Toward quantitative and rigorous conclusions from heavy ion collisions

Scott Pratt, Michigan State University

MADAI Collaboration

Models and Data Analysis Initiative

http://madai.us

Cyber-enabled Discovery and Innovation

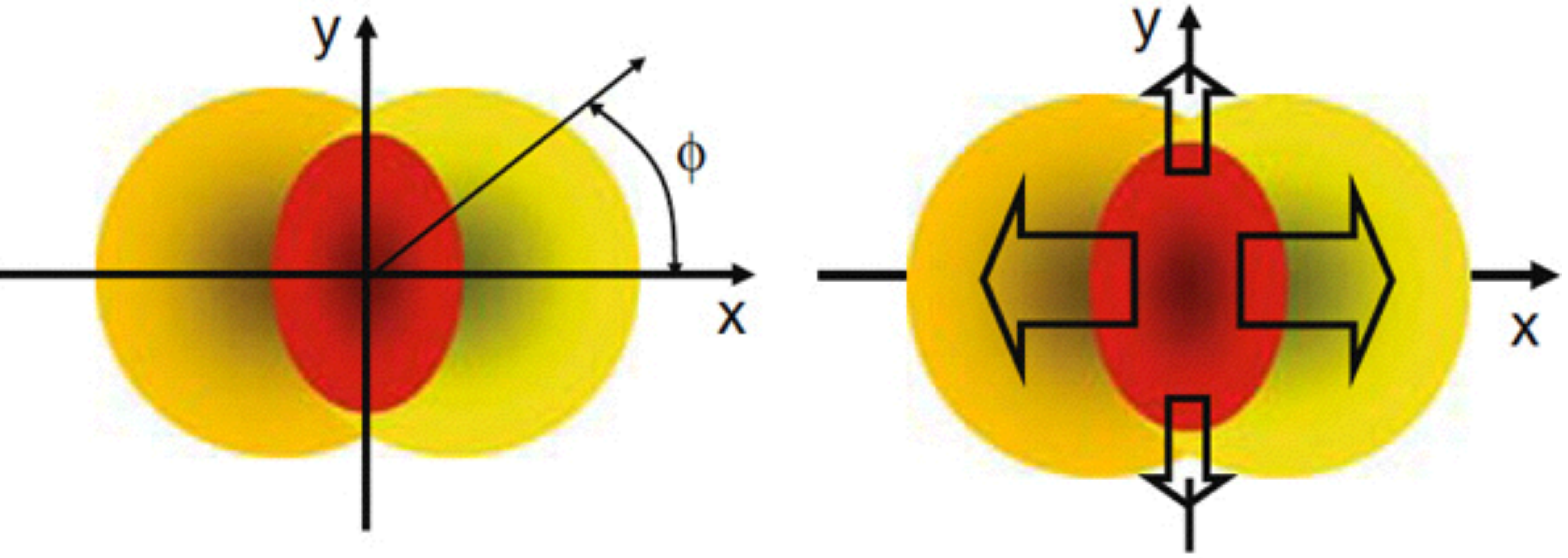
MICHIGAN STATE UNIVERSITY Duke UNIVERSITY THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL renci

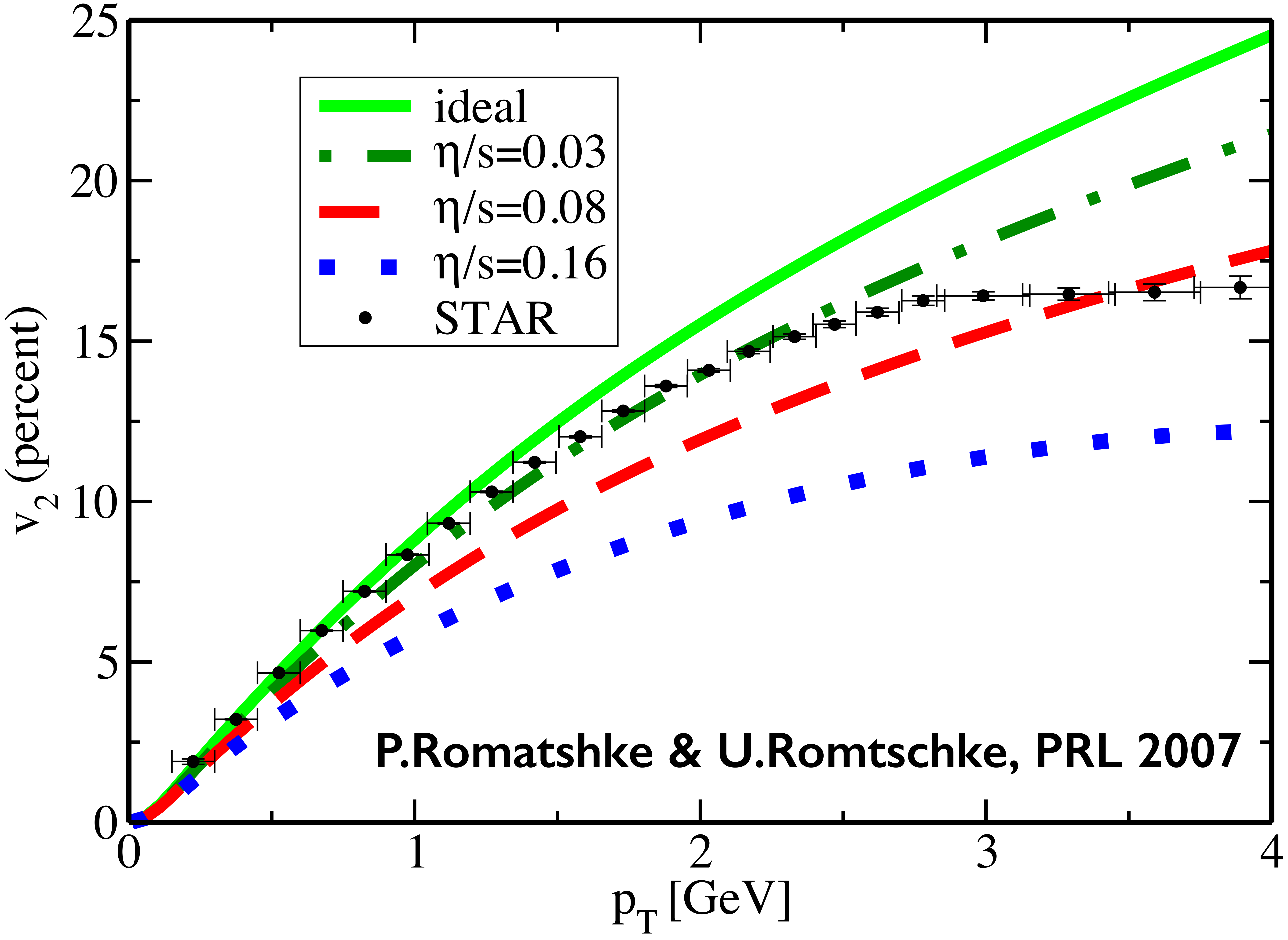
1st MADAI Collaboration Meeting, SANDIA 2010

How this was done before (*v*2 and η/s)

Study single parameter vs. single observable

$$v_2 \equiv \langle \cos 2\phi \rangle$$





PROBLEM

v2 depends on ....

- viscosity
- saturation model
- pre-thermal flow
- Eq. of State
- T-dependence of η/s
- initial T_{xx}/T_{zz}
- . . . .

Correct Way (MCMC)

- Simultaneously vary *N* model parameters x_i
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{\left(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})}\right)^2}{2\sigma_a^2}\right\}$$

- Use all observables y_a
- Obtain representative sample of posterior

Difficult Because…

I. Too Many Model Runs
Requires running model $\sim 10^6$ times

II. Many Observables
Could be hundreds of plots,
each with dozens of points
Complicated Error Matrices

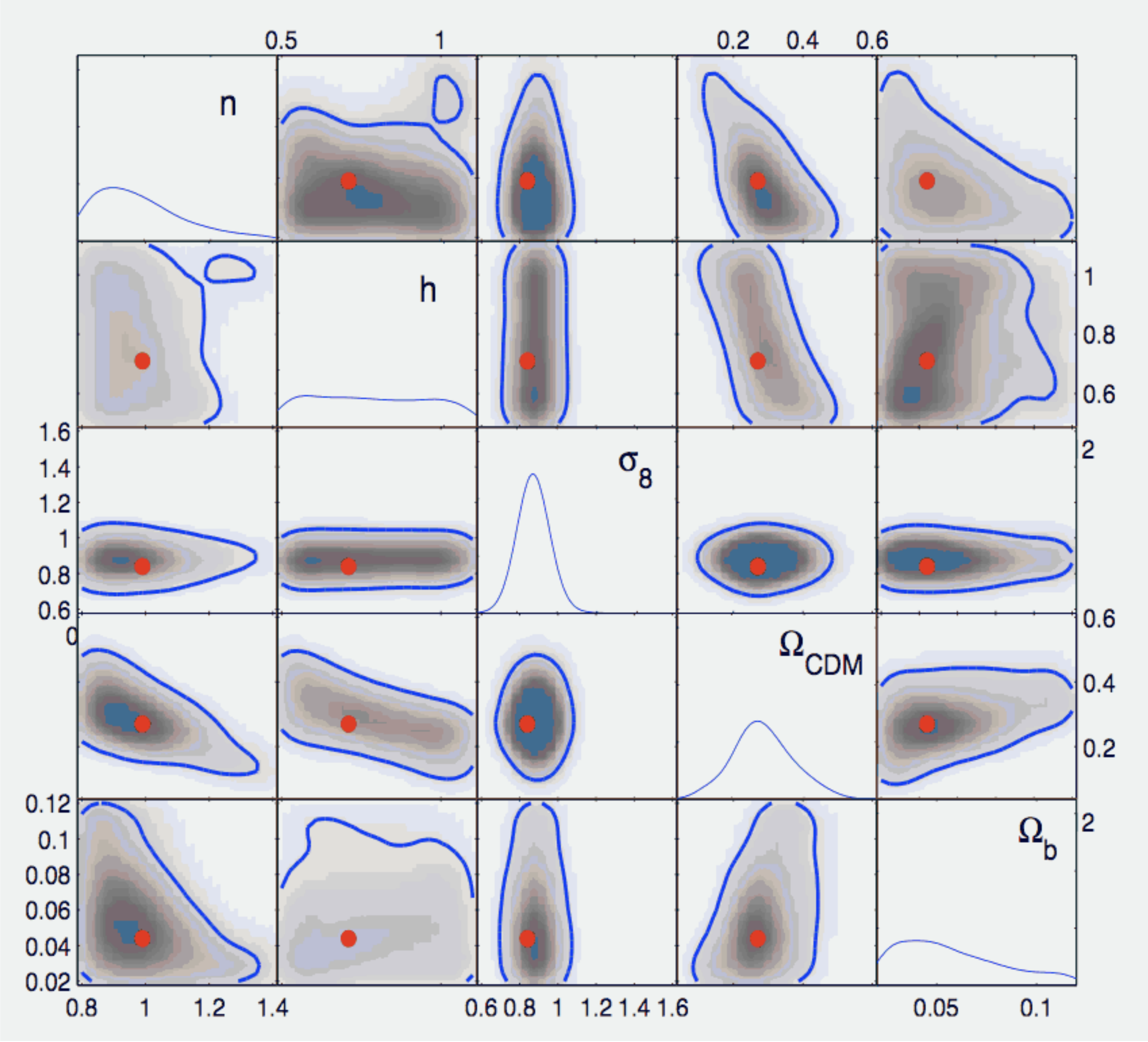
Model Emulators

1. **Run the model ~1000 times**
   Semi-random points (LHS sampling)

2. **Determine Principal Components**
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

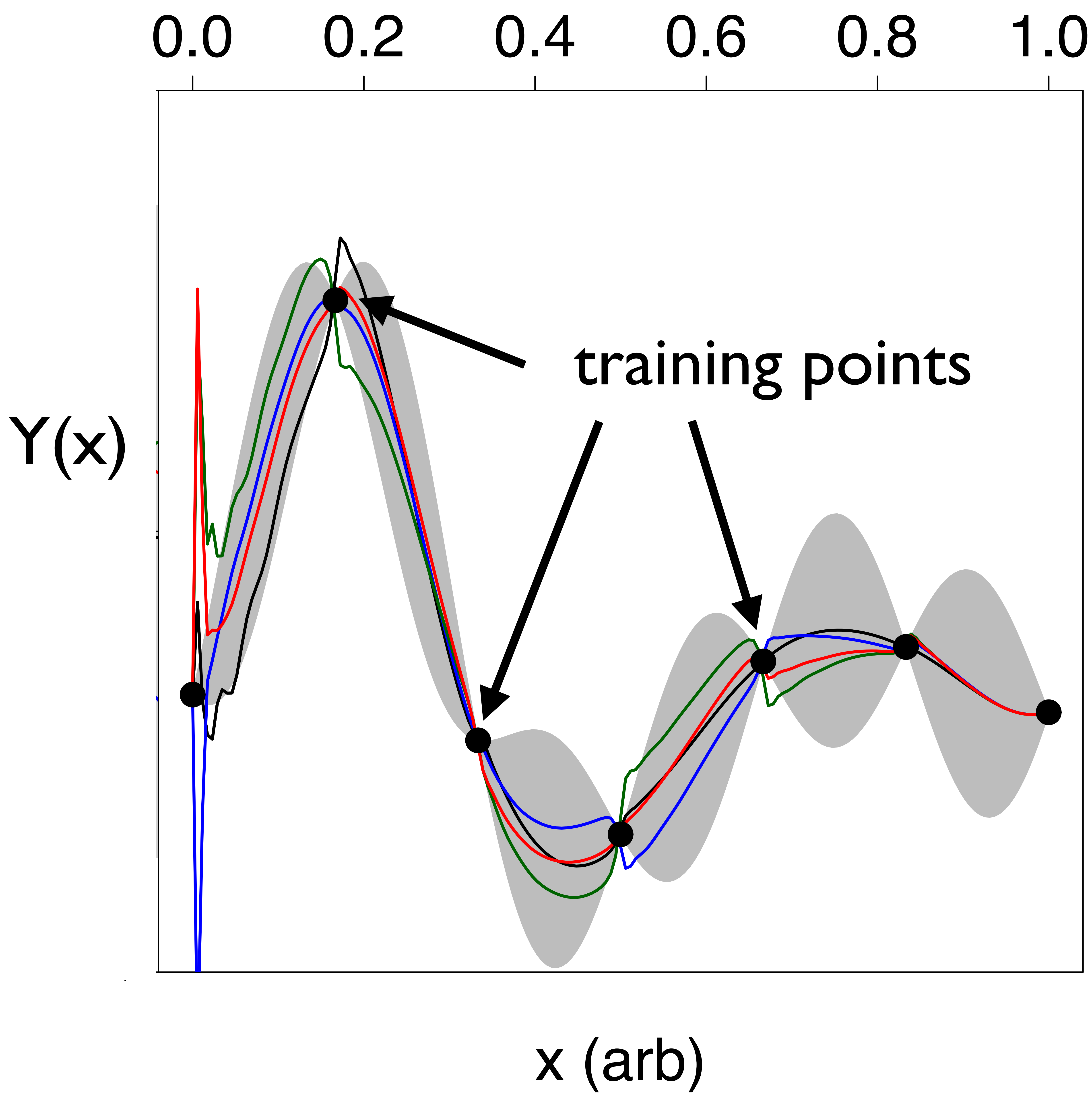
3. **Emulate z_a (Interpolate) for MCMC**
   Gaussian Process…

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\frac{1}{2}\sum_a (z_a^{(\mathrm{emulator})}(\mathbf{x}) - z_a^{(\mathrm{exp})})^2\right\}$$



S. Habib,K.Heitman,D.Higdon,C.Nakhleh&B.Williams, PRD(2007)

Emulator Algorithms



- Gaussian Process
  - Reproduces training points
  - Assumes localized Gaussian covariance
  - Must be trained, i.e. find “hyper parameters”
- Other methods also work

14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- π, K, p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- v_2 Weighted by p_t

Initial State Parameters

$$\epsilon(\tau = 0.8\text{fm}/c) = f_{\text{wn}}\epsilon_{\text{wn}} + (1 - f_{\text{wn}})\epsilon_{\text{cgc}},$$

$$\epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_B\right)\right\} + (A \leftrightarrow B)$$

$$\epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_{\text{max}}\right)\right\}$$

$$T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\text{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

Equation of State and Viscosity

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

2 parameters for EoS, 2 for η/s

DATA Distillation

1. Experiments reduce PBs
   to 100s of plots

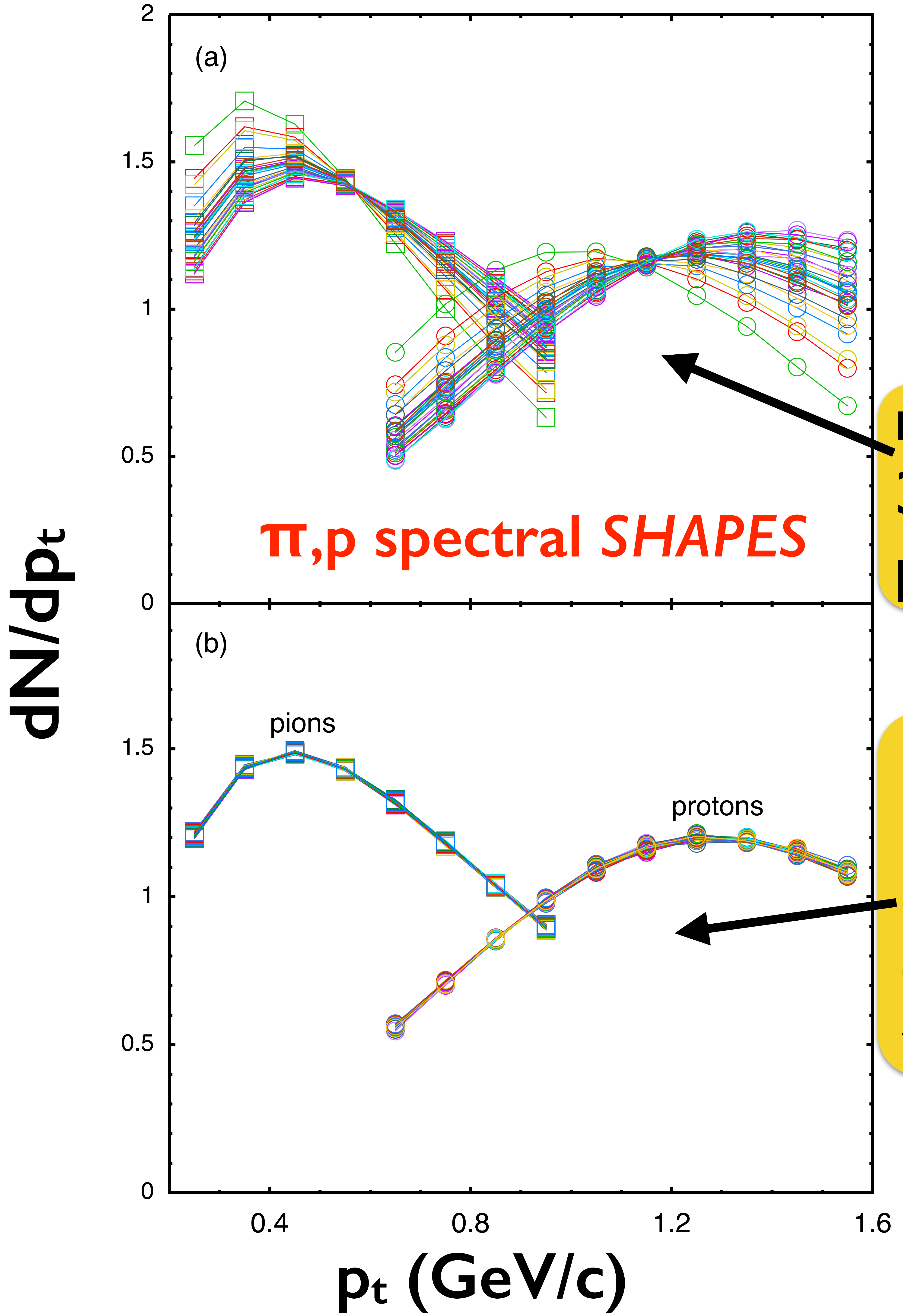
2. Choose which data to analyze
   Does physics *factorize*?

3. Reduce plots to a few
   representative numbers, y_a

4. Transform to principal
   components

Checking the Distillation

Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$



model spectra from 30 random points in parameter prior

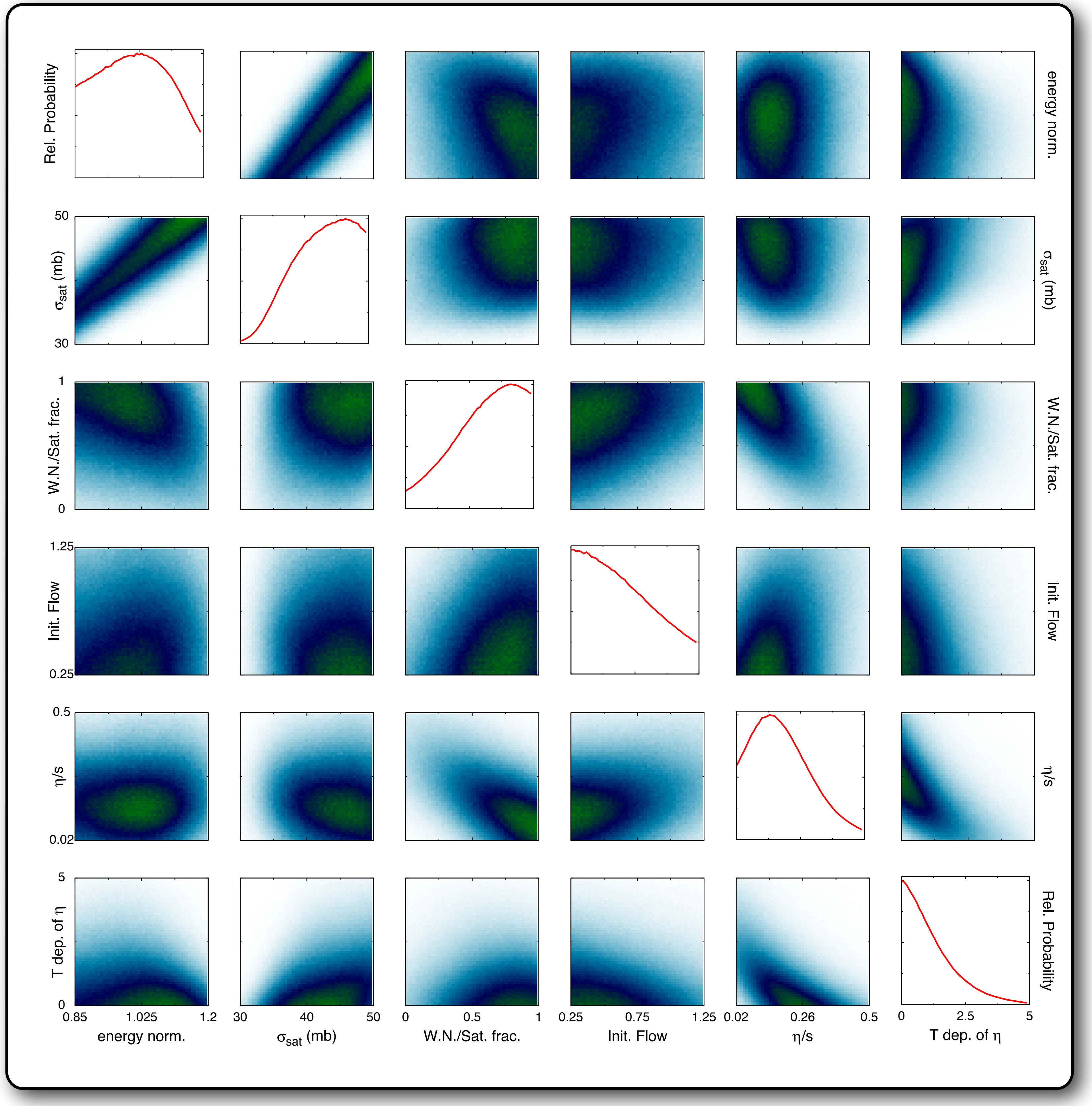
74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV

44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV

Two Calculations

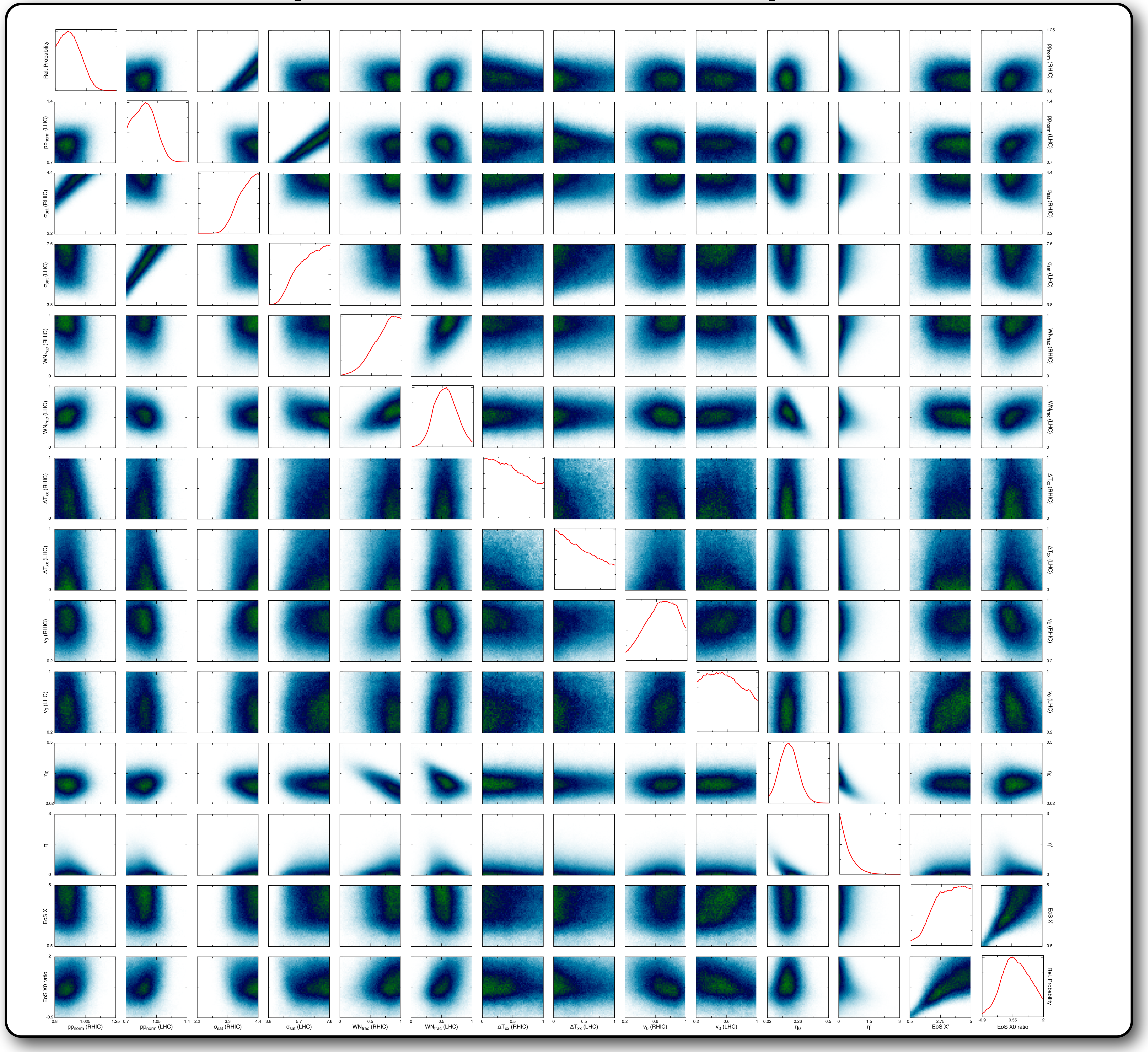
1. J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, ArXiv:1303.5769

RHIC Au+Au Data
6 parameters

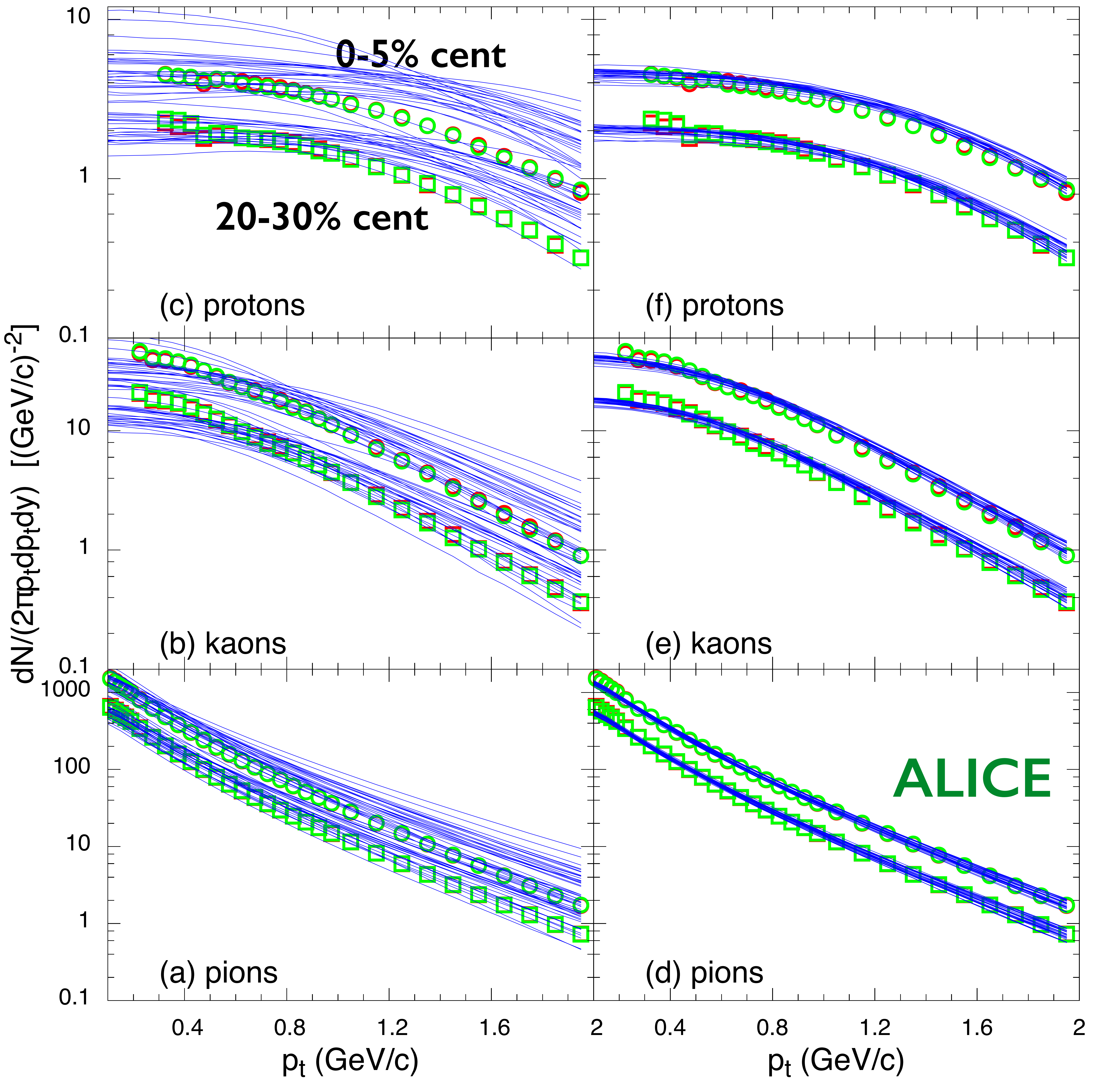


2. S.P., E.Sangaline, P.Sorensen & H.Wang, in progress

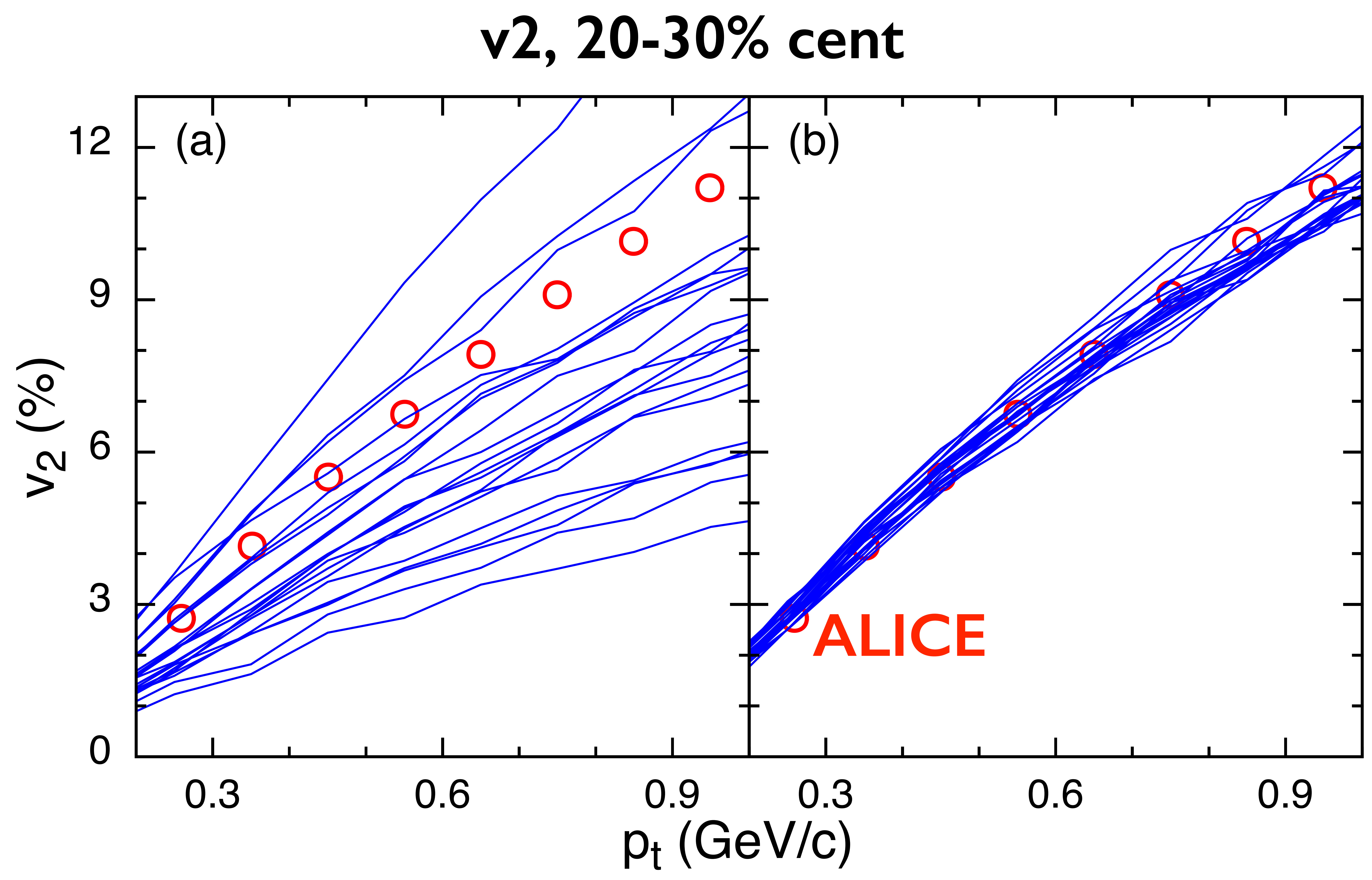
RHIC Au+Au and LHC Pb+Pb Data
14 parameters, include Eq. of State



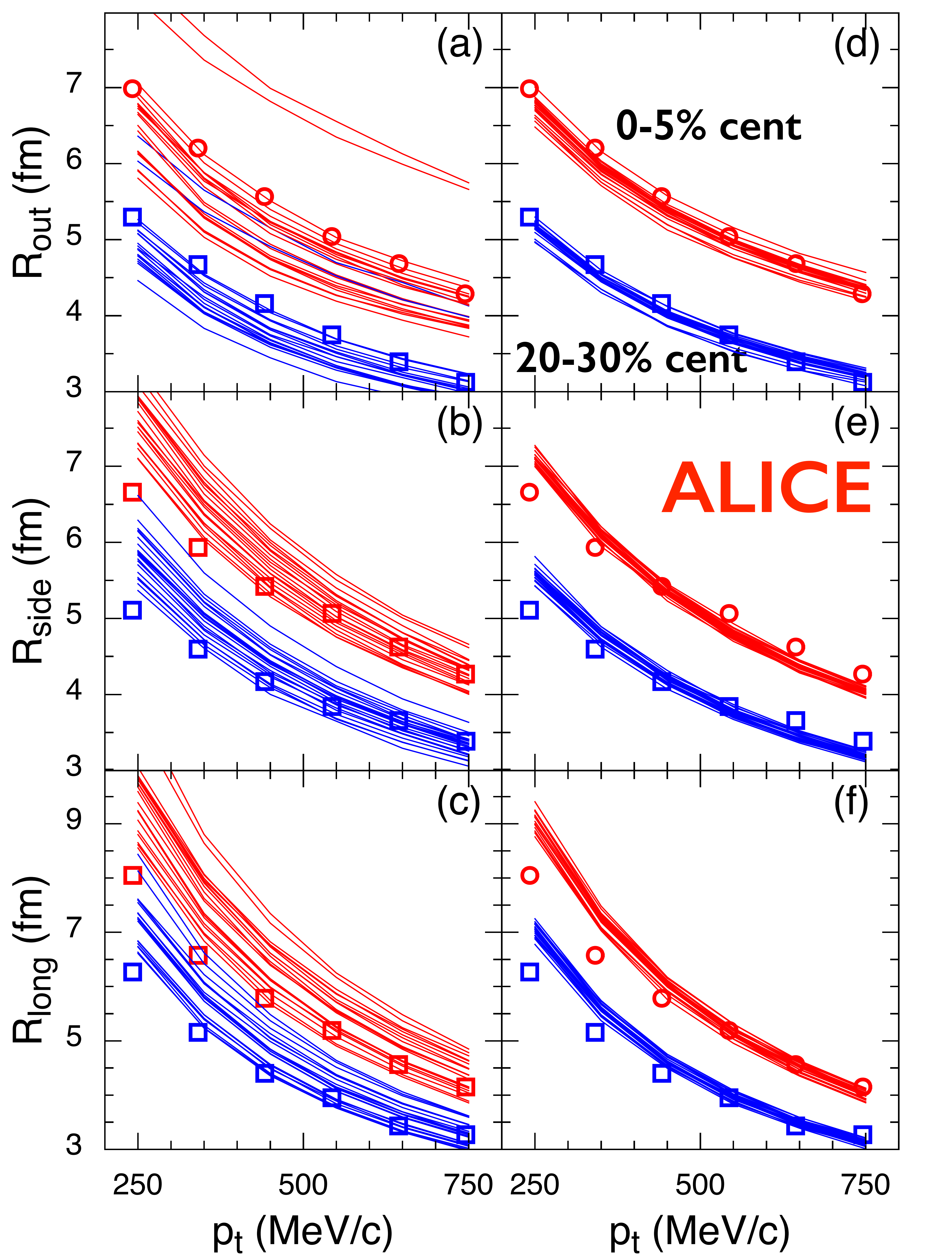
Sample Spectra from Prior and Posterior



Sample V2 from Prior and Posterior

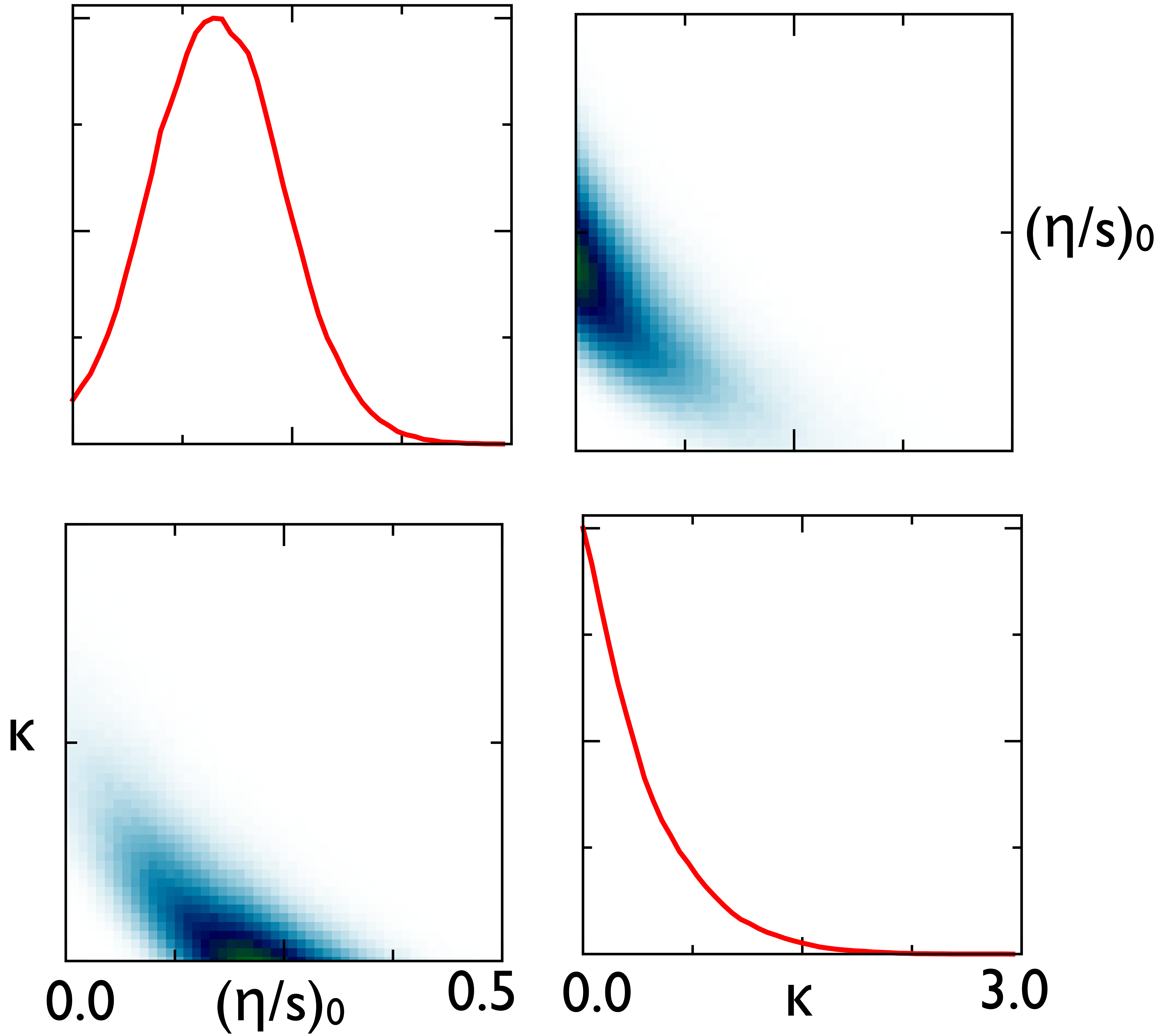


Sample HBT from Prior and Posterior

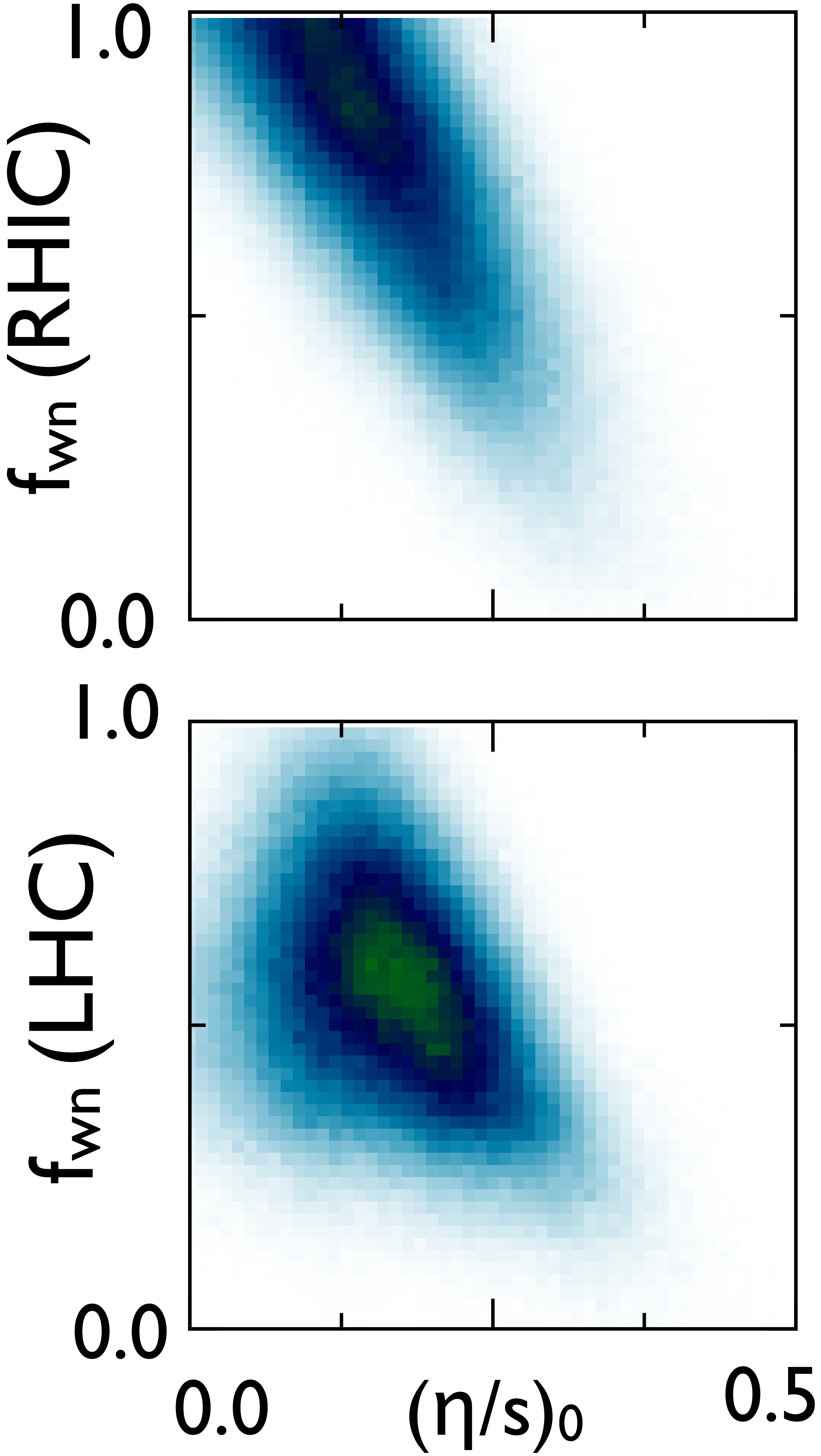


η/s(T)

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$



η/s vs saturation picture



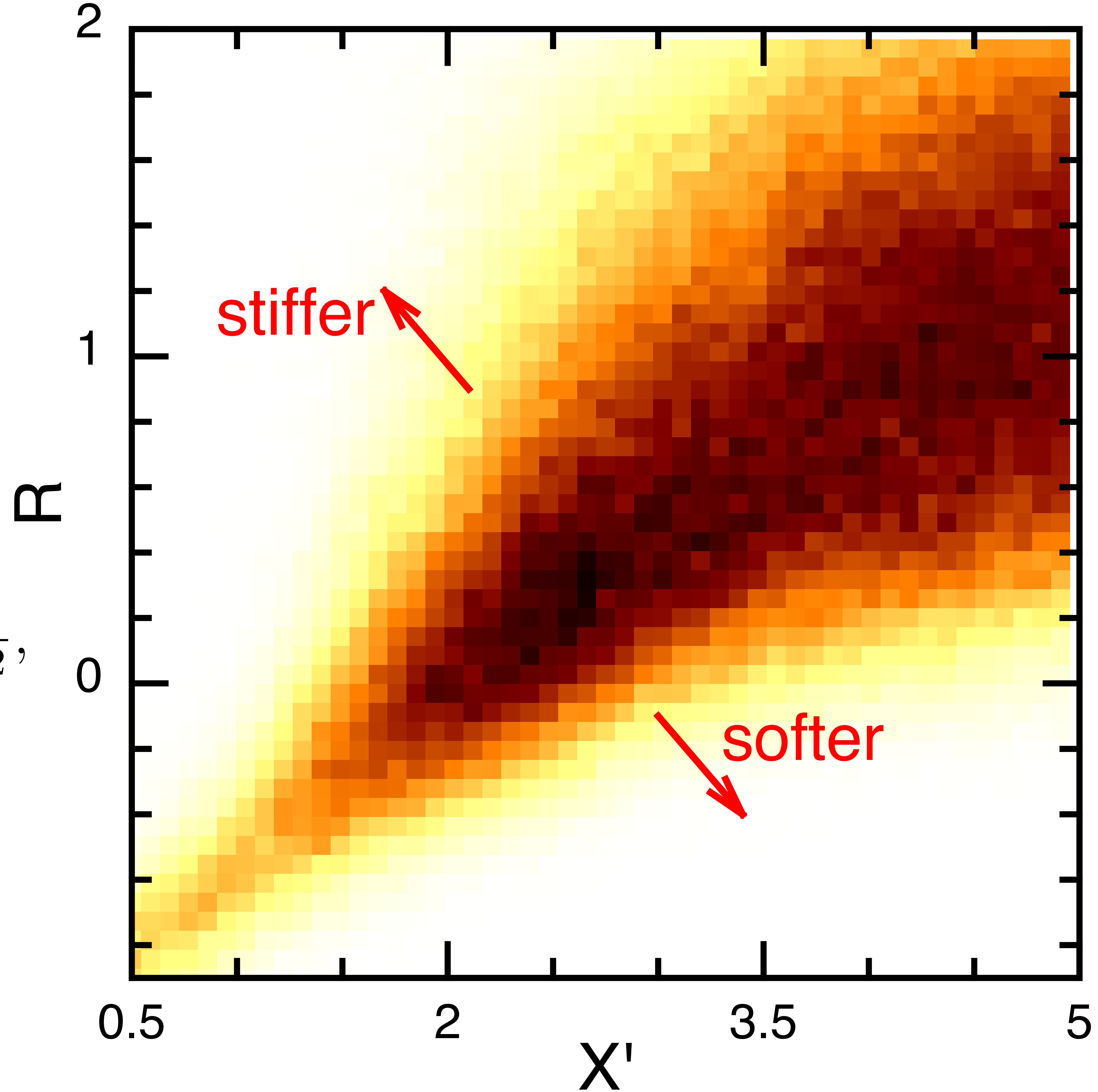
See Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right)\frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

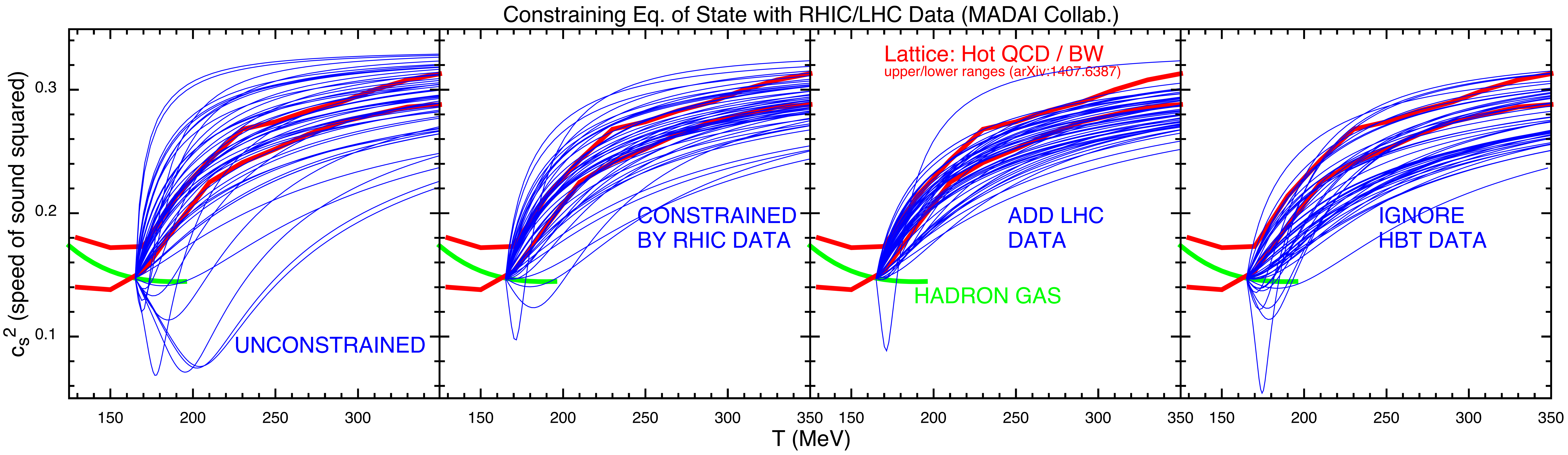
$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

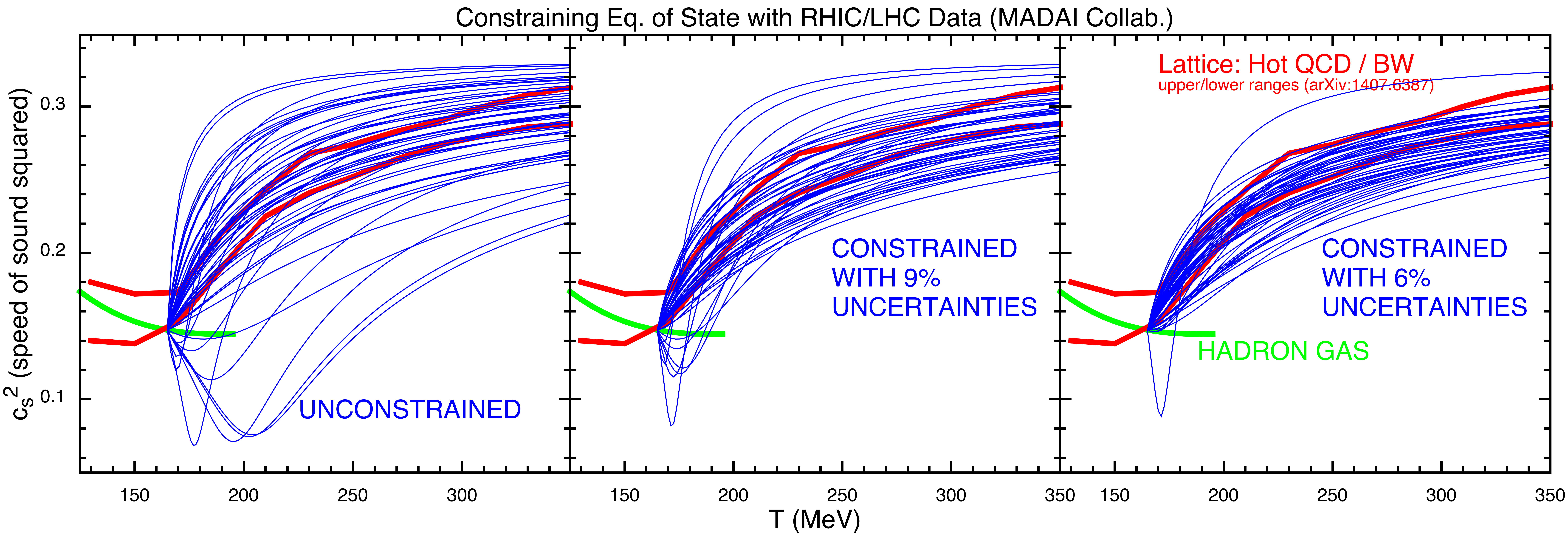


Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
CONSTRAINED
BY DATA
HADRON GAS
UNCONSTRAINED
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

Which observables constrain the EoS?



Sensitivity to Uncertainty



SUMMARY

- Robust
- Emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from RHIC & LHC data
- Other parameters not as well constrained
- Heavy-Ion Physics can be a Quantitative Science!!!!

NEAR FUTURE

- **Improve models (will lead to more parameters)**
  - hadronization uncertainties
  - bulk viscosity
  - more realistic cascade
    Bose enhancement, better cross sections
  - 3D corrections
  - lumpy IC

- **Better statements of uncertainty**
  - Requires cooperation, both experimenters and theorists

- **Extend to different analyses**
  - Initial state studies
  - Jet Physics
  - …

BEAM ENERGY SCAN

- **Improve models (MANY more parameters)**
  - 3D Initial Conditions
    baryon stopping, initial flow and rotation,
    initial temperatures, corona
  - Paramterize IC
  - Density Dependent EoS
  - Mean-field for hadronic Boltzmann

- **Statistics may require rethinking**
  - $N_{parameters} \sim 50$

- **Should be able to determine *P(ρ,T)***

If you're interested…

1. Tools are readily extended
2. Download software and tutorial from http://madai.us
3. Talk to me (prattsc@msu.edu)
   or Evan Sangaline (esangaline@gmail.com)

Made possible by contributions from DOE, NSF, Chris Coleman-Smith, John Novak, Kevin Novak, Evan Sangaline, Paul Sorensen, Joshua Vredevoogd, Hui Wang, Robert Wolpert, and viewers like you.

Additional slide: Charge BFs and charge susceptibilities

S.P., C. Ratti and W.McCormack, arXiv:1409.2164