Lattice QCD and the search for the critical point

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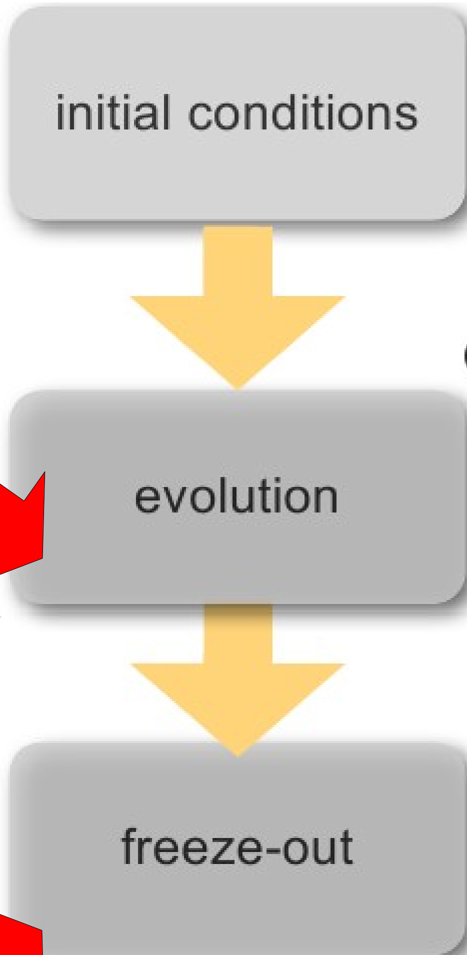
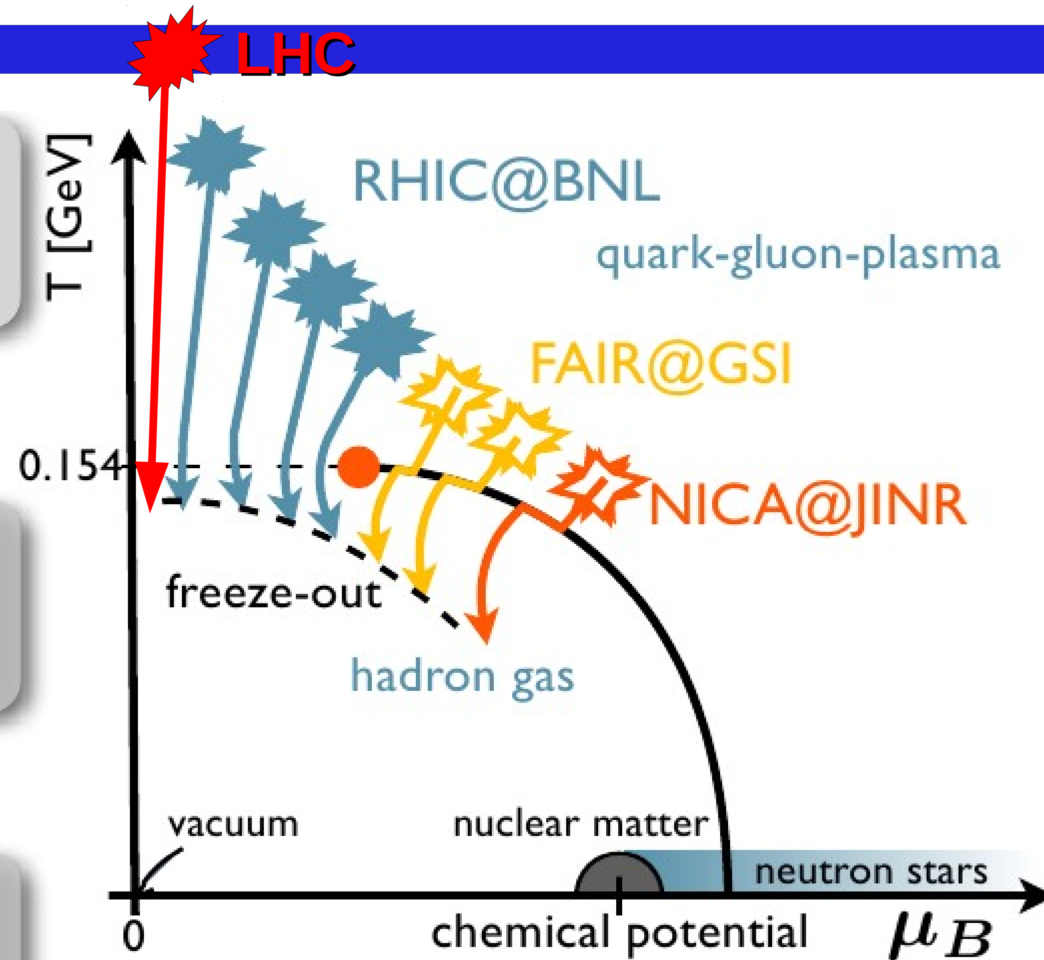
OUTLINE

– the QCD critical point
– EoS at non-zero baryon chemical potential
– cumulant ratios of conserved charge fluctuations
– freeze-out conditions from QCD
– power of Taylor expansions

Exploring the QCD phase diagram





controlled by the **QCD equation of state**, T_c

expectation: **freeze-out** close to QCD transition line

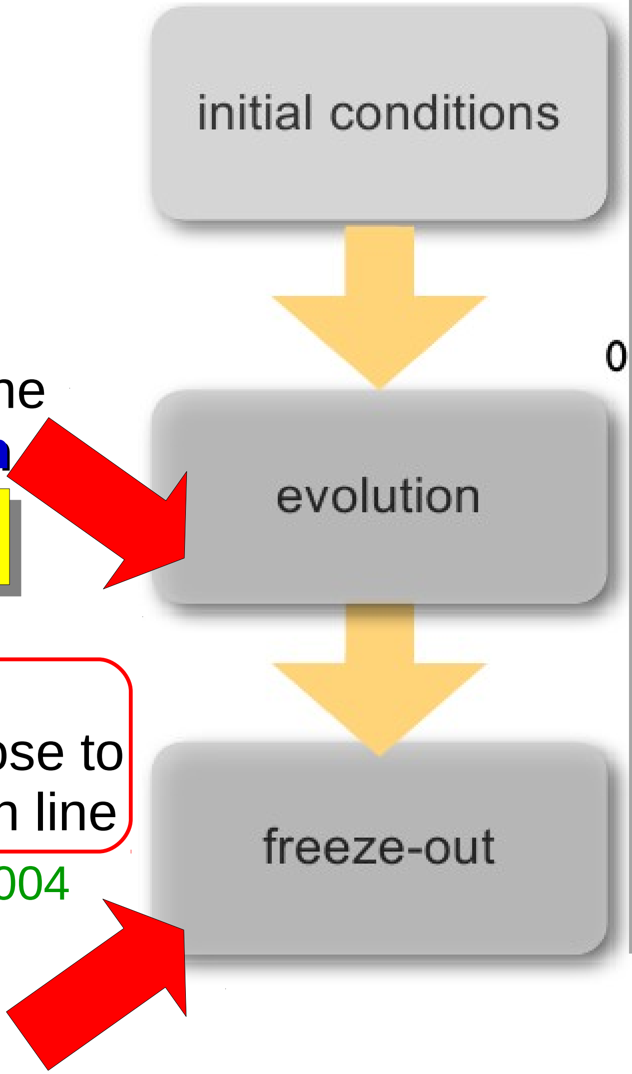
PBMetalPLB2004

observable consequences:
freeze-out pattern of mesons and baryons, controlled by T_f, μ_B, μ_S

– LHC: establish contact with the QCD PHASE transition

– RHIC: locate/provide evidence for the QCD critical point

Exploring the QCD phase diagram



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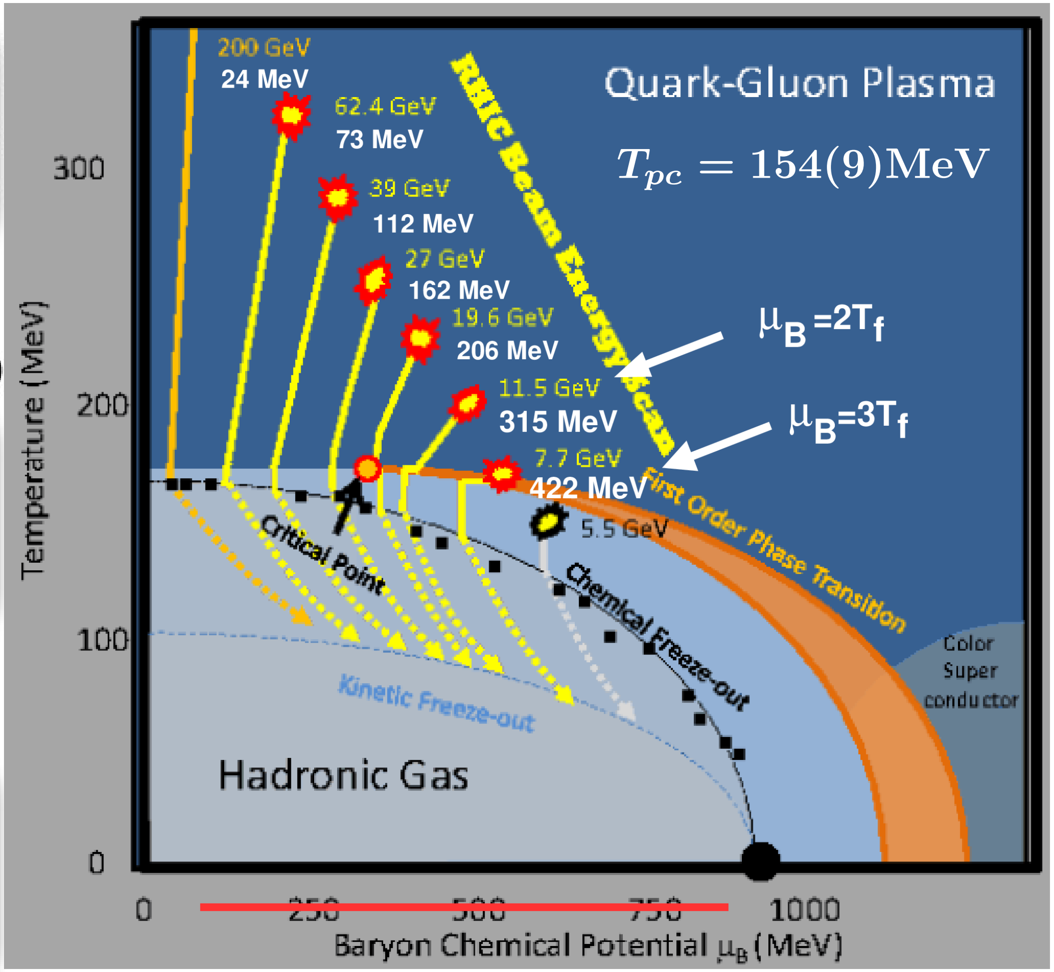

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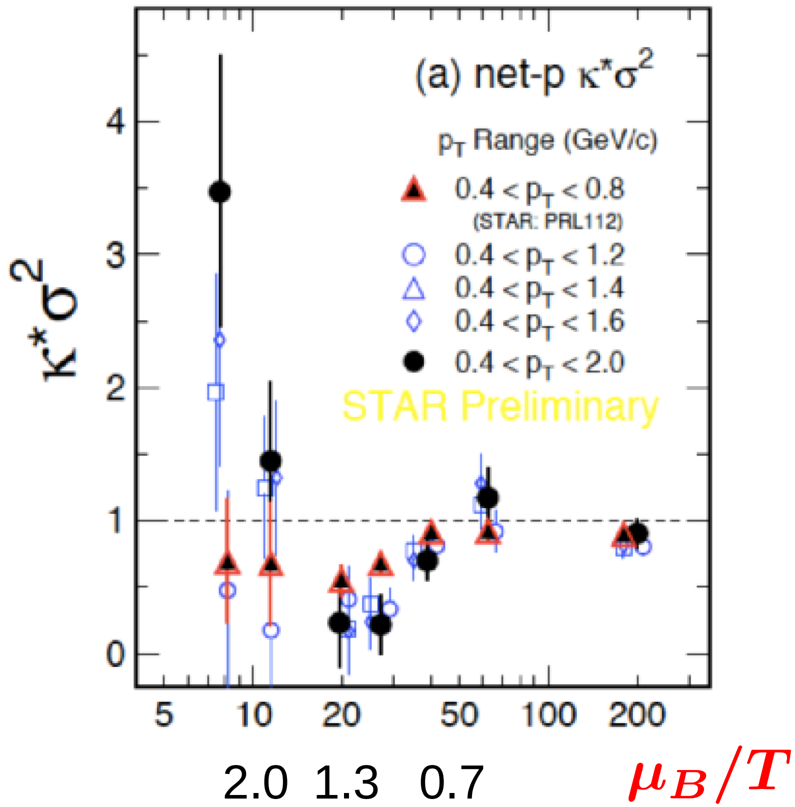

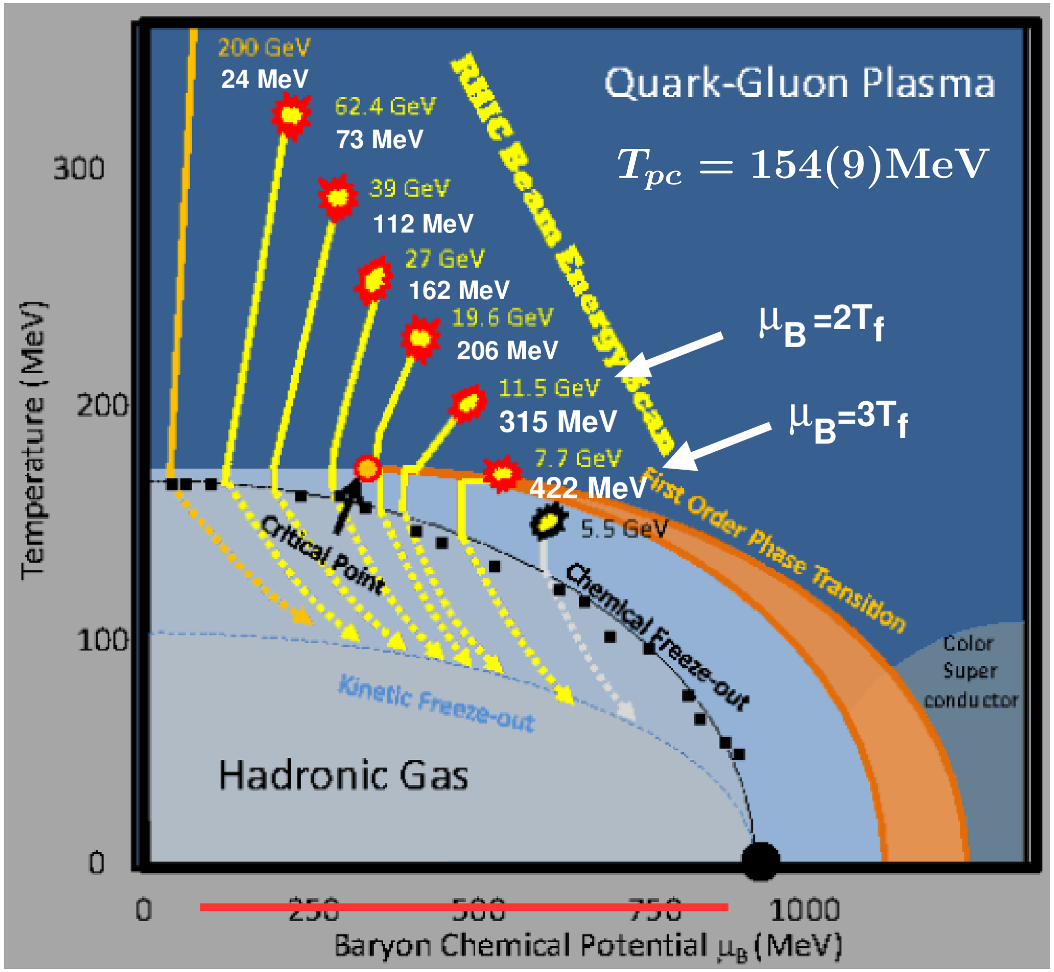

observable consequences:

conserved charge fluctuation

controlled by T_f, μ_B, μ_S

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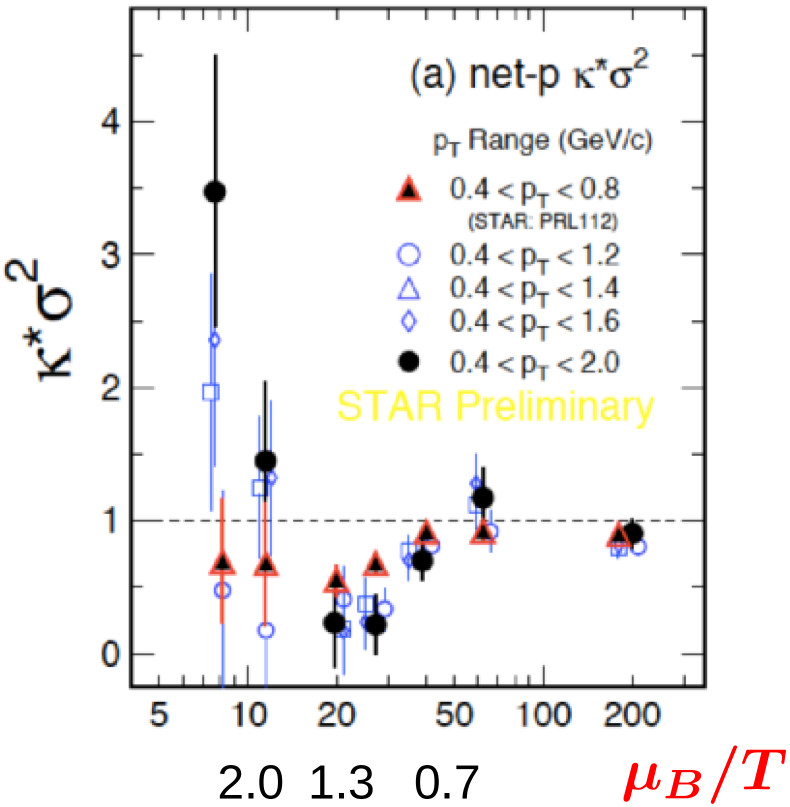
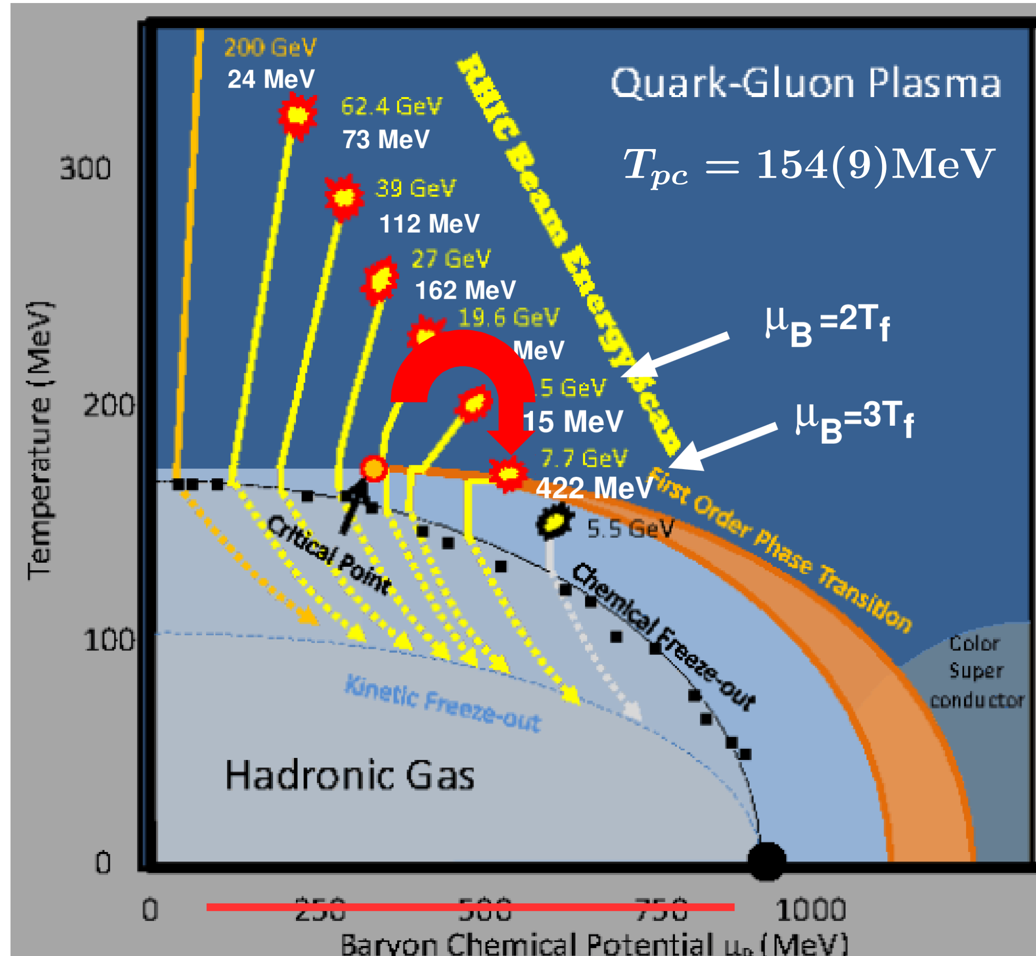




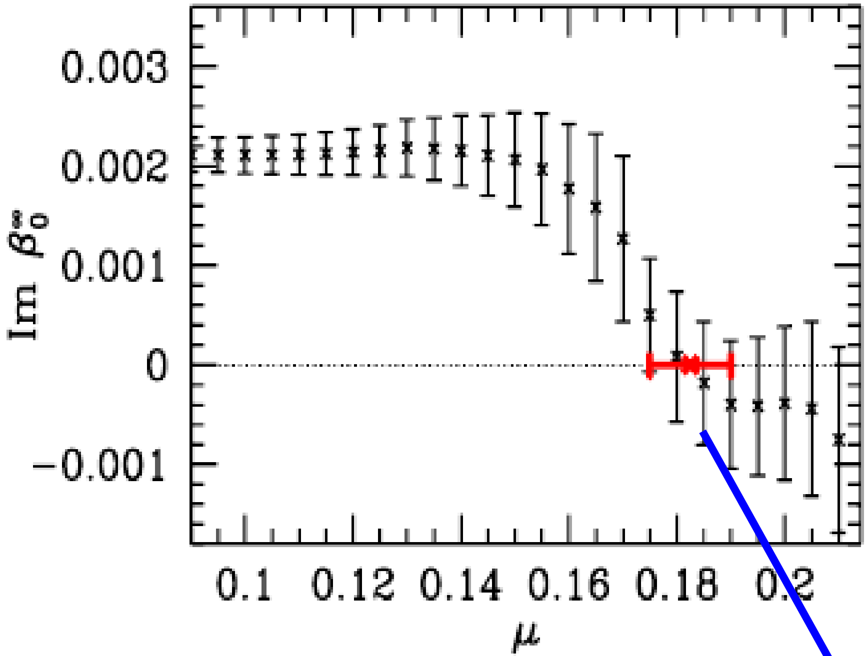
observable consequences:

conserved charge fluctuation

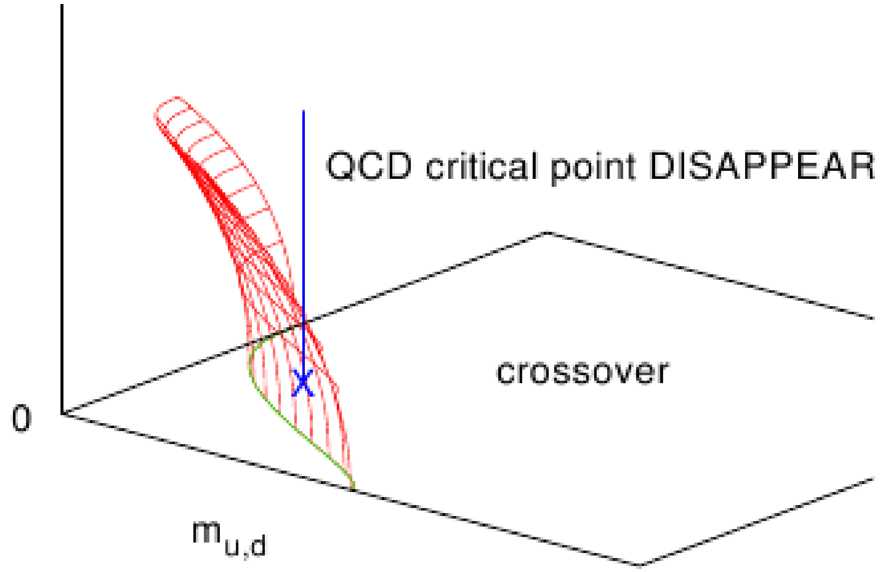
controlled by T_f, μ_B, μ_S

Where is the critical point?

LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude

critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 10 years no progress along this line

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T,\mu_B)$ to be monotonically growing with μ_B/T

⇕

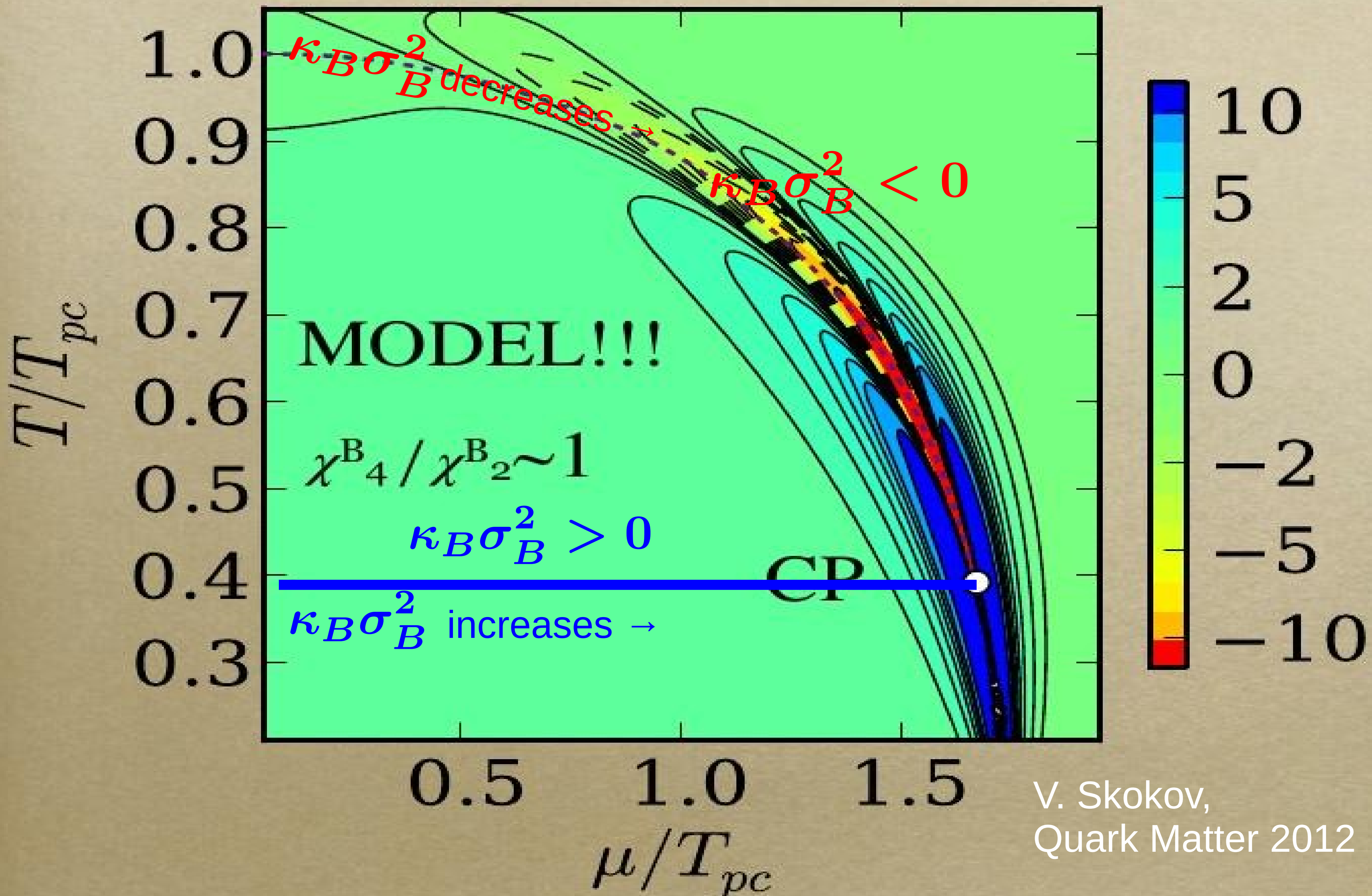
at T_{CP} : $\kappa_B \sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

– Taylor expansion can not be used close to the critical point

Chiral model and negative χ^B_4/χ^B_2:



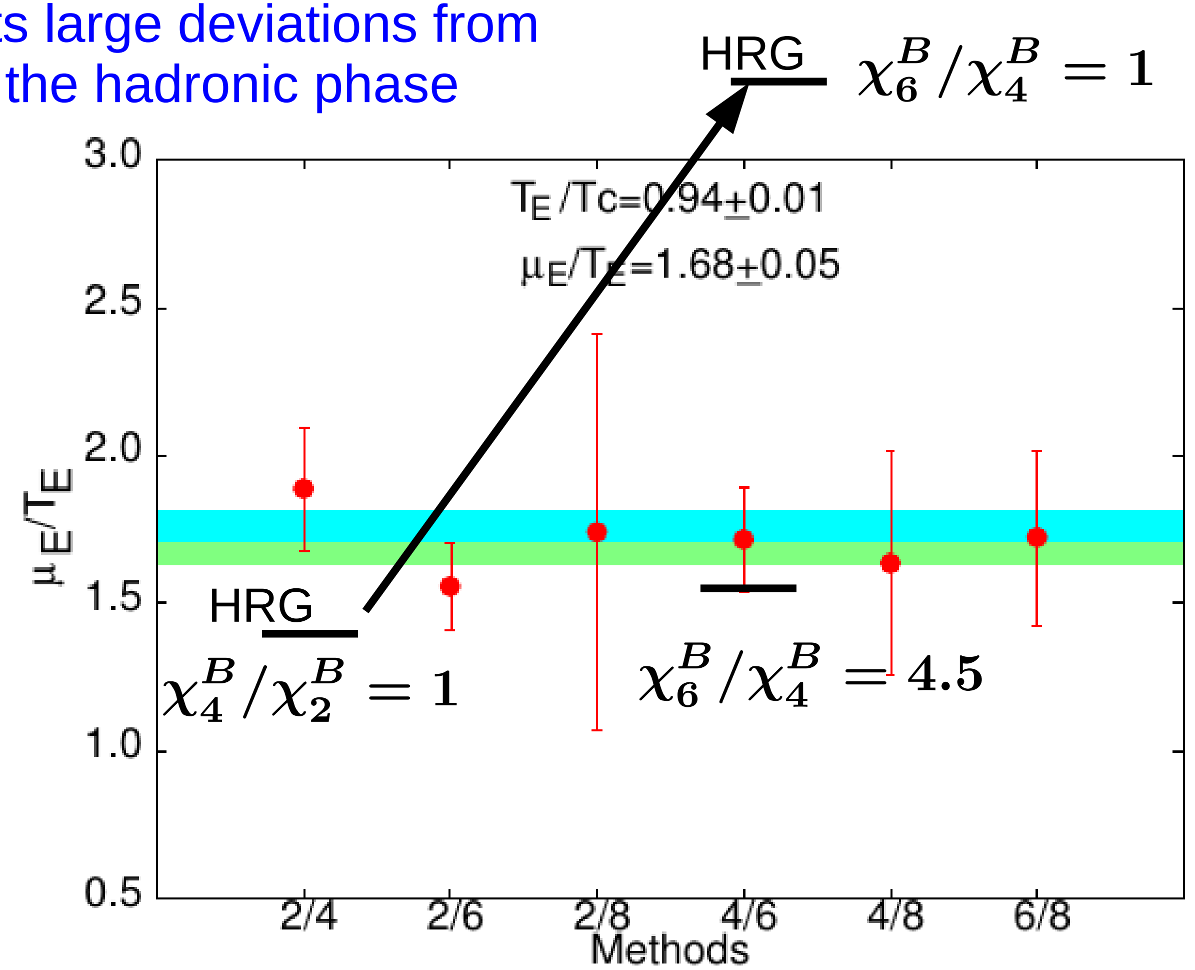
V. Skokov,
Quark Matter 2012

Estimates of the radius of convergence

a challenging prediction from susceptibility series for standard staggered fermions:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

suggests large deviations from HRG in the hadronic phase



huge deviations from HRG in 6th order cumulants!

S. Datta et al., PoS Lattice2013 (2014) 202

suggests a critical point for $\mu_B/T < 2$

at present, we cannot rule it out!

BNL-Bielefeld-CCNU

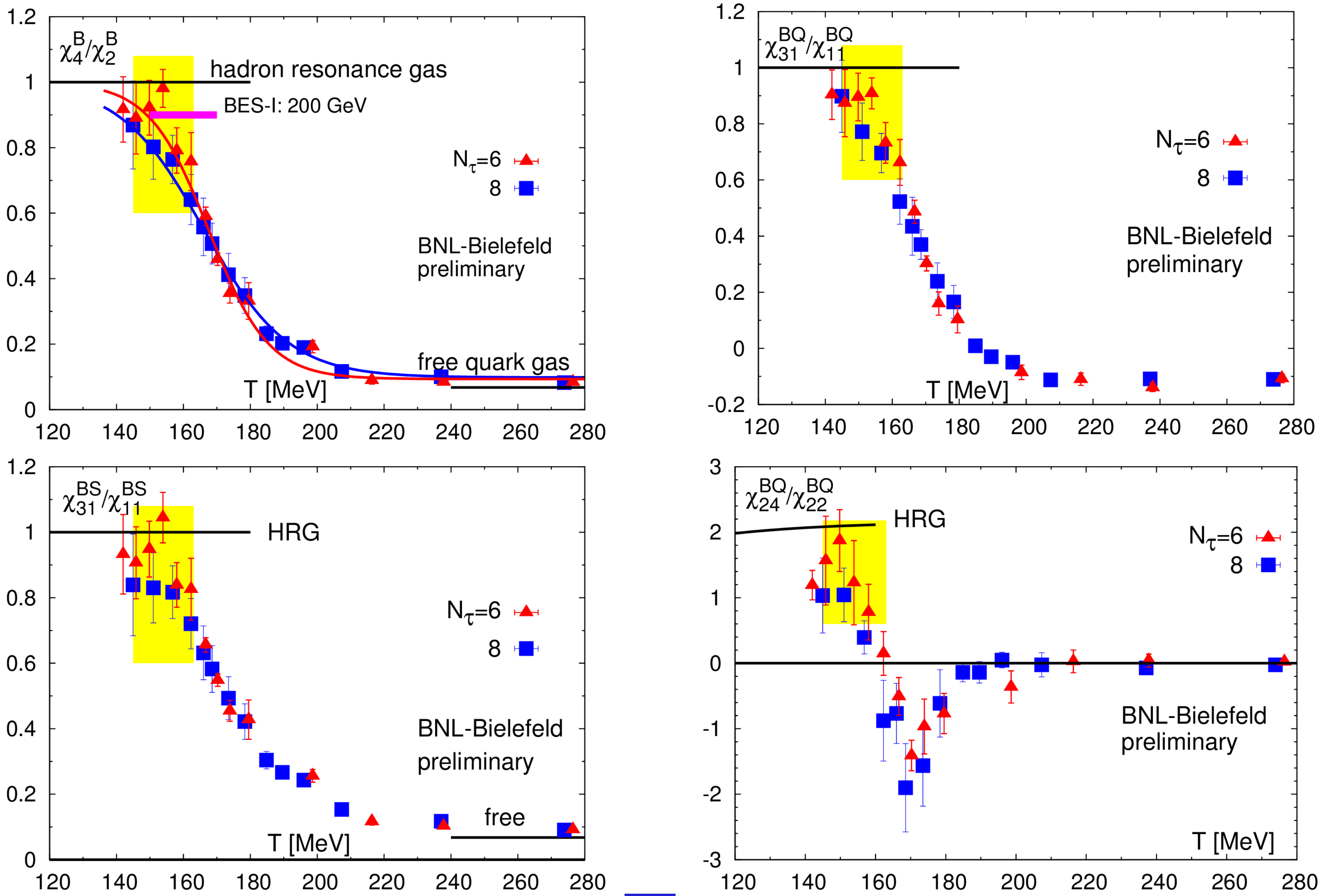
Taylor expansion of the EoS and critical point

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$
$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu=0}$$

– valid up to radius of convergence: μ_c (critical point?)

Some 4th and 6th order cumulants



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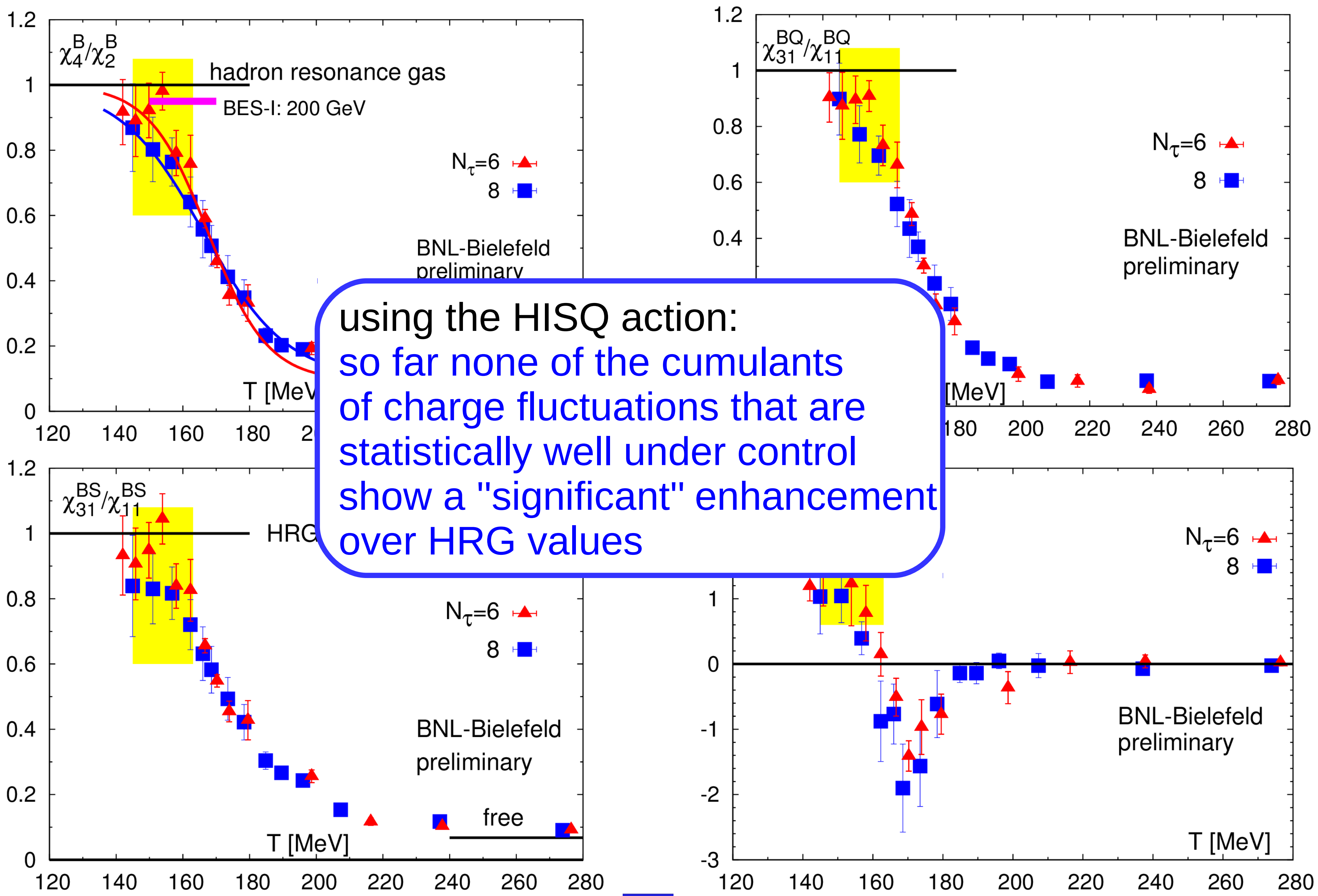

using the HISQ action:
so far none of the cumulants of charge fluctuations that are statistically well under control show a "significant" enhancement over HRG values

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

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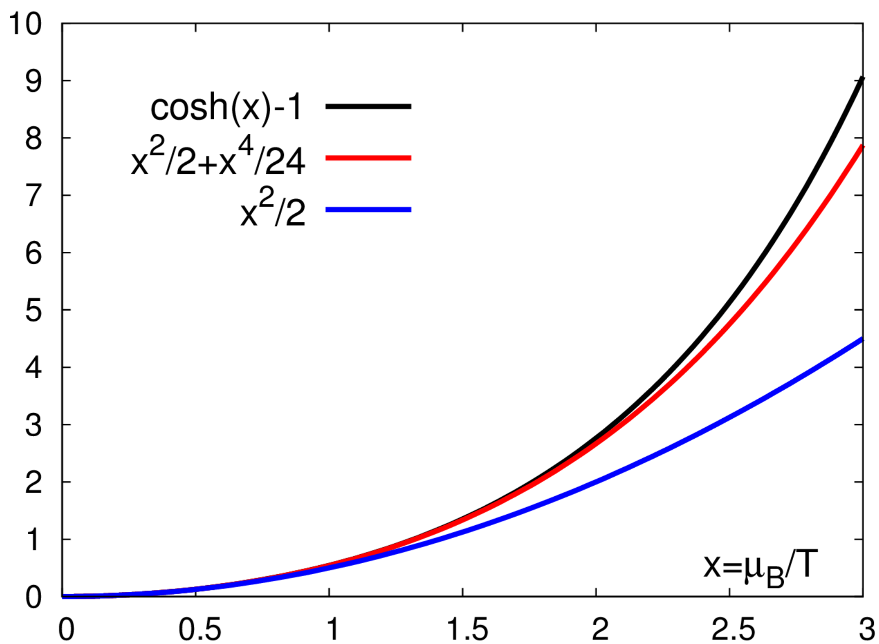
An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

How good is an $\mathcal{O}((\mu_B/T)^4)$ expansion in a HRG?

- deviation is less than 3% at $\mu_B/T = 2$



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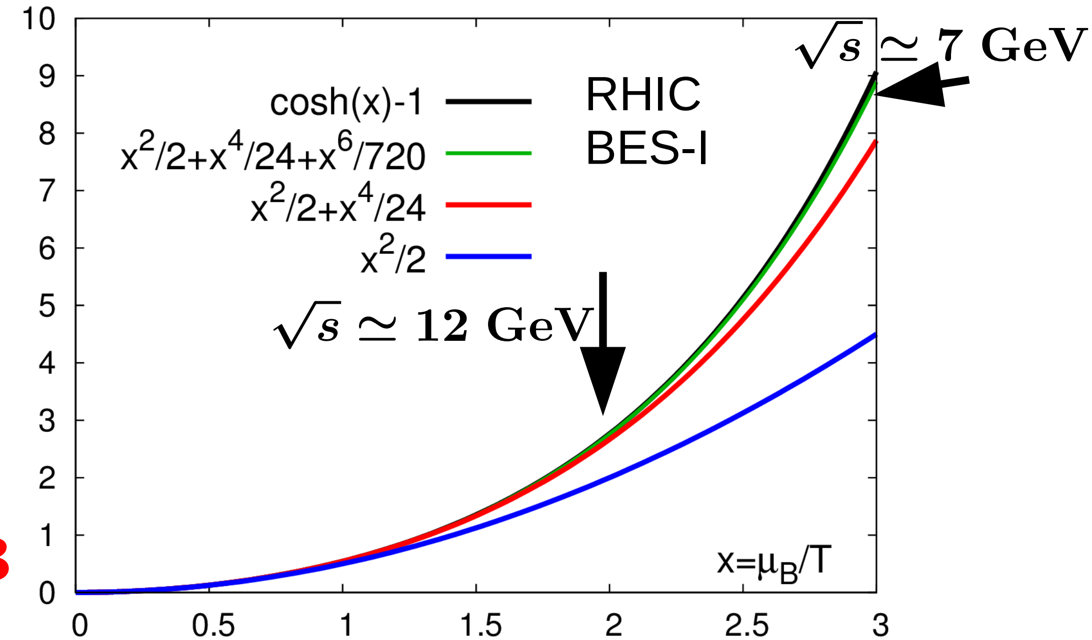
An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

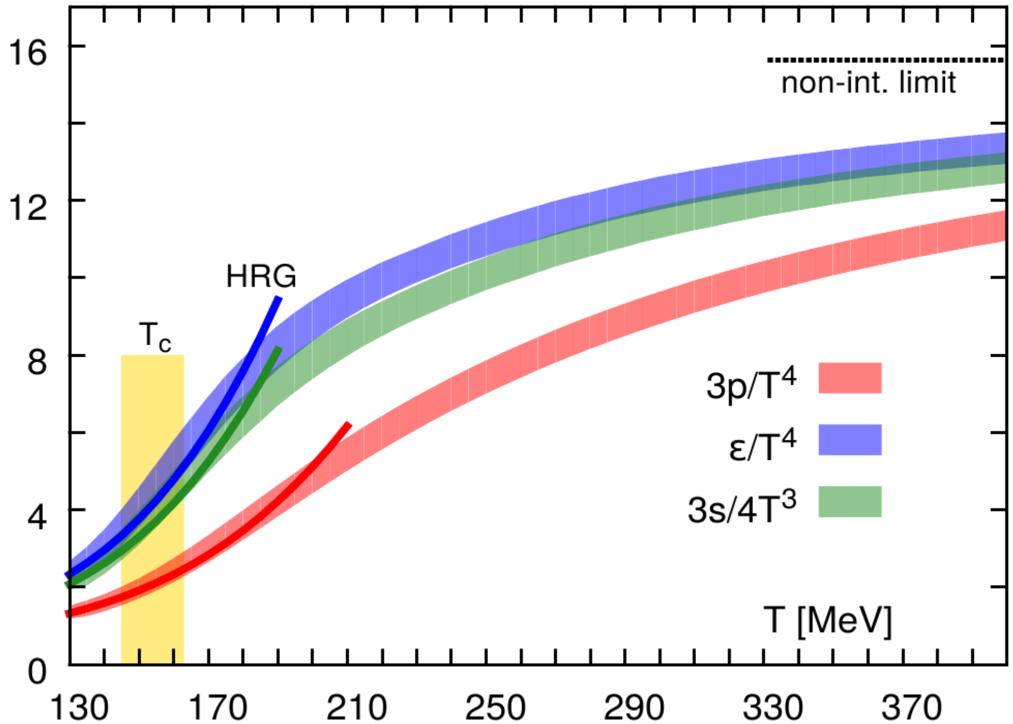
How good is an $\mathcal{O}((\mu_B/T)^4)$ expansion in a HRG?

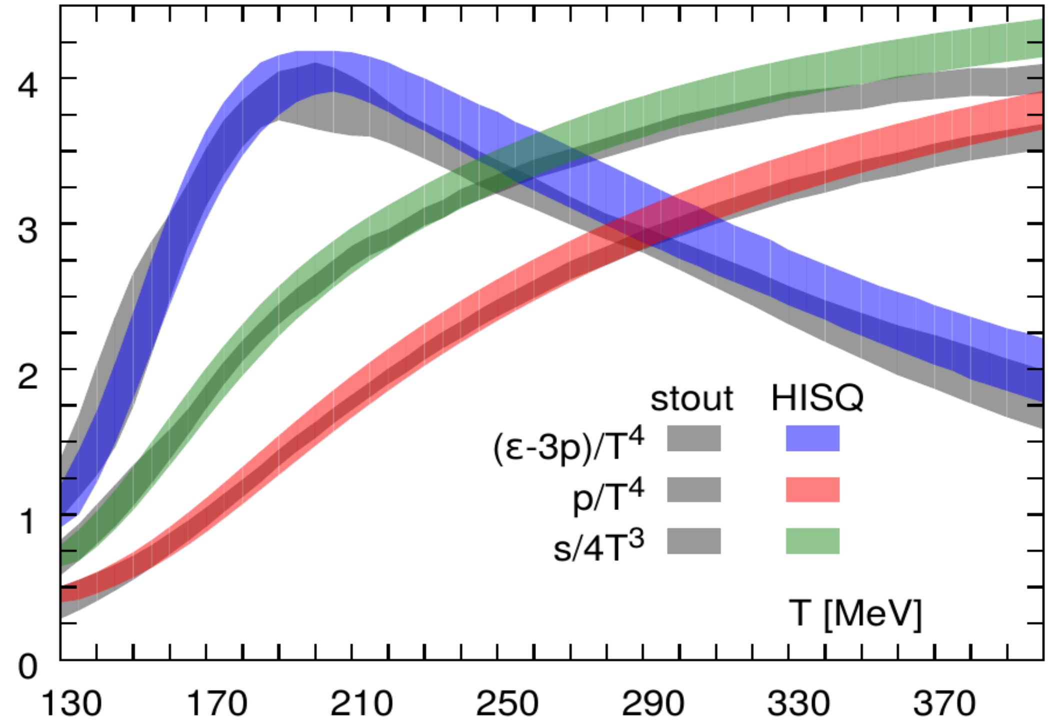
– an $\mathcal{O}((\mu_B/T)^6)$ expansion is almost perfect up to $\mu_B/T = 3$



Equation of state of (2+1)-flavor QCD

pressure, **entropy** & **energy** density





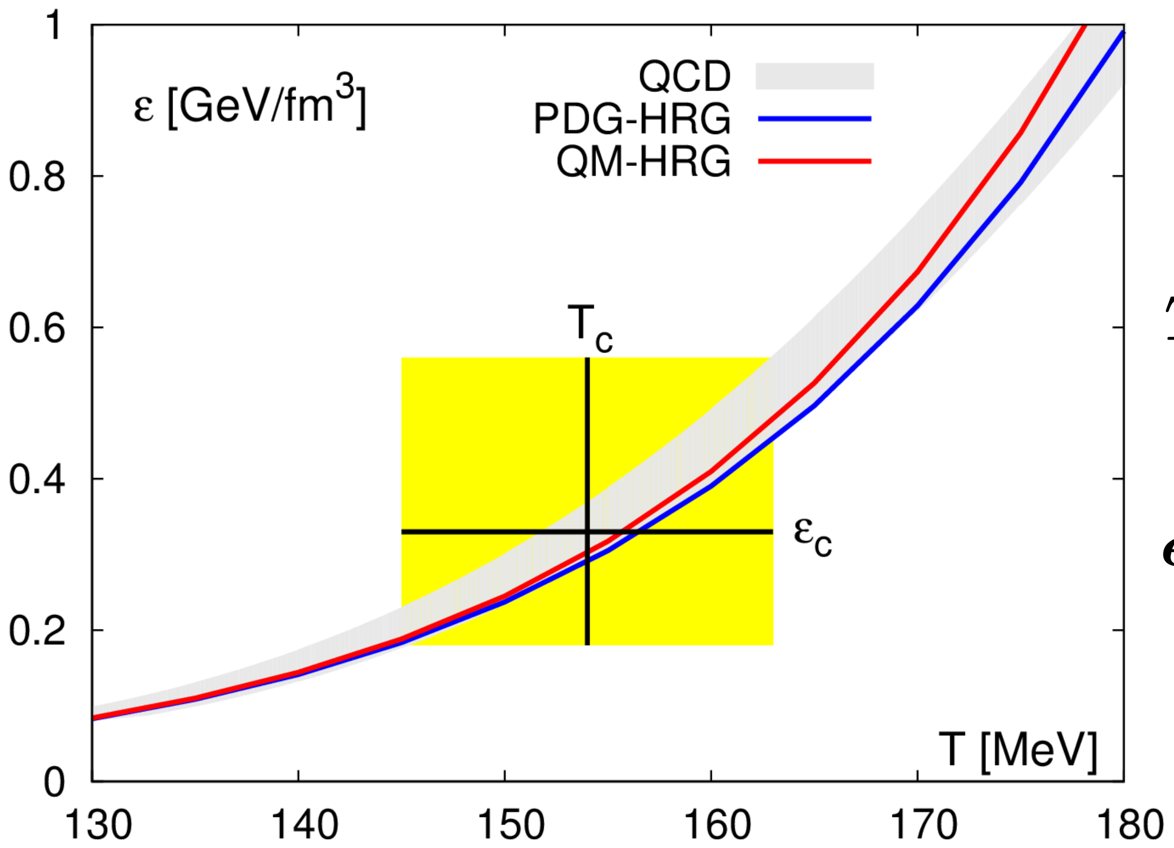
A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

– improves over earlier hotQCD calculations:
  A. Bazavov et al., Phys. Rev. D80, 014504 (2009)
– consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

– up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

Crossover transition parameters

PDG: Particle Data Group hadron spectrum
QM: Quark model hadron spectrum



$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm^3}$$

compare with:

$$\epsilon^{\mathrm{nucl.\ mat.}} \simeq 150\ \mathrm{MeV/fm^3}$$

$$\epsilon^{\mathrm{nucleon}} \simeq 450\ \mathrm{MeV/fm^3}$$

A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

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variance of net-baryon number distribution

kurtosis*variance

$\kappa_B \sigma_B^2$

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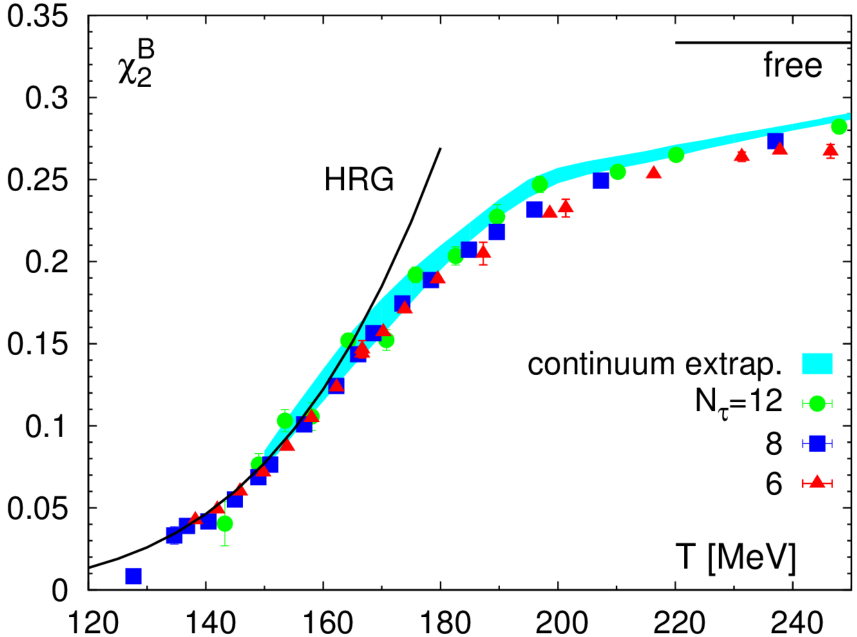

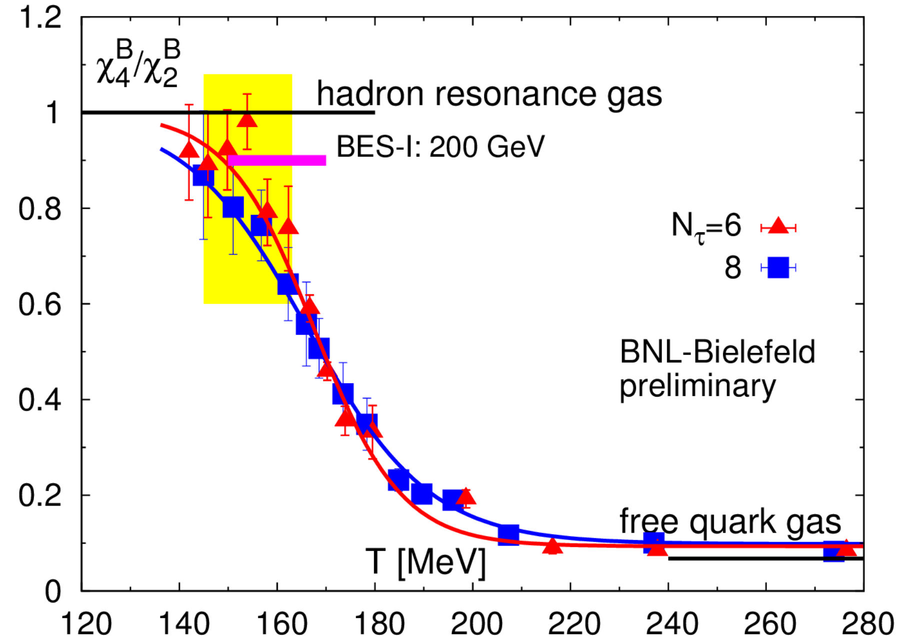

leading order correction agrees well with HRG in crossover region

~20% deviations from HRG in crossover region

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

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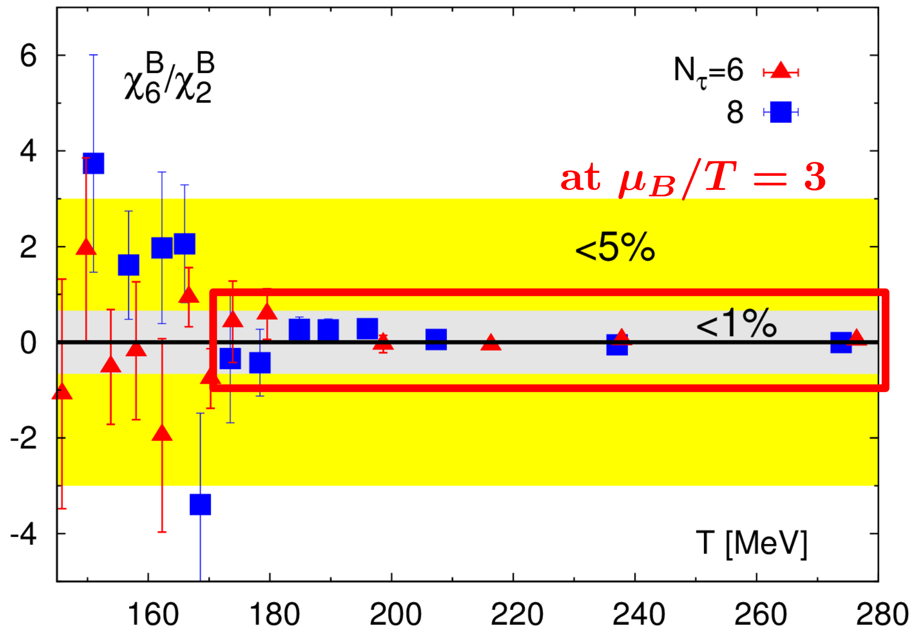
estimating the $\mathcal{O}((\mu_B/T)^6)$ correction: $\sim \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6$



bands: magnitude of 6th order contribution relative to total of 0th, 2nd and 4th order

for $\mu_B/T \leq 2$:

$\mathcal{O}((\mu_B/T)^6)$ corrections to P/T^4

contribute less than 1% for T>170 MeV

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

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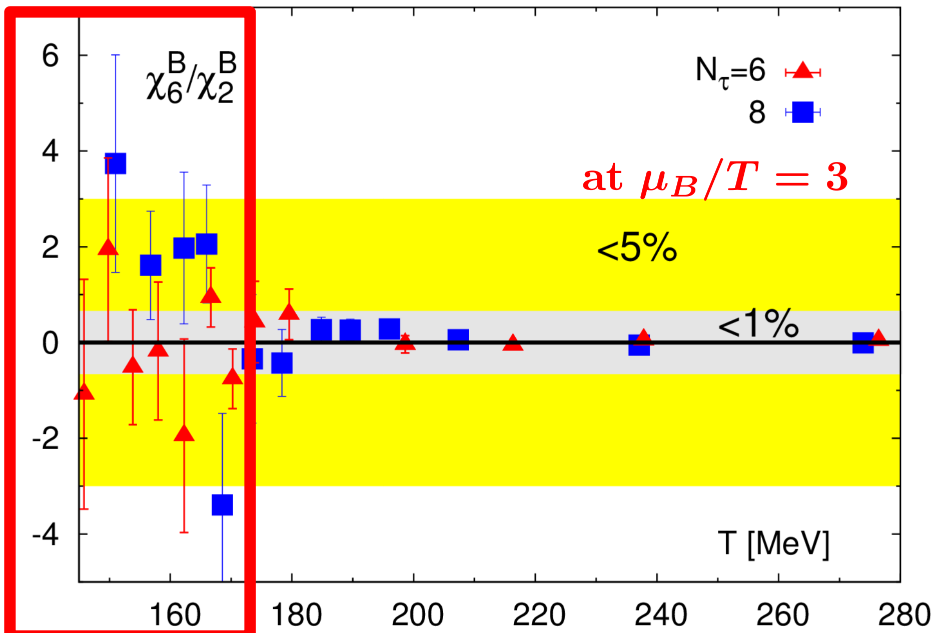

bands: magnitude of 6th order contribution relative to total of 0th, 2nd and 4th order

for $\mu_B/T \leq 2$:

$\mathcal{O}((\mu_B/T)^6)$ corrections to P/T^4

contribute less than 1% for T>170 MeV and less than ~ 5% for 150 MeV < T < 170 MeV

crucial: control 6th order cumulants in and below the crossover region

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

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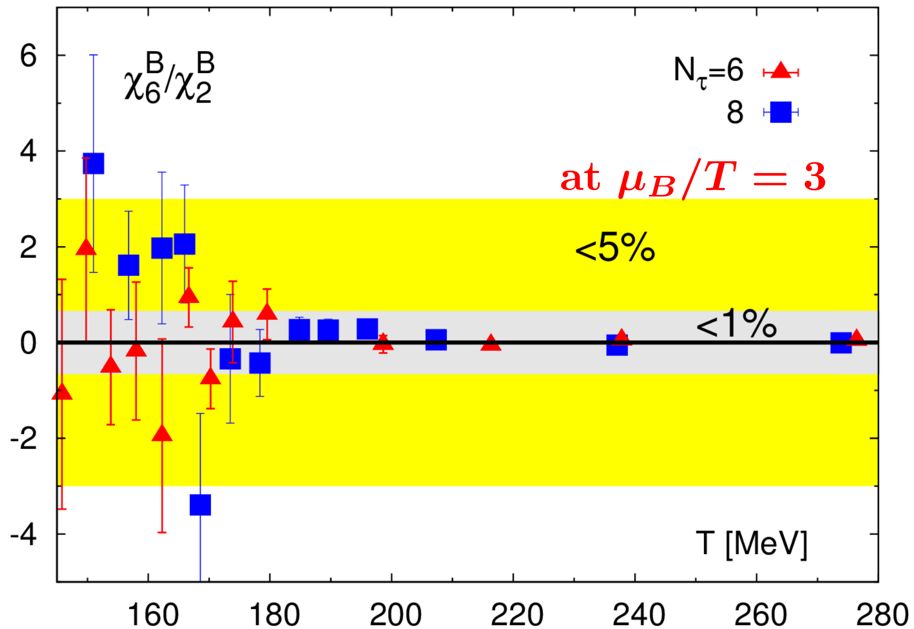
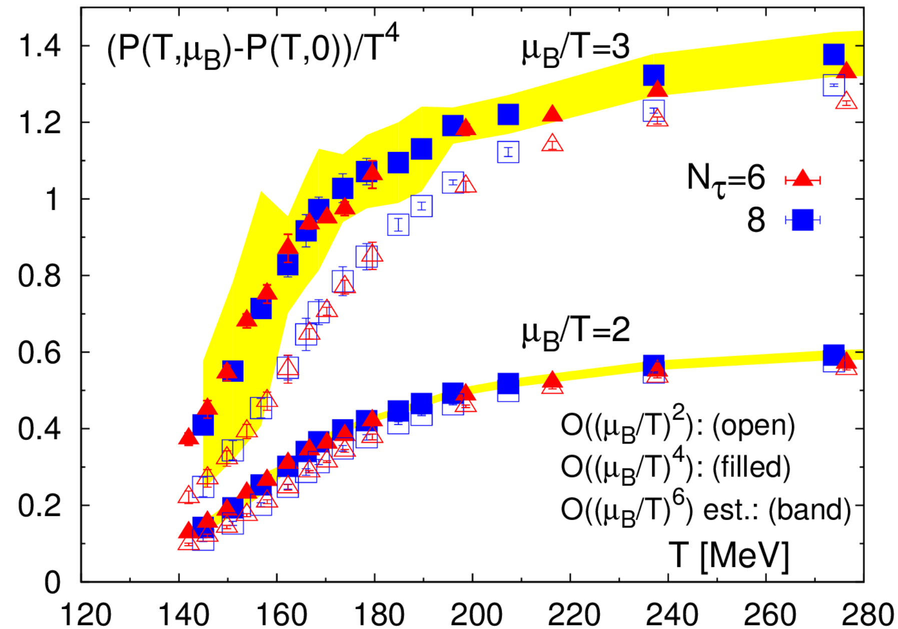
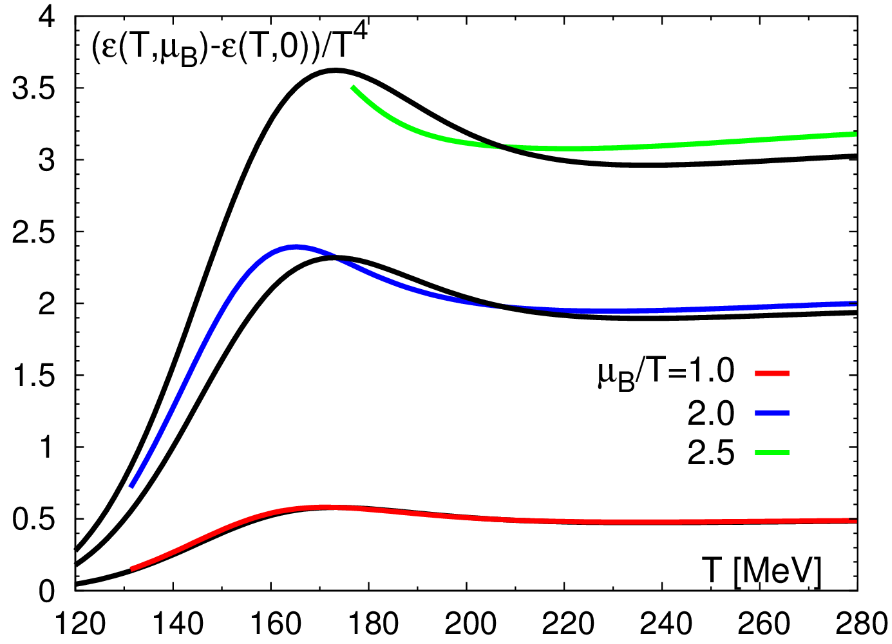
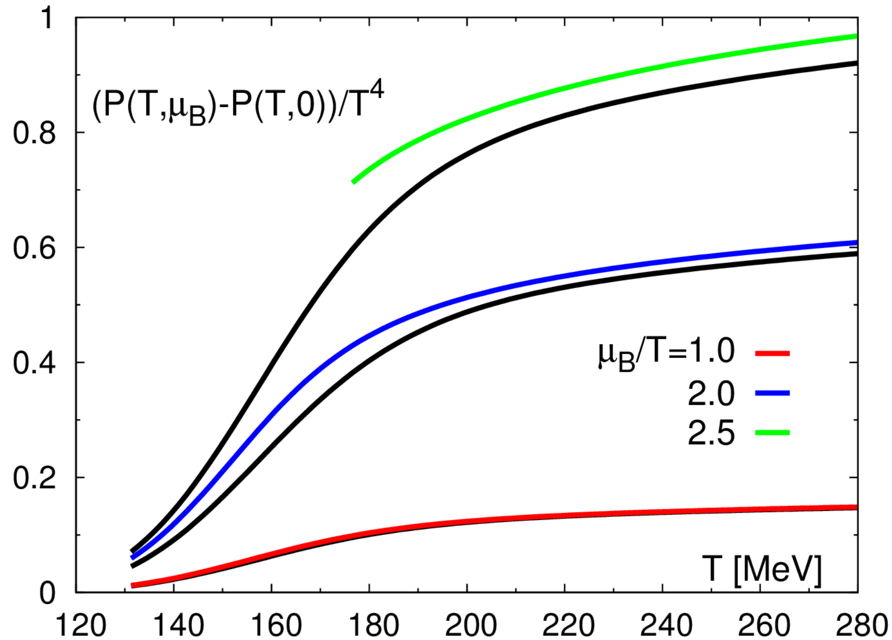




⟹ The EoS is well controlled for $\mu_B/T \leq 2$

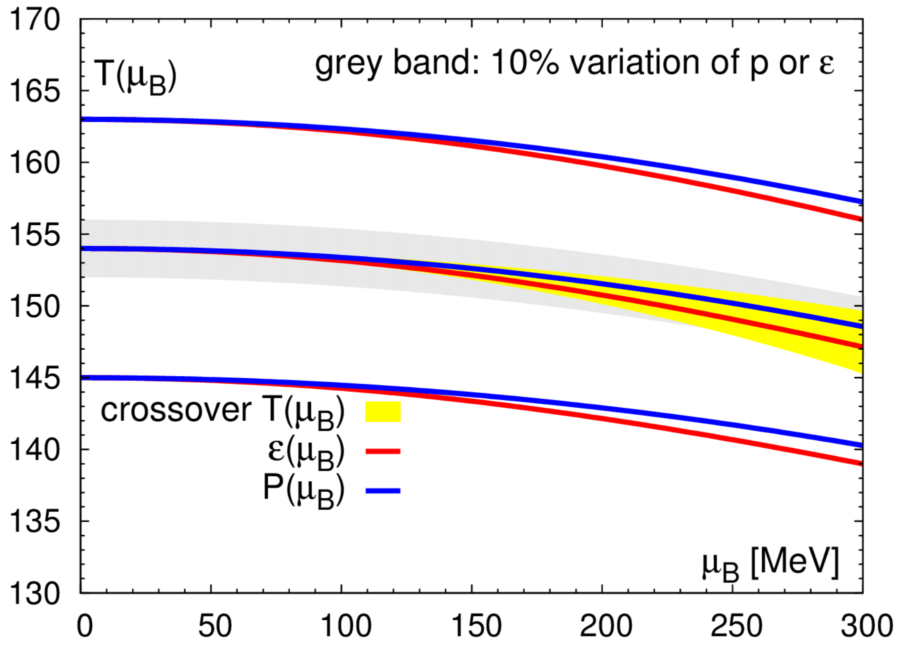
Lines of constant thermodynamics and freeze-out





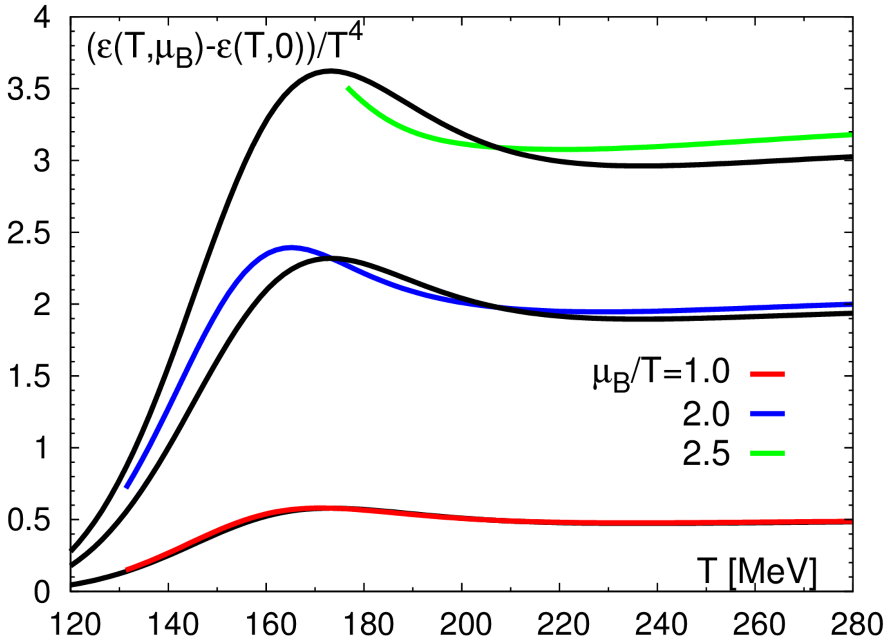
black lines: $\mathcal{O}(\mu_B^2)$

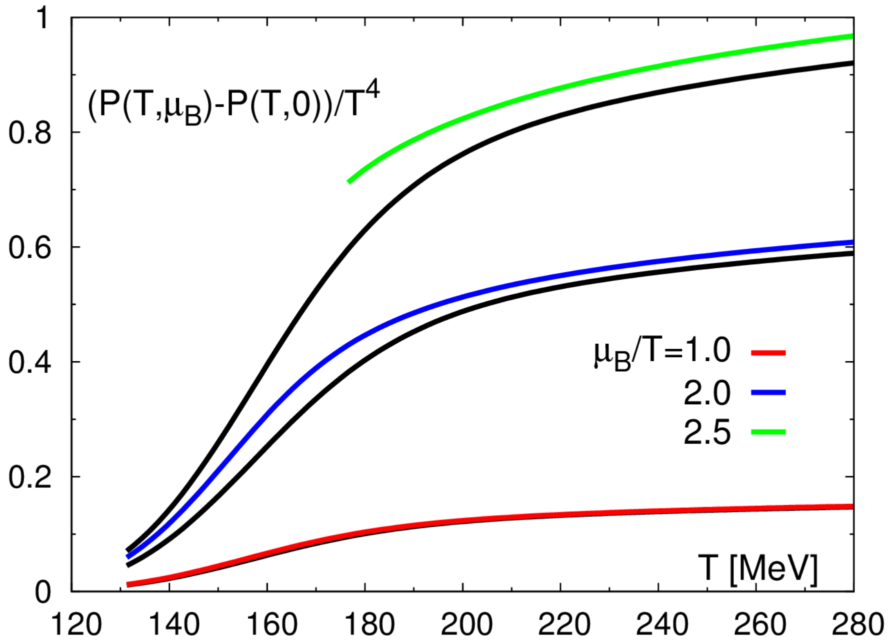
colored lines: $\mathcal{O}(\mu_B^4)$



$\mathcal{O}(\mu_B^4)$ is shown only when the estimated $\mathcal{O}(\mu_B^6)$ contribution is smaller than 5%

Lines of constant thermodynamics and freeze-out





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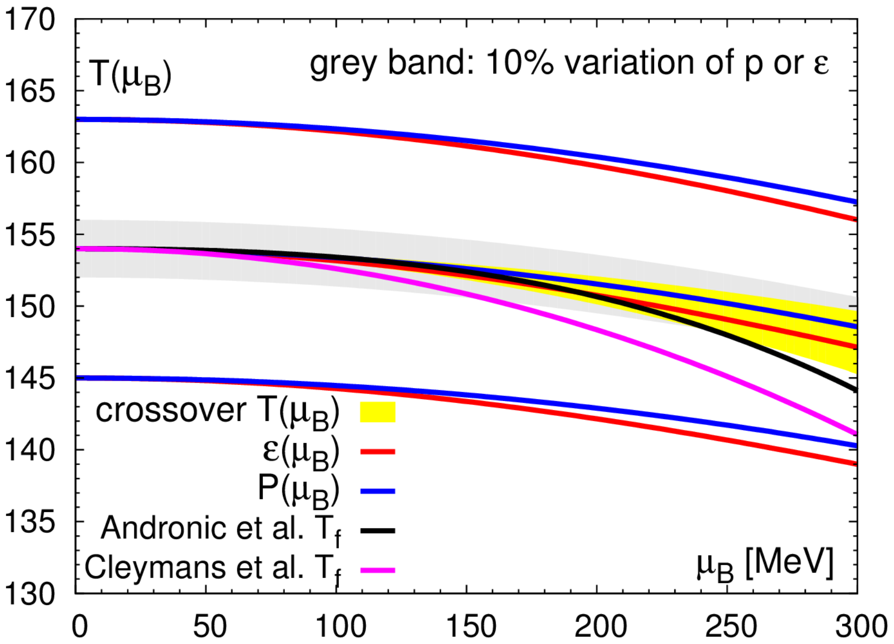


energy density and pressure decrease on the commonly used phenomenological freeze-out curves, but stay approximately constant on the crossover line for

$$\mu_B/T \lesssim 2$$

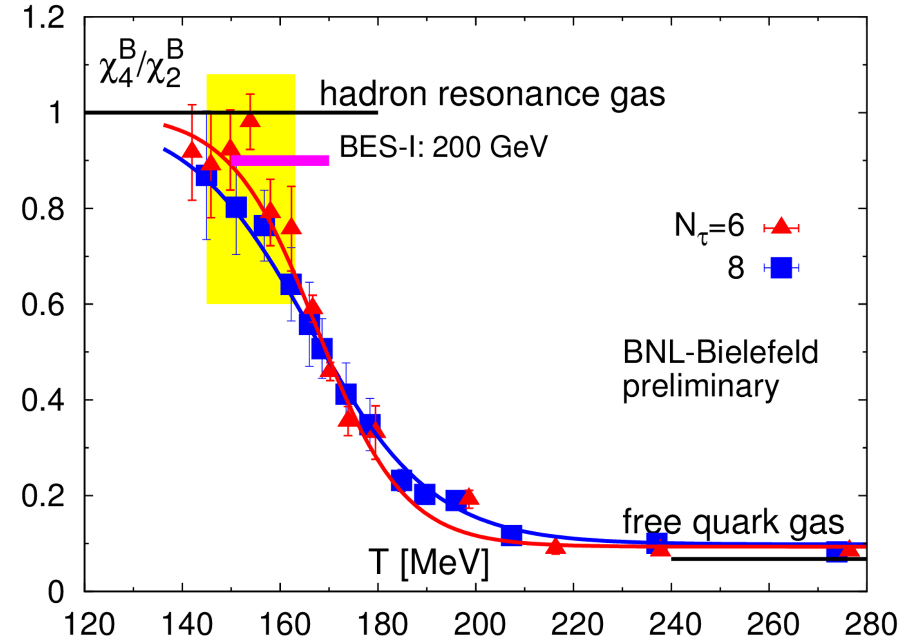
Conserved charge fluctuations and freeze-out

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

kurtosis*variance

$$(\kappa_B\sigma_B^2)_{\mu_B/T=0}$$

controls also leading terms in several ratios of conserved charge fluctuations



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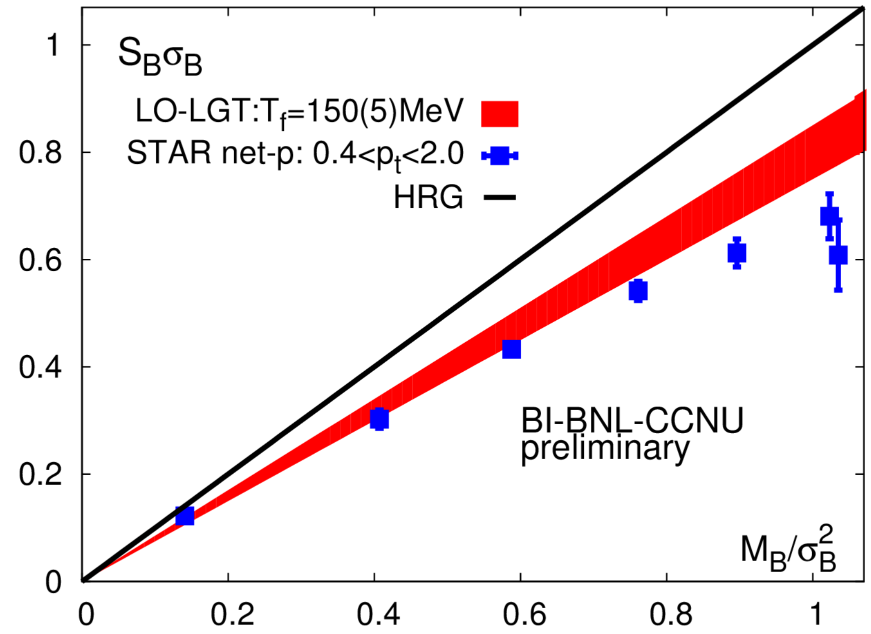
$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



warning: net-proton ≠ net-baryon

M.Kitazawa et al, PR C86 (2012) 024904
A.Bzdak et al., PR C87 (2013) 014901

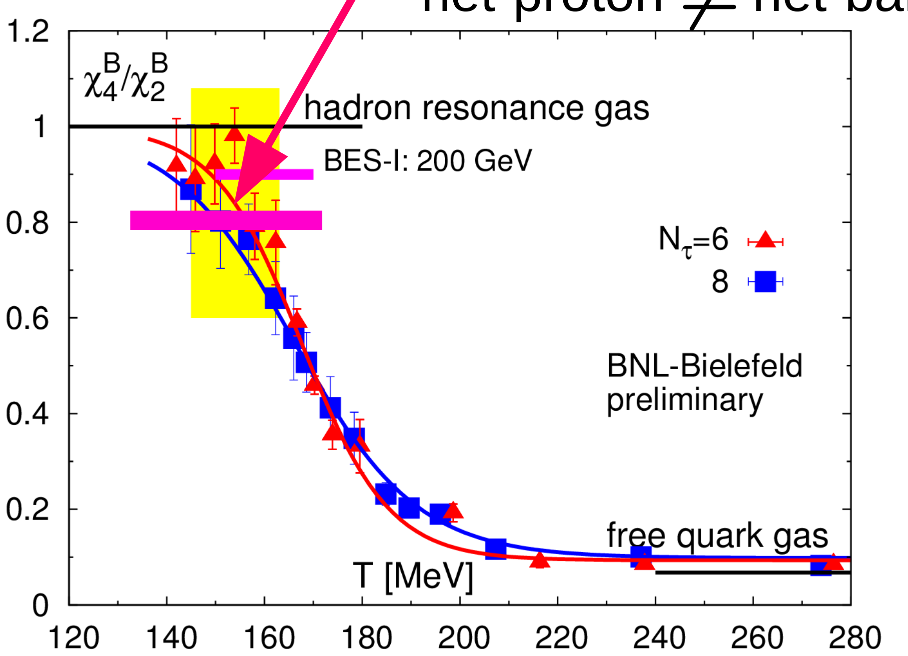
Conserved charge fluctuations and freeze-out

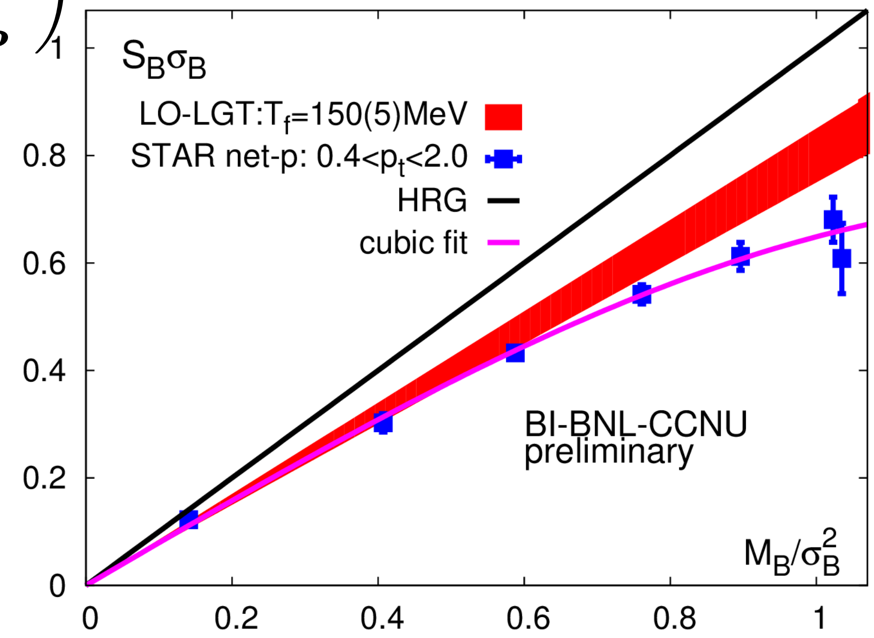
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

$$S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

fit: $S_P\sigma_P = 0.79(2)\frac{M_P}{\sigma_P^2} - 0.15(3)\left(\frac{M_P}{\sigma_P^2}\right)^3$

warning:
net-proton ≠ net-baryon





warning: net-proton ≠ net-baryon

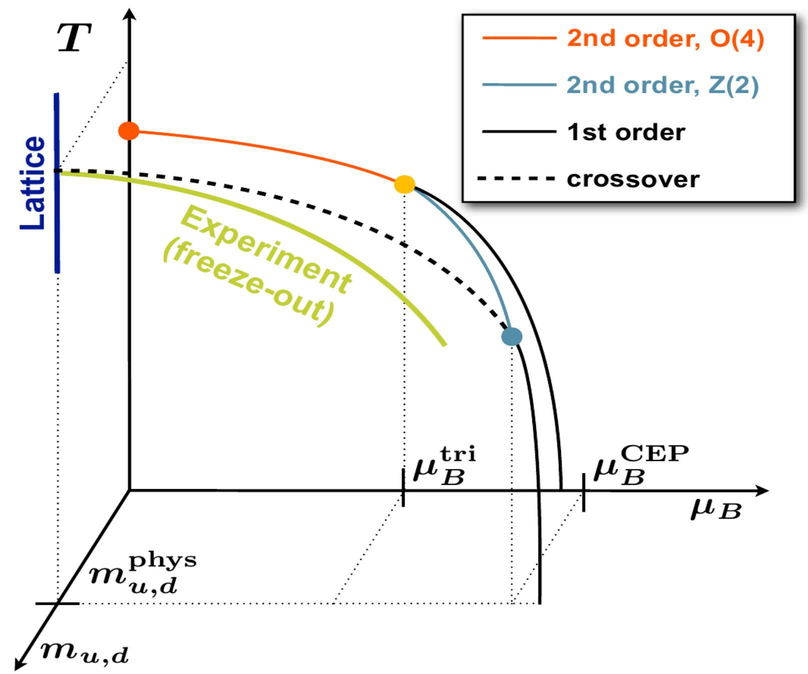
M.Kitazawa et al, PR C86 (2012) 024904
A.Bzdak et al., PR C87 (2013) 014901

Conserved charge fluctuations and freeze-out

Next order: depends on 6th order cumulants and requires knowledge on the parametrization of the freeze-out curve, eg.

$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_f \left(\frac{\mu_B}{T}\right)^2\right)$$

ratio of cumulants on "a line" in the (T, μ_B) plane



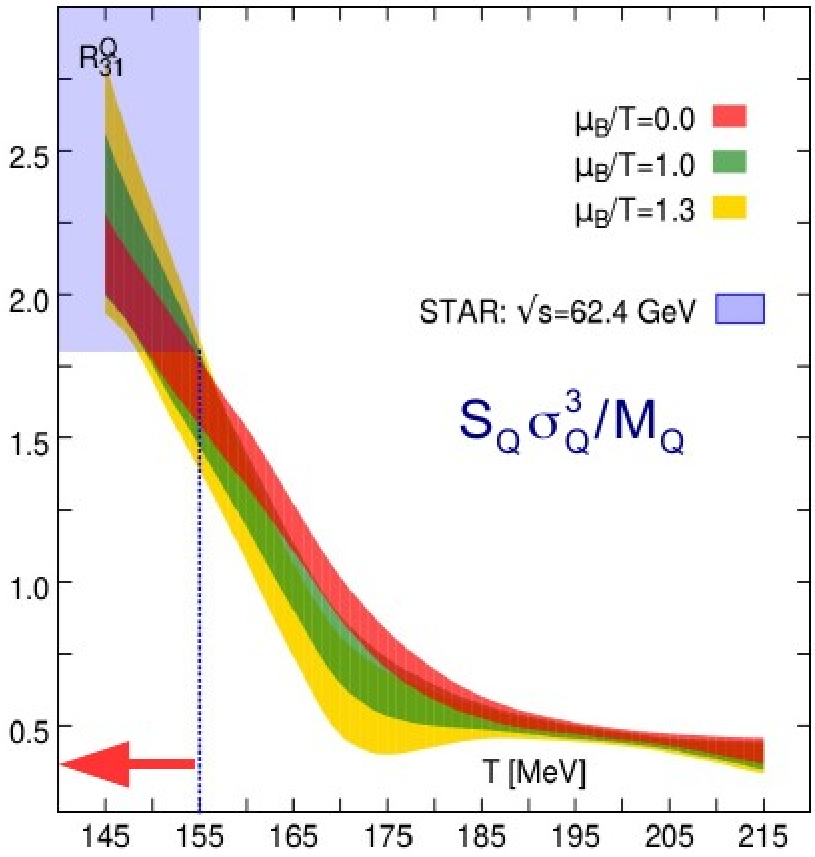
$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

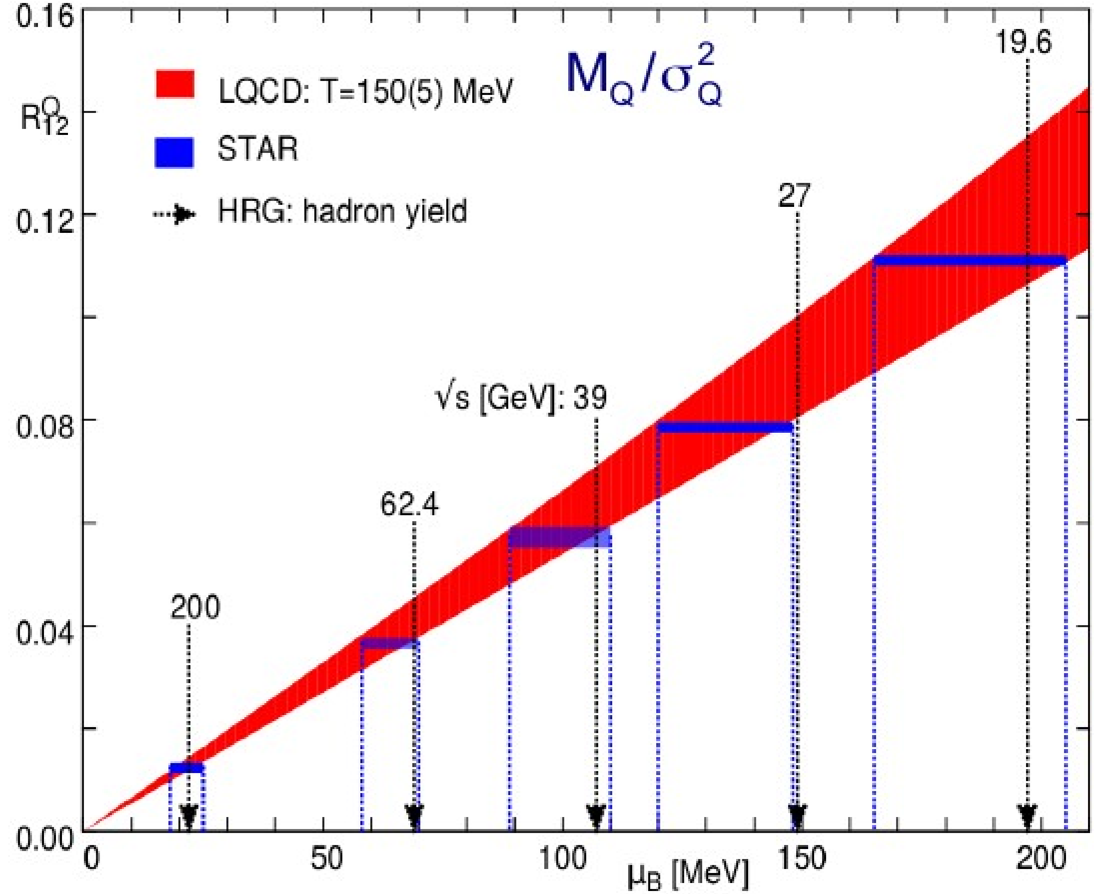
$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\equiv \left(\frac{\chi_4^B}{\chi_2^B}\right)_{T_f(0)} - \kappa_f T_f(0) \left(\frac{\chi_4^B}{\chi_2^B}\right)' \left(\frac{\mu_B}{T}\right)^2$$

Freeze-out parameter from conserved charge fluctuations

cumulant ratios of electric charge fluctuations





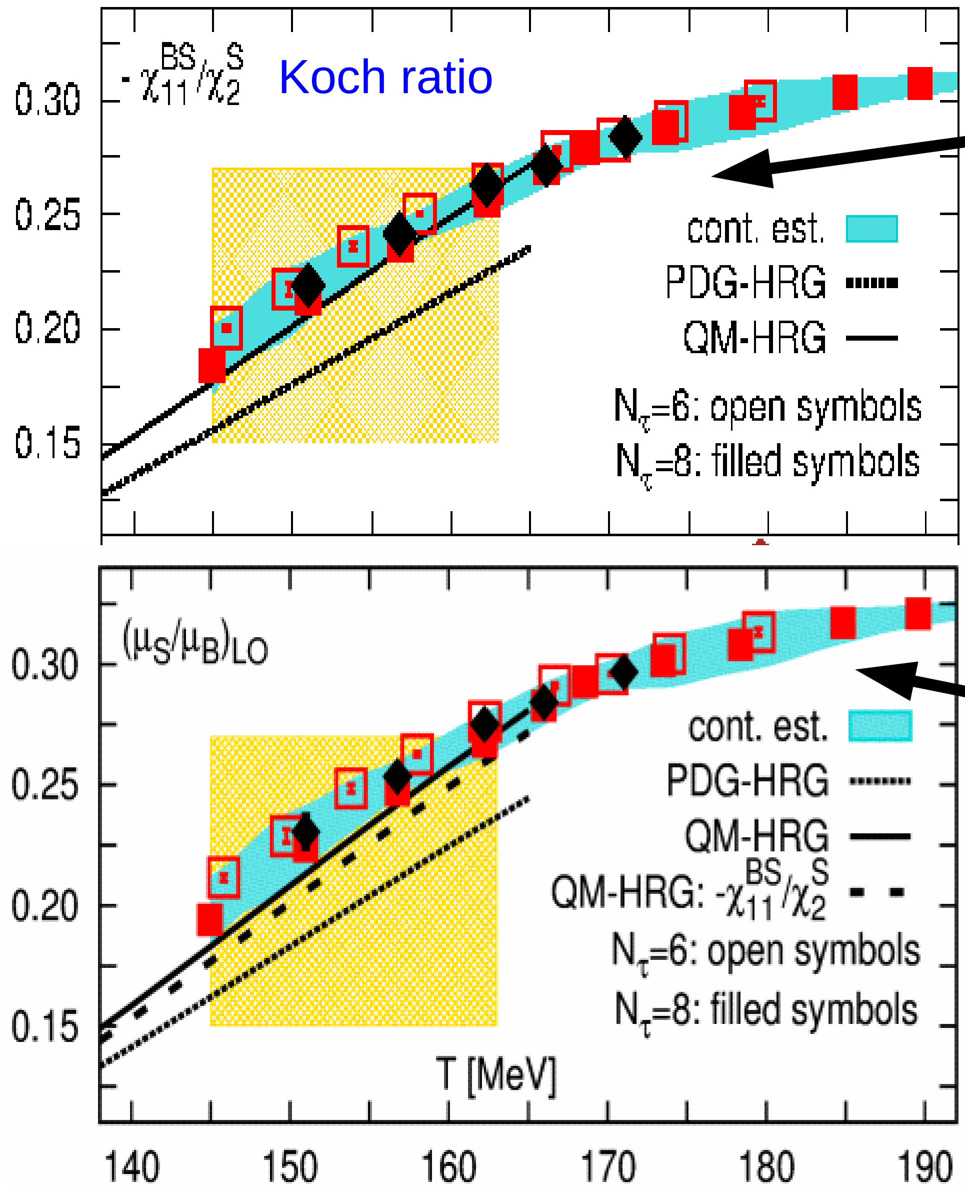
constraints freeze-out temperature

$$T_f \simeq (150 \pm 5)\ \mathrm{MeV}$$

determines freeze-out chemical potential

BI-BNL, PRL 109, 192302 (2012)
S. Mukherjee, M. Wagner, PoS CPOD2013 (2013) 039

Strangeness vs. baryon chemical potential



A. Bazavov et al., PRL 113, 072001 (2014), arXiv:1404.6511

enhanced
strangeness-baryon correlation over strangeness fluctuations

strangeness neutrality
enforces relation between chemical potentials

$$\langle n_S \rangle = 0$$
$$= \chi_2^S \hat{\mu}_S^2 + \chi_{11}^{BS} \hat{\mu}_S \hat{\mu}_B + \mathcal{O}(\mu^4)$$

$$\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)$$

HRG provides good guidance for thermal conditions at freeze-out. However,

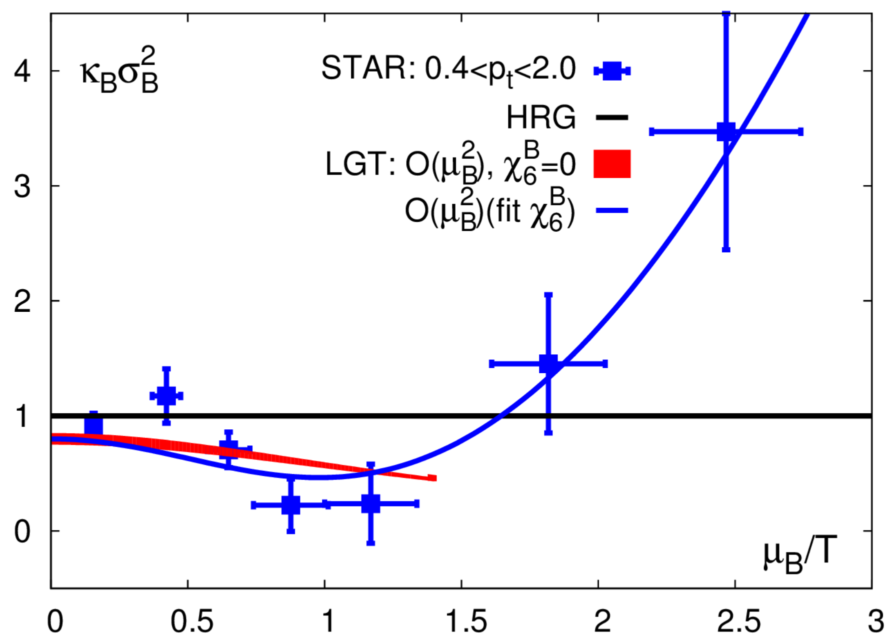
HRG is not QCD

we need/want a self-consistent determination of freeze-out parameters based on QCD

Kurtosis*variance on the freeze-out line

$$\kappa_B \sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2} \simeq \frac{\chi_4^B}{\chi_2^B}\left(1 - \frac{1}{2}\left(\frac{\chi_4^B}{\chi_2^B} - \frac{\chi_6^B}{\chi_4^B}\right)\left(\frac{\mu_B}{T}\right)^2\right)$$

$\frac{\chi_6^B}{\chi_4^B}$ changes sign in crossover region



ansatz: $\frac{\chi_6^B}{\chi_4^B} = a_6 + b_6 \left(\frac{\mu_B}{T}\right)^2$

consistent treatment requires knowledge of T-dependence of

$$\frac{\chi_4^B}{\chi_2^B}, \frac{\chi_6^B}{\chi_4^B}$$

on the freeze-out line

To do list

What is needed to understand equilibrium properties of conserved charge fluctuations on the freeze-out line?

– accurate lattice QCD results on 6th (and 8th) order cumulants of conserved charge fluctuations

– self-consistent determination of freeze-out parameters within QCD: $T_f(\mu_B),\ \mu_B,\ [\mu_S(\mu_B),\ \mu_Q(\mu_B)]$

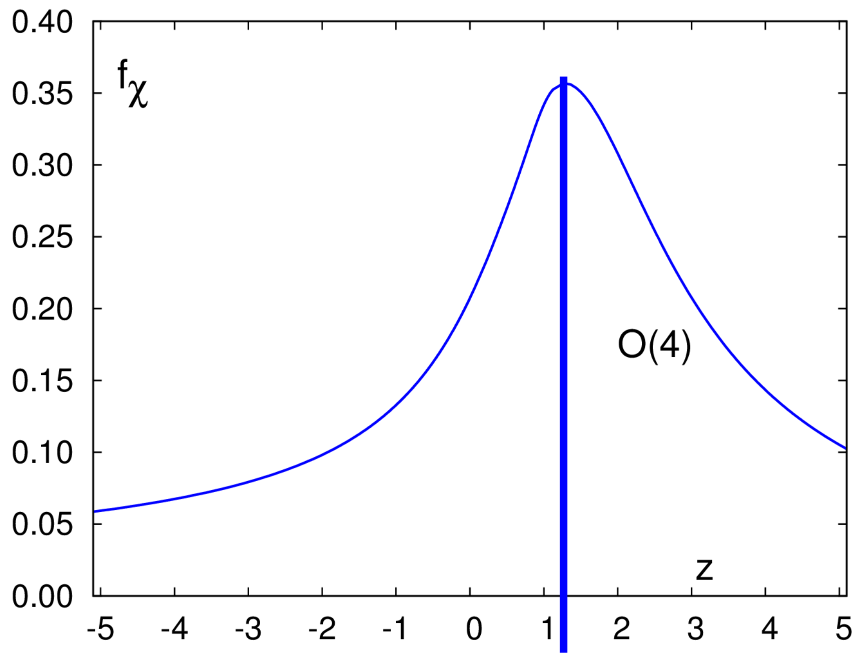
– Quantify influence of finite-V, acceptance, p $\neq$ B etc. in close interaction with experiment and HI-phenomenology

What can be done about "locating the critical point"?

– use 6th (and 8th) order cumulants to put bounds on its location

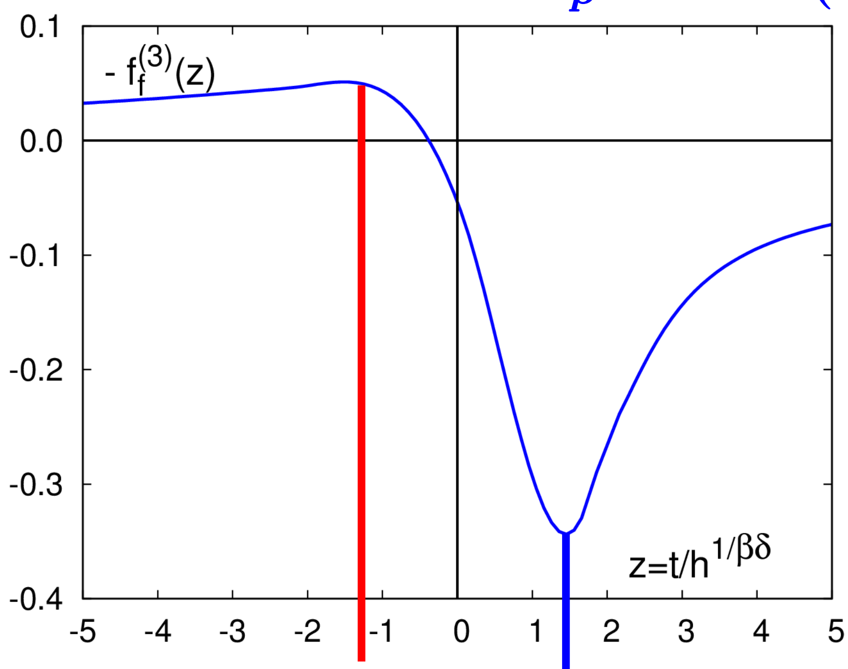
– keep working on new simulation techniques

By-product: EoS in the entire parameter range accessible to the RHIC BES-II



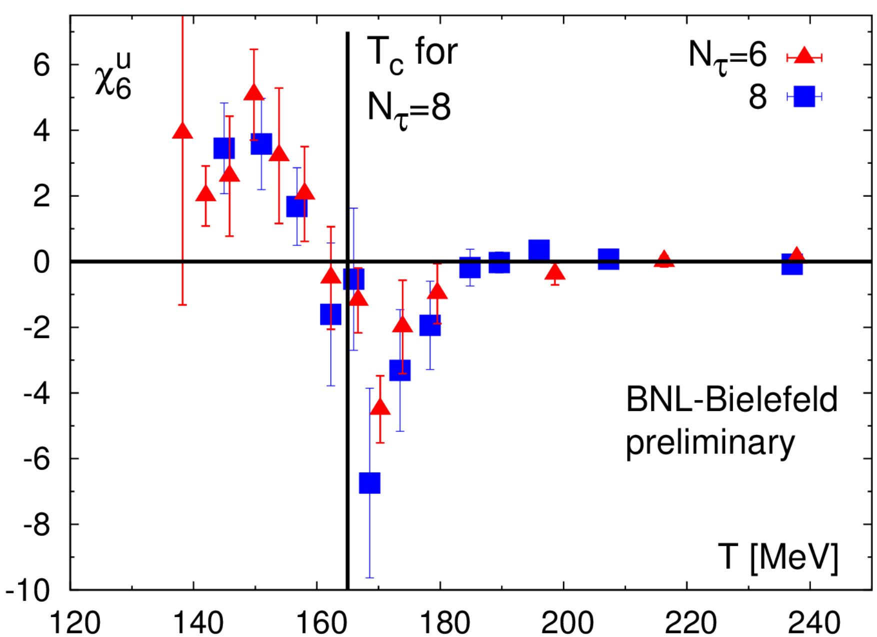
$z_p = 1.33(5)$

The peak in the scaling function that determines the location of the chiral crossover transition as seen by the chiral susceptibility is at (almost) the same temperature, at which the 6th order quark number susceptibility has its minimum – if contributions from regular terms are small!!



$z_- \simeq -1.50$ $z_+ \simeq 1.48$

6-th order net "up-ness" fluctuations



qualitatively as expected

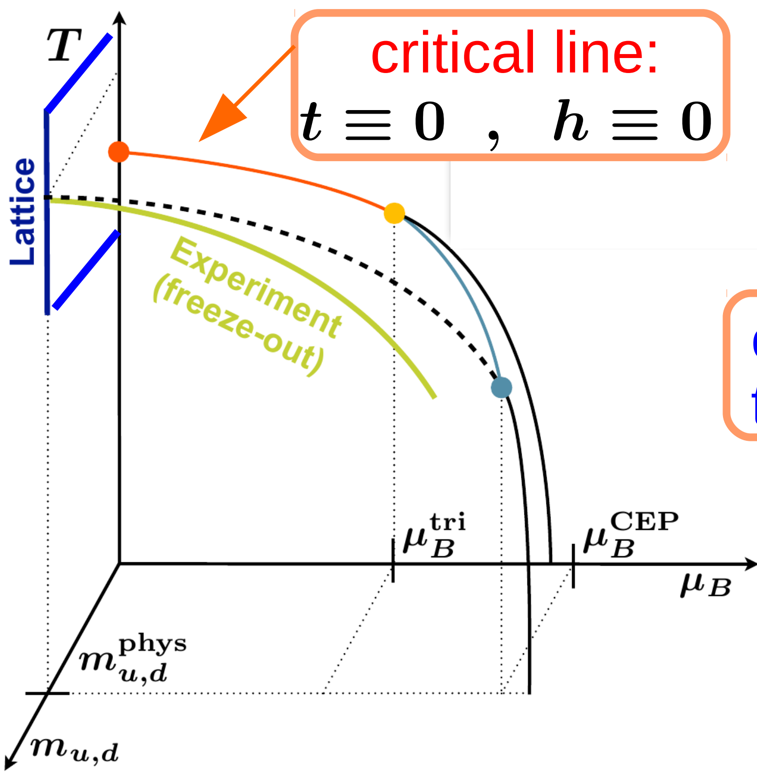
Chiral Transition at small μ_B/T

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal O(4) scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\vec{\mu}) = -h_0\ h^{1+1/\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2 \quad , \quad h \sim \frac{m_q}{T_c}$$



controls curvature of the critical line

suppressing dependence on strange quark chemical potential

$$\chi_n^q = \frac{\partial^n P/T^4}{\partial(\mu_q/T)^n} \sim \kappa_q^n f_f^{(n)}(t/h^{1/\beta\delta})$$

O(4) Scaling in QCD: Curvature of the critical line

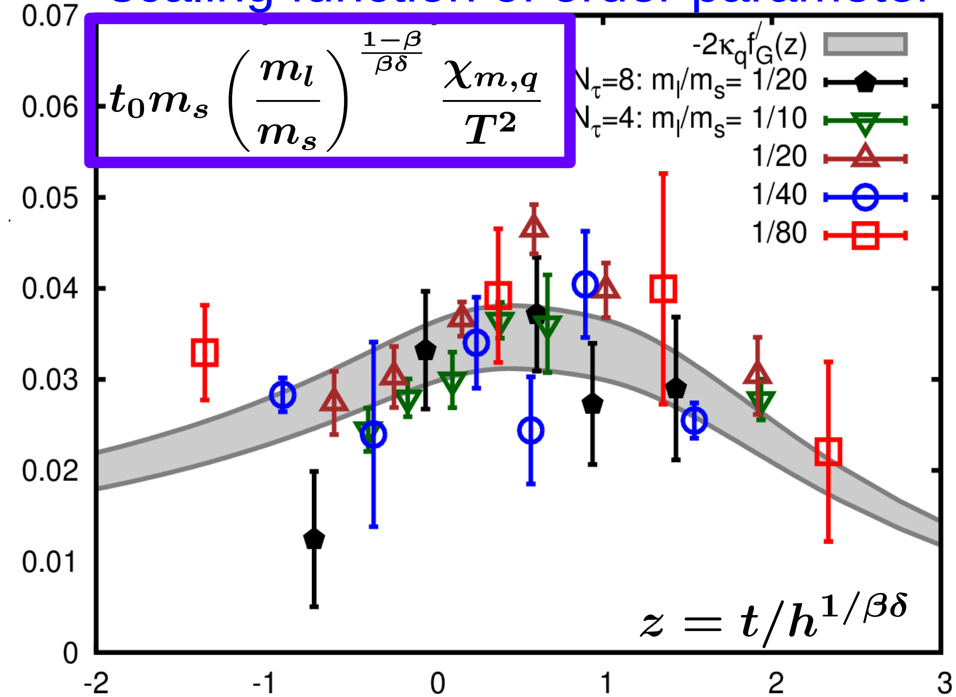
$$M_b \equiv \frac{m_s \langle \bar\psi\psi \rangle}{T^4} = h^{1/\delta} f_G(z)$$

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2 \langle \bar\psi\psi \rangle / T^3}{\partial(\mu_q/T)^2}$$

$$= \frac{2\kappa_q T}{t_0 m_s} h^{(\beta-1)/\delta\beta} f'_G(z)$$

$$\kappa_B = \kappa_q/9 = 0.0066(7)$$

scaling function of order parameter

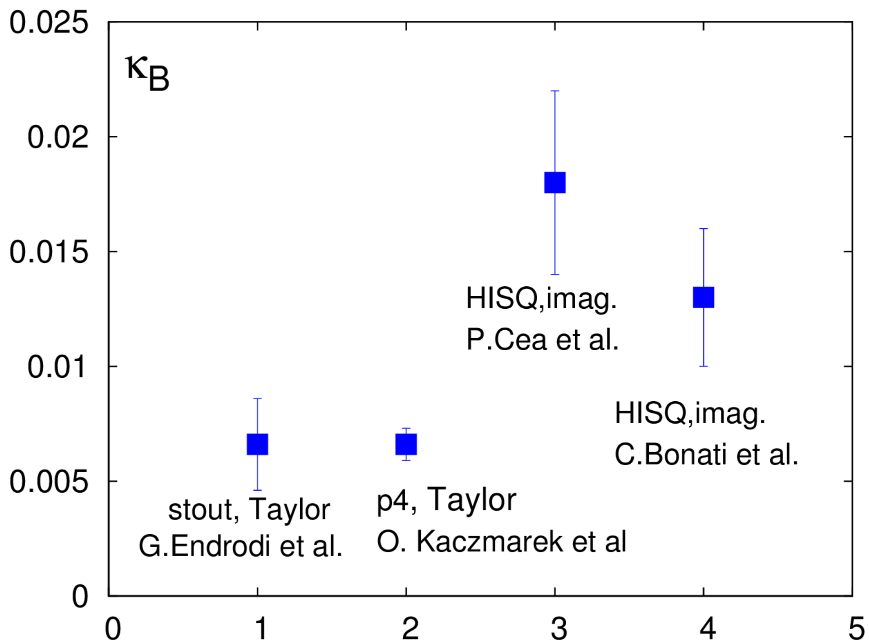


p4-action: $N_\tau = 4$

Bielefeld-BNL, Phys. Rev. D83, 014504 (2011)

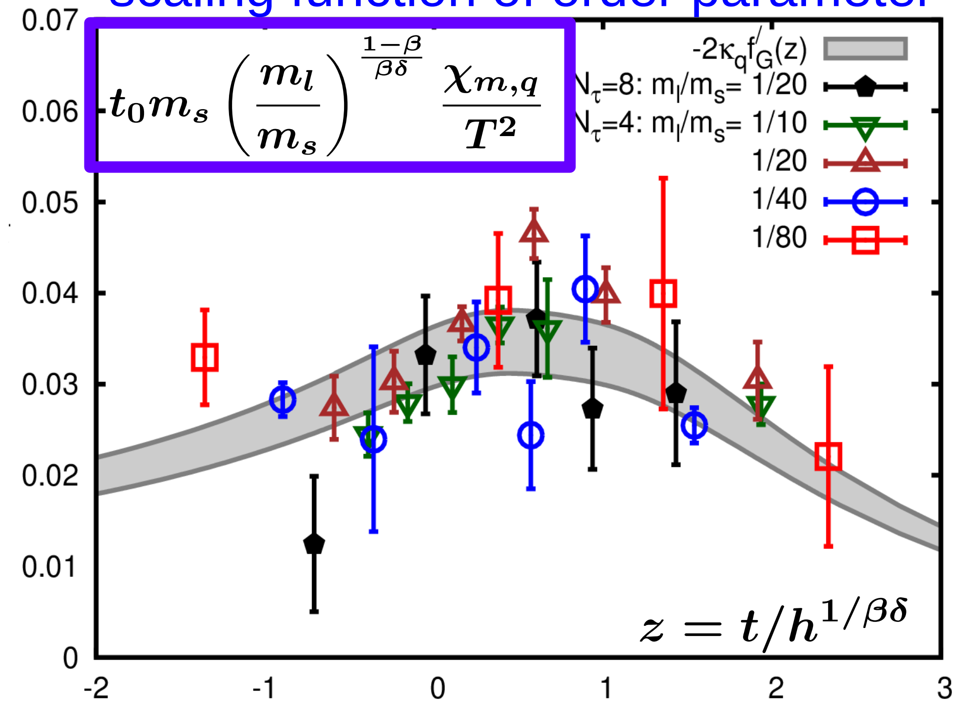
O(4) Scaling in QCD: Curvature of the critical line

summary of current values
for the curvature term:



$$0.006 \lesssim \kappa_B \lesssim 0.018$$

scaling function of order parameter



p4-action: $N_\tau = 4$

Bielefeld-BNL, Phys. Rev. D83, 014504 (2011)

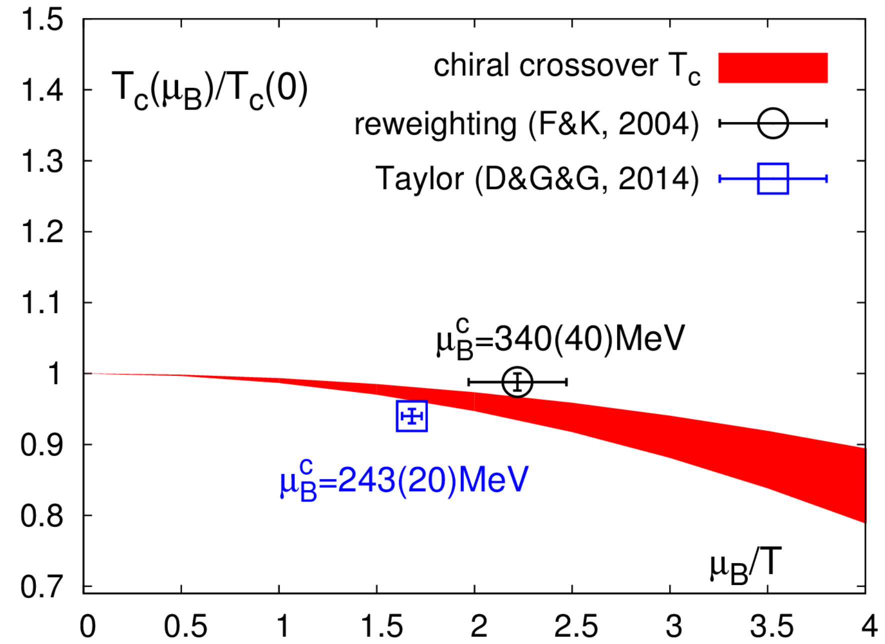
→ the crossover temperature changes by (4-12) MeV

for $0 \leq \mu_B/T \leq 2$

or $15\ \mathrm{GeV} < \sqrt{s} < \infty$

Critical Point searches

lattice QCD



reweighting:
Z. Fodor, S. Katz,
JHEP 04, 204 (2004)

Taylor expansion:
S. Datta, R.V. Gavai, S. Gupta,
PoS Lattice 2013 (2014) 202