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in collaboration with

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Can Heavy Quarkonia be used as a thermometer for QGP?

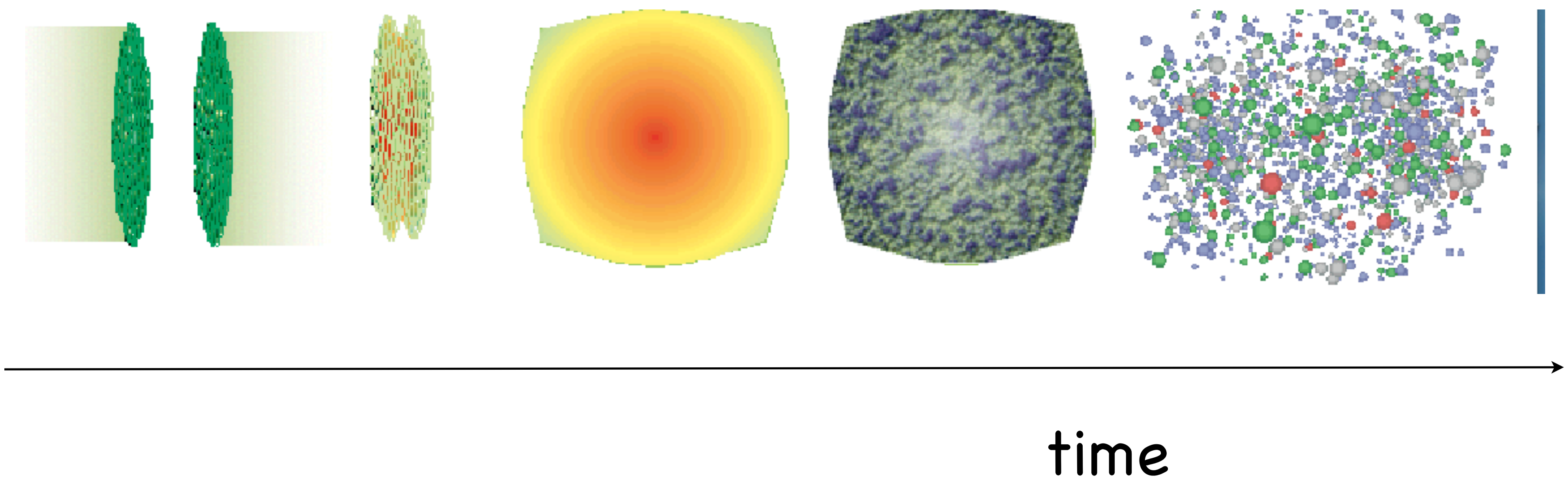
.............The application of ***sequential melting picture*** to the rapidly evolving fireball implicitly assumes the ***adiabatic evolution of heavy quarkonium states***. Here we shall show that the validity of this assumption is far from warranted.................

News

In February 2010, scientists at Brookhaven National Laboratory's Relativistic Heavy Ion Collider on Long Island, New York, USA, announced that they had smashed together gold ions at nearly the speed of light, briefly forming an exotic state of matter known as a quark-gluon plasma. During the experiment the plasma reached temperatures of around 4 trillion°C, some 250,000 times hotter than the centre of the Sun.

www.guinnessworldrecords.com/world-records/10000/highest-man-made-temperature

Heavy Ion Collisions



Two Heavy nuclei colliding with each other at a very high energy.

Pb-Pb collision in LHC and Au-Au collisions in RHIC

Two important probes for deconfined medium

1. Heavy quarkonia suppression

PHYSICS LETTERS B

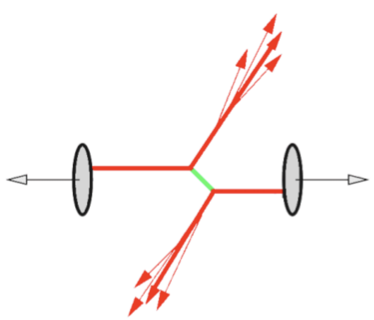
J/Ψ **SUPPRESSION BY** QUARK-GLUON PLASMA FORMATION

If high energy heavy ion collisions lead to the formation of a hot quark-gluon plasma, then colour screening prevents $c\bar{c}$ binding in the deconfined interior of the interaction region......

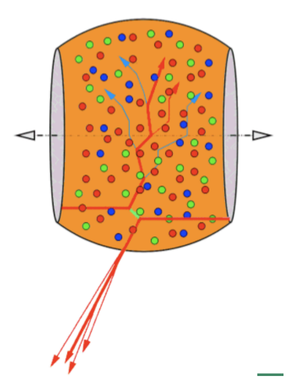
T. Matsui and H. Satz (1986)

2. Suppression of parton jets in the medium

Parton jets propagating through a dense medium will lose pat of their energy and momentum. Therefore a suppression of jets is expected in QGP.



in vacuum



in medium

Quarkonium states in vacuum

Heavy quarkonium bound states can be described using non relativistic Schroedinger equation

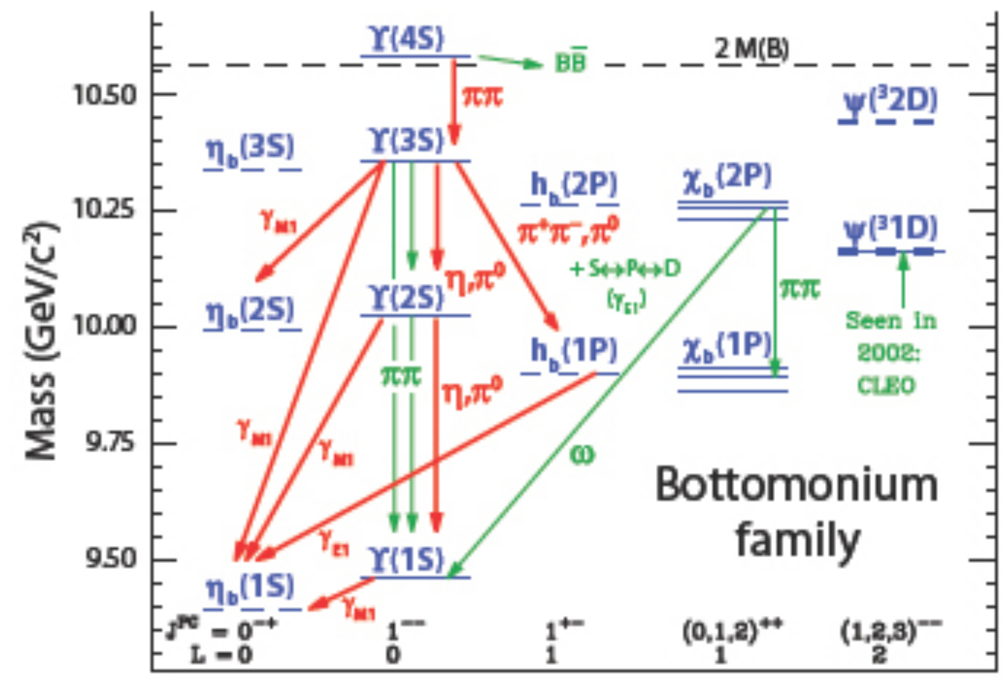
$$\Big(-\frac{1}{2\mu}\nabla^2 + V(r)\Big)\Phi(r) = E\Phi(r)$$

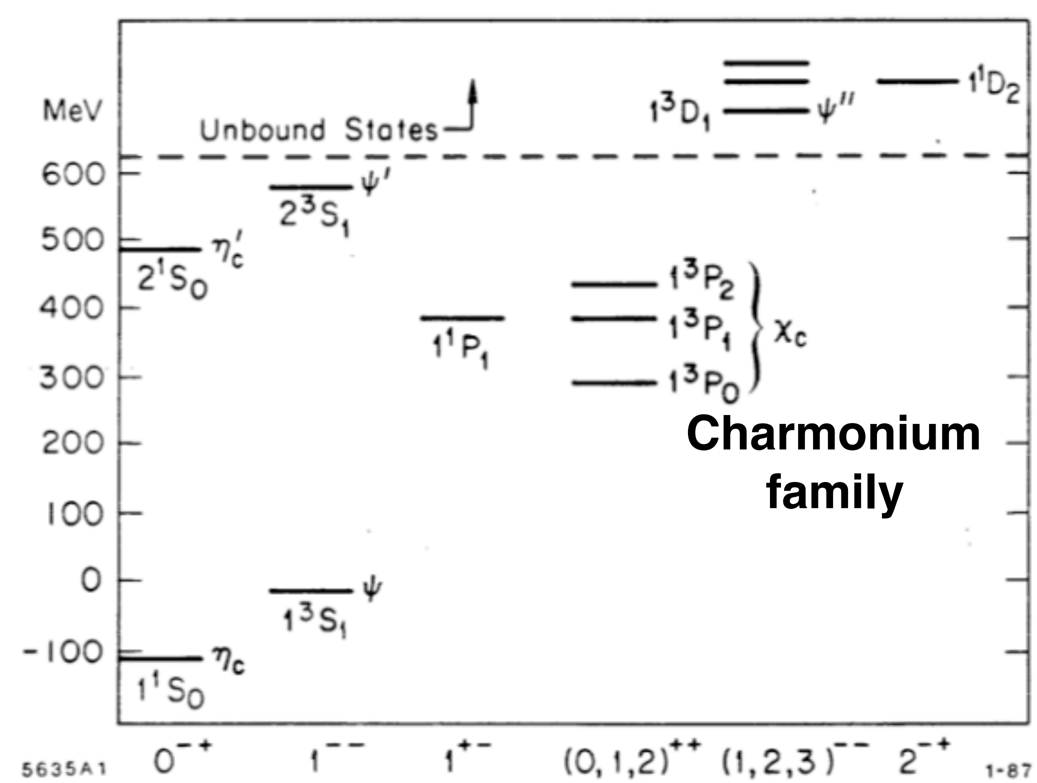
$$V(r) = \sigma r - \frac{\alpha}{r}$$

$$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{m}{2}$$

States	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
M[GeV]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
ΔE[GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
ΔM[GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
r[fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

Source: [Quark Matter and Nuclear Collisions: A Brief History of Strong Interaction Thermodynamics; H. Satz]

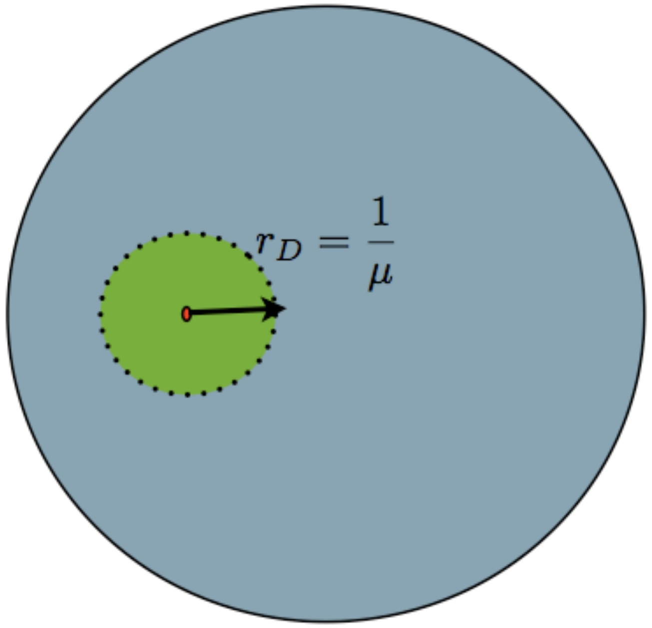




Sequential suppression in static plasma

Effective potential

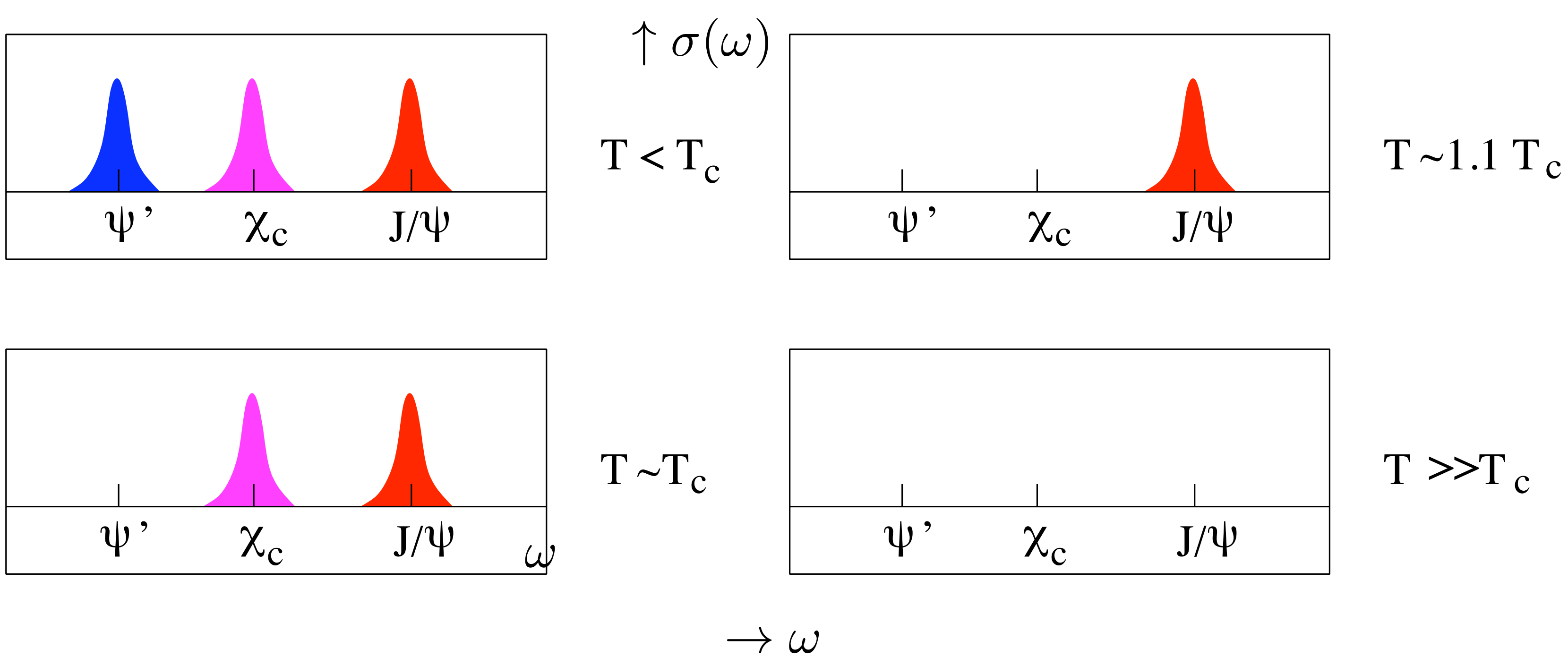
$$V(r,T) = -\frac{\alpha}{r}e^{-\mu(T)r} + \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r})$$
$$\approx -\frac{\alpha}{r}e^{-\mu(T)r}; \qquad T > T_c$$



$r_D(T) = \frac{1}{\mu(T)}$

Debye screening

Spectral function



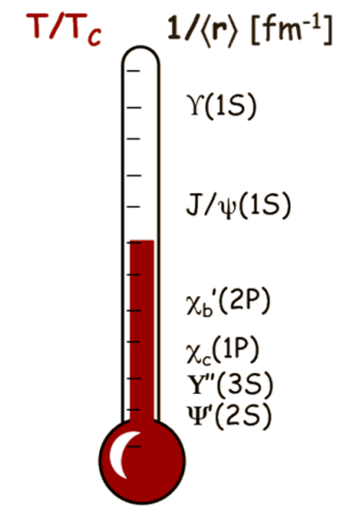
Applied in heavy ion collision

Sequential suppression

With increasing temperature,
Debye radius decreases

$$\mu^2 = \frac{4}{3} g^2 T^2 \quad \text{where} \quad \mu = \frac{1}{r_D}$$

Different bound states have different
dissociation temperatures which shows
a sequential suppression pattern.



QGP thermometer

Nuclear modification factor

$$R_{AA} = \frac{\text{number of specific quarkonium states in nucleus-nucleus collision } (N_{AA})}{\text{number of the same species in p-p collision } (N_{pp}).\ n_{binary}}$$

$= 0$ or 1

n_{binary}: number of p-p collision per nucleus.

Nuclear modification factor

For charmonium

$$R_{\mathrm{AA}}^{J/\Psi} = 0.545 \pm 0.032(\mathrm{stat.}) \pm 0.083(\mathrm{syst.})$$

For bottomonium

$$\begin{aligned}
\mathrm{R}_{\mathrm{AA}}(\Upsilon(1\mathrm{S})) &= 0.56 \pm 0.08\,(\mathrm{stat.}) \pm 0.07\,(\mathrm{syst.})\,, \\
\mathrm{R}_{\mathrm{AA}}(\Upsilon(2\mathrm{S})) &= 0.12 \pm 0.04\,(\mathrm{stat.}) \pm 0.02\,(\mathrm{syst.})\,, \\
\mathrm{R}_{\mathrm{AA}}(\Upsilon(3\mathrm{S})) &= 0.03 \pm 0.04\,(\mathrm{stat.}) \pm 0.01\,(\mathrm{syst.})
\end{aligned}$$

Double ratios

$$\begin{aligned}
\frac{\Upsilon(2\mathrm{S})/\Upsilon(1\mathrm{S})|_{\mathrm{PbPb}}}{\Upsilon(2\mathrm{S})/\Upsilon(1\mathrm{S})|_{\mathrm{pp}}} &= 0.21 \pm 0.07\,(\mathrm{stat.}) \pm 0.02\,(\mathrm{syst.})\,, \\
\frac{\Upsilon(3\mathrm{S})/\Upsilon(1\mathrm{S})|_{\mathrm{PbPb}}}{\Upsilon(3\mathrm{S})/\Upsilon(1\mathrm{S})|_{\mathrm{pp}}} &= 0.06 \pm 0.06\,(\mathrm{stat.}) \pm 0.06\,(\mathrm{syst.})
\end{aligned}$$

CMS Collaboration, arXiv:1208.2826

Wait.....

A thermometer needs some time to feel the temperature

Quarkonia has to reach thermal equilibrium (Default in lattice calculation) with the medium.

Is it really the case in Heavy Ion Collision?

Remember the QGP temperature falls from 500 MeV to 170 in less than 10 fm/C

Does quarkonia get enough time to feel the temperature ?

Look at the time scales....

Typical size of a bound bottomonium state $r_{\mathrm{rms}} \approx 0.3 - 0.75 \ fm$

The characteristic velocity $v \sim 0.3c$

The duration for an orbit corresponding to the non relativistic quarkonium $\tau \approx 5 - 10 \ fm/c$

This is comparable to the life time of QGP at LHC

at least this amount of time is needed for quarkonia to realize that they are in a specific eigenstate.

Can we expect that quarkonia get enough time to be thermalized in the medium ..so as to be described through a sequential suppression picture?

There is even more serious issue...QGP is evolving!

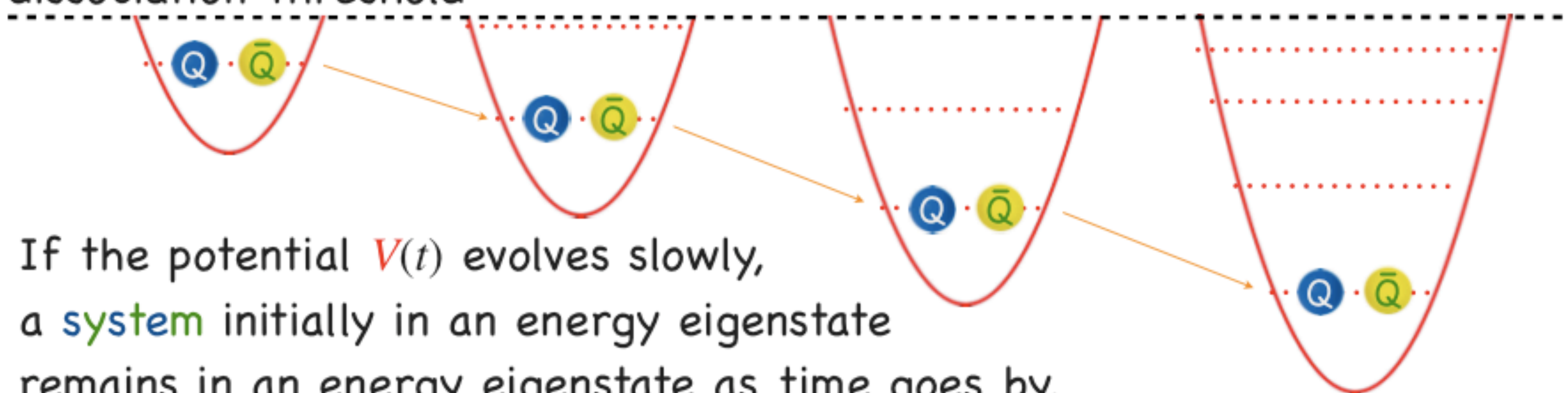
If we consider that the quarkonia can be described through an effective potential,

We have to consider a potential which is changing with time

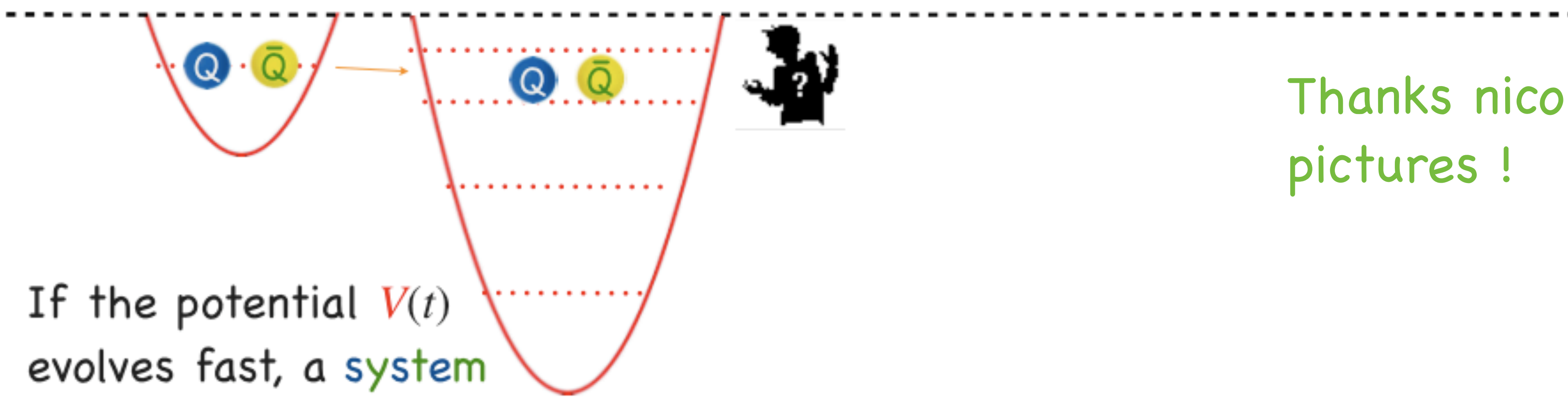
Because the medium temperature falls off with time

Then it is crucial to know how fast the potential changes!

Slow vs rapid evolution (adiabatic or non-adiabatic)



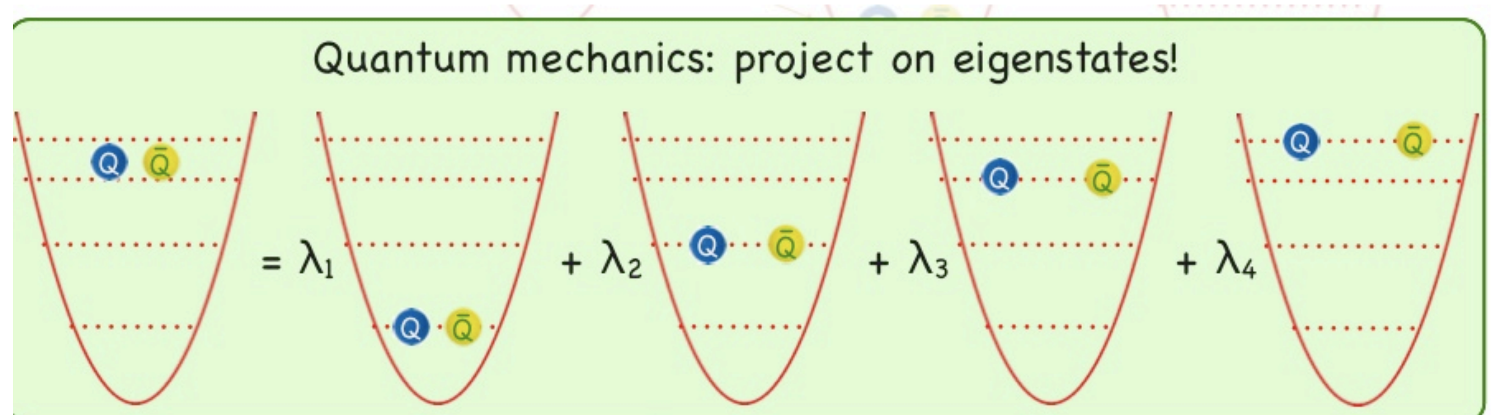
If the potential $V(t)$ evolves slowly, a system initially in an energy eigenstate remains in an energy eigenstate as time goes by.



If the potential $V(t)$ evolves fast, a system initially in an eigenstate cannot follow the change...

Thanks nicolas for drawing such nice pictures !

See your favorite text books...Griffiths, Messiah and others on quantum mechanics!



Sequential suppression says that a specific quarkonium states is either alive or melted.

The state which survives at the beginning of QGP will live until the late electroweak decay.

And those states which are melted can not reappear except for the fact of regeneration through the recombination of uncorrelated quarks and antiquarks

1. By statistical regeneration at the phase boundary (P. Braun-Munzinger and J. Stachel, Phys. Lett. B 490, 196(2000)).
2. Coalescence of q and $\bar{q}$ in the evolving fireball (Ralf Rapp et al, Phys. Rev. Lett. 92, 212301 (2004)).

Usual description of quarkonia suppression:

Color screening(Sequential suppression) + Regeneration

?

For the medium produced in heavy ion collision

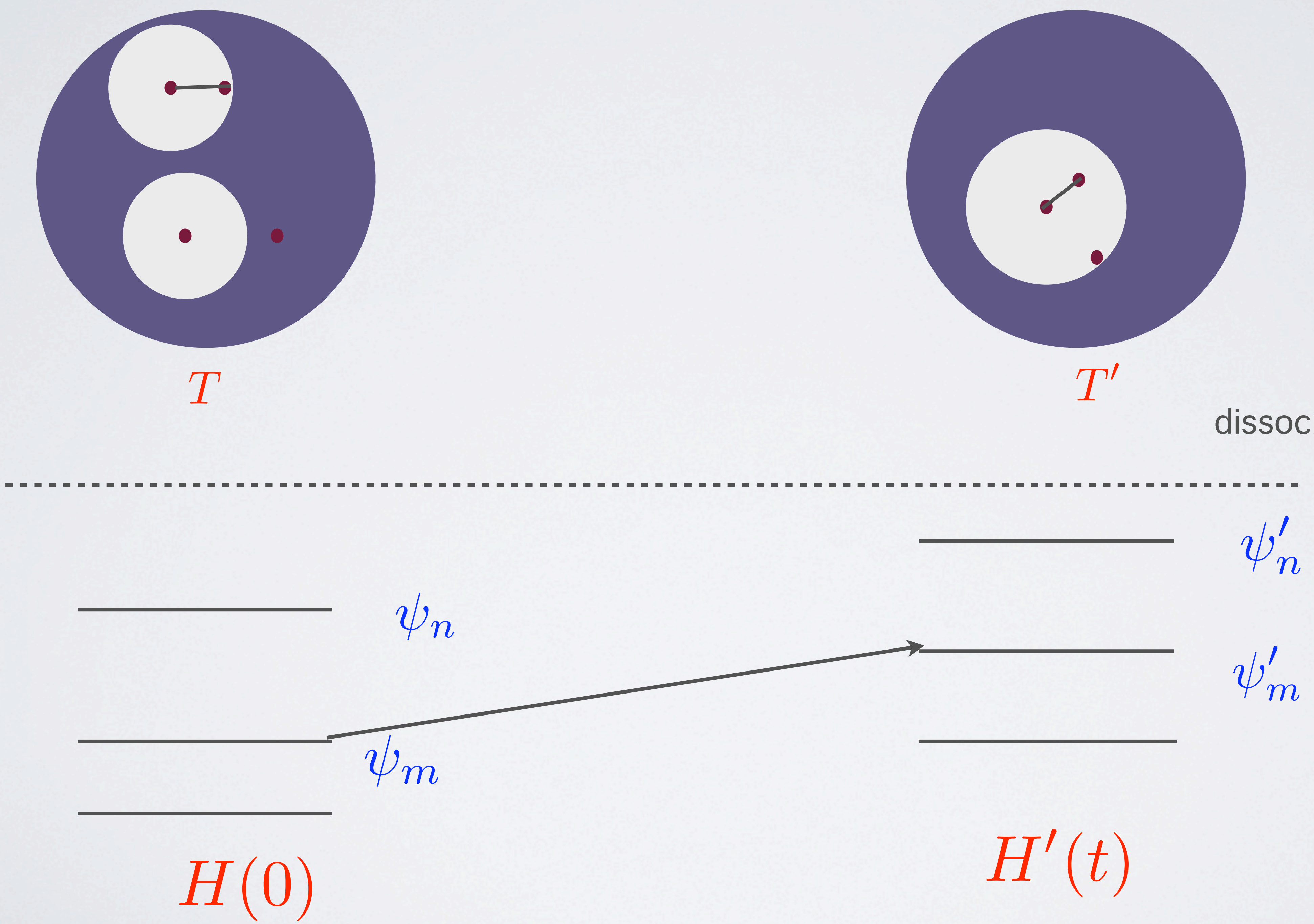
(Keeping aside the issues of regeneration from uncorrelated quark anti-quark in the medium)

Sequential suppression of quarkonium states applied for the evolving medium produced in heavy ion collision is based on two ingredients.

1. Sequential suppression pattern in the initial stage.

2. Adiabatic evolution of quarkonium bound states in the medium

Adiabatic approximation



a system initial at $|\psi_i(0)\rangle$ evolving due to a time dependent Hamiltonian.

The final state

$$|\phi(t)\rangle = e^{i\gamma_i(t)} e^{i\theta_i t} |\psi_i(t)\rangle \qquad \text{if} \qquad \frac{|\langle\psi_m(t)|\dot{H}(t)|\psi_n(t)\rangle|}{(E_m - E_n)^2} \ll 1$$

adiabatic condition

That also could realized through the comparison of two different time scales

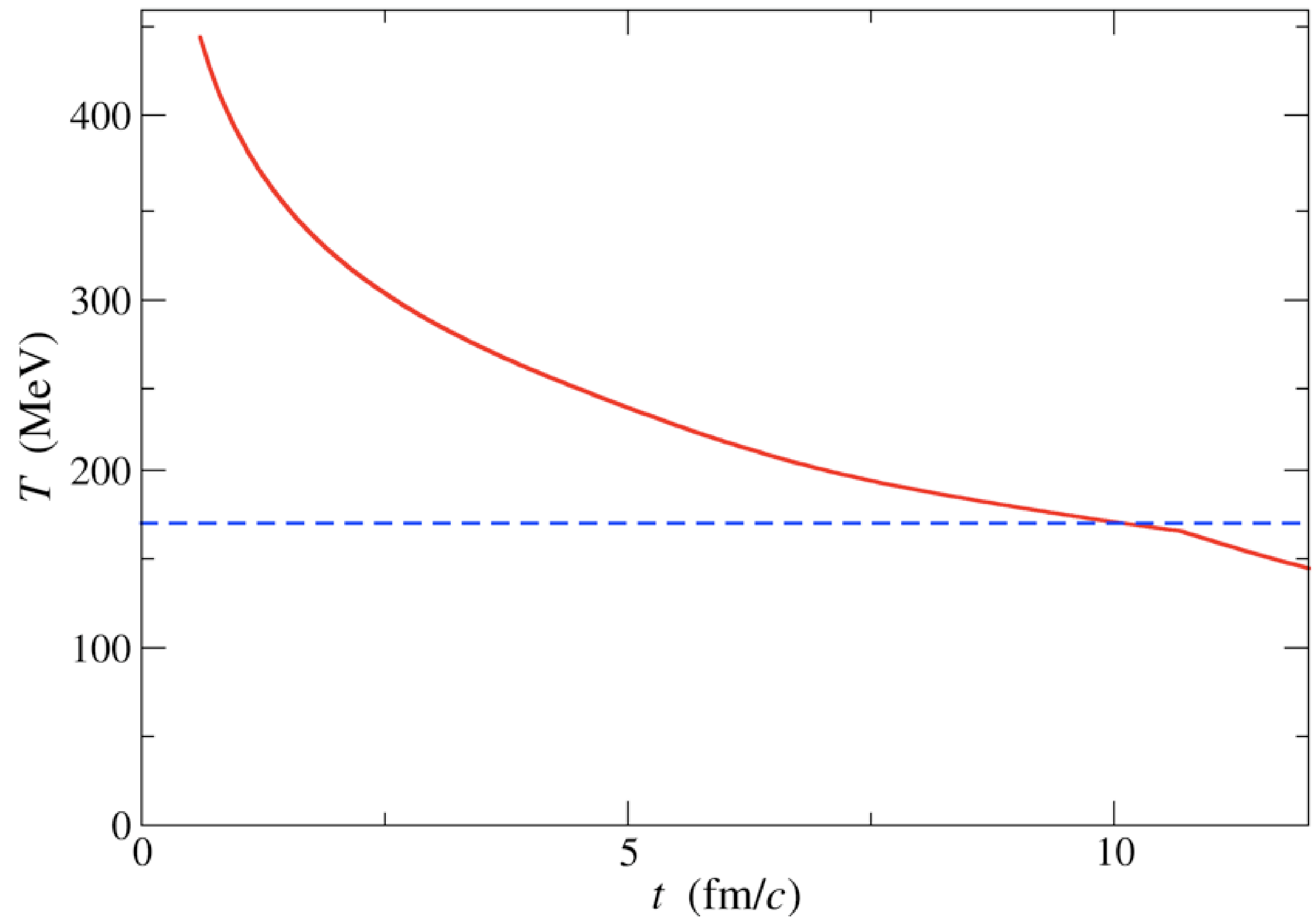
$$\frac{\tau}{\tau_m} \ll 1$$

time scale related to the energy gap of the bound state

time scale associated with the medium evolution

Rapidly evolving medium

Change of temperature in medium
produced in heavy ion collision



the rate of change of temperature
greater than 30 MeV per fm/C

at the initial stage
$\sim 5o$ MeV per fm/C

the characteristic energy gap for bottomonium 100-300 MeV

The ratio

$$V(r) \sim \frac{\frac{4}{3}\alpha_s(T)}{r}\,\mathrm{e}^{-A\sqrt{1+N_f/6}\,T g_{2\,\mathrm{loop}}(T)\,r}$$

Kaczmarek & Zantow, Phys. Rev. D **71** (2005) 114510

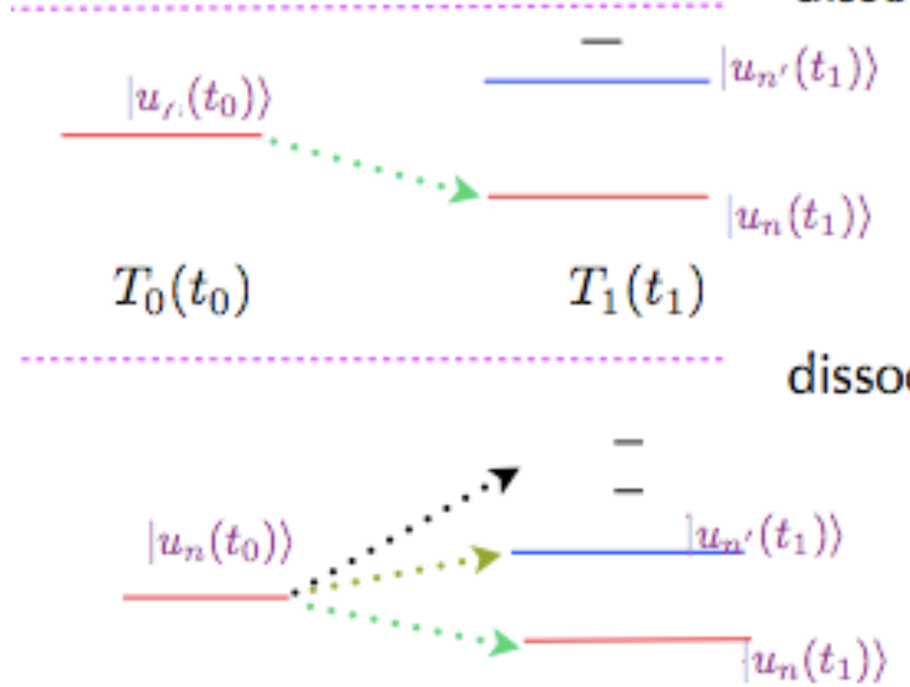
$$\frac{|\langle\psi_m|\dot{H}(t)|\psi_n\rangle|}{(E_m - E_n)^2} \sim \frac{|\langle\psi_m(t)|\dot{T}\frac{dV}{dT}|\psi_n(t)\rangle|}{(E_m - E_n)^2} \sim \frac{(80 - 160MeV)^2}{(100 - 350MeV)^2}$$

For the bottomonia states,
the ratio can be in some cases is smaller than 0.1
but for other channels larger than 1.

Nirupam Dutta, Nicolas Borghini arXiv:1206.2149

Reshuffling picture

dissociation threshold



no transition from n to n' if

$$\frac{\left|\langle n'(t)|\,\dot{\mathrm{V}}(t)\,|n(t)\rangle\right|}{\left[E_n(t)-E_{n'}(t)\right]^2} \ll 1,$$

dissociation threshold

"Reshuffling"

Therefore, the reshuffling picture is the fate of heavy quarkonia in the medium produced in heavy ion collision.

So far we have learnt

The medium produced in heavy ion collisions is evolving rapidly that can keep quarkonia from adiabatic evolution.

As a result if we start with let say $\Upsilon(2S)$, it will have projection to the ground $\Upsilon(1S)$ and as well as to excited states like $\Upsilon(3S)$ and others....

(Invites a dynamical framework with real time!)

The simple sequential-melting picture is then blurred by the rapid evolution of the QGP, and the role of quarkonia as straightforward thermometers becomes questionable...

Let me be constructive now..........

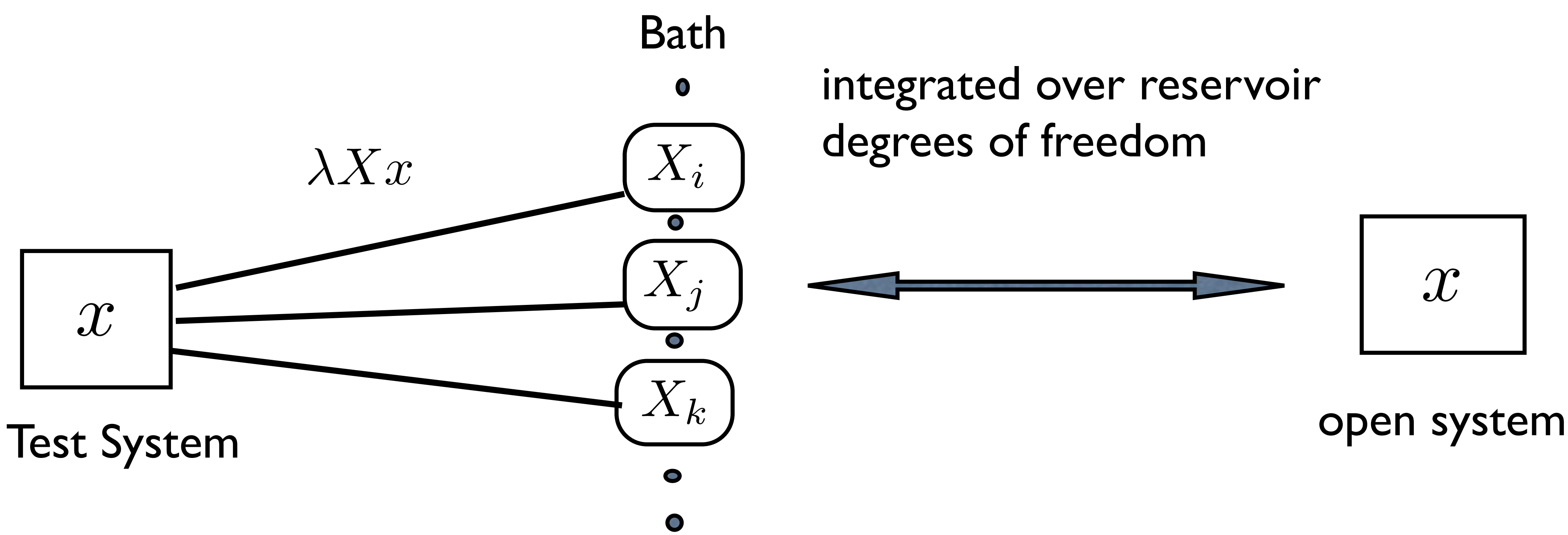
Proposing possible remedy in next couple of slides

1. By using approach based on open quantum system

2. Approaching through effective potential which is time dependent as the medium evolves (obviously considering quarkonia can sense the temperature?).

open quantum system

Quarkonia ➡ few internal degrees of freedom: "small system"



Quark-gluon plasma ➡ many degrees of freedom

$$V_I = \vec{d}.\vec{E} \rightarrow \lambda X x$$

- The dipolar approximation is equivalent to the bilinear interaction.

Simple model

Quarkonia, a color dipole interacting with the chromoelectric field of the medium.

The interaction between the the quarkonia and the Field

$$V_I = \vec{d}.\vec{E}$$ (at first approximation)

In the quantized form of chromoelectric field, the interaction is with the gluonic modes with a continuum frequency span.

The challenge is given...

Quarkonia is interacting with many degrees of freedom of QGP

usual recipes

- Feynman-Vernon Theory of Influence Functional.

(Restricted to some simple cases as far as calculability is concerned.)

- Master equation approach.

(Perturbative approach appropriate within weak coupling)

- Monte-Carlo wave function.

(Numerical Solution of Stochastic Schrödinger Equation.)

Wave function based approach

Heavy quarkonia initially at $|\Psi_m\rangle$ in vacuum.

The probability of being in the state $|\Psi_n\rangle$ after a while (when there is a medium).

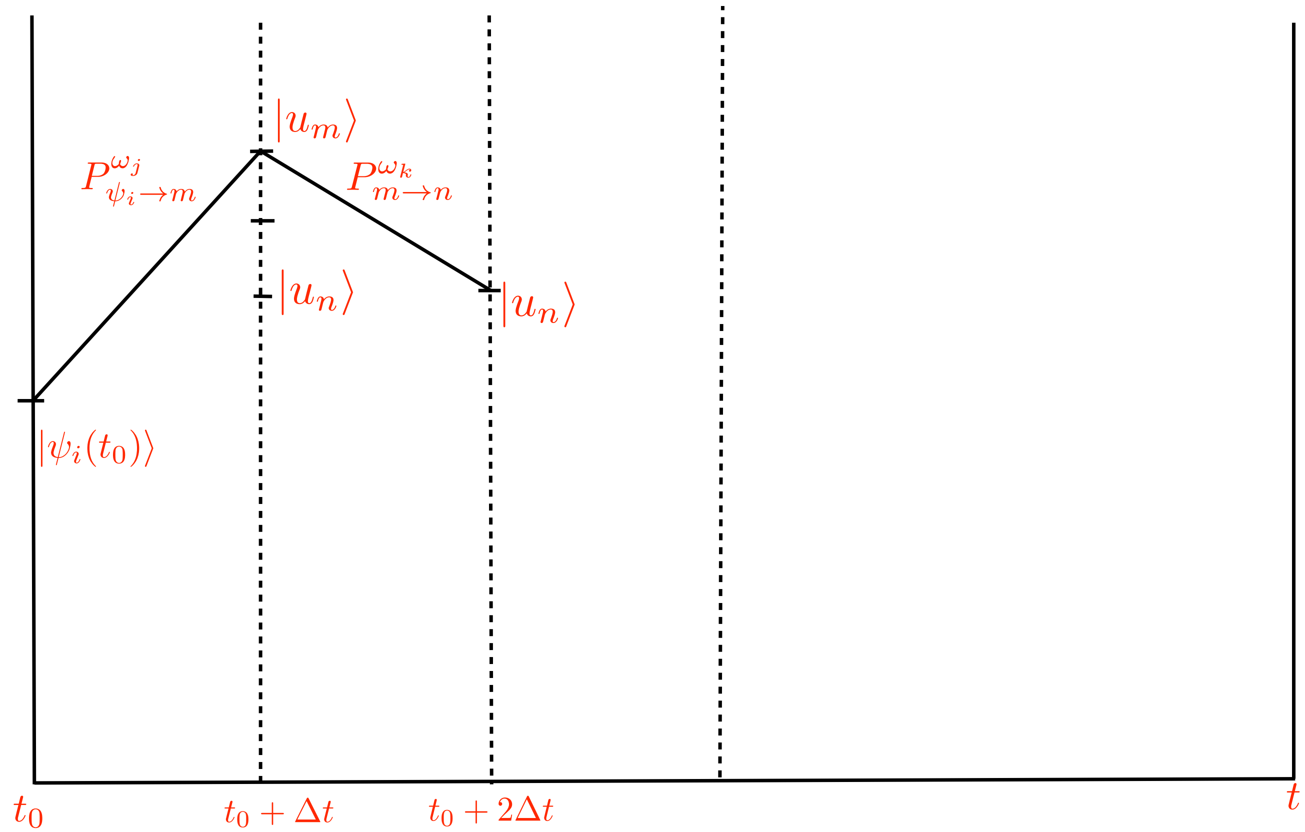
$$P_{n,m} = \int \Psi_n(Q'_t)\Psi^\star_n(Q_t)\exp[i(S_0(Q) - S_0(Q'))]\underline{F(Q,Q')}\Psi^\star_m(Q_\tau)\Psi_m(Q'_\tau)dQ_\tau dQ_\tau dQ_t dQ'_t \mathcal{D}Q\mathcal{D}Q'$$

$F(Q,Q')$ encodes the medium effect, known as *influence functional.*

Wave function based approach is **cost effective**
and helps to **overcome drawbacks** of other approaches.

Technical digression...

A system initially at $|\Psi_i(t_0)\rangle$



Coarse-grained interaction scenario

$$P^{seq.1}_{i\to n} = \sum_m P^{\omega_k}_{i\to m} P^{\omega_k}_{m\to n};$$

$$P^{seq.2}_{i\to n} = \sum_m P^{\omega_k}_{i\to m} P^{\omega_j}_{m\to n};$$

$$P^{seq.3}_{i\to n} = \sum_m P^{\omega_j}_{i\to m} P^{\omega_j}_{m\to n}$$

$$P_{i\to n} = P^{seq.1}_{i\to n} + P^{seq.2}_{i\to n} + ... + P^{seq.p}_{i\to n} + ...$$

chasing the challenge.....

A system initially in the state $|\psi_0(t_0)\rangle$ evolves by interacting with a medium. What will be the probability of having the system in the state $|u_i\rangle$ at some instant t during the evolution?

$$P_{i\to f} = \int_\omega \int_{\omega'} \int_{\omega''} ....d\omega d\omega' d\omega'' ....P_{i\to m}(\omega) f(\omega)$$
$$P_{m\to n}(\omega') f(\omega') P_{n\to q}(\omega'') f(\omega'')....$$

$$P^k_{m\to n} = \int dX_i dX_f dY_i dY_f u_m(X_i) u^\star_m(Y_i)$$
$$\Big(\int \mathcal{D}X\mathcal{D}Y \exp\big(iS[X]\big) \exp\big(-iS[Y]\big) F^k[X,Y]\Big) u^\star_n(X_f) u_n(X_f)$$
$$= \Bigg| \sum_{m\prime,n\prime} \int dx_i dx_f dX_i dX_f \phi^k_{m\prime}(x_i) u_m(X_i)$$
$$\Big(\mathcal{D}x\mathcal{D}X \exp i\big(S[x] + S[X] + S_I[x,X]\big)\Big) \phi^{k\star}_{n'}(x_f) u^\star_n(X_f) \Bigg|^2$$

- The simplest case is a Harmonic Oscillator (test system) in a Medium.

$$L = L_0 + L_B + L_I$$

$$L_0 = \frac{1}{2}M\dot{X}^2 - \frac{1}{2}M\omega_0^2 X^2$$

- The Medium is modeled as a collection of a large number of Harmonic Oscillators.

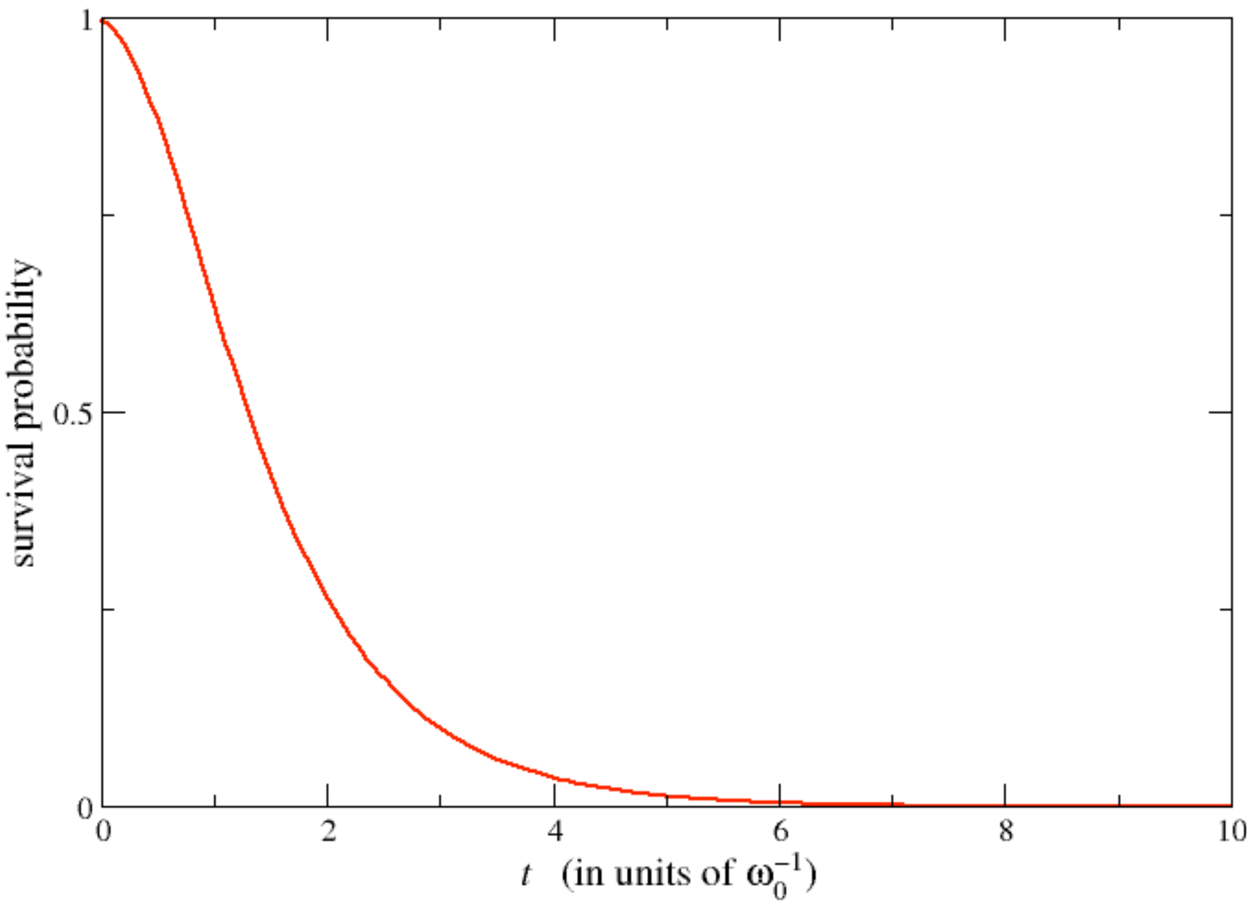
$$L_B = \frac{1}{2}\sum_j m\left(\dot{x}_j^2 - \omega_j^2 x_j^2\right)$$

- The interaction of each oscillator of the medium with the test system is bilinear.

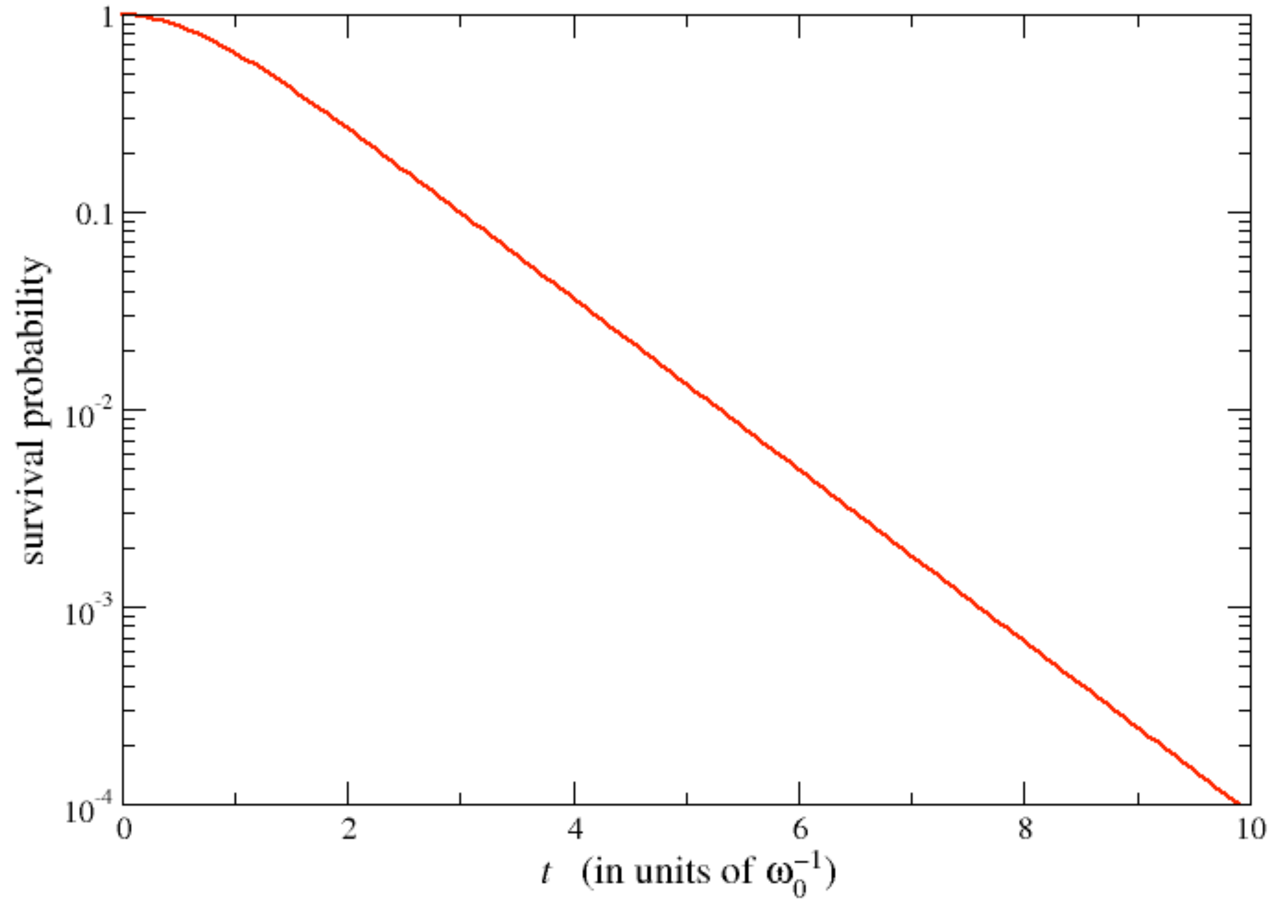
$$L_I = \lambda X x$$

Results from exploratory studies

Linear Plot



Log Plot



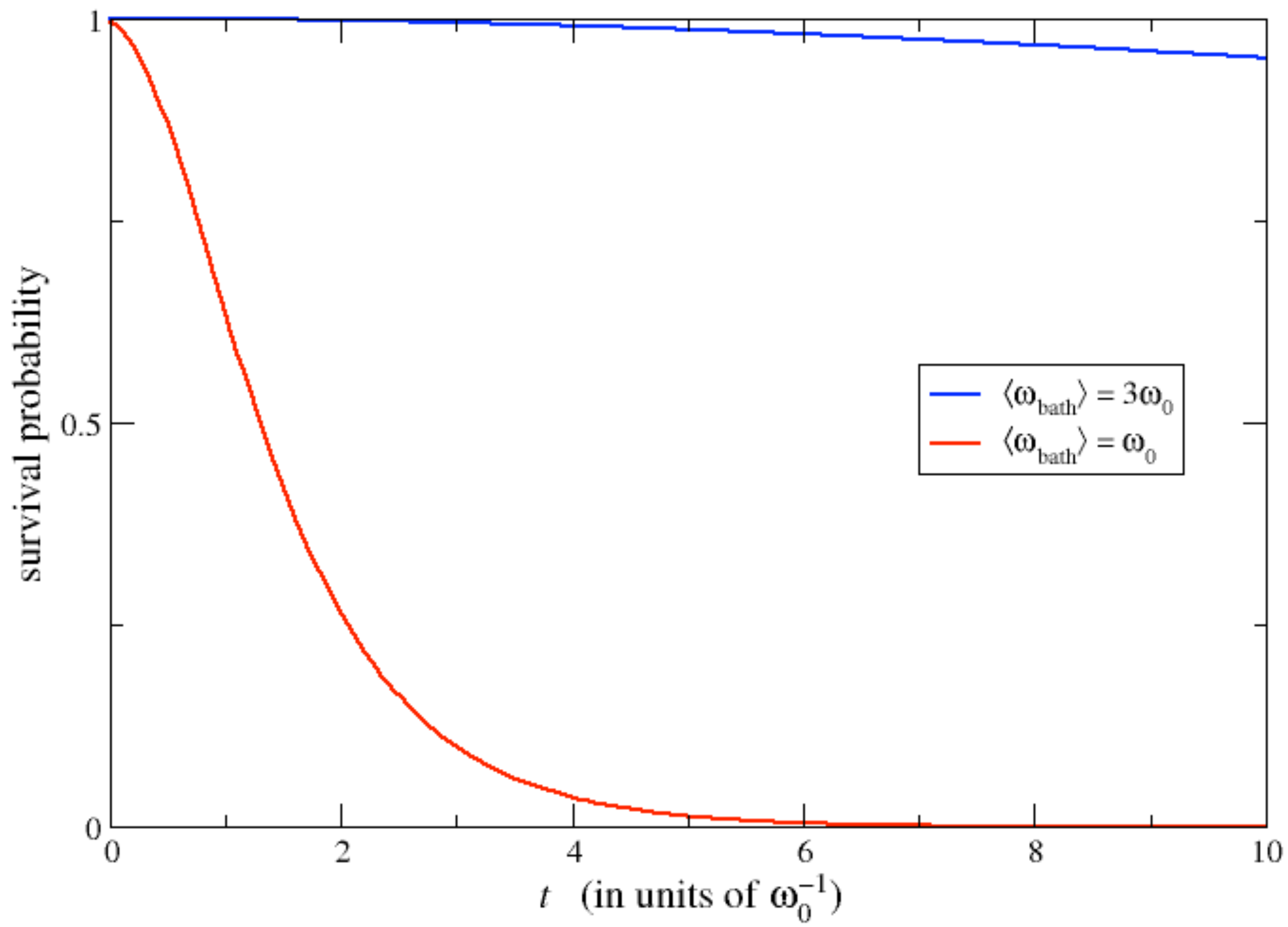
Effect of Bath modes with frequencies comparable to ω_0

reminding results from master equation approach!

Results from exploratory studies

Blue line: considering distant modes

Linear plot

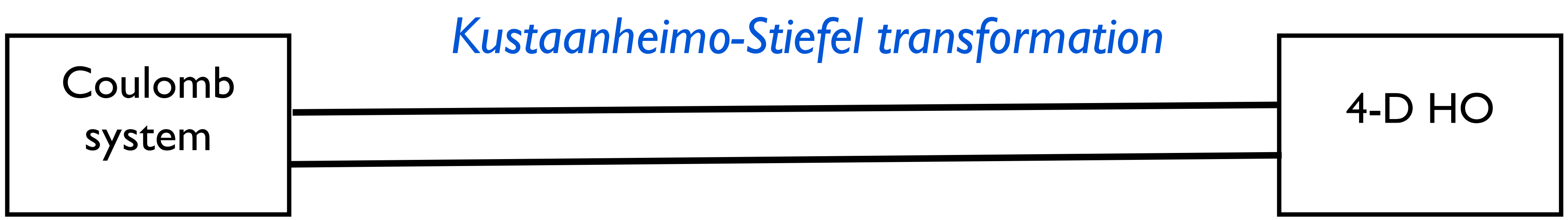


Red line: considering comparable modes

Generalization for quarkonia

Correspondence between oscillator and coulomb system

- Generalization of the test system to a 4-D constrained Harmonic Oscillator, which corresponds to the real system (quarkonium).



We already have studied 3-D Isotropic oscillator.

Work in Progress

We are quite sure that the results will not differ much at the qualitative level.

Recent attempts...

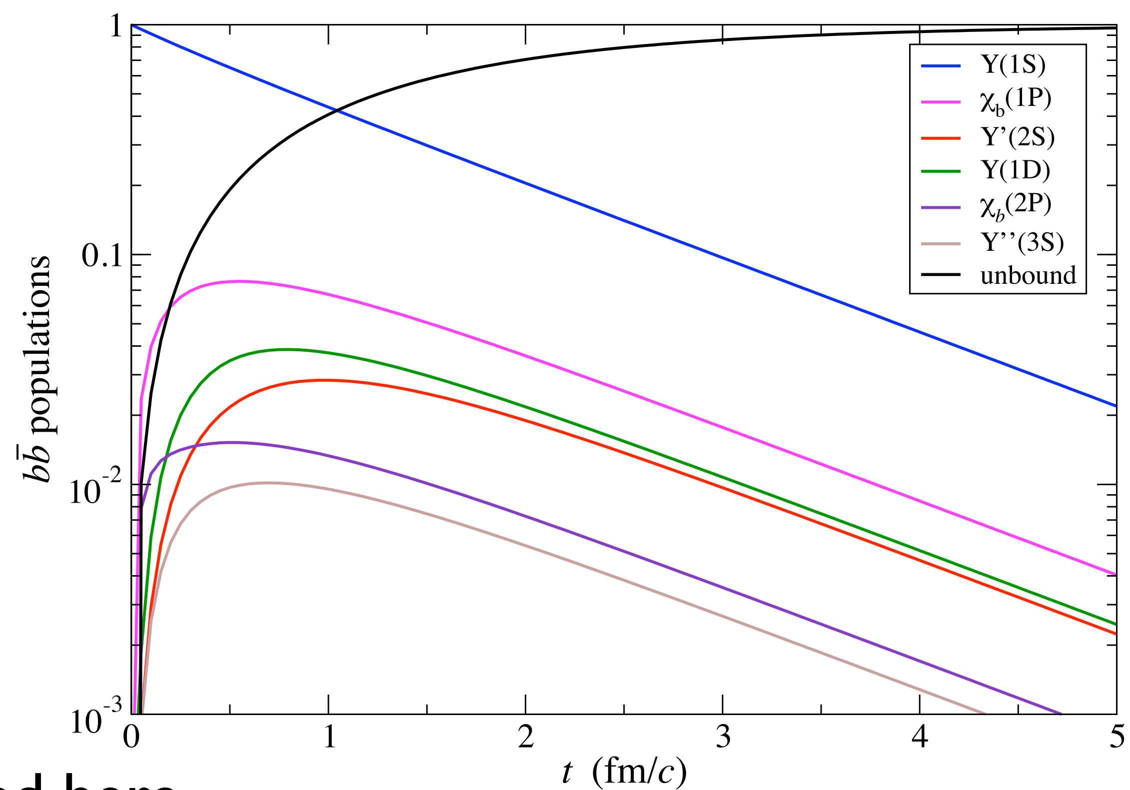
Master equation approach

$$\frac{d}{dt}\rho = -i[H,\rho]$$

$$\rho^s = Tr_R \rho$$

$$H = H_S + H_R + V_I$$

evolution of bottomonia states at $5T_c$



dipolar interaction $V_I = R\ S$ is considered here

Eur.Phys.J. C72 (2012) 2000
by N. Borghini. and C. Gombeaud.

The plot has been made by preparing bottomonium initially in the ground state

After a transient regime, the various bound states evolve together

(contrary to sequential pattern)

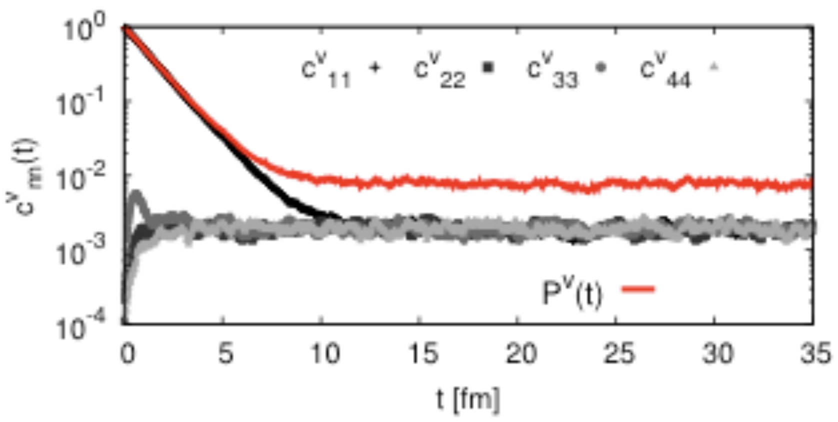
They also have shown the thermalization time $\tau_{bb} \approx 8\ \mathrm{fm/C}$ at $2T_c$

equilibration

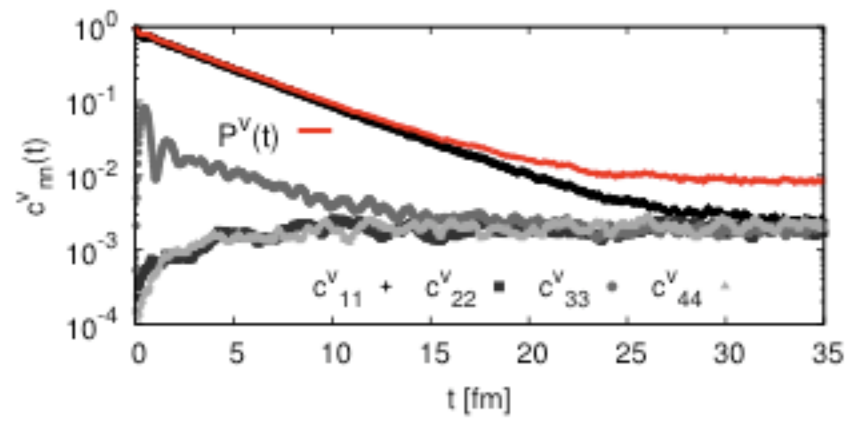
Recent attempts…

Stochastic Schroedinger equation

$$i\frac{d}{dt}\langle\Psi_{q\bar{q}}(\mathbf{r},t)\rangle = \Big(-\frac{\nabla^2}{m_q} + 2m_q + V(\mathbf{r}) - \frac{i}{2}\Gamma(\mathbf{r},\mathbf{r})\Big)\langle\Psi_{q\bar{q}}(\mathbf{r},t)\rangle$$



using Lattice QCD parameter set



using perturbative QCD parameter set

Phys.Rev. D85 (2012) 105011 by A. Rothkopf and Y. Akamatsu

some other contemporary approaches based on open quantum system framework :

Clint Young and Kevin Dusling. Quarkonium above deconfinement as an open quantum system. Phys.Rev., C87:065206, 2013.

Nirupam Dutta. Heavy quarkonia in quark gluon plasma as open quantum systems. PhD thesis, Universitaetsbibliothek, University of Bielefeld, Germany, 2013.

Jean-Paul Blaizot et al, Heavy quark bound states in a quark-gluon plasma: dissociation and recombination arXiv:1503.03857

Method of Lewis-Riesenfeld invariant :

Quarkonia can be described by time dependent effective potential !
(Relying on the assumption that the time scale of medium evolution is bigger than the thermalization time of quarkonia.)

Method (constructing invariant for the system):

Introducing a quantum mechanical invariant

$$\frac{dI}{dt} = \frac{\partial I}{\partial t} - i[I, H(t)]$$

A proper choice of Invariant can therefore serve the purpose.

time dependent screening potential can be mapped as a harmonic oscillator with time dependent frequency (K-S transformation)

Regeneration of excited states

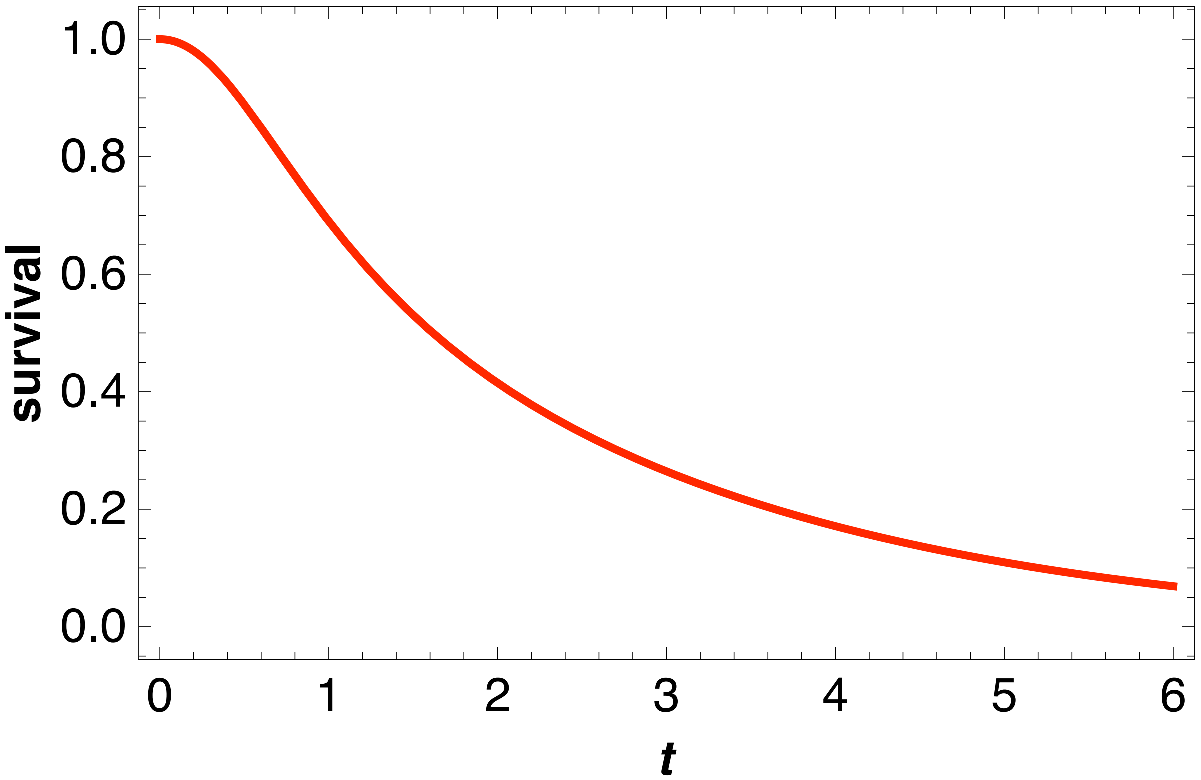
$$I = \frac{1}{2}[(\frac{1}{\rho^2})x^2 + (\rho p - M\dot{\rho}x)^2]$$

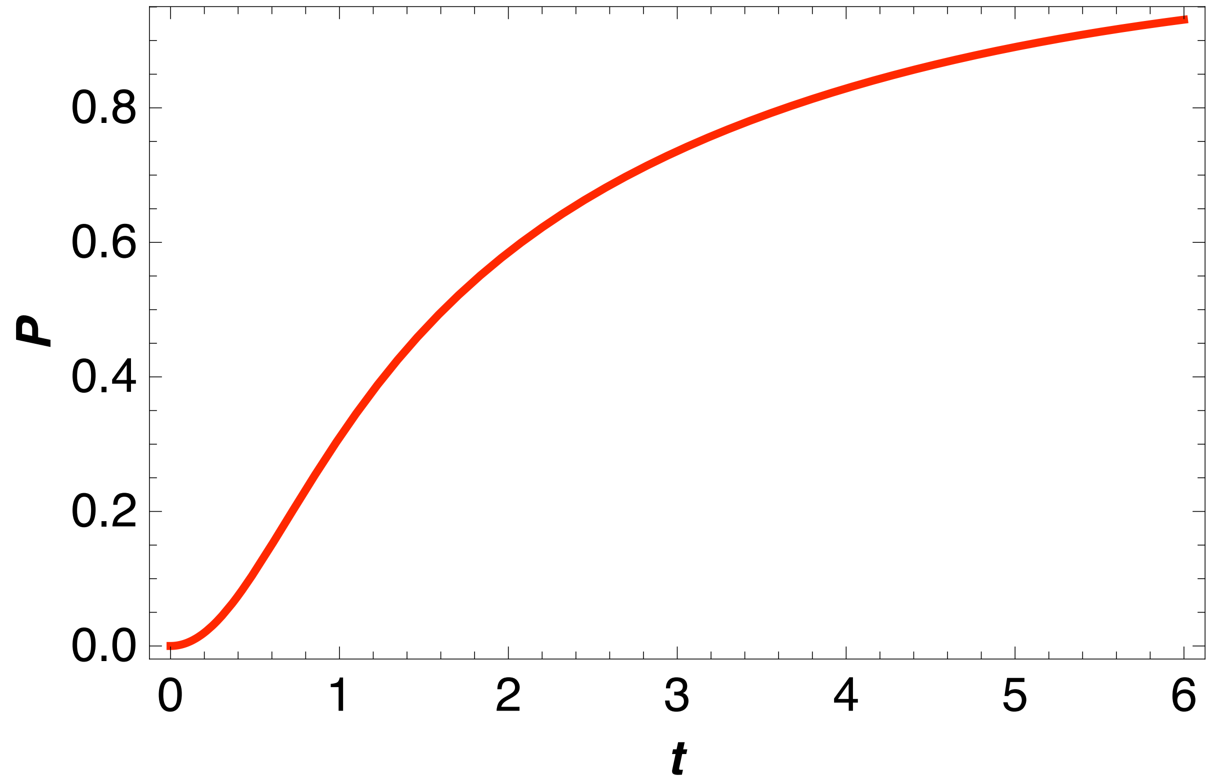
$$T_{00} = (4\Omega_2)^{\frac{1}{4}}\left[\left(\frac{1}{\rho} + \Omega_2\rho\right)^2 + M^2\dot{\rho}^2\right]^{-\frac{1}{4}}$$

$$M^2\ddot{\rho} + \omega^2(t)\rho - \frac{1}{\rho^3} = 0$$

$$P_{0m} = \frac{m!}{2^m[(m/2)!]^2}\left(\frac{\cosh\delta - 1}{\cosh\delta + 1}\right)^{m/2}\left(\frac{2}{\cosh\delta + 1}\right)^{\frac{1}{2}}$$

$$M^2\dot{\rho}^2 + \omega^2\rho^2 + \frac{1}{\rho^2} = 2\omega cosh\delta$$





These are exploratory studies...we are still approaching towards more realistic scenario.

For realistic prediction of quarkonia suppression...

Quarkonia production in the primordial stage

Suppression in QGP and recombination through uncorrelated quark anti quark pair

Non-adiabatic transition among different states (reshuffling) due to rapid medium evolution

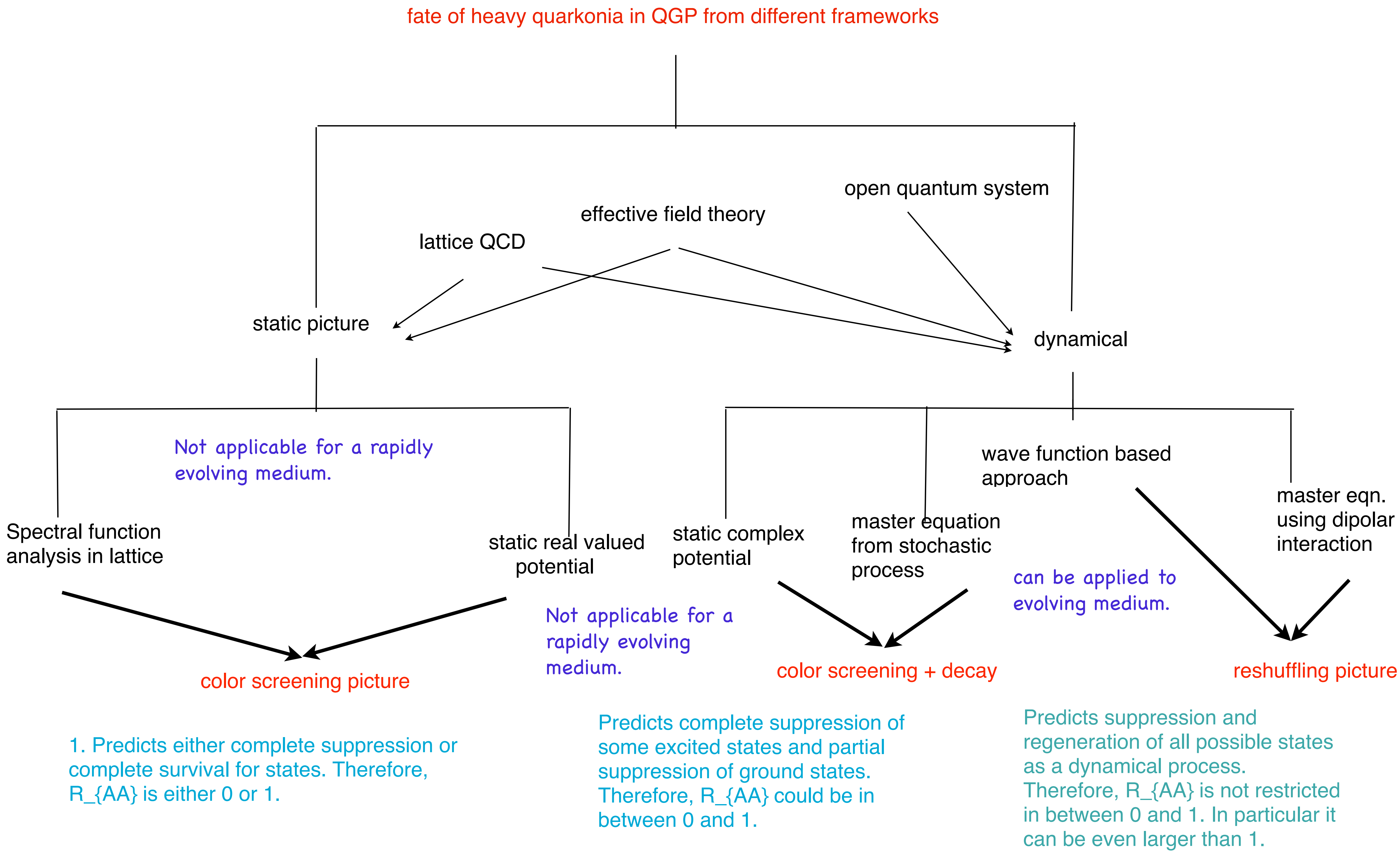
Combined in a single rate equation

Different channels are now coupled due to non adiabatic transition to different states(reshuffling)

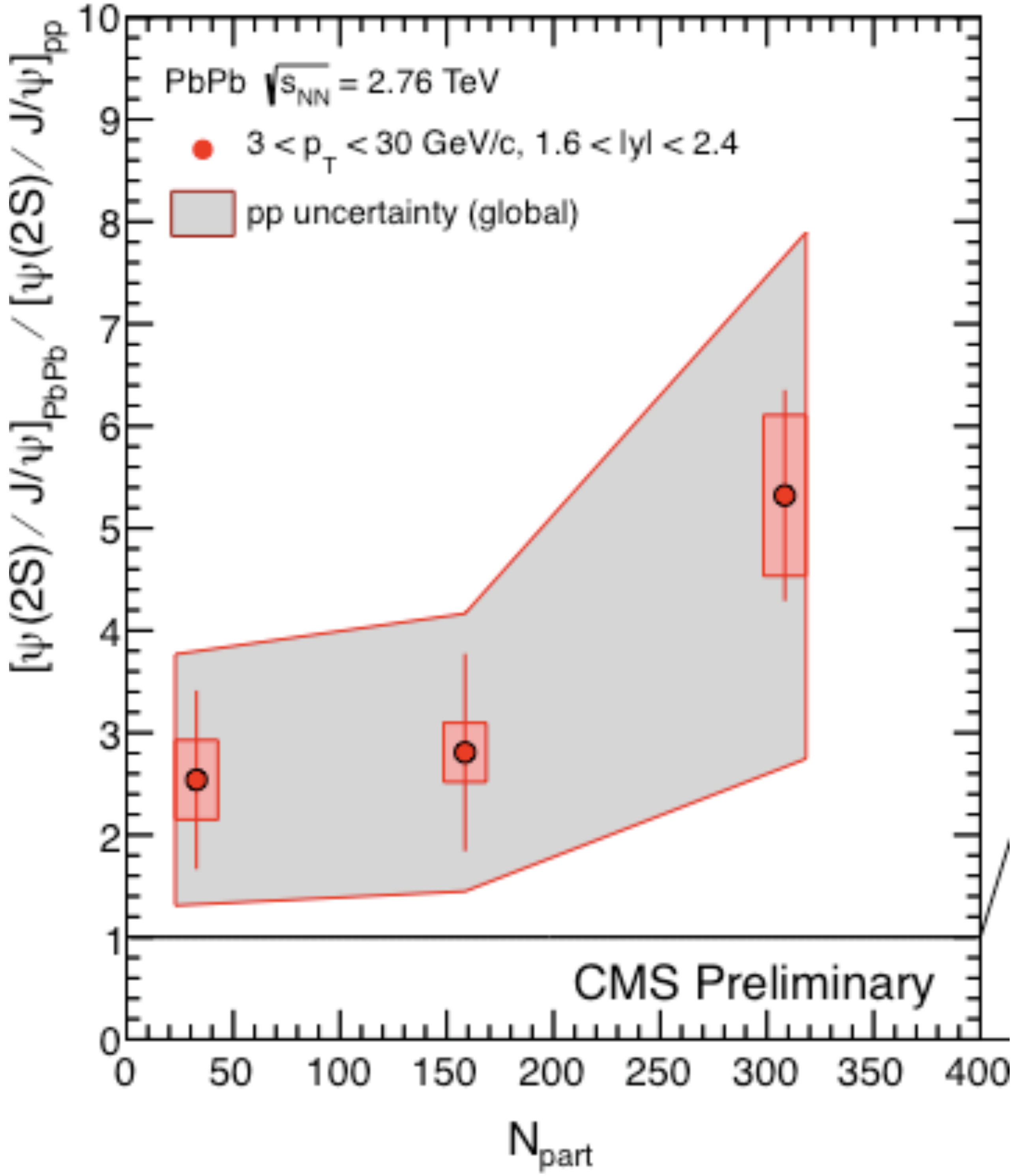
Different from the usual linear rate equation

Connection with experimental results

Fate of heavy quarkonia from different approaches



A *surprising* preliminary



$\Psi(2S)$ is less suppressed than J/Ψ

Surprise!   but still not confirmed
as the statistics is very low.

PRL 113, 262301 (2014) BY CMS COLLABORATION

conclusion

- Sequential suppression of heavy quarkonia is questionable in heavy ion collisions.

- Reshuffling among the bound states is the fate of heavy quarkonia in a medium created in heavy ion collisions.

- Thermalization time of quarkonium states could be crucial for a medium of very short persistence.

- Realtime dynamics is important to incorporate these issues.

- Modeling through open quantum system provides a way to deal with the problem.

conclusion

We need a proper initial condition for quarkonium states.

We need to know the interaction of quarkonia with dynamically evolving quark gluon plasma.

The frame work will enable us to describe the dynamics of heavy quarkonia in order to employ them for a convincing probe of the medium produced in Heavy Ion Collision.