Stony Brook University | The State University of New York

Event-shape fluctuations and flow correlations in HI collisions

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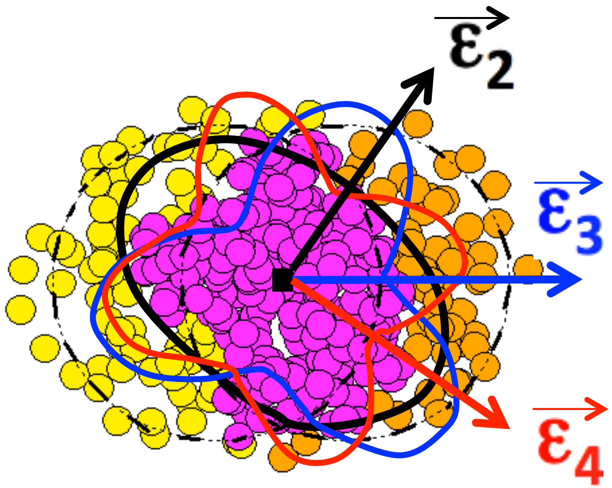
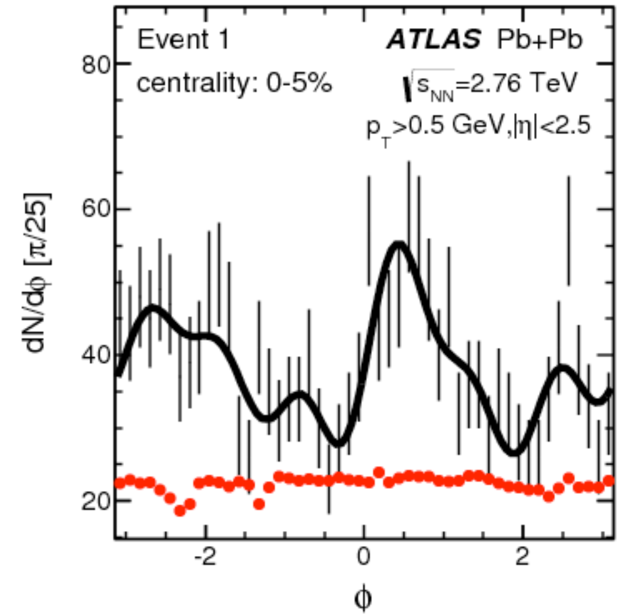
5th School of Collective Dynamics in High Energy Collisions

Geometry and harmonic flow



Collective expansion



$$\vec{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi}\rangle}{\langle r^n\rangle}$$

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n \cos n(\phi - \Phi_n)$$

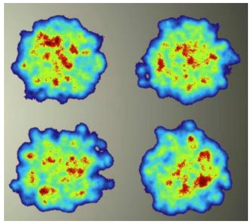
$$\vec{v}_n \equiv v_n e^{in\Phi_n}$$

- Probes: initial geometry and transport properties of QGP
  - How $(\varepsilon_n, \Phi_n^*)$ are transferred to (v_n, Φ_n)?
  - What is the nature of final state (non-linear) dynamics?
  - What is the nature of longitudinal flow dynamics?

Event-by-event observables

Many little bangs

1104.4740, 1209.2323,1203.5095 ,1312.3572



$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1{,}2{,}...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle -$ $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2 \rangle \langle v_n^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle -$ $\langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$	yes

Event-plane correlators

- Angular component can be expanded into a Fourier series

$$\frac{dN_{\text{evts}}}{d\Phi_1 d\Phi_2 ... d\Phi_l} \propto \sum_{c_n=-\infty}^{\infty} a_{c_1,c_2,...,c_l} \cos(c_1\Phi_1 + c_2\Phi_2 ... + c_l\Phi_l)$$

$$a_{c_1,c_2,...,c_l} = \langle \cos(c_1\Phi_1 + c_2\Phi_2 + ... + c_l\Phi_l) \rangle$$

- Φ_n has n-fold symmetry, thus correlation should be invariant under

$$\Phi_n \to \Phi_n + 2\pi/n$$ or appear in multiple of $n\Phi_n$

- invariant under global rotation by any θ: $\Sigma_k \Phi_k = \Sigma_k(\Phi_k + \theta)$

- So the physical quantities are:

$$\langle \cos(c_1\Phi_1 + 2c_2\Phi_2 ... + lc_l\Phi_l) \rangle, c_1 + 2c_2 ... + lc_l = 0$$

Cumulants

Moments → Cumulants

- Two-particle cumulants

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c \rightarrow \langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

- Three-particle cumulants

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

$$\downarrow$$

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$

- Higher-order cumulants obtained recursively

Cumulants for $p(v_n)$

- Observables: $$X = e^{in\phi}$$ $$\langle X \rangle_c = \langle e^{in\phi} \rangle = 0$$
- Moments

$$\langle X_n X_{-n} \rangle = \langle \cos n(\phi_1 - \phi_2) \rangle = \langle v_n^2 \rangle$$ + finite number& non-flow

$$\langle X_n X_{-n} X_n X_{-n} \rangle = \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle = \langle v_n^4 \rangle$$

....

- Cumulants

$$c_n\{2\} = \langle X_n X_{-n} \rangle_c = \langle \cos n(\phi_1 - \phi_2) \rangle_c = \langle v_n^2 \rangle$$

$$c_n\{4\} = \langle X_n X_{-n} X_n X_{-n} \rangle_c = \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle_c = \langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2$$

$$c_n\{6\} = ... = \langle v_n^6 \rangle - 9\langle v_n^2 \rangle \langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3$$

$$c_n\{8\} = ... = \langle v_n^8 \rangle - 16\langle v_n^6 \rangle \langle v_n^2 \rangle - 18\langle v_n^4 \rangle^2 + 144\langle v_n^4 \rangle \langle v_n^2 \rangle^2 - 144\langle v_n^2 \rangle^4$$

....

- Define:

$$v_n\{2\} = c_n\{2\}^{1/2} \qquad v_n\{4\} = (-c_n\{4\})^{1/4}$$

$$v_n\{6\} = \left(\frac{1}{4} c_n\{6\}\right)^{1/6} \qquad v_n\{8\} = \left(-\frac{1}{33} c_n\{8\}\right)^{1/8}$$

Cumulants for p(Φ_n,Φ_m…)

- Example

$$\langle\cos(2\phi_1+2\phi_2-4\phi_3)\rangle = \langle v_2 v_2 v_4 \cos(2\Phi_2+2\Phi_2-4\Phi_4)\rangle$$
$$= \langle v_2^2 v_4 \cos 4(\Phi_2-\Phi_4)\rangle$$

- In general for mixed-harmonics:

$$\langle\cos(\Sigma_{i_1=1}^{c_1}\phi_{i_1}+\Sigma_{i_2=1}^{c_2}2\phi_{i_2}+...+\Sigma_{i_l=1}^{c_l}l\phi_{i_l})\rangle$$
$$=\langle v_1^{c_1}v_2^{c_2}...v_l^{c_l}\cos(c_1\Phi_1+2c_2\Phi_2+...+lc_l\Phi_l)\rangle$$

it is a correlation involving c_1+c_2+..+c_l particles $\Sigma_k k c_k = 0$

- Moment is the same as cumulants for mixed-harmonics, i.e

$$\langle\cos(2\phi_1+2\phi_2-4\phi_3)\rangle_c = \langle\cos(2\phi_1+2\phi_2-4\phi_3)\rangle$$

all other terms vanishes, since for any other partition the Σ of coefficient ≠0
Such as

$$\langle\cos(2\phi_1+2\phi_2)\rangle = \langle\cos(2\phi_1-4\phi_3)\rangle = .. = 0$$

Cumulants for $p(v_n, v_m \ldots, \Phi_n, \Phi_m \ldots)$

- Example, combining $\cos(2\phi_1 + 2\phi_2 - 4\phi_3)$ and $\cos(2\phi_1 - 2\phi_2)$

$$\begin{aligned}\langle\cos(2\phi_1 + 2\phi_2 - 4\phi_3 + 2\phi_4 - 2\phi_5)\rangle &= \langle v_2^2 v_4 v_2^2 \cos(2\Phi_2 + 2\Phi_2 - 4\Phi_4 + 2\Phi_2 - 2\Phi_2)\rangle \\ &= \langle v_2^4 v_4 \cos 4(\Phi_2 - \Phi_4)\rangle\end{aligned}$$

- Corresponding cumulants:

$$\begin{aligned}&\langle\cos(2\phi_1 + 2\phi_2 - 4\phi_3 + 2\phi_4 - 2\phi_5)\rangle_c \\ &= \langle v_2^2 v_2^2 v_4 \cos 4(\Phi_2 - \Phi_4)\rangle - \langle v_2^2\rangle\langle v_2^2 v_4 \cos 4(\Phi_2 - \Phi_4)\rangle\end{aligned}$$

probes $p(v_2, \Phi_2, \Phi_4)$ distribution

- Can be generalized into other mixed-correlators

Cumulants for $p(v_n,v_m,...)$

- Example, combining $\cos(4\phi_1 - 4\phi_2)$ and $\cos(2\phi_1 - 2\phi_2)$

$$\langle\cos(2\phi_1 - 2\phi_2 + 4\phi_3 - 4\phi_4)\rangle$$
$$= \langle v_2^2 v_4^2 \cos(2\Phi_2 - 2\Phi_2 + 4\Phi_4 - 4\Phi_4)\rangle = \langle v_2^2 v_4^2\rangle$$

- Corresponding cumulants,

$$\langle\cos(2\phi_1 - 2\phi_2 + 4\phi_3 - 4\phi_4)\rangle_c = \langle v_2^2 v_4^2\rangle - \langle v_2^2\rangle\langle v_4^2\rangle$$

probes $p(v_2,v_4)$ distribution

- Other examples

$$\langle\cos(2\phi_1 - 2\phi_2 + 3\phi_3 - 3\phi_4)\rangle_c = \langle v_2^2 v_3^2\rangle - \langle v_2^2\rangle\langle v_3^2\rangle$$

probes $p(v_2,v_3)$ distribution

1312.3572

Event-by-event observables

Many little bangs

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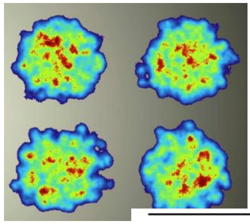

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\text{evts}}} \frac{dN_{\text{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

	pdf's	cumulants
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k = 1,2,...$
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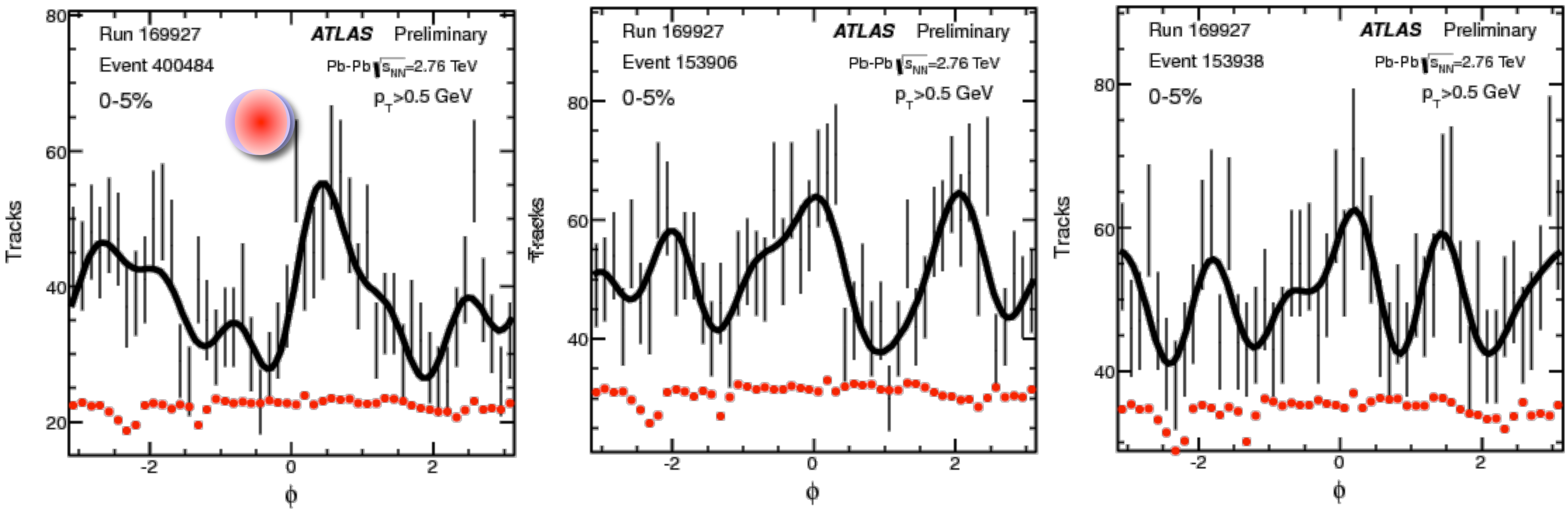
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2 \rangle + 2\langle v_n^2 \rangle \langle v_m^2 \rangle \langle v_l^2 \rangle -$ $\langle v_n^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_m^2 v_l^2 \rangle \langle v_n^2 \rangle - \langle v_l^2 v_n^2 \rangle \langle v_m^2 \rangle$
	...	Obtained recursively as above
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle -$ $\langle v_l^2 \rangle \langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n \Phi_n + c_m m \Phi_m + ...) \rangle$ $\sum_k k c_k = 0$

Experimental reality



$$\frac{dN}{d\phi} \propto 1 + 2\sum_n \mathrm{v}_n^{\mathrm{obs}} \cos n\left(\phi - \Phi_n^{\mathrm{obs}}\right)$$

Obtain $p(\Phi_n,\Phi_m)$ from $p(\Phi_n^{obs},\Phi_m^{obs})$

Obtain $p(v_n)$ from $p(v_n^{obs})$

Need to remove non-flow:

final number effects, resonance, jets, momentum conservation..

What we know about flow fluctuation? $p(v_n)$

Expectation for v_n fluctuations

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0809.2949

$$\vec{\varepsilon}_n = (\varepsilon_x, \varepsilon_y)$$

$$\vec{\varepsilon}_n = \vec{\varepsilon}_n^{\,0} + \vec{\Delta}_n^{fluc}$$

$$p(\vec{\varepsilon}_n) \propto \exp\left(\frac{-(\vec{\varepsilon}_n - \vec{\varepsilon}_n^{\,0})^2}{2\delta_{\varepsilon_n}^2}\right)$$

$\vec{\varepsilon}_n^{\,0} \rightarrow$ *Mean Geometry*

$\delta_{\varepsilon_n} \rightarrow$ *Fluctuations*

$$\vec{v}_n \propto \vec{\varepsilon}_n$$

$$\vec{v}_n = (v_n \cos n\Phi_n, v_n \sin n\Phi_n)$$

$$\vec{v}_n = \vec{v}_n^{\,0} + \vec{p}_n^{fluc}$$

$$p(\vec{v}_n) \propto \exp\left(\frac{-(\vec{v}_n - \vec{v}_n^{\,0})^2}{2\delta_n^2}\right)$$

$\vec{v}_n^{\,0} \rightarrow$ *Mean Geometry*

$\delta_n \rightarrow$ *Fluctuations*

Expectation for v_n fluctuations

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$$\vec{\varepsilon}_n = \vec{\varepsilon}_n^{\,0} + \vec{\Delta}_n^{\,\mathrm{fluc}}$$

$$p(\vec{\varepsilon}_n) \propto \exp\left(\frac{-(\vec{\varepsilon}_n - \vec{\varepsilon}_n^{\,0})^2}{2\delta_{\varepsilon_n}^2}\right)$$

$\vec{\varepsilon}_n^{\,0} \rightarrow Mean\,Geometry$

$\delta_{\varepsilon_n} \rightarrow Fluctuations$

$$\vec{v}_n \propto \vec{\varepsilon}_n$$

$$\vec{v}_n = \vec{v}_n^{\,0} + \vec{p}_n^{\,\mathrm{fluc}}$$

$$p(\vec{v}_n) \propto \exp\left(\frac{-(\vec{v}_n - \vec{v}_n^{\,0})^2}{2\delta_n^2}\right)$$

$\vec{v}_n^{\,0} \rightarrow Mean\,Geometry$

$\delta_n \rightarrow Fluctuations$

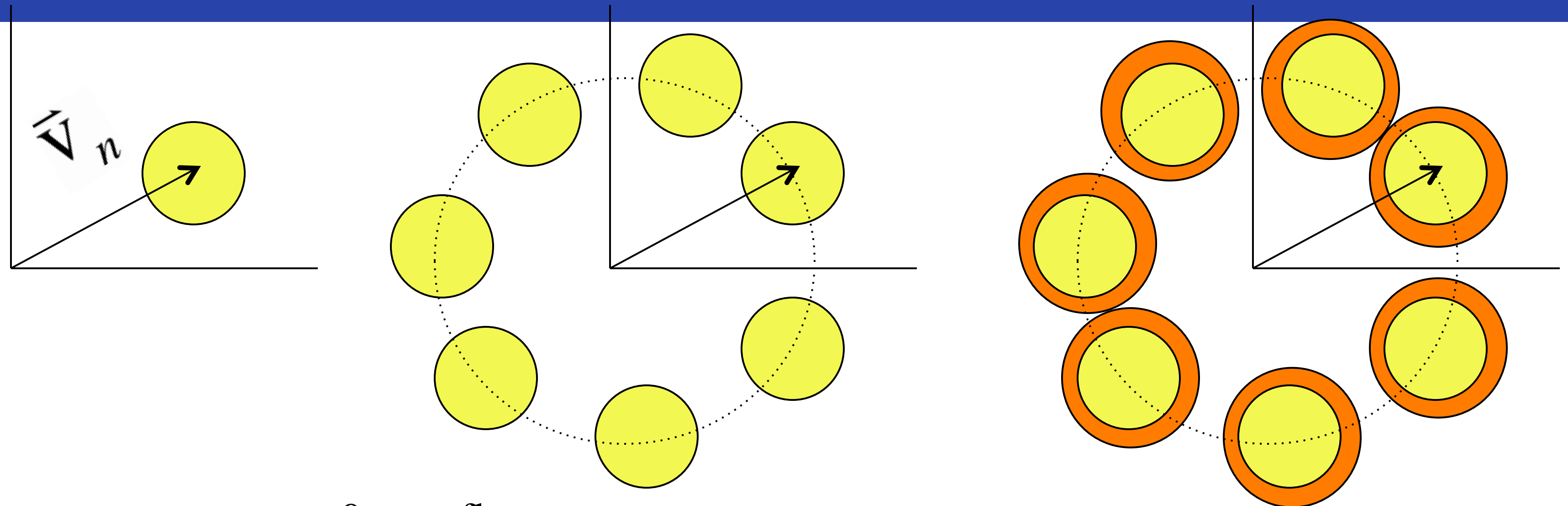
$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^0)^2)}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^0}{\delta_n^2}\right)$$

$$\vec{v}_n^{\,\mathrm{obs}} = \vec{v}_n + \vec{p}_n^{\,\mathrm{smear}}$$

Finite number & nonflow

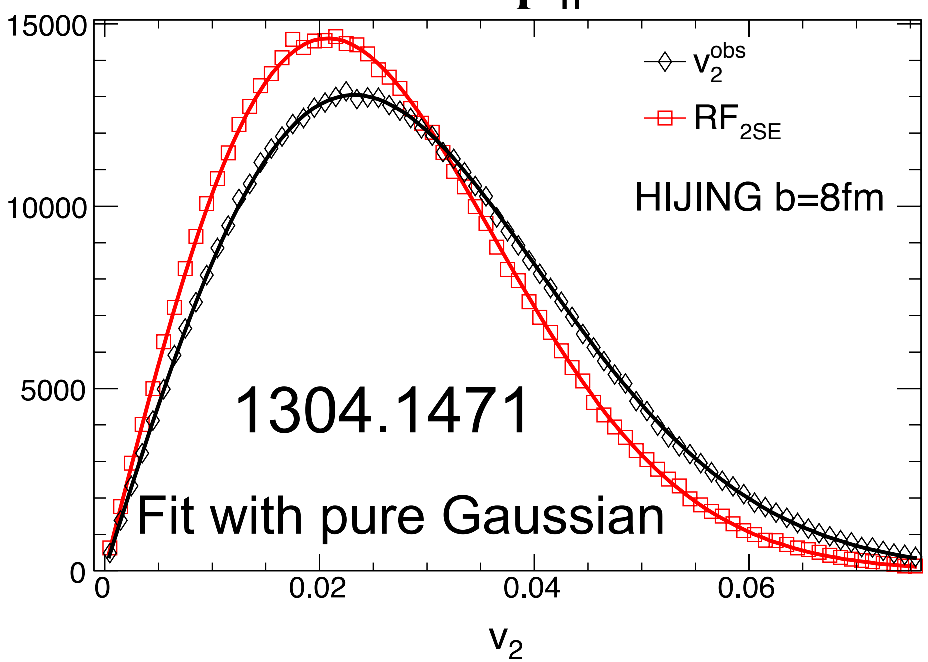
The key is response function: $p(v_n^{\mathrm{obs}} | v_n)$

Flow vector distributions



$$\vec{v}_n = \vec{v}_n^{\,0} + \vec{p}_n^{\,fluc}$$

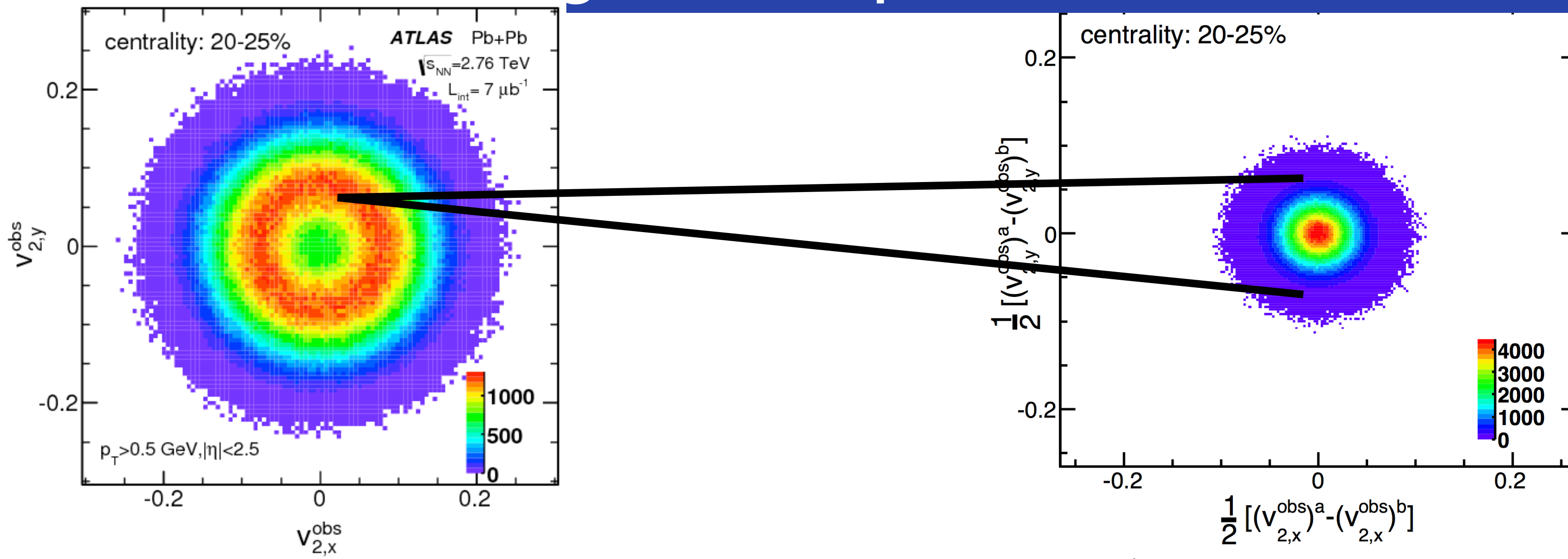
$$\vec{v}_n^{\,obs} = \vec{v}_n^{\,0} + \vec{p}_n^{\,fluc} + \vec{p}_n^{\,smear}$$



nonflow/noise $\vec{p}_n^{\,smear}$ is gaussian!

Checked in hijing

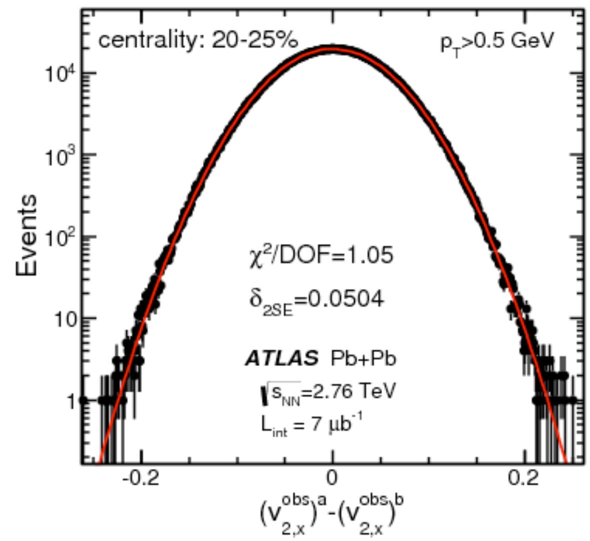
Obtaining the response function



$$\vec{v}_n^{\,\mathrm{obs}} = (\vec{v}_n^{\,\mathrm{obs,F}} + \vec{v}_n^{\,\mathrm{obs,B}})/2$$
$$= \mathrm{nonflow} + \mathrm{noise} + \vec{v}_n$$

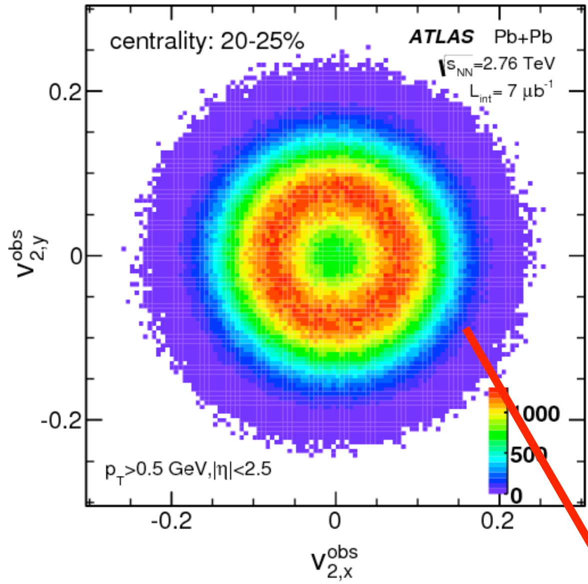
$$\vec{\delta}_n^{\,\mathrm{RF}} = (\vec{v}_n^{\,\mathrm{obs,F}} - \vec{v}_n^{\,\mathrm{obs,B}})/2$$
$$= \mathrm{nonflow} + \mathrm{noise}$$

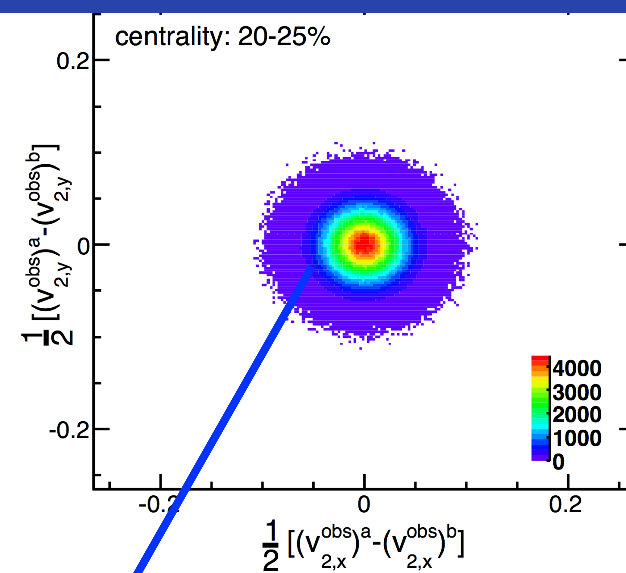
$$p(\vec{v}_n^{\,\mathrm{obs}}) = p(\vec{v}_n) \otimes p(\vec{\delta}_n^{\,\mathrm{RF}})$$



Obtain $p(v_n)$ via unfolding

Obtaining the response function



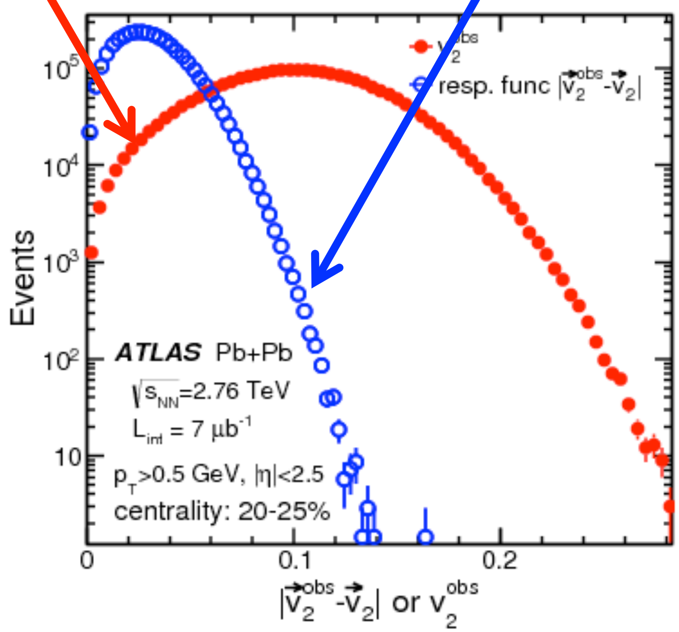


$$\vec{v}_n^{\,obs} = (\vec{v}_n^{\,obs,F} + \vec{v}_n^{\,obs,B})/2$$
$$= \text{nonflow} + \text{noise} + \vec{v}_n$$

$$\vec{\delta}_n^{\,RF} = (\vec{v}_n^{\,obs,F} - \vec{v}_n^{\,obs,B})/2$$
$$= \text{nonflow} + \text{noise}$$

$$p(\vec{v}_n^{\,obs}) = p(\vec{v}_n) \otimes p(\vec{\delta}_n^{\,RF})$$

Obtain $p(v_n)$ via unfolding



Cumulants for $p(v_n)$

- Observables: $X = e^{in\phi}$ $\langle X\rangle_c = \langle e^{in\phi}\rangle = 0$
- Moments

$$\langle X_n X_{-n}\rangle = \langle \cos n(\phi_1 - \phi_2)\rangle = \langle v_n^2\rangle$$
$$\langle X_n X_{-n} X_n X_{-n}\rangle = \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4)\rangle = \langle v_n^4\rangle$$
$$....$$

- Cumulants

$$c_n\{2\} = \langle X_n X_{-n}\rangle_c = \langle \cos n(\phi_1 - \phi_2)\rangle_c = \langle v_n^2\rangle$$
$$c_n\{4\} = \langle X_n X_{-n} X_n X_{-n}\rangle_c = \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4)\rangle_c = \langle v_n^4\rangle - 2\langle v_n^2\rangle^2$$
$$c_n\{6\} = ... = \langle v_n^6\rangle - 9\langle v_n^2\rangle\langle v_n^4\rangle + 12\langle v_n^2\rangle^3$$
$$c_n\{8\} = ... = \langle v_n^8\rangle - 16\langle v_n^6\rangle\langle v_n^2\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^4\rangle\langle v_n^2\rangle^2 - 144\langle v_n^2\rangle^4$$
$$....$$

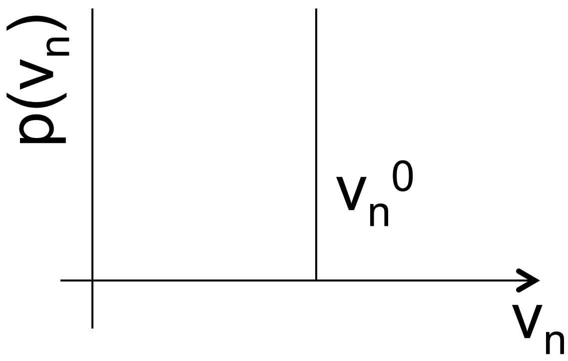
Rely on Large cancellation to remove finite N and non-flow

→is or is not straightforward to cancel systematics?

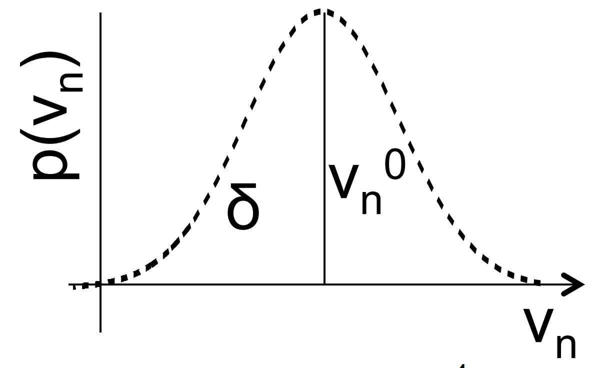
→should different terms are treated separate measurement?

Cumulants for azimuthal correlations

No fluctuation



Gaussian fluctuation



Same answer!:

$$c_n\{2\} = \left(v_n^0\right)^2 + 2\delta^2 \quad c_n\{4\} = -\left(v_n^0\right)^4$$

$$c_n\{6\} = 4\left(v_n^0\right)^6 \quad c_n\{8\} = -33\left(v_n^0\right)^8$$

- Define

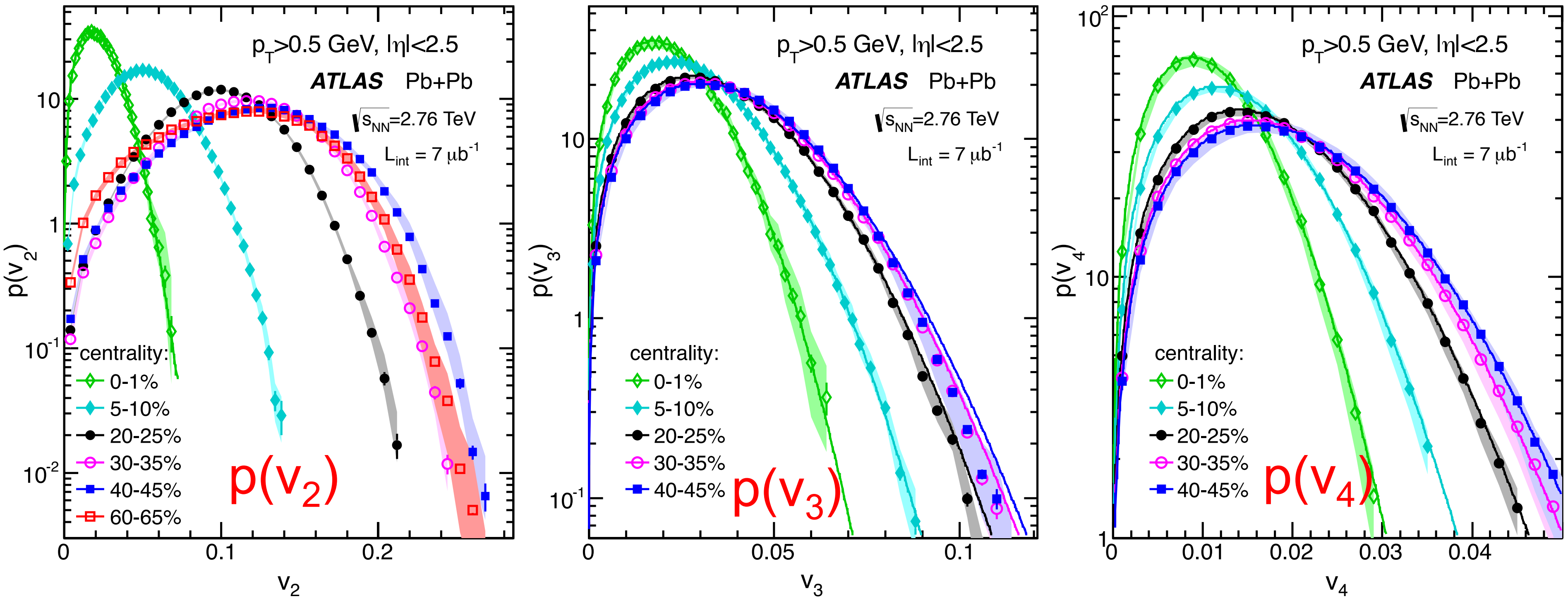
$$v_n\{2\} = c_n\{2\}^{1/2} \qquad v_n\{4\} = \left(-c_n\{4\}\right)^{1/4}$$

$$v_n\{6\} = \left(\frac{1}{4}c_n\{6\}\right)^{1/6} \quad v_n\{8\} = \left(-\frac{1}{33}c_n\{8\}\right)^{1/8}$$

- Gaussian fluctuation: $v_n\{4\} = v_n\{6\} = v_n\{8\} = .. = v_n^0$

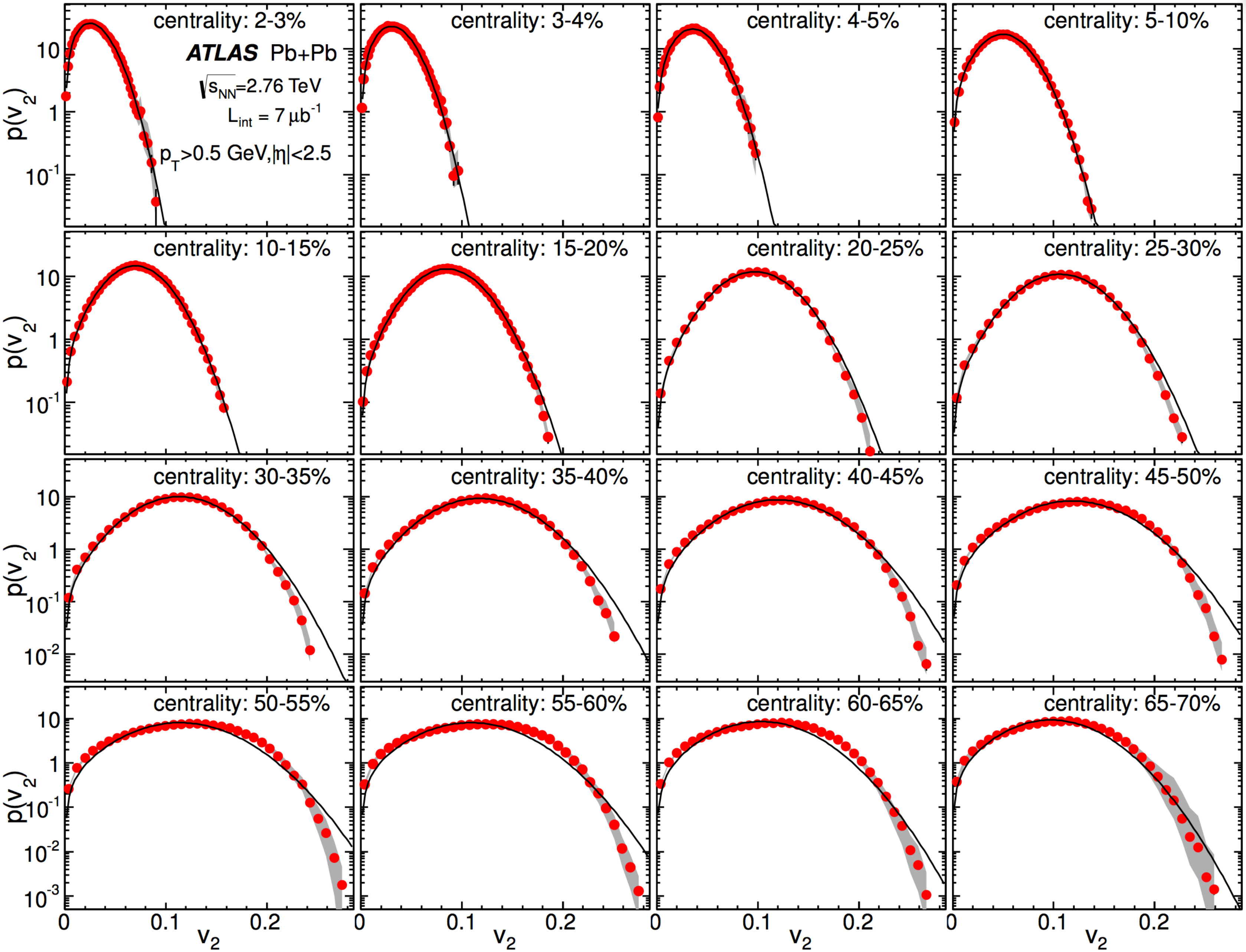
Higher-order cumulants suppress non-flow **because** non-flow is Gaussian!!
Is cumulants just mathmatical construct? **What if non-flow is non-Gaussian?**

$p(v_2)$, $p(v_3)$ and $p(v_4)$ distributions



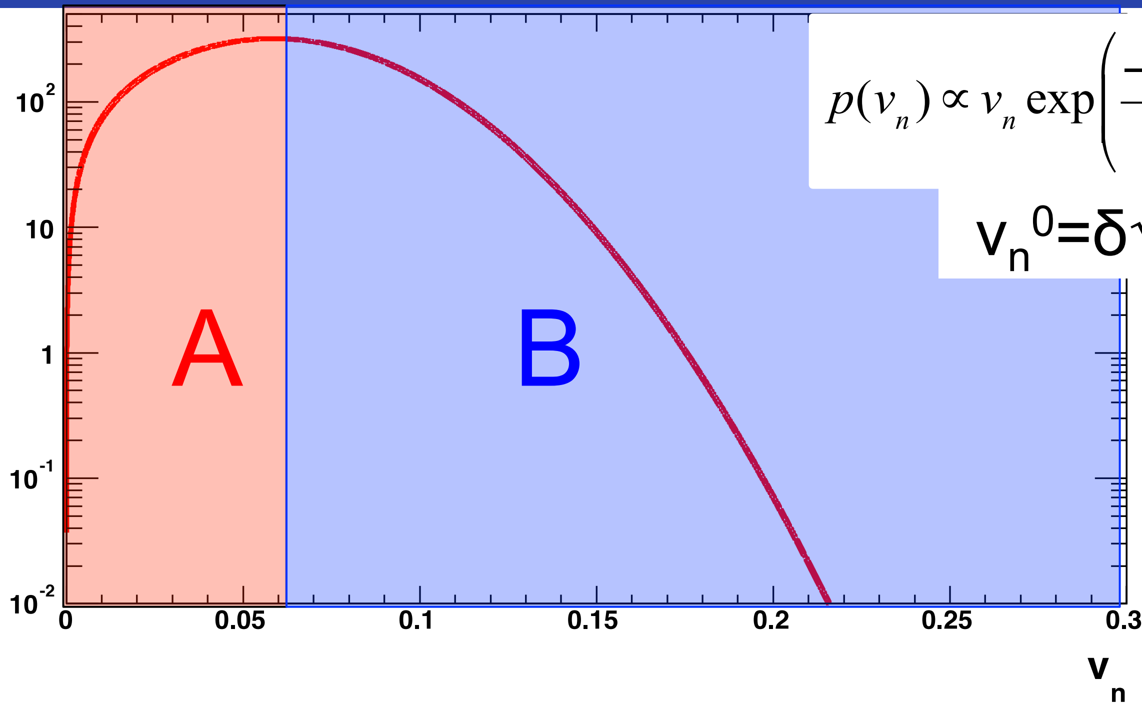
$$v_n\{4\}^4 = 2\langle v_n^2\rangle^2 - \langle v_n^4\rangle \neq 0 \quad \text{for} \quad n = 2, 3$$

- The non-zero $v_n\{4,6..\}$ either due to
  - average geometry such as v_2^{RP} or
  - non-Gaussianness in the flow fluctuation or
  - non-Gaussianness in non-flow such as p+Pb system.



Furthermore $p(v_2)$ is also non-B-G in the distribution tail

Are cumulants sensitive to non gaussian?



$$p(v_n) \propto v_n \exp\left(\frac{-(v_n^2 + (v_n^0)^2)}{2\delta_n^2}\right) I_0\left(\frac{v_n v_n^0}{\delta_n^2}\right)$$

$v_n^0 = \delta\sqrt{2}$ as example

- Divide B-G distri. to 2 equal parts, and calculate cumulants separately.

	$v_n\{2\}$	$v_n\{4\}$	$v_n\{6\}$	$v_n\{8\}$
all	1.414	1	1	1
A	0.851	0.759	0.746	0.744
B	1.809	1.690	1.701	1.701

In units of δ

- The non-BG is reflected by difference of 4,6 particle cumulants

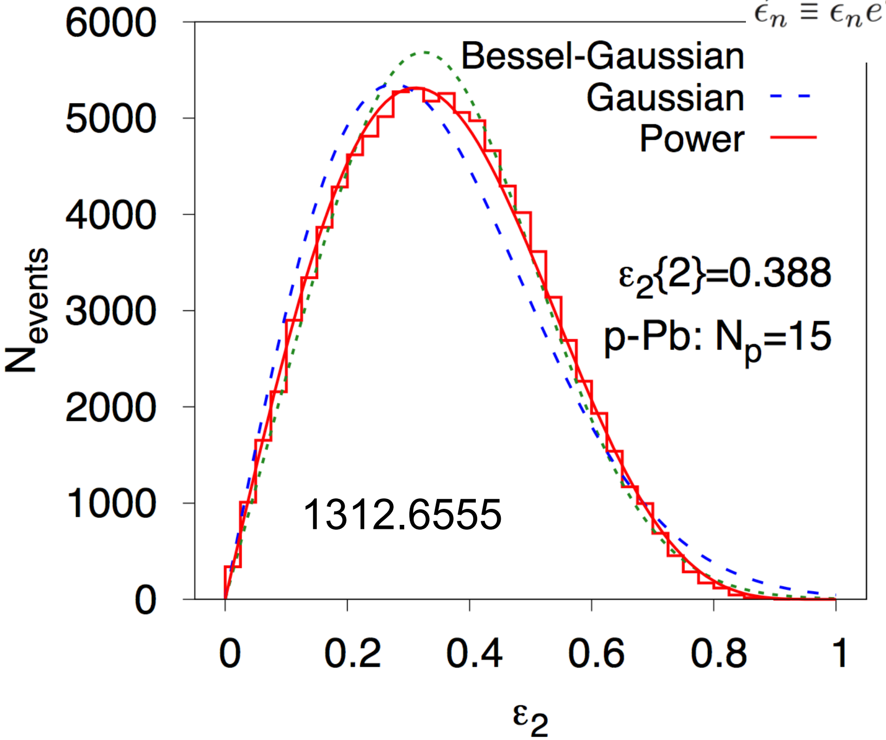
Cumulants not very sensitive to details of p(vn)?

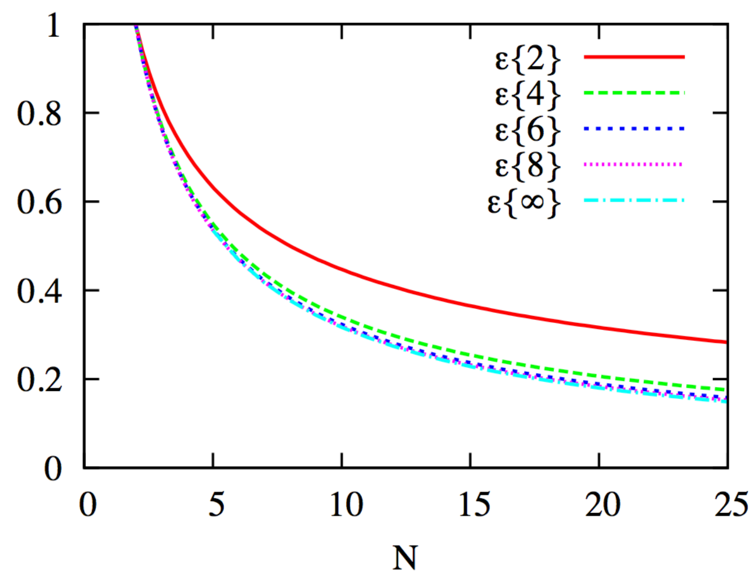
Small system

- Eccentricity distri. not gaussian, due to smaller number sources

$$\vec{\epsilon}_n \equiv \epsilon_n e^{in\Phi_n^*} \equiv -\frac{\langle r^n e^{in\phi}\rangle}{\langle r^n\rangle}$$

$$P(\varepsilon) = 2\alpha\varepsilon(1-\varepsilon^2)^{\alpha-1}$$



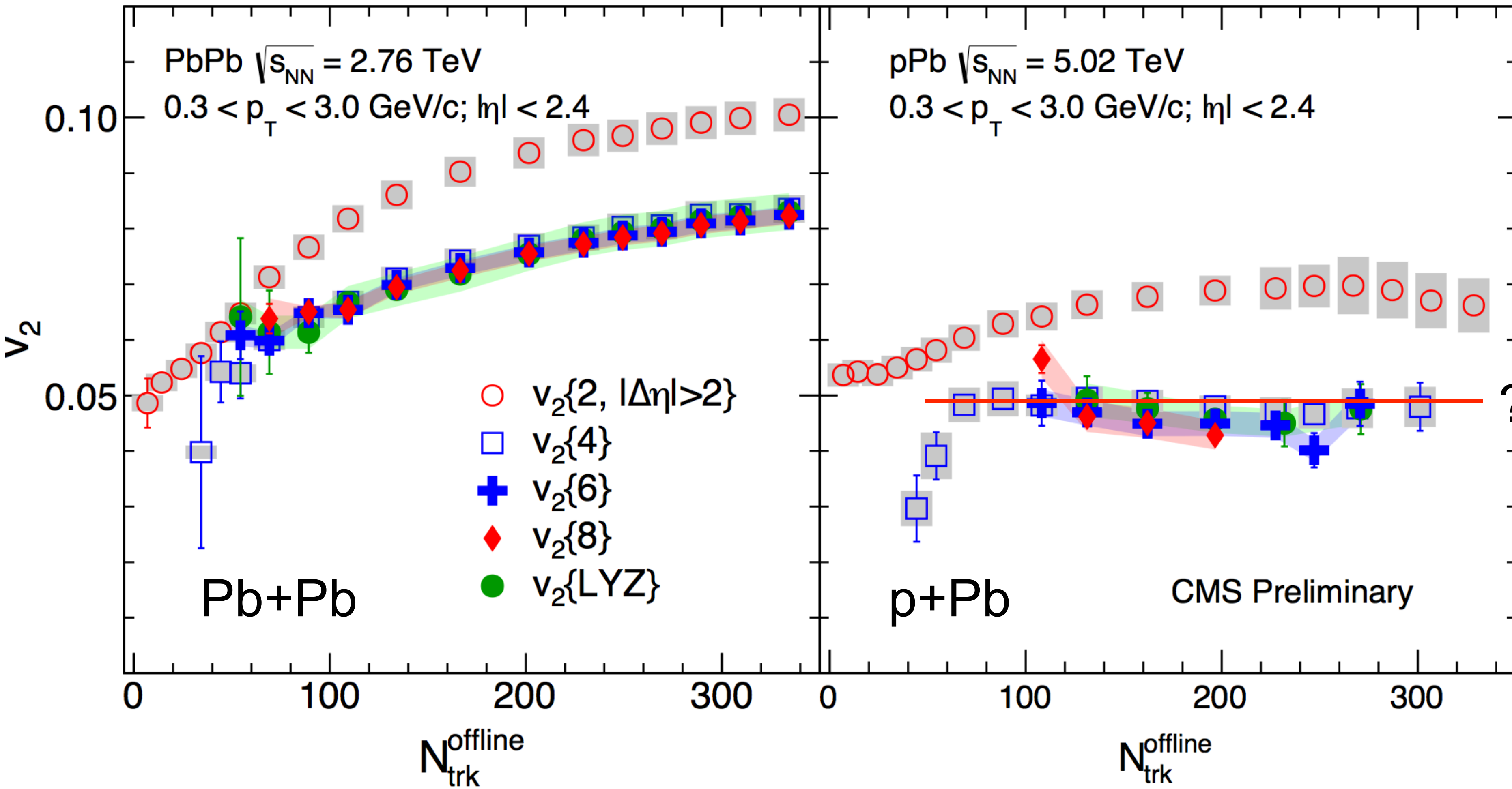


- The non-zero $v_2\{4,6,8\ldots\}$ suggest the $p(v_2)$ distri. is non-Gaussian?

$$v_n\{4\} = \left(2\langle v_n^2\rangle^2 - \langle v_n^4\rangle\right)^{1/4} \longleftrightarrow$$

a 4% difference gives a $v_n\{4\}$ value of about 45% of $v_n\{2\}$

Multi-particle correlation in p+Pb

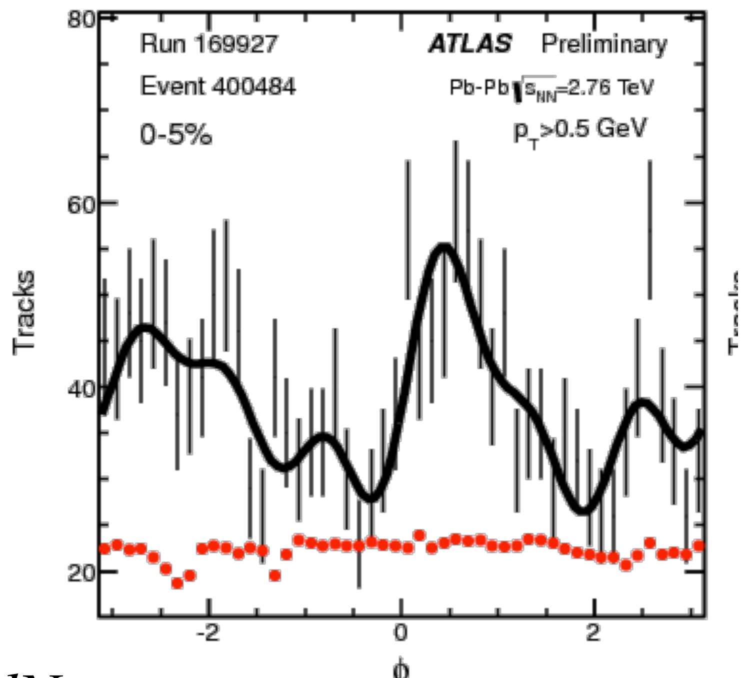


- What is the meaning of $v_2\{4,6,8,\}$ in p+Pb collisions?
- Why non-Gaussian component are not increasing with multiplicity?

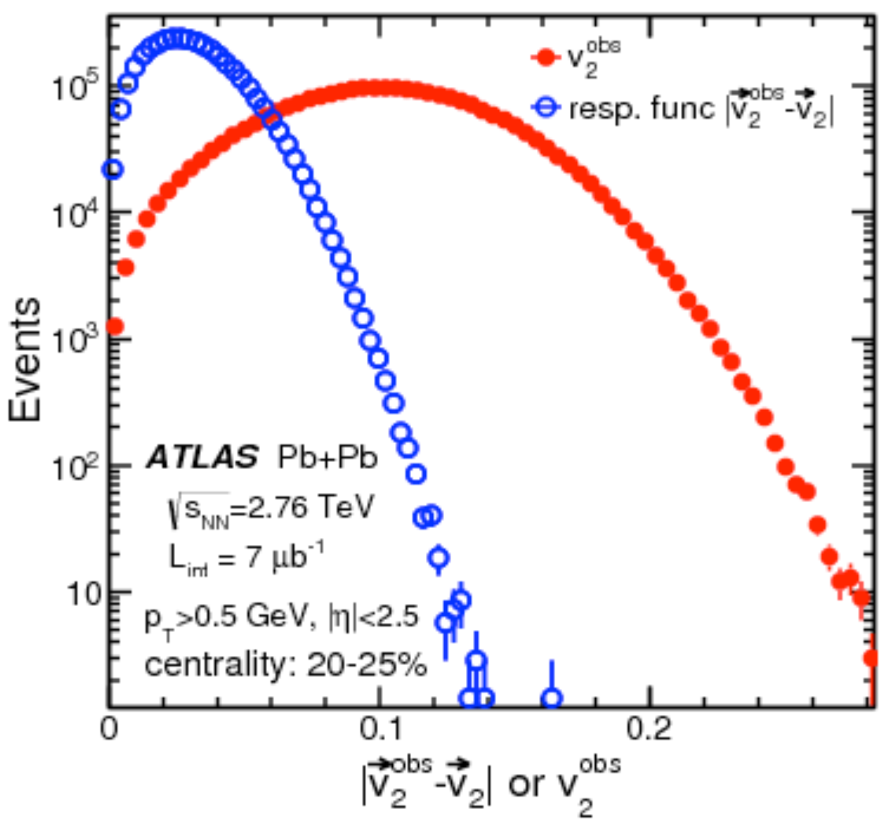
Connection between $p(v_n)$ and $v_n\{2k\}$

- $v_n\{2k\}$ removes all Gaussian sources, it removes non-flow only because it is nearly Gaussian, but in this case, one can just calculate them directly from $p(v_n^{obs})$ distribution

$$\langle 2k\rangle = \left\langle \cos\left(\sum_{j=1}^{k} n(\phi_{2j}-\phi_{2j+1})\right)\right\rangle \overset{?}{=} \langle v_n^{2k}\rangle = \int v_n^{2k} p(v_n) dv_n$$

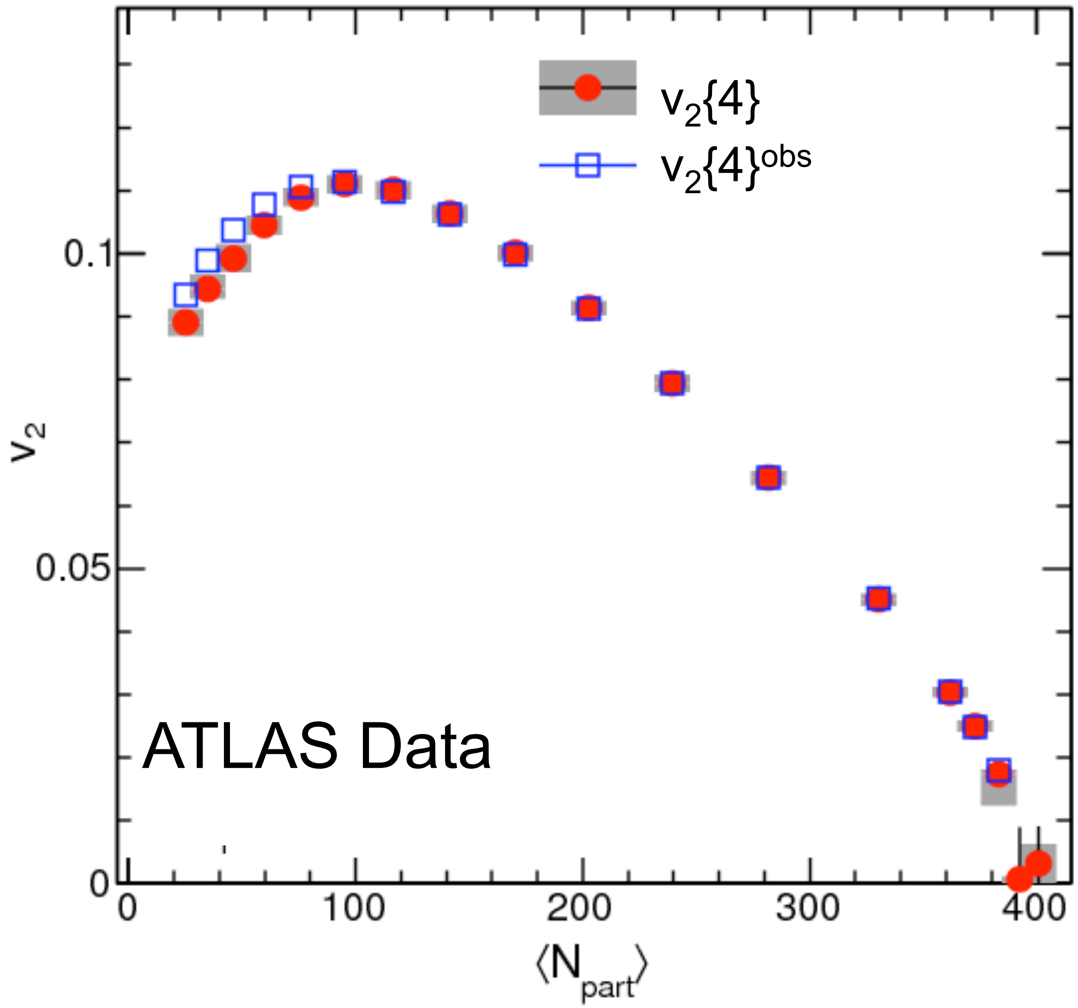


$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n^{obs} \cos n\left(\phi - \Phi_n^{obs}\right)$$



Effect of non-flow

- Additional Gaussian smearing won't change higher-order cumulants

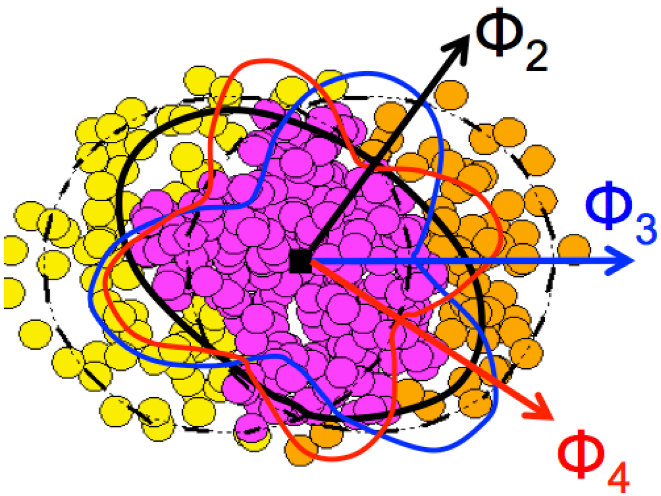


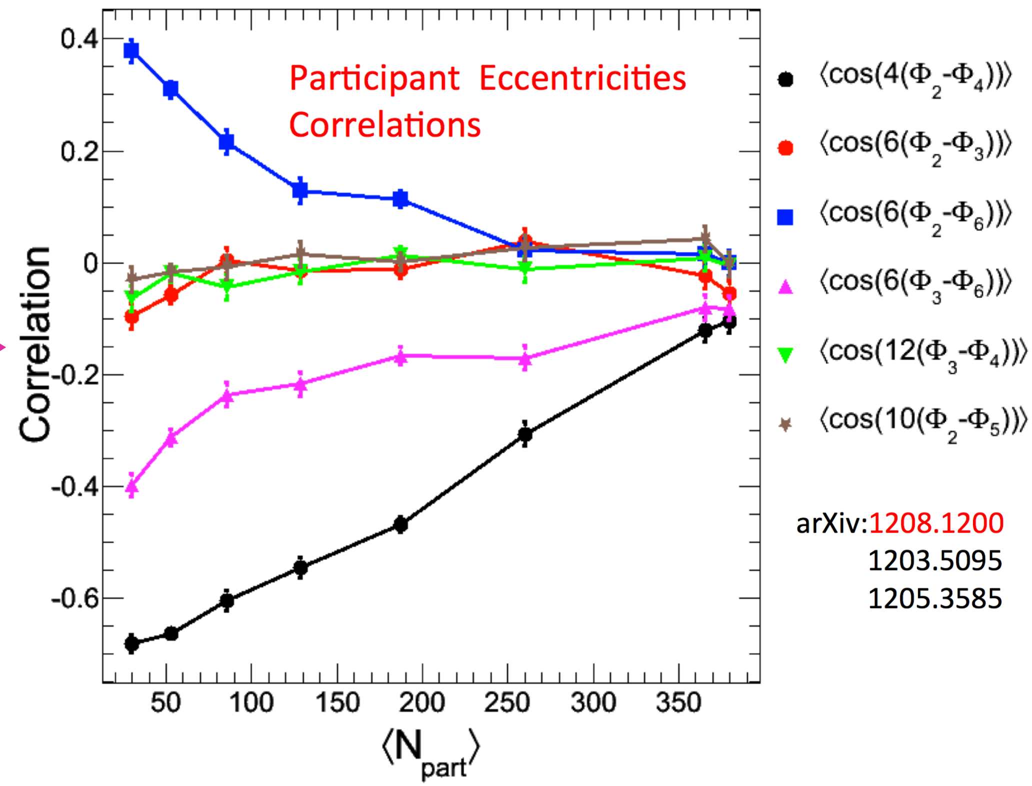
Same $v_2\{4\}$ value from either $p(v_2)$ or $p(v_2^{obs})$ distribution

Event-plane correlations $p(\Phi_n, \Phi_m \ldots)$

Event-plane correlation

- Correlations exist in the initial geometry



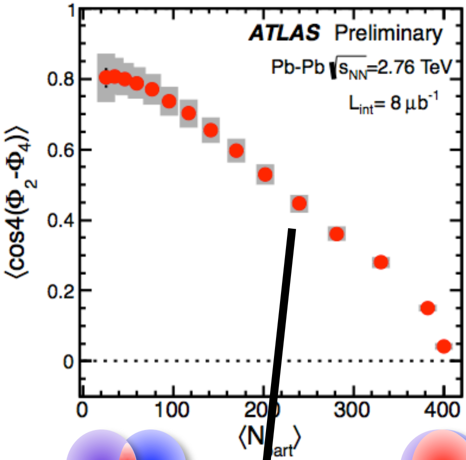


- Also generated during hydro evolution: non-linear mixing, e.g.

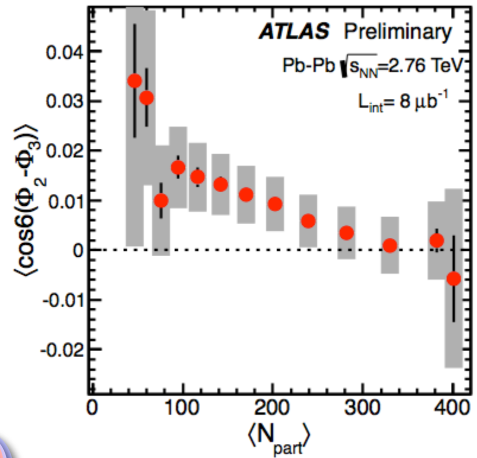
$$v_4 e^{-i4\Phi_4} \propto \varepsilon_4 e^{-i4\Phi_4^*} + c v_2^2 e^{-i4\Phi_2} + \ldots$$

Event-plane correlation results

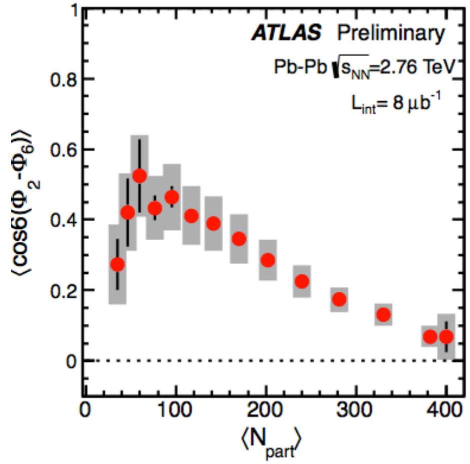
$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$



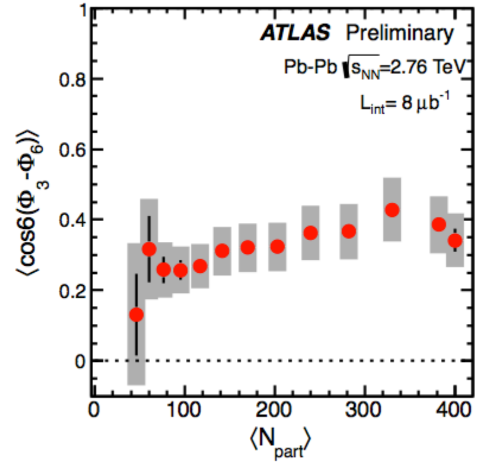
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$

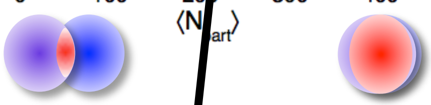


$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$



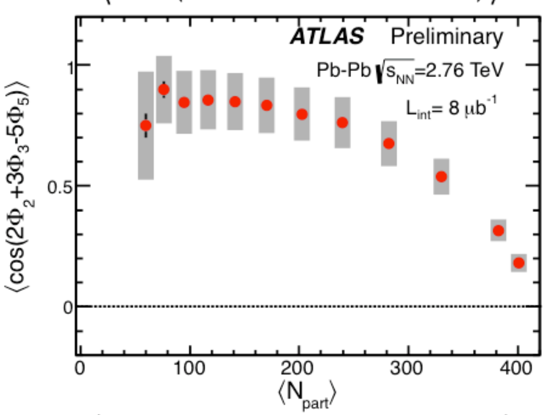
$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$





$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

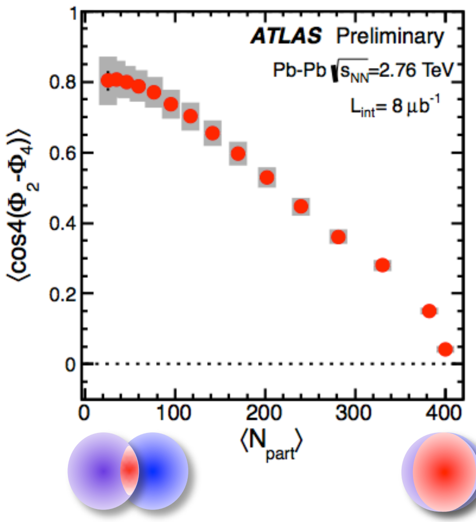
$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



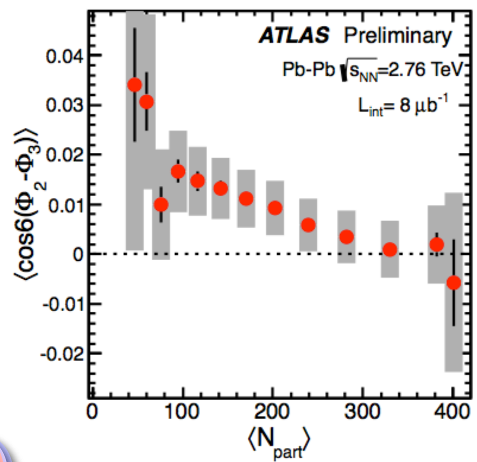
Teaney & Yan

Event plane correlation results

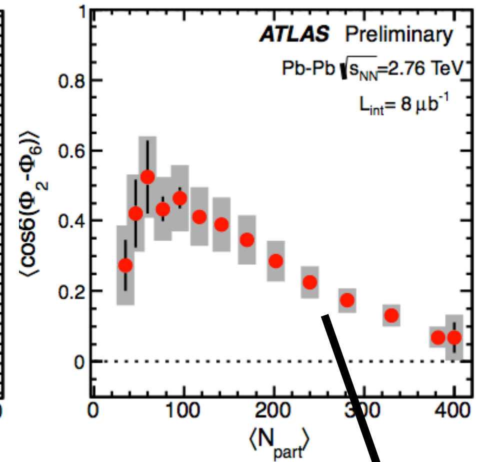
$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$



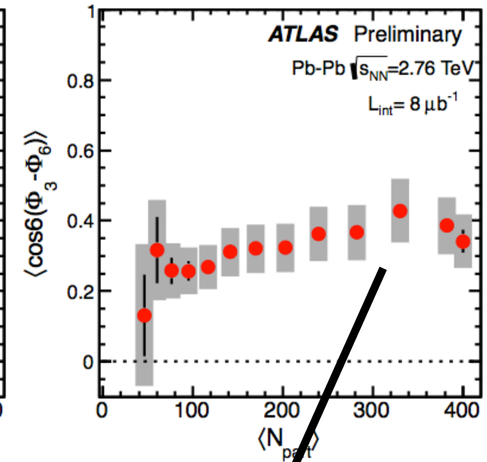
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$



$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$

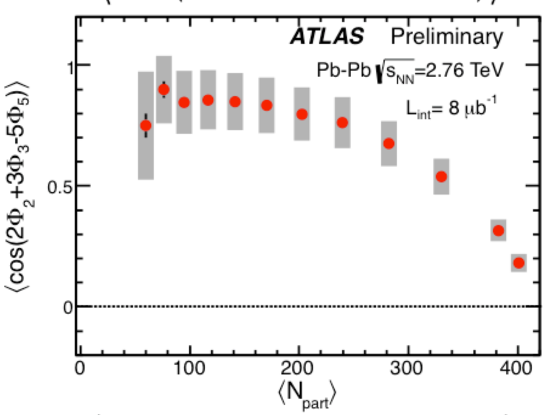


$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + \ldots$$

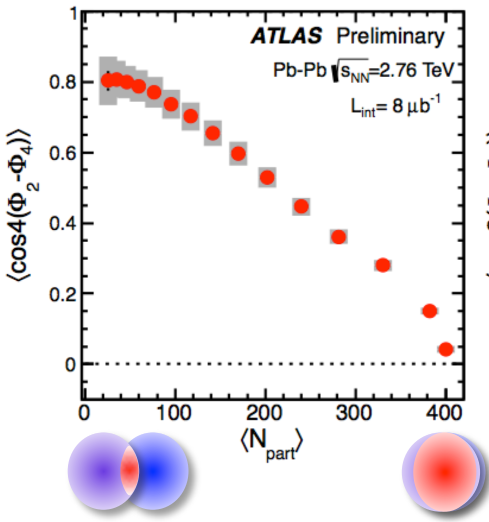
$\langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



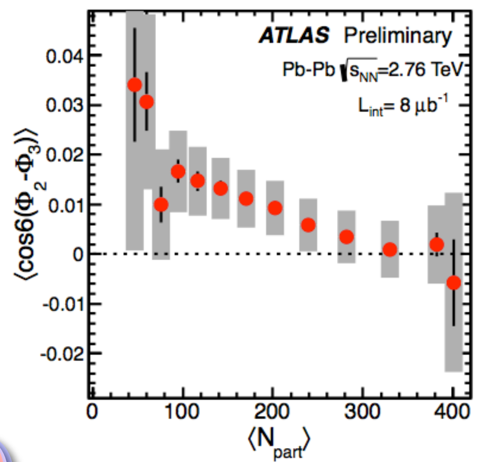
Teaney & Yan

Event plane correlation results

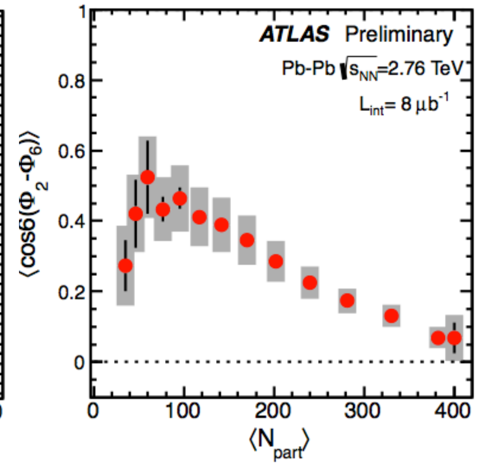
$\langle \cos 4(\Phi_2 - \Phi_4) \rangle$



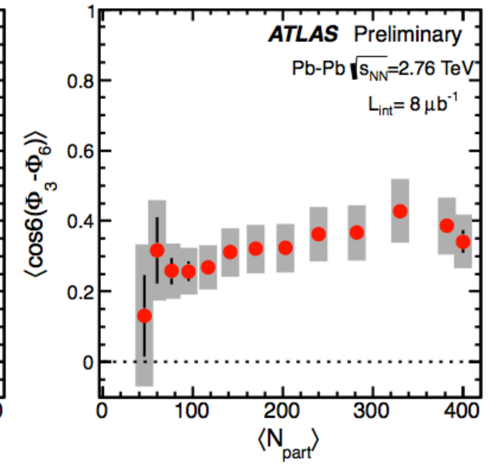
$\langle \cos 6(\Phi_2 - \Phi_3) \rangle$



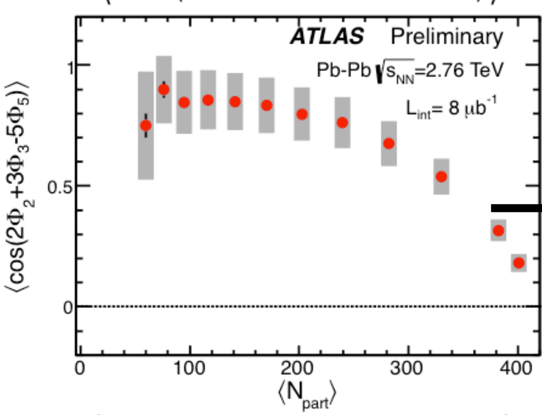
$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$



$\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



$\langle \mathbf{cos}(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle$



$$v_5 e^{i5\Phi_5} \propto \varepsilon_5 e^{i5\Phi_5^*} + c v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$

Teaney & Yan

How $(\varepsilon_n, \Phi_n^*)$ are transferred to (v_n, Φ_n)?

- Flow response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}$$

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- Flow response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{-i2\Phi_2} \propto \epsilon_2 e^{-i2\Phi_2^*}, \quad v_3 e^{-i3\Phi_3} \propto \epsilon_3 e^{-i3\Phi_3^*}$$

- Higher-order flow arises from EP correlations., e.g. :

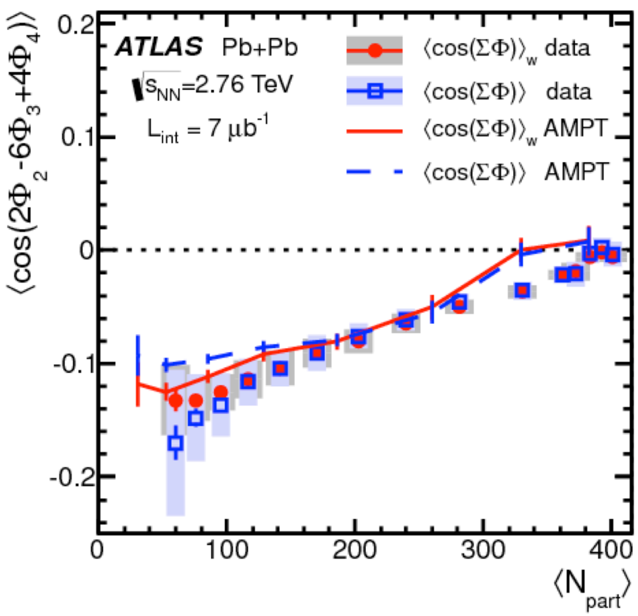
$$v_4 e^{i4\Phi_4} \propto \varepsilon_4 e^{i4\Phi_4^*} + c v_2^2 e^{i4\Phi_2} + \ldots$$

Ollitrault, Luzum, Teaney, Li, Heinz,Chun….

$$v_5 e^{i5\Phi_5} \propto \varepsilon_5 e^{i5\Phi_5^*} + c v_2 v_3 e^{i(2\Phi_2+3\Phi_3)} + \ldots$$

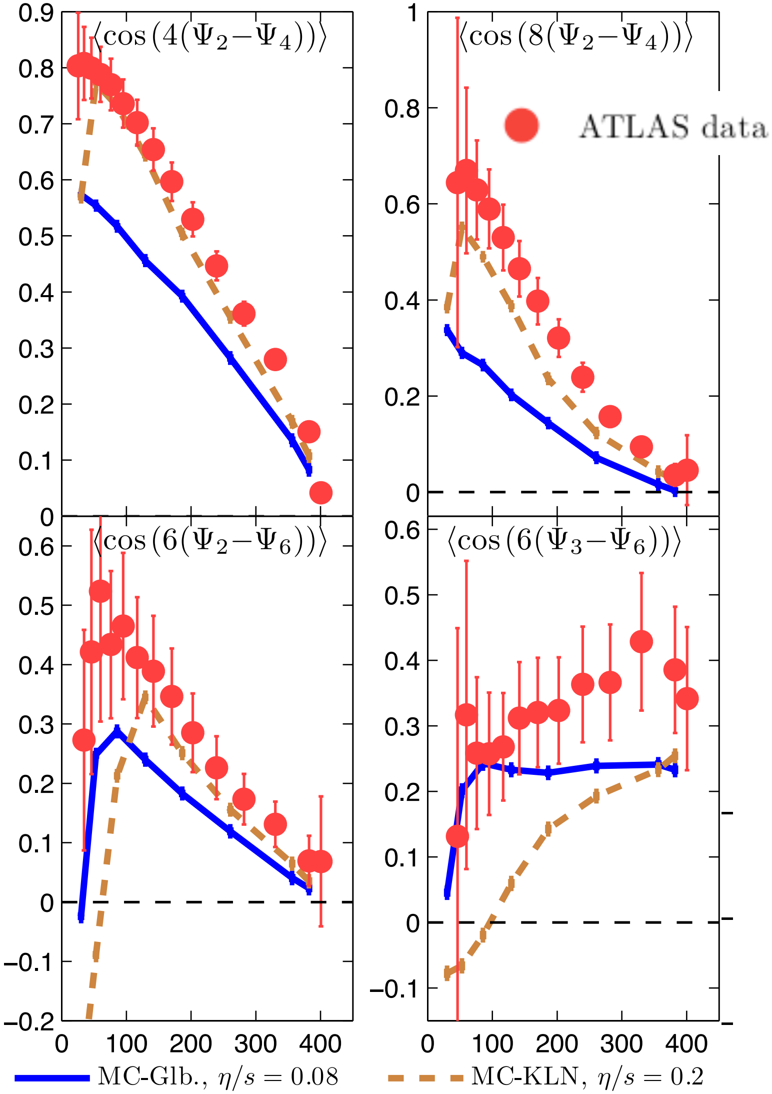
$$v_6 e^{i6\Phi_6} \propto \varepsilon_6 e^{i6\Phi_6^*} + c_1 v_2^3 e^{i6\Phi_2} + c_2 v_3^2 e^{i6\Phi_3} + c_3 v_2 \varepsilon_4 e^{i\left(2\Phi_2+4\Phi_4^*\right)} \ldots$$

- Some correlators lack no intuitive explanation e.g. 2-3-4 correlation
  - Although described by EbyE hydro and AMPT



Compare with EbE hydro calculation

Initial geometry + hydrodynamic Zhe & Heinz 1208.1200



Compare with EbE hydro calculation

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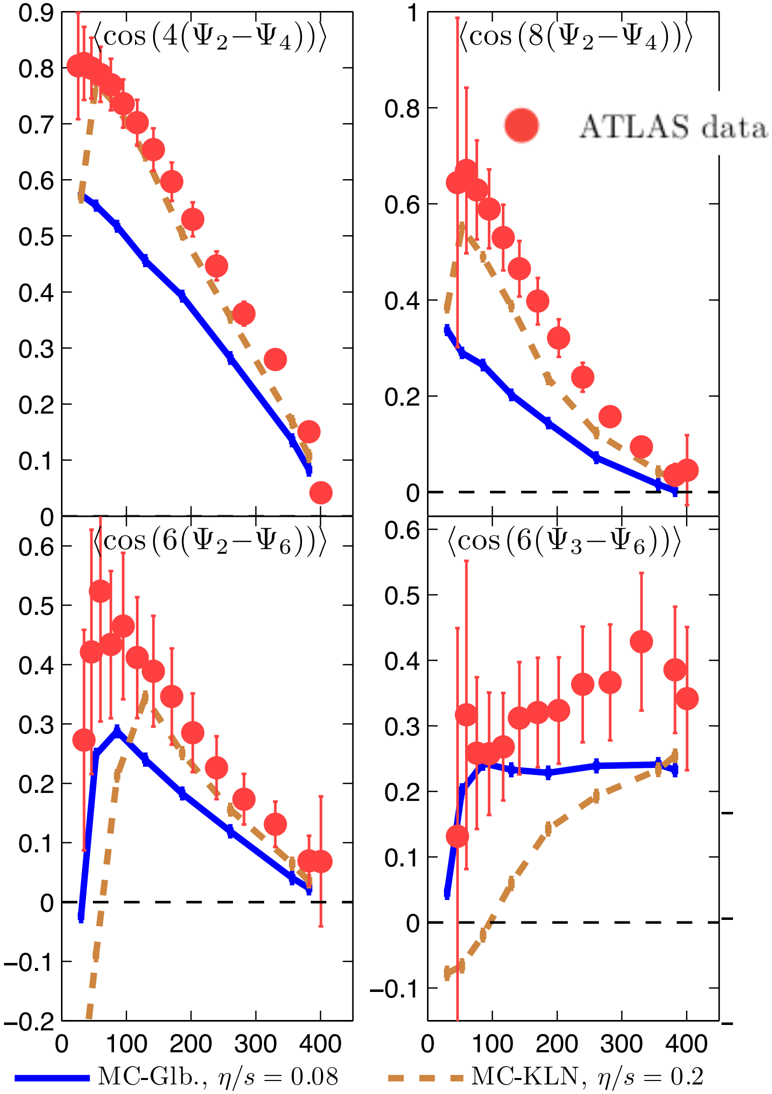
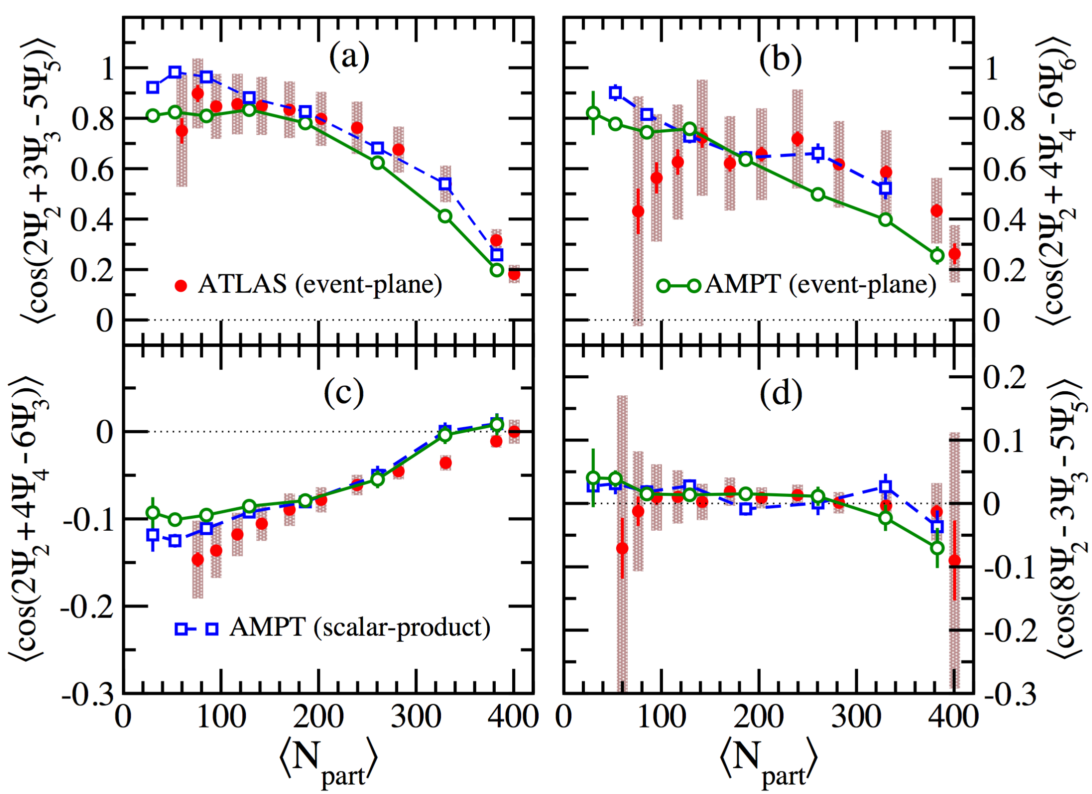


Initial geometry + transport 1307.0980 Bhalerao,et.al.



EbyE hydro and transport models reproduce features in the data

What is the origin of mode-mixing? example

- Hadrons freezeout from exponential distribution of the flow field

$$E\frac{d^3N}{d^3\vec{p}} \approx \frac{g}{(2\pi)^3}\int_\Sigma \exp(-\frac{p\cdot u(x)}{T})p\cdot d^3\sigma(x)$$

- Flow field u(x) has a harmonic modulation driven by geometry

$$u(\phi) = u_0(1+2\sum \beta_n \cos(\phi - \Phi_n))$$

- Quadratic term in saddle-point expansion leads to mode-mixing

$$e^{-p_\mathrm{T}u(\phi)} \approx 1 - p_\mathrm{T}u(\phi) + 1/2p_\mathrm{T}^2u^2(\phi)...$$

Borghini, Ollitrault 2005
Teaney, Yan 2012
Lang, Borghini 2013

$$v_2(p_\mathrm{T}) \approx I(p_\mathrm{T})\beta_2, v_3(p_\mathrm{T}) \approx I(p_\mathrm{T})\beta_3$$

$$I(p_t) \equiv \frac{\bar{u}_\mathrm{max}}{T}(p_t - m_t\bar{\mathrm{v}}_\mathrm{max})$$

$$v_4(p_\mathrm{T}) \approx I(p_\mathrm{T})\beta_4 + \frac{I(p_\mathrm{T})^2}{2}\beta_2^2 \longrightarrow v_2^2$$

$$v_5(p_\mathrm{T}) \approx I(p_\mathrm{T})\beta_5 + I(p_\mathrm{T})^2\beta_2\beta_3 \longrightarrow v_2v_3$$

$$v_6(p_\mathrm{T}) \approx I(p_\mathrm{T})\beta_6 + \frac{I(p_\mathrm{T})^3}{6}\beta_2^3 + \frac{I(p_\mathrm{T})^2}{2}\beta_3^2 + I(p_\mathrm{T})^2\beta_2\beta_4$$

v_2^3, v_2^2, v_2v_4

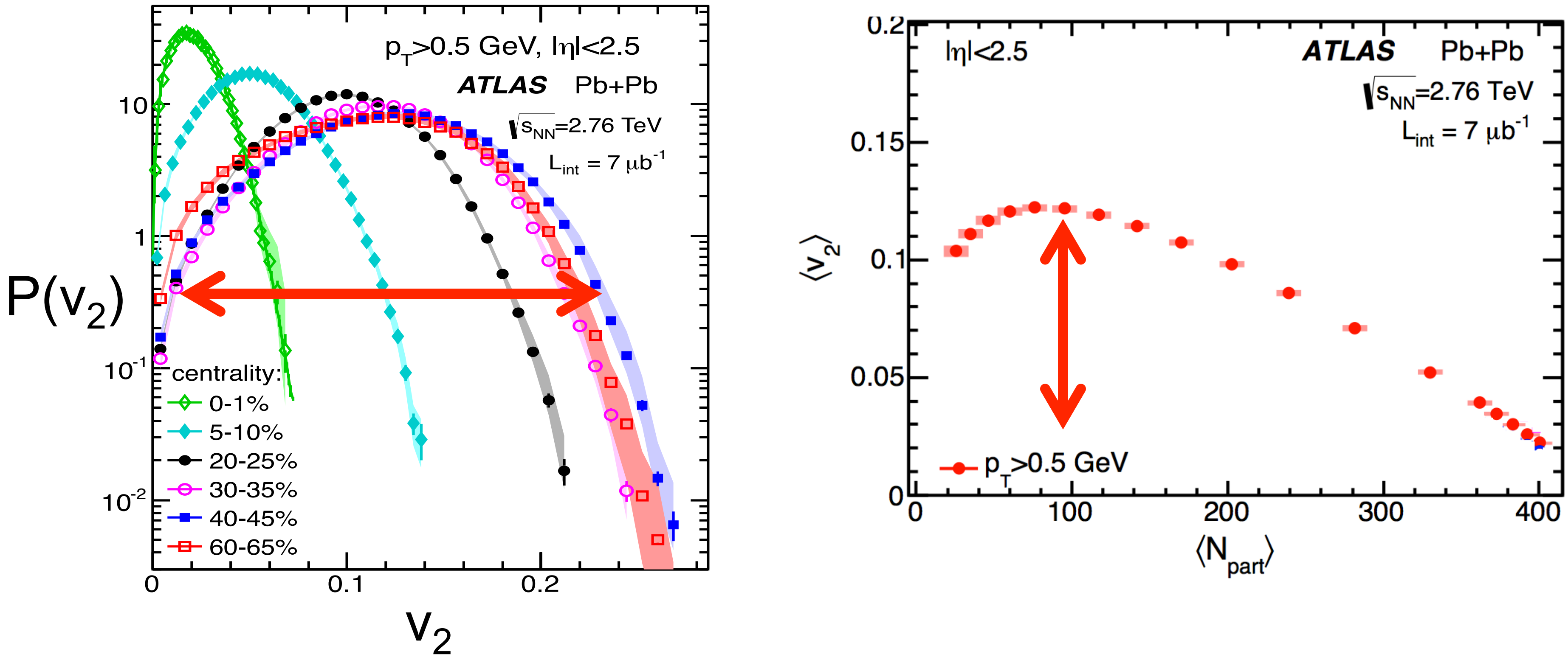
Event-shape selection technique

Can we do better?

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

Can we do better?

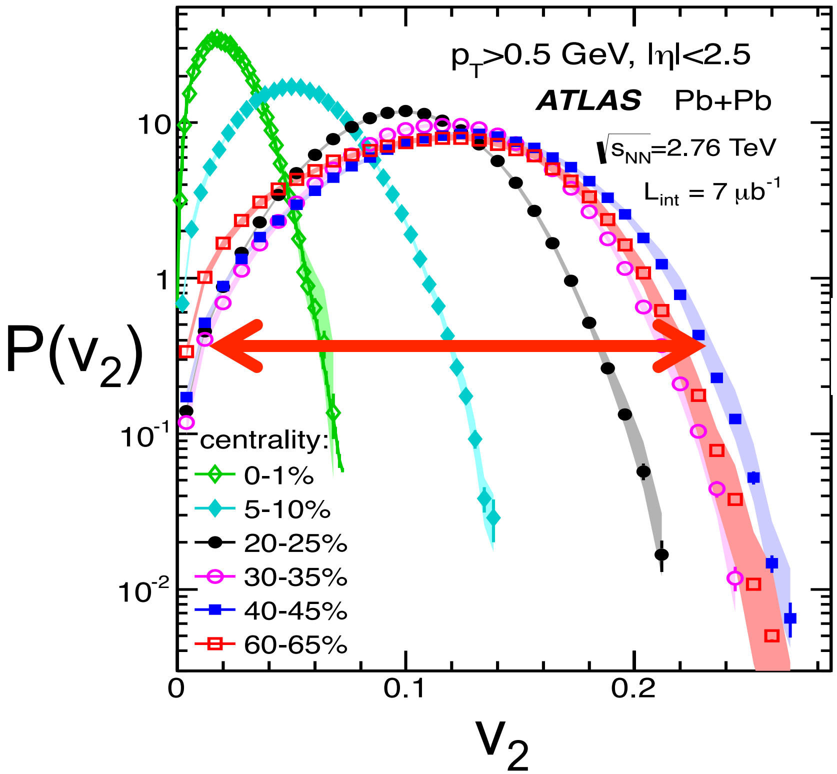
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{evts}} \frac{dN_{evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

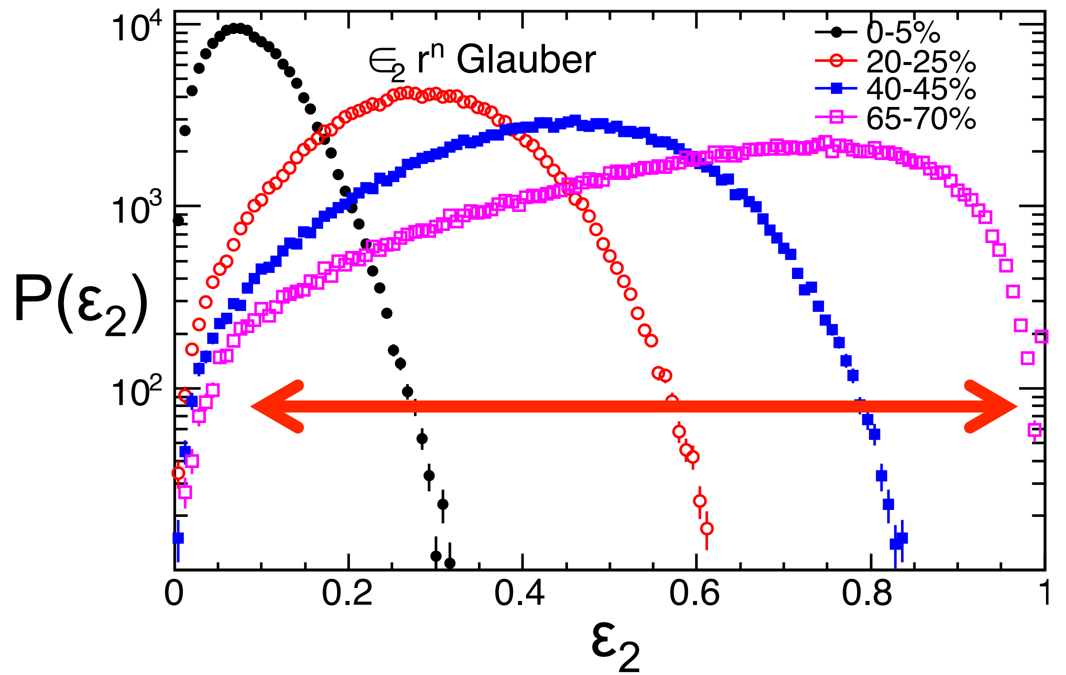


- More variation in v2 within one centrality than variation of mean v2 across all centralities

Can we do better?

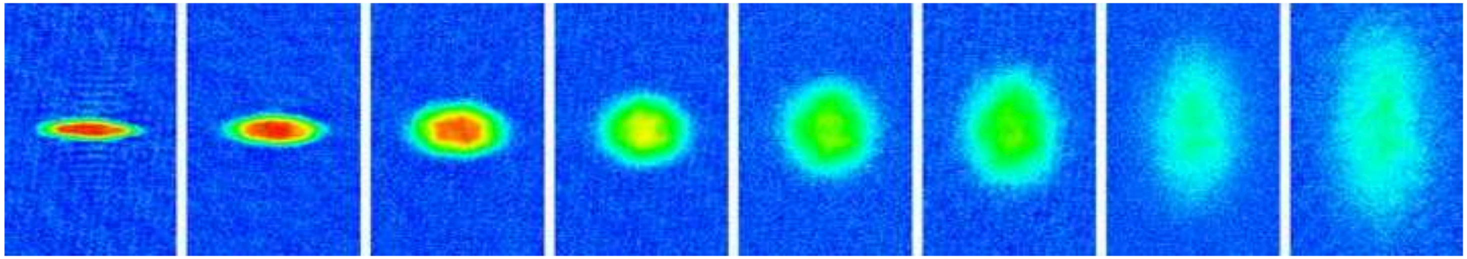
$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\mathrm{evts}}} \frac{dN_{\mathrm{evts}}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$





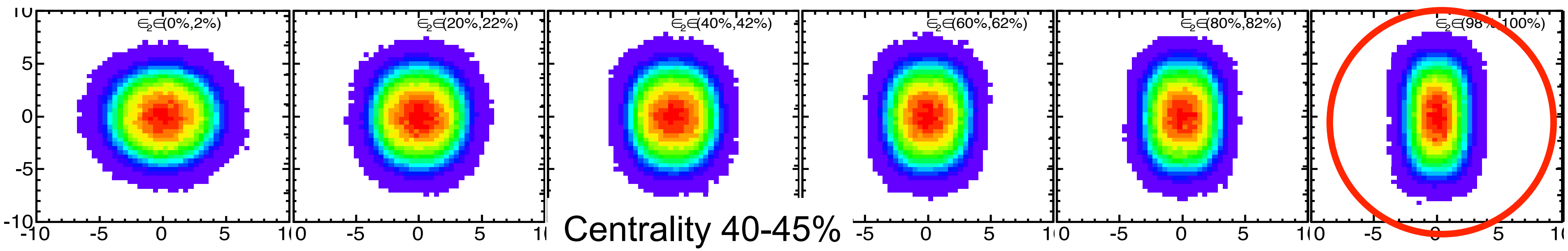
- More variation in v2 within one centrality than variation of mean v2 across all centralities
- Study the variation of vn at fixed centrality but varying event-geometry: “event-shape-selected vn measurements

Ideal case: selecting on eccentricity



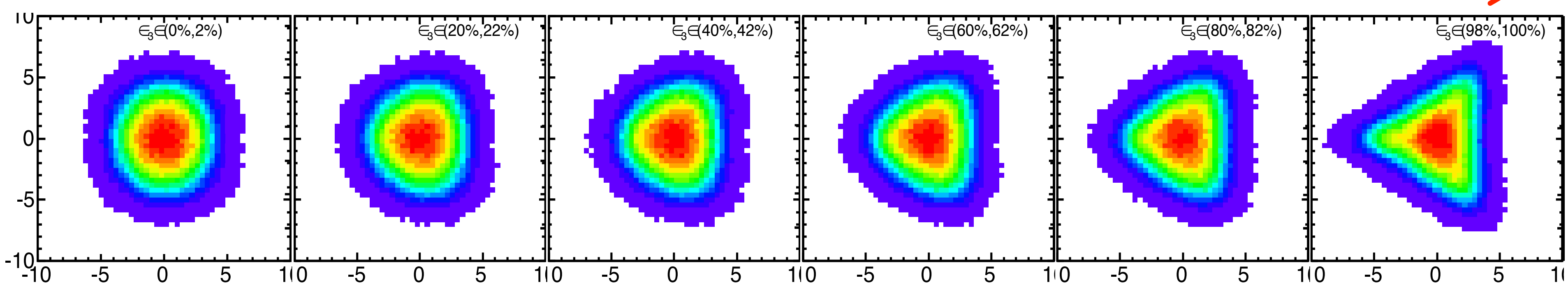
What is the radial flow profile?

Increasing ε_2



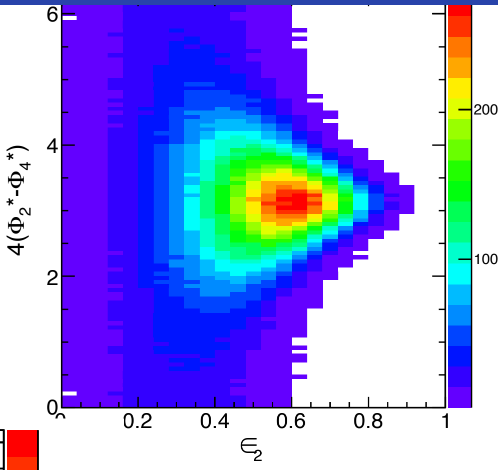
Centrality 40-45%

Increasing ε_3



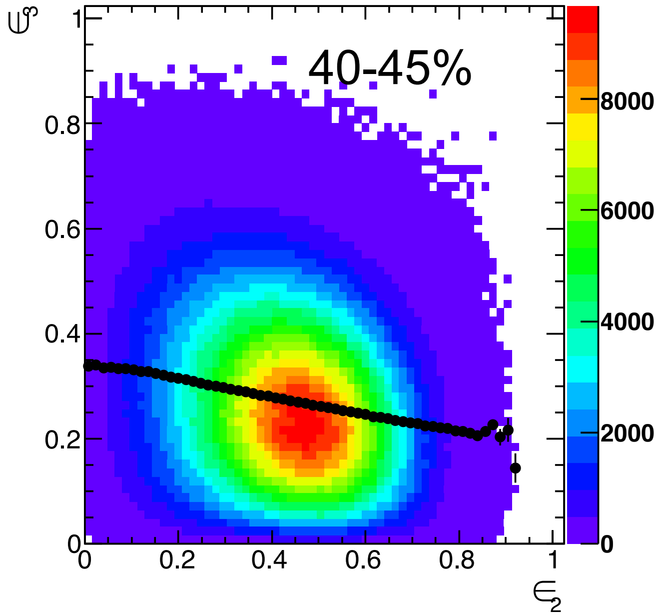
Hidden correlations at fixed-centrality

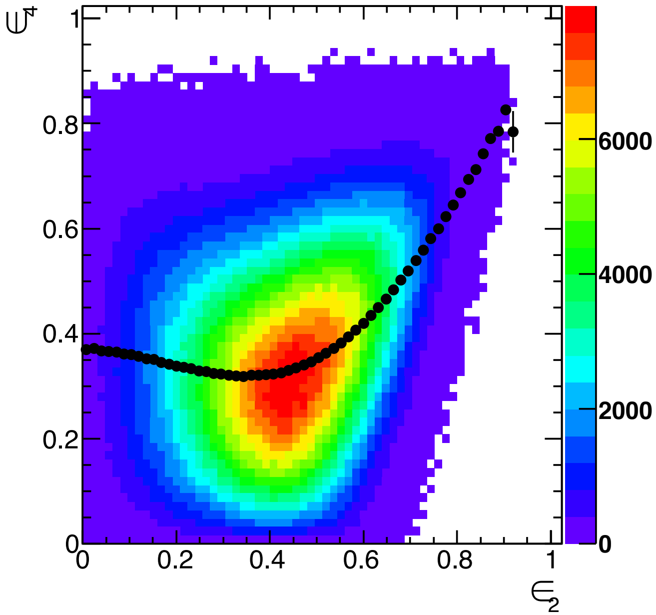
- Evolution of v_n correlated with v_m via
  - Non-linearities from hydro evolution and freeze-out
  - But also initial correlation



anti-correlation between ε_2 and ε_3.

positive-correlation between ε_2 and ε_4.





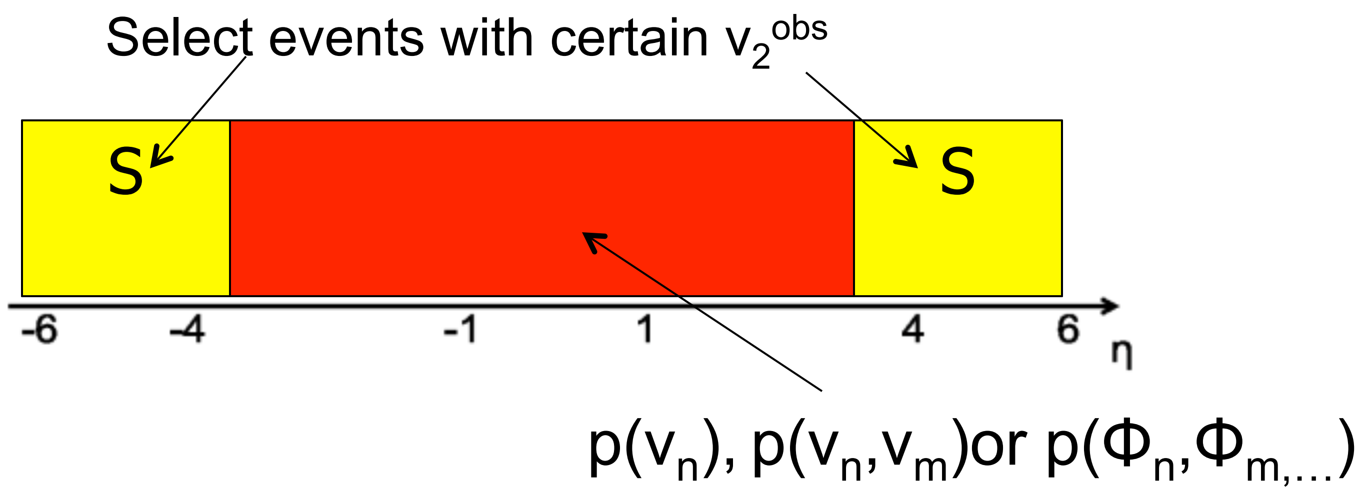
1311.7091

- Naturally studied via event-shape selection technique
  - E.g. select events with different v_2 and study v_n. in FIXED centrality

Event-shape selection technique

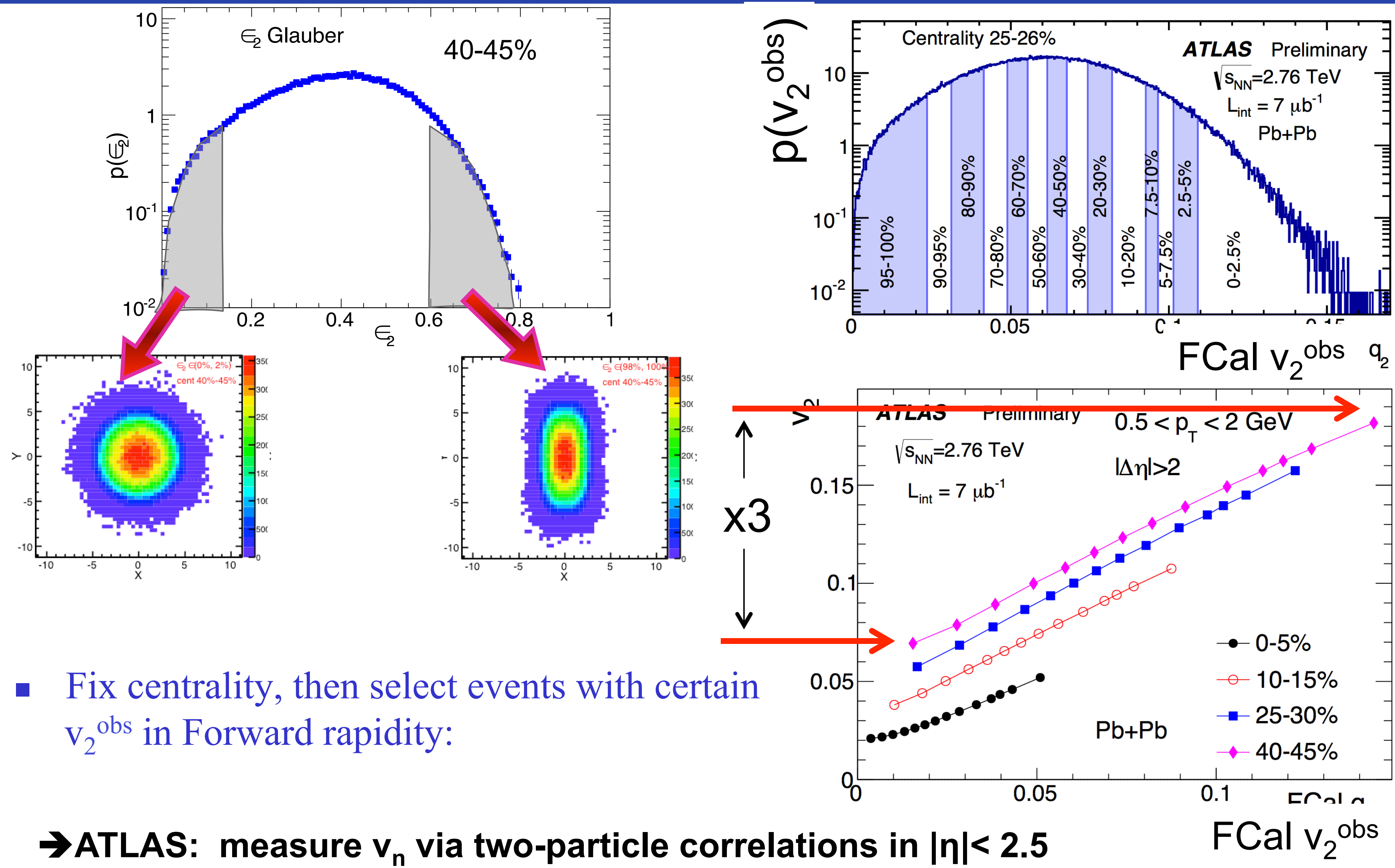
Schukraft, Timmins, and Voloshin, arXiv:1208.4563

Huo, Mohapatra, JJ arxiv:1311.7091



$$\vec{q}_n = \frac{1}{\Sigma w}(\Sigma w \cos n\phi_n, \Sigma w \sin n\phi_n), w = p_T \ , \qquad q_n = |\vec{q}_n| \text{ or } v_n^{obs}$$

More info by selecting on event-shape

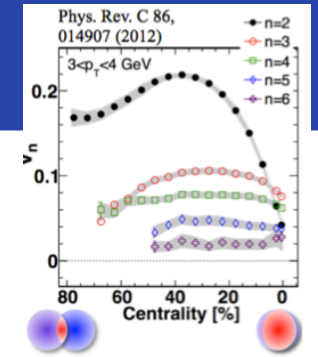


- Fix centrality, then select events with certain v_2^{obs} in Forward rapidity:

➔ATLAS: measure v_n via two-particle correlations in |η|< 2.5

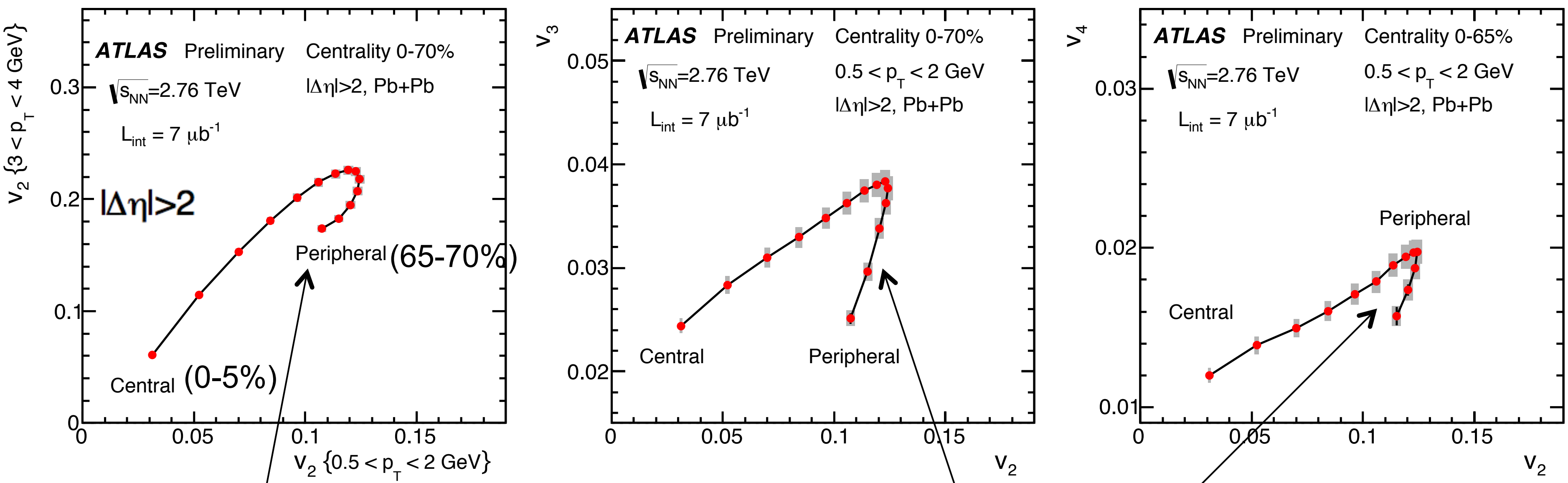
Vary ellipticity by a factor of 3!

v_n-v_2 correlations: centrality dependence



- First correlation without event v_2-selection, 5% steps

v_2 (higher p_T) v_3 v_4



"Boomerang" reflects stronger viscous damping at higher p_T and peripheral

"Boomerang" reflects reflects different centrality dependence, which is also sensitive to the viscosity effect.

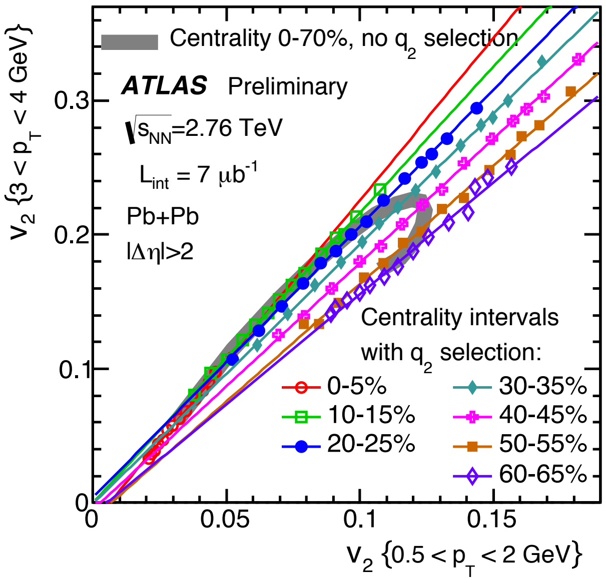
S. Mohapatra

v_n-v_2 correlations: within fixed centrality

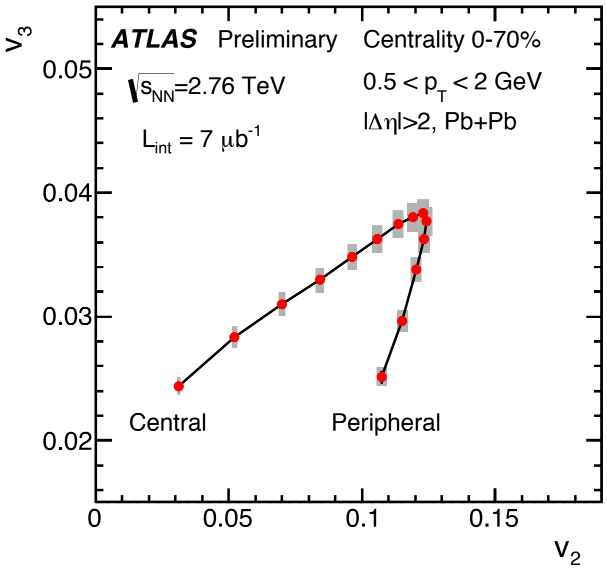
- Fix system size and vary the ellipticitiy!

Probe $p(v_n,v_2)$

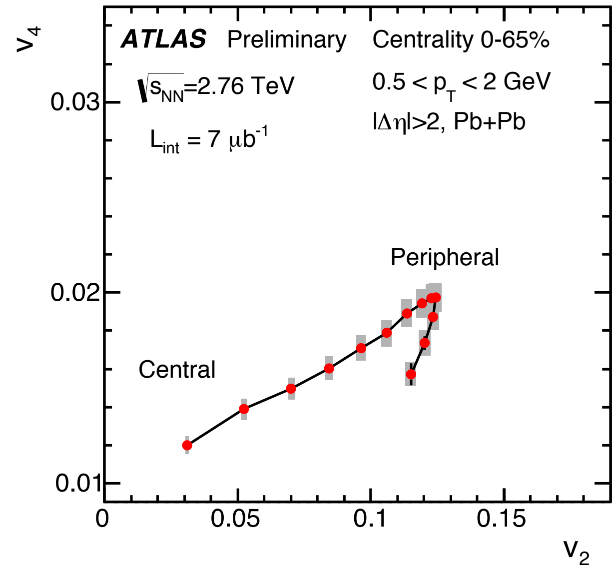
v_2 (higher p_T)



v_3



v_4

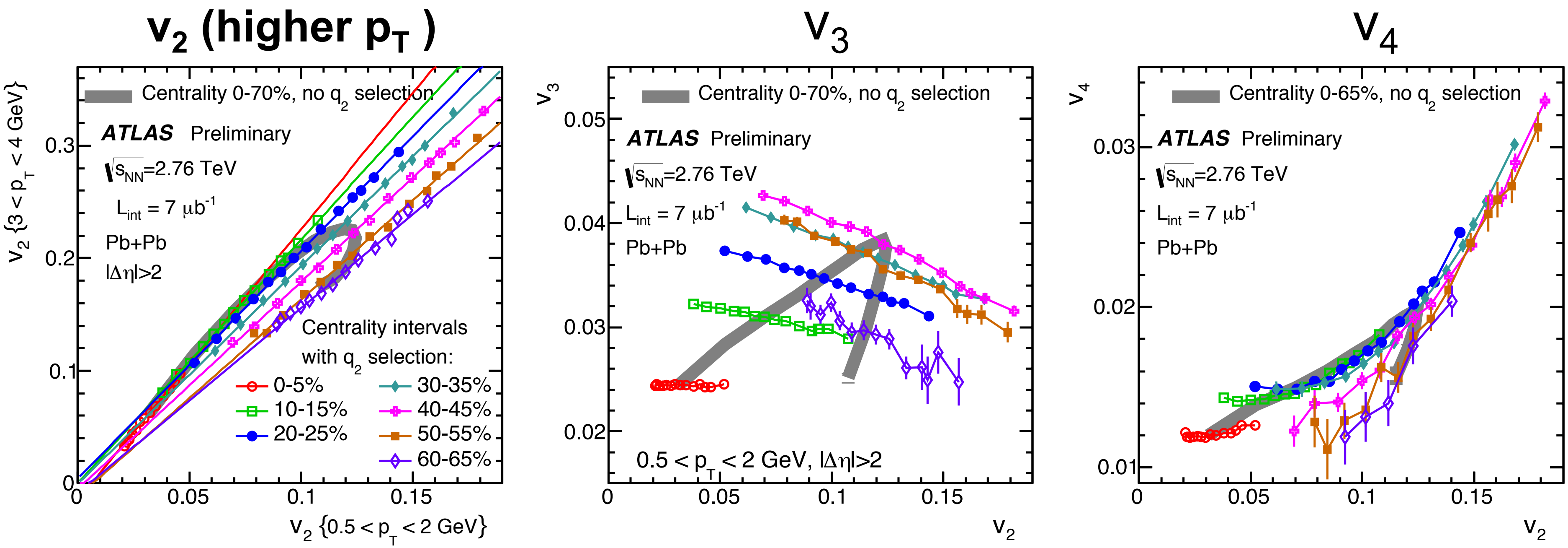


Linear correlation for forward v_2-selected bin → viscous damping controlled by system size, not shape

v_n-v_2 correlations: within fixed centrality

- Fix system size and vary the ellipticitiy!
- Overlay ε_3-ε_2 and ε_4-ε_2 correlations, rescaled

Probe $p(v_n,v_2)$



Linear correlation for forward v_2-selected bin→viscous damping controlled by system size, not shape

Clear anti-correlation,

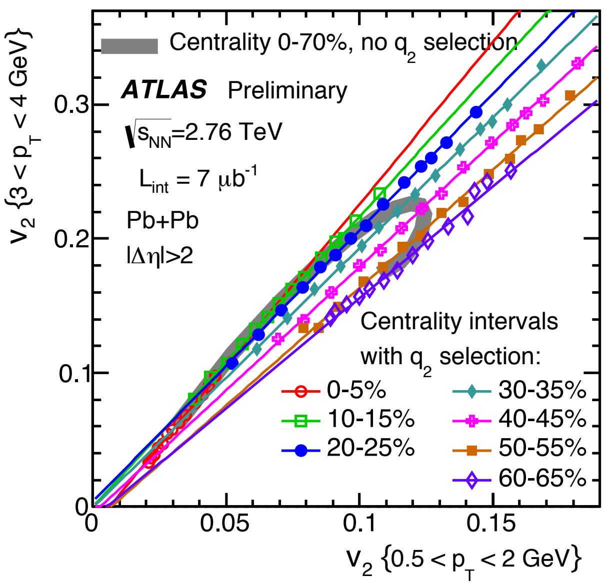
quadratic rise from non-linear coupling to v_2^2

v_n-v_2 correlations: within fixed centrality

- Fix system size and vary the ellipticitiy!
- Overlay ε_3-ε_2 and ε_4-ε_2 correlations, rescaled

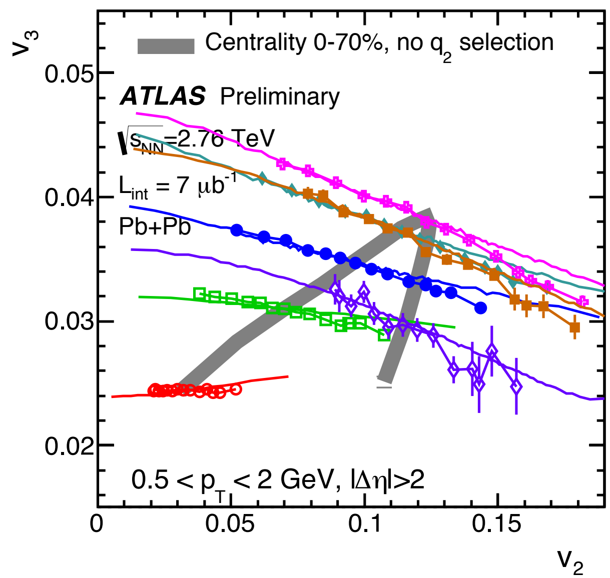
Probe $p(v_n,v_2)$

v_2 (higher p_T)



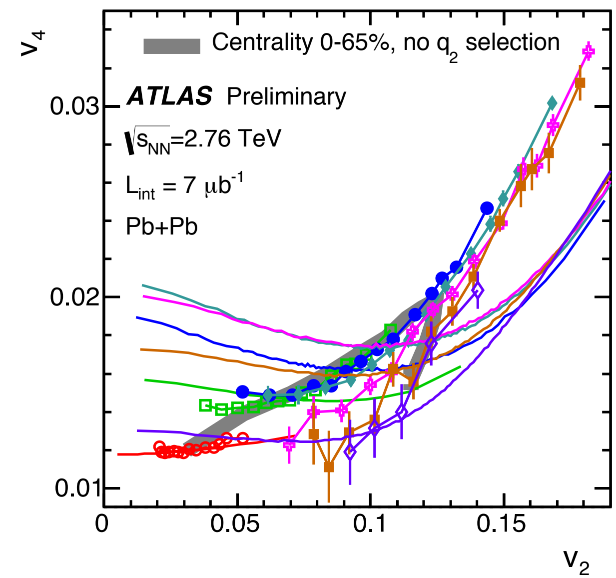
Linear correlation for forward v_2-selected bin→viscous damping controlled by system size, not shape

v_3



Clear anti-correlation, mostly initial geometry effect!!

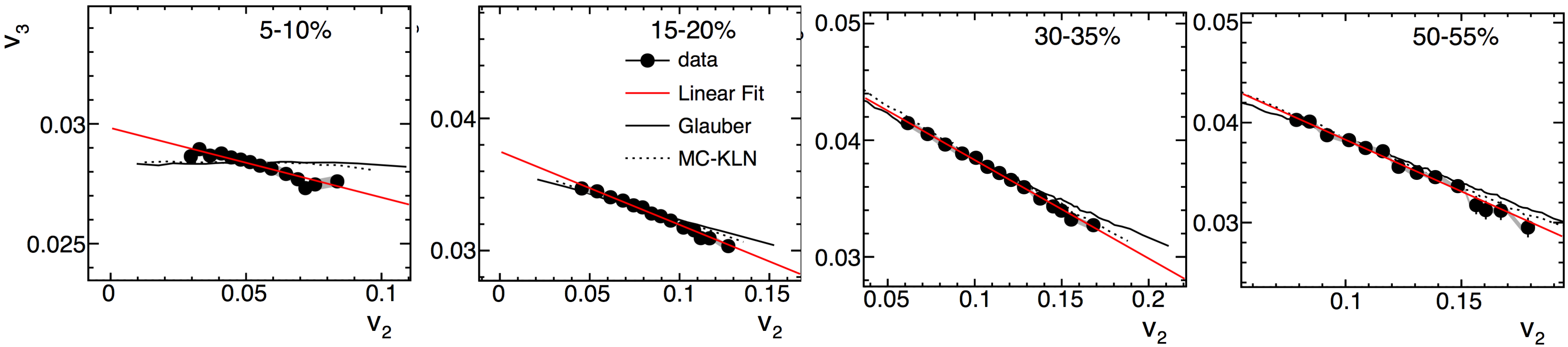
v_4



quadratic rise from non-linear coupling to v_2^2 initial geometry do not work!!

Initial geometry describe v_3-v_2 but fails v_4-v_2 correlation

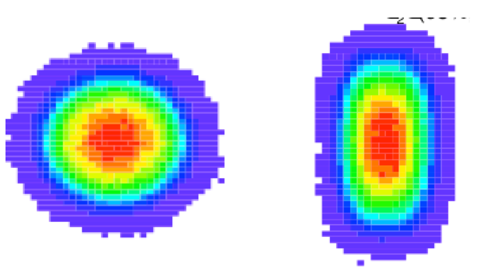
Anti-correlation between v3 and v2

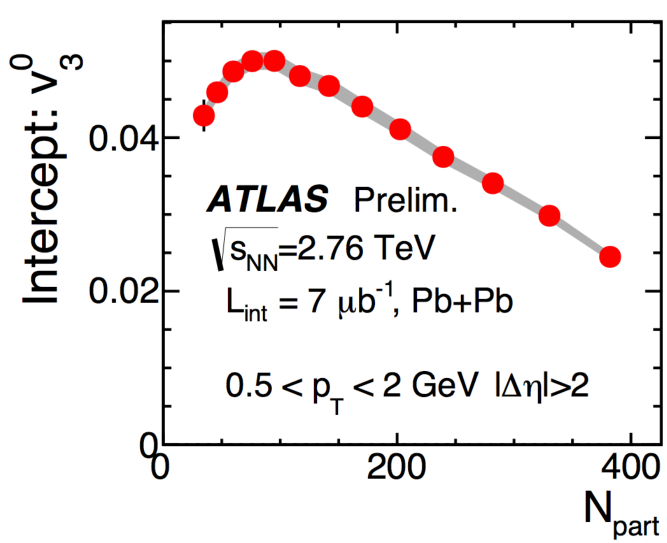


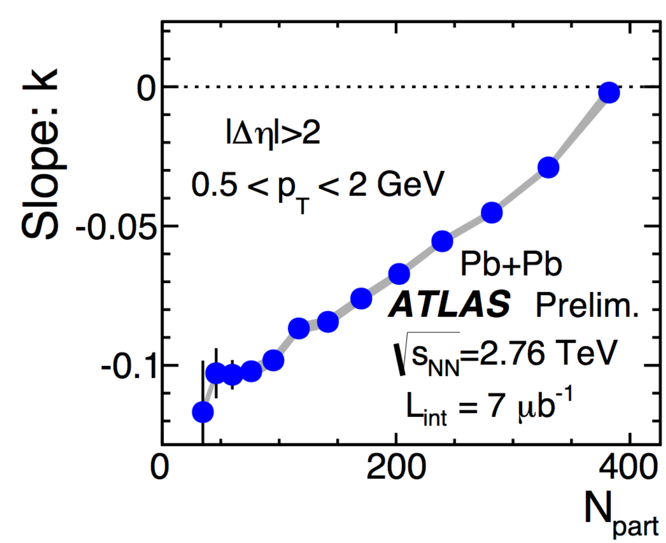
Can be used to fine tune initial geometry models!

- Quantified by a linear fit and extract the intercept and slope

$$v_3 = k v_2 + v_3^0$$



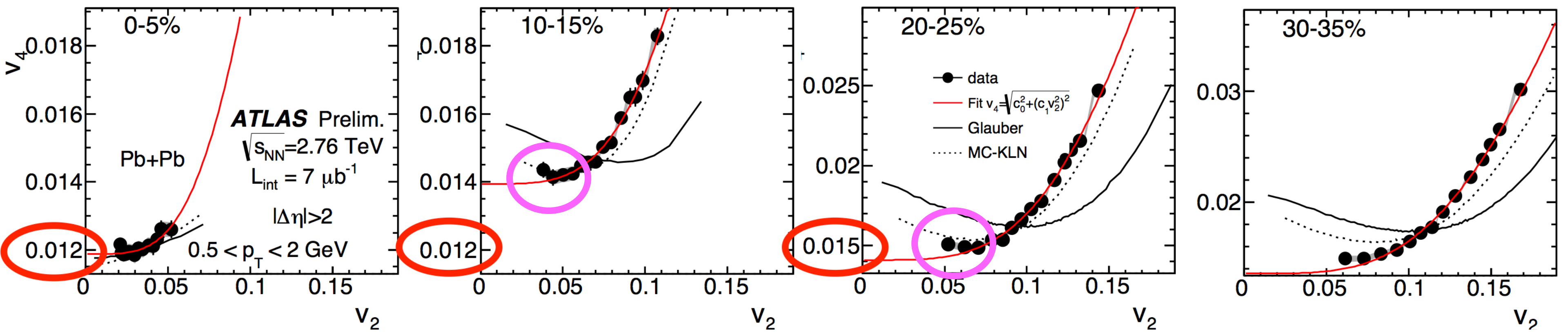




Events with zero ε_2 has larger average ε_3 → larger v_3.

linear (ε_4) and non-linear (v_2^2) component of v_4

- v_4-v_2 correlation for fixed centrality bin $v_4 e^{i4\Phi_4} = c_0 e^{i\Phi_4^*} + c_1\left(v_2 e^{i2\Phi_2}\right)^2 \Rightarrow$ Fit by $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$



- Fit $v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$ to separate linear (ε_4) and non-linear (v_2^2) component

linear (ε_4) and non-linear (v_2^2) component of v_4

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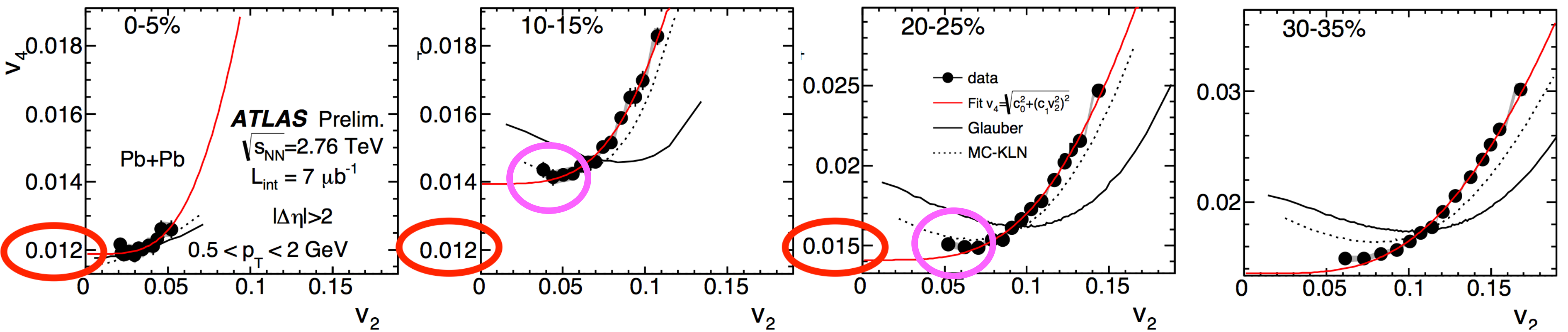

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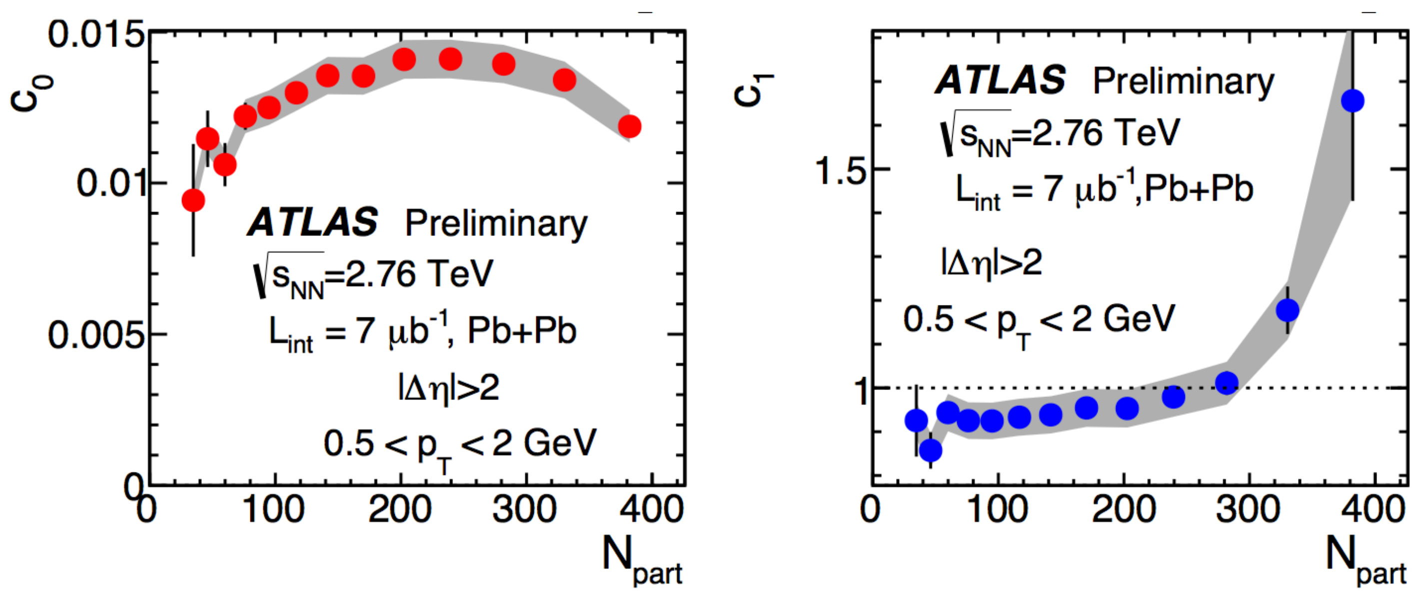
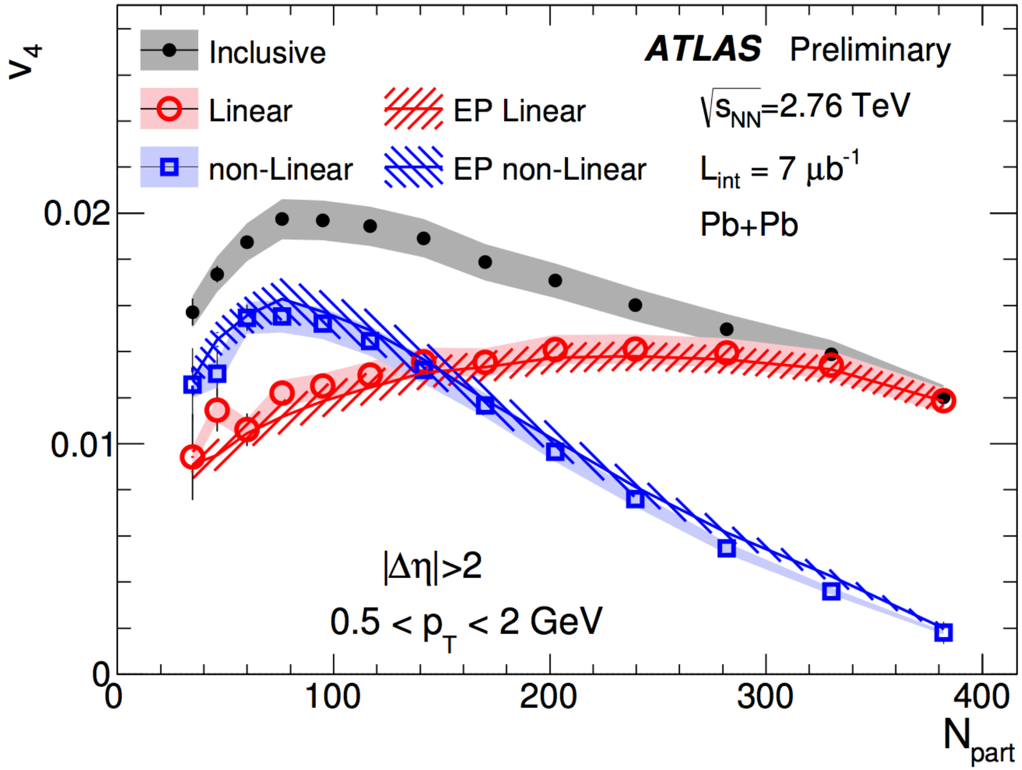
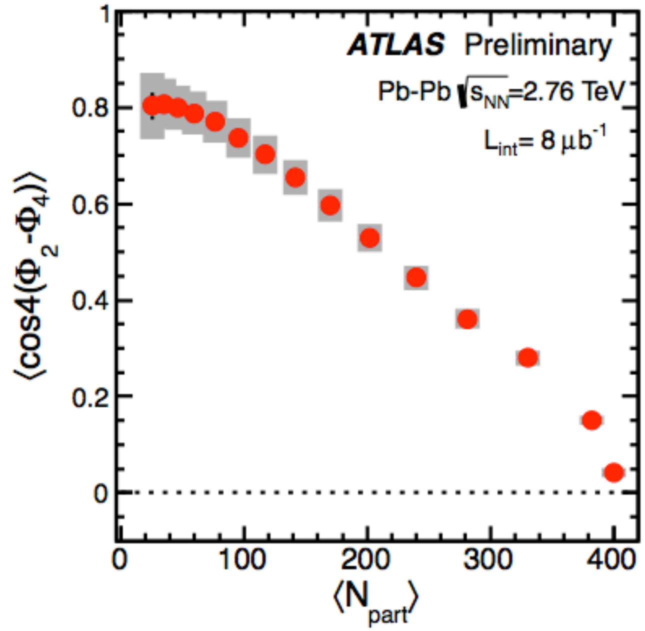


v4 decomposition compare with EP correlation

- Leading non-linear term is enough $v_4 e^{i4\Phi_4} = c_0 e^{i4\Phi_4^*} + c_1 v_2^2 e^{i4\Phi_2}$





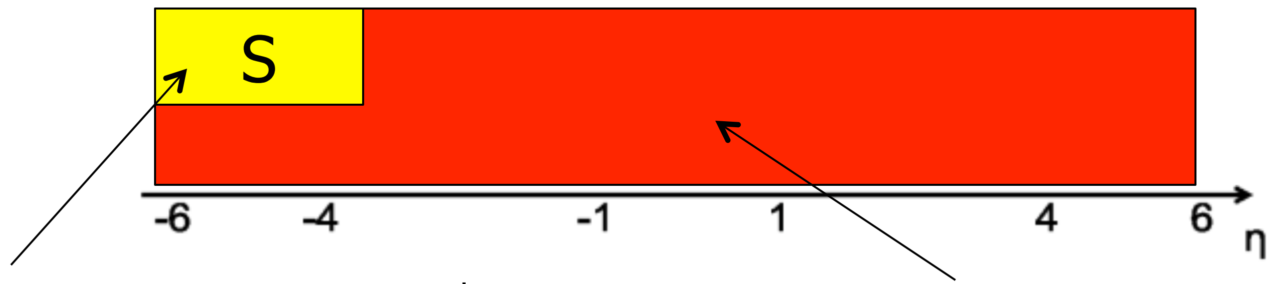
- If so, can also predict L and NL component from EP correlations
  - Good agreement is seen!

$$v_4^{NL} = v_4 \langle \cos 4(\Phi_2 - \Phi_4) \rangle , \quad v_4^{L} = \sqrt{v_4^2 - (v_4^{NL})^2}$$

What about select on one side?

Schukraft, Timmins, and Voloshin, arXiv:1208.4563

Huo, Mohapatra, JJ arxiv:1311.7091

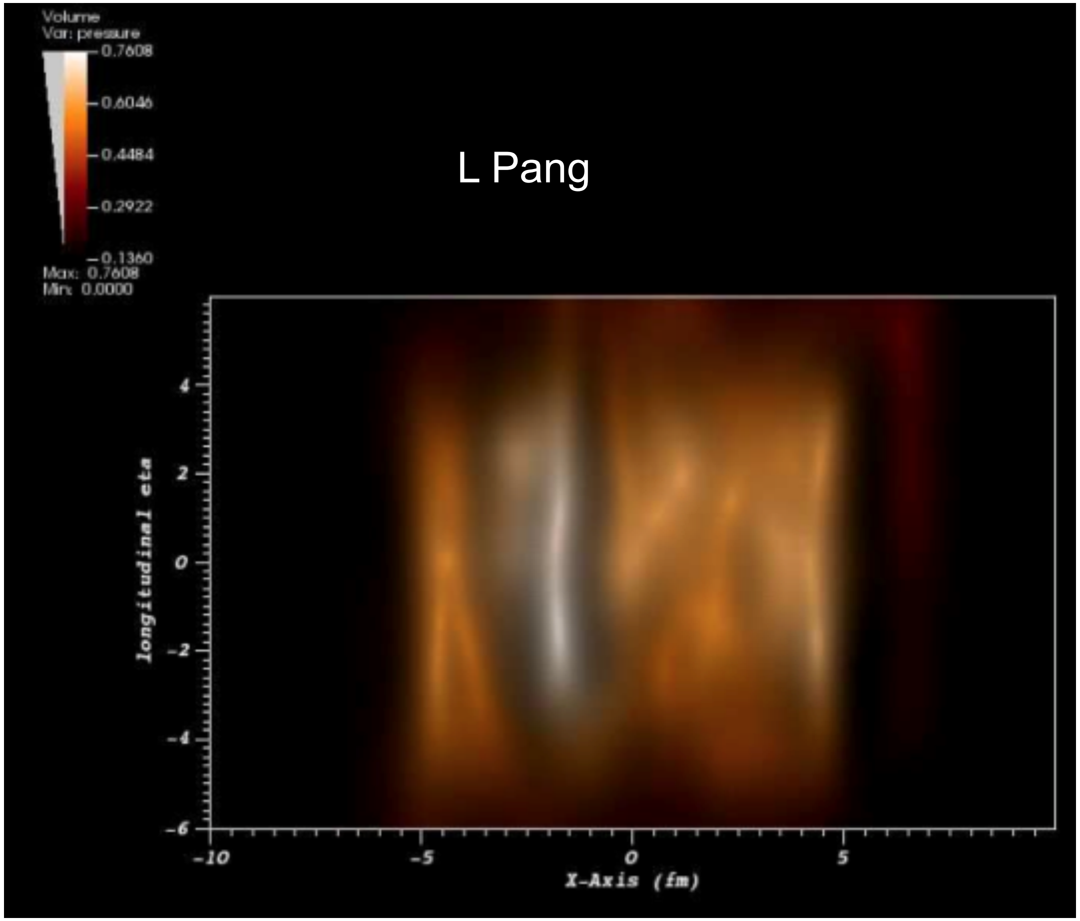


Select events with certain v_2^{obs}

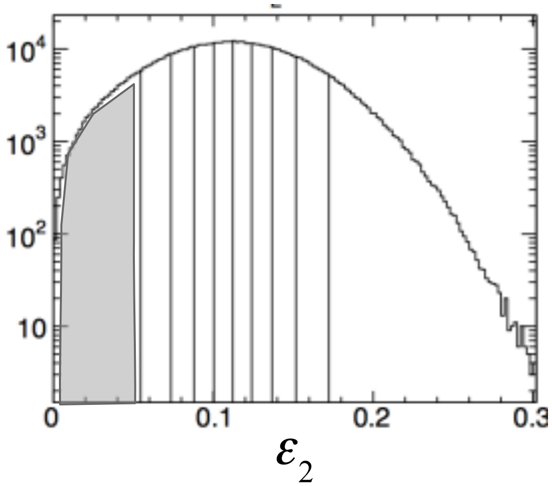
$p(v_n)$, $p(v_n,v_m)$or $p(\Phi_n,\Phi_{m,\ldots})$

AMPT model

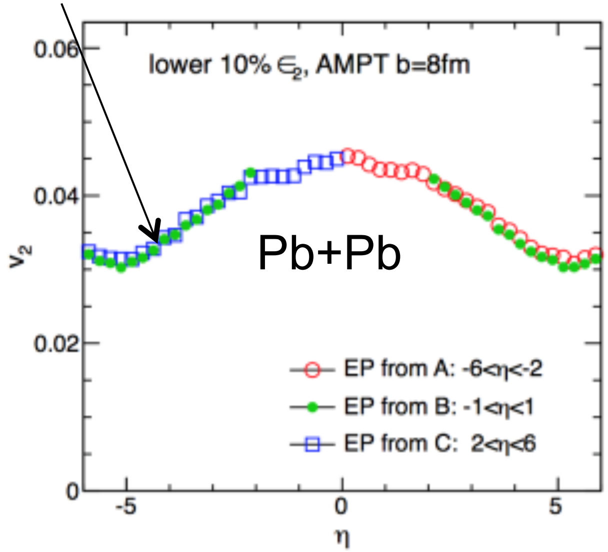
- AMPT model: Glauber+HIJING+transport
  - Has fluctuating geometry and collective flow
  - Longitudinal fluctuations and initial flow



$v_2(\eta)$: select on ε_2



Flow suppressed



1311.7091

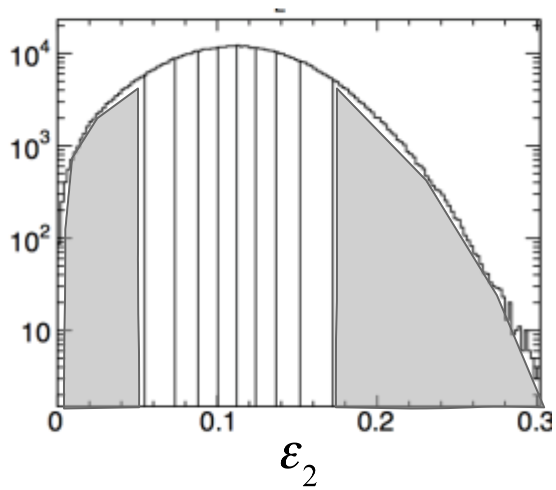
$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

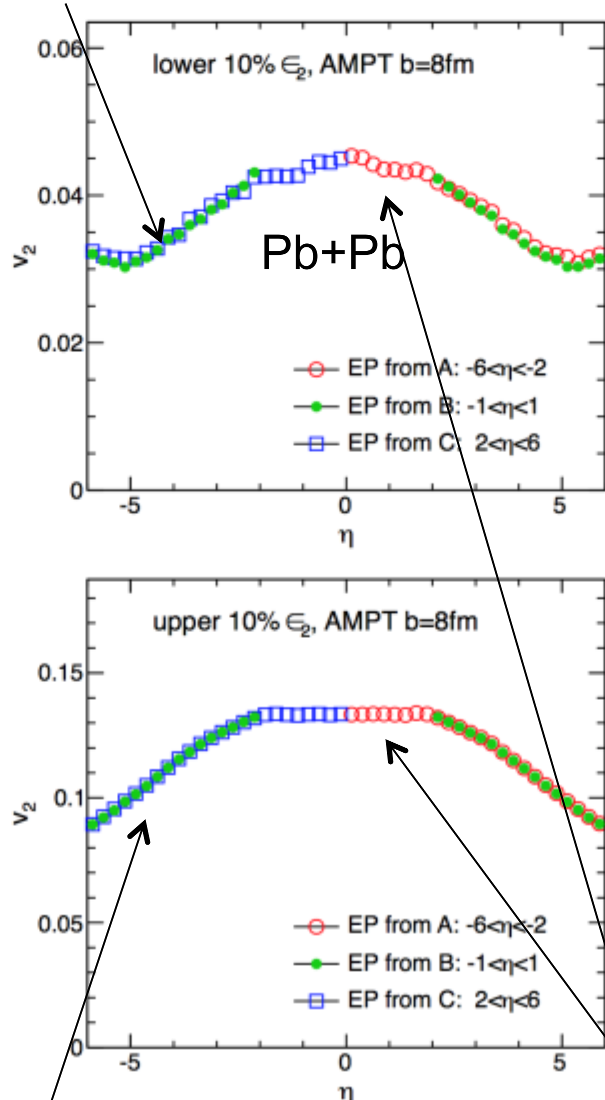
$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

$v_2(\eta)$: select on ε_2

1311.7091



Flow suppressed



$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

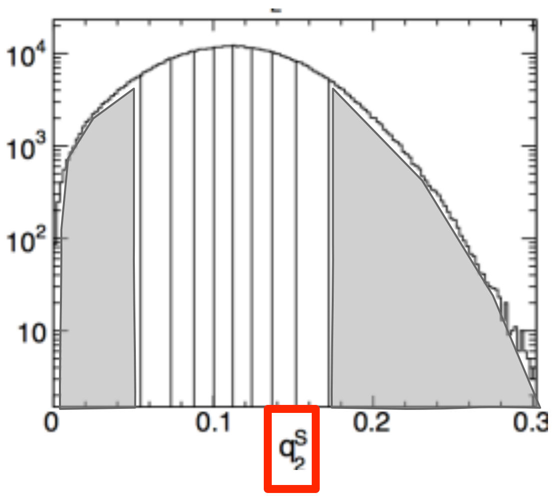
Flow enhanced

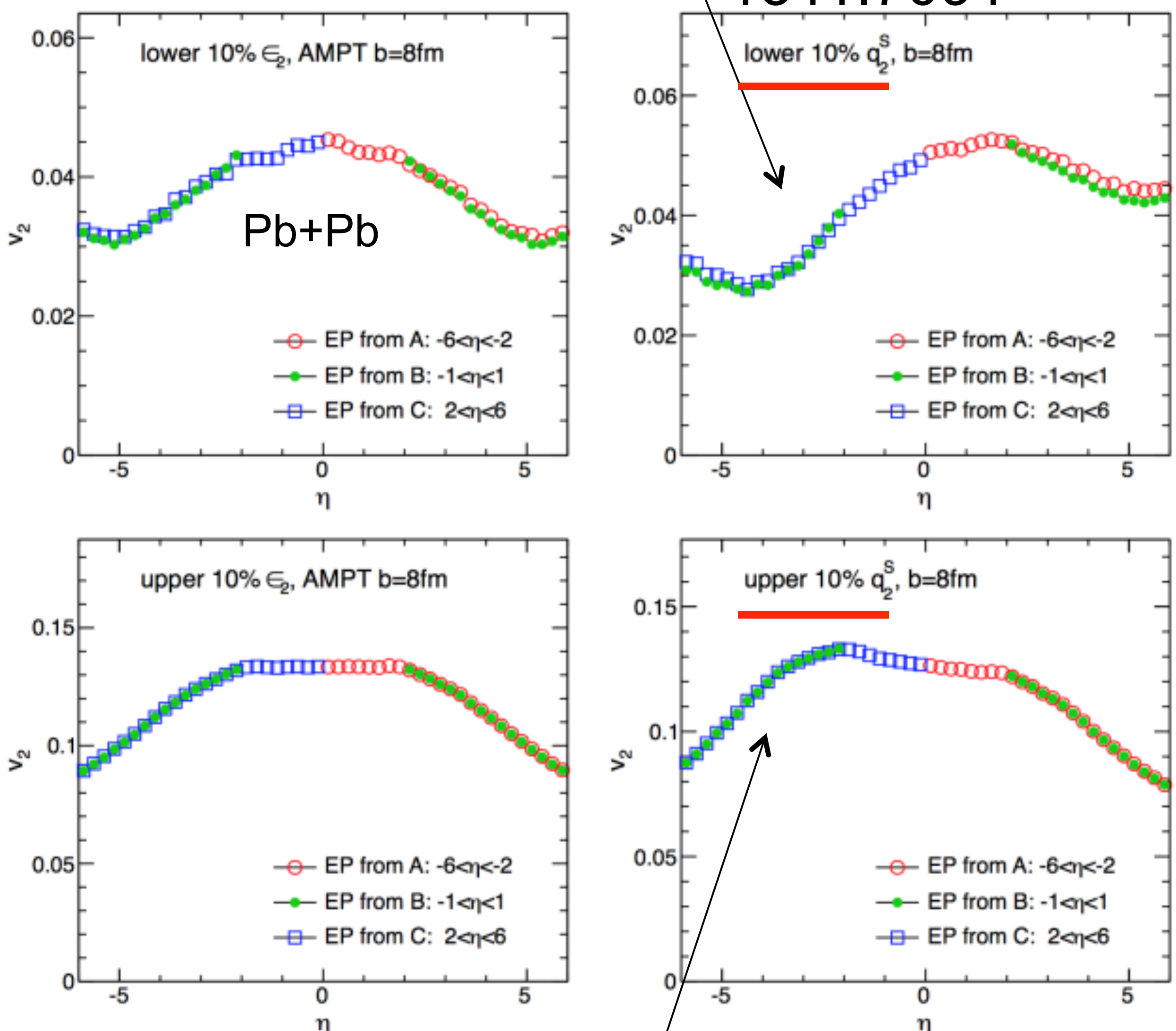
Symmetric distribution expected

$v_2(\eta)$: compare with selection on q_2

Suppression of flow in the selection window

1311.7091





$v_2(\eta)|_{\eta>0}$ when EP in $-6<\eta<-2$

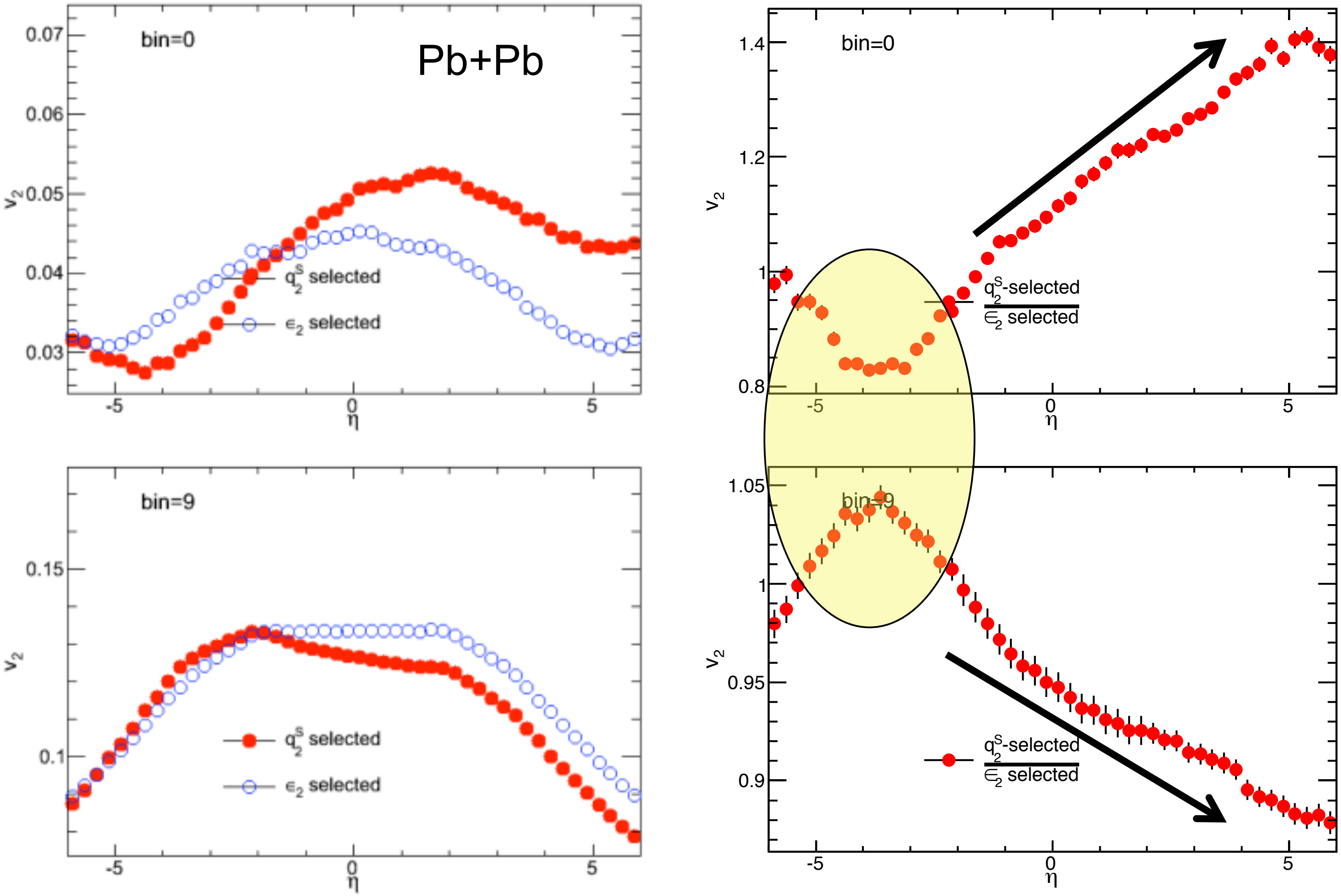
$v_2(\eta)|_{\eta<0}$ when EP in $2<\eta<6$

$v_2(\eta)|_{|\eta|>2}$ when EP in $|\eta|<1$

enhancement of flow in the selection window

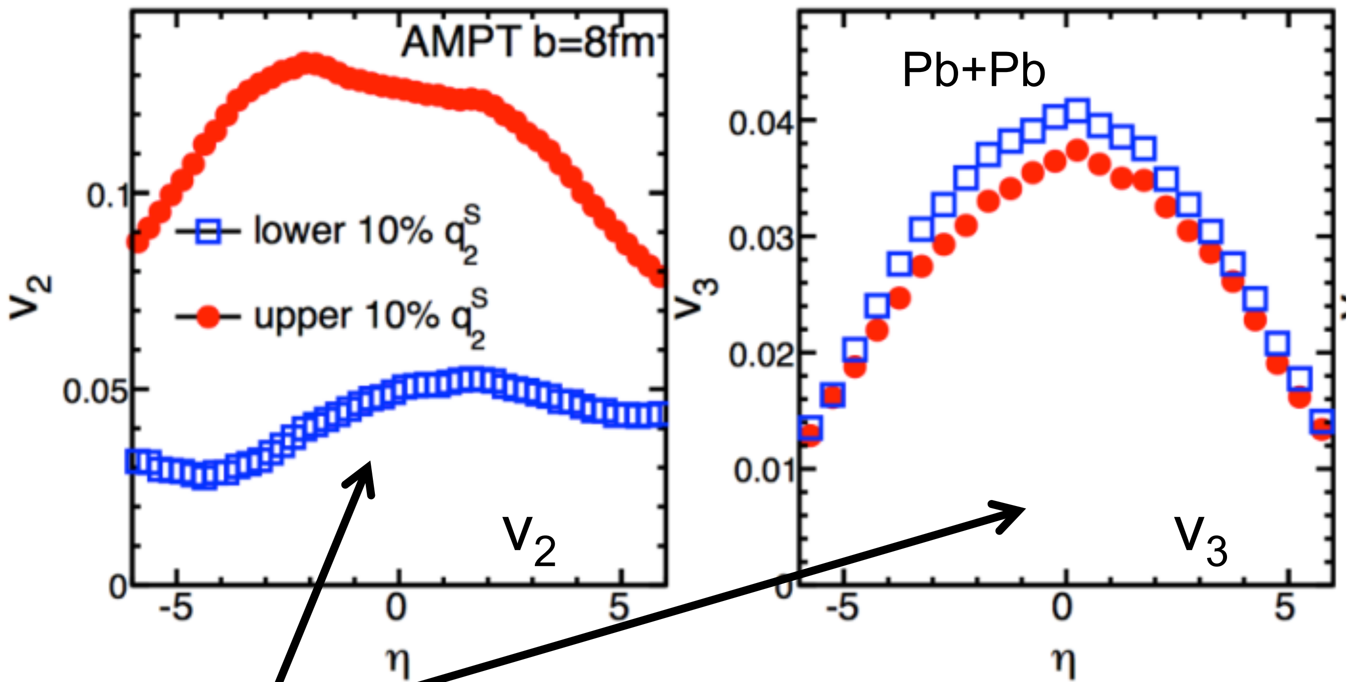
What is the origin of $v_2(\eta)$ asymmetry?

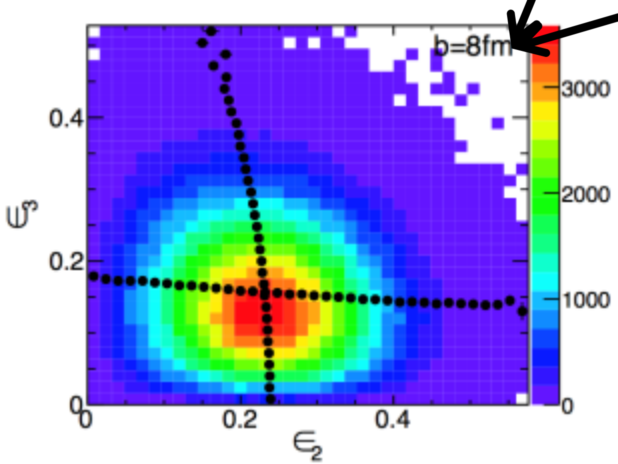
- Suppression/enhancement of flow in the selected window 1311.7091
- Decreasing response to flow selection outside the selection window



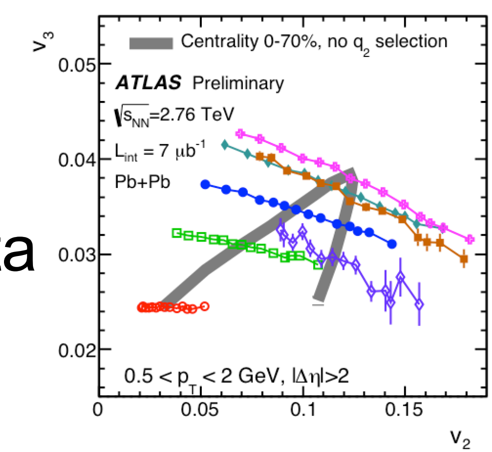
Dependence of $v_3(\eta)$ on q_2 in fixed centrality

- v_3 anti-correlated with v_2 → reflection of $p(\varepsilon_2,\varepsilon_3)$





Similar to data



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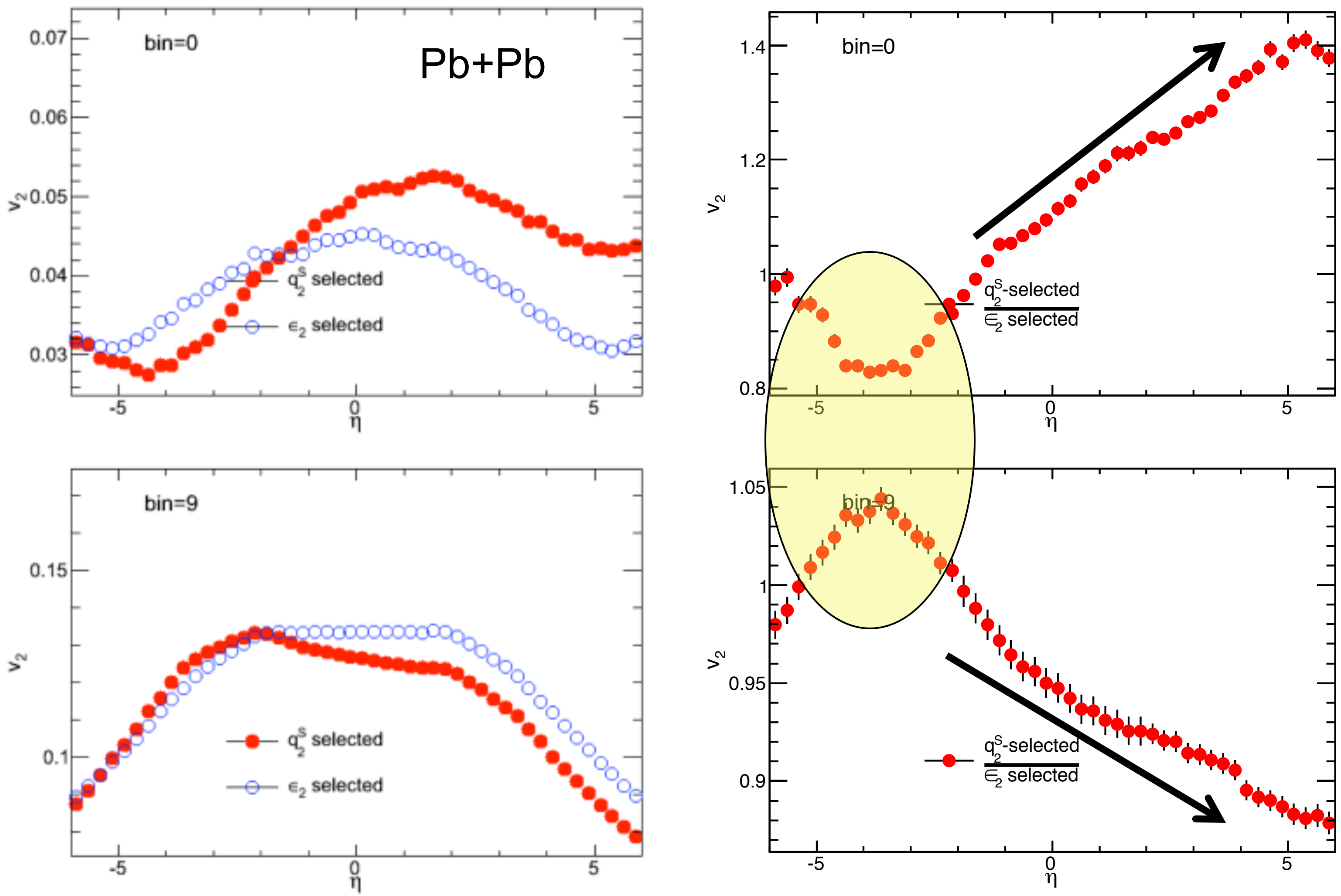
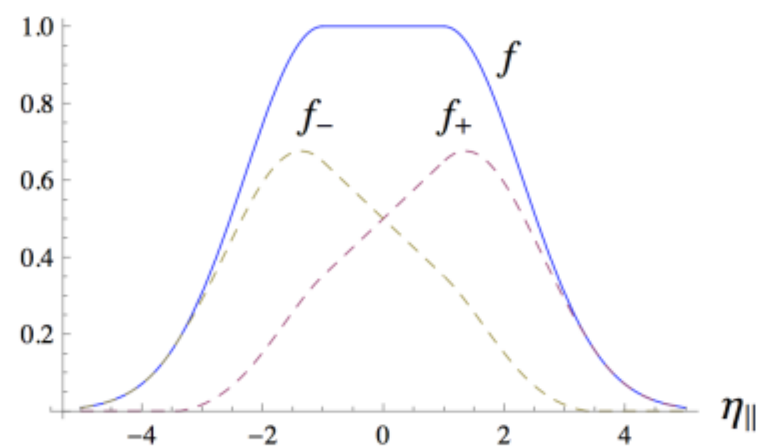


Longitudinal particle production

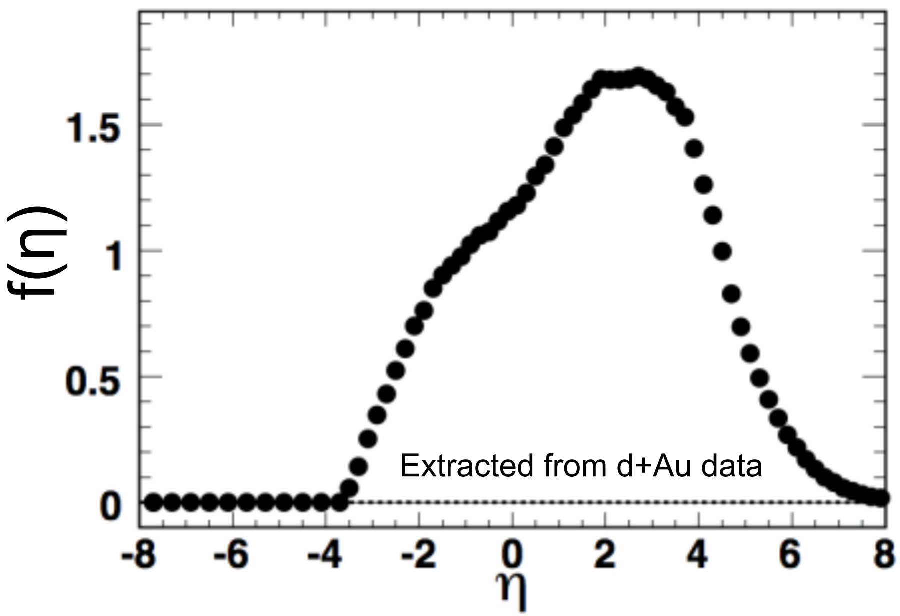
wounded nucleon model — Bialas, Bzdak, Zalewski, Wozniak…. STAR/PHOBOS

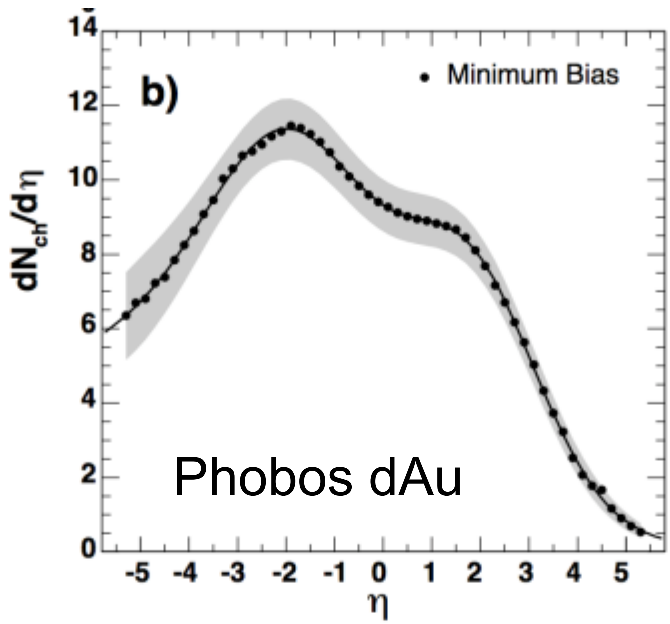
$$dN/d\eta \propto f^{F}(\eta)N^{F}_{\mathrm{part}} + f^{B}(\eta)N^{B}_{\mathrm{part}}$$

- Assumes that after the collision of two nuclei, the secondary particles are produced by independent fragmentation of wounded nucleons

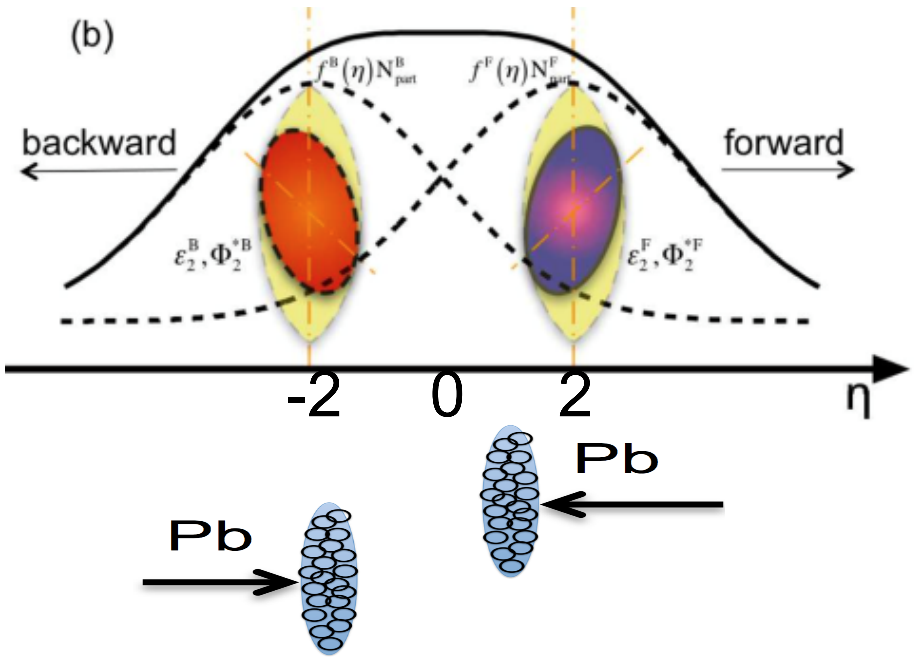


Emission function of one wounded nucleon

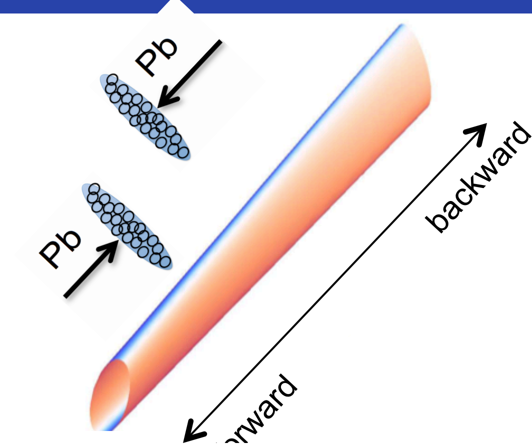


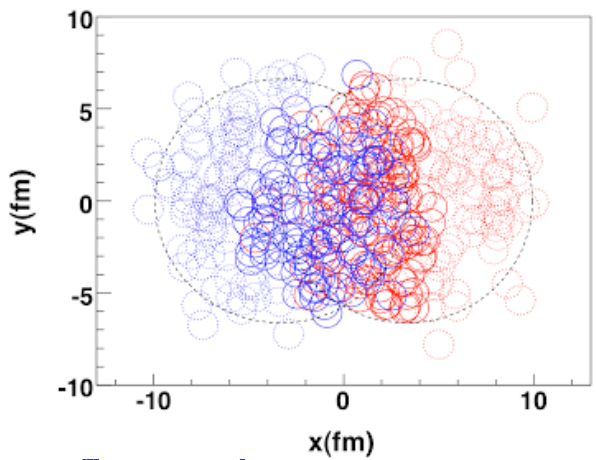


Flow longitudinal dynamics



1011.3354, 1403.6077



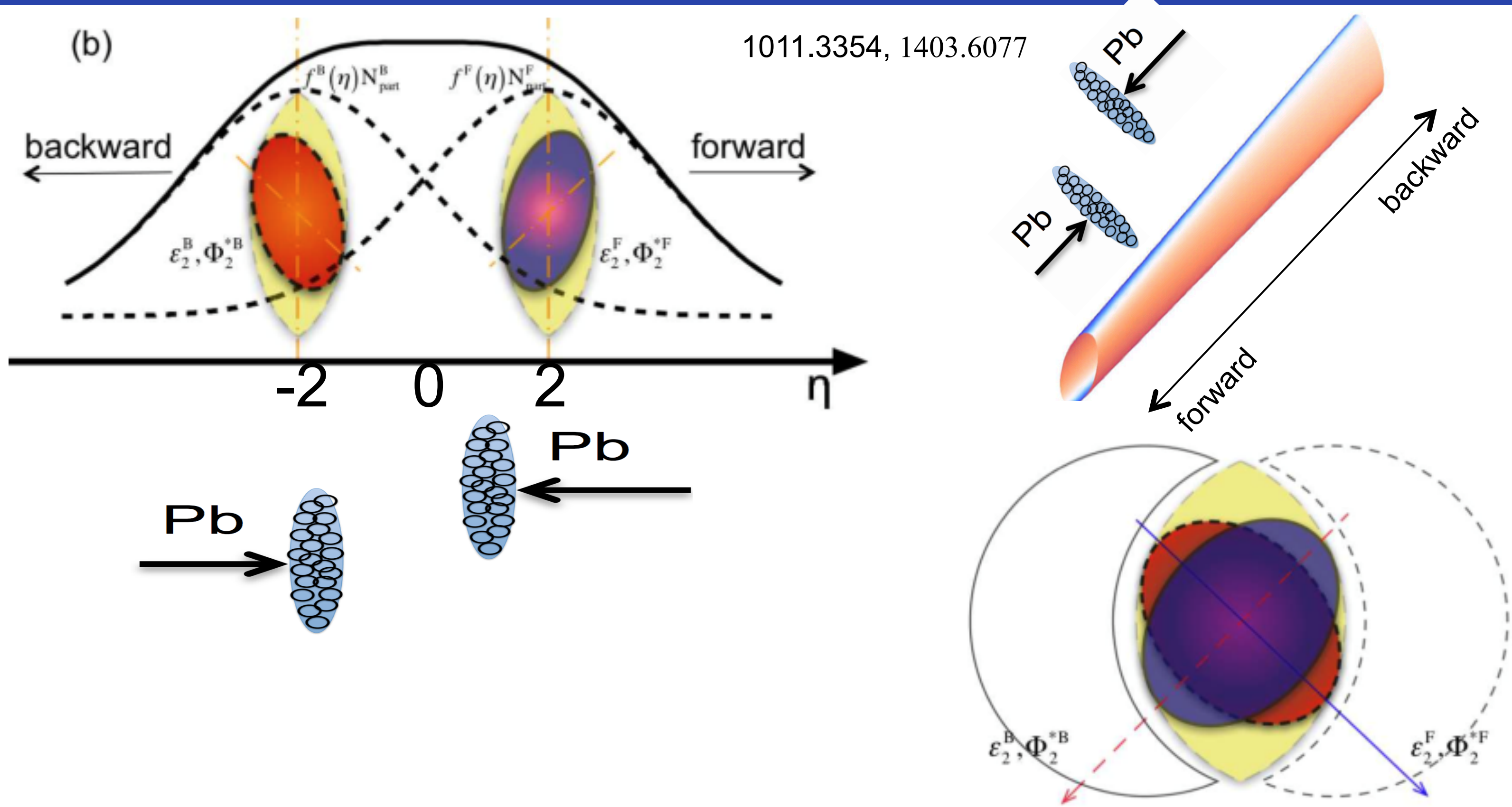


- Shape of participants in two nuclei not the same due to fluctuation

$$\varepsilon^{F}_{m},\Phi^{*F}_{m} \quad \varepsilon^{B}_{m},\Phi^{*B}_{m} \quad \varepsilon_{m},\Phi^{*}_{m} \qquad N^{F}_{part},N^{B}_{part},N_{part} \qquad \varepsilon^{F}_{n},\Phi^{*F}_{n} \neq \varepsilon^{B}_{n},\Phi^{*B}_{n}$$

- Particles are produced by independent fragmentation of wounded nucleons, emission function f (η) not symmetric in η→ Wounded nucleon model

Flow longitudinal dynamics

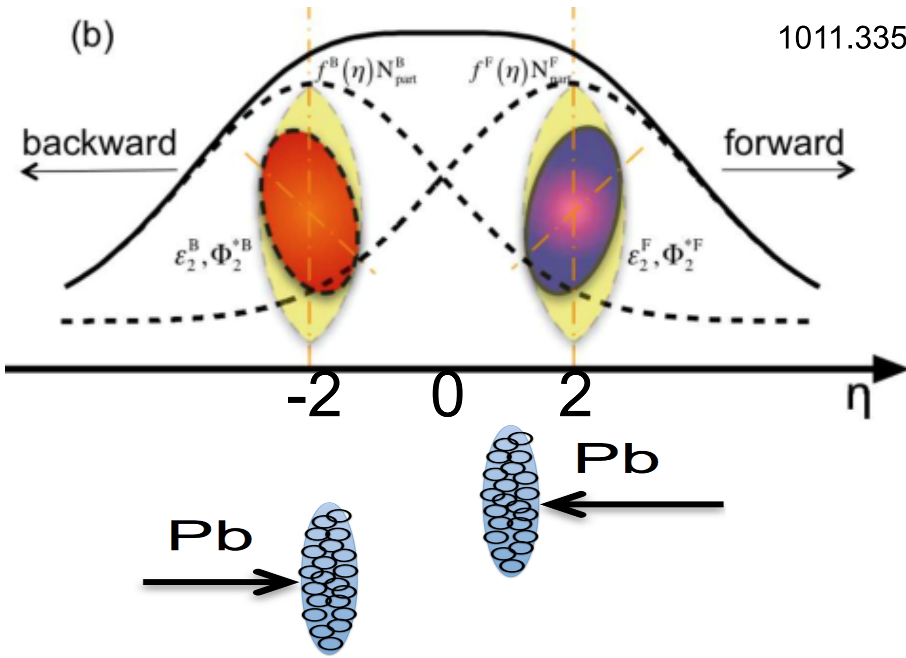


- Shape of participants in two nuclei not the same due to fluctuation

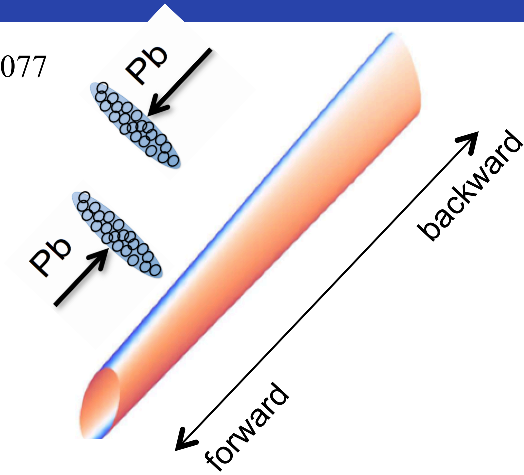
$$\varepsilon_m^{F},\Phi_m^{*F} \quad \varepsilon_m^{B},\Phi_m^{*B} \quad \varepsilon_m,\Phi_m^{*} \qquad N_{part}^{F},N_{part}^{B},N_{part} \qquad \varepsilon_n^{F},\Phi_n^{*F} \neq \varepsilon_n^{B},\Phi_n^{*B}$$

- Particles are produced by independent fragmentation of wounded nucleons, emission function f (η) not symmetric in η→ Wounded nucleon model

Flow longitudinal dynamics



1011.3354, 1403.6077



$$\varepsilon_n^{F}, \Phi_n^{*F} \neq \varepsilon_n^{B}, \Phi_n^{*B}$$

- Eccentricity vector interpolates between $\vec{\epsilon}_n^{F}$ and $\vec{\epsilon}_n^{B}$

$$\vec{\epsilon}_n^{tot}(\eta) \approx \alpha(\eta)\vec{\epsilon}_n^{F} + (1-\alpha(\eta))\vec{\epsilon}_n^{B} \equiv \epsilon_n^{tot}(\eta) e^{in\Phi_n^{*tot}(\eta)}$$

α(η) determined by f(η)

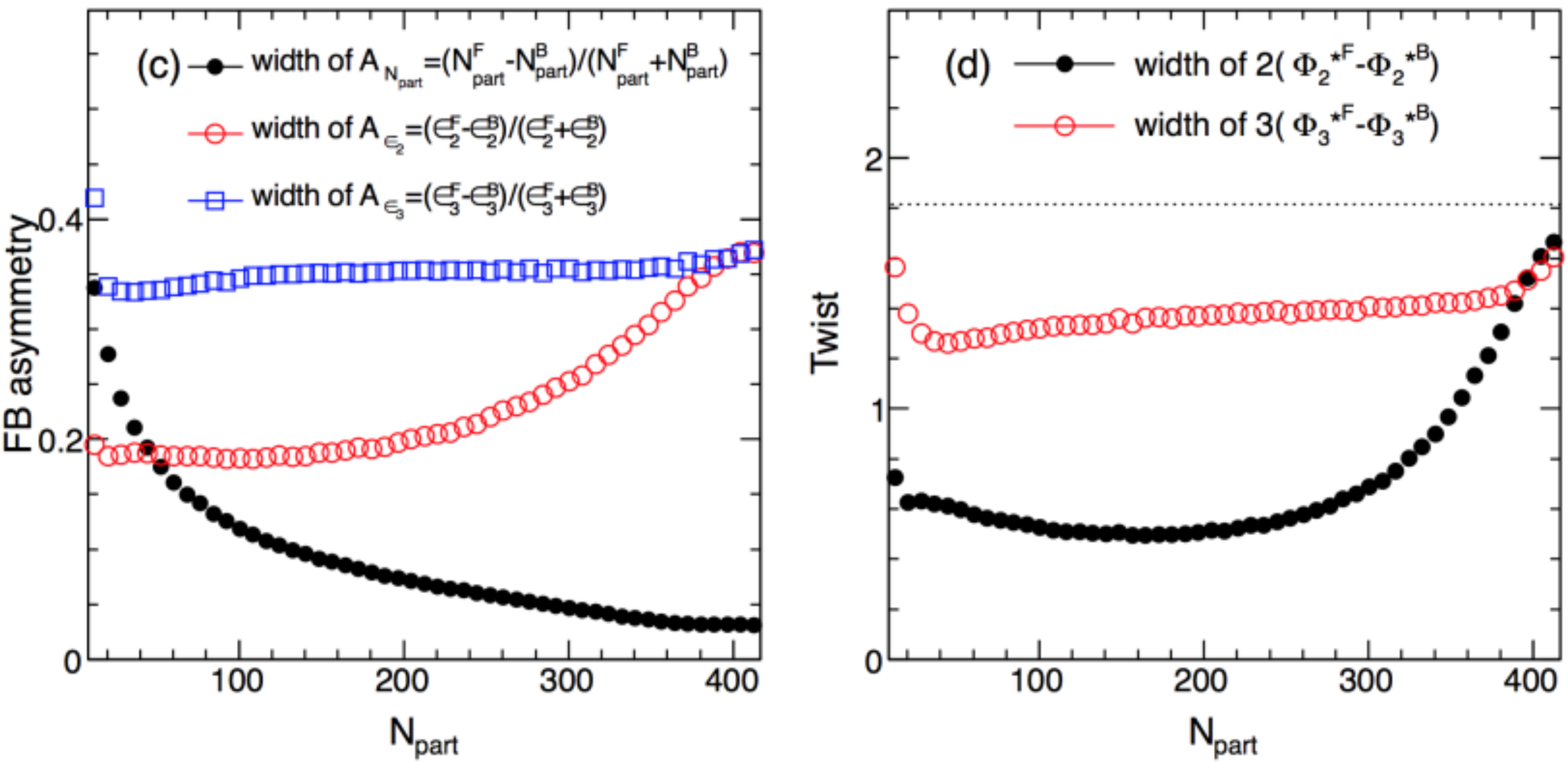
Asymmetry:	$\varepsilon_n^{F} \neq \varepsilon_n^{B}$
Twist:	$\Phi_n^{*F} \neq \Phi_n^{*B}$

- Hence $\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{F} + (1-\alpha(\eta))\vec{\epsilon}_n^{B}\right]$ for n=2,3
  - Picture verified in AMPT simulations, magnitude estimated 1403.6077

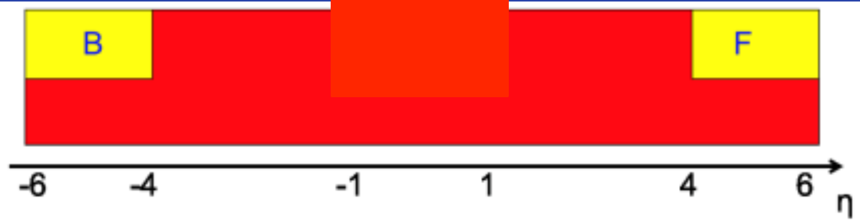
P. Huo Poster

FB eccentricity fluctuations from Glauber

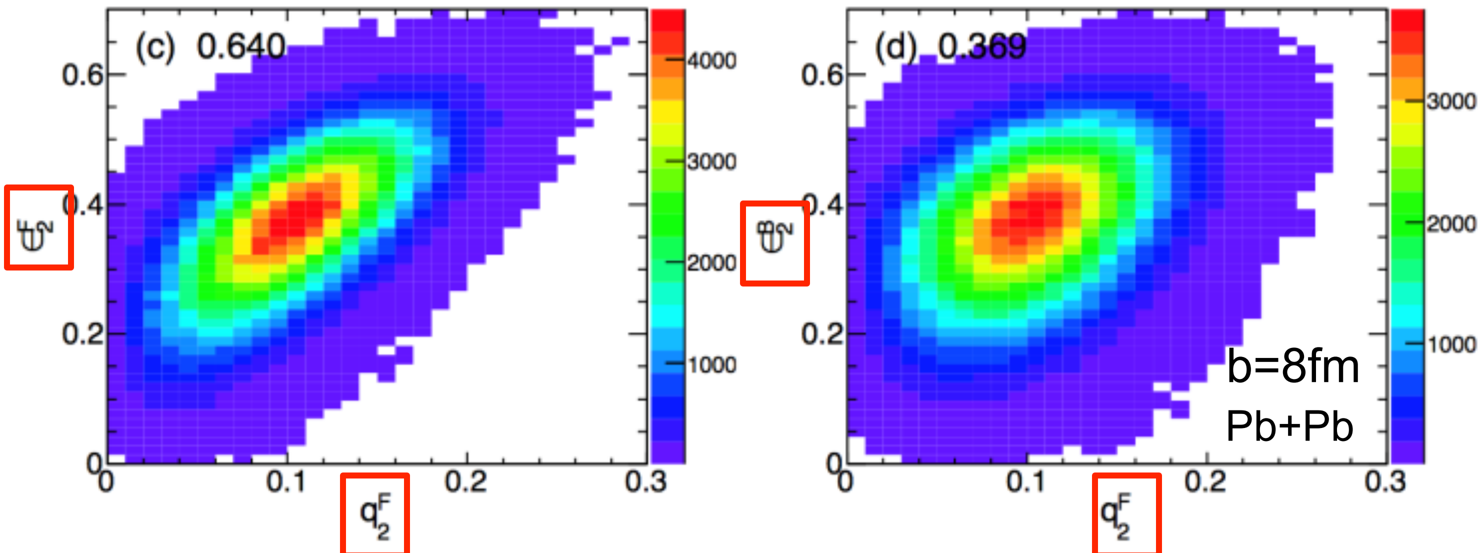
- Significant EbyE FB asymmetry: $\varepsilon_n^{F} \neq \varepsilon_n^{B}$
- Significant EbyE twist: $\Phi_n^{*F} \neq \Phi_n^{*B}$



What AMPT tell us?

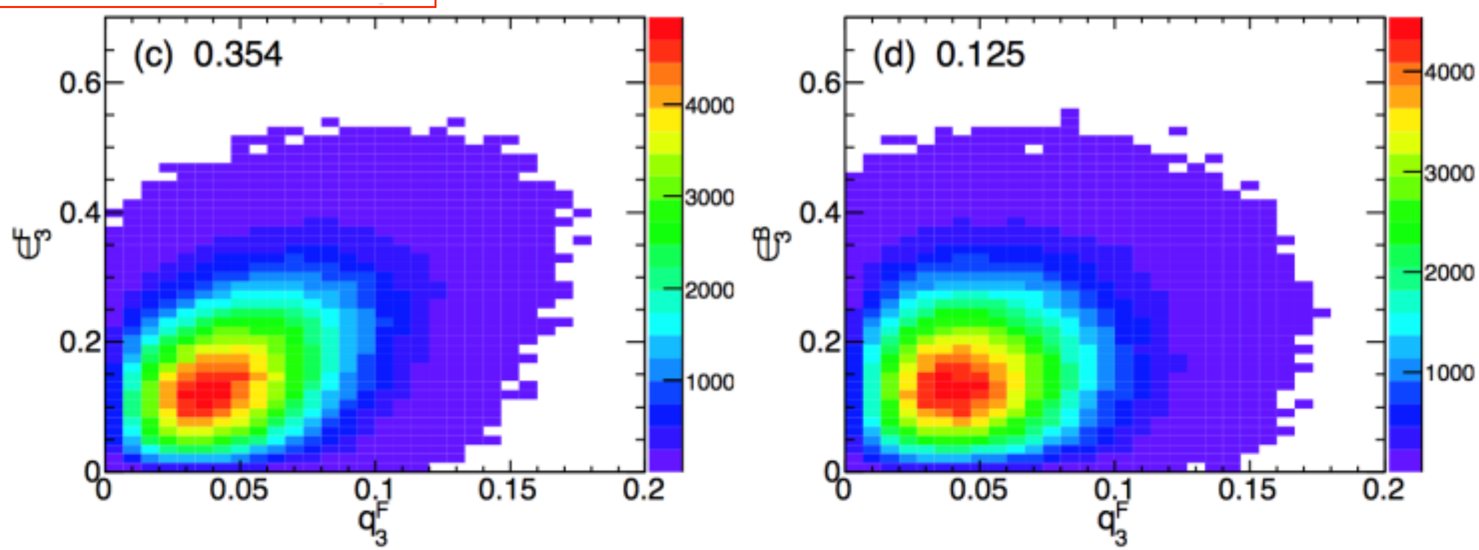


ε_2^F more correlated with q_2^F than q_2^B



FB asymmetry survives

ε_3^F more correlated with q_3^F than q_3^B



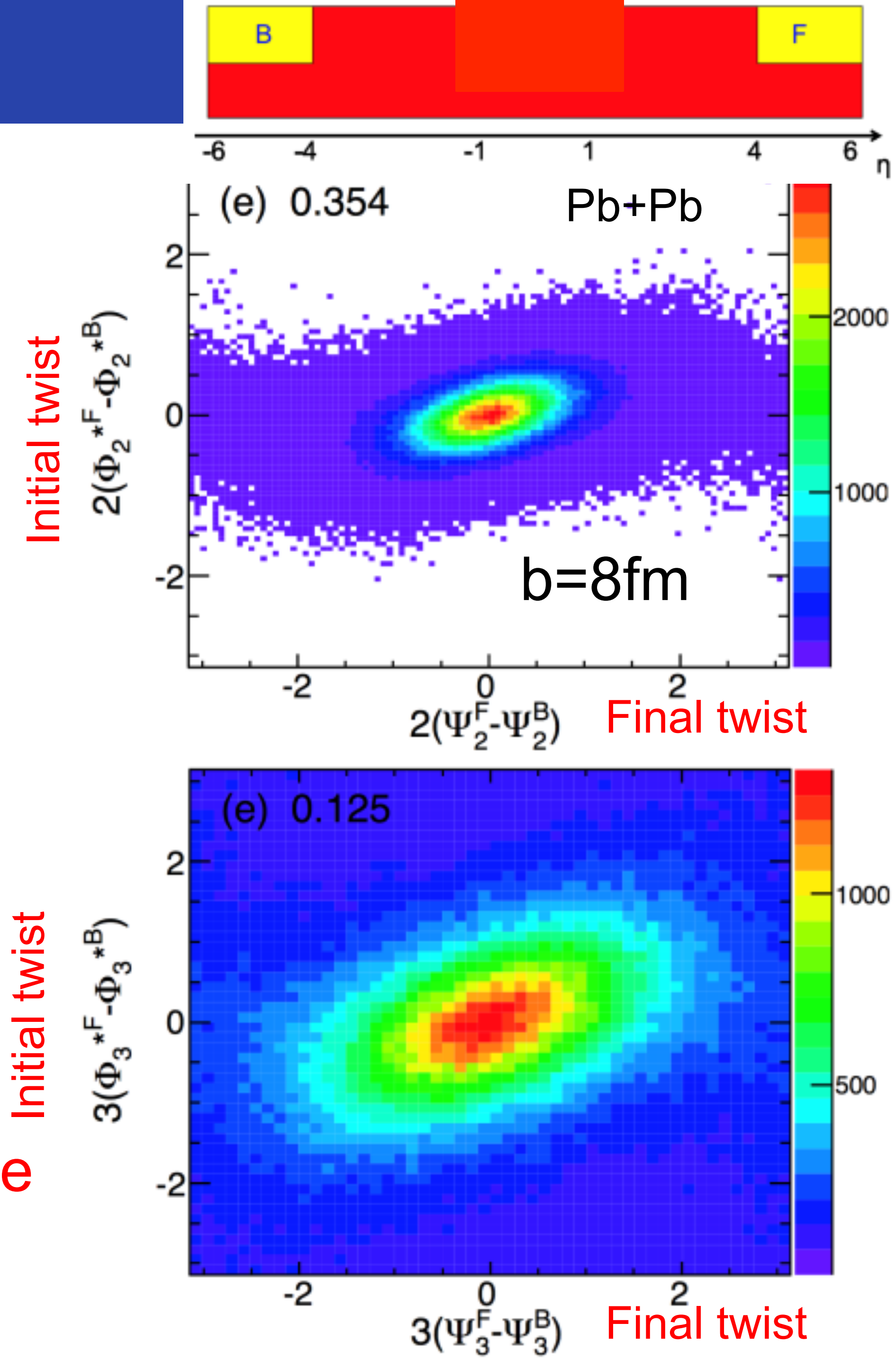
What AMPT tell us?

- Twist in initial geometry appears as twist in the final state flow
  - Participant plane angles:

$$\Phi_n^{*F} \quad \Phi_n^{*B}$$

  - Final state event-plane angles

$$\Psi_n^F \quad \Psi_n^B$$

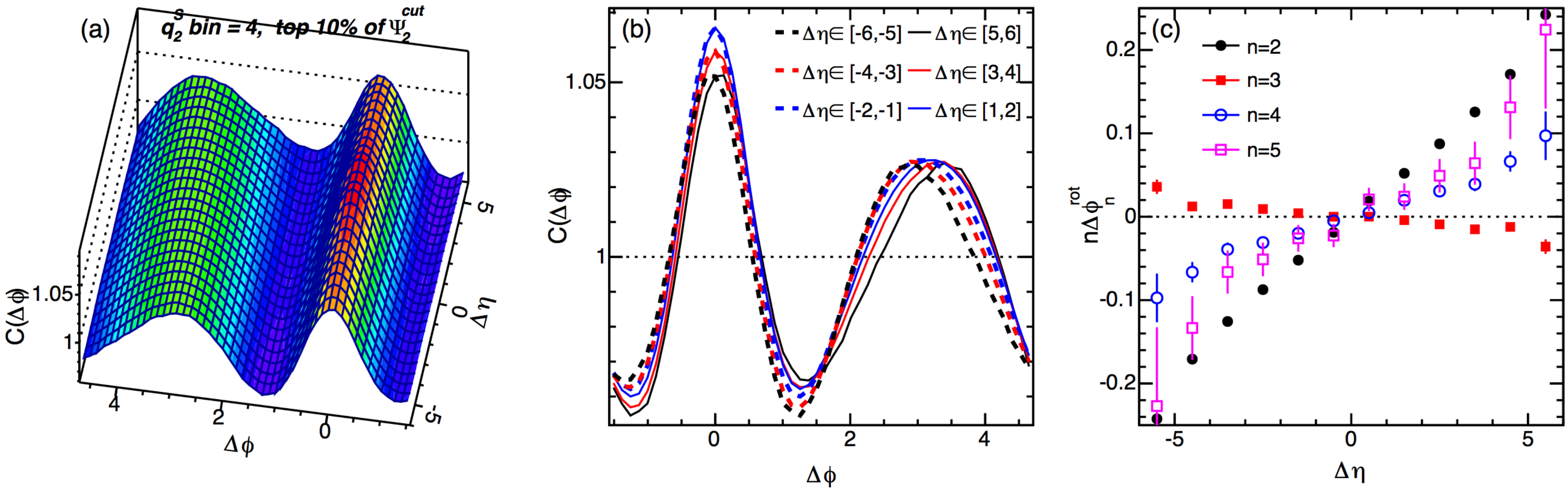


Initial twist survives to final state

Twist seen in simple 2PC analysis

- NO event-plane determination! Just select twist in large η and check correlation at center-rapidity.

$$C(\Delta\phi, \Delta\eta) \propto 1 + 2\sum v_n^{\mathrm{a}} v_n^{\mathrm{b}} \cos\left(n\Delta\phi - n\Delta\Phi_n^{\mathrm{rot}}\right)$$



- Though twist is enforced on q_2, twist also seen for higher order v_n
- Non-linear mixing to the higher order harmonics!! .

$$v_4 e^{-i4\Phi_4} \propto \varepsilon_4 e^{-i4\Phi_4^*} + c v_2 v_2 e^{-i4\Phi_2} + \ldots$$

$$v_5 e^{-i5\Phi_5} \propto \varepsilon_5 e^{-i5\Phi_5^*} + c v_2 v_3 e^{-i(2\Phi_2 + 3\Phi_3)} + \ldots$$

Implications

- System <u>not</u> boost-invariant EbyE not only for dN/dη, but also flow
- Longitudinal decorrelation effects breaks the factorization, despite being initial state effects. $V_{n\Delta}(\eta_1,\eta_2) \neq v_n(\eta_1)v_n(\eta_2)$
- Decorrelation effects much stronger in pA, dA, HeA and Cu+Au system

Summary-I

- Event-shape fluctuations contains a lot of information

$$p(v_n, v_m, ..., \Phi_n, \Phi_m, ...) = \frac{1}{N_{\rm evts}} \frac{dN_{\rm evts}}{dv_n dv_m ... d\Phi_n d\Phi_m ...}$$

- Three complementary methods:

Strong fluctuation within fixed centrality!

	pdf's	cumulants	event-shape method
Flow-amplitudes	$p(v_n)$	$v_n\{2k\},\ k=1,2,...$	NA
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$	yes
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v_m^2\rangle\langle v_l^2\rangle -$ $\langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2\rangle\langle v_n^2\rangle - \langle v_l^2 v_n^2\rangle\langle v_m^2\rangle$	yes
	...	Obtained recursively as above	yes
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2\rangle\langle v_n^{c_n} v_m^{c_m} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$	yes

Summary-II

- Rich patterns forward/backward EbyE flow fluctuations:

$$\vec{v}_n(\eta) \approx c_n(\eta)\left[\alpha(\eta)\vec{\epsilon}_n^{\,\mathrm{F}} + (1-\alpha(\eta))\vec{\epsilon}_n^{\,\mathrm{B}}\right]$$

Event-shape selection and event twist techniques

- New avenue to study initial state fluctuations, particle production and collective expansion dynamics.