

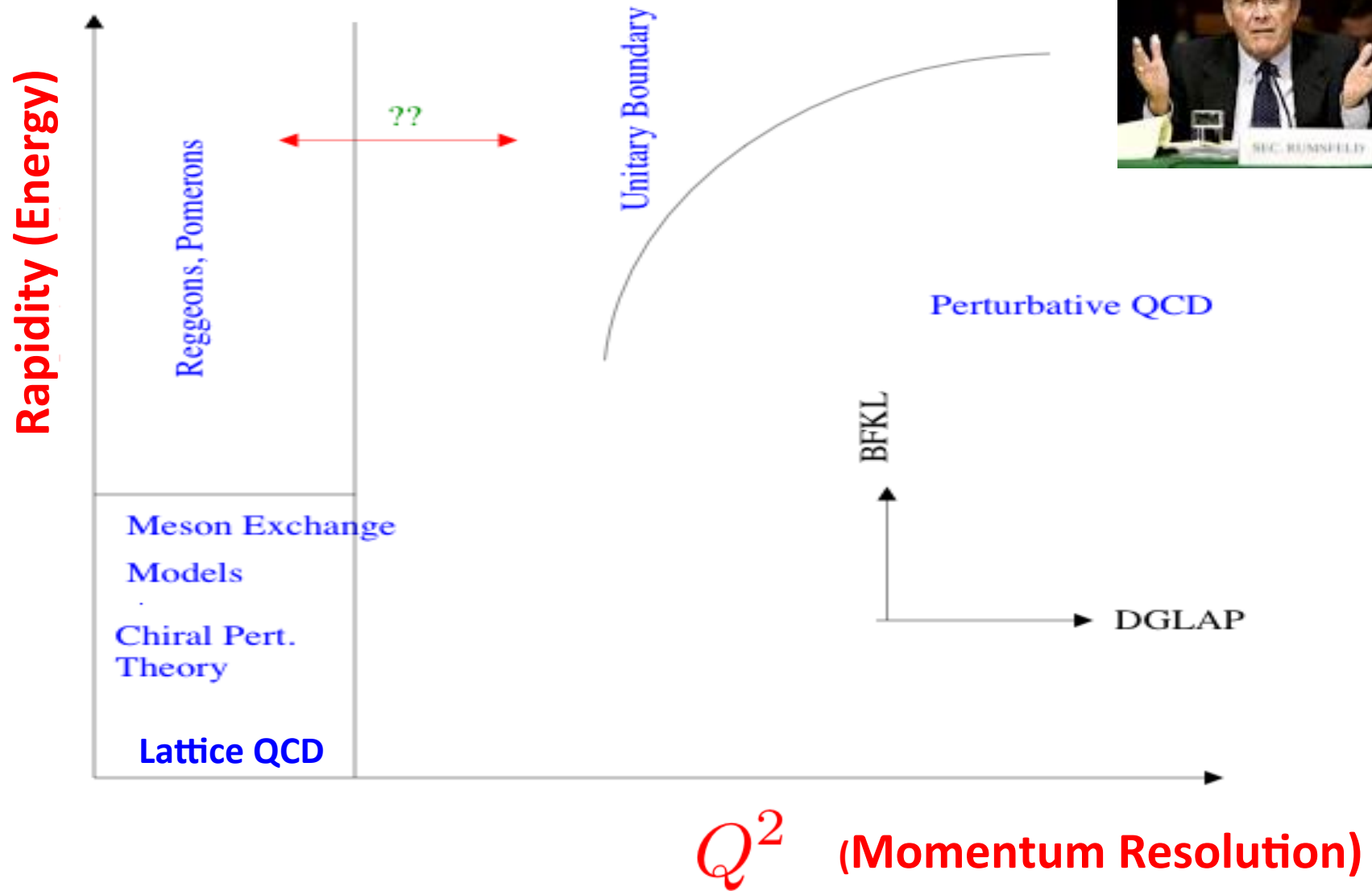


Multi-particle production and thermalization  
in hadron-hadron collisions

Raju Venugopalan  
Brookhaven National Laboratory

Berkeley Summer School, June 9-12, 2014

# QCD: Known Knowns and Known Unknowns



# QCD: Known Knowns and Known Unknowns

## Known knowns in QCD:

- ◆ **Perturbative QCD: precision physics for large  $Q^2$  – rare processes (also weak coupling techniques in finite T and  $\mu_B$  QFT)**
- ◆ **Lattice QCD: Quantitative description of (mostly) hadron ground state properties. (see Prof. Fodor's talk)**
- ◆ **Chiral perturbation theory: low energy meson and baryon interactions**

# QCD: Known Knowns and Known Unknowns

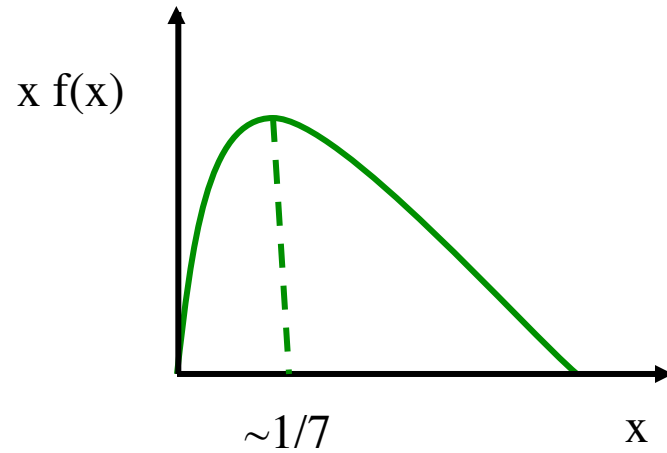
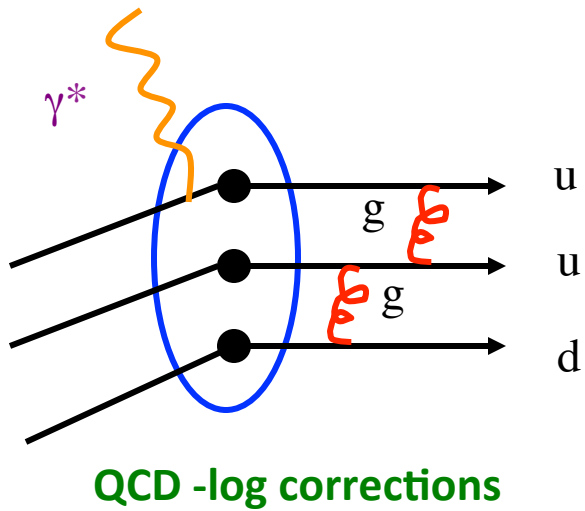
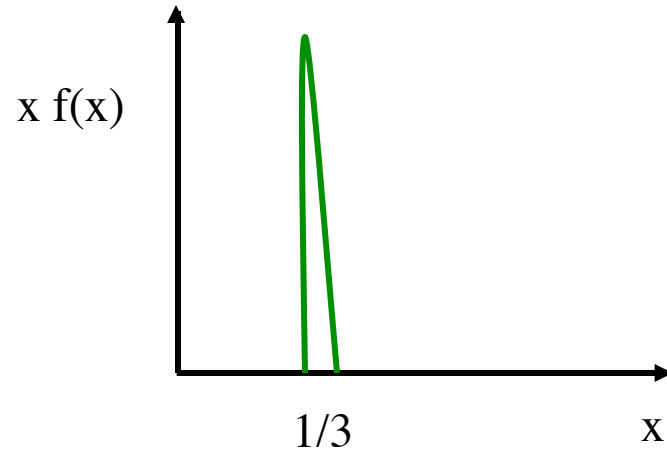
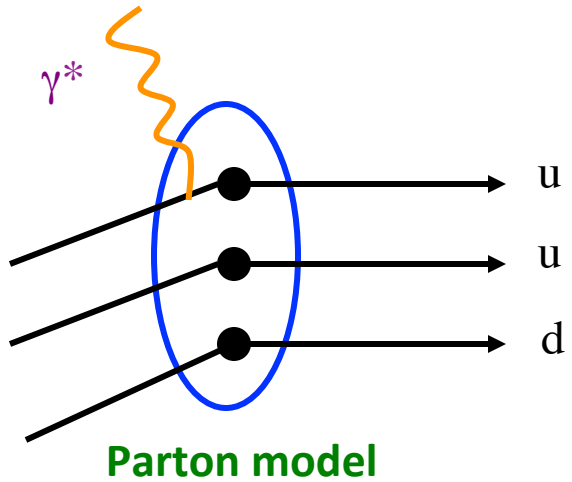
## Known unknowns in QCD:

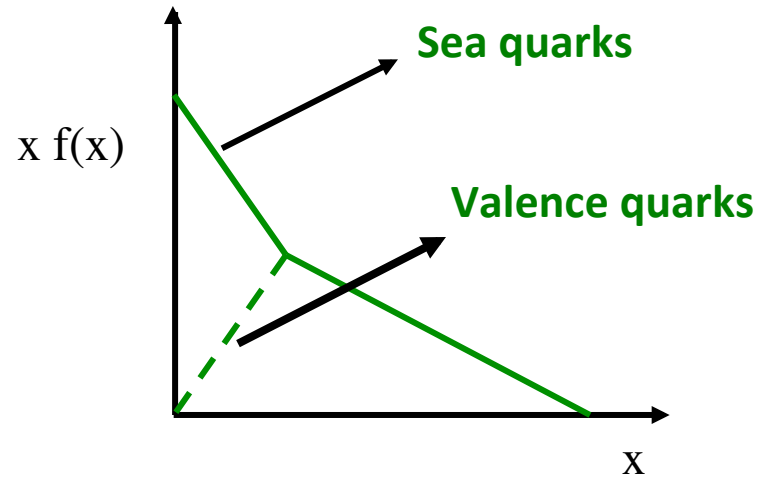
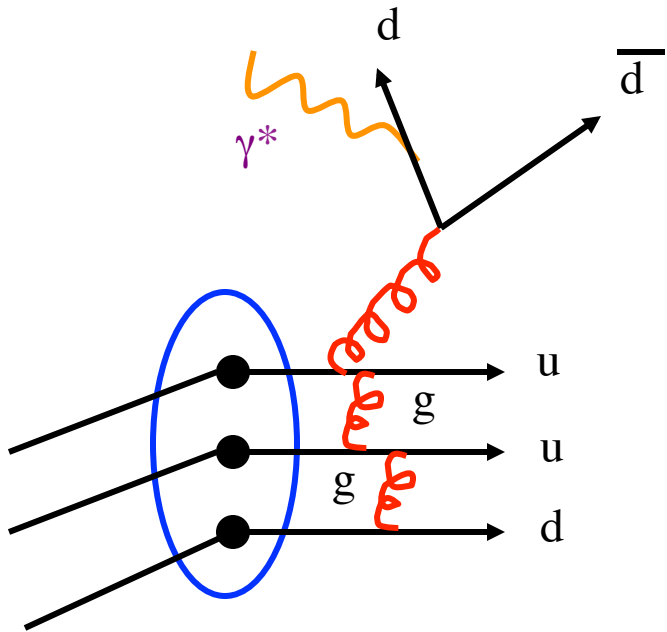
- ◆ The bulk of elastic, inelastic and diffractive cross-sections in QCD (sometimes called “soft” physics – though includes scales of a few GeV).
- ◆ Fragmentation/hadronization is not understood—though useful and successful parametrizations exist.
- ◆ Stringy models (**PYTHIA,DPM,AMPT,EPOS**) successfully parametrize a lot of data and loosely capture features of the underlying theory.
- ◆ However, they *cannot be derived* in any limit from QCD, and require further ad hoc assumptions and parameters when applied In extreme environments

# What we need

- **An effective theory to describe the varied phenomena of multi-particle production in high energy collisions**
- **Smoothly matches to QCD in appropriate kinematic limits**
- **The rest of my talk will briefly outline the elements of such a theory.**
- **The theory has much predictive power—however, it is least effective when the physics is sensitive to the infrared scales that govern chiral symmetry breaking and confinement.**

# The proton at high energies



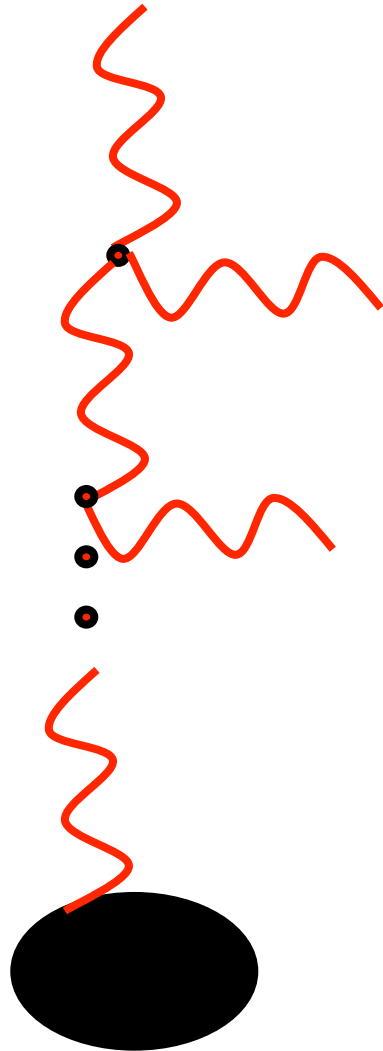


**“x-QCD”- small x evolution**

$$\int_0^1 \frac{dx}{x} (xq(x) - x\bar{q}(x)) = 3 \longrightarrow \text{\# of valence quarks}$$

$$\int_0^1 \frac{dx}{x} (xq(x) + x\bar{q}(x)) \rightarrow \infty \longrightarrow \text{\# of quarks}$$

# Bremsstrahlung -linear QCD evolution



Each rung of the ladder gives

$$\alpha_S \int \frac{dk_t^2}{k_t^2} \int \frac{dx}{x} \equiv \alpha_S \ln \left( \frac{x_0}{x} \right) \ln \left( \frac{Q^2}{Q_0^2} \right)$$

If only transverse momenta are  
ordered from target to projectile:

$$k_{T1}^2 \ll k_{T2}^2 \ll \dots Q^2$$

Sum leading logs in  $Q^2$  (DGLAP evolution)

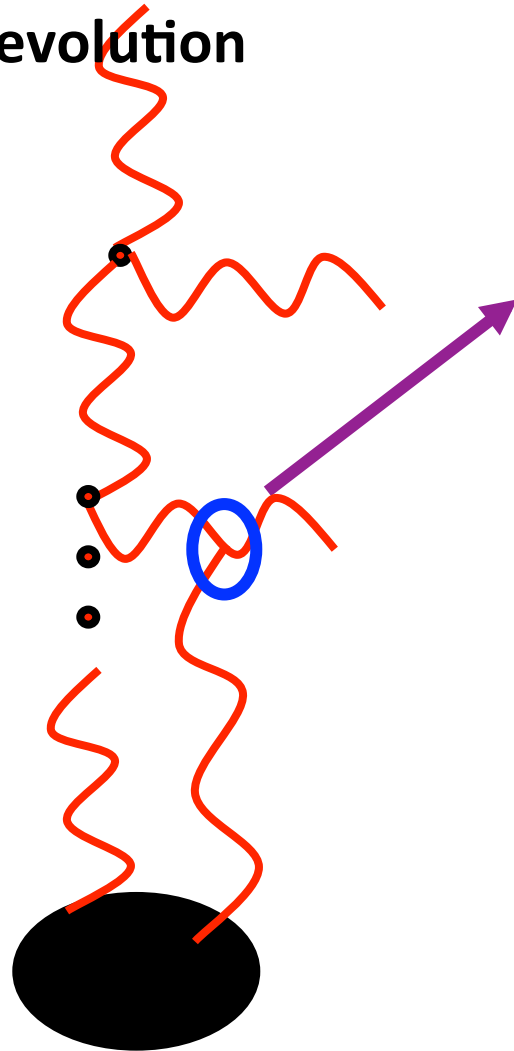
Conversely,  $x_0 \gg x_1 \dots \gg x$

Sum leading logs in  $x$  (BFKL evolution)

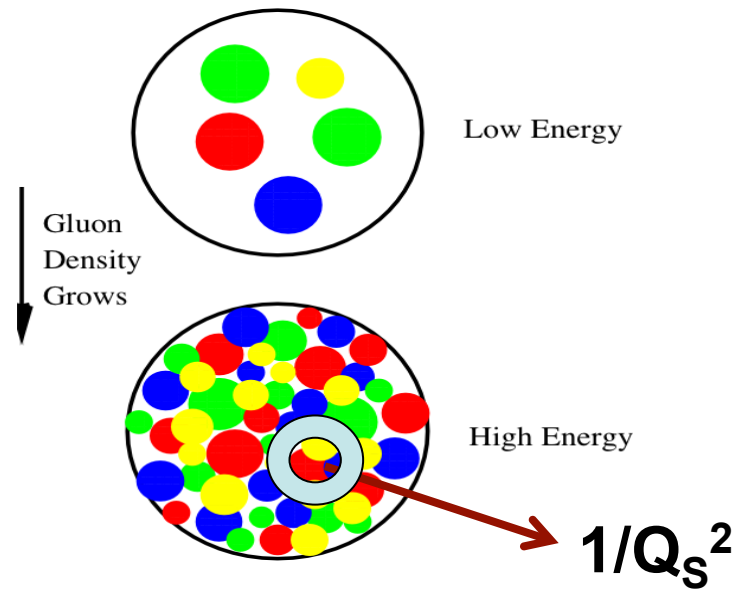
Both DGLAP and BFKL give rapid growth of gluon density at small  $x$



# Bremsstrahlung -linear QCD evolution



# Gluon recombination and screening -non-linear QCD evolution



Proton becomes a dense many body system at high energies

# Parton Saturation

Gribov, Levin, Ryskin (1983)  
Mueller, Qiu (1986)

- Competition between attractive bremsstrahlung and repulsive recombination and screening effects

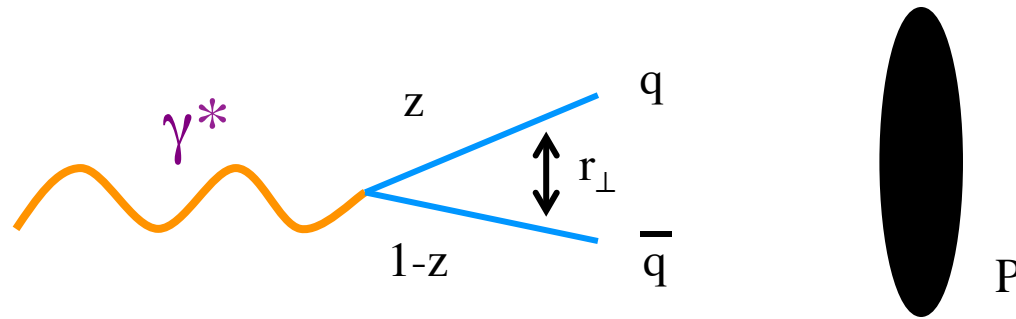
Maximum phase space density ( $f = 1/\alpha_s$ ) =>

$$\frac{1}{2(N_c^2 - 1)} \frac{x G(x, Q^2)}{\pi R^2 Q^2} = \frac{1}{\alpha_s(Q^2)}$$

This relation is saturated for

$$Q = Q_s(x) \gg \Lambda_{\text{QCD}} \approx 0.2 \text{ GeV}$$

# Parton Saturation: Golec-Biernat --Wusthoff dipole model



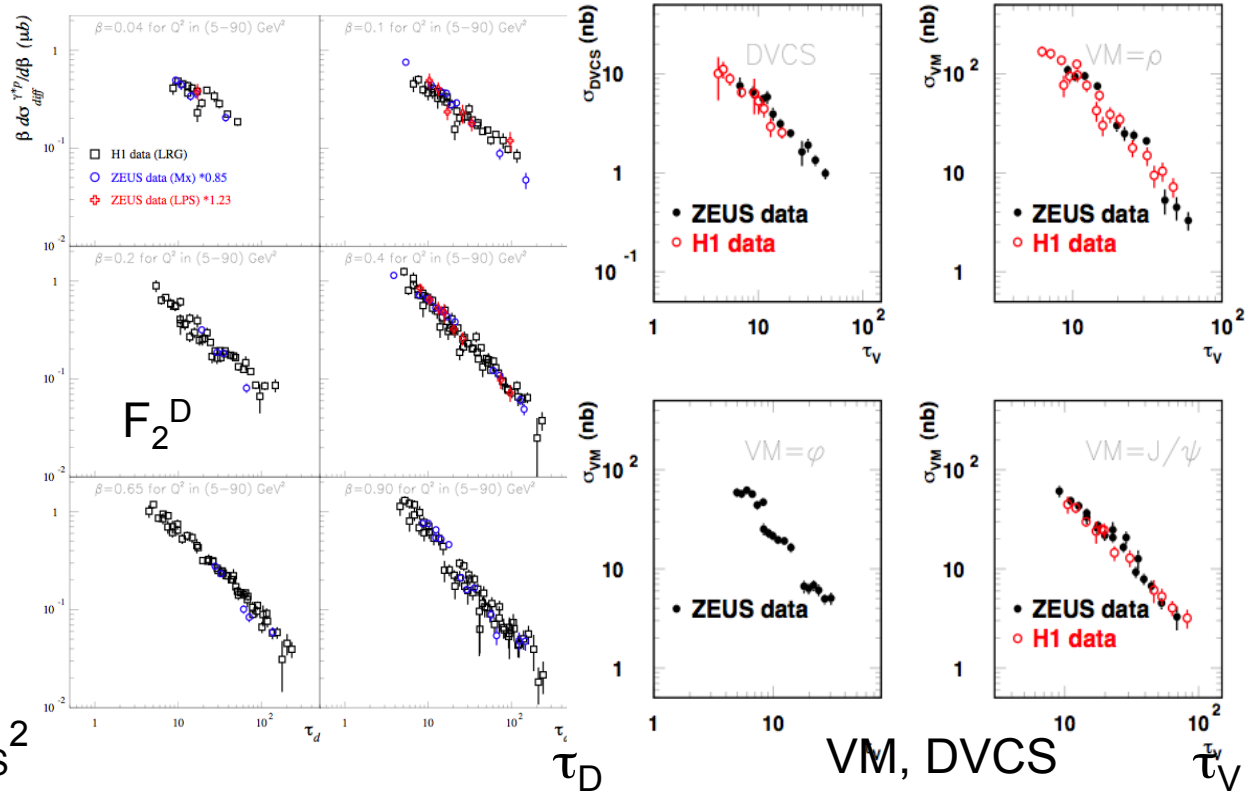
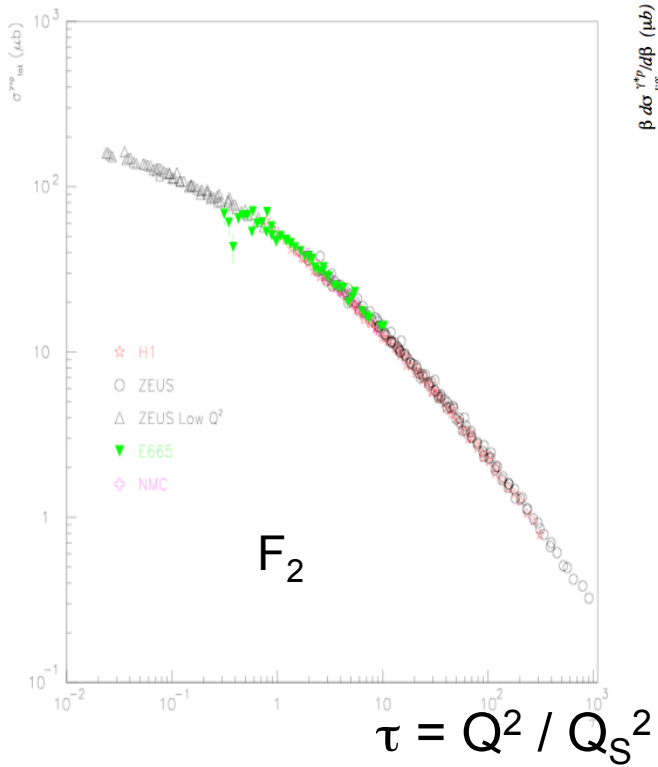
$$\sigma_{T,L}^{\gamma^*,P} = \int d^2 r_\perp \int dz |\psi_{T,L}(r_\perp, z, Q^2)|^2 \sigma_{q,\bar{q},P}(r_\perp, x)$$

$$\sigma_{q\bar{q}P}(r_\perp, x) = \sigma_0 \left[ 1 - \exp\left(-r_\perp^2 Q_s^2(x)\right) \right] \quad Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

Parameters:  $Q_0 = 1 \text{ GeV}$ ;  $\lambda = 0.3$ ;  $x_0 = 3 \cdot 10^{-4}$ ;  $\sigma_0 = 23 \text{ mb}$

# Evidence from HERA for geometrical scaling

Golec-Biernat, Stasto, Kwiecinski

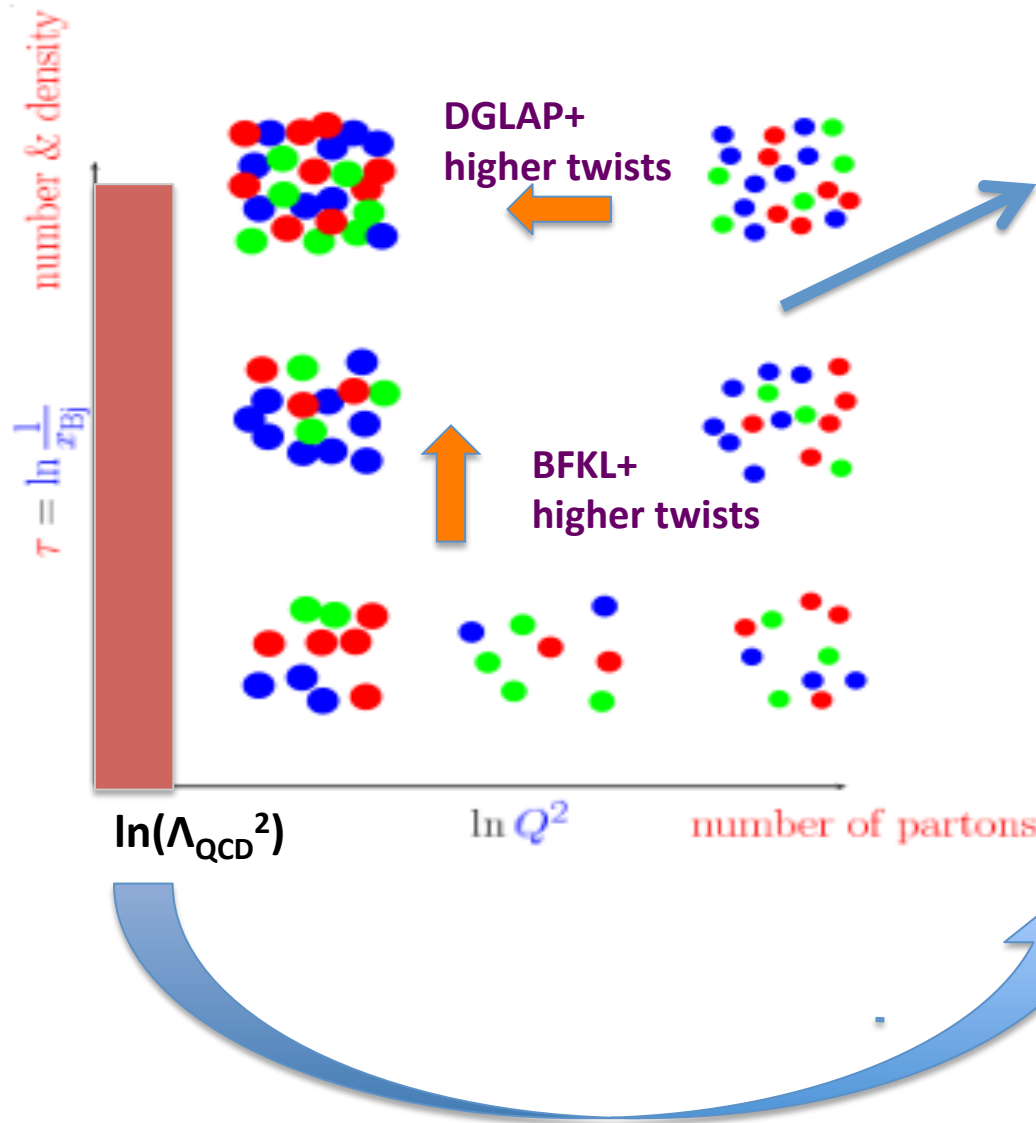


Marquet, Schoeffel hep-ph/0606079

❖ Scaling seen for  $F_2^D$  and VM, DVCS for same  $Q_S$  as  $F_2$

Gelis et al., hep-ph/0610435

# Many-body dynamics of universal gluonic matter



How does this happen ? What are the right degrees of freedom ?

How do correlation functions of these evolve ?

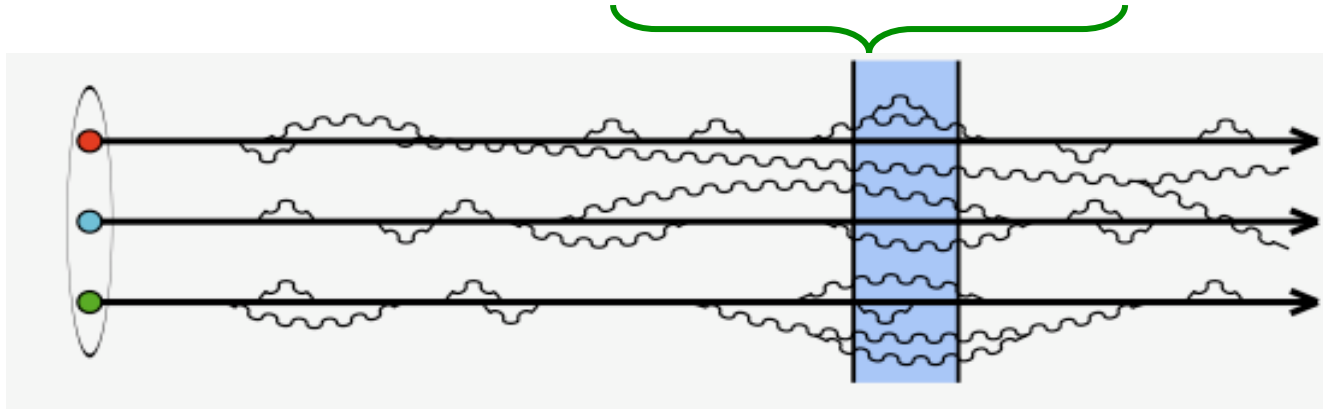
Is there a universal fixed point for the RG evolution of d.o.f

Does the coupling run with  $Q_s^2$  ?

How does saturation transition to chiral symmetry breaking and confinement

# The high energy nuclear wavefunction in QFT

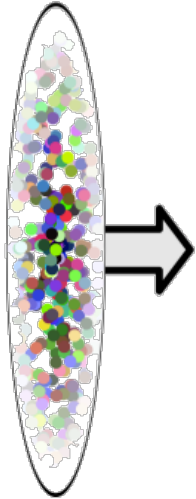
$$|A\rangle = |qqq\dots q\rangle + \dots + |qqq\dots qgg\dots g\rangle$$



- ❖ At high energies, interaction time scales of fluctuations are **dilated** well beyond typical hadronic time scales
- ❖ Lots of short lived (gluon) fluctuations now seen by probe -- proton/nucleus -- **dense many body system of (primarily) gluons**
- ❖ Fluctuations with lifetimes much longer than interaction time for the probe function as **static color sources** for more short lived fluctuations

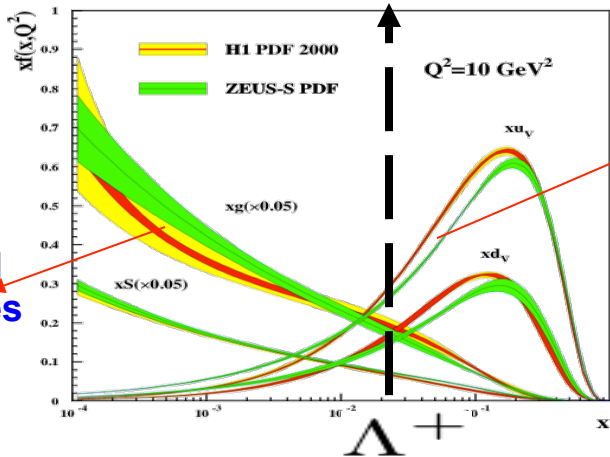
Nuclear wave function at high energies is a **Color Glass Condensate**

# The nuclear wavefunction at high energies



$$|A\rangle = |qqq\dots q\rangle + \dots + |qqq\dots q\underbrace{gg\dots gg}_{\text{Higher Fock components}}\rangle$$

Higher Fock components dominate  
multiparticle production-  
construct Effective Field Theory



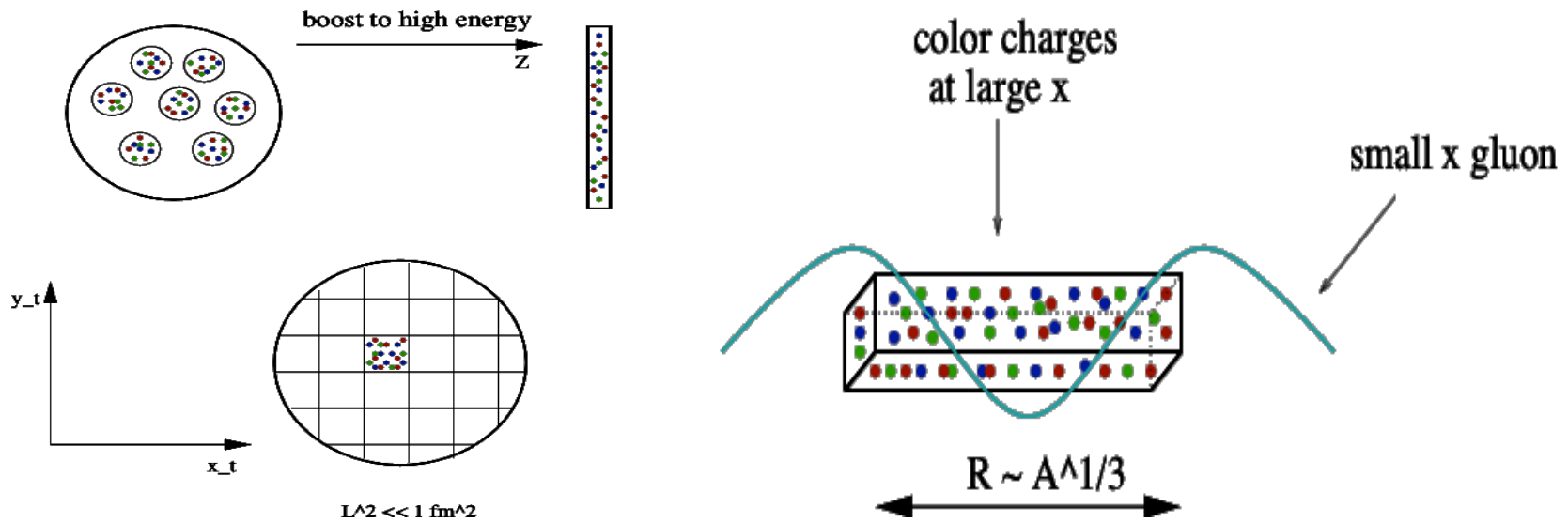
Dynamical Wee modes

Valence modes are static sources for wee modes

Born--Oppenheimer LC separation natural for EFT.

RG eqns describe evolution of wavefunction with energy

# What do sources look like in the IMF ?



$$\lambda_{\text{wee}} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} \gg \lambda_{\text{val.}} \equiv \frac{Rm_p}{P^+} \Rightarrow x \ll A^{-1/3}$$

**Wee partons “see” a large density of color sources at small transverse resolutions**



# Effective Field Theory on Light Front

Susskind  
Bardacki-Halpern

Poincare group on LF  $\longleftrightarrow$  Galilean sub-group  
of 2D Quantum Mechanics  
isomorphism

Eg., LF dispersion relation  $P^- = \frac{P_\perp^2}{2P^+}$

Energy  $\swarrow$   $\searrow$  Momentum  
Mass

Large  $x$  ( $P^+$ ) modes: static LF (color) sources  $\rho^a$   
Small  $x$  ( $k^+ \ll P^+$ ) modes: dynamical fields  $A_\mu^a$

McLerran, RV

CGC: Coarse grained many body EFT on LF

$$\langle P | \mathcal{O} | P \rangle \longrightarrow \int [d\rho^a][dA^{\mu,a}] W_{\Lambda^+}[\rho] e^{iS_{\Lambda^+}[\rho,A]} \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$  non-pert. gauge invariant “density matrix”  
defined at initial scale  $\Lambda_0^+$

RG equations describe evolution of  $W$  with  $x$

JIMWLK, BK

# Classical field of a large nucleus

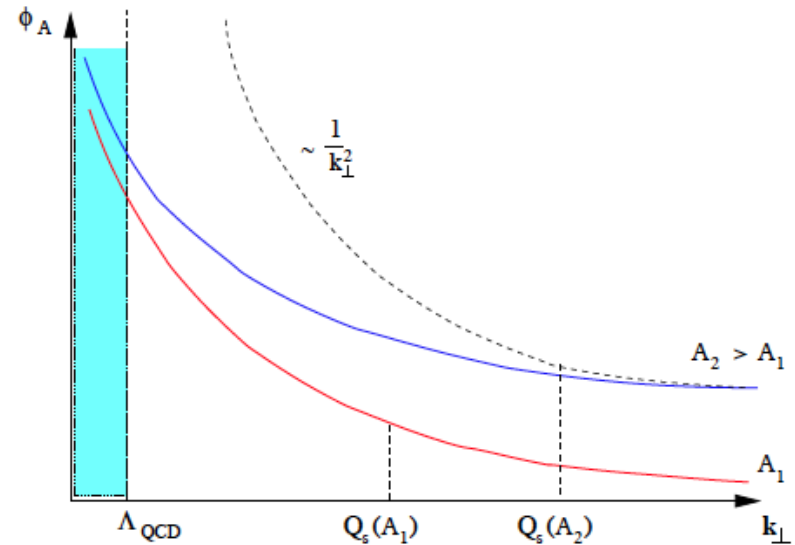
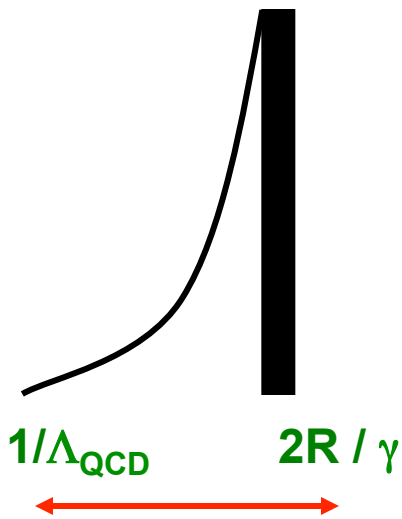
$$\langle AA \rangle_\rho = \int [d\rho] A_{\text{cl.}}(\rho) A_{\text{cl.}}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus,  $A \gg 1$ ,

$$W_{\Lambda^+} = \exp \left( - \int d^2 x_\perp \left[ \frac{\rho^a \rho^a}{2 \mu_A^2} - \frac{d_{abc} \rho^a \rho^b \rho^c}{\kappa_A} \right] \right)$$

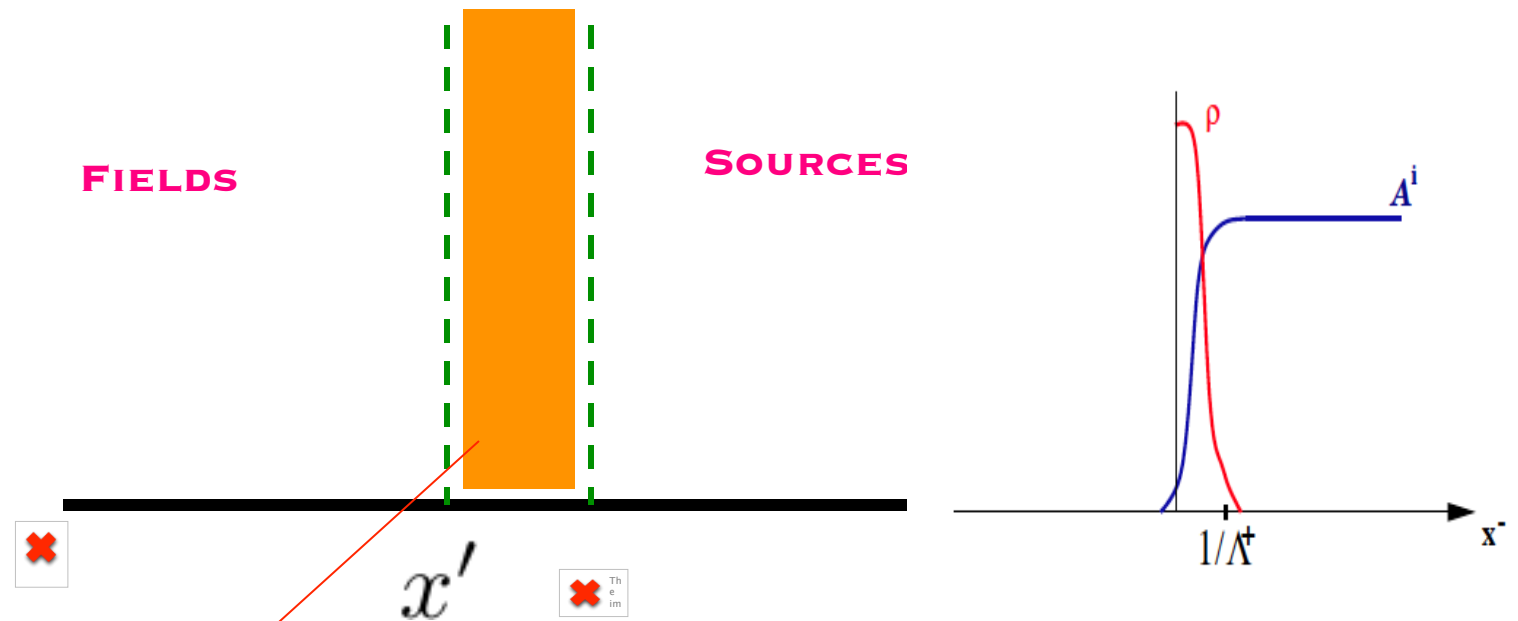
$A_{\text{cl}}$  from  $\longrightarrow (D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+} \delta(x^-) \rho^a(x_\perp)$

McLerran, RV  
Kovchegov  
Jeon, RV



Wee parton dist. :  $\frac{1}{\Lambda_{\text{QCD}}} e^{-\lambda \Delta Y / 2}$  determined from RG

# Quantum evolution of classical theory: Wilson RG



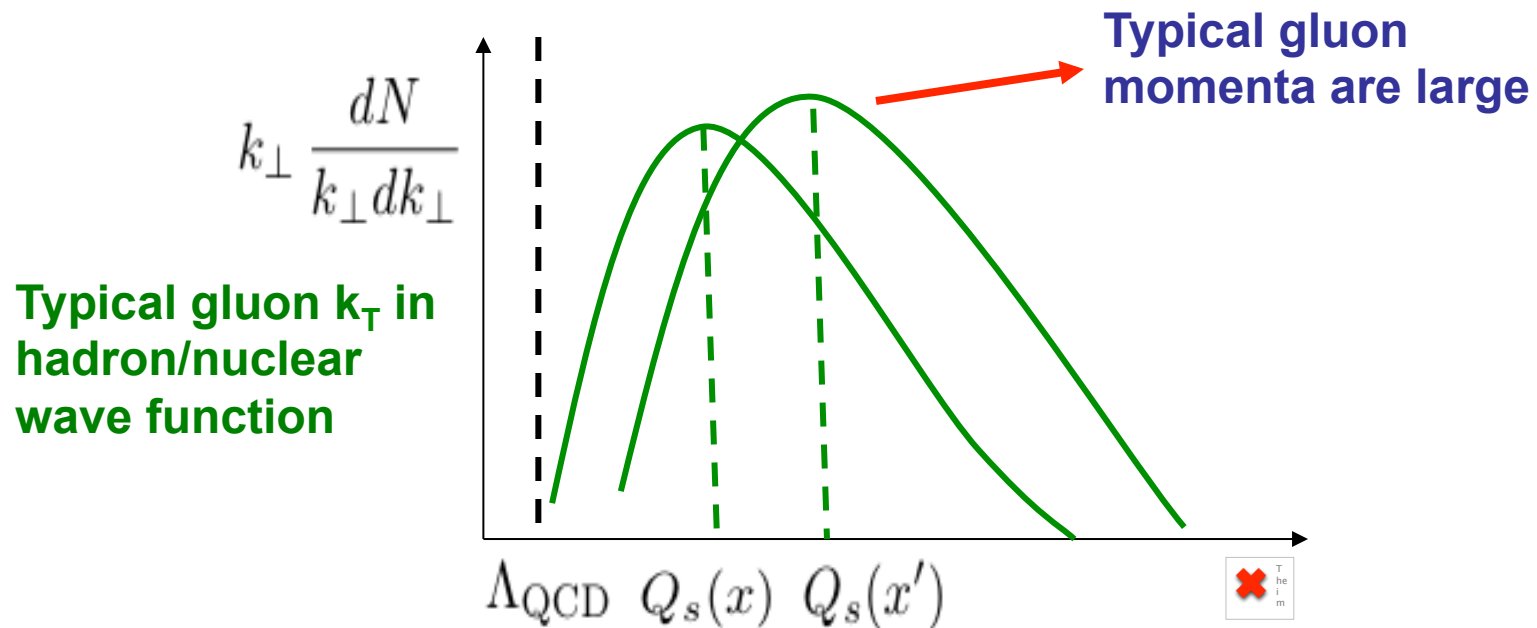
Integrate out  $\rightarrow$   
Small fluctuations  $\Rightarrow$  Increase color charge of sources

Wilsonian RG equations describe evolution of all N-point correlation functions with energy

JIMWLK

Jalilian-marian, Iancu, McLerran, Weigert, Leonidov, Kovner

# Saturation scale grows with energy

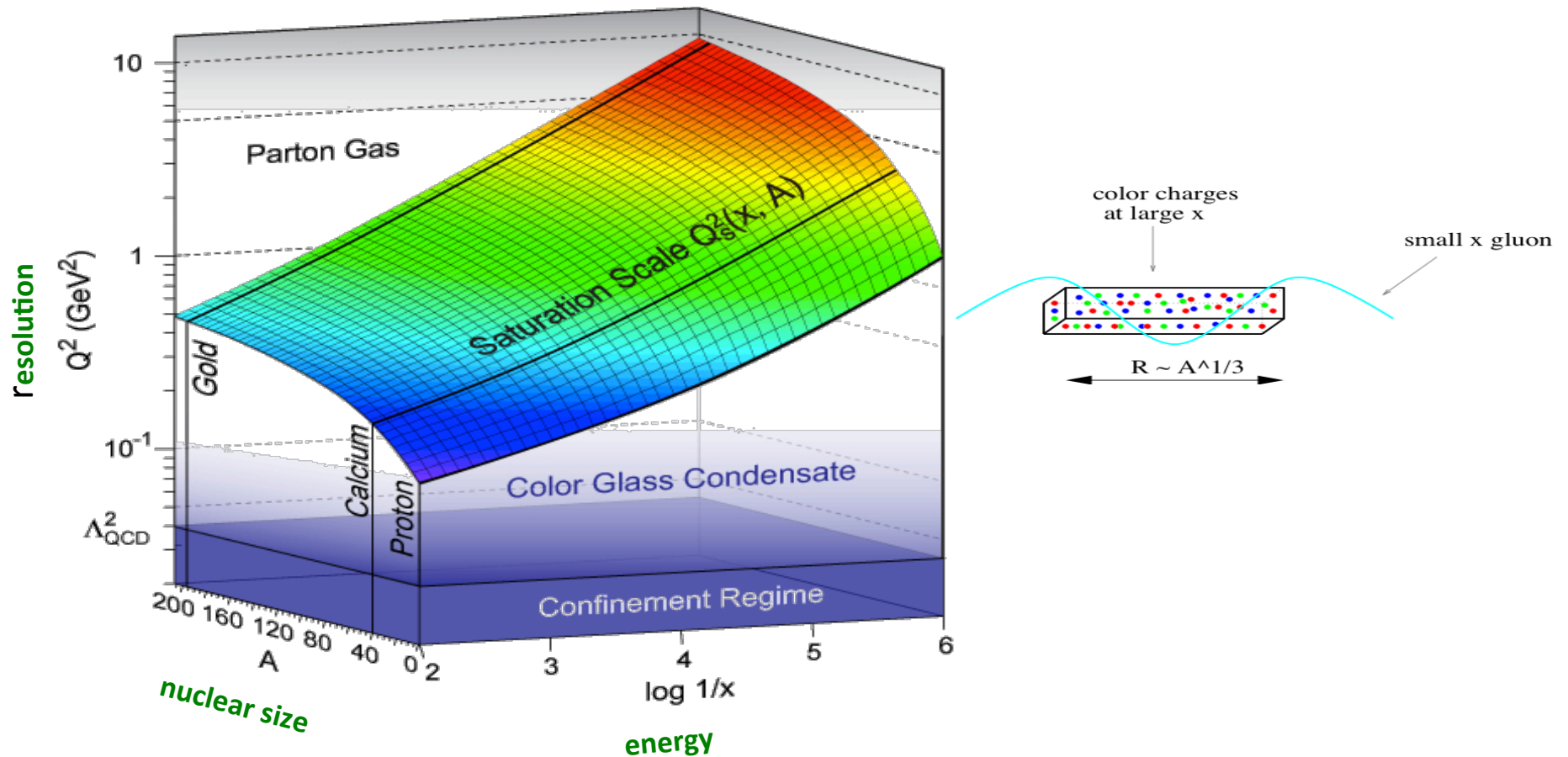


**Bulk of high energy cross-sections:**

- a) obey dynamics of novel non-linear QCD regime
- b) Can be **computed systematically** in weak coupling

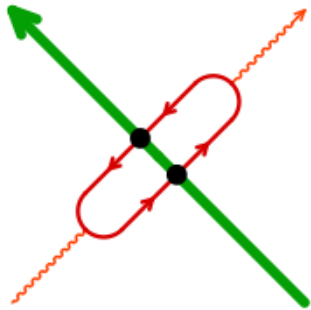
# Many-body high energy QCD: The **Color Glass Condensate**

Gelis, Iancu, Jalilian-Marian, RV:  
Ann. Rev. Nucl. Part. Sci. (2010), arXiv: 1002.0333



Dynamically generated semi-hard “saturation scale” opens window for systematic weak coupling study of non-perturbative dynamics

# Inclusive DIS: dipole evolution



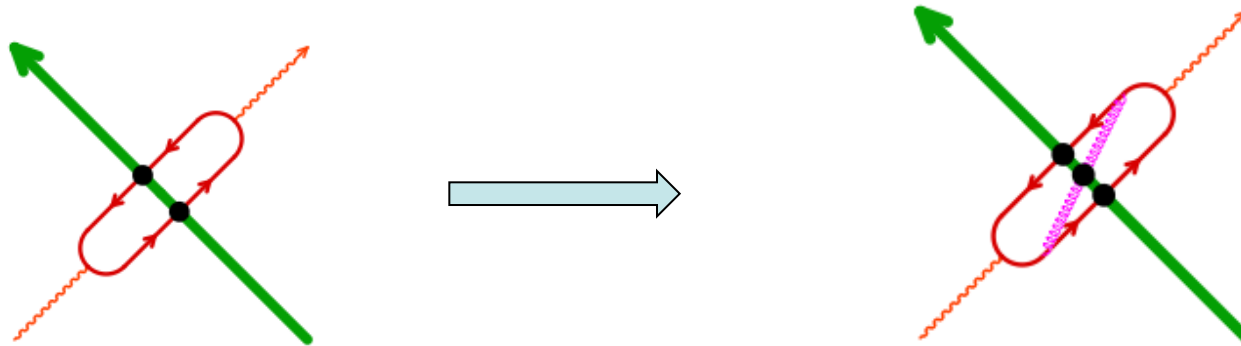
$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_{\perp} |\psi(z, r_{\perp})|^2 \sigma_{\text{dipole}}(x, r_{\perp})$$

$$\sigma_{\text{dipole}}(x, r_{\perp}) = 2 \int d^2 b \int [D\rho] W_{\Lambda^+}[\rho] T\left(b + \frac{r_{\perp}}{2}, b - \frac{r_{\perp}}{2}\right)$$



$$1 - \frac{1}{N_c} \text{Tr} \left( V \left( b + \frac{r_{\perp}}{2} \right) V^{\dagger} \left( b - \frac{r_{\perp}}{2} \right) \right)$$

# Inclusive DIS: dipole evolution



**B-JIMWLK eqn. for dipole correlator**

$$\frac{\partial}{\partial Y} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y = -\frac{\alpha_S N_c}{2\pi^2} \int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \text{Tr}(V_x V_y^\dagger) - \frac{1}{N_c} \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y$$

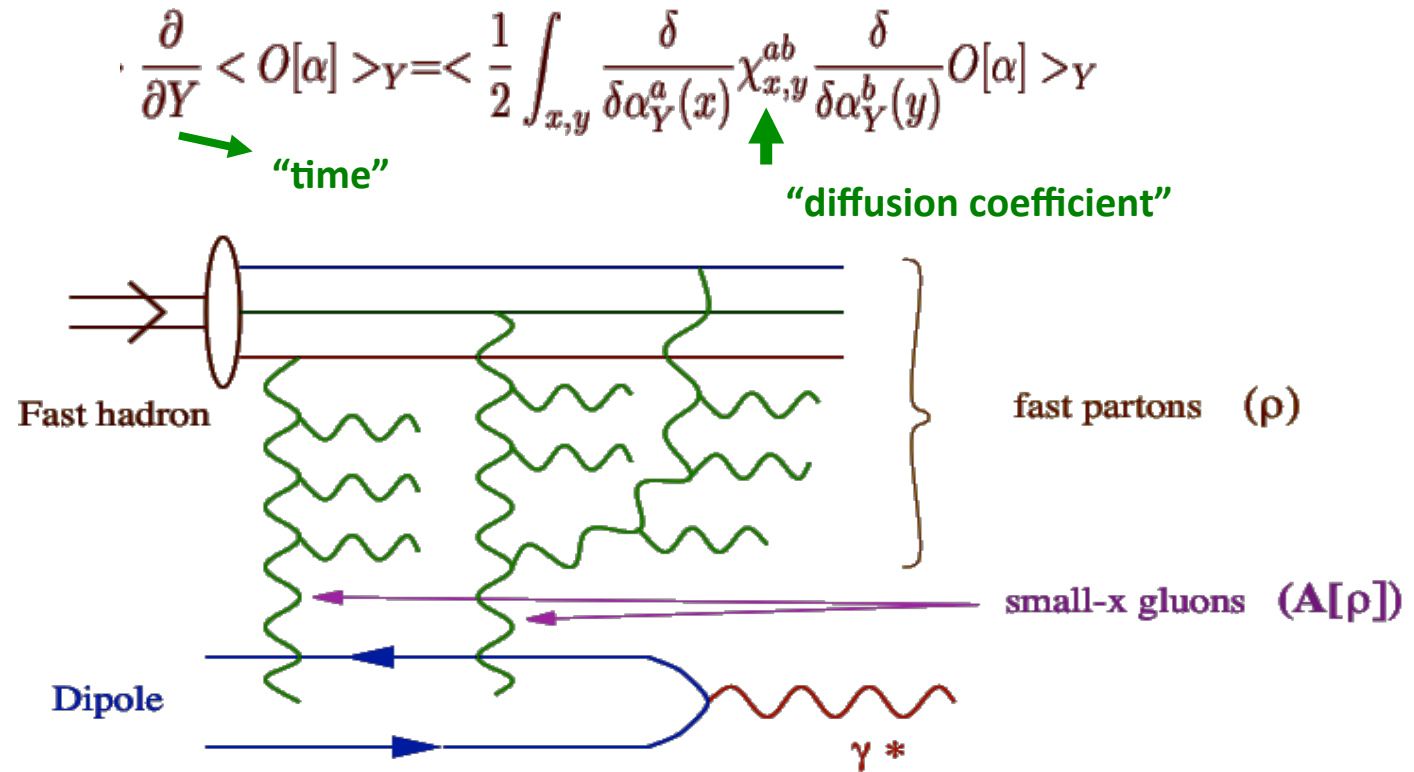
**Dipole factorization:**

$$\langle \text{Tr}(V_x V_z^\dagger) \text{Tr}(V_z V_y^\dagger) \rangle_Y \longrightarrow \langle \text{Tr}(V_x V_z^\dagger) \rangle_Y \langle \text{Tr}(V_z V_y^\dagger) \rangle_Y \quad N_c \rightarrow \infty$$

**Resulting closed form eqn. is the Balitsky-Kovchegov eqn.**

**Widely used in phenomenological applications**

# CGC Effective Theory: B-JIMWLK hierarchy of correlators



At high energies, the d.o.f that describe the frozen many-body gluon configurations are novel objects: **dipoles, quadrupoles, ...**

**Universal – appear in a number of processes in p+A and e+A;  
how do these evolve with energy ?**



# Solving the B-JIMWLK hierarchy

- ❑ JIMWLK includes multiple scatterings & leading log evolution in  $x$
- ❑ Expectation values of Wilson line correlators at small  $x$  satisfy a Fokker-Planck eqn. in functional space Weigert (2000)
- ❑ This translates into a hierarchy of equations for  $n$ -point Wilson line correlators
- ❑ As is generally the case, Fokker-Planck equations can be re-expressed as Langevin equations – in this case for Wilson lines

Blaizot, Iancu, Weigert  
Rummukainen, Weigert

# B-JIMWLK hierarchy: Langevin realization

Numerical evaluation of Wilson line correlators on 2+1-D lattices:

$$\langle \mathcal{O}[U] \rangle_Y = \int D[U] W_Y[U] \mathcal{O}[U] \longrightarrow \frac{1}{N} \sum_{U \in W} \mathcal{O}[U]$$

Langevin eqn:

$$\partial_Y [V_x]_{ij} = [V_x i t^a]_{ij} \left[ \int d^2 y [\mathcal{E}_{xy}]_k [\xi_y]_k + \sigma_x^a \right]$$

Gaussian random variable

$$\mathcal{E}_{xy}^{ab} = \left( \frac{\alpha_S}{\pi^2} \right)^{1/2} \frac{(x-y)_k}{(x-y)^2} [1 - U_x^\dagger U_y]^{ab}$$

“square root” of JIMWLK kernel

$$\sigma_x^a = -i \left( \frac{\alpha_S}{2\pi^2} \int d^2 z \frac{1}{(x-z)^2} \text{Tr}(T^a U_x^\dagger U_z) \right)$$

“drag”

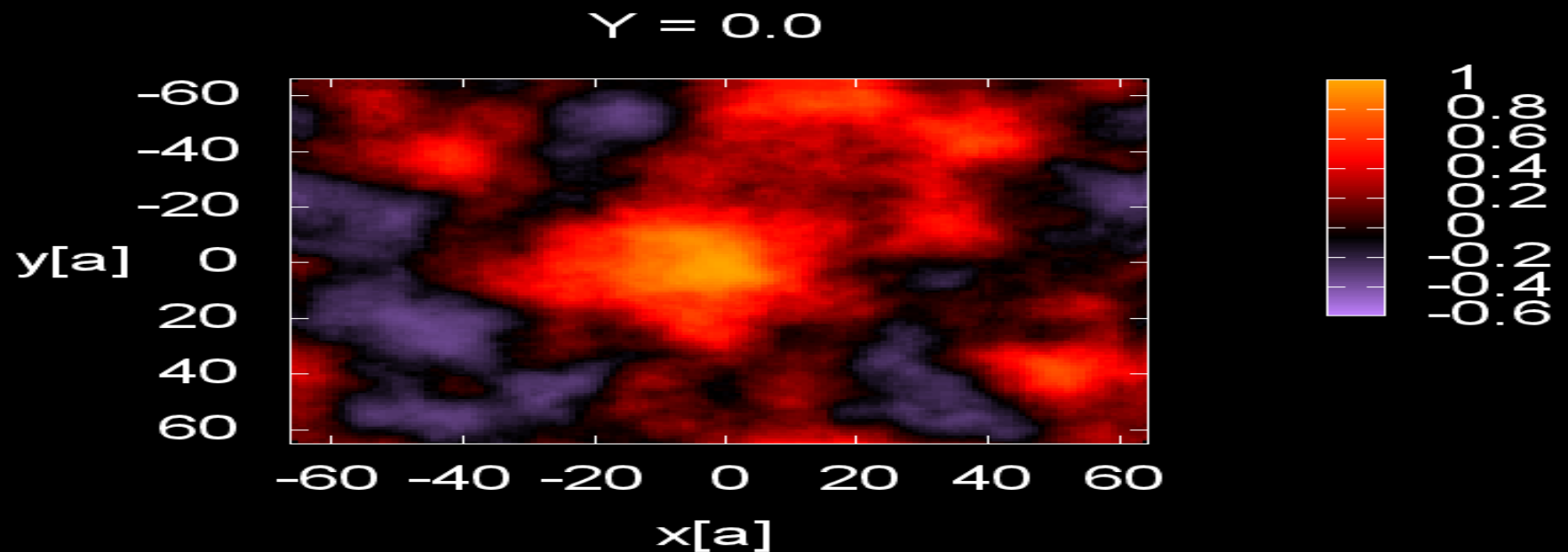
- ❑ Initial conditions for V's from the MV model
- ❑ Daughter dipole prescription for running coupling

# Functional Langevin solutions of JIMWLK hierarchy

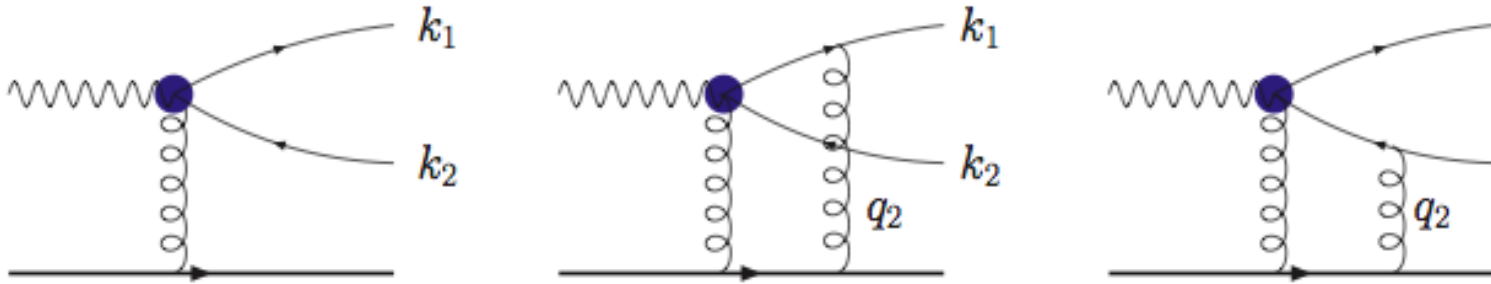
Rummukainen, Weigert (2003)

Dumitru, Jalilian-Marian, Lappi, Schenke, RV, PLB706 (2011)219

- ✓ *We are now able to compute all  $n$ -point correlations of a theory of strongly correlated gluons and study their evolution with energy!*



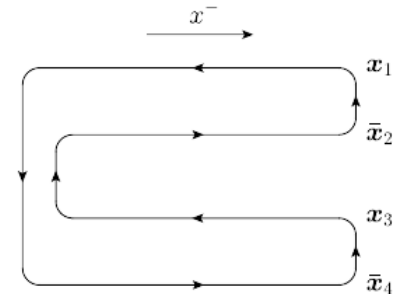
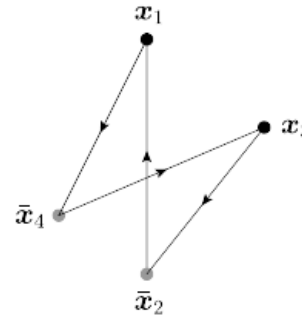
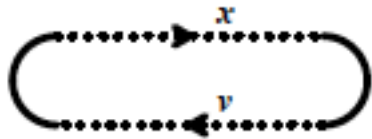
# Semi-inclusive DIS: quadrupole evolution



Dominguez, Marquet, Xiao, Yuan (2011)

$$\frac{d\sigma^{\gamma_{T,L}^* A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp} \cdot (x-\bar{x})} e^{ik_{2\perp} \cdot (y-\bar{y})} [1 + Q(x,y;\bar{y},\bar{x}) - D(x,y) - D(\bar{y},\bar{x})]$$

# Semi-inclusive DIS: quadrupole evolution



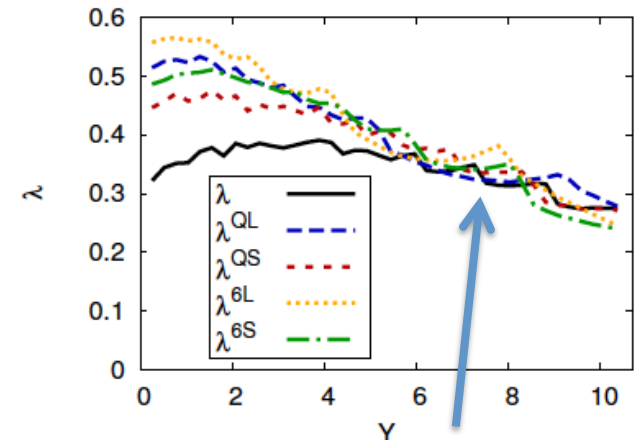
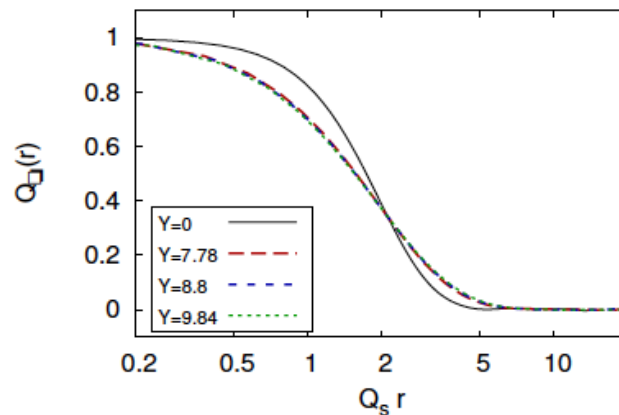
$$D(x, y) = \frac{1}{N_c} \langle \text{Tr}(V_x V_y^\dagger) \rangle_Y$$

$$Q(x, y; \bar{y}, \bar{x}) = \frac{1}{N_c} \langle \text{Tr}(V_x V_{\bar{x}}^\dagger V_{\bar{y}} V_y^\dagger) \rangle_Y$$

RG evolution provides fresh insight into multi-parton correlations

Dumitru, Jalilian-Marian, Lappi, Schenke, RV: arXiv:1108.1764

Quadrupoles, like  
Dipoles, exhibit  
Geometrical Scaling

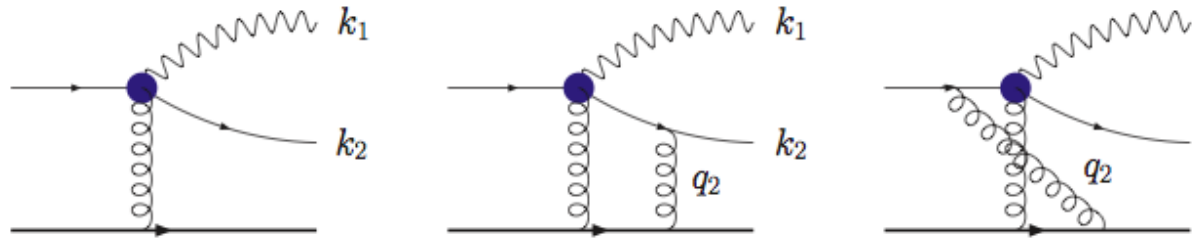


Iancu, Triantafyllopoulos, arXiv:1112.1104

Rate of energy evolution of  
dipole and quadrupole  
saturation scales

# Universality: Di-hadrons in p/d-A collisions

Jalilian-Marian, Kovchegov (2004)  
 Marquet (2007), Tuchin (2010)  
 Dominguez, Marquet, Xiao, Yuan (2011)  
 Strikman, Vogelsang (2010)

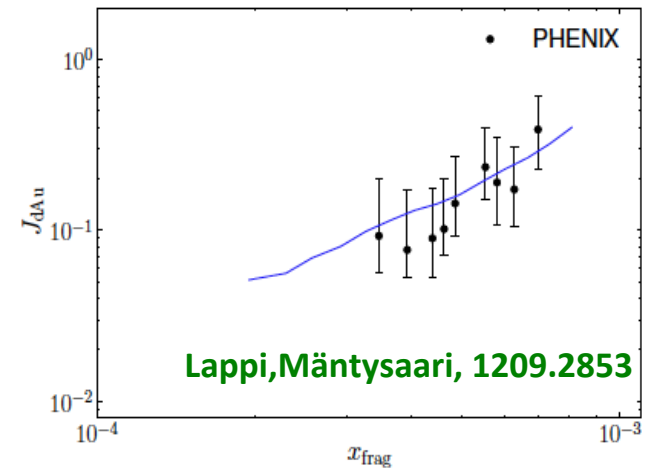
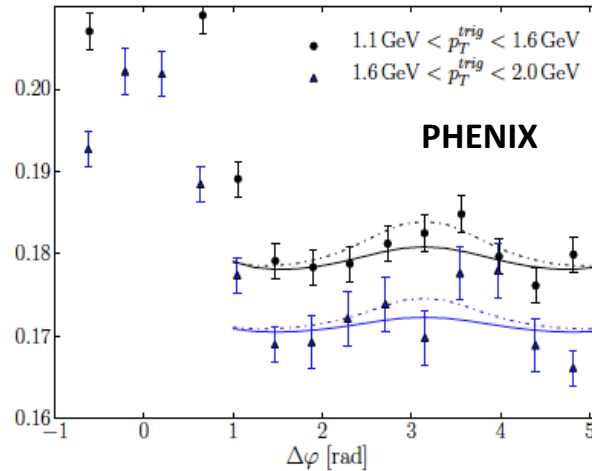
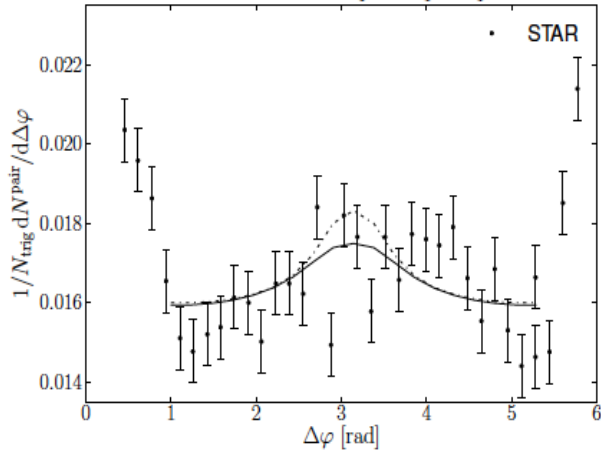


$$\frac{d\sigma^{qA \rightarrow qgX}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp} \cdot (x-\bar{x})} e^{ik_{2\perp} \cdot (y-\bar{y})} [S_6(x,y,\bar{x},\bar{y}) - S_4(x,y,v) - \dots]$$

$$\frac{N_c}{2C_F} \left\langle Q(x,y,\bar{y},\bar{x}) D(y,\bar{y}) - \frac{D(x,\bar{x})}{N_c} \right\rangle \quad \frac{N_c}{2C_F} \left\langle D(x,y) D(\bar{y},\bar{x}) - \frac{D(x,\bar{x})}{N_c} \right\rangle$$

Forward-forward di-hadrons sensitive to both **dipole** and **quadrupole** correlators

d + Au,  $2.4 < y_1, y_2 < 4$ ,  $1 \text{ GeV} < p_T^{\text{ass}} < p_T^{\text{trig}}$ ,  $p_T^{\text{trig}} > 2 \text{ GeV}$

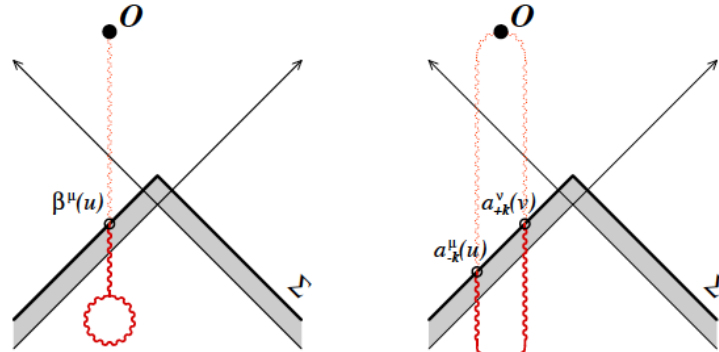


Recent computations (Stasto, Xiao, Yuan + Lappi, Mäntysaari) include Pedestal, Shadowing (color screening) and Broadening (multiple scattering) effects in CGC

# RG evolution for 2 nuclei

Gelis,Lappi,RV (2008)

Log divergent contributions  
crossing nucleus 1 or 2:



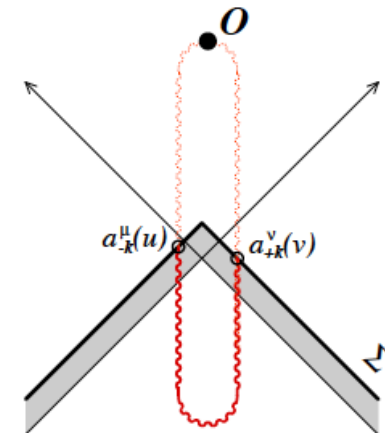
$$\mathcal{O}_{\text{NLO}} = \left[ \frac{1}{2} \int_{\vec{u}, \vec{v}} \mathcal{G}(\vec{u}, \vec{v}) \mathcal{T}_u \mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u}) \mathcal{T}_u \right] \mathcal{O}_{\text{LO}}$$

$\mathcal{G}(\vec{u}, \vec{v})$  and  $\beta(\vec{u})$  can be computed on the initial Cauchy surface

$$\mathcal{T}_u = \frac{\delta}{\delta A(\vec{u})} \quad \text{linear operator on initial surface}$$

Contributions across both nuclei are finite-no log  
divergences => factorization

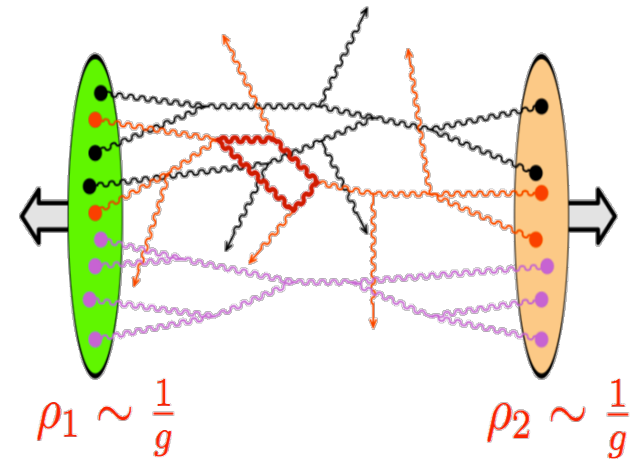
$$\mathcal{O}_{\text{NLO}} = \left[ \ln \left( \frac{\Lambda^+}{p^+} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda^-}{p^-} \right) \mathcal{H}_2 \right] \mathcal{O}_{\text{LO}}$$



# Factorization + temporal evolution in the Glasma

$$T_{\text{LO}}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F_{\lambda}^{\nu} \quad \mathcal{O}\left(\frac{Q_S^4}{g^2}\right)$$

$\epsilon=20\text{-}40 \text{ GeV}/\text{fm}^3$  for  $\tau=0.3 \text{ fm}$  @ RHIC



**NLO terms are as large as LO for  $\alpha_s \ln(1/x)$ :  
small  $x$  (leading logs) and strong field ( $gp$ ) resummation**

Gelis,Lappi,RV (2008)

$$\langle T^{\mu\nu}(\tau, \underline{\eta}, x_{\perp}) \rangle_{\text{LLog}} = \int [D\rho_1 d\rho_2] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] T_{\text{LO}}^{\mu\nu}(\tau, x_{\perp})$$

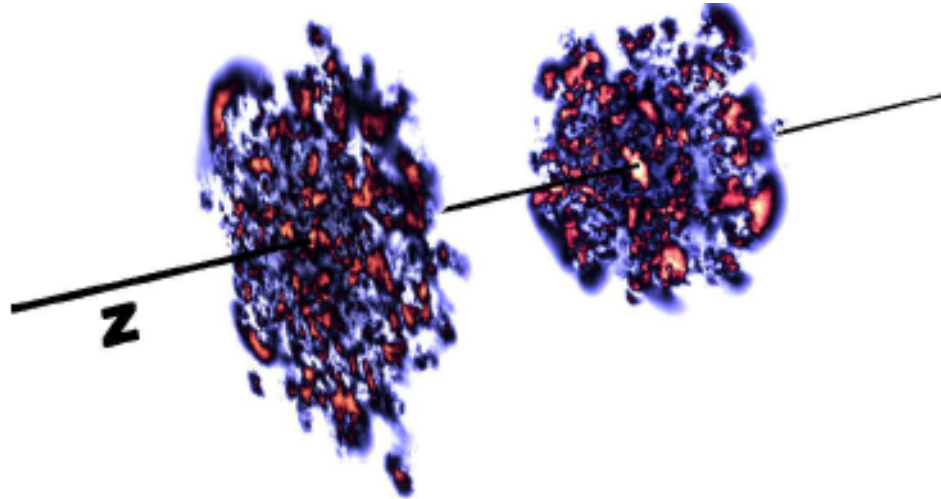
$$Y_1 = Y_{\text{beam}} - \eta; Y_2 = Y_{\text{beam}} + \eta$$

**Glasma factorization => universal “density matrices  $W$ ”  $\otimes$  “matrix element”**



# Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon “shock” waves



Systematic framework: Quantum field theory in presence of strong time dependent color sources.

For inclusive quantities, *initial value problem in the Schwinger-Keldysh formalism.*

In QCD, important and subtle issues: factorization, renormalization, universality

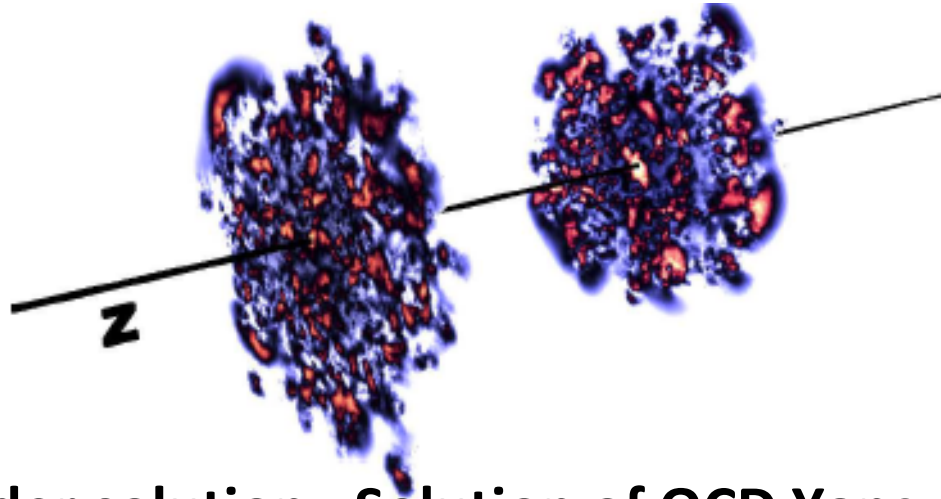
Gelis, Venugopalan (2006)

Gelis, Lappi, Venugopalan (2008,2009)

Jeon (2014)

# Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon “shock” waves



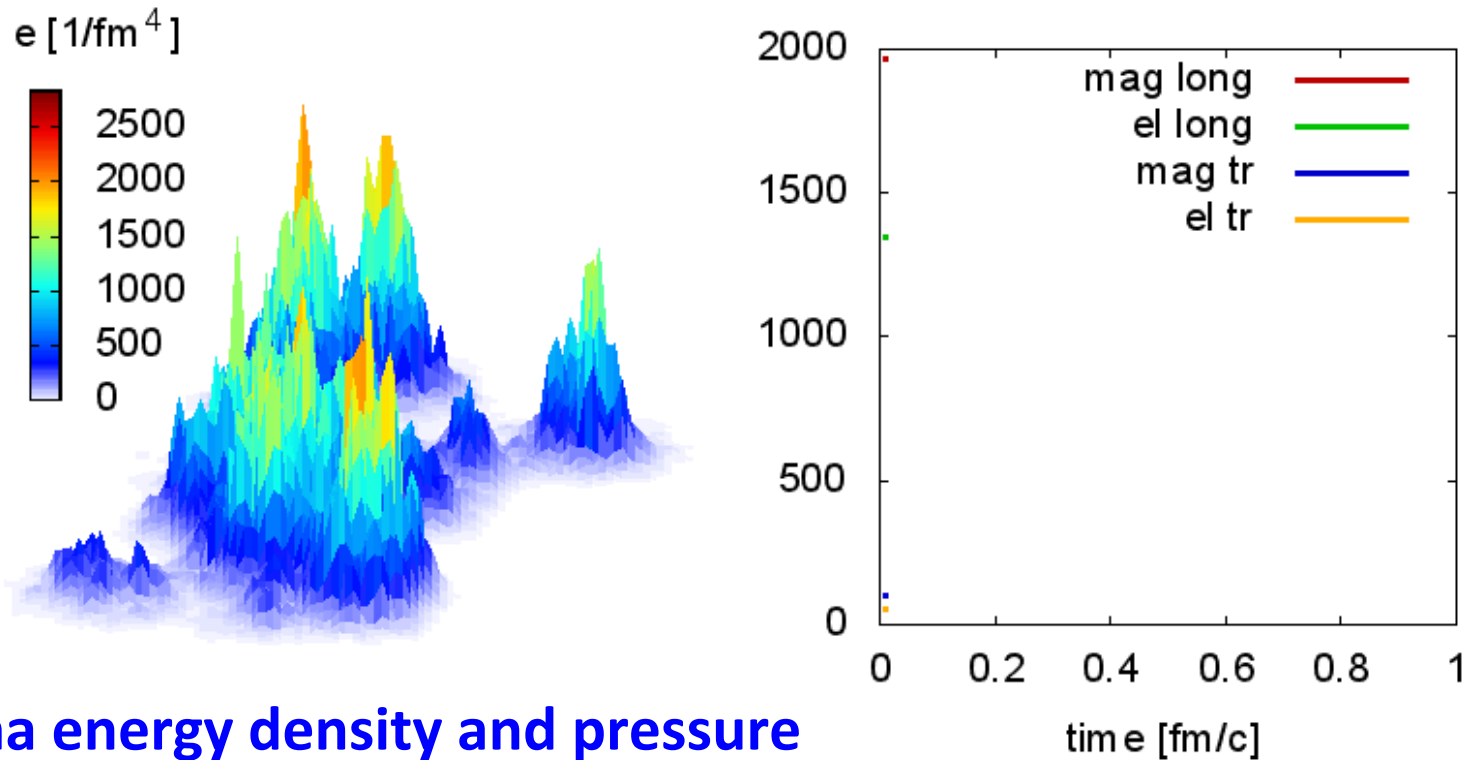
Leading order solution: Solution of QCD Yang-Mills eqns

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+} \rho_A^a(x_\perp) \delta(x^-) + \delta^{\nu-} \rho_B^a(x_\perp) \delta(x^+)$$

$$x^\pm = t \pm z$$

$$F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\mu,a} + g f^{abc} A^{\mu,b} A^{\nu,c}$$

# $T^{\mu\nu}$ from Yang-Mills dynamics



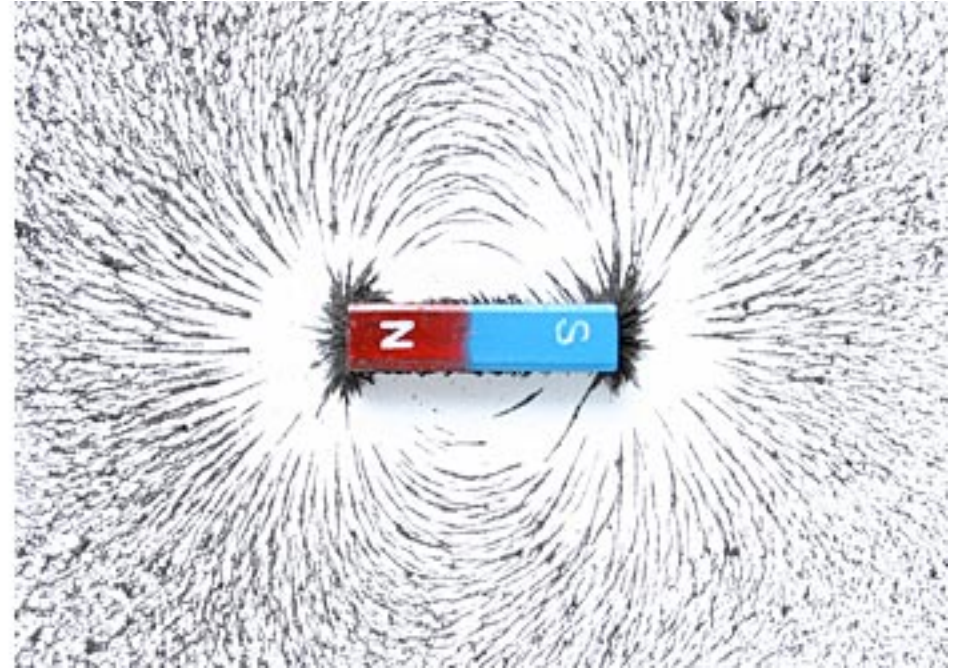
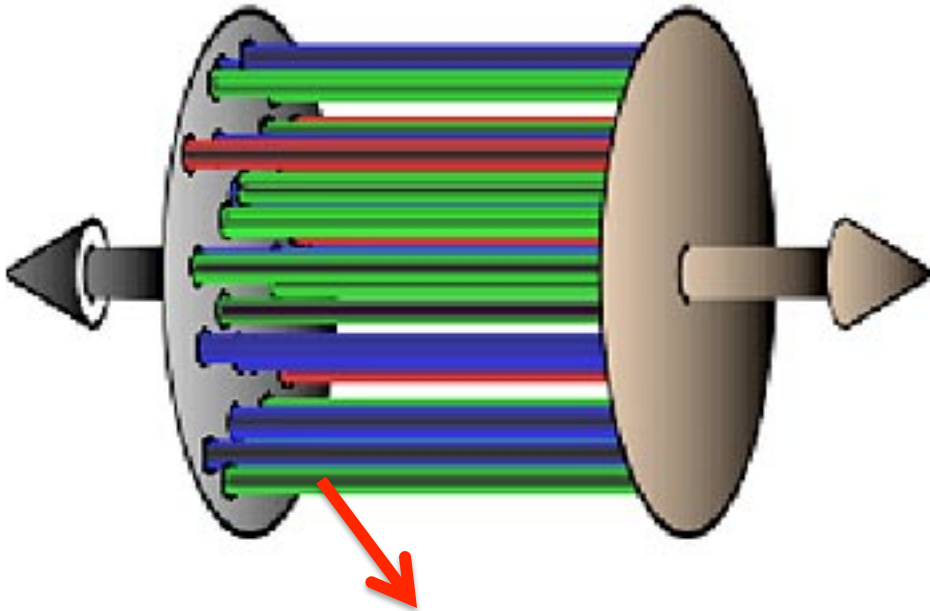
## Glasma energy density and pressure

$$T_{\mu\nu}(\tau = 0) = \frac{1}{2}(B_z^2 + E_z^2) \times \text{diag}(1, 1, 1, -1)$$

Initial longitudinal pressure is negative:

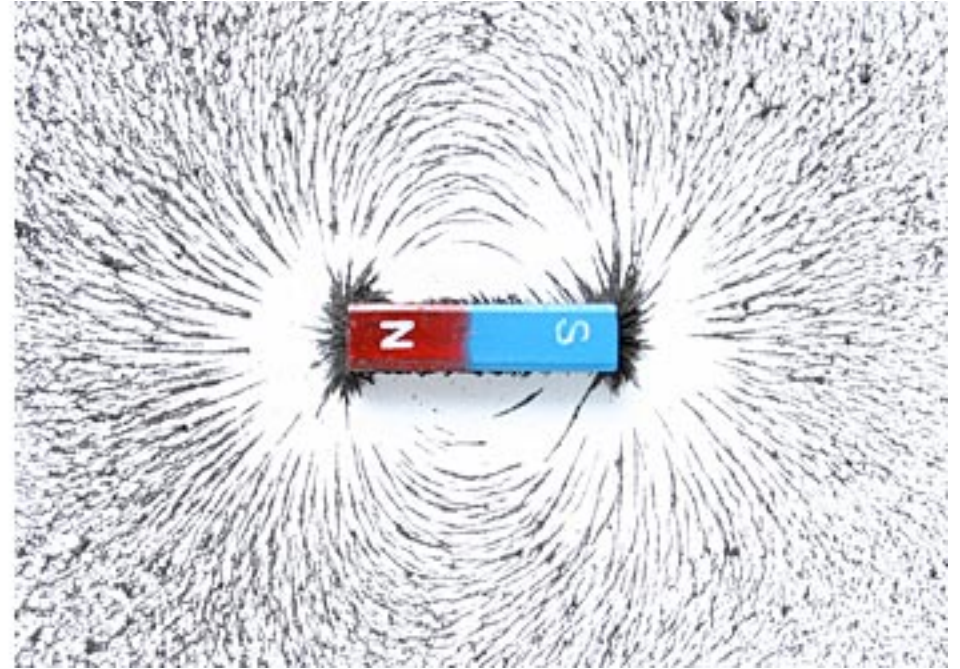
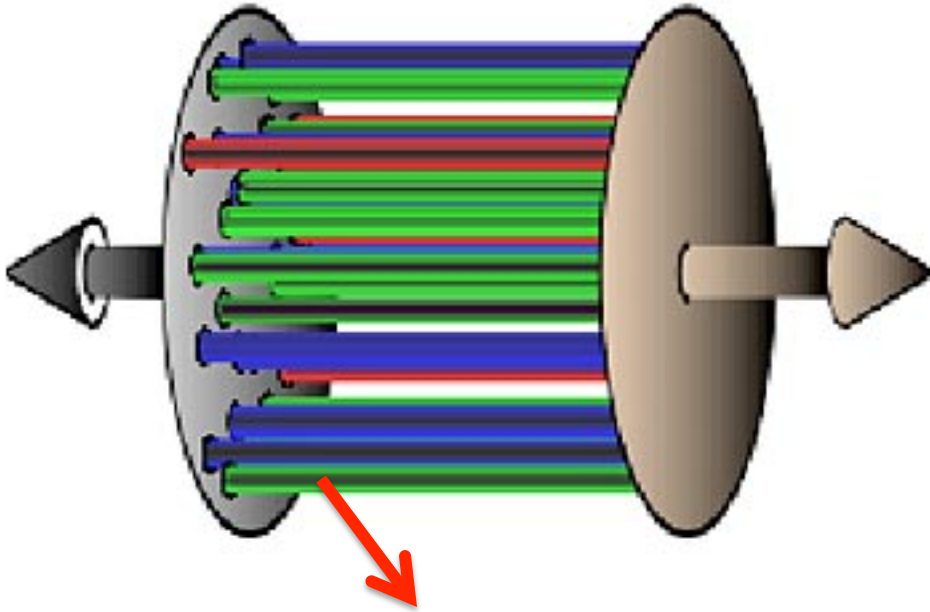
Goes to  $P_L = 0$  from below with time evolution

## Imaging the force fields of QCD



Solns. of QCD Yang-Mills eqns. demonstrate that each of these color “flux tubes” stretching out in rapidity is of transverse size  $1/Q_s \ll 1$  fm

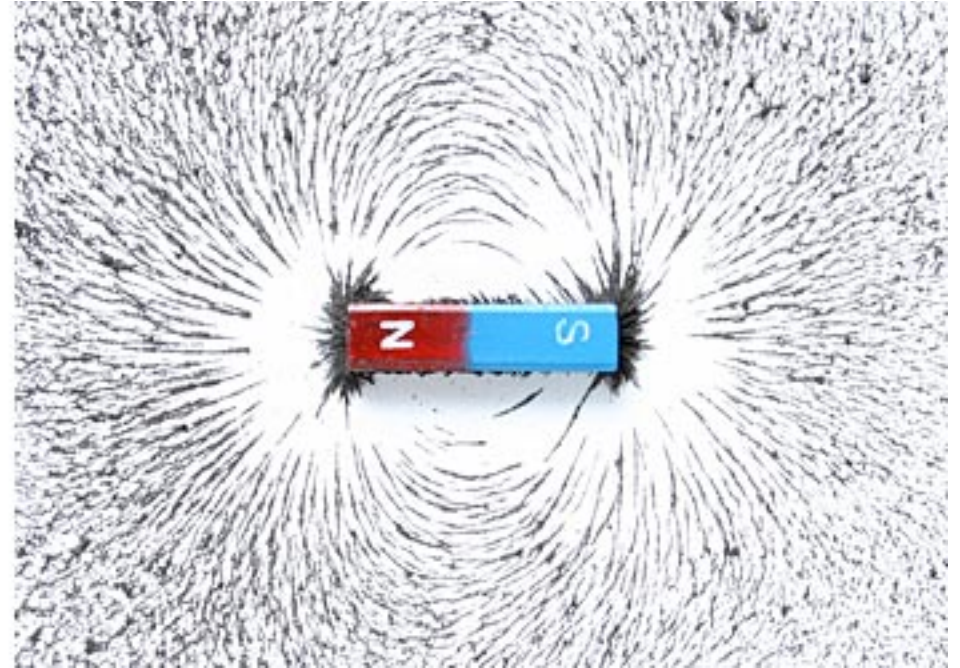
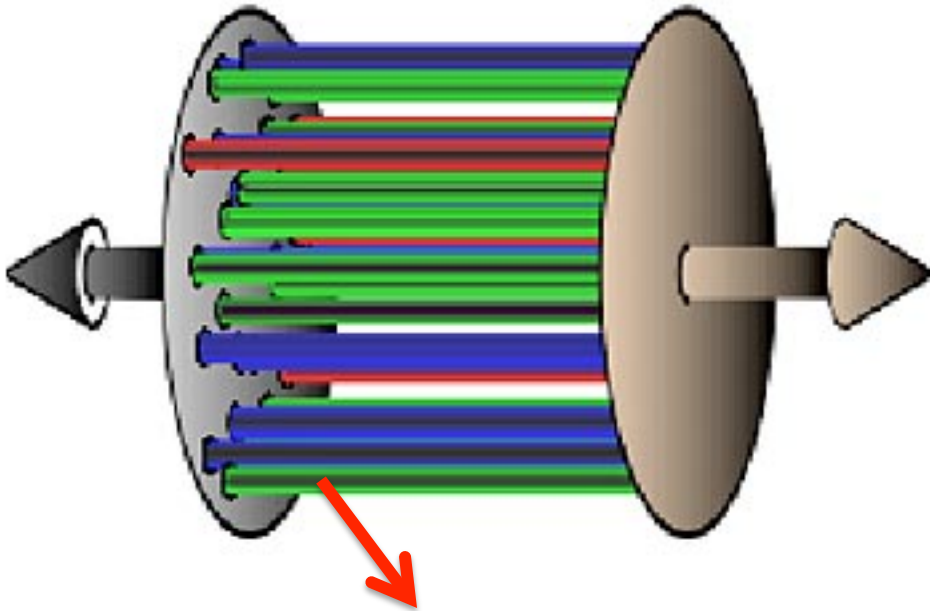
## Imaging the force fields of QCD



Solns. of QCD Yang-Mills eqns. demonstrate that each of these color “flux tubes” stretching out in rapidity is of transverse size  $1/Q_s \ll 1$  fm

**Multiparticle dynamics is controlled by sub-nucleon QCD scales**

## Imaging the force fields of QCD



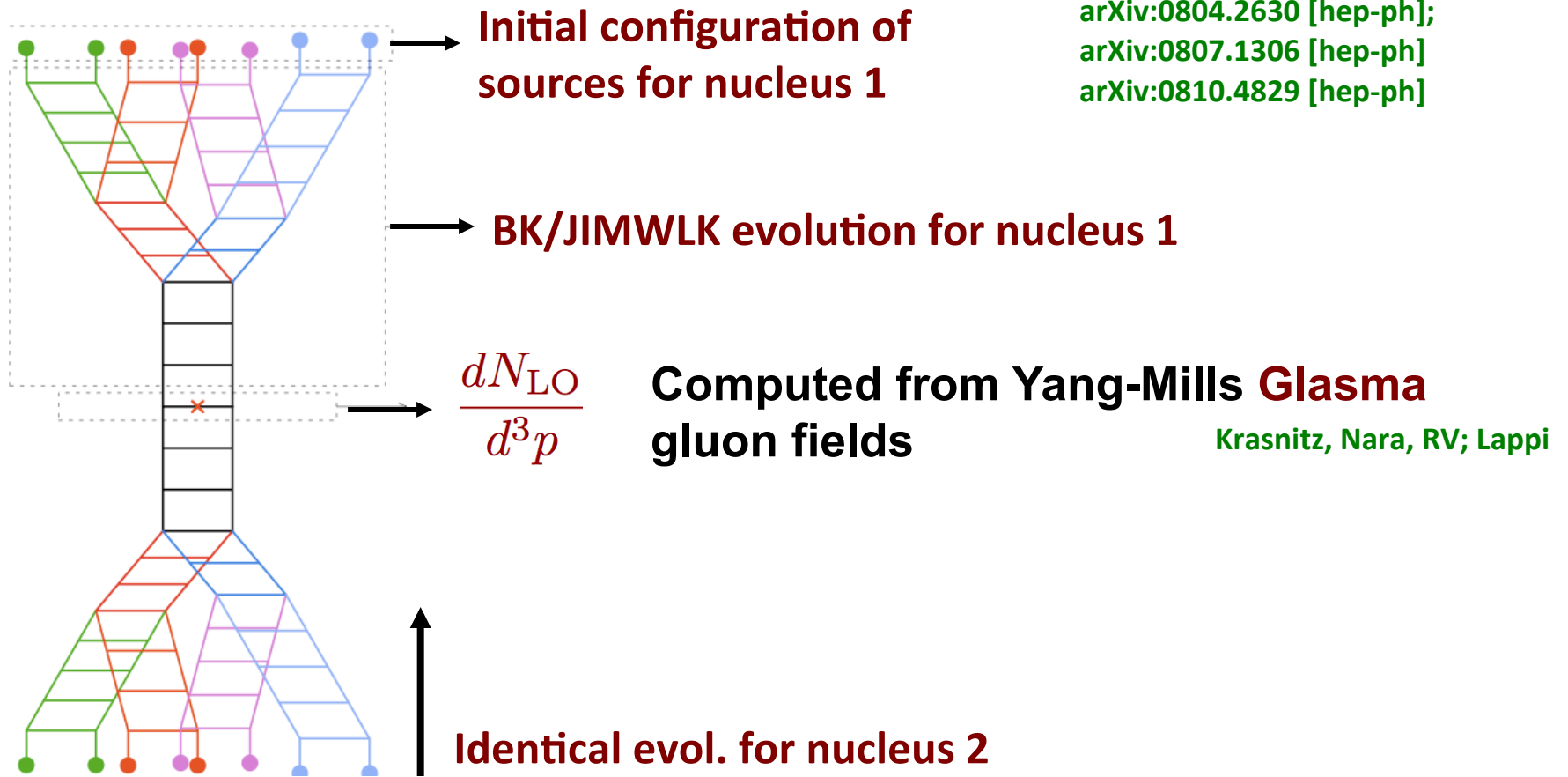
Solns. of QCD Yang-Mills eqns. demonstrate that each of these color “flux tubes” stretching out in rapidity is of transverse size  $1/Q_s \ll 1$  fm

**Multiparticle dynamics is controlled by sub-nucleon QCD scales**

There are  $\sim \pi R^2 Q_s^2$  flux tubes – multiplicity,  $dn/d\eta \approx \pi R^2 Q_s^2 / \alpha_s$

# Single inclusive gluon production

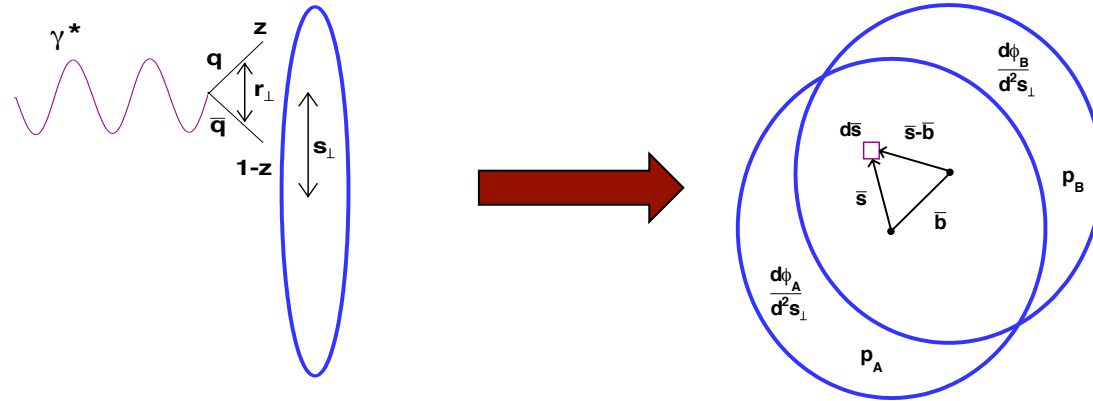
Gelis,Lappi,RV  
arXiv:0804.2630 [hep-ph];  
arXiv:0807.1306 [hep-ph]  
arXiv:0810.4829 [hep-ph]



- ◆ Full JIMWLK+YM evolution feasible Lappi, PLB 703 (2011)209
- ◆ In practice: approximations of varying rigor

# Extracting lumpy glue in the proton-IPSat model

Bartels, Golec-Biernat, Kowalski  
 Kowalski, Teaney  
 Kowalski, Motyka, Watt

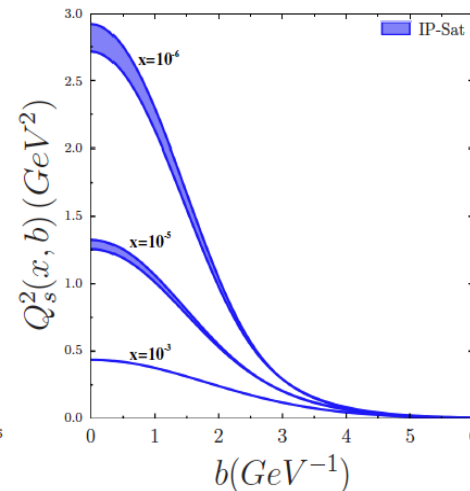
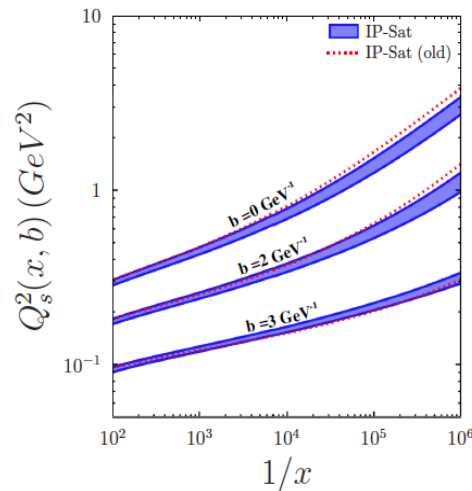


$$\frac{d\sigma_{\text{dip}}^p}{d^2b_{\perp}}(r_{\perp}, x, b_{\perp}) = 2\mathcal{N}(r_{\perp}, x, b_{\perp}) = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r_{\perp}^2 \alpha_s(\tilde{\mu}^2) x g(x, \tilde{\mu}^2) T_p(b_{\perp}) \right) \right]$$

$$T_p(b_{\perp}) = e^{-\frac{b_{\perp}^2}{2BG}}$$

Average gluon radius of the proton extracted from HERA diffractive data

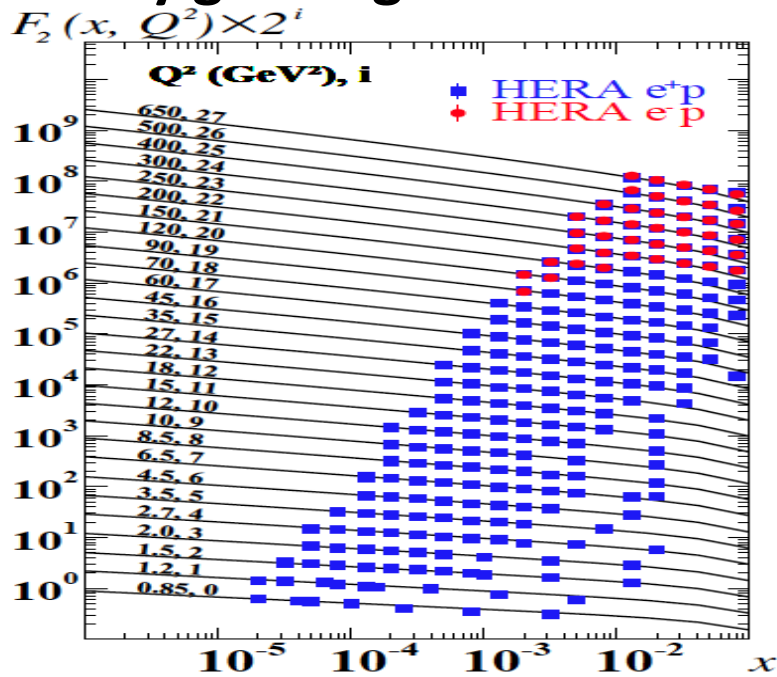
$$\tilde{\mu}^2 = \mu_0^2 + \frac{4}{r_{\perp}^2}$$





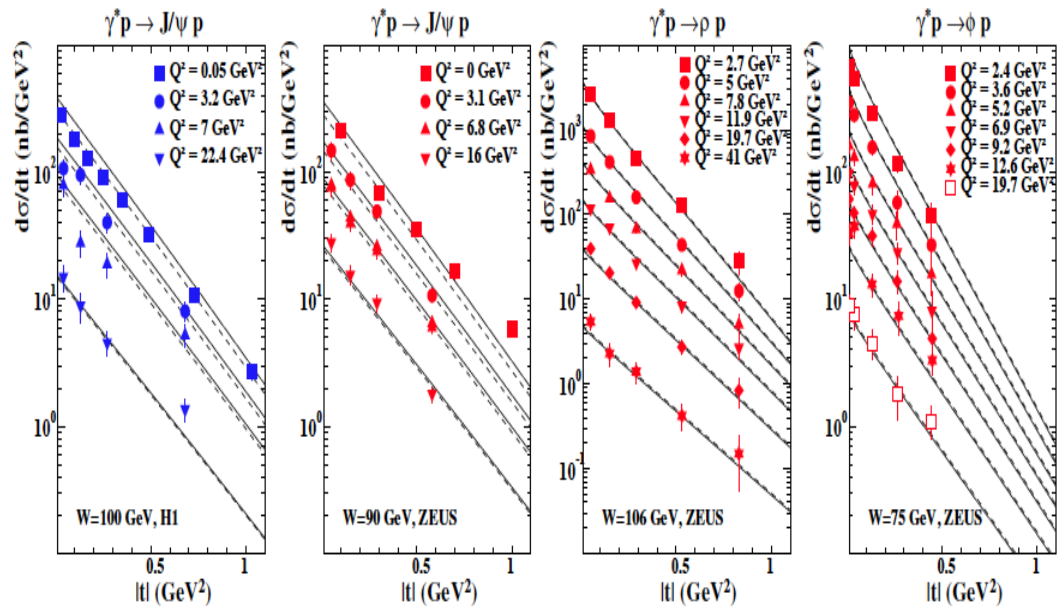
# Extracting lumpy glue in the proton-IPSat model

Very good agreement of IPSat model with combined HERA data



Inclusive DIS off proton

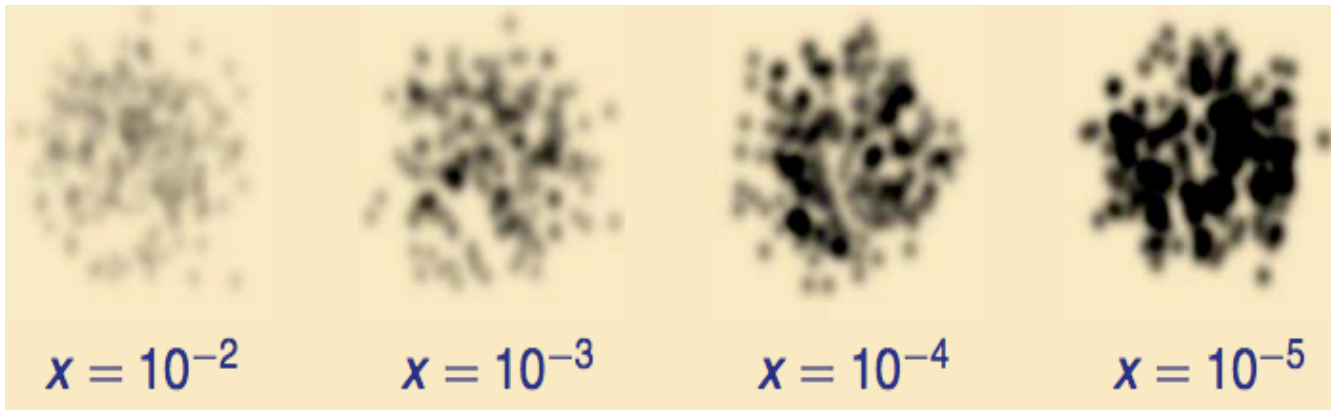
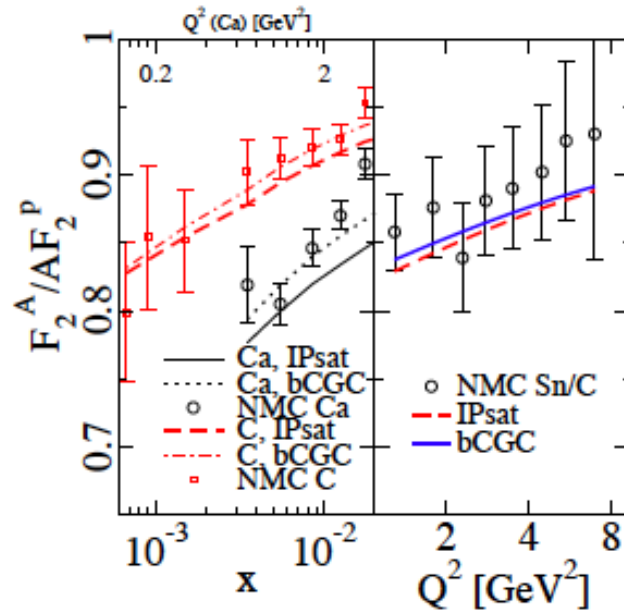
Rezaiean, Siddikov, Van der Klundert, RV:1212.2974



Exclusive DIS off proton

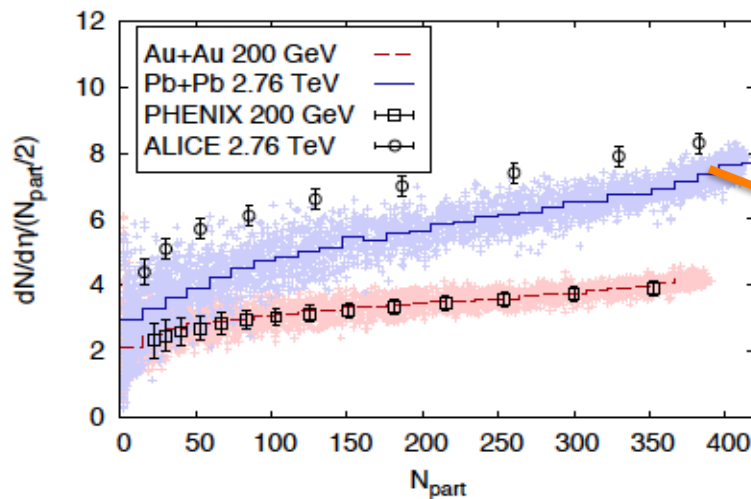
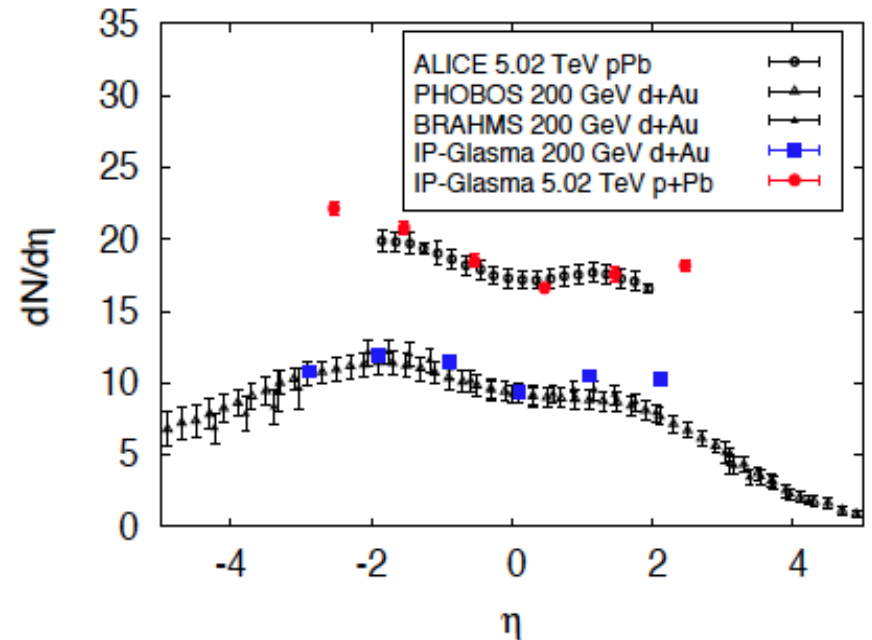
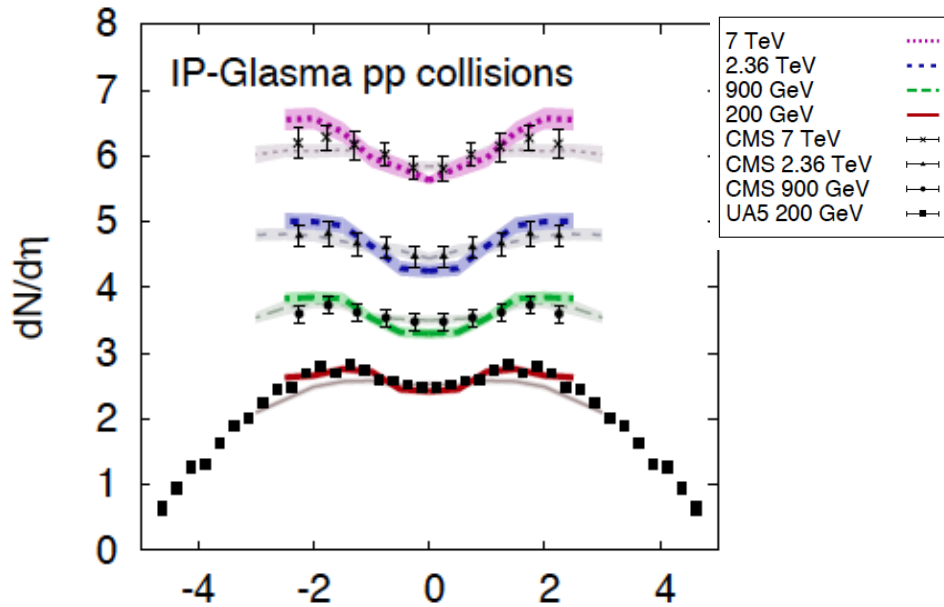
# Lumpy nuclei: constrained by (limited) DIS data

Kowalski, Lappi, RV, PRL (2008)



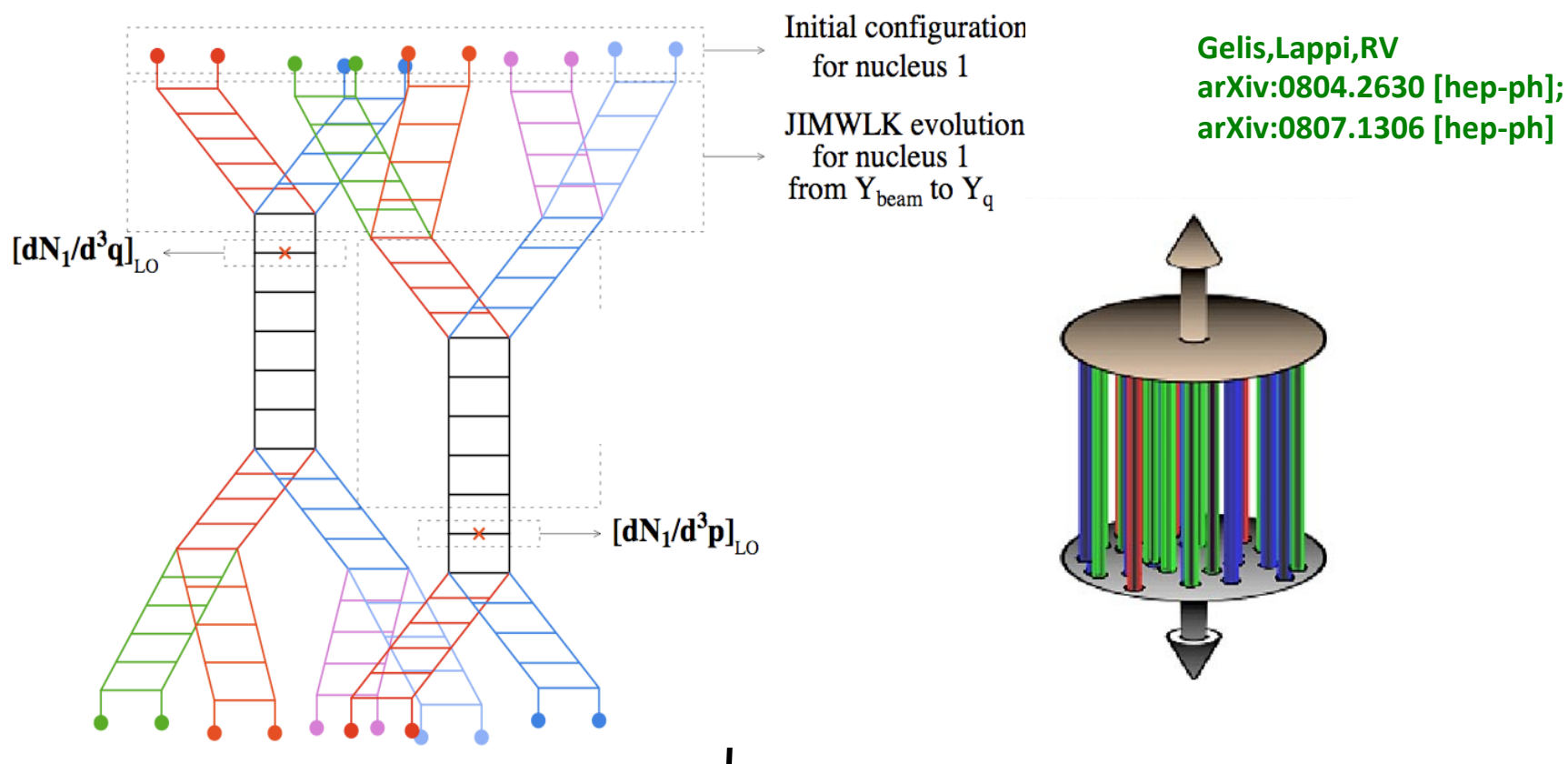
# Multiplicities from Yang-Mills dynamics

Schenke, Tribedy, RV:1311.3636



15% (30 %) shortfall for large  $N_{part}$  (low  $N_{part}$ ) at LHC possibly due to the  $\sim 70\%$  greater  $\eta/s$

# High multiplicity events: two particle correlations



◆ Full YM+JIMWLK evolution – not available yet

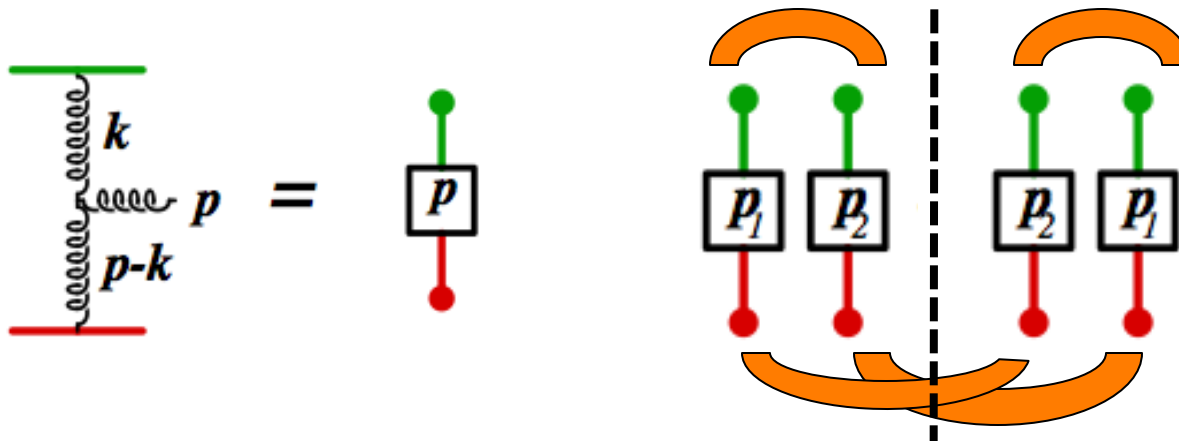
◆ Approximations:

I) BK Gaussian truncation approximation -evolution but no rescattering

II) YM results for MV model: rescattering but no evolution

Dusling,Gelis,Lappi,RV:0911.2720; Lappi,Srednyak,RV:0911.2068;  
Kovchegov,Wertepny: 1212.1195

## 2-particle correlations



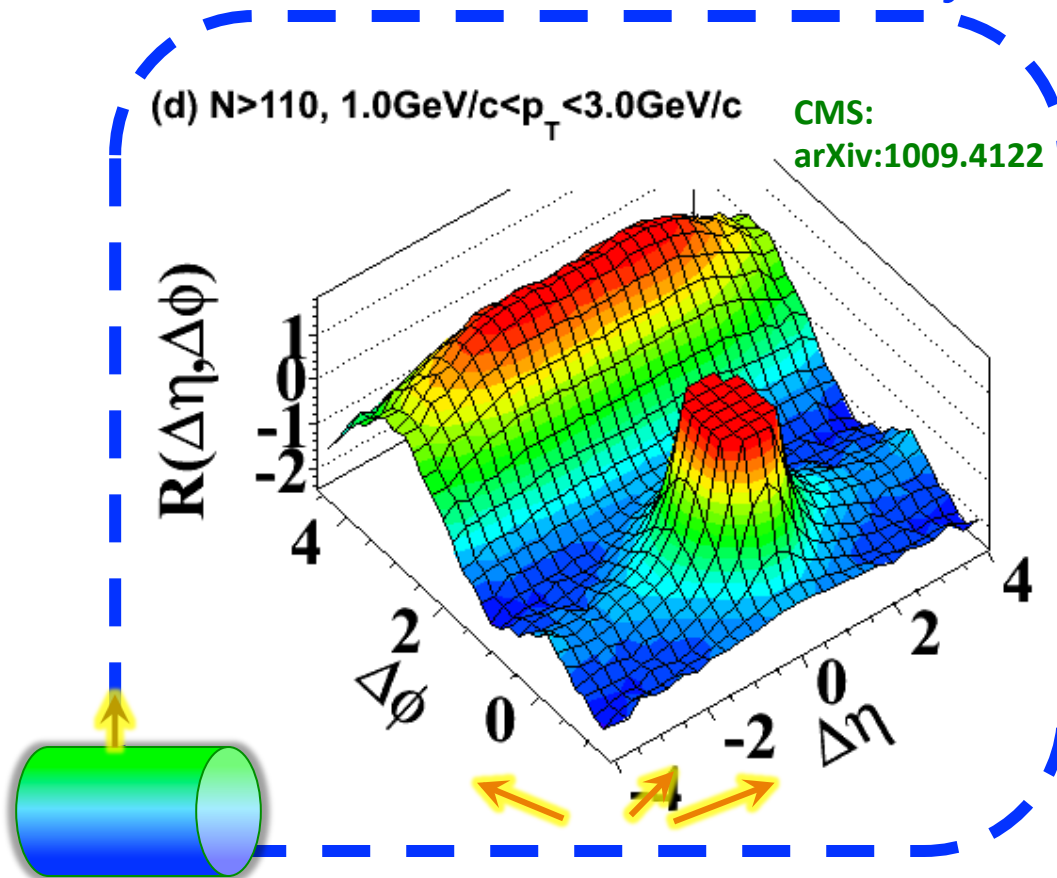
Dumitru, Gelis, McLerran, RV  
Dusling, Fernandez-Fraile, RV

**Glasma flux tube picture:** two particle correlations  
proportional to ratio  $1/Q_s^2 / S_T$

Only certain color combinations of “dimers” give leading contributions

# Two particle correlations: the ridge

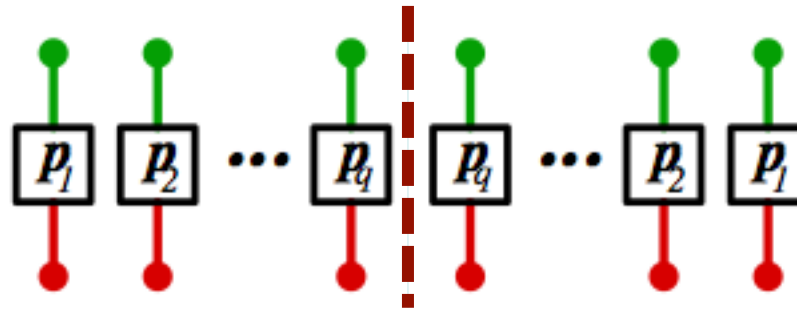
*“Discovery”*



◆ To be discussed in the next lecture...

# 2-particle n-particle correlations

Gelis, Lappi, McLerran



Multiplicity distribution: Leading combinatorics of dimers gives the negative binomial distribution

$$P_n^{\text{N.B.}}(\bar{n}, k) = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\bar{n}^n k^k}{(\bar{n} + k)^{n+k}}$$

$$k = \zeta \frac{(N_c^2 - 1) Q_S^2 S_\perp}{2\pi}$$

$k = 1$  : Bose-Einstein

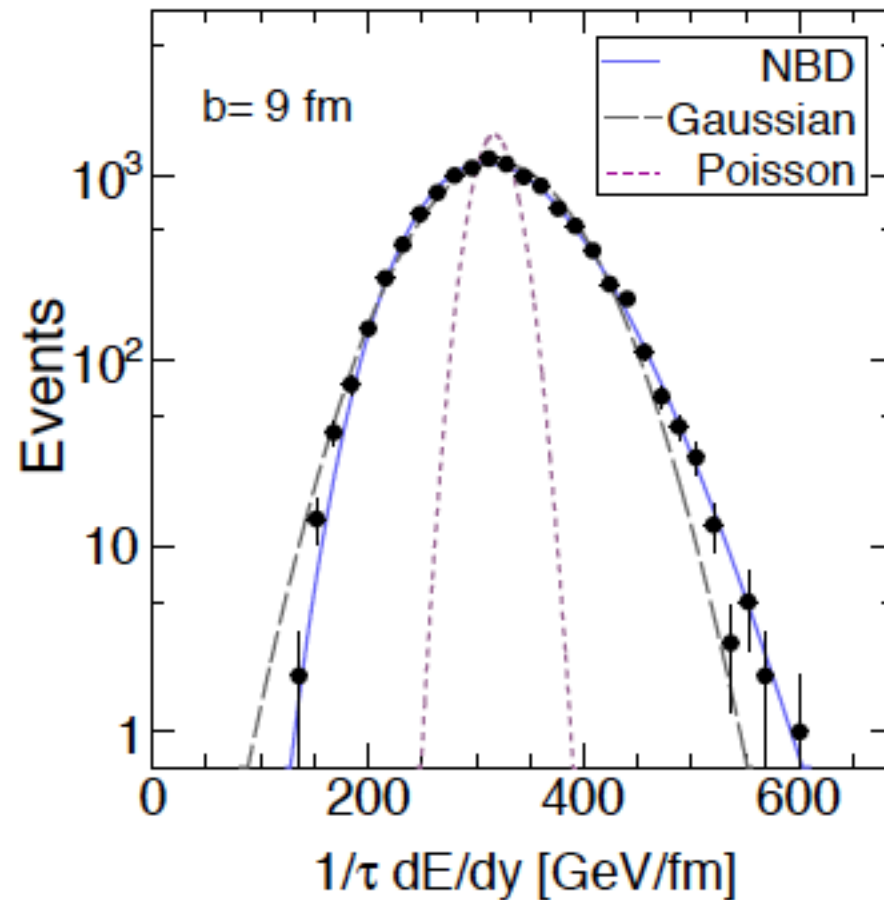
$k = \infty$  : Poisson

Non.pert.constant-can be computed in Yang-Mills simulations

Lappi, Srednyak, RV, 0911.2068  
Schenke, Tribedy, RV, 1206.6805

# Negative Binomial Distributions from non-perturbative Yang-Mills dynamics

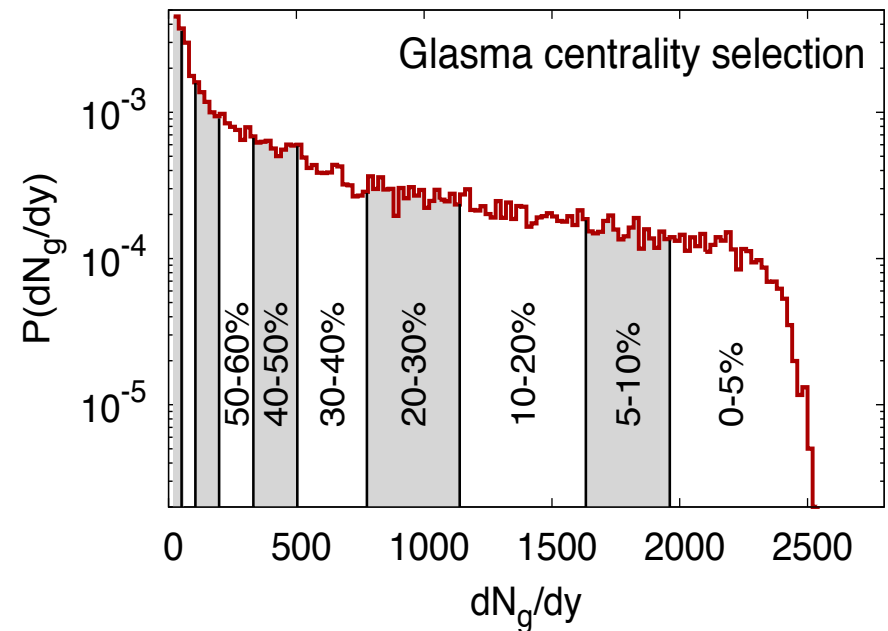
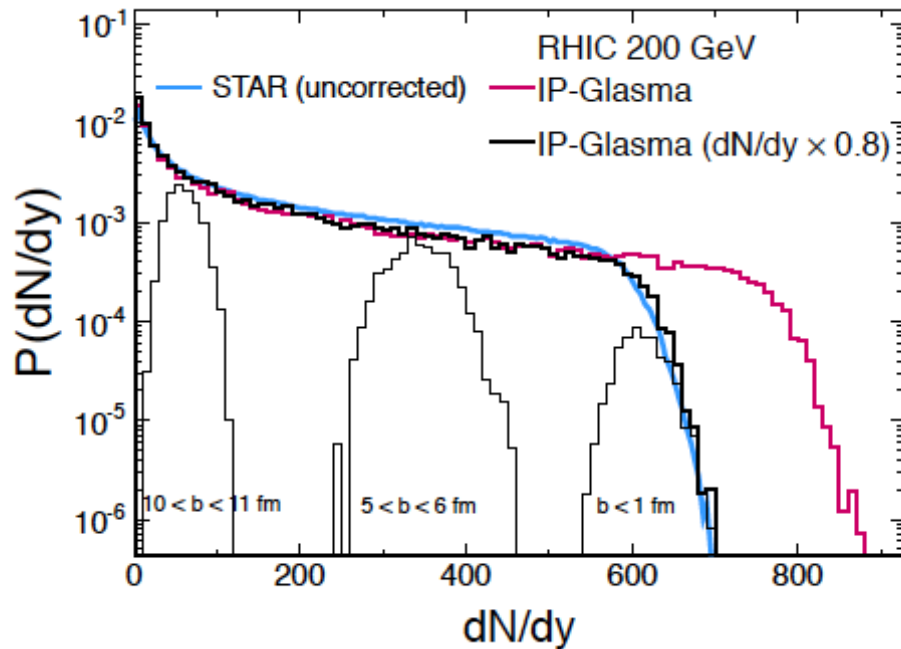
Schenke, Tribedy, RV:1202.6646





# NBDs in heavy ion collisions

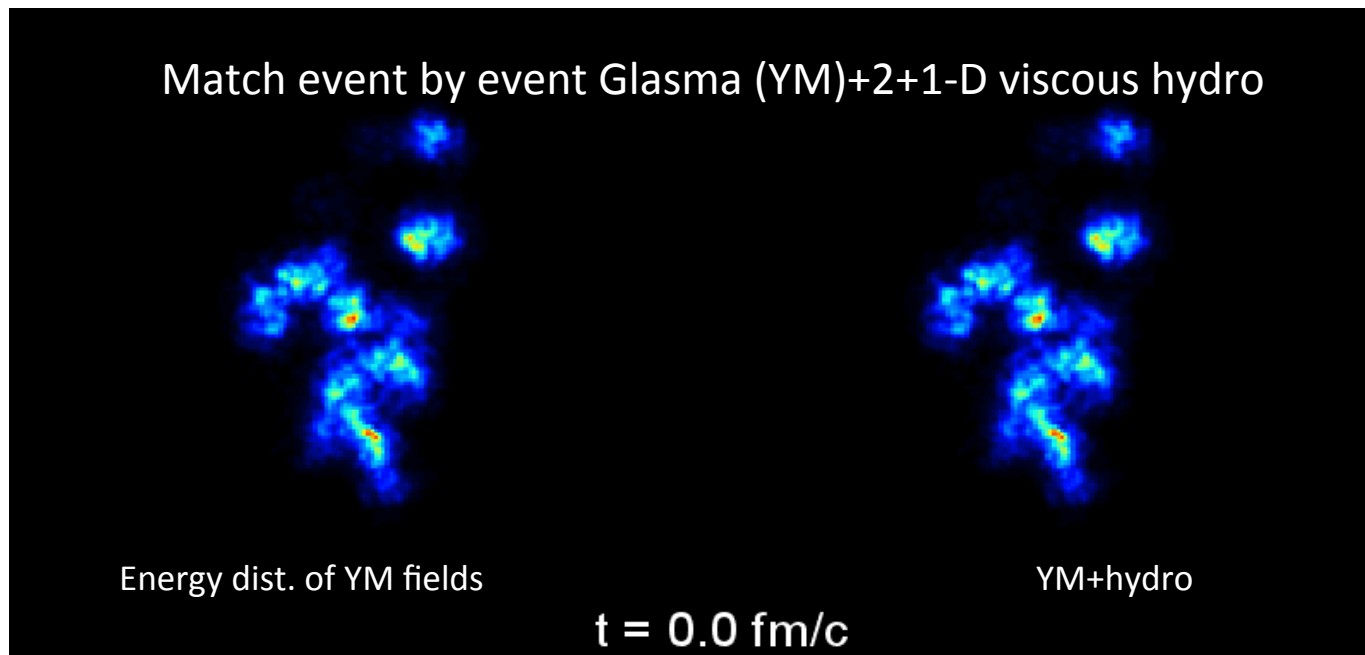
Schenke, Tribedy, RV: arXiv:1206.6805



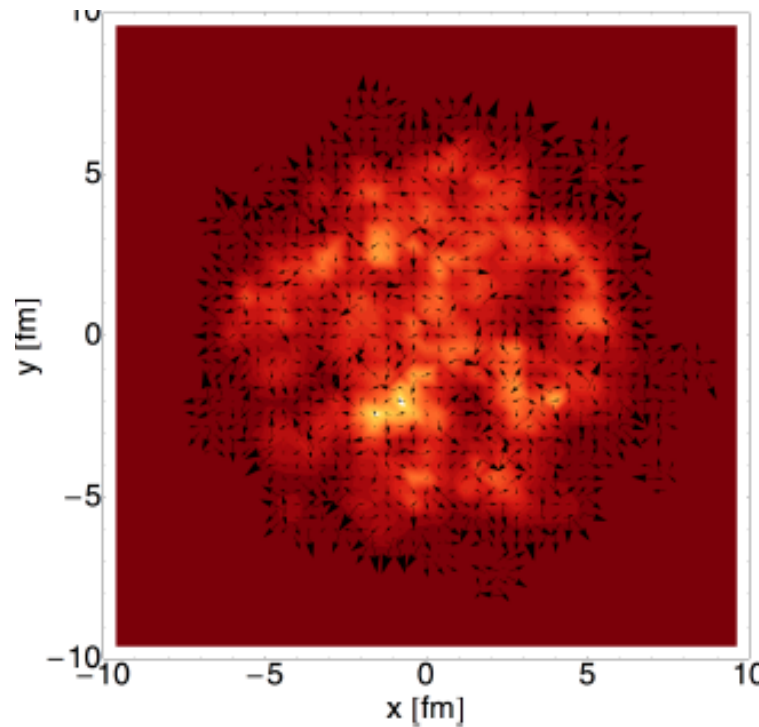
**Only model of heavy ion collisions where  
multiplicity dist./centrality selection is not an external input**

# Matching boost invariant Yang-Mills to hydrodynamics

State of the art phenomenology: Solve relativistic viscous hydrodynamic equations with Glasma (Yang-Mills) initial conditions



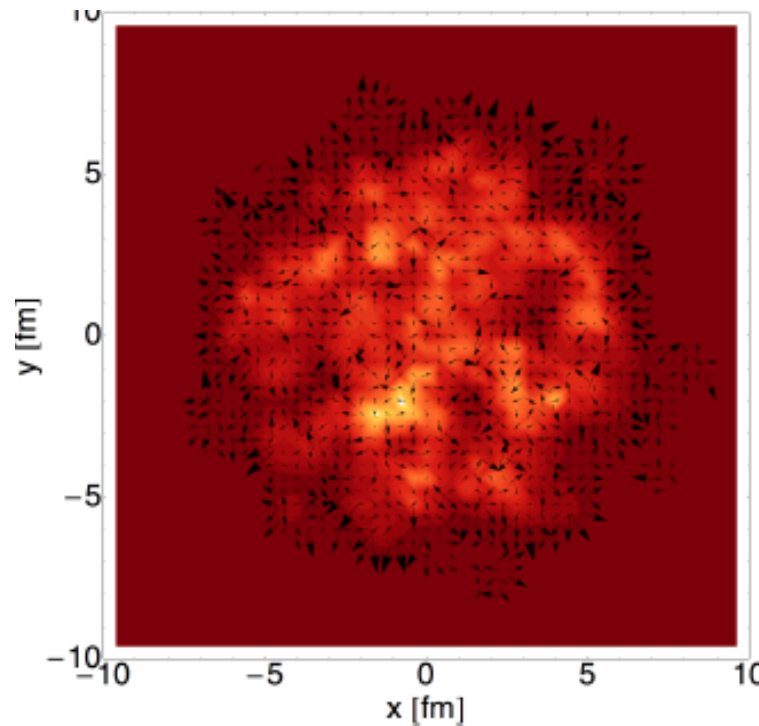
# Matching boost invariant Yang-Mills to hydrodynamics



Energy density  
and  $(u_x, u_y)$   
at  $\tau = 0.4 \text{ fm}/c$

Energy density and  $(u_x, u_y)$  from  $u_\mu T^{\mu\nu} = \varepsilon u^\nu$

# Matching boost invariant Yang-Mills to hydrodynamics



Energy density  
and  $(u_x, u_y)$   
at  $\tau = 0.4 \text{ fm}/c$

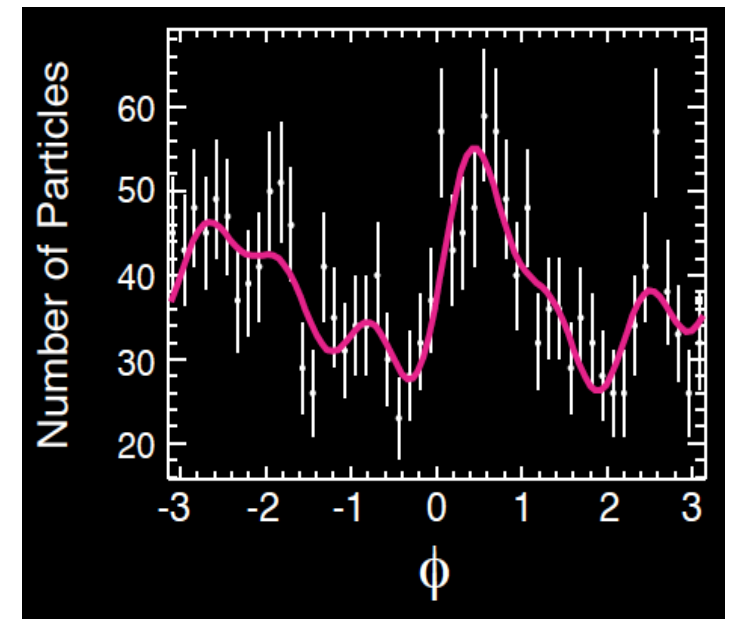
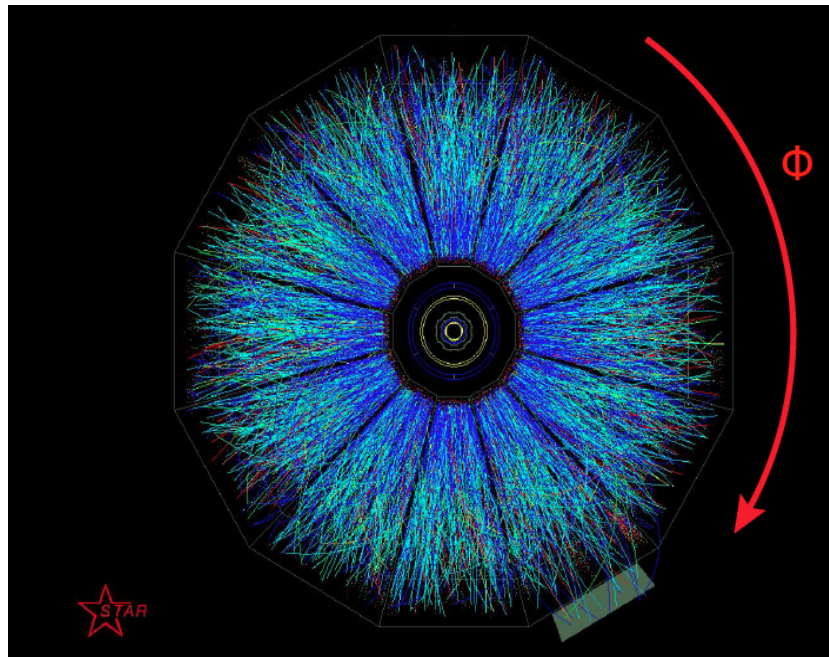
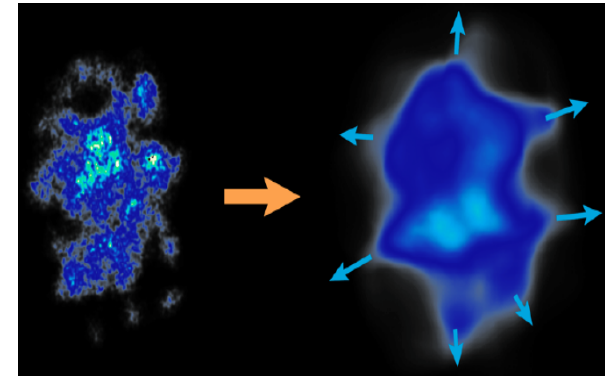
Matching to viscous hydro is “brutal” :  
assume very rapid isotropization at initial hydro time

Large systematic uncertainty: how does isotropization/  
thermalization occur on times  $< 1 \text{ fm}/c$  ?

# Heavy ion phenomenology in weak coupling

Hydrodynamics: efficient translation of spatial anisotropy into momentum anisotropy

$$\frac{dN}{d\phi} = \frac{N}{2\pi} (1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + 2v_3 \cos(3\phi) + 2v_4 \cos(4\phi) + \dots)$$



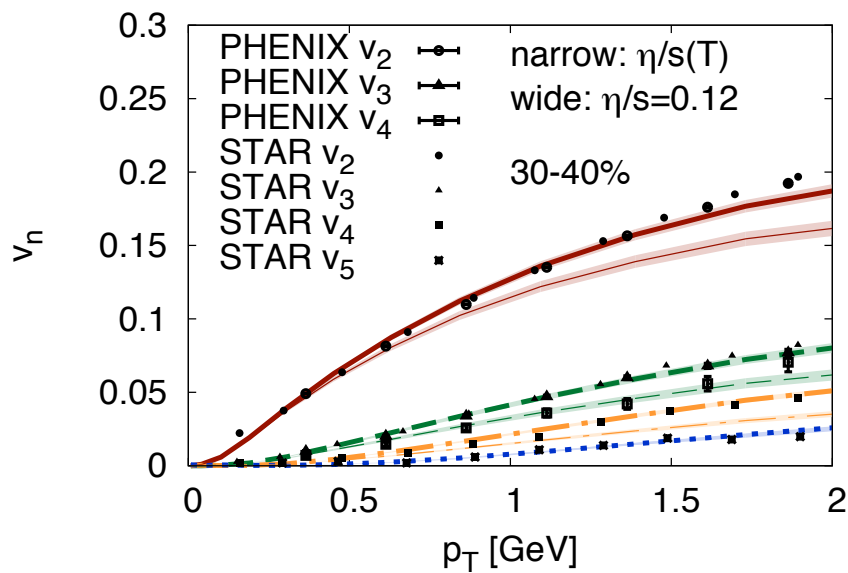
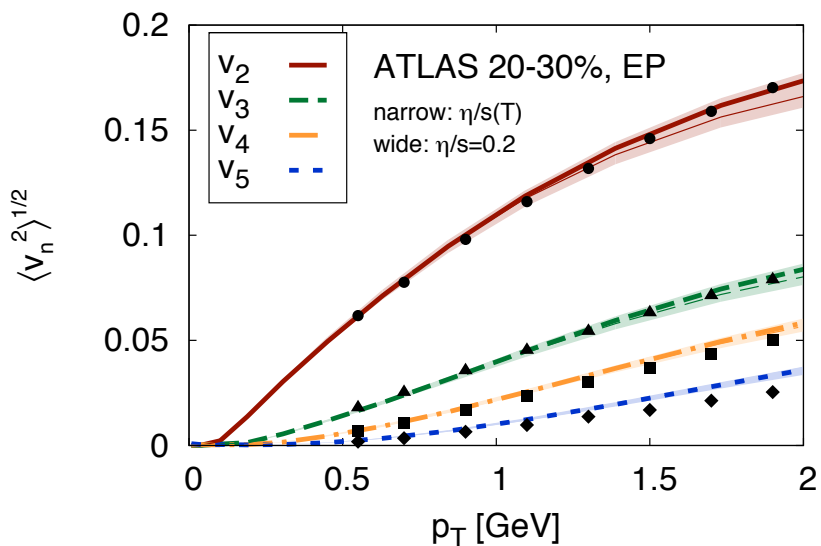
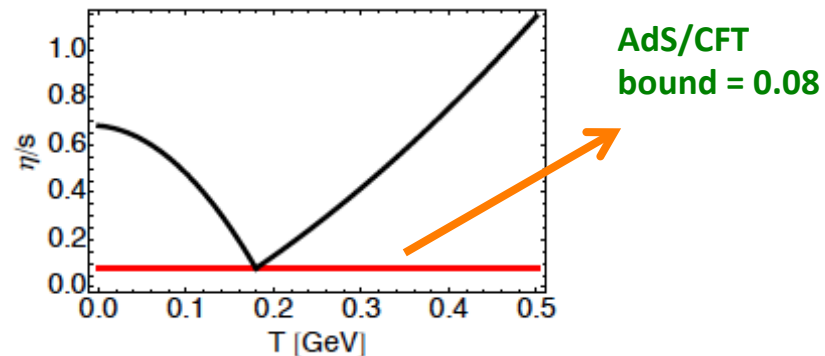
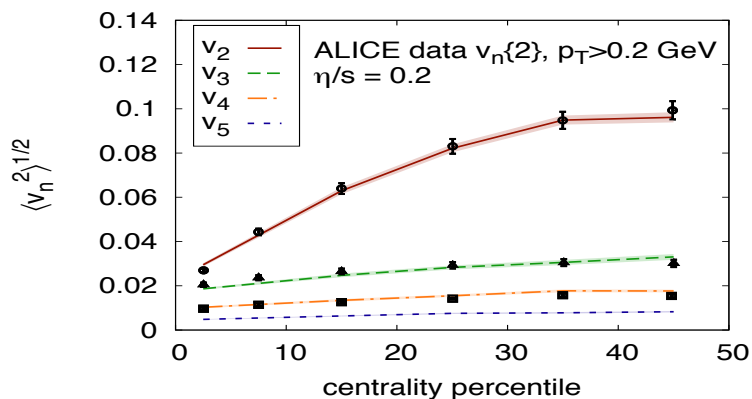
**MUSIC: 3+1-D event-by-event viscous relativistic hydro model**

Schenke, Jeon, Gale

# Heavy ion phenomenology in weak coupling

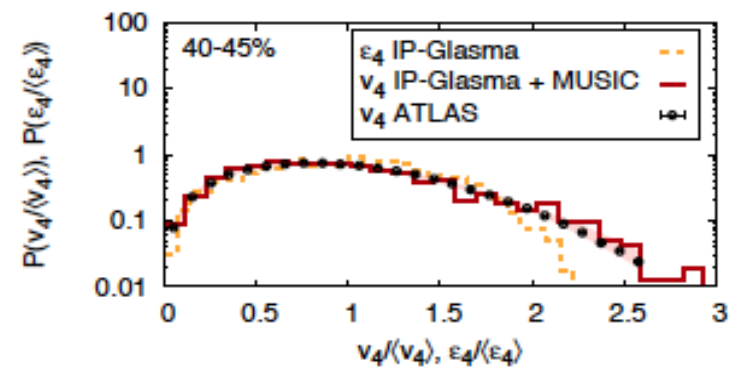
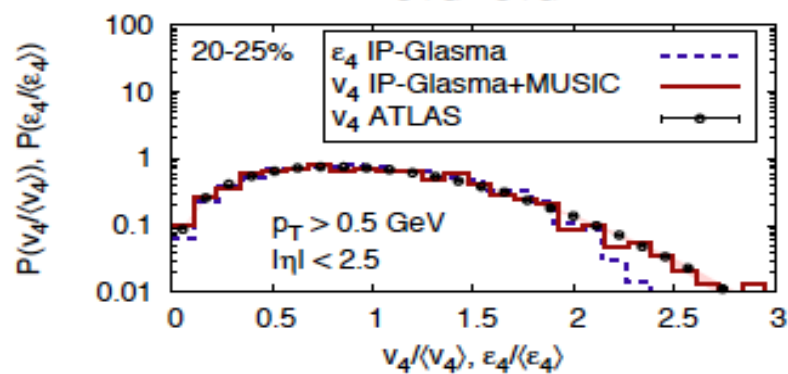
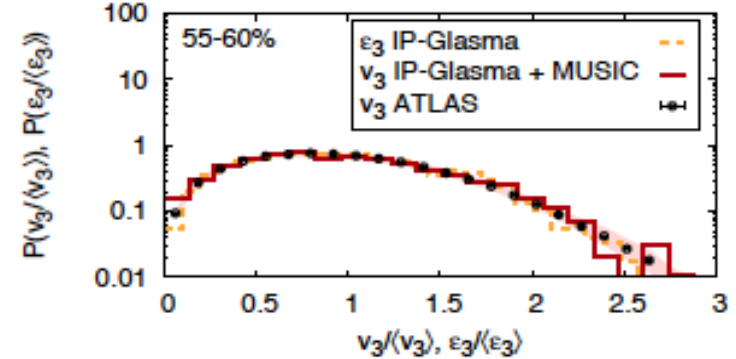
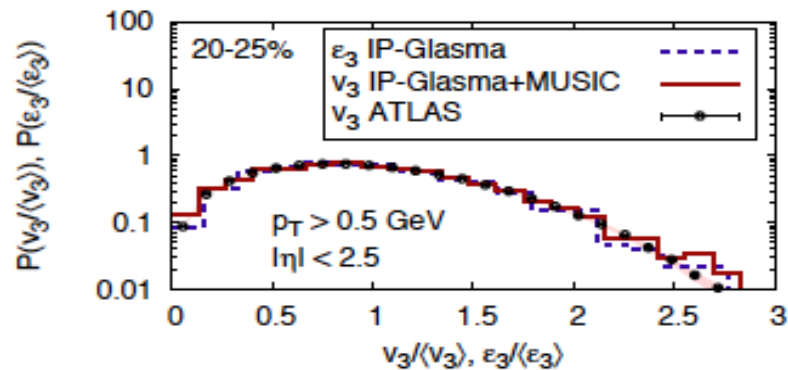
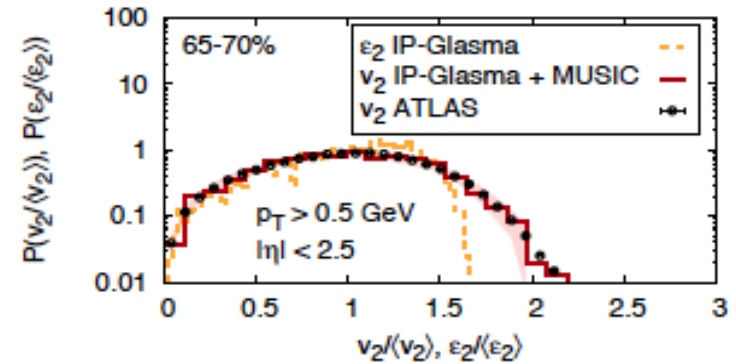
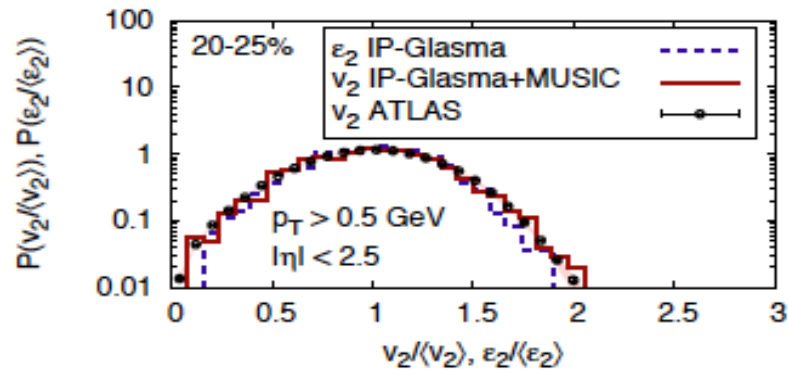
Gale, Jeon, Schenke, Tribedy, Venugopalan, PRL (2013) 012302

Results from the IP-Glasma +MUSIC model:



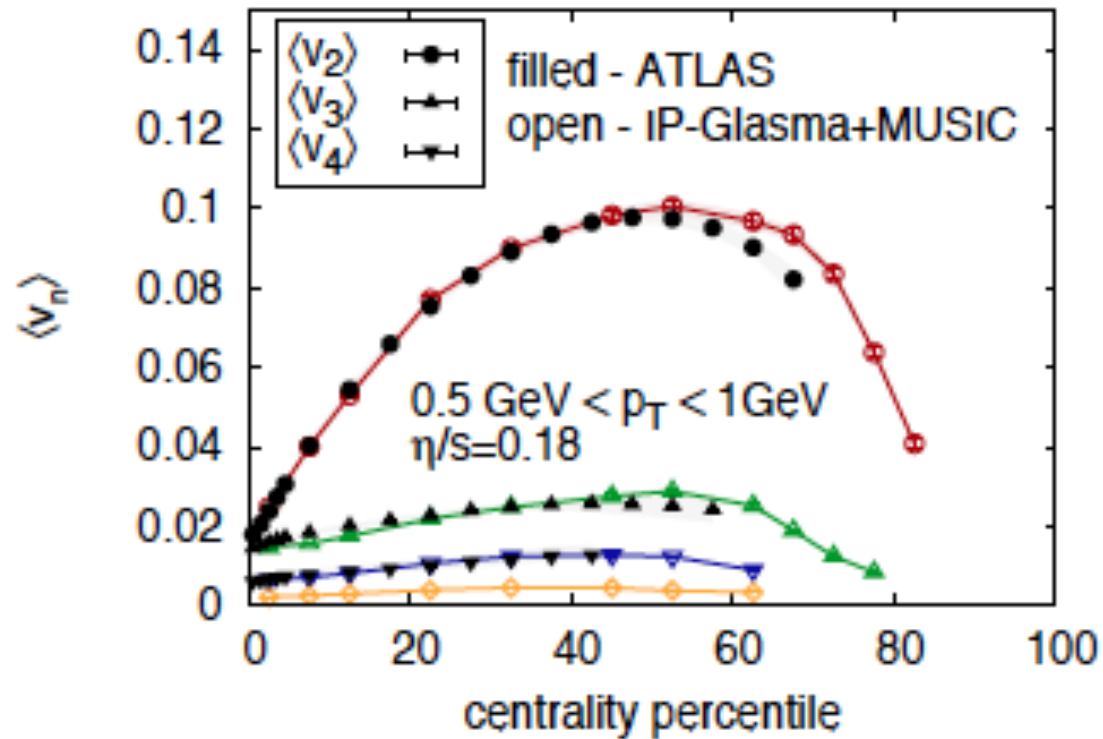
RHIC data require lower average value of  $\eta/s$  relative to LHC

# Heavy ion phenomenology in weak coupling



# Heavy ion phenomenology in weak coupling

Schenke, Venugopalan, arXiv:1405.3605

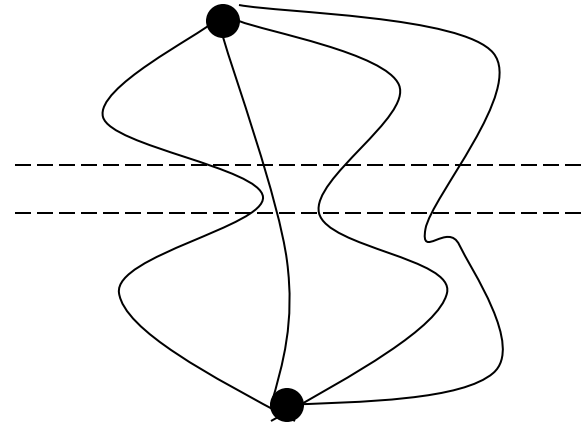


Remarkable agreement of IP-Glasma+MUSIC with data out to fairly peripheral overlap geometries...



**BACKUP SLIDES**

## PATH INTEGRAL:



Coarse graining  $\rightarrow$  Box of size  $1/p_t$  in transverse plane

### Sum over spins in box:

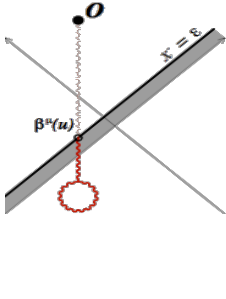
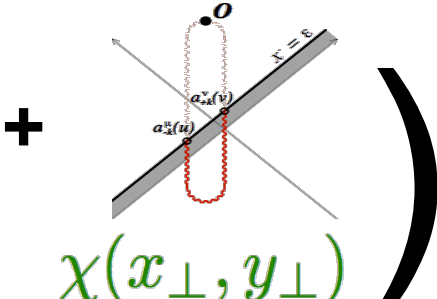
$$\sum_l v_l^{(k)} \sum_{m=-l}^l |l, m\rangle \langle l, m| \rightarrow \int d^3l e^{-2l^2/k}$$

### Classical color/spin density:

$$l^a = (\Delta x_\perp)^2 \frac{1}{g} \rho^a(x_\perp) \Rightarrow 2 \frac{l^2}{k} = \frac{\pi R^2}{g^2 A} (\Delta x_\perp)^2 \rho^a \rho^a$$

## JIMWLK RG evolution for a single nucleus:

$$\mathcal{O}_{\text{NLO}} = \left( \text{Diagram 1} + \text{Diagram 2} \right) \mathcal{O}_{\text{LO}}$$

$$= \ln \left( \frac{\Lambda^+}{p^+} \right) \mathcal{H} \mathcal{O}_{\text{LO}} \quad (\text{keeping leading log divergences})$$

$$\begin{aligned} \langle \mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}} \rangle &= \int [d\tilde{\rho}] W[\tilde{\rho}] [\mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}}] \\ &= \int [d\tilde{\rho}] \left\{ \left[ 1 + \ln \left( \frac{\Lambda^+}{p^+} \right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\text{LO}} \end{aligned}$$

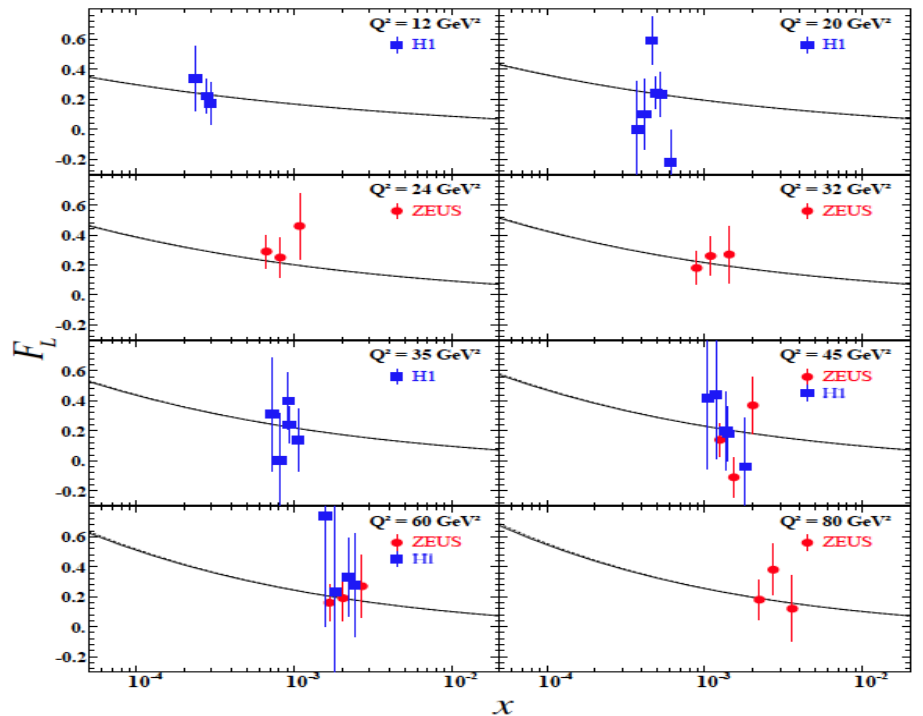
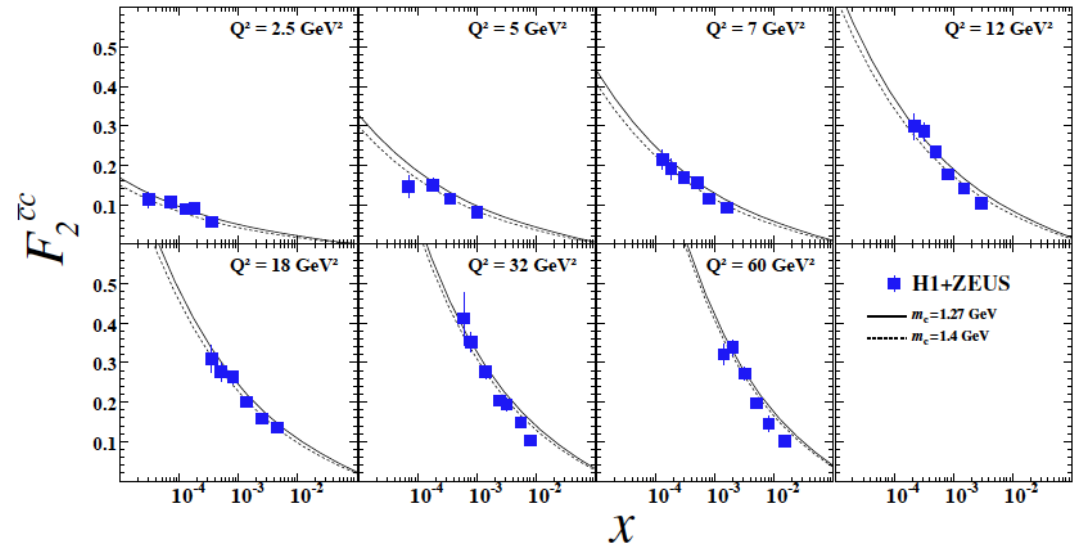
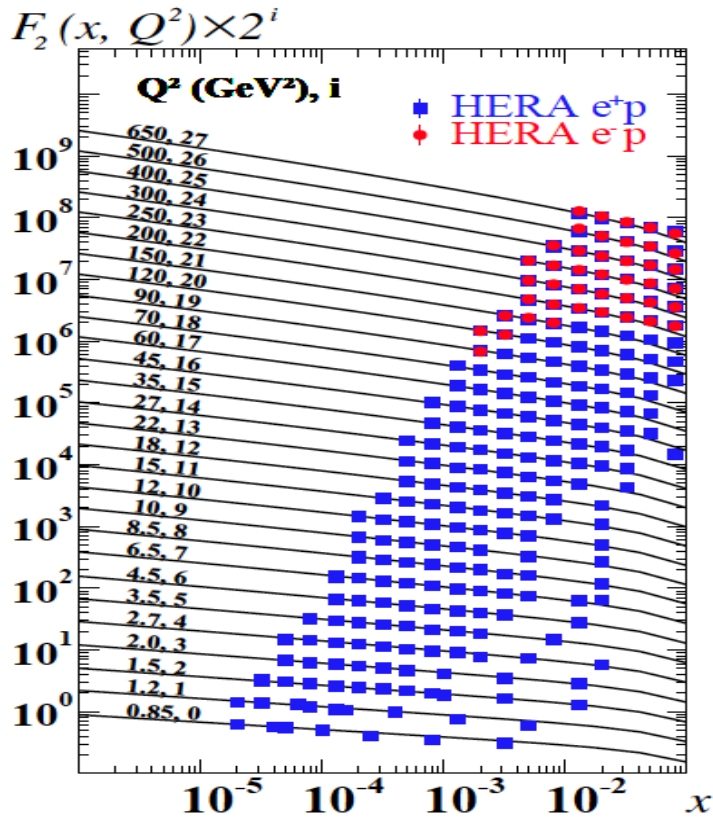
LHS independent of  $\Lambda^+ \Rightarrow$

$$\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H} W[\tilde{\rho}]$$

JIMWLK eqn.

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

# Inclusive DIS: dipole evolution a la IP-Sat



(Few) parameters fixed by  
 $\chi^2 \sim 1$  fit to combined  
 (H1+ZEUS) red. cross-section

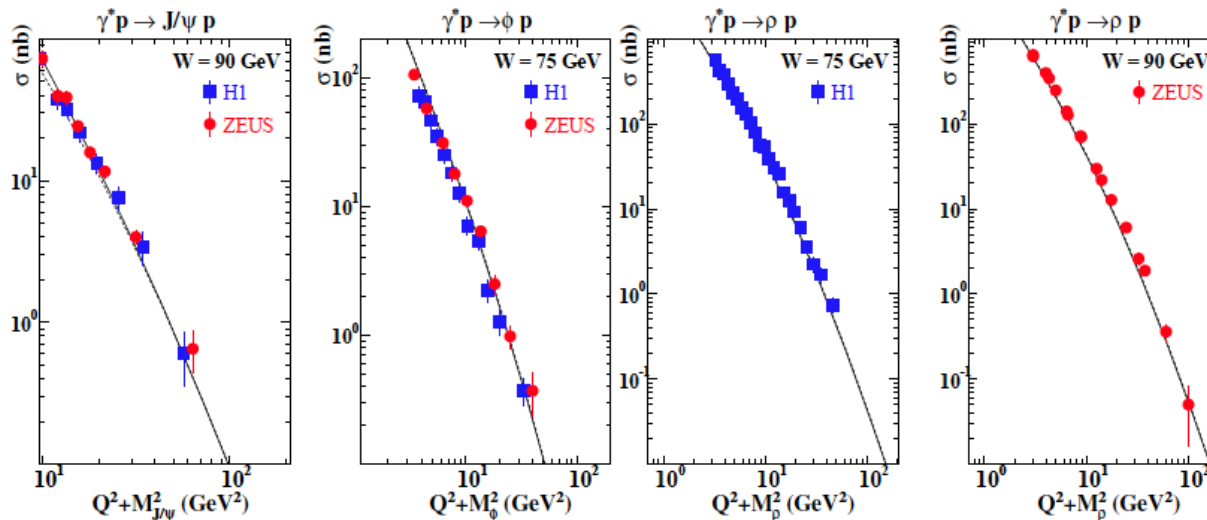
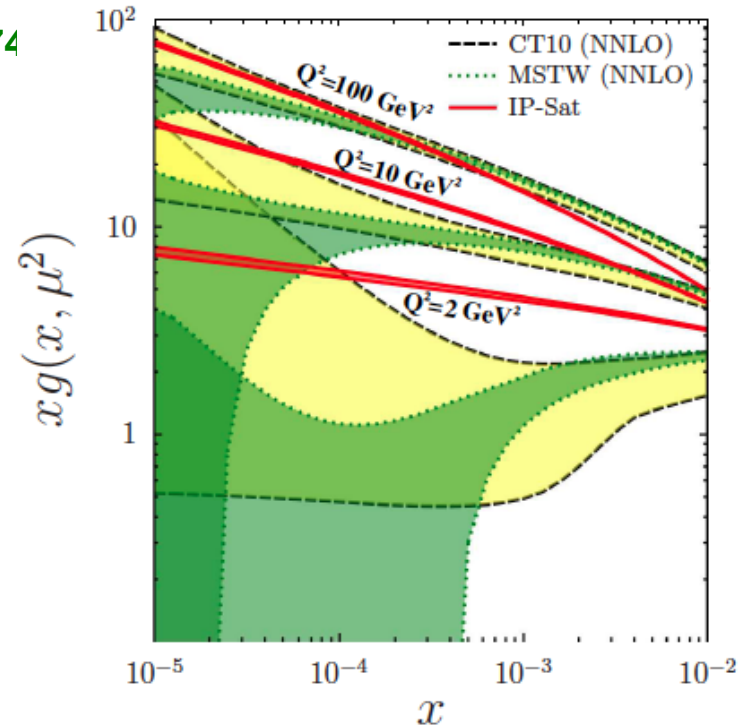
Rezaiean, Siddikov, Van de Klundert, RV: 1212.2974

# Inclusive DIS: dipole evolution a la IP-Sat

Rezaiean, Siddikov, Van de Klundert, RV: 1212.2974

More stable gluon dist. at small  $x$  relative to NNLO pdf fits

Exclusive Vector meson production:



Comparable quality fits for energy ( $W$ ) and  $t$ -distributions