Fluid dynamics and fluctuations in the QCD phase diagram

Marlene Nahrgang

Duke University

RBRC Workshop, Theory and Modeling for the Beam Energy Scan

In collaboration with Christoph Herold (Suranaree University of Technology)

Duke University

DAAD

The goal...

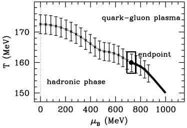
... is to understand the phase structure and the phase diagram of QCD theoretically and experimentally.

Make the connection between QCD thermodynamics (LQCD) and heavy-ion collisions.

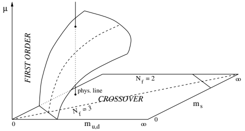
https://news.uic.edu/collider-reveals-sharp-change-from-quark-soup-to-atoms

From the theory side...

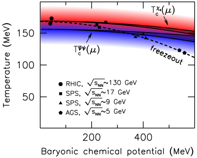
- Lattice QCD calculations:



Wuppertal-Budapest (2002)



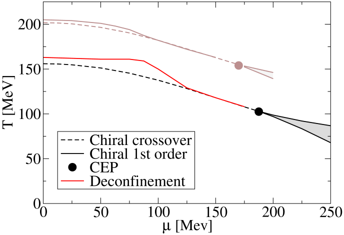
F. Karsch et al. (2004)



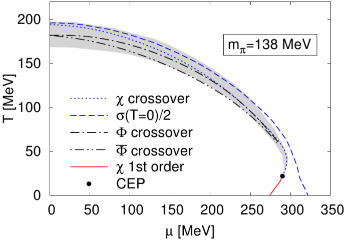
Wuppertal-Budapest (2011)

+ newer approaches to circumvent the sign problem!

- Functional methods of QCD:



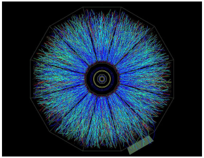
C. Fischer, J. Luecker, PLB**718** (2013)



T. Herbst, J. Pawlowski, BJ. Schaefer PRD**88** (2013)

From the experimental side...

- One of the main goals of heavy-ion collisions is to understand the phase structure of hot and dense strongly interacting matter.

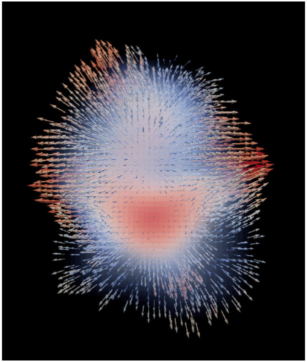


- Can we experimentally produce a deconfined phase with colored degrees of freedom?
- What are the properties of this phase?
- What is the nature of the phase transition between deconfined and hadronic phase?

Dynamics of heavy-ion collisions

Systems created in heavy-ion collisions

- are short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical



plot by H. Petersen, madai.us

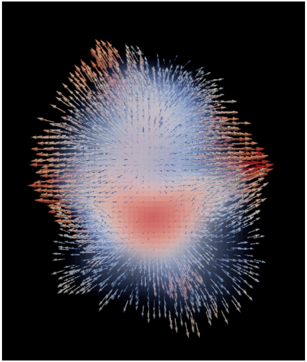
Indications that we might still be able to learn about thermodynamic properties:

- success of fluid dynamics ($\Rightarrow$ local thermalization) with input from LQCD (EoS)
- success of statistical model and HRG analysis of particle yields and fluctuations

Dynamics of heavy-ion collisions

Systems created in heavy-ion collisions

- are short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical



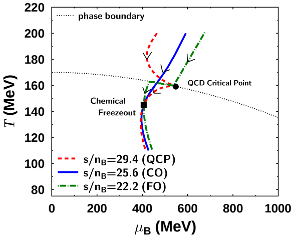
plot by H. Petersen, madai.us

Indications that we might still be able to learn about thermodynamic properties:

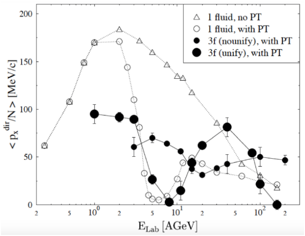
- success of fluid dynamics ($\Rightarrow$ local thermalization) with input from LQCD (EoS)
- success of statistical model and HRG analysis of particle yields and fluctuations

Phase transitions in fluid dynamics

- Conceptually, studying phase transitions in fluid dynamics is really simple!
- $\Rightarrow$ Just need to know the equation of state and transport coefficients!



C. Nonaka, M. Asakawa PRC**71** (2005)

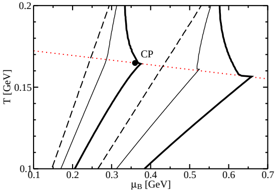


J. Brachmann et al. PRC**61** (2000)

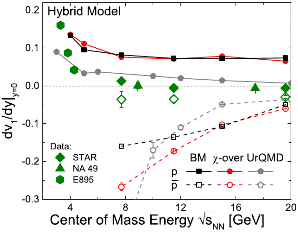
-
-

Phase transitions in fluid dynamics

- Conceptually, studying phase transitions in fluid dynamics is really simple!
- ⇒ Just need to know the equation of state and transport coefficients!



M. Bluhm, B. Kampfer CPOD 2006



J. Steinheimer et al. PRC**89** (2014)

- No clear sensitivity on the equation of state in observables.
- BUT at the phase transition: fluctuations matter! Including fluctuations in fluid dynamics is more challenging...

Fluctuations at the phase transition

At a critical point

- correlation length of fluctuations of the order parameter diverges $\xi \to \infty$
- fluctuations of the order parameter diverge: $\langle \Delta\sigma^n \rangle \propto \xi^\alpha$ with higher powers of divergence for higher moments
- mean-field studies in Ginzburg-Landau theories, beyond mean-field: renormalization group
- relaxation time diverges $\Rightarrow$ critical slowing down!

$\Rightarrow$ fluctuations in equilibrated systems!

... and a first-order PT:

- at T_c coexistence of two stable thermodynamic phases
- metastable states above and below $T_c \Rightarrow$ supercooling and -heating
- nucleation and spinodal decomposition in nonequilibrium
- domain formation and large inhomogeneities

$\Rightarrow$ fluctuations in nonequilibrium!

... but also at the crossover:

- remnant of the $O(4)$ universality class in the chiral limit.

$\Rightarrow$ fluctuations in equilibrated systems!

Ref. by many people: M. Stephanov, E. Shuryak, K. Rajagopal, L. Csernai, J. Randrup, I. Mishustin, C. Sasaki, B. Friman, K. Redlich et al.

Nonequilibrium chiral fluid dynamics (NχFD)

IDEA: combine the dynamical propagation of fluctuations at the phase transition with fluid dynamical expansion!

(model-independent is nice, but in the end some real input is needed...)

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T_{\rm q}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu} \,, \quad \partial_\mu N_{\rm q}^\mu = 0$$

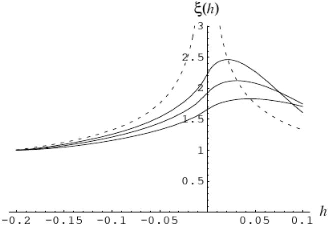
$\Rightarrow$ includes a stochastic source term!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC **84** (2011); PLB **711** (2012); JPG **40** (2013)
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013); NPA**925** (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG **41** (2014)

Dynamical slowing down

Phenomenological equation:

$$\frac{d}{dt} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{eq}(t)})$$

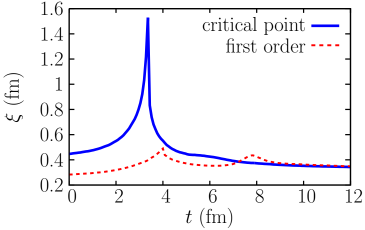


B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Input from the dynamical universality class.

In NχFD:

$$\xi^2 = 1/m_\sigma^2 = (\frac{d^2 V_{eff}}{d\sigma^2})^{-1}$$



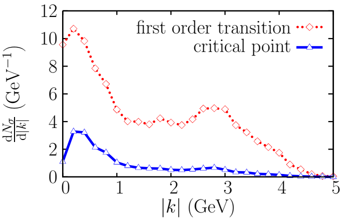
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

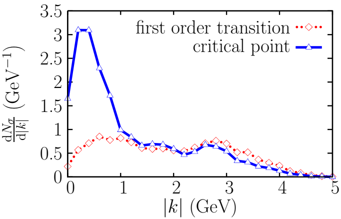
Definition of ξ in inhomogeneous systems: involves averaging!

$\Rightarrow$ Similar magnitude of $\xi/\xi_0 \sim 1.5 - 2$!

Dynamics versus equilibration

- Fluctuations of the order parameter:



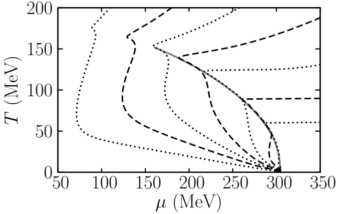


- Strong enhancement of the intensities for a first-order phase transition during the evolution.
- Strong enhancement of the intensities for a critical point scenario after equilibration.

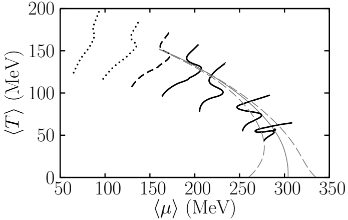
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

Trajectories and isentropes at finite μ_B

Isentropes in the PQM model

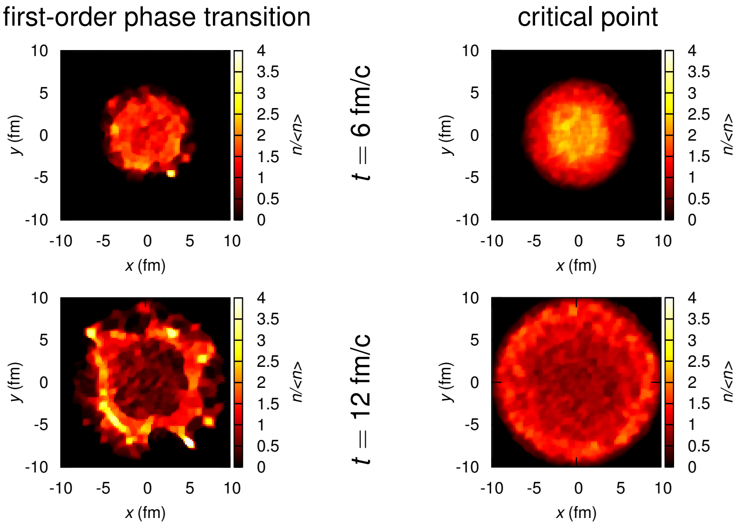


Fluid dynamical trajectories



- Fluid dynamical trajectories similar to the isentropes in the crossover region.
- No significant features in the trajectories left of the critical point.
- Right of the critical point: trajectories differ from isentropes and the system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

Bubble formation in net-baryon density

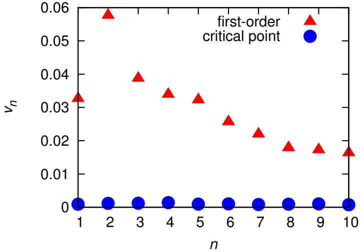


C. Herold, MN, I. Mishustin, M. Bleicher, arxiv:1304.5372

Bubble formation in net-baryon density

Fourier-decomposition of $n_B(x, y)$
$\rightarrow$ quantifies strong enhancement of first-order PT versus critical point/crossover.

not (yet) in momentum space!

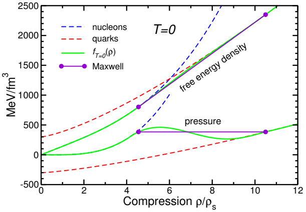


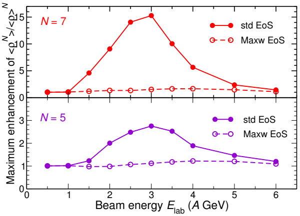
Can we expect experimental evidences for the first-order phase transition from bubble formation?

- Do the irregularities survive when a realistic hadronic phase is assumed?
- A strong pressure could transform the coordinate-space irregularities into momentum-space Fourier-coefficients of baryon-correlations $\Rightarrow$ enhanced higher flow harmonics at a first-order phase transition? Very eos dependent!

Comparison

- Nonequilibrium construction of the EoS from QGP and hadronic matter:

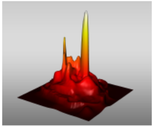




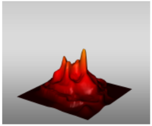
J. Steinheimer, J. Randrup, PRL **109** (2012), PRC **87** (2013)

- Significant amplification of **initial** density irregularities

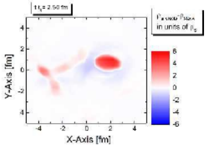
With phase transition:



Without phase transition:



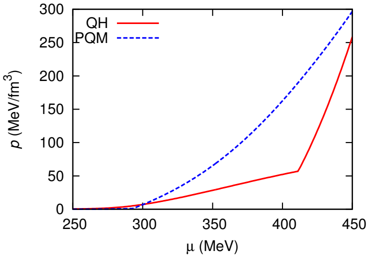
Density enhancement:

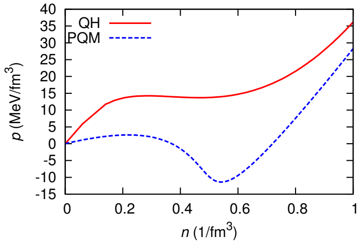


plot by V. Koch

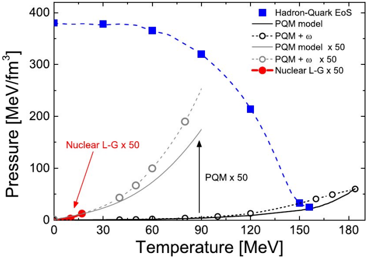
- BUT: deterministic evolution of the system ⇒ No inhomogeneities for smooth initial conditions!

EoS: PQM versus QH





- Below μ_c, $p \approx 0$ in PQM, while it still decreases in HQ model and $p < 0$ can arise in PQM!
- Several eos lead to similar pressures at $\mu_B \approx 0$, but differ at large μ_B.
- With coexistence between dense quark matter and compressed nuclear matter (HQ-EoS) : $\partial p_c / \partial T < 0$
- From effective models, like PNJL, PQM etc.: $\partial p_c / \partial T > 0$



J. Steinheimer, J. Randrup, V. Koch PRC**89** (2014)

SU(3) chiral quark-hadron model

- Hadronic SU(3) non-linear sigma model including quark degrees of freedom

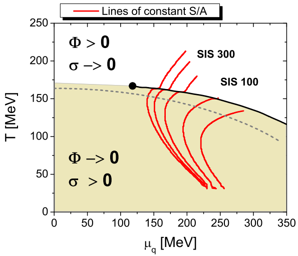
$$\mathcal{L} = \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i)\psi_i + 1/2(\partial_\mu \sigma)^2 - U(\sigma, \zeta, \omega) - \mathcal{U}(\ell)$$

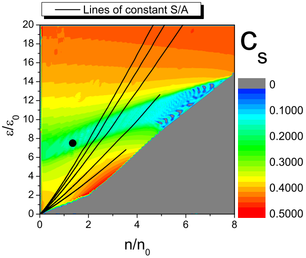
and effective masses generated by

$$M_q = g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell)$$
$$M_B = g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{qB}\ell^2$$

V. Dexheimer, S. Schramm, PRC**81** (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC**88** (2013)

- hadrons are included as quasi-particle degrees of freedom
- yields a realistic structure of the phase diagram and phenomenologically acceptable results for saturated nuclear matter:

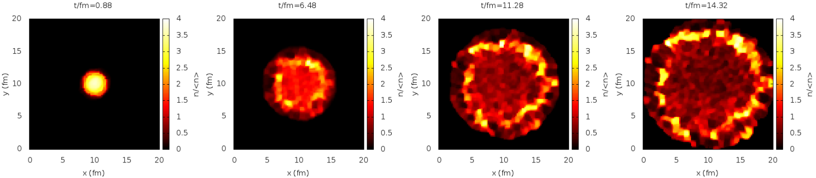




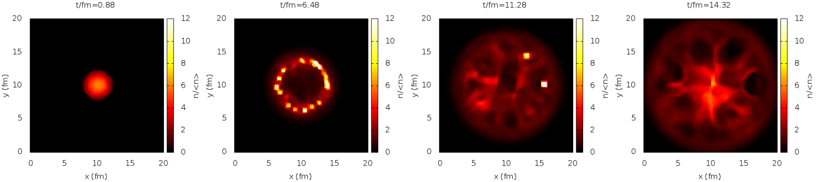
J. Steinheimer, V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm, H. Stoecker, PRC**81** (2010)

PQM vs. QH model - stability of droplets

PQM EoS



QH EoS

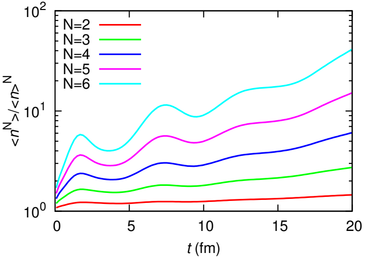


- Dynamical and stochastic droplet formation at the phase transition and subsequent decay in the hadronic phase.

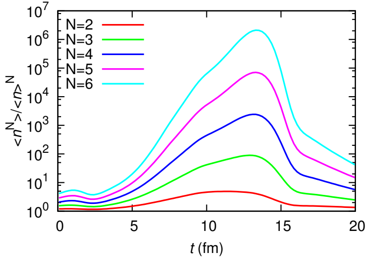
PQM vs. QH model - moments of netbaryon density

Define normalized moments of the net-baryon density distribution as:

$$\langle n^N \rangle = \int \mathrm{d}^3 x n(x)^N P_n(x) \quad \text{with} \quad P_n(x) = \frac{n(x)}{\int \mathrm{d}^3 x n(x)}$$



PQM EoS



QH EoS

- Infinite increase in the PQM.
- Increase in the HQ model around the phase transition followed by a rapid decrease due to pressure in the hadronic phase!
- REMEMBER: We started with smooth initial conditions and all inhomogeneities are formed dynamically!

And the critical point?

- At $\mu_B \neq 0$ σ mixes with the net-baryon density n (and e and $\vec{m}$)
- In a Ginzburg-Landau formalism:

$$V(\sigma, n) = \int \mathrm{d}^3x (\sum_m (a_m \sigma^m + b_m n^m) + \sum_{m,l} c_{m,l} \sigma^m n^l) - h\sigma - jn$$

- $V(\sigma, n)$ has a flat direction in $(a\sigma, bn)$ direction
- Equations of motion (including symmetries in $V(\sigma, n)$):

$$\partial_t^2 \sigma = -\Gamma \delta V / \delta\sigma + \ldots$$

$$\partial_t n = \gamma \vec{\nabla}^2 \delta V / \delta n + \ldots$$

- two time scales (with $D \to 0$ at the critical point)

$$\omega_1 \propto -i\Gamma a$$

$$\omega_2 \propto -i\gamma D \vec{q}^2$$

- The diffusive mode becomes the critical mode in the long-time dynamics. These fluctuations need to be included at the critical point!

H. Fuji, M. Ohtani PRD**70** (2004); M. Stephanov, D. Son PRD**70** (2004)

Fluid dynamical fluctuations

- Already in equilibrium there are thermal fluctuations
- The fast processes, which lead to local equilibration also lead to noise!

Stochastic viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu} \qquad \text{with} \quad \langle \Xi^{\mu\nu} \rangle = 0$$

$$N^{\mu} = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} + \mathrm{I}^{\mu} \qquad \text{with} \quad \langle \mathrm{I}^{\mu} \rangle = 0$$

The two formulations differ when one calculates correlation functions!

In linear response theory the retarded correlator

- $\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$ gives the viscosities and
- $\langle N^{\mu}(x) N^{\mu}(x') \rangle$ the charge conductivities

via the dissipation-fluctuation theorem (Kubo-formula)!

When dissipation is included also fluctuations need to be included!

P. Kovtun, J.Phys. A**45** (2012); C. Chafin and T. Schäfer, PRA**87** (2013); P. Romatschke and R. E. Young, PRA**87** (2013);
K. Murase, T. Hirano, arXiv:1304.3243; C. Young et al. arxiv:1407.1077

Fluid dynamical fluctuations

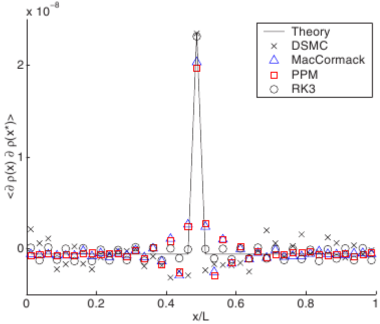
- In second-order fluid dynamics the relaxation equations become (e.g. for $\pi^{\mu\nu}$):

$$u^\gamma \partial_\gamma \pi^{\mu\nu} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\mathrm{NS}}}{\tau_\pi} + I^{\mu\nu}_\pi + \xi^{\mu\nu}$$

- With the white noise approximation: $\langle \xi^{\mu\nu}_\pi \xi^{\alpha\beta}_\pi \rangle = 2T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x - x')$
- In a numerical treatment $\rightarrow$ discretization: $\langle \xi^2 \rangle \propto \frac{1}{\Delta V}$
- $\Rightarrow$ large fluctuations from cell to cell - can a fluid dynamical code handle this?

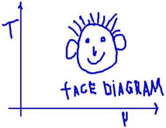
Example:
non-relativistic Navier-Stokes equations with fluctuations, one-dimensional, dilute gas, periodic boundary conditions

J. Bell, A. Garcia, S. Williams, PRE**76** (2007)



Summary

- Fluctuation data from heavy-ion collisions at finite μ_B can only be understood with dynamical models of the phase transition!
- In NχFD, effects like critical slowing down and droplet formation can be observed.
- PQM-like EoS do not include pressure in hadronic phase, droplets remain stable.
- In HQ-like EoS: droplets form dynamically at the phase transition, then decay.
- Next steps: particle production in NχFD and (net-baryon) fluid dynamical fluctuations.



Challenges for the BES II

- Need good dynamical models.
- Need good input.
- Need good observables.
- Need good data.

Challenges for the BES II

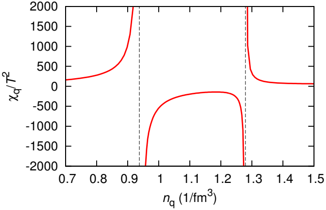
- Need good dynamical models.
  Initial state, coupling to FD, propagation of fluctuations, coupling to hadrons, ...
- Need good input.
  Equation of state, phase transition dynamics, transport coefficients, ...
- Need good observables.
  Large scale simulations, sensitivity analysis, statistical tools, ...
- Need good data.
  Efficiency corrected, smaller error bars, 14.5 GeV, different particle species, ...

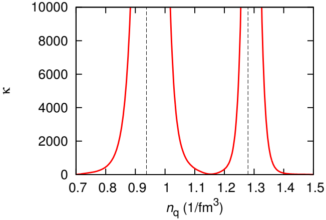
backup

Chiral model with dilatons

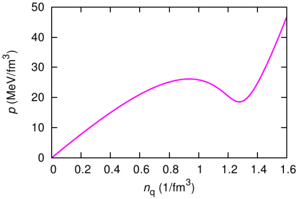
- Compare to a dilaton effective quark-meson model C. Sasaki, I. Mishustin PRC**85** (2012)

Susceptibilities along the spinodals:

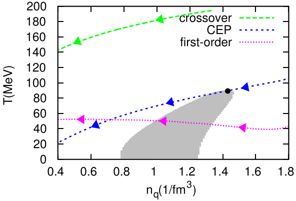




pressure vs density



fluid dynamical trajectories



- Improvement over the PQM EoS!