Introduction to and Recent Progress in Lattice QCD

Z. Fodor

University of Wuppertal

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thermodynamics and T=0 physics

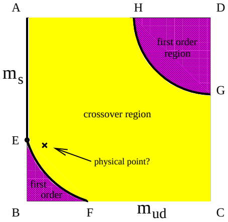
examples to reach the physical limit (physical mass & continuum)

SFB TR55

Outline

1. Nature of the transition
2. Transition temperature
3. Equation of state
4. Non-vanishing chemical potential
5. Fluctuation
6. Summary

Phase diagram and its uncertainties



physical quark masses: important for the nature of the transition

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

intermediate quark masses: we have an analytic cross over (no χPT)

F.Karsch et al., Nucl.Phys.Proc. 129 ('04) 614; G.Endrodi et al. PoS Lat'07 182('07);

de Forcrand, S. Kim, O. Philipsen, Lat'07 178('07)

continuum limit is important for the order of the transition:

n_f=3 case (standard action, N_t=4): critical $m_{ps}\approx$300 MeV

different discretization error (p4 action, N_t=4): critical $m_{ps}\approx$70 MeV

the physical pseudoscalar mass is just between these two values

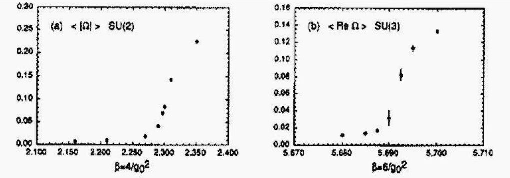
Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies

Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)

existence of a transition between confining and deconfining phases:

Polyakov loop exhibits rapid variation in a narrow range of β



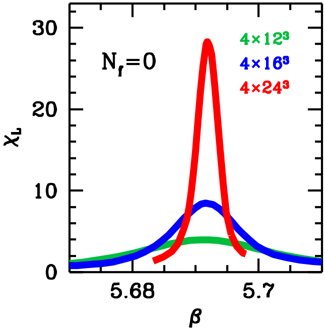
- theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

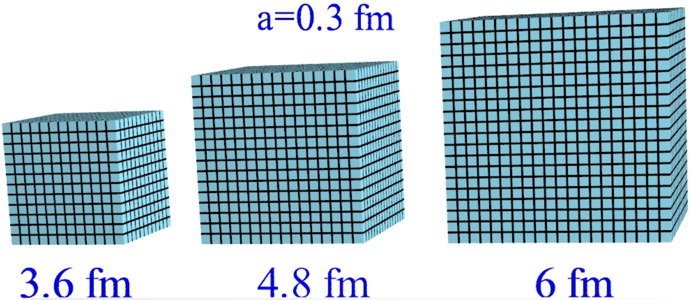
Finite size scaling in the quenched theory

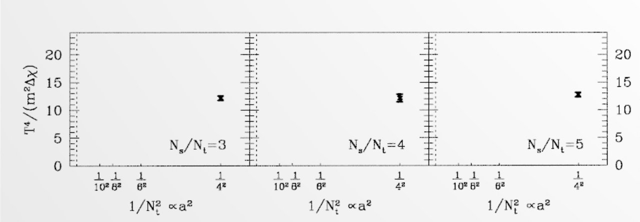
look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

Approaching the continuum limuit





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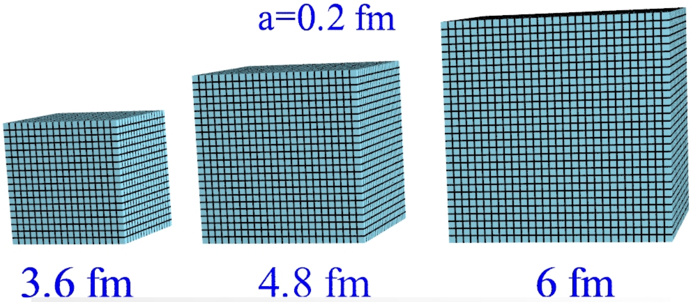

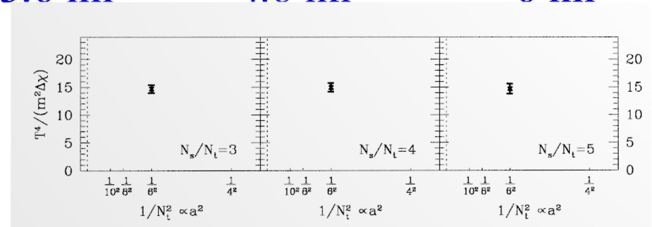

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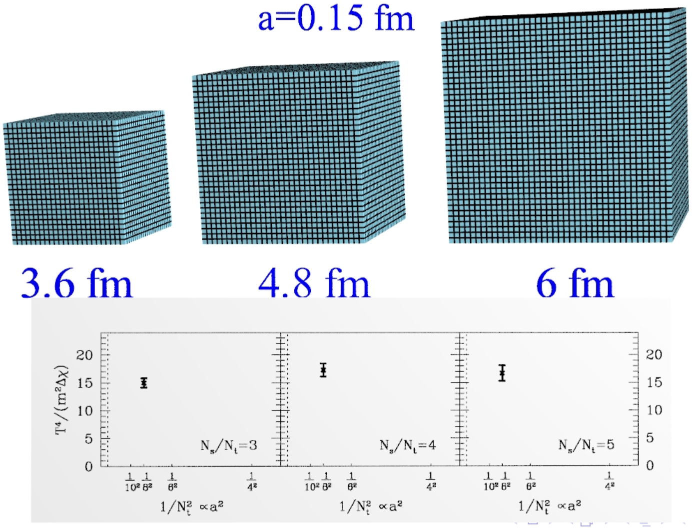

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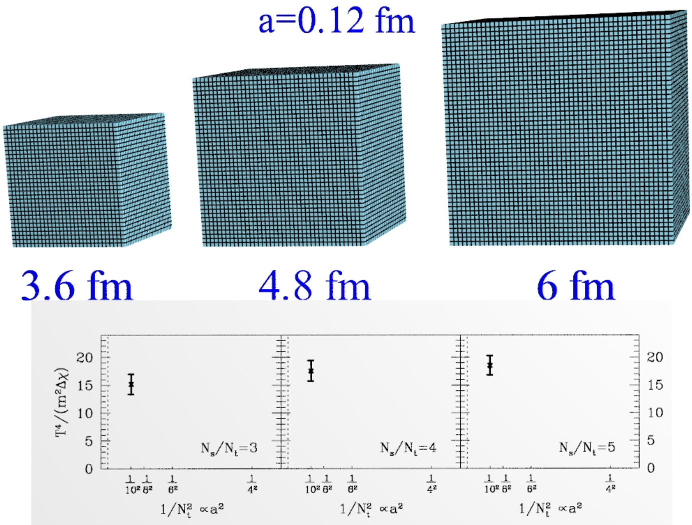

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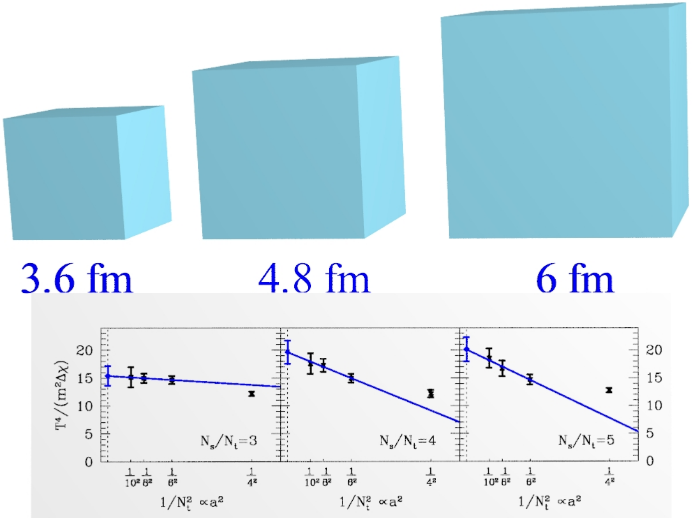
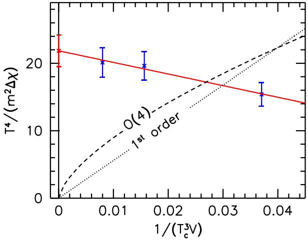


The nature of the QCD transition: analytic

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

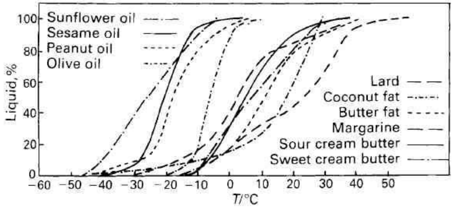
the QCD transition is a cross-over

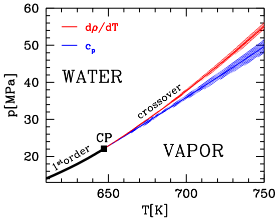
Wuppertal-Budapest Collaboration: Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

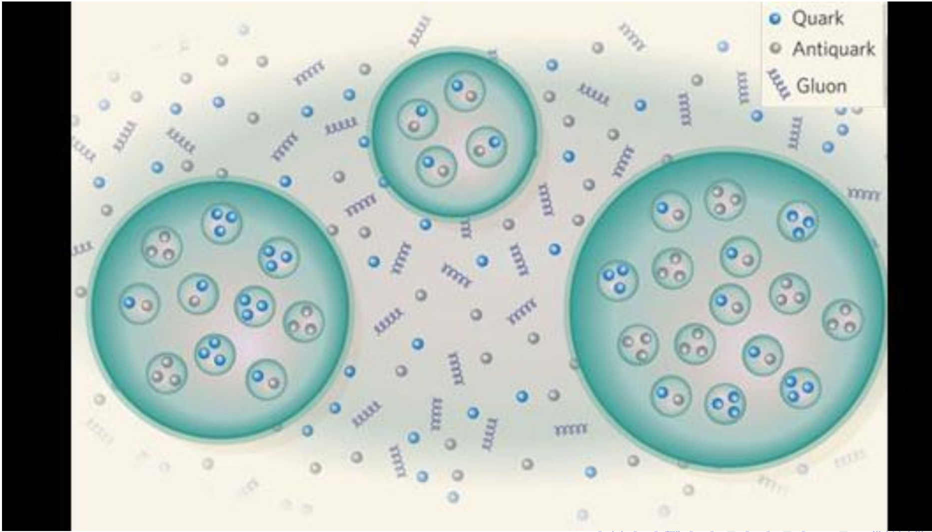
analytic transition (cross-over) ⇒ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition



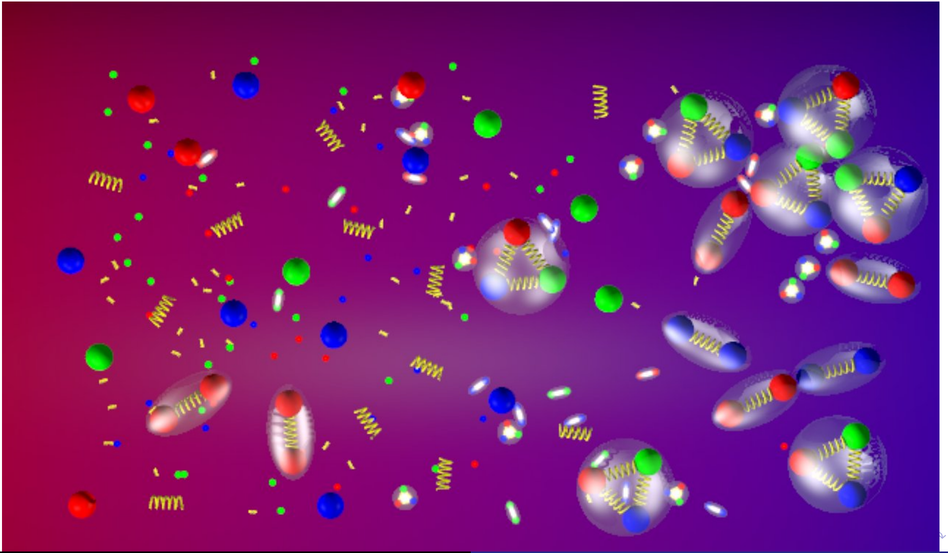


above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values ⇒ physical difference

Possible first order scenario with critical bubbles



Reality: smooth analytic transition (cross-over)



Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Collaboration:

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Bielefeld-Brookhaven-Riken-Columbia merged with MILC: ‘hotQCD’

Wuppertal-Budapest group: WB

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

‘chiral T_c’: ≈40 MeV; ‘confinement T_c’: ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π
in 2006 freeze out: 172 MeV → dramatic differences in physics:
need for strongly interacting hadronic matter

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

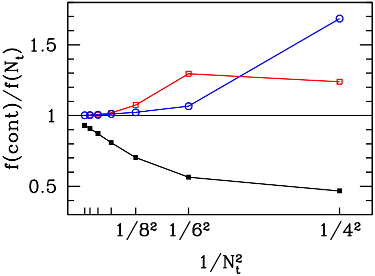
various types of discretization errors $\Rightarrow$ we improve on them (costs)

we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$<T_c$

Examples for improvements, consequences

how fast can we reach the continuum pressure at T=∞?



p4 action is essentially designed for this quantity T$\gg T_c$

asqtad designed mostly for T=0 physics (but good at high T, too)

stout-smeared one-link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

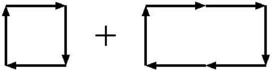
one can improve on the action (expensive) or observable level

Choice of the action

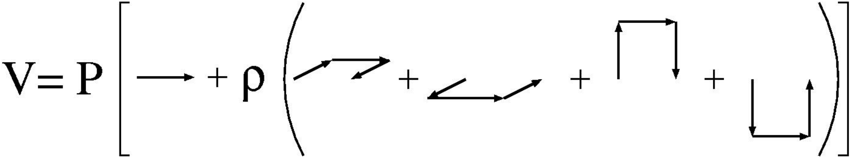
no consensus: which action offers the most cost effective approach

Aoki, Fodor, Katz, Szabo, JHEP 0601, 089 (2006) [arXiv:hep-lat/0510084]

WB choice: tree-level $O(a^2)$-improved Symanzik gauge action



multi-level (stout) smeared improved staggered/Wilson/overlap fermions



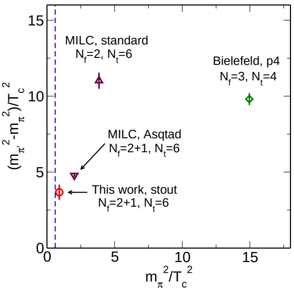
best known way to improve on taste symmetry violation

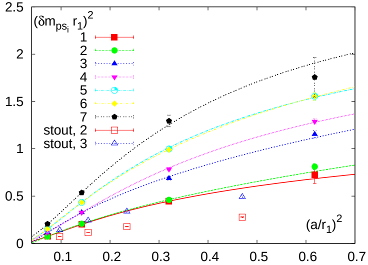
Chiral symmetry/pions

Wuppertal-Budapest: JHEP 0601 (2006) 089. [hep-lat/0510084]

transition temperature for remnant of the chiral transition:
balance between the f's of the chirally broken & symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

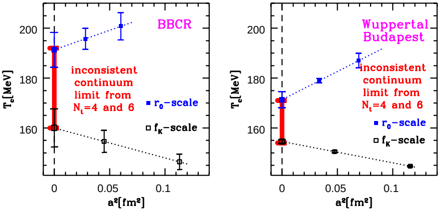
staggered QCD: 1 ($\frac{3}{16}$) pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact ⇒ splitting disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact





Consequences of the non-scaling behaviour

for large 'a' no proper a^2 scaling (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the $a \to 0$ limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition



N_t=4,6: inconsistent continuum limit
N_t=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same T_c
signal: extrapolation is safe, we are in the a^2 scaling regime

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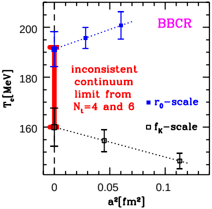

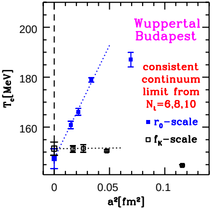

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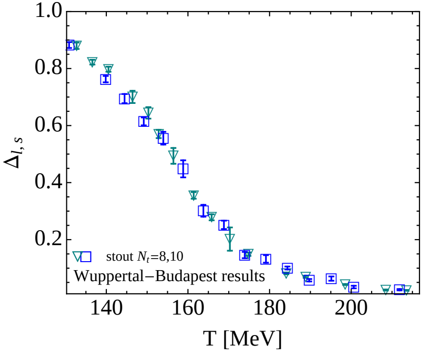
progress in the transition temperature

Wuppertal-Budapest: physical quark masses ($m_s/m_{ud} \approx$28)
gauge configs: N_t=8,10 in 2006 $\Rightarrow$ N_t=12 in 2009 $\Rightarrow$ N_t=16 in 2010

hotQCD 2009: realistic quark masses (m_s/m_{ud} =10)
Phys.Rev. D85 (2012) 054503: physical quark masses (m_s/m_{ud} =20)



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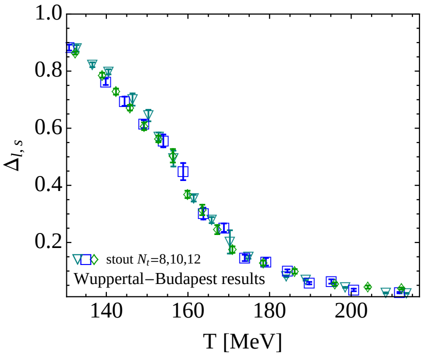

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Wuppertal-Budapest JHEP 1009 '10 73

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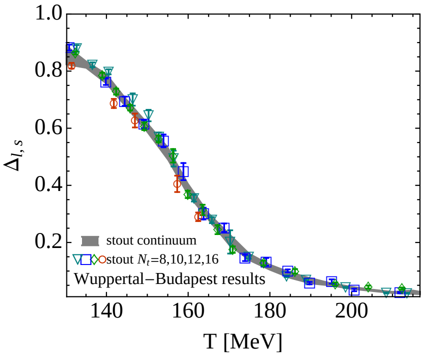

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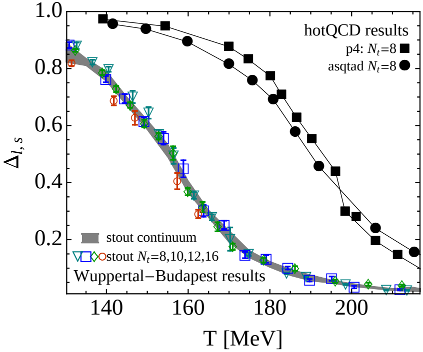

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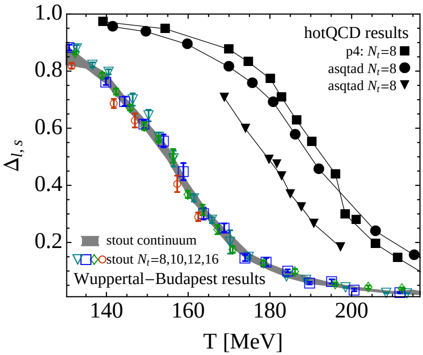

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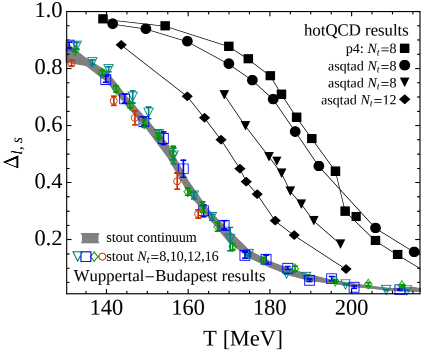

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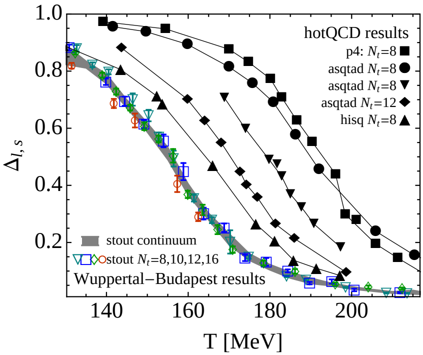


from this observable

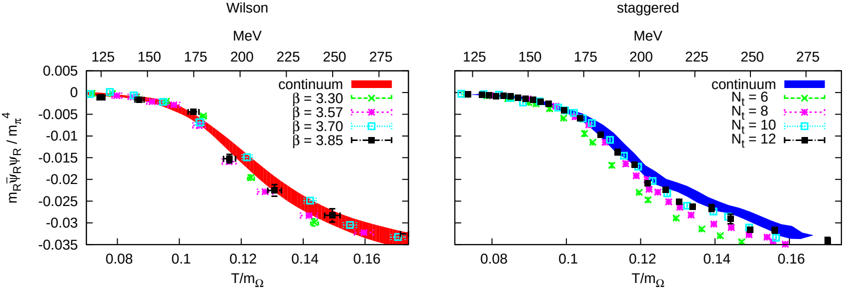
Wuppertal-Budapest
T_c=157(4)

hotQCD: hisq N_t=12
T_c=154(9)

T>0 transition with Wilson fermions S. Borsanyi et al., JHEP 1208 (2012) 126

staggered formalism has four quarks $\Rightarrow$ rooting
Wilson fermions are cleaner than staggered (more expensive)

T=1/N_t·a instead of "a" we change N_t fixed scale approach
N_s up to 64, transition up to N_t=20 with M_π ≈545 MeV



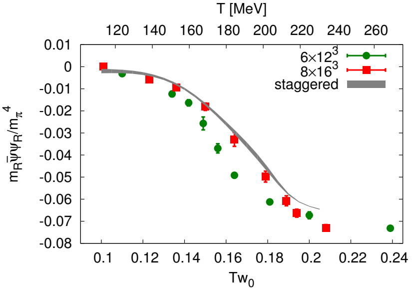
continuum extrapolated result matched with our staggered predicition
consistent picture $\Longrightarrow$ huge importance: credibility & feasibility

T>0 transition with overlap fermions

S. Borsanyi et al., PLB 713 (2012) 342

Wilson fermions are cleaner than staggered (more expensive)
overlap fermions ($\gg$ expensive): correct chiral properties (chiral T_c)

N_f=2 with $M_\pi \approx$350 MeV and N_t=6,8 (exploratory: a$\to$0 later)
chiral condensate (strange susceptibility and Polyakov loop)

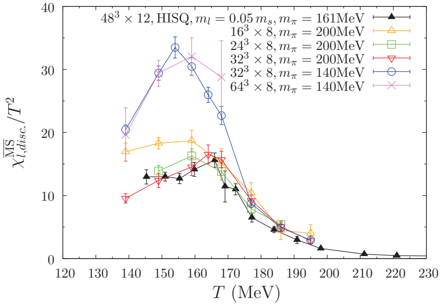


continuum extrapolated result should be matched with staggered
consistent picture $\Longrightarrow$ huge importance: credibility

T>0 transition with DW fermions T. Bhattacharya et al. (hotQCD) 1402.5175

Wilson fermions are cleaner than staggered (more expensive)
domain wall ($\gg$ expensive): better chiral properties (chiral T_c)

N_f=2+1 with M_π $\approx$140 MeV and N_t=8 (exploratory: a$\to$0 later)
light quark chiral susceptibility

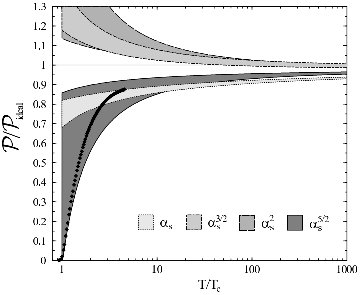


non-continuum result but matches nicely the staggered (T_c=155 MeV)
consistent picture $\Longrightarrow$ huge importance: credibility

Equation of state: difficulties at high temperatures

lattice results for the EoS extend upto a few times T_c

perturbative series "converges" only at asymptotically high T



applicability ranges of perturbation theory and lattice don't overlap
it was believed to be "impossible" to extend the range for lattice QCD

The standard technique is the integral method

$\bar{p}$=T/V·log(Z), but Z is difficult
$\Rightarrow$ $\bar{p}$ integral of (∂log(Z)/$\partial\beta$,∂log(Z)/∂m)

substract the T=0 term, the pressure is given by:
$p(T)$=$\bar{p}(T)$-$\bar{p}(T = 0)$

back of an envelope estimate:

T_c≈150–200 MeV, m_π=135 MeV
try to reach T=20·T_c for N_t=8 (a=0.0075 fm)

$\Rightarrow N_s > 4/m_\pi \approx 6/T_c = 6\cdot20/\text{T} = 6\cdot20\cdot N_t \approx 1000$

$\Rightarrow$ completely out of reach

Practical solution for the problem

G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, arXiv:0710.4197

a. substract successively:

$p(T)=\bar{p}(T)-\bar{p}(T=0)=[\bar{p}(T)-\bar{p}(T/2)]+[\bar{p}(T/2)-\bar{p}(T/4)]+...$

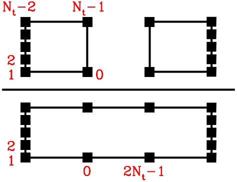
$\Longrightarrow$ for substractions at most twice as large lattices are needed (physical reason: there are no new UV divergencies at finite T)

b. instead of the integral method calculate:

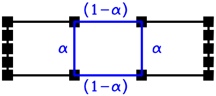
$\bar{p}(T)-\bar{p}(T/2)=T/(2V)\cdot\log[Z^2(N_t)/Z(2N_t)]$

and introduce an interpolating partition function $Z(\alpha)$

$$\frac{Z^2(N_t)}{Z(2N_t)} =$$



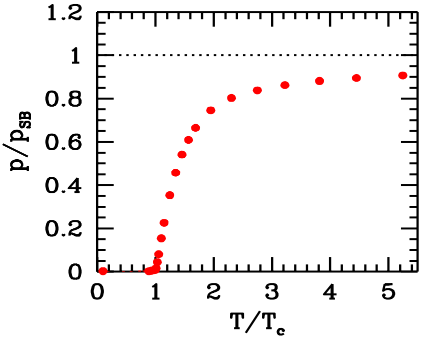
$$\overline{Z}(\alpha) =$$



define $\bar{Z}(\alpha)=\int \mathcal{D}U \exp[-\alpha S_{1b}-(1-\alpha)S_{2b}] \Rightarrow Z^2(N_t)=\bar{Z}(0), \quad Z(2N_t)=\bar{Z}(1)$

one gets directly for $\bar{p}(T)-\bar{p}(T/2)=T/(2V)\cdot\log[Z^2(N_t)/Z(2N_t)]$

$T/(2V)\int_0^1 d\log[\bar{Z}(\alpha)]/d\alpha\cdot d\alpha=T/(2V)\int_0^1\langle S_{1b}-S_{2b}\rangle\alpha\cdot d\alpha$



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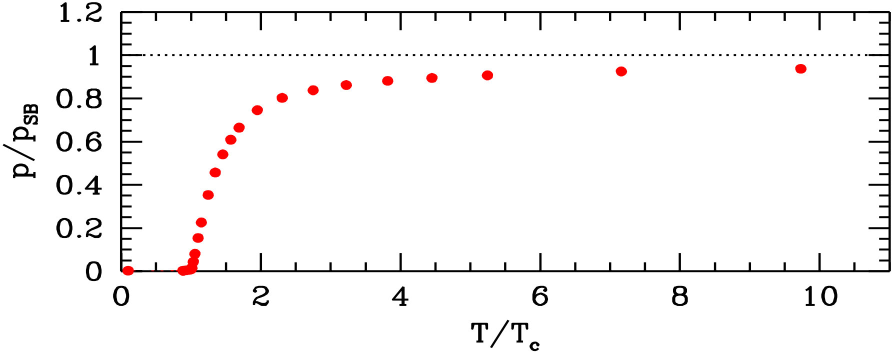

define $\bar{Z}(\alpha)=\int \mathcal{D}U \exp[-\alpha S_{1b}-(1-\alpha)S_{2b}] \Rightarrow Z^2(N_t)=\bar{Z}(0), \quad Z(2N_t)=\bar{Z}(1)$

one gets directly for $\bar{p}(T)-\bar{p}(T/2)=T/(2V)\cdot\log[Z^2(N_t)/Z(2N_t)]$

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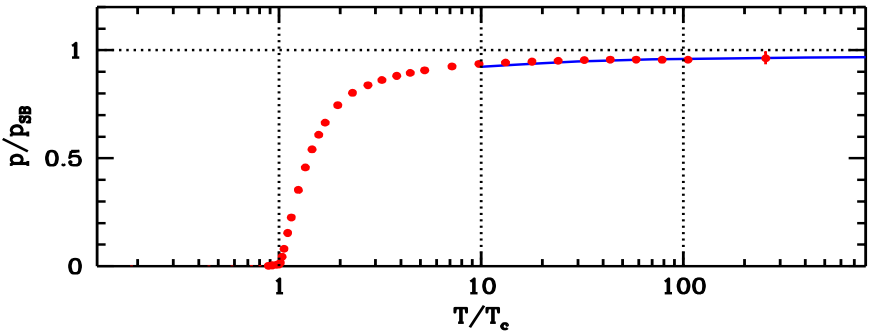

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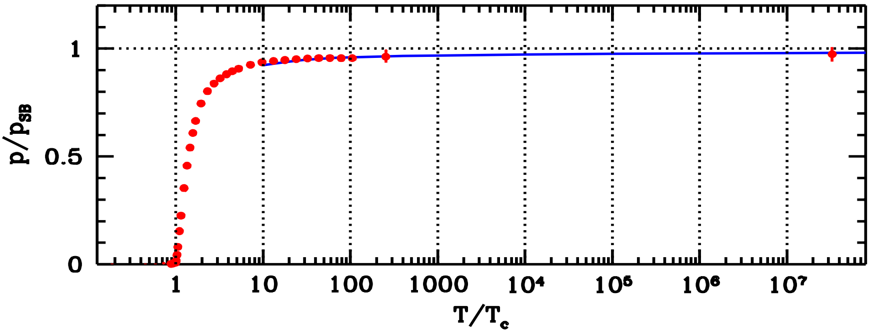

long awaited link between lattice thermodynamics and pert. theory

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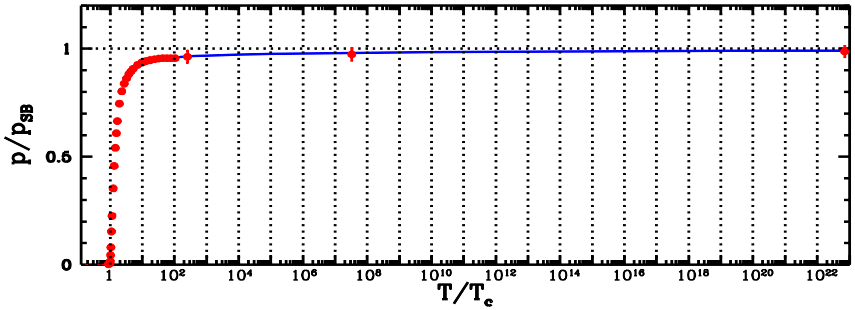


long awaited link between lattice thermodynamics and pert. theory

Equation of state for $T \gg T_c$ S. Borsanyi et al., JHEP 1207 (2012) 056

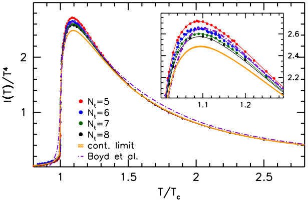
high temperatures are/were not accessable by lattice simulations

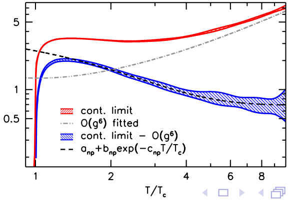
1. earlier T/T_c approx 3-4, but T_c is smaller & LHC energy larger
2. difficult to make connection between lattice and perturbation theory

perturbative series converges only slowly (comes from pure gauge)

solution: technique based on “no new divergencies appear at T>0”

continuum result for the pure gauge EoS up to $1000 \cdot T_c$ (*full QCD)
low T region, around T_c, up to $5T_c$ and $T \gg T_c$
description for all T (different theoretical rigor and accuracy)





Equation of state: Wuppertal-Budapest: JHEP 0601 (2006) 089; 1011 (2010) 77; PLB 370 (2014) 99

Integral method: J. Engels et al., Phys. Lett. B252 (1990) 625
on the lattice the dimensionless pressure is given by

$$p^{\text{lat}}(\beta, m_q) = (N_t N_s^3)^{-1} \log \mathcal{Z}(\beta, m_q)$$

not accessible using conventional algorithms, only its derivatives

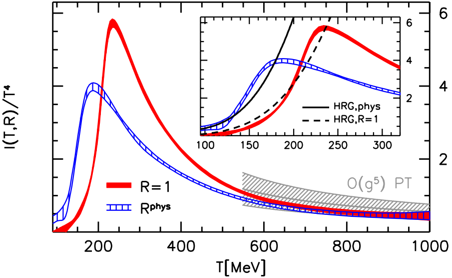
$$p^{\text{lat}}(\beta, m_q) - p^{\text{lat}}(\beta^0, m_q^0) = (N_t N_s^3)^{-1} \int_{(\beta^0, m_q^0)}^{(\beta, m_q)} \left(d\beta \frac{\partial \log \mathcal{Z}}{\partial \beta} + dm_q \frac{\partial \log \mathcal{Z}}{\partial m_q} \right)$$

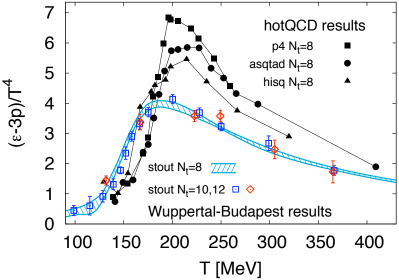
first term: gauge action & second term: chiral condensate

the pressure has to be renormalized: subtraction at T=0 (or T>0)

T≠0 simulations can't go below T≈100 MeV (lattice spacing is large)
physical HRG gives here 5% contribution of SB ⇒
path starts at M_π ⇒ distorted HRG no contribution at our T

Equation of state: I(T)=ϵ-3p Borsanyi et al., JHEP 1011 (2010) 77



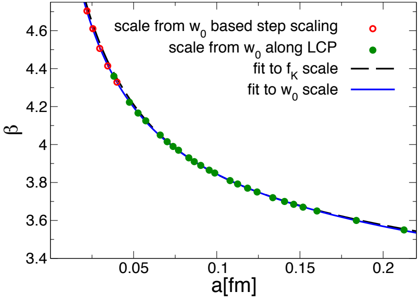


two pion masses: $M_\pi \approx 720$ MeV (R=1) and $M_\pi = 135$ MeV (R^{phys})
good agreement with the HRG model up to the transition region
quark mass dependence disappears for high T
good agreement with perturbation theory

comparison with the published results of the hotQCD collaboration
discrepancy: higher peak ≈70, 50, 40%

Comparison of LCPs given by f_K (old) and w_0 (new)

old LCP: fixing f_K/M_π and m_s/m_{ud} to their physical values
new LCP: using w_0 and the step scaling method



difference is included in the systematic error of EoS
since we know the m_q dependence of the EoS it is also included

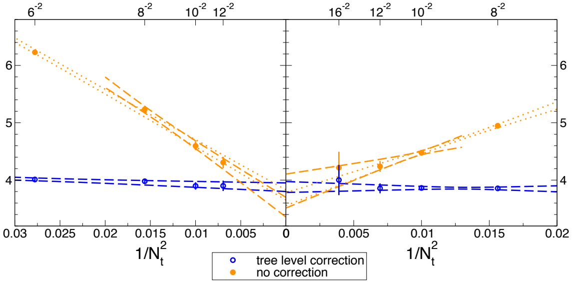
Special care for the peak

main discrepancy with hotQCD is the height of the peak

a. extend the lattice spacing set to N_t=16

b. completely independent cross check: use a new action

other action, other parameters, other LCP



complete agreement between the two actions

with/without tree level improvement or including coarse points

Finite renormalization for the pressure

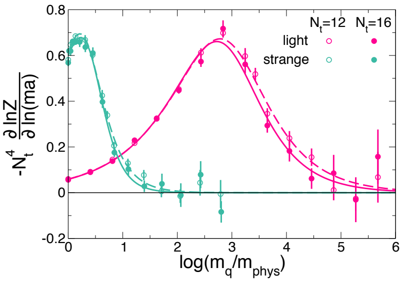
in large homogenous systems $p/T^4 = N_t^3/N_s^3 logZ$

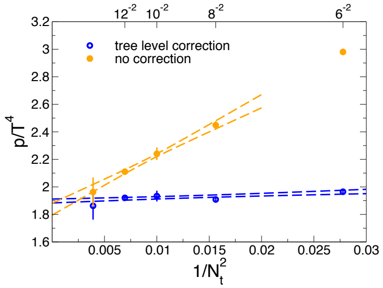
Z is hard to determine: calculate derivatives and integrate

our choice: integrate in the quark masses along fixed β

for each N_t they correspond to $T_* = 214$ MeV at the physical point

(starting point: infinitely heavy m_q deep in the confined phase)





gives perfect agreement with hardon resonance gas model

Error analysis

continuum results need fully controlled systematic error analyses
considered various fit methods (each could be correct)

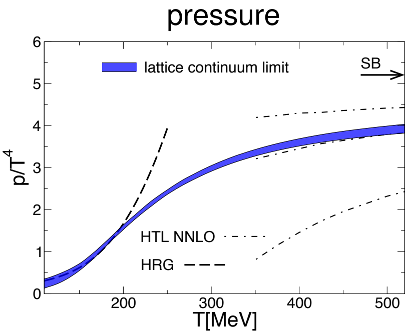
a. four basic types of continuum extrapolation
(with/without i. tree level improvement ii. a^4 term)
b. two continuum extrapolation ranges (with/without N_t=6)
c. seven ways of subtraction (direct or interpolations)
d. two scale setting methods (f_K or w_0)
e. eight options choosing among various spline functions for $\epsilon - 3p$

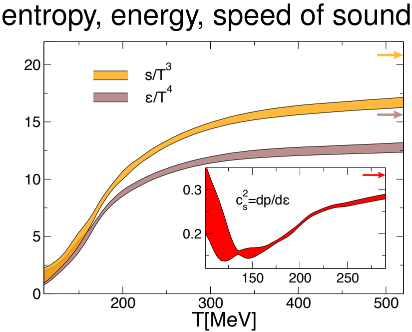
$\Rightarrow$ 4·2·7·2·8=896 methods

calculate the goodness of fit Q and/or various other weights (AIC)
construct a histogram weighted by these weights

Final results & HRG comparison

S. Borsanyi et al. Wuppertal-Budapest Coll., Phys. Lett. B730 (2014) 99

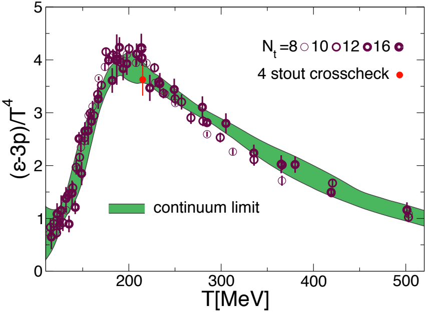




perfect agreement with HRG (also for the energy, entrupy, etc.)
HTL: 3 different renormalization scales (πT, $2\pi T$, $4\pi T$)

Trace anomaly continuum result

all of our point with various lattice spacings
using our second, independent action gives the same height
error is obtained with our hystogram technique using 896 methods

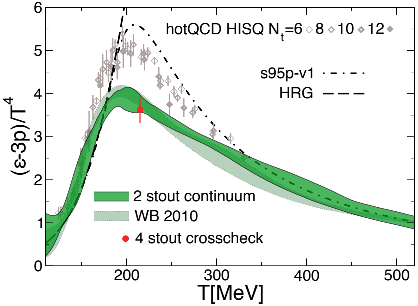


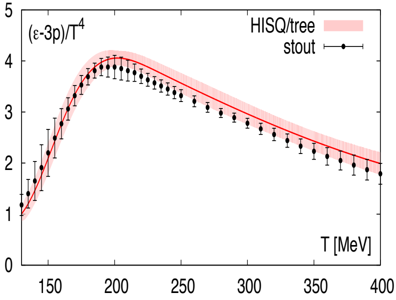
the results are unchanged since 2005 (very economic solution)

Trace anomaly continuum result

all of our point with various lattice spacings
comparison with hotQCD (which is the basis of s95p-v1)

published result ⇓ Quark Matter'14 ⇓





long standing discrepancy (since 2005) finally disappeared

Finite chemical potential: the sign problem

at μ=0 the fermion matrix is γ_5 hermitian: $M^\dagger=\gamma_5 M \gamma_5$
easy to check $\Longrightarrow$ eigenvalues: either real or conjugate pairs
det(M) is real, which is not true any more for non-vanishing μ

importance sampling (algorithms) for complex det(M) does not work

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g) \det(M[U'])/\det(M[U])\right]$$

sign problem $\Rightarrow$ from 2001 new methods to go to μ>0

Fodor-Katz: multiparameter reweighting (hep-lat/0104001, PLB)
Bielefeld-Swansee: det(M) Taylor expanded (hep-lat/0204010, PRD)
de Forcrand-Philipsen: imaginary μ (hep-lat/0205016, Nucl.Phys.B)
D'Elia-Lombardo: imaginary μ (hep-lat/0209146, PRD)

the three methods look different, they are essentially the same

Overlap improving multi-parameter reweighting

one wants to calculate the following path integral

$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha, U)] \det M(U, \alpha)$$

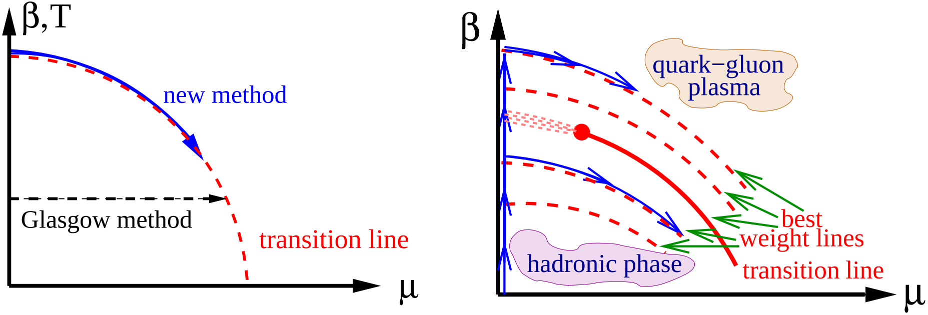
α: parameter set (gauge coupling, mass, chemical potential)
for some parameters α_0 importance sampling can be done

$$Z(\alpha) = \int [dU] \exp[-S_{bos}(\alpha_0, U)] \det M(U, \alpha_0)$$
$$\{\exp[-S_{bos}(\alpha, U) + S_{bos}(\alpha_0, U)] \det M(U, \alpha) / \det M(U, \alpha_0)\}$$

first line: measure; curly bracket: observable (will be measured)
e.g. transition configurations are mapped to transition ones

reweighting factor (ratio of the determinants) can be expressed by the eigenvalues of the (reduced) fermion matrix: closed formula for any μ

Compare with Glasgow (Ferrenberg-Swendsen)



Glasgow method ⇒ multiparameter reweighting
single parameter (μ) ⇒ two parameters (μ and β)
purely hadronic ⇒ transition configurations

map transition configurations to transition ones

Equivalence of the methods (formal/numerical)

(recent lattice review at μ=0 and μ>0: Fodor-Katz 0908.3341)

det(M) can be given by the eigenvalues of M' (transformed) at μ=0

$$\det M(\mu) = e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i)$$

observable at μ>0 or μ_I is given by the observable and λ_i at μ=0

$$Pl(\beta,\mu) = \langle Pl \exp[\Delta\beta Pl] e^{-3V\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i) \rangle$$

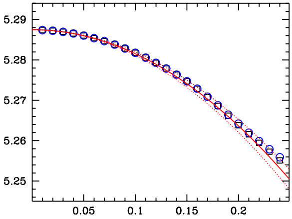
det(M) or Pl(β,μ) can be trivially Taylor expanded (Bielefeld-Swansee)
termination of the series & stochastic determination of the coefficients
$\Longrightarrow$ do not expect this method to work for as large μ as the full one

det(M)$>$0 for imaginary μ: impartance sampling still works
determine the phase line $T_c(\mu_I)$ (e.g. use a quadratic/quartic fit)
plug real μ into the same quadratic/quartic function: $c_2\mu^2$+ $c_4\mu^4$
formally: numerical determination of the (μ^2,μ^4) Taylor coefficients

Equivalence of the methods (formal/numerical)

$\Rightarrow$ for moderate μ Taylor and μ_I agree with reweighting

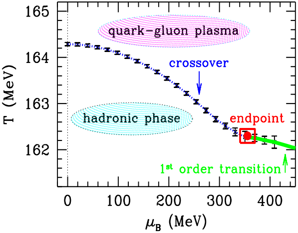
take n_f=2 setting of de Forcrand-Philipsen: $\beta_c(\mu)$ upto 4 digits

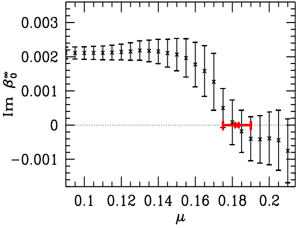


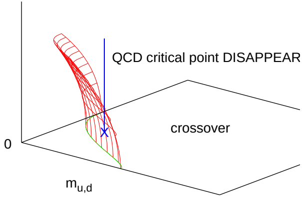
solid/dotted: imaginary μ & error; box: reweighting; circle: Taylor
for larger μ values higher order terms are getting more important

what to choose (depends on the question):
for this particular case imaginary μ has the largest CPU demand;
next one is reweighting; cheapest is Taylor (does not work for large μ)

Critical endpoint discussion (controversy?)







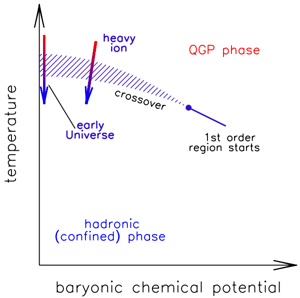
all results are from coarse lattices (a=0.3 fm, read our abstract!)

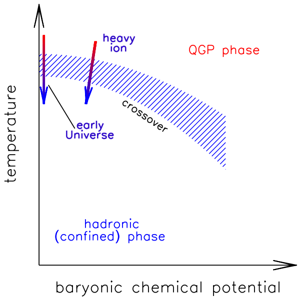
deForcrand-Philipsen: leading order $\Rightarrow$ not stronger, slightly weaker
same from reweighting: $\mu_I/T \approx 1$–3 (μ_{crit}: result of the higher orders)

Taylor & radius of convergence (!) only a lower bound: Lee-Yang

full answer (all the way to the continuum) needs much more CPU

Possible scenarios





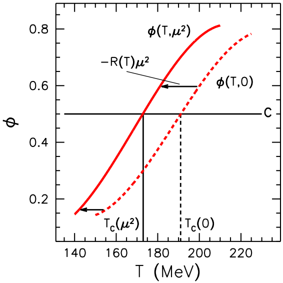
phase diagram with a transition growing stronger
even turning into a first-order phase transition at a critical endpoint

weakening transition and no critical endpoint

here we calculate the first non-trivial term: physical mass & a$\rightarrow$0
(we do not expect any conclusion to the critical endpoint)

The curvature

we change μ and look at the transition curve
it shifts to the left, we look at its value of a fixed C

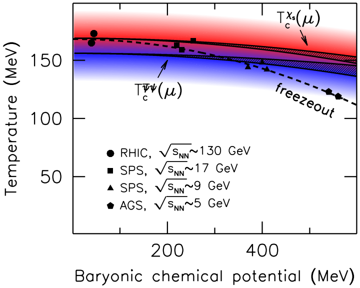


the dimensionless curvature is defined as $\kappa(T) = -T_c(\mu = 0) \cdot R(T)$

$d\kappa/dT$ at T_c tells if the transition is broadening or narrowing
(a point below T_c has a larger or smaller curvature)

Continuum prediction for the curvature: full result

G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, JHEP 1104 2011 001



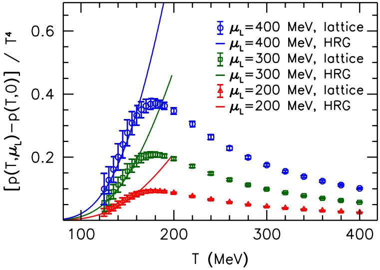
dashed line: freeze-out curve from experiments

lower solid line: T_c from the chiral condensate
upper solid line: T_c from the strange susceptibility

bands (red and blue) indicate the widths of the transition lines
the widths remain in this order approximately the same

Equation of state for $\mu > 0$ S. Borsanyi et al., JHEP 1208 (2012) 053

non-vanishing chemical potential is difficult (sign problem)
recent techniques: reweighting, Taylor, imaginary chemical potential
determine the EoS up to μ^2: physical quark masses & a$\rightarrow$0



for low temperatures good agreement with the HRG model
curvature from the EoS is somewhat larger than chiral condensate
full parametrization is provided

high statistics in the Taylor method

determining the T dependence needs 10-times more statistics than just one single temperature point
this gives more than just the inflection point
a clear signal for broadening or shrinking can be seen

$a \rightarrow 0$ could have been done with present resources

the Taylor procedure gives only the leading order term(s) in μ
N_t=4 unimproved staggered experience [Fodor-Katz'01, Fodor-Katz'04]
the leading order terms are insensitive to the critical point $\Rightarrow$
evaluation of the whole determinant, we need all the terms in μ

our action (smeared improved): μ-dependent decomposition works
for p4, asqtad or hisq no such eigenvalue structure (det) is known

(it gives certainly more information than just the leading order terms)

memory/CPU requirements for full determinants

N_t=4 & N_s=8,10,12 needed 1 GB memory & 25 CPU years (in '04)
memory requirements grow as N_t^6, CPU requirements as N_t^9

accumulate the same statistics (shown by the first CPU row)
to reach the same μa: exponentially more configs are needed
'05 observation: applicability range $\propto V^{-0.35}$ and $\mu a \propto V^{-0.25}$
$\Rightarrow$ additional increase of the statistics (second CPU$^+$ row)

N_t	4	6	8	10	12
memory [GB]	1	11	64	250	750
'04 CPU [kyears]	0.025	1	13	95	500
'04 CPU$^+$ [kyears]	0.025	1	18	150	1000
machine [year]	cluster	cluster	2 BG/P	15 BG/P	100 BG/P

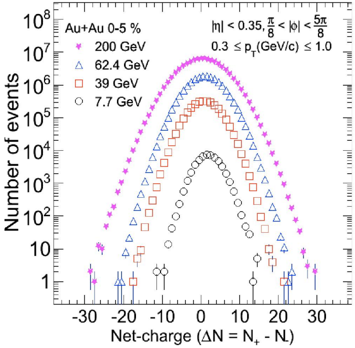
$\Rightarrow$ N_t=6,8,10: our present resources are not enough for that

Motivation

- The deconfined phase of QCD can be reached in the laboratory
- Need for unambiguous observables to identify the transition
  $\rightarrow$ fluctuations of conserved charges
  (baryon number, electric charge, strangeness)
  (Jeon and Koch, 2000, Asakawa, Heinz, Müller, 2000)
- These observables are sensitive to the microscopic structure of matter
- A rapid change of these observables in the vicinity of T_c provides an unambiguous signal for deconfinement
- They can be measured on the lattice as combinations of quark number susceptibilities

Fluctuations in experiments

what fluctuates in a heavy-ion collision?
we have a fixed number of conserved charges (Z=82, A=207)?



imposing kinematical constraints:
consider particles coming only from
a small part of the whole system

charges from subvolumes
will fluctuate
from one event to the other

small enough subvolumes to be a grand canonical ensemble
yet large enough to behave like an ensemble

Fluctuations on the lattice

grand canonical ensemble $\Longrightarrow$ fluctuations
derivatives of the partition function (respect to various μ-s)

$$\frac{\chi_{lmn}^{BSQ}}{T^{l+m+n}} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

one can define the usual moments

$$\text{mean}: \ M = \chi_1 \qquad \text{variance}: \ \sigma^2 = \chi_2$$
$$\text{skewness}: \ S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \ \kappa = \chi_4/\chi_2^2$$

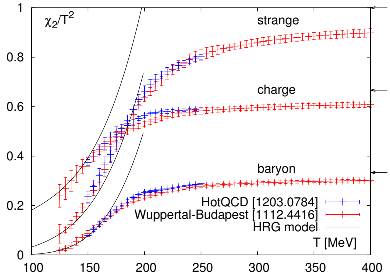
serious limitation: we do not know the volume of the subsystem
with the moments one defines volume independent ratios

$$S\sigma = \chi_3/\chi_2 \quad ; \quad \kappa\sigma^2 = \chi_4/\chi_2$$
$$M/\sigma^2 = \chi_1/\chi_2 \quad ; \quad S\sigma^3/M = \chi_3/\chi_1$$

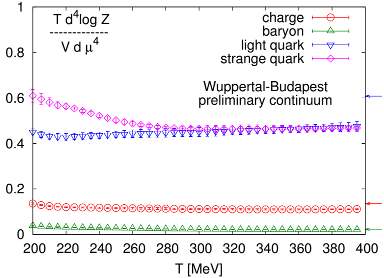
Previous results (prior the SFB's prolongation)

WB papers: 1112.4416 for second and 1210.6901 for fourth moments

compare published continuum results of Wuppertal-Budapest and hotQCD collaborations

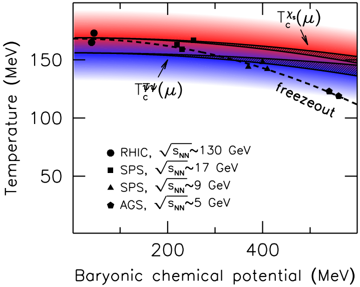


Fourth moments at high temperatures



Relate lattice and experiments

lattice QCD predicted a phase diagram (at least for small μ)
continuum result with physical quark masses
reflecting all the features of the cross-over



can we read off the temperature and baryonic chemical potential directly from experiments $\Longrightarrow$ thermometer/baryometer

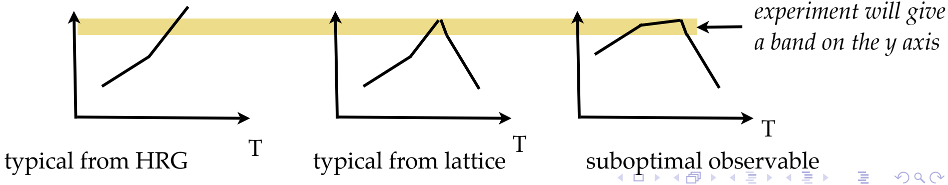
Thermometer/baryometer

older idea, new formulations Gupta et al. Science 332 (2011) 1525, Karsch CEJP 10 (2012) 1234

before freeze-out the system is described with a time-dependent temperature and baryo, charge and strange chemical potentials

assume/test: after freeze-out net bayron, charge and strangeness reflects a system in equilibrium at the freeze-out tempearture

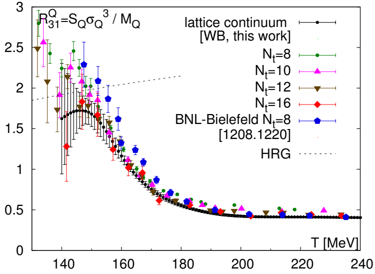
since the fluctuations T and μ dependent, one can compare experimental measurements and lattice predictions to get T and μ use ratios to eliminate the volume dependence (V is unknown)

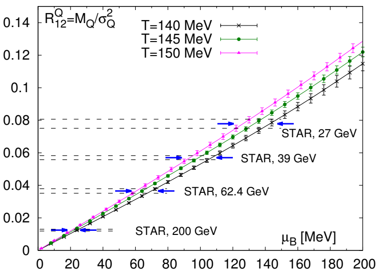


Charge fluctuations: good baryometer

Borsanyi et al. Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 062005 (2013)

skewness (third moment) and variance (second moment) ratios for Q



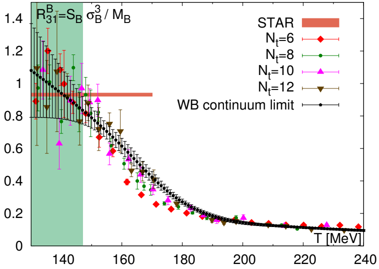


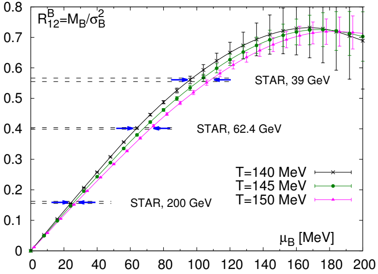
one can directly read off the temperature and chemical potential needed: experimental measurements (possibly precise)

for the first time give T_f and μ_b based on ab-initio method

Baryon fluctuations are good thermo/baryometer

skewness (third moment) and variance (second moment) ratios for B

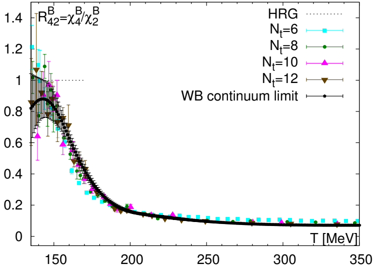




independent way to determine T and μ
baryon fluctuation are noisier but have less cut-off effects
ordering of the temperatures for Q and B are opposite
non-trivial cross-check for the unambiguity of T_f and μ_f

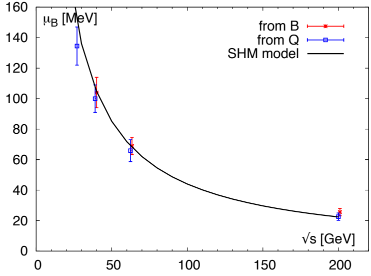
Kurtosis for baryonic fluctuations

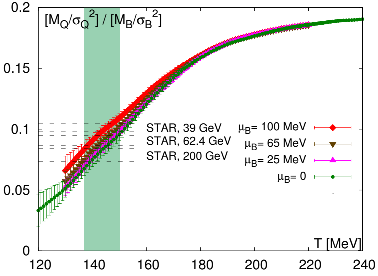
fourth moment ratio for B: $\kappa\sigma^2 = \chi_4/\chi_2$



independent determination of the freeze out temperature
essentially flat in the hadronic phase (no sensitivity)
if the freeze out happens above 150 MeV we can measure it
otherwise only upper bound for the temperature

Combining baryon and charge fluctuations



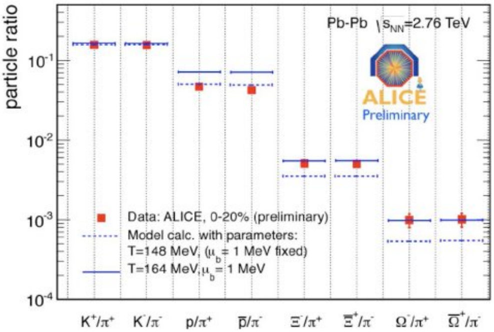


two independent ways to determine μ: complete agreement
non-trivial consistency check (can lattice be applied?) for μ_f

devide the two R_{12}-s: volume factor cancel separately
far easier to obtain both for lattice and experiment
since it does not involve skewness and kurtosis
$\Longrightarrow$ narrow temperature band instead of upper limit

Statistical hadronization models

fit to a gas of free hadrons (statistical model)
show inconistent results for strange / non-strange
yield ratios at the LHC give a difference of about 16 MeV

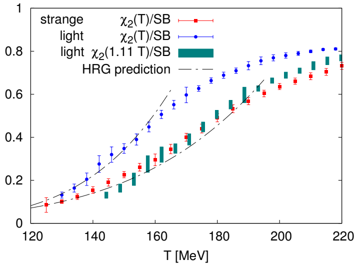


protons support a freeze out temperature of 148 MeV
Ω-s support a freeze out temperature of 164 MeV

Differences between strange and light quarks

Bellwied et al., Wuppertal-Budapest Coll. Phys.Rev.Lett. 111, 202302 (2013)

already since 2006 we observe higher T_cs for strange than for light
compare light and strange quark susceptibilities



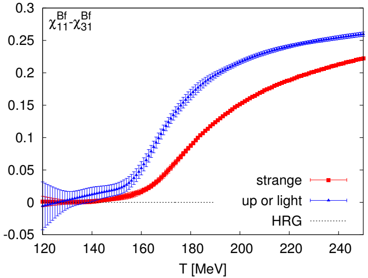
striking observation is an approximate scaling relation
$\chi_2^L(T \cdot x)$=$\chi_2^s(T)$ rescaling factor x=1.11 is preferred
independently how we determine the transition temperature
$(x-1) \cdot T_c \simeq 15$ MeV higher for s than for the light quarks

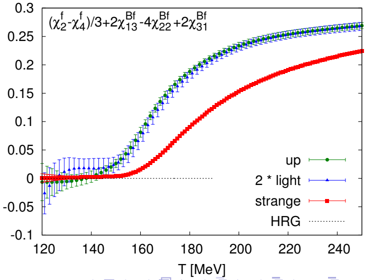
Linear combinations of cumulants

model-dependent but enlightening approach:
compare HRG and lattice ⇒ where do they deviate
two interesting quantities:

$v_1^f = \chi_{11}^{Bf} - \chi_{31}^{Bf}$ and $v_2^f = \frac{1}{3}(\chi_2^f - \chi_4^f) + 2\chi_{13}^{Bf} - 4\chi_{22}^{Bf} + 2\chi_{31}^{Bf}$

they are constructed in a way to be zero in the HRG model
we obseve a separation between the light and strange sectors

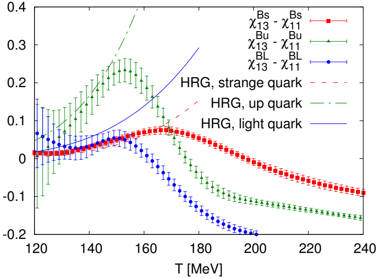


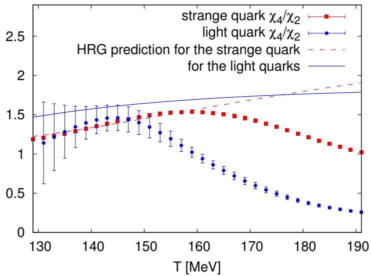


Volume independent measure

even more sensitive to the flavor content: $w^f = \chi_{13}^{Bf} - \chi_{11}^{Bf}$
e.g. hadronic phase: contributions only from hadrons
more than one quark of flavor f (=u or =s or =L)

clear peaks and deviation from the HRG prediction





better for experiments (volume independent but expensive) χ_4^f/χ_2^f
seperation between the kinks is again 15 MeV

Summary