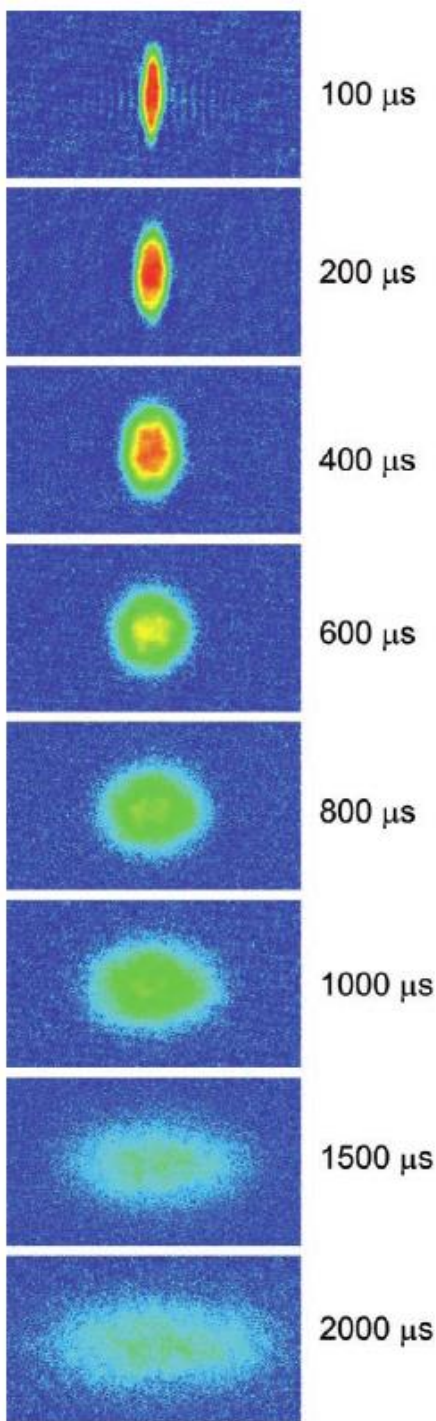
Typical energy  $\sim T \sim 10^{-16} \text{ MeV}$

Intrinsic energy quantum  $1/(mr^2) \sim 10^{-20} \text{ MeV}$ , negligible.

**Cold Lithium atoms are actually “hotter” than the hot QGP.**

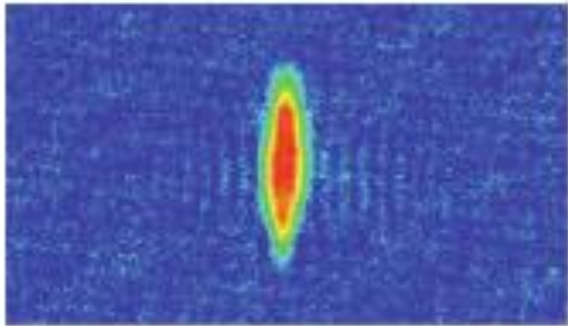
$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\epsilon}{1 + \epsilon} \sim 10^{-5}$$

The observed large  $v_2$  is indeed due to strong interactions.



# Is quantum $v_2$ real?

- It should be... but need experiment to verify
- Would be neat to verify QM and uncertainty principle



## Cold atom experiment

- Need trap size **x100 smaller**
- Or need **nano-Kelvin** temperature

Proposing a cold atom quantum simulator for high-energy nuclear physics

# Control the interaction

- Hydrodynamics is only an assumption
- Is initial QM  $v_2$  important after hydro evolution?
- When does hydro sets in and takes over?
- Will the initial QM  $v_2$  be washed out by hydro?
- Current hydro implementation is classical
- Need to incorporate QM into hydro: quantum hydrodynamics

# Shooting fast atoms through trap

- jet-quenching partonic energy loss mechanisms are far from clear. A very active and extensive field
- Can we gain insights from cold atoms?
- Shoot fast atoms through cold atom system

PHYSICAL REVIEW A 85, 053643 (2012)

## Probing strongly interacting atomic gases with energetic atoms

Yusuke Nishida

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(Received 26 October 2011; revised manuscript received 9 April 2012; published 29 May 2012)*

We investigate properties of an energetic atom propagating through strongly interacting atomic gases. The operator product expansion is used to systematically compute a quasiparticle energy and its scattering rate both in a spin-1/2 Fermi gas and in a spinless Bose gas. Reasonable agreement with recent quantum Monte Carlo

- External hard probes under full control



# Summary

- **Particle correlations are a powerful tool to study pp, pA, AA collisions**
- **Unambiguous signal of strongly interacting QGP from high- $p_T$  jet-quenching data.**
- **Low  $p_T$  anisotropic flow data indicate hydrodynamic behavior of sQGP. Extracting transport properties (such as  $\eta/s$ ) from measured data still need extra effort. Initial anisotropy may not be neglected.**
- **There should be indispensable information at intermediate  $p_T$  from jet-medium interactions (not discussed in this lecture). Need creative mind and novel approaches.**