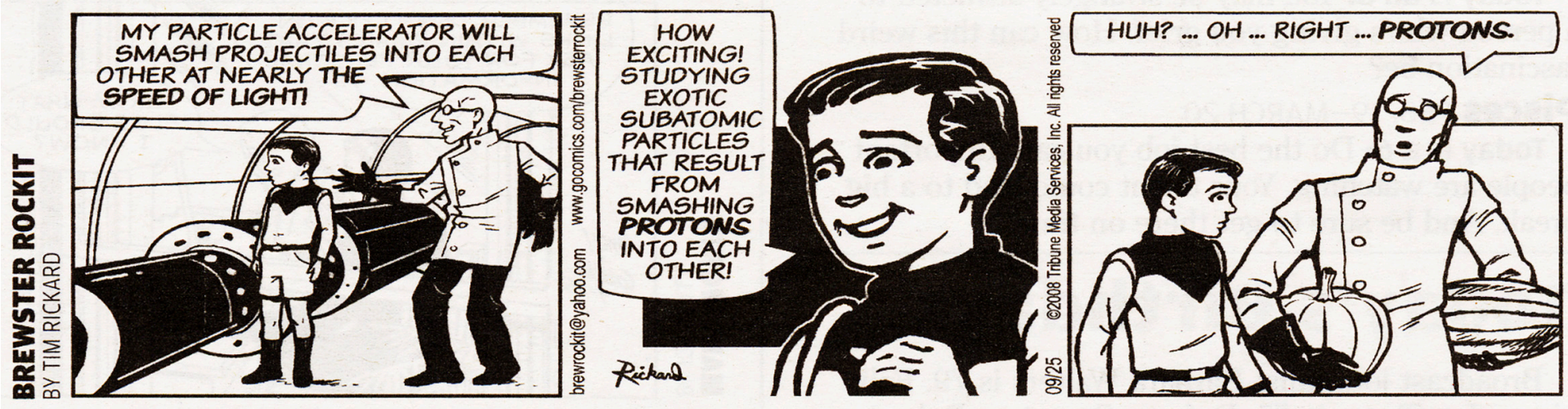
Z. Chajęcki & MAL PRC **78** 064903 (2008)
Z. Chajęcki & MAL arXiv:0807.3569 [nucl-th]
Z. Chajecki arXiv:0901.4078 [nucl-ex]
Z. Chajęcki & MAL, to be published



Oh, right… protons

(in the soft sector)

Mike Lisa
Ohio State University

Outline

- Why collide watermelons rather than seeds [reminder]

- Results from 2-pion correlations in p+p collisions
  - femtoscopic correlations
  - nonfemtoscopic correlations (EMCICs*)

Phase I

- Re-examining multiplicity-evolution of pT spectra, considering evolution of available phasespace (EMCICs**)
  - postulate of unchanging parent

Phase II

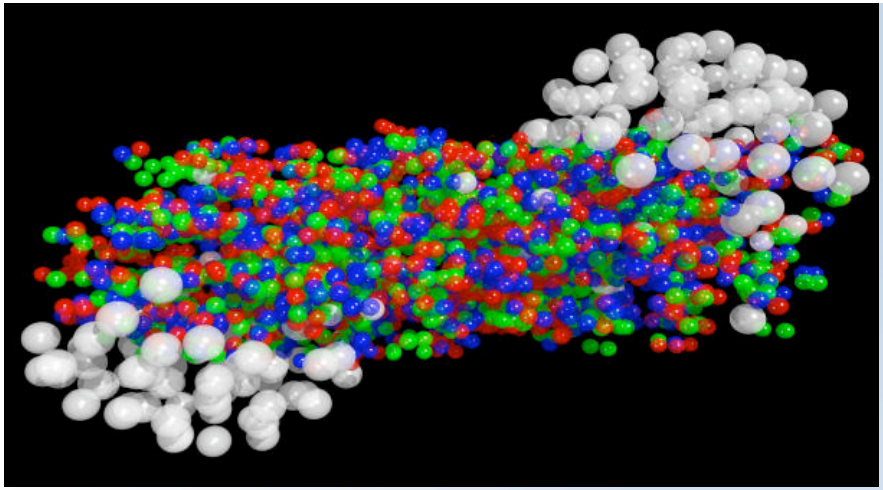
- *consistent* treatment of EMCICs *and* bulk in femtoscopy *and* spectra

- Summary

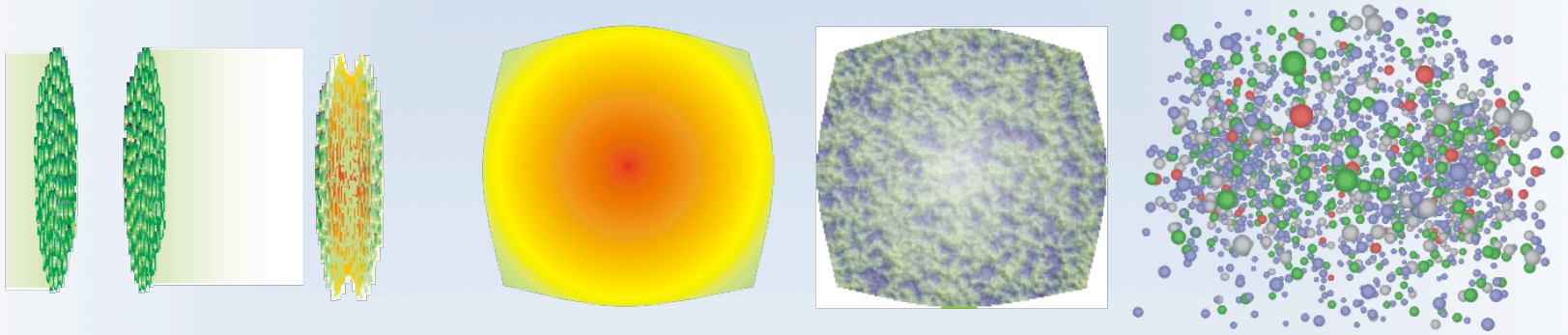
* EMCIC = "Energy and Momentum Induced Correlation"

** EMCIC = "Energy and Momentum Induced Constraint"

paradigms



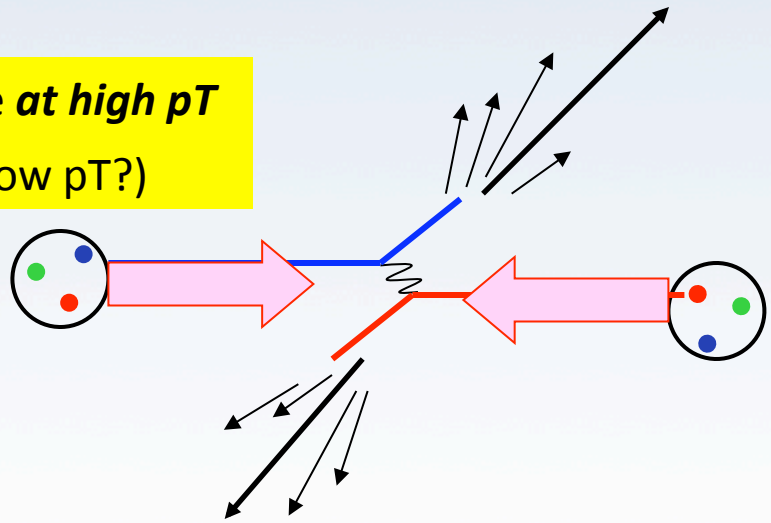
A+A → a *system*

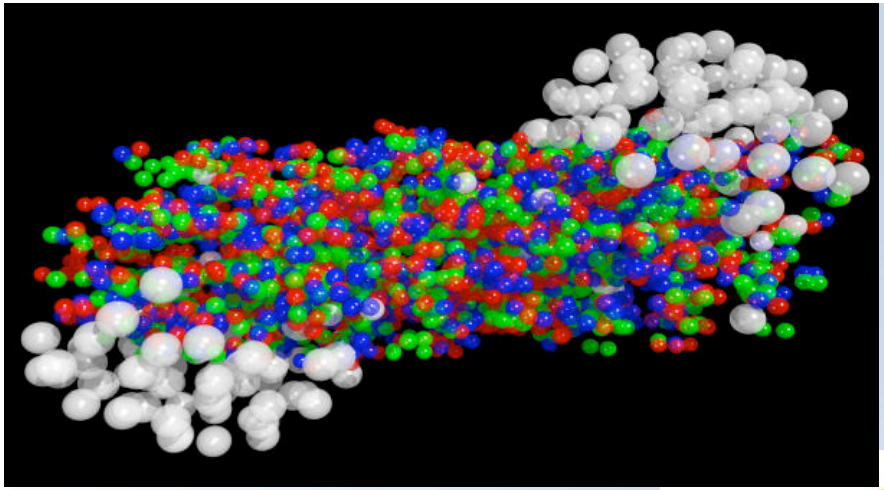


"Clean" p+p– a crucial reference ***at high pT***

(do we understand/care about low pT?)

p+p: a *process*





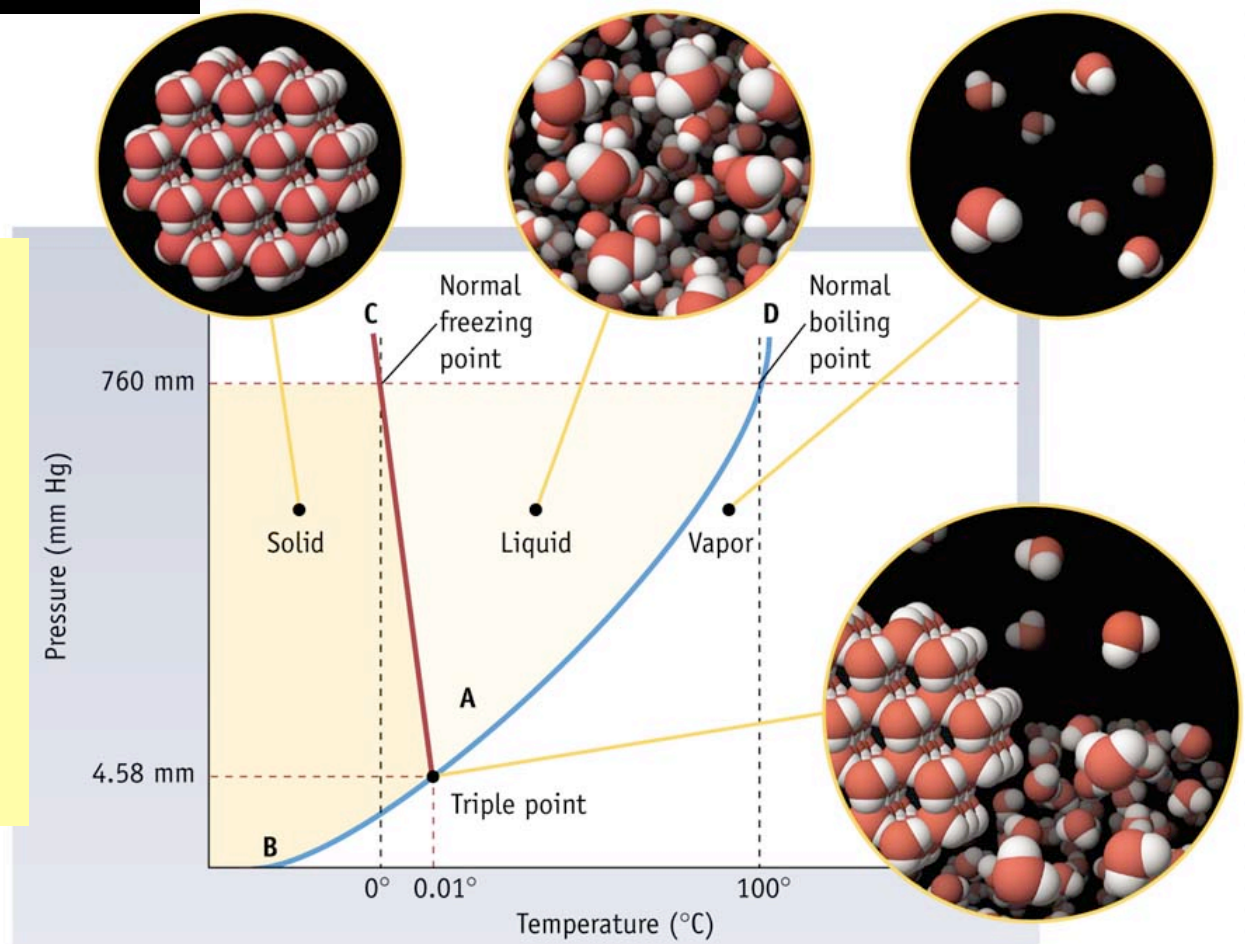
H.I.C. – a system

bulk physics

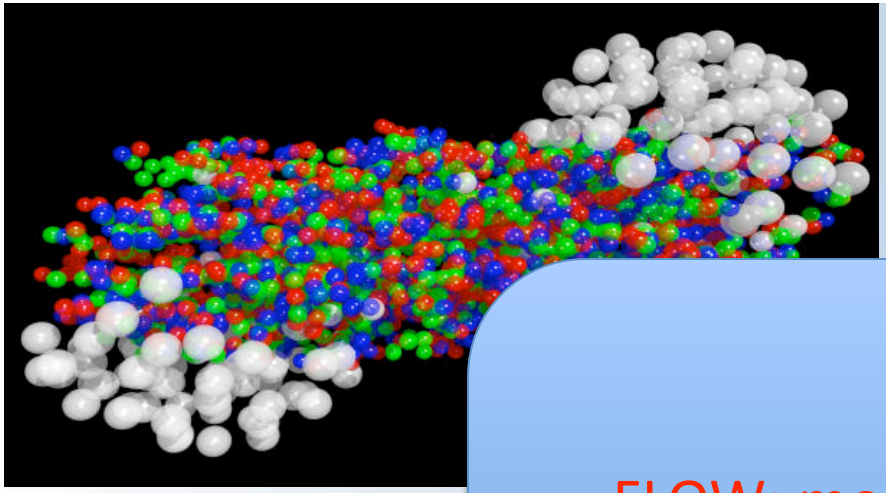
- superfluids
- superconductors
- metal/insulator
- …

Only for large *system*

- can't melt one H_2O molecule!



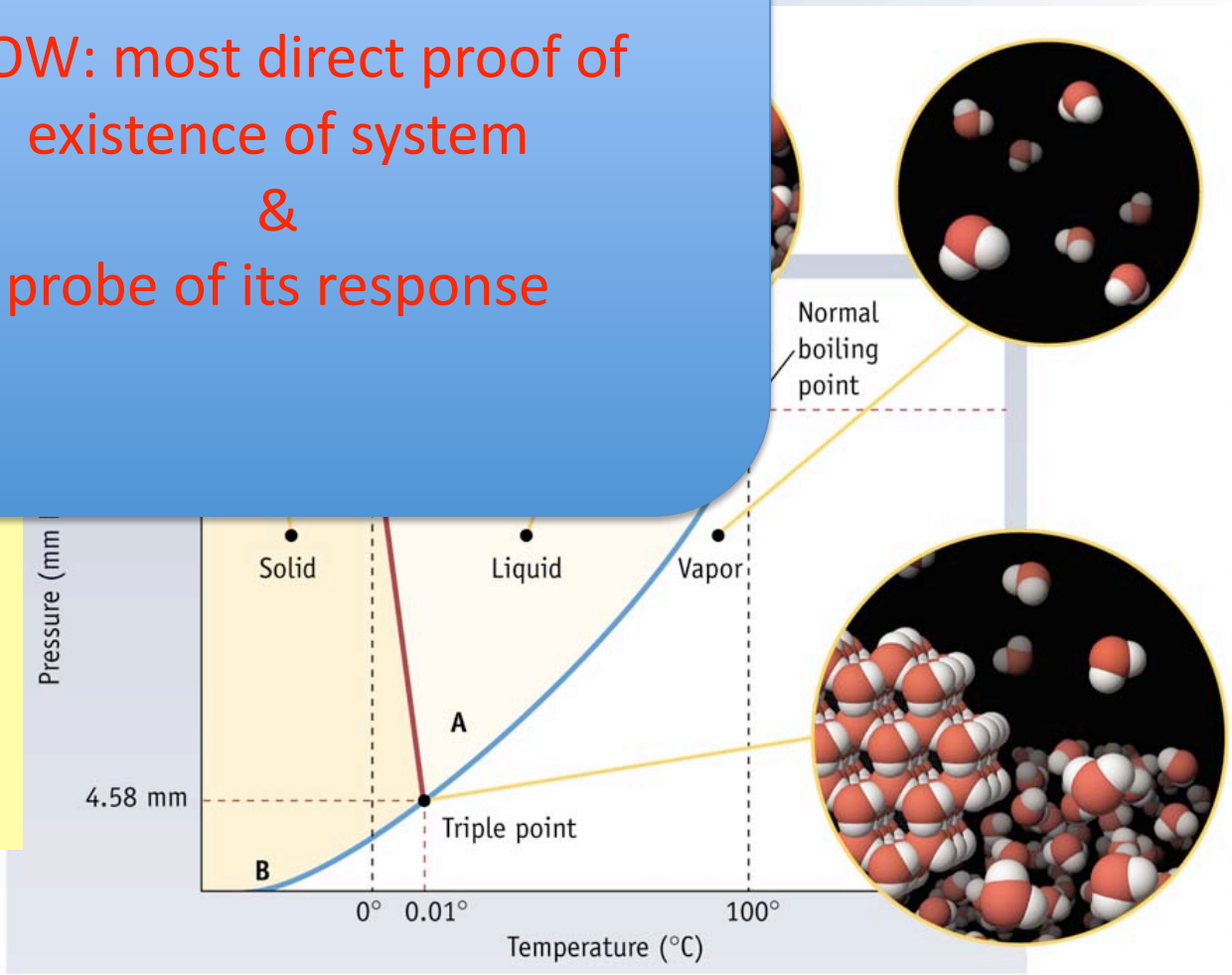
H.I.C. – a system



FLOW: most direct proof of existence of system
&
probe of its response

bulk physics
- superfluids
- superconductors
- metal/insulator
- …

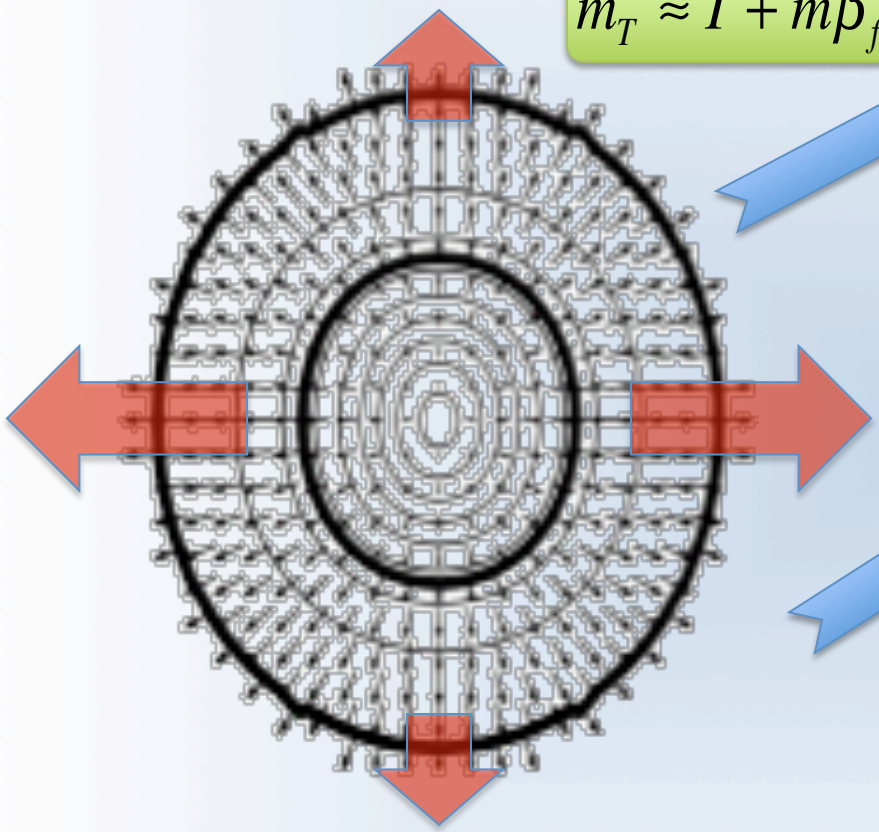
Only for large *system*
- can't melt one H_2O molecule!

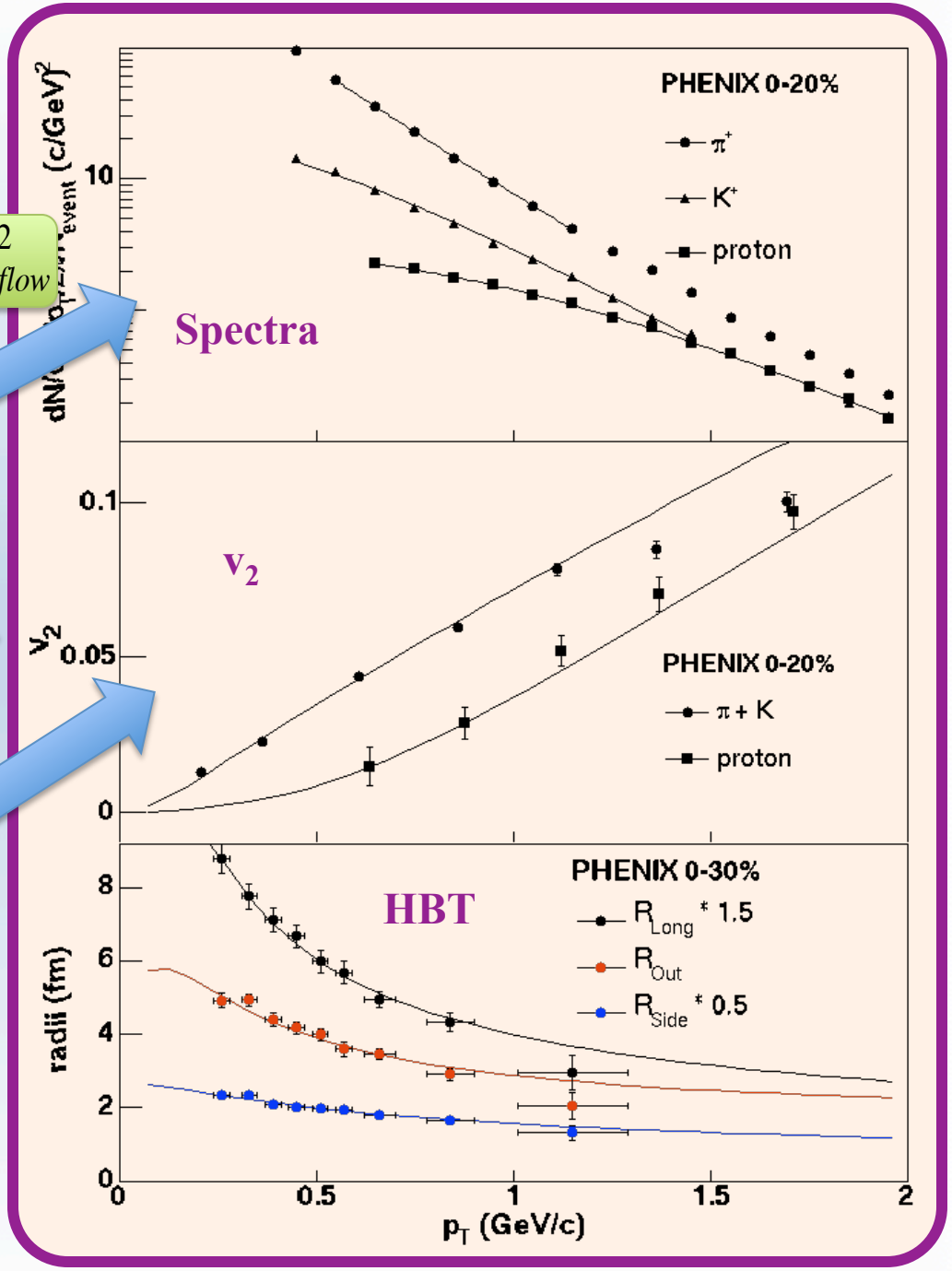


Explosive flow revealed through ***specific fingerprints*** on soft-sector observables

calculable in hydrodynamics or toy "blast wave" models

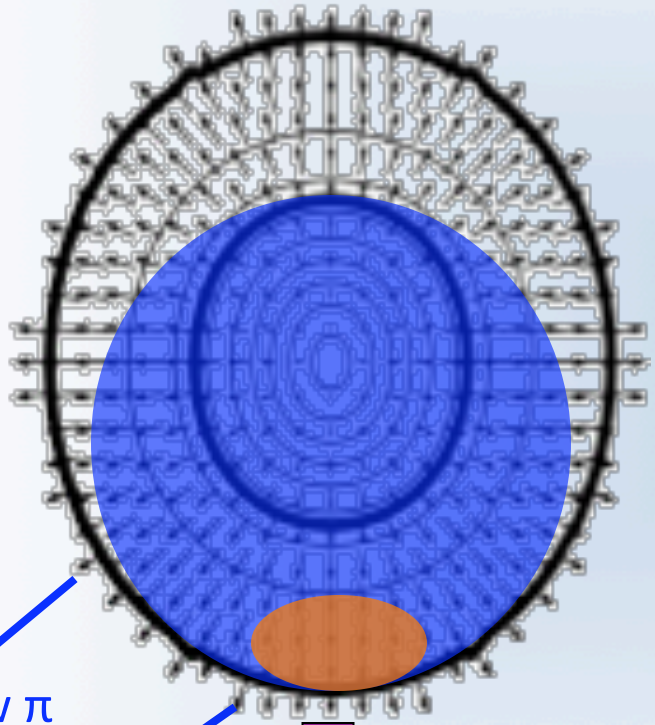
$$m_T \approx T + m\beta_{flow}^2$$





Explosive flow revealed through ***specific fingerprints*** on soft-sector observables

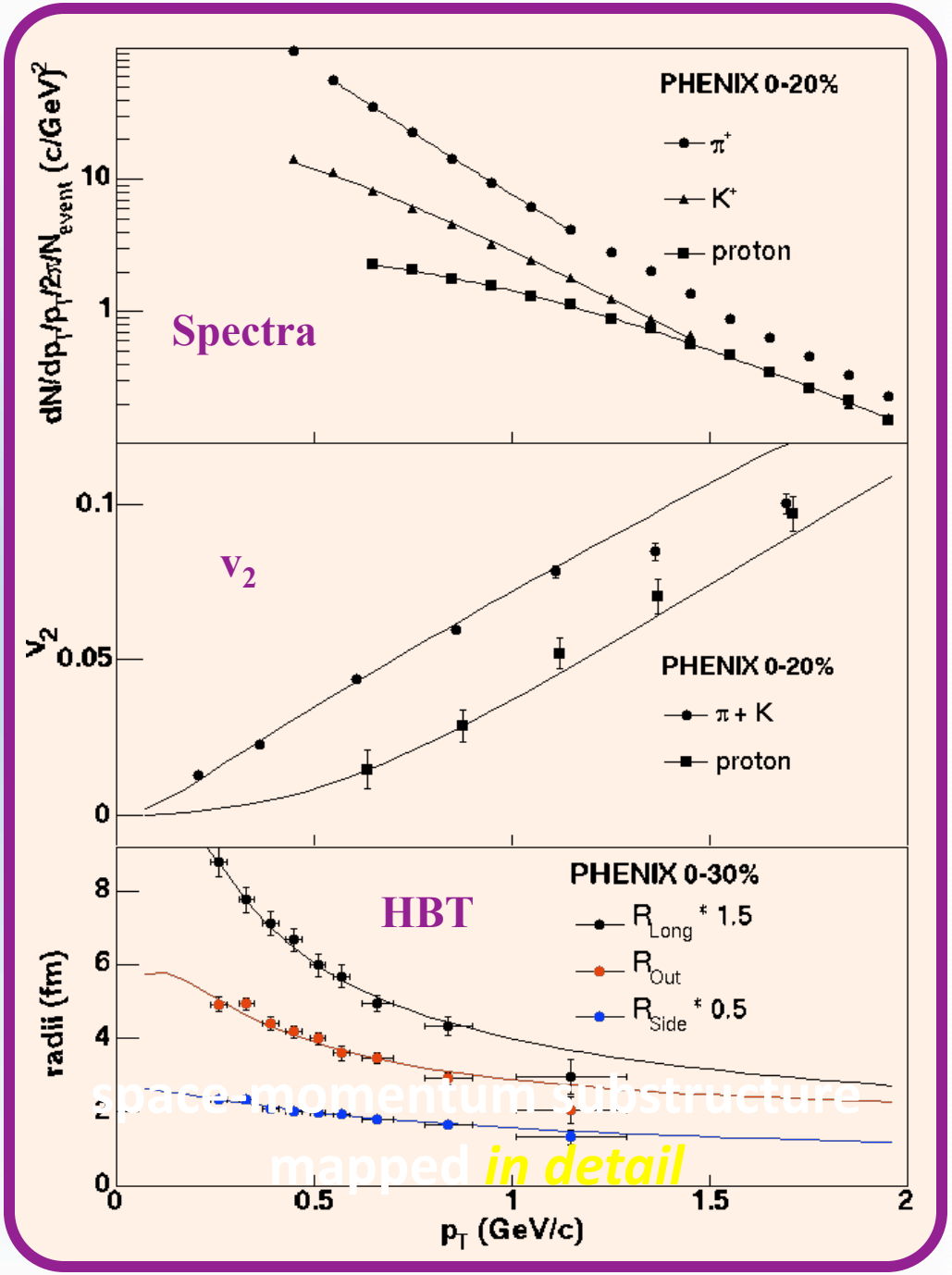
calculable in hydrodynamics or toy "blast wave" models



slow π

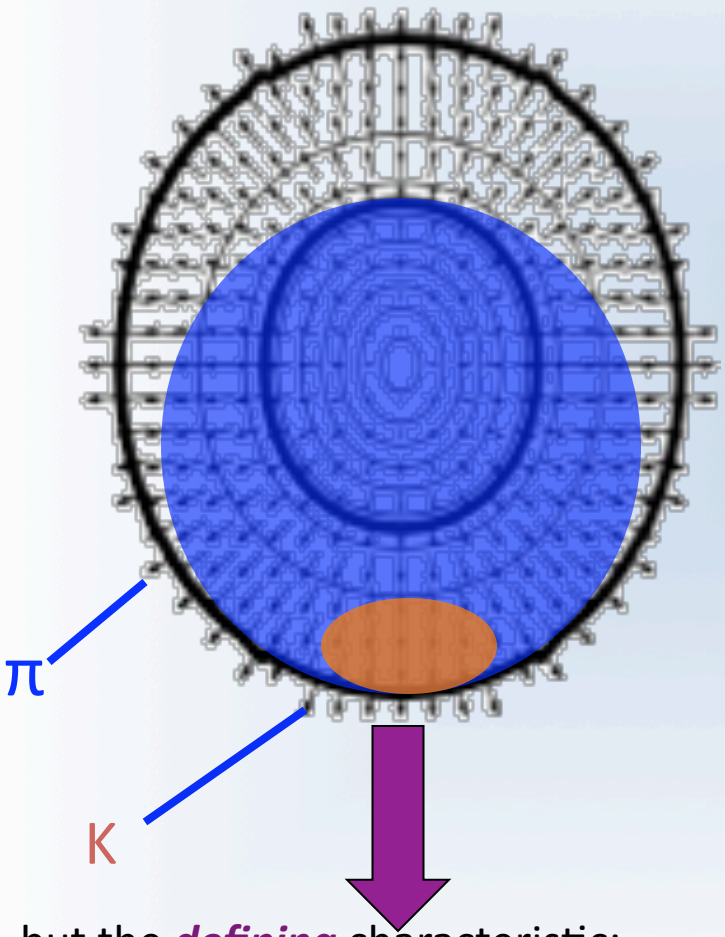
fast π

but the ***defining*** characteristic:
correlated position and boost direction

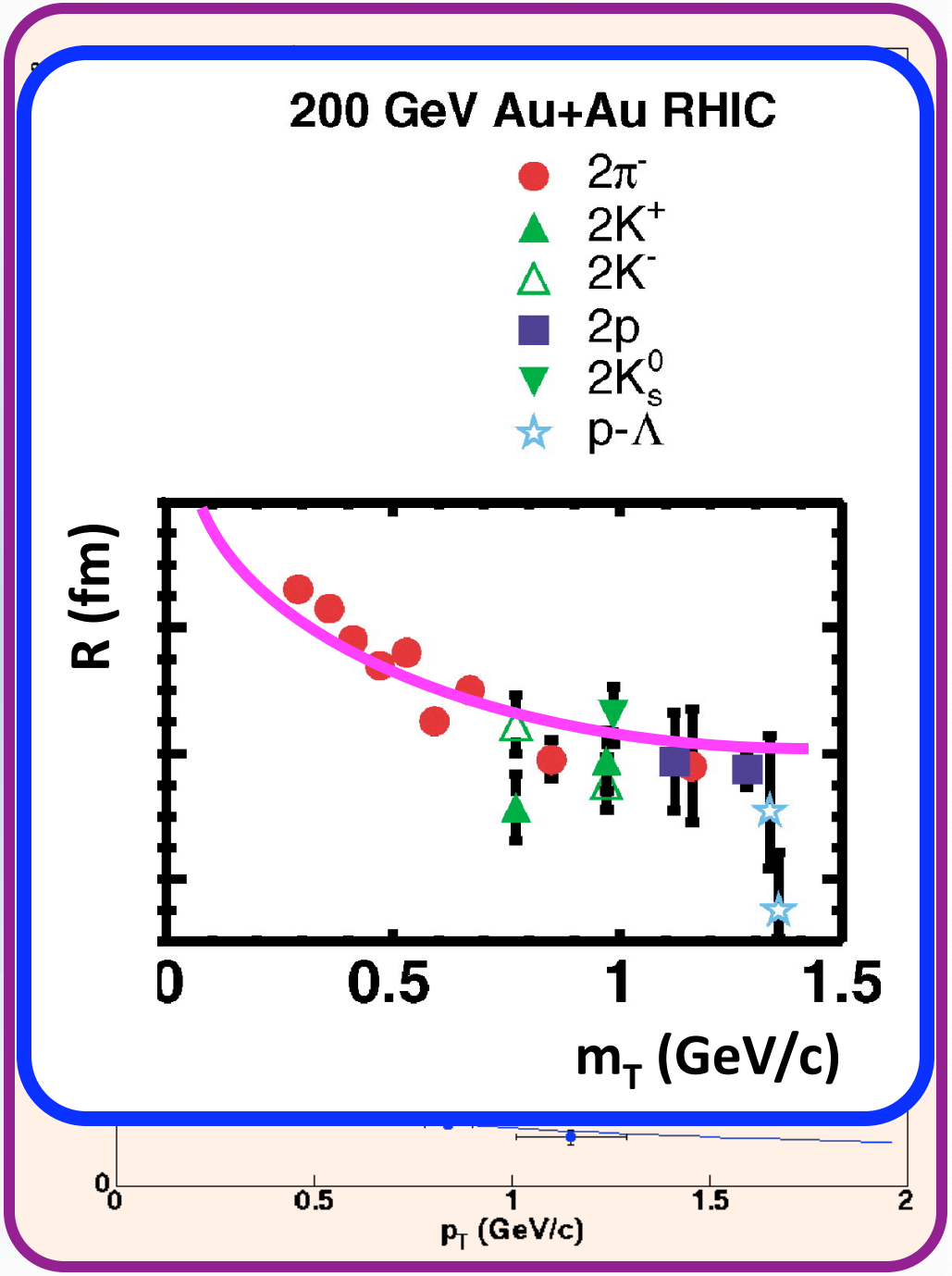


Explosive flow revealed through ***specific fingerprints*** on soft-sector observables

calculable in hydrodynamics or toy "blast wave" models

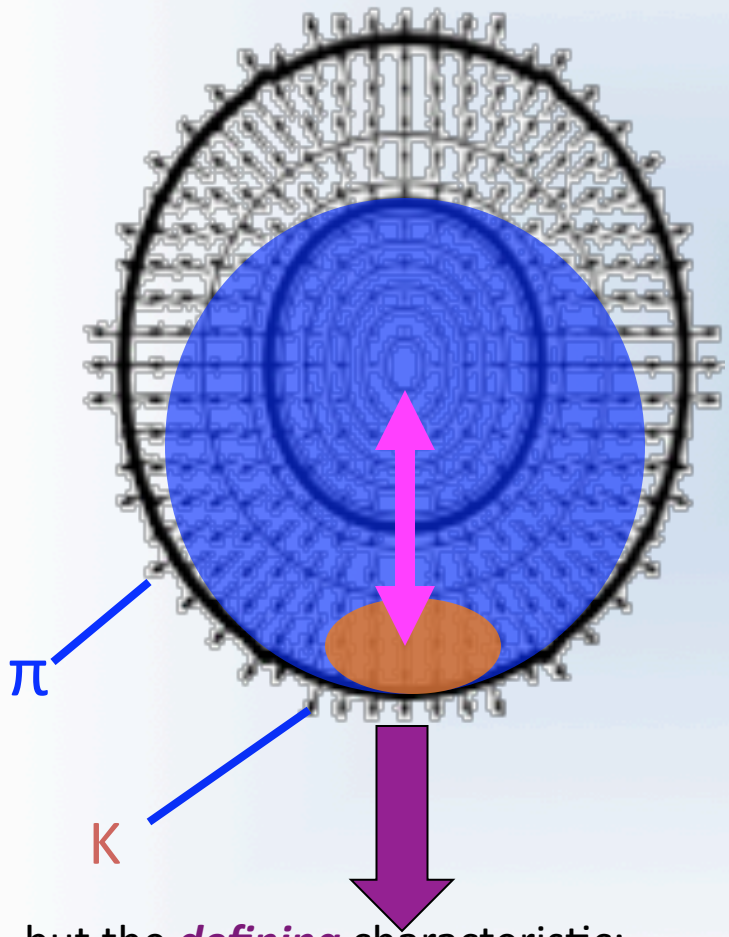


but the ***defining*** characteristic:
correlated position and boost direction

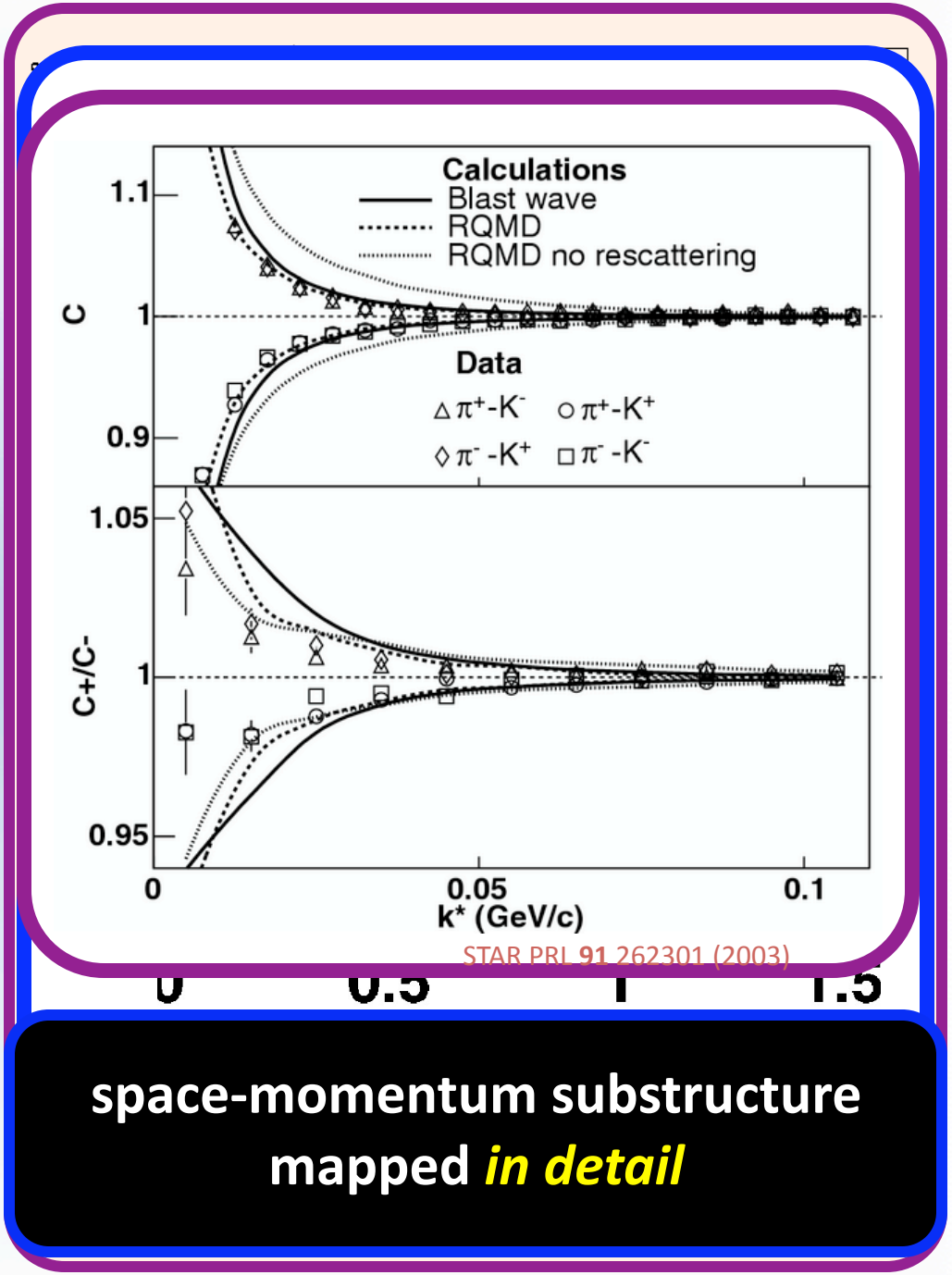


Explosive flow revealed through ***specific fingerprints*** on soft-sector observables

calculable in hydrodynamics or toy "blast wave" models



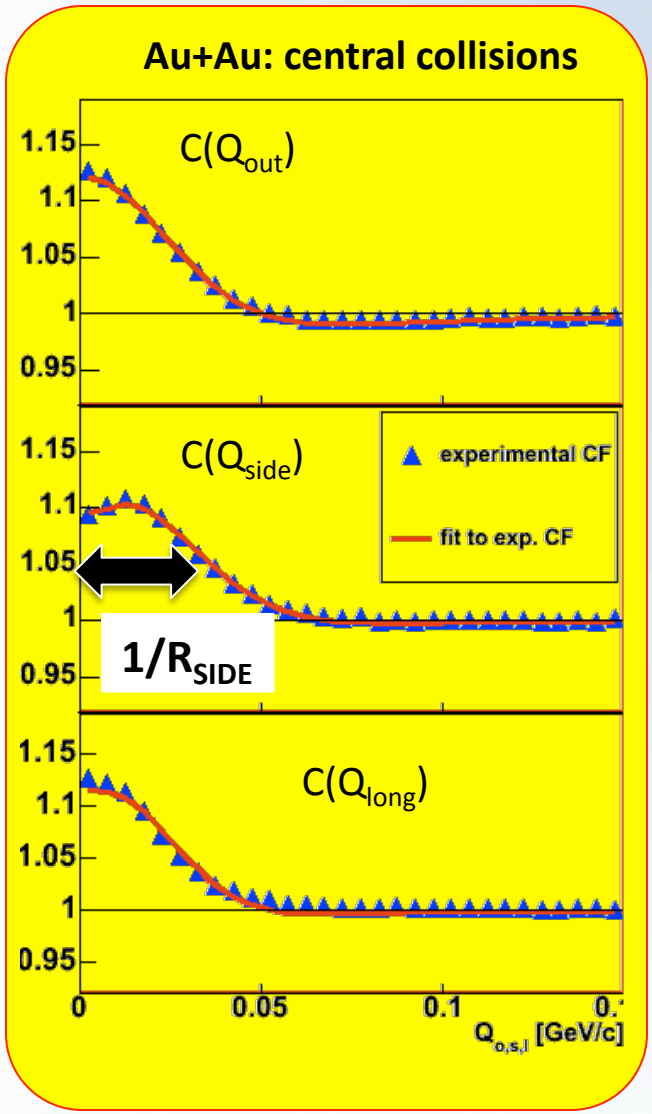
but the ***defining*** characteristic:
correlated position and boost direction



STAR PRL **91** 262301 (2003)

space-momentum substructure mapped *in detail*

Obtaining 3D radii from 3D correlation functions

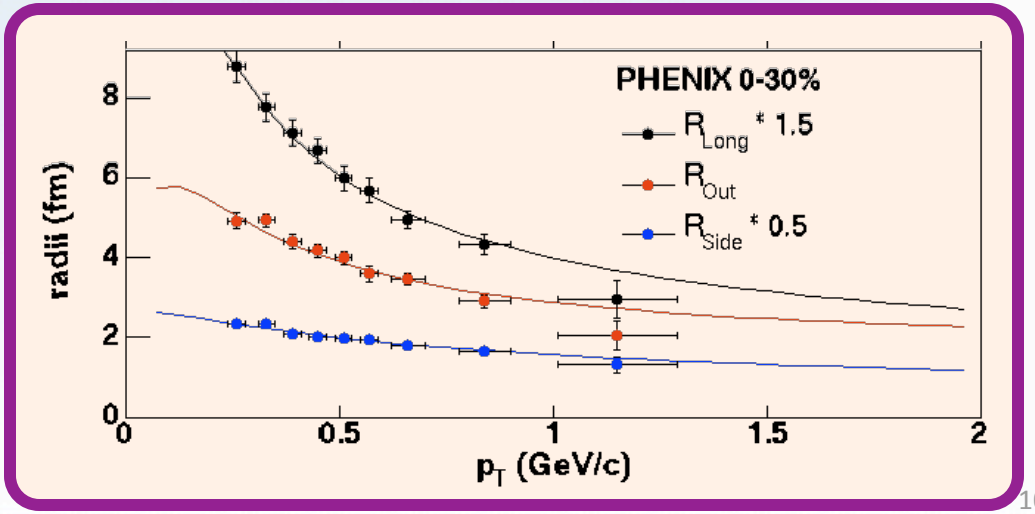


$$C(\vec{q}) = N \cdot \left[1 + \lambda \cdot \left(K_{coul}(\vec{q}) \cdot \left\{ 1 + e^{-\left(q_o^2 R_o^2 + q_s^2 R_s^2 + q_l^2 R_l^2\right)} \right\} - 1 \right) \right]$$

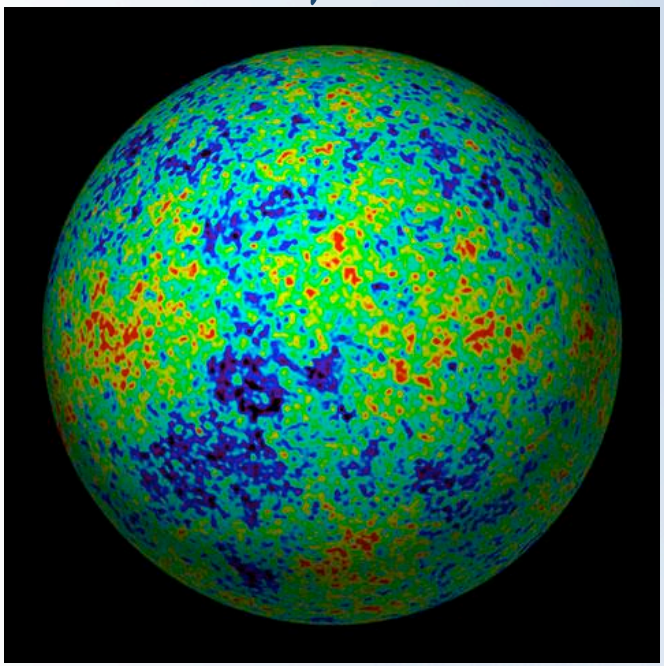
typical "Gaussian" fitting function

- Au+Au: "Gaussian" radii capture bulk scales
  - (resonance tails from imaging)
- $R(p_T)$ consistent with explosive flow

"set of zero measure" of full 3D correlation fctn



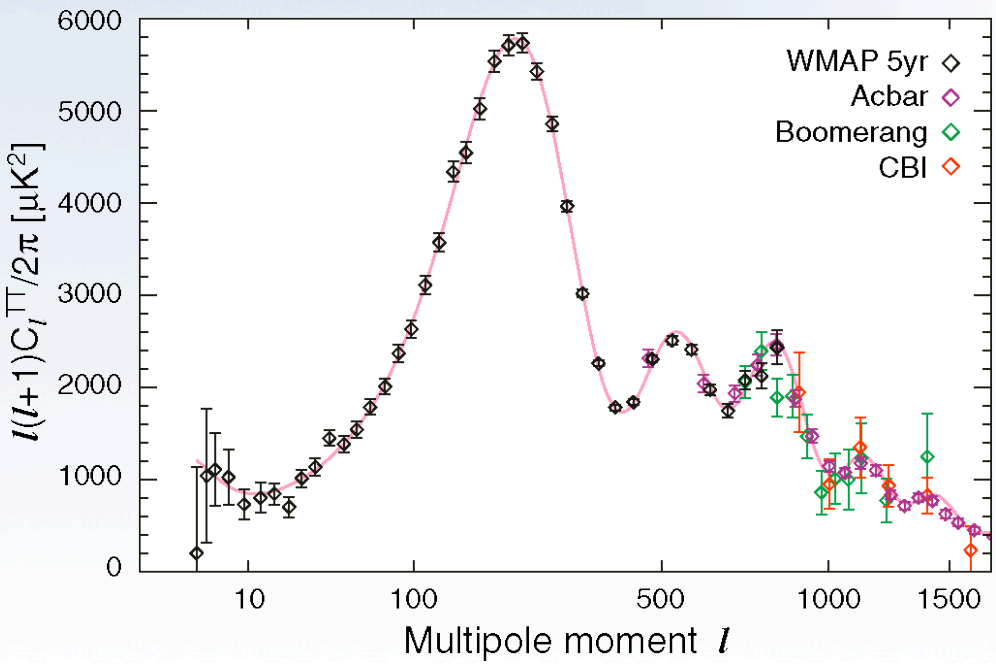
Spherical harmonic representation of 3D data



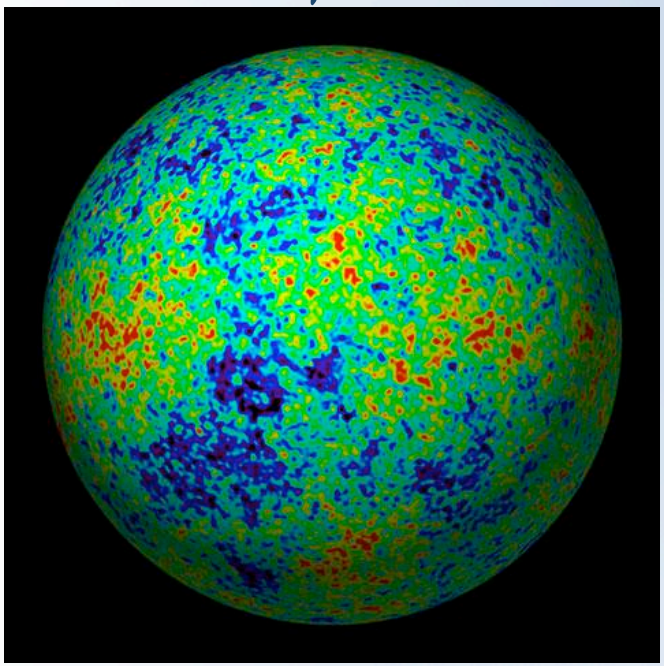
$$a_{l,m} \equiv \int d\Omega \cdot T(\theta,\phi) \cdot Y^*_{l,m}(\theta,\phi)$$

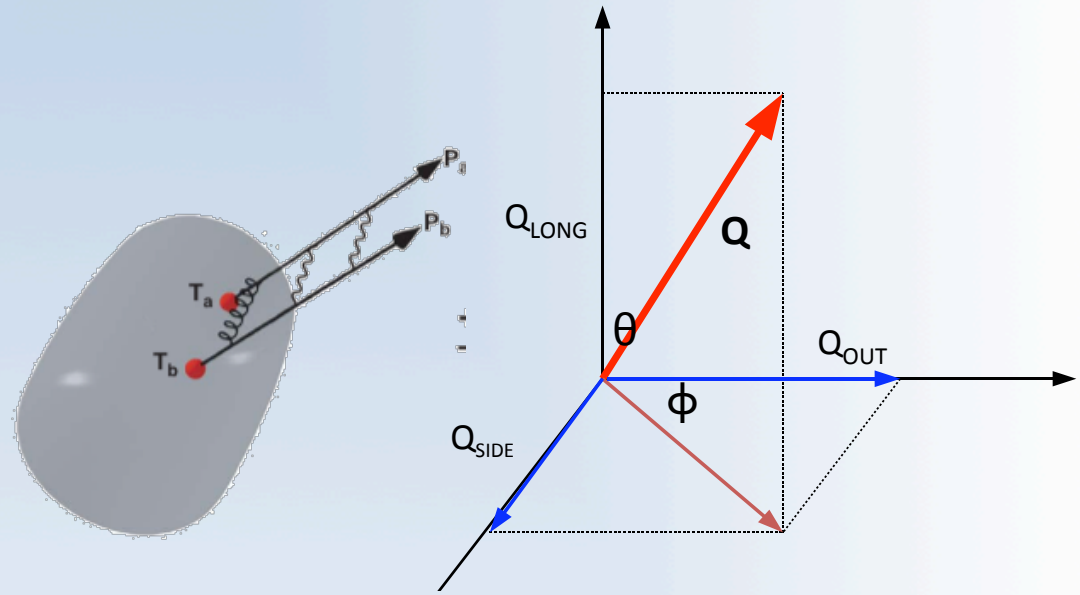
$$C_l^{TT} \equiv \left\langle \left| a_{l,m} \right|^2 \right\rangle_m$$

(average over m ⊠ no "special" direction)



Spherical harmonic representation of 3D data





$$a_{l,m} \equiv \int d\Omega \cdot T(\theta,\phi) \cdot Y^*_{l,m}(\theta,\phi)$$

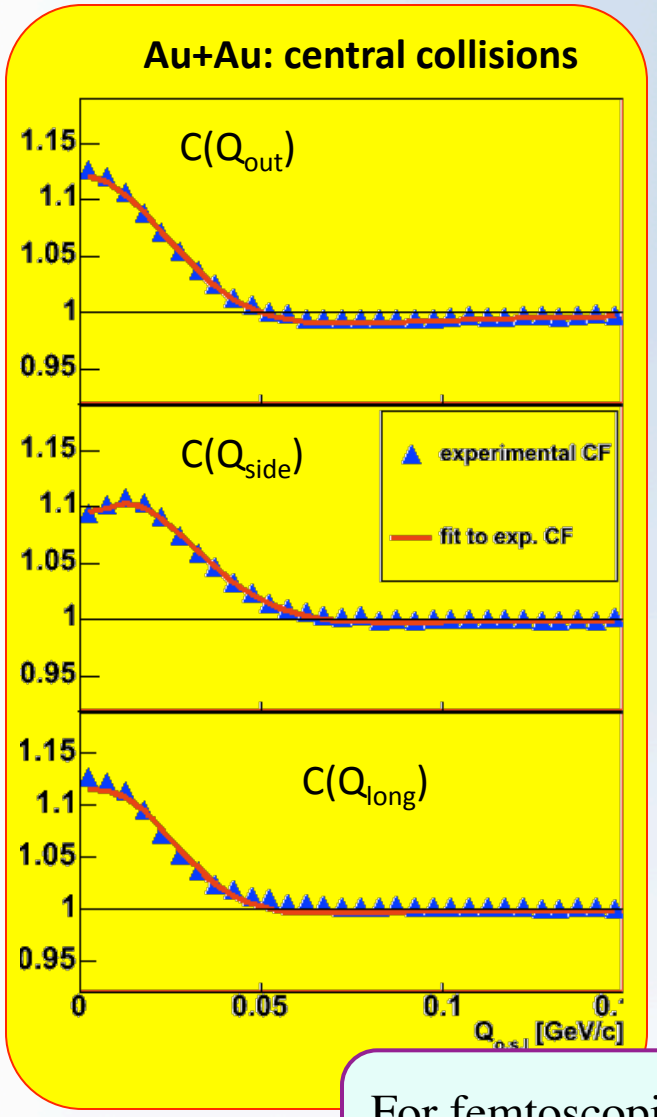
$$C_l^{TT} \equiv \left\langle \left| a_{l,m} \right|^2 \right\rangle_m$$

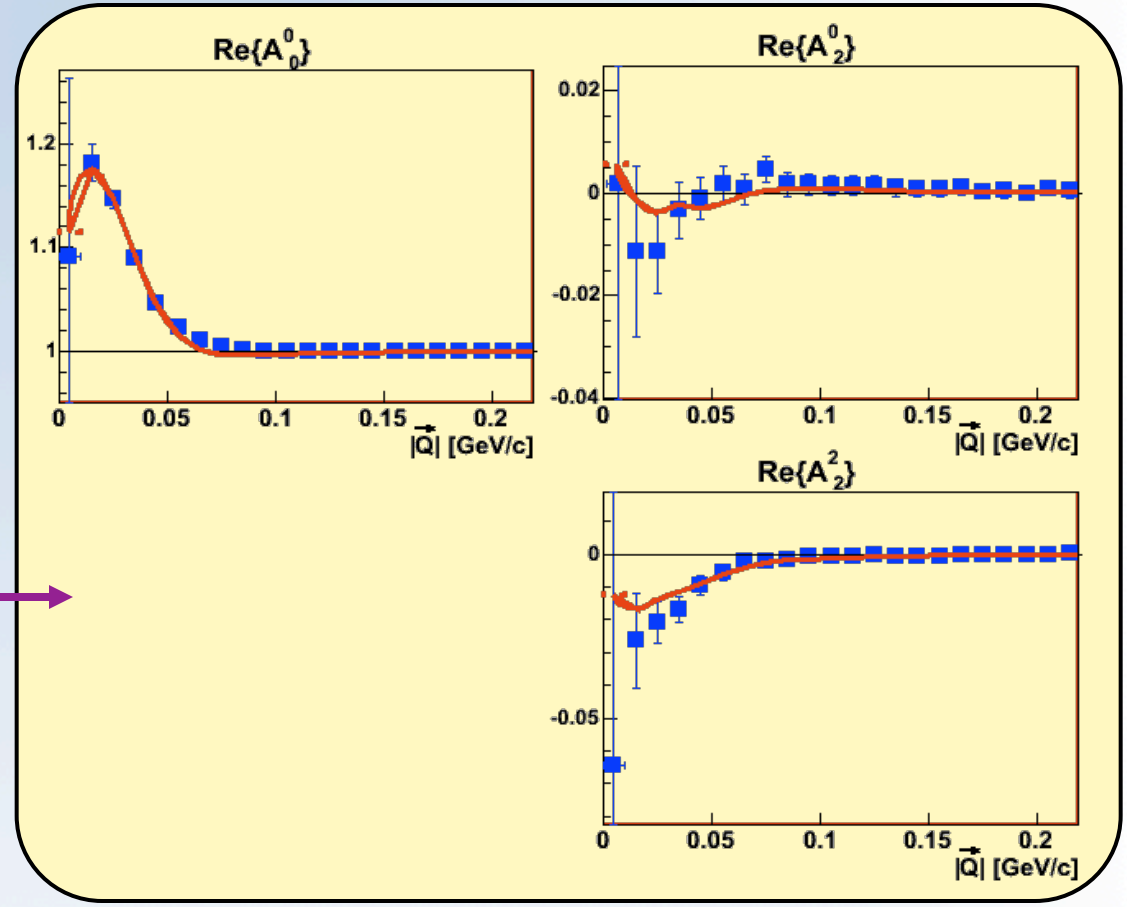
(average over m ⊠ no "special" direction)

$$A_{l,m}\left(\left|\vec{Q}\right|\right) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_i^{bins} Y^*_{l,m}(\theta_i,\phi_i) C\left(\left|\vec{Q}\right|, \cos\theta_i, \phi_i\right)$$

Z. Chajecki & MAL, PRC **78** 064903 (2008)

Spherical harmonic representation of 3D data





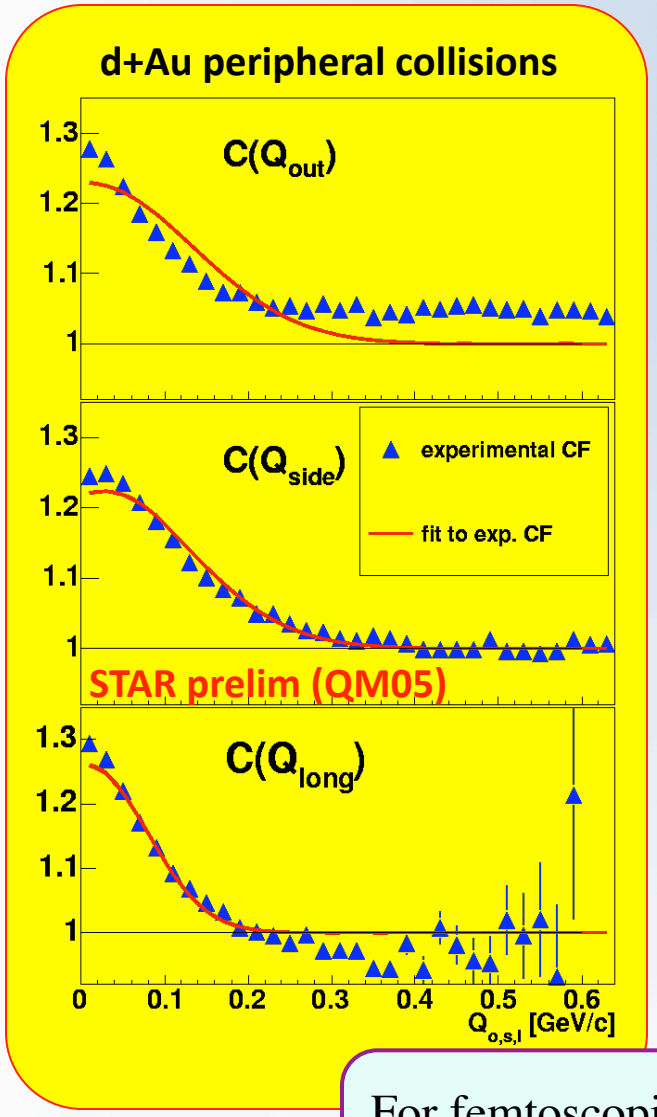
$$A_{l,m}\left(\left|\vec{Q}\right|\right) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_{i}^{bins} Y^{*}_{l,m}(\theta_i,\phi_i)\, C\left(\left|\vec{Q}\right|, \cos\theta_i, \phi_i\right)$$

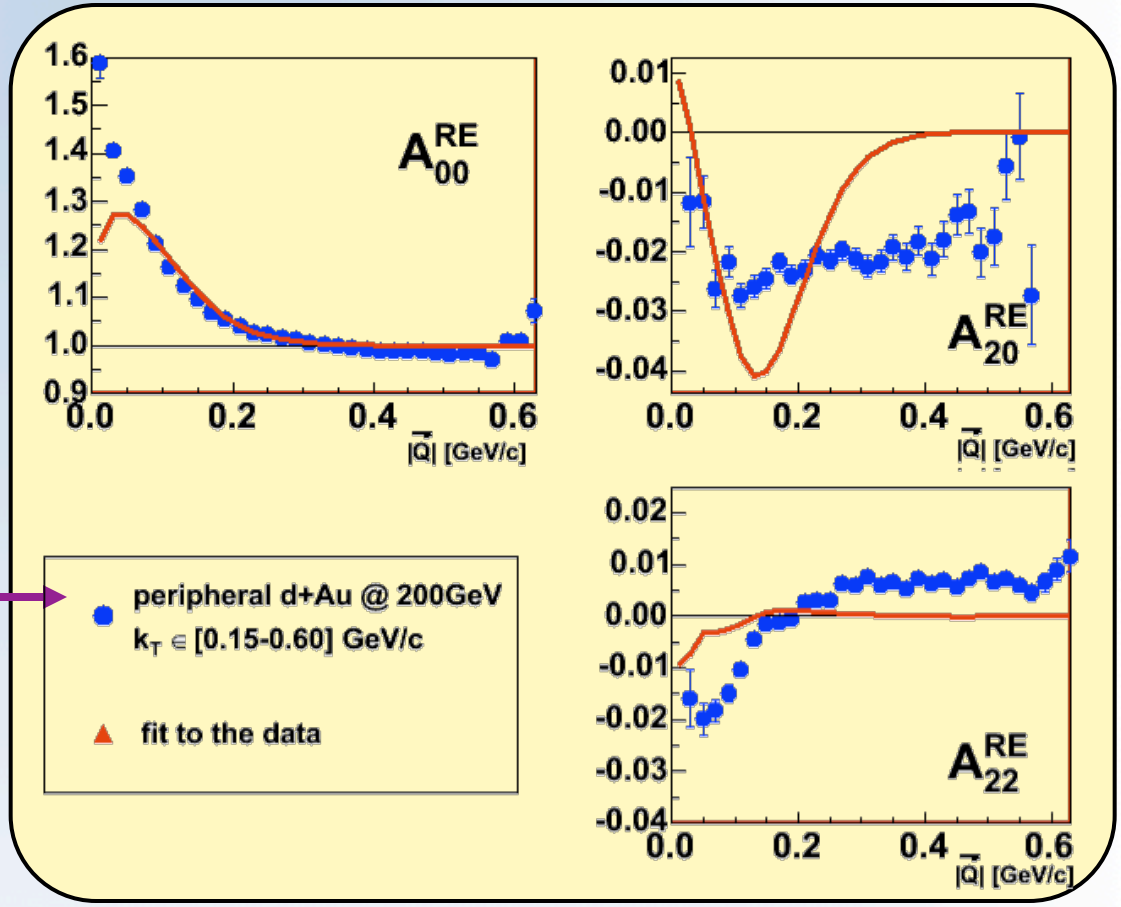
Z. Chajecki & MAL, PRC **78** 064903 (2008)

For femtoscopic correlations:

$$C(\vec{q};|\vec{q}|\to\infty) = C(|\vec{q}|\to\infty) \Rightarrow \mathrm{A}^{m}_{\ell\neq 0}(|\vec{q}|\to\infty) = 0$$

Spherical harmonic representation of 3D data





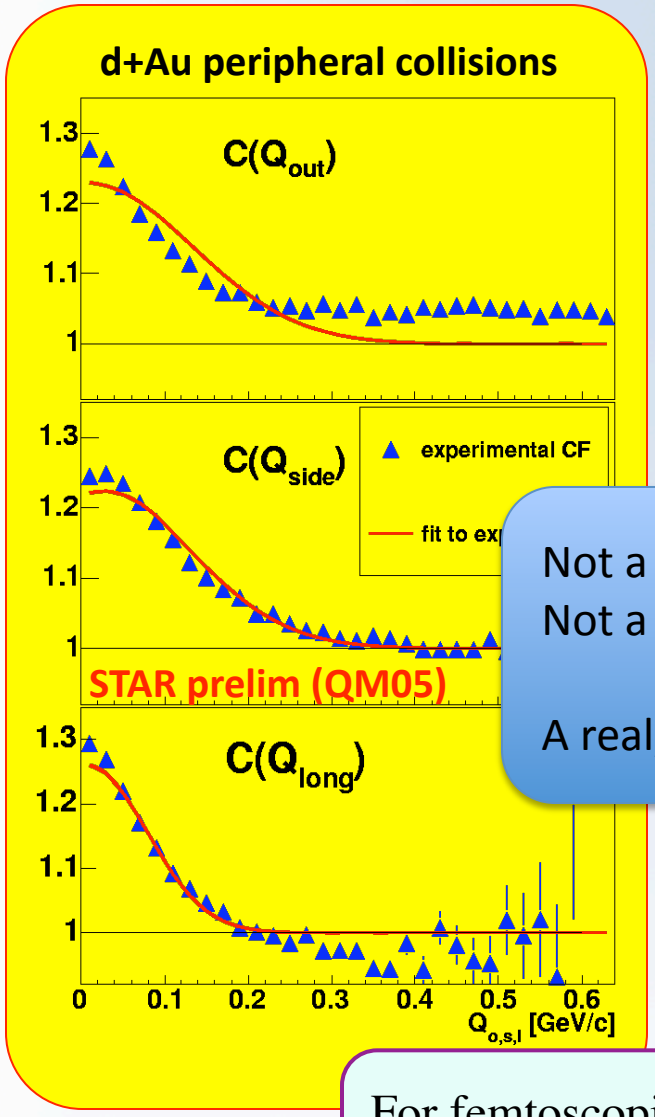
$$A_{l,m}\left(\left|\vec{Q}\right|\right) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_{i}^{bins} Y^{*}_{l,m}(\theta_i,\phi_i)\, C\left(\left|\vec{Q}\right|, \cos\theta_i, \phi_i\right)$$

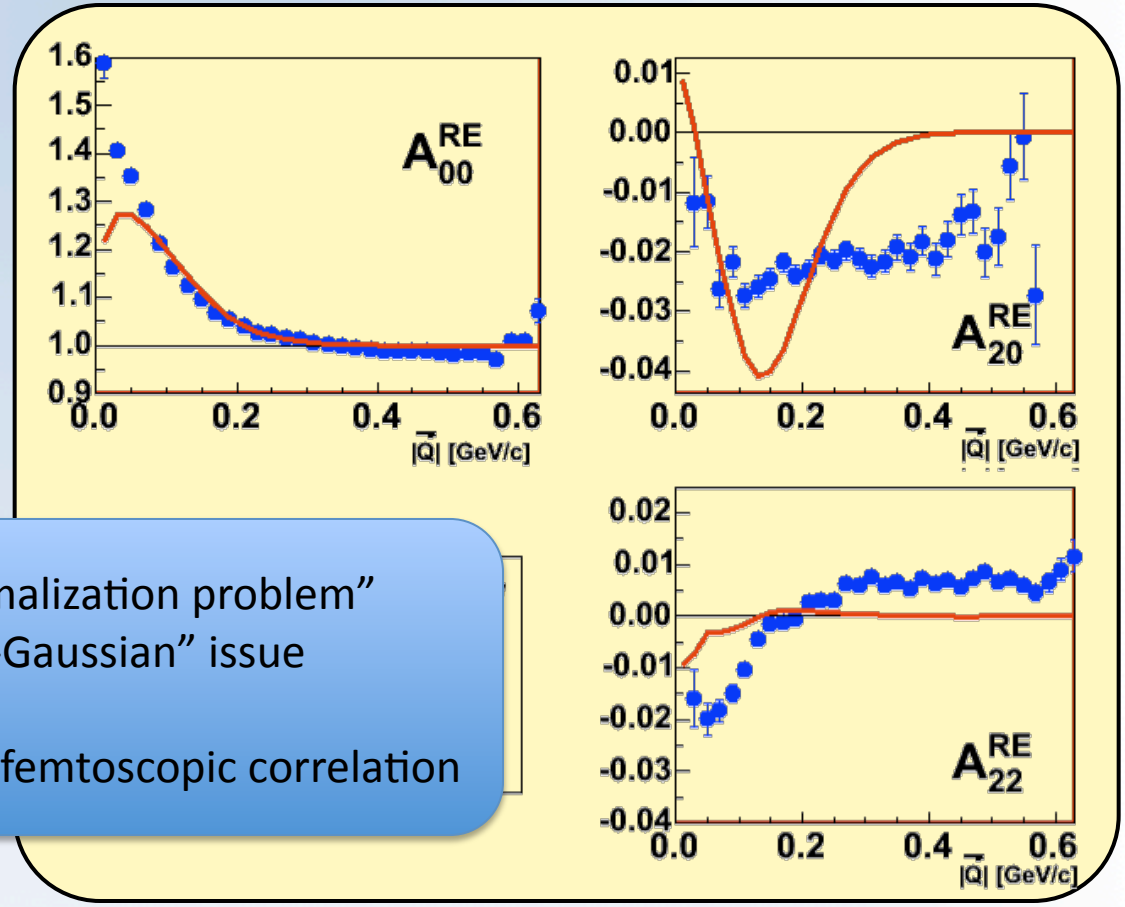
Z. Chajecki & MAL, PRC **78** 064903 (2008)

For femtoscopic correlations:

$$C(\vec{q};|\vec{q}| \to \infty) = C(|\vec{q}| \to \infty) \Rightarrow \mathrm{A}^{m}_{\ell\neq 0}(|\vec{q}| \to \infty) = 0$$

Spherical harmonic representation of 3D data





Not a "normalization problem"
Not a "non-Gaussian" issue

A real, non-femtoscopic correlation

$$A_{l,m}\left(\left|\vec{Q}\right|\right) = \frac{\Delta_{\cos\theta}\Delta_{\phi}}{\sqrt{4\pi}} \sum_{i}^{bins} Y^{*}_{l,m}(\theta_i,\phi_i)\, C\left(\left|\vec{Q}\right|, \cos\theta_i, \phi_i\right)$$

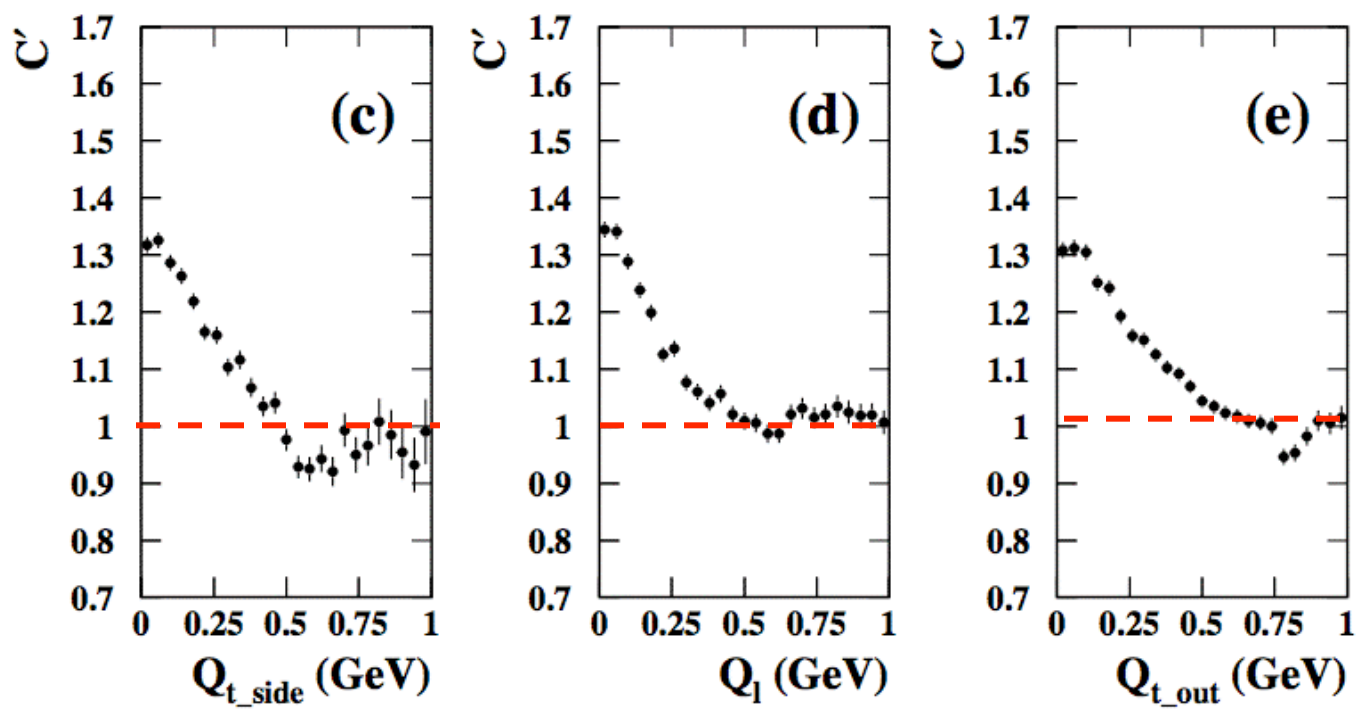
Z. Chajecki & MAL,PRC in press, arXiv:0803.0022 [nucl-th]

For femtoscopic correlations:

$$C(\vec{q};|\vec{q}|\to\infty) = C(|\vec{q}|\to\infty) \Rightarrow \mathrm{A}^{m}_{\ell\neq 0}(|\vec{q}|\to\infty) = 0$$

We are not alone...

Non-femto correlations in B-E analysis through the years:



Q_x<0.2 GeV/c

OPAL, CERN-PH-EP/2007-025
(submitted to Eur. Phys. J. C.)

NA22, Z. Phys. C71 (1996) 405

CLEO PRD32 (1985) 2294

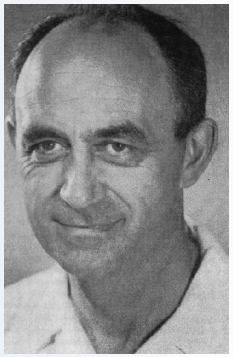
non-femto "large-Q" behaviour - various approaches

- ignore it
- various ad-hoc parameterizations
- divide by $\pi^+\pi^-$ (only semi-successful, and only semi-justified)
- divide by MonteCarlo PYTHIA, tuning until tail is matched (similar to ad-hoc)

- Can we understand it in terms of simplest-possible effect–
  Energy and **M**omentum **C**onservation **I**nduced **C**orrelations (**EMCICs**)?
  – Z. Chajecki & MAL, PRC **78** 064903 (2008)

- see also
  – pT conservation effects on v2 [Danielewicz, Ollitrault & Borghini]
  – pT conservation on 3-particle "conical emission" observables [Borghini]
  – p and E conservation effects on single particle spectra [Chajecki & MAL]

energy-momentum conservation in n-body states



spectrum of kinematic quantity α (angle, momentum) given by

$$f(\alpha) = \frac{d}{d\alpha}\left(|M|^2 \cdot R_n\right)$$

where

$M =$ matrix element describing interaction

$(M = 1 \rightarrow$ all spectra given by phasespace$)$

---

n-body Phasespace factor R_n

$$R_n = \int^{4n} \delta^4\left(P - \sum_{j=1}^{n} p_j\right) \prod_{i=1}^{n} \delta\left(p_i^2 - m_i^2\right) d^4 p_i$$

where

$P =$ total 4-momentum of n-particle system

$p_i =$ 4-momentum of particle i

$m_i =$ mass of particle i

statistics: "density of states"

$$\delta\left(p_i^2 - m_i^2\right) d^4 p_i = \frac{|\vec{p}_i|^2}{E_i} d|\vec{p}_i| \cdot d(\cos\theta_i) \cdot d\phi_i$$

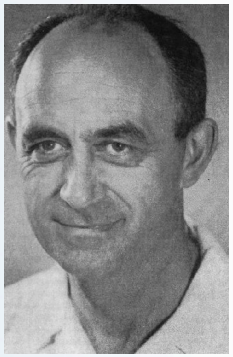
larger particle momentum ⇒ more available states

$P_{\boxtimes}$ conservation

$$\delta^4\left(P - \sum_{j=1}^{n} p_j\right)$$

Induces "trivial" correlations (i.e. even for M=1)

Example of use of total phase space integral



- In absence of "physics" in *M* : (i.e. phase-space dominated)

$$\frac{\Gamma(p\bar{p} \to \pi\pi\pi)}{\Gamma(p\bar{p} \to \pi\pi\pi\pi)} = \frac{R_3(1.876;\pi,\pi,\pi)}{R_4(1.876;\pi,\pi,\pi,\pi)}$$

- single-particle spectrum (e.g. p_T):

$$W(p_i) = d^3 p_i \cdot \bar{S}_n(p_i) R_n$$

Hagedorn

- "spectrum of events":

In limit where "α"="event" = collection of momenta $\vec{p}_i$

$$\text{"spectrum of events"} = f(\alpha) = \frac{d}{d\alpha} R_n$$

$$\to \text{Prob}_{\text{event } \alpha} \propto \frac{d^{3n}}{\prod_{i=1}^{n} dp_i^3} R_n$$

F. James, CERN REPORT 68-15 (1968)

Average matrix element - factorization

Probability for an n-particle final state:

$$P_n \propto \int \cdots \int \prod_{i=1}^{n} \delta\left(p_i'^2 - m^2\right) d^4 p_i' \times \delta^4\left(\sum_{j=1}^{n} p_j' - p_1 - p_2\right) S\left(p_1' \ldots p_n' \mid p_1, p_2\right)$$

$$\equiv \bar{S}_n \underbrace{\int \cdots \int \prod_{i=1}^{n} \delta\left(p_i'^2 - m^2\right) d^4 p_i' \times \delta^4\left(\sum_{j=1}^{n} p_j' - p_1 - p_2\right)}_{R_n}$$

$$\equiv \bar{S}_n R_n$$

Single-particle spectrum

$$W\left(p_1'\right) d^3 p_1' \propto d^3 p_1' \int \cdots \int \delta\left(p_1'^2 - m^2\right) dp_{01} \prod_{i=2}^{n} \delta\left(p_i'^2 - m^2\right) d^4 p_i' \times$$

$$\delta^4\left(\sum_{j=1}^{n} p_j' - p_1 - p_2\right) S\left(p_1' \ldots p_n' \mid p_1, p_2\right)$$

$$\equiv d^3 p_1' \cdot \bar{S}_n\left(p_1'\right) R_F$$

R. Hagedorn, *Relativistic Kinematics* 1963

Correlations arising (only) from conservation laws (PS constraints)

$$\tilde{f}(p_i) = 2E_i \frac{dN}{d^3 p_i}$$

single-particle "parent" distribution
w/o P.S. restriction

what we measure

$$\tilde{f}_c(p_1,\ldots,p_k) \equiv \left(\prod_{i=1}^{k} \tilde{f}(p_i)\right) \cdot \frac{\int \left(\prod_{i=k+1}^{N} \frac{d^3 p_i}{2E_i} \tilde{f}(p_i)\right) \delta^4\left(\sum_{i=1}^{N} p_i - P\right)}{\int \left(\prod_{i=1}^{N} \frac{d^3 p_i}{2E_i} \tilde{f}(p_i)\right) \delta^4\left(\sum_{i=1}^{N} p_i - P\right)}$$

no other correlations

$$= \left(\prod_{i=1}^{k} \tilde{f}(p_i)\right) \cdot \frac{\int \left(\prod_{i=k+1}^{N} d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i)\right) \delta^4\left(\sum_{i=1}^{N} p_i - P\right)}{\int \left(\prod_{i=1}^{N} d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i)\right) \delta^4\left(\sum_{i=1}^{N} p_i - P\right)}$$

k-particle distribution (k<N) **with P.S. restriction**

Simplification for "large" N-k (1)

Numerator is the probability distribution of a sum of many (N-k) uncorrelated vectors

(i.e. the probability that they will add up to $P-\Sigma_{i=1}^{k} p_i$)

If (N-k) big →Multivariate Central Limit Theorem

$$\sum_{i=k+1}^{N} p_i - \left(P - \sum_{i=1}^{k} p_i \right)$$

$$= \left(\prod_{i=1}^{k} \tilde{f}(p_i) \right) \cdot \frac{\int \left(\prod_{i=k+1}^{N} d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^{N} p_i - P \right)}{\int \left(\prod_{i=1}^{N} d^4 p_i \delta(p_i^2 - m_i^2) \tilde{f}(p_i) \right) \delta^4 \left(\sum_{i=1}^{N} p_i - P \right)}$$

Denominator is "just" a constant normalization (indep p_i)

Simplification for "large" N-k (2)

Numerator is the probability distribution of a sum of many (N-k) uncorrelated vectors

(i.e. the probability that they will add up to $P-\Sigma_{i=1}^{k} p_i$)

If (N-k) big →Multivariate Central Limit Theorem

$$\left(\prod_{i=1}^{k}\tilde{f}(p_i)\right)\cdot\frac{\int\left(\prod_{i=k+1}^{N}d^4p_i\delta(p_i^2-m_i^2)\tilde{f}(p_i)\right)\delta^4\left(\sum_{i=1}^{N}p_i-P\right)}{\int\left(\prod_{i=1}^{N}d^4p_i\delta(p_i^2-m_i^2)\tilde{f}(p_i)\right)\delta^4\left(\sum_{i=1}^{N}p_i-P\right)}$$

$$\xrightarrow{\text{large } N-k}$$

$$\left(\prod_{i=1}^{k}\tilde{f}(p_i)\right)\cdot\left(\frac{N}{N-k}\right)^2\exp\left[-\left(\sum_{i=1}^{k}\left(p_i^{\mu}-\langle p^{\mu}\rangle\right)\right)\frac{b_{\mu\nu}}{2(N-k)}\left(\sum_{i=1}^{k}\left(p_i^{\nu}-\langle p^{\nu}\rangle\right)\right)\right]$$

averages $\langle X\rangle\equiv\int d^4p\cdot X\cdot\delta\left(p^2-m^2\right)\tilde{f}(p)$

covariance matrix $\left(b^{-1}\right)_{\mu\nu}=\langle p_{\mu}p_{\nu}\rangle-\langle p_{\mu}\rangle\langle p_{\nu}\rangle$

Fortunately, diagonal covariance matrix!

$$\left(b^{-1}\right)_{\mu\nu} = \left\langle p_\mu p_\nu \right\rangle - \left\langle p_\mu \right\rangle \left\langle p_\nu \right\rangle$$

- ✓ Work in global C.O.M. frame: $\left\langle p^\mu \right\rangle = \delta_{\mu,0} \left\langle E \right\rangle$
- ✓ No elliptic flow, etc: $\left(b^{-1}\right)_{1,2} = \left\langle p_x p_y \right\rangle = 0$
- ✓ No directed flow, etc: $\left(b^{-1}\right)_{1,3} = \left(b^{-1}\right)_{2,3} = 0$
- ✓ On-shell means momentum and energy obviously correlated... but covariance (second moment) vanishes. For $i \neq 0$:

$$\left(b^{-1}\right)_{0,i} = \left\langle E p_i \right\rangle - \left\langle E \right\rangle \left\langle p_i \right\rangle = \left\langle E p_i \right\rangle = \frac{\int dE \int d^3p \cdot E\tilde{f}(p)\delta\left(p^2 - m\right) \cdot p_i}{\int dE \int d^3p \cdot \tilde{f}(p)\delta\left(p^2 - m\right)} = 0$$

since $E\tilde{f}(p)\delta\left(p^2 - m\right)$ is even and p_i is odd

Using central limit theorem ("large N-k")*

k-particle distribution in N-particle system

$$\tilde{f}_c(p_1,\ldots,p_k) = \left(\prod_{i=1}^{k} \tilde{f}(p_i)\right)\left(\frac{N}{N-k}\right)^2 \exp\left(-\sum_{\mu=0}^{3} \frac{\left(\sum_{i=1}^{k}\left(p_{i,\mu} - \langle p_\mu \rangle\right)\right)^2}{2(N-k)\sigma_\mu^2}\right)$$

where

$$\sigma_\mu^2 = \langle p_\mu^2 \rangle - \langle p_\mu \rangle^2$$

$$\langle p_\mu \rangle = 0 \quad \text{for} \quad \mu = 1,2,3$$

N.B.
relevant later

$$\langle p_\mu^2 \rangle \equiv \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}(p)}_{\substack{\text{unmeasured}\\ \text{parent distrib}}} \neq \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}_c(p)}_{\text{measured}}$$

–Danielewicz *et al*, PR**C38** 120 (1988)
–Borghini, Dinh, & Ollitraut PR**C62** 034902 (2000)
–Borghini Eur. Phys. J. C30:381-385, (2003)
–Chajecki & MAL, PRC **78** 064903 (2008)

* "large": N > ~10

Effects on single-particle distribution

$$\tilde{f}_c(p_i) = \tilde{f}(p_i)\left(\frac{N}{N-1}\right)^2 \exp\left(-\sum_{\mu=0}^{3}\frac{\left(p_{i,\mu} - \langle p_\mu \rangle\right)^2}{2(N-1)\sigma_\mu^2}\right)$$

$$= \tilde{f}(p_i)\left(\frac{N}{N-1}\right)^2 \exp\left(-\frac{1}{2(N-1)}\left(\frac{p_{x,i}^2}{\langle p_x^2 \rangle} + \frac{p_{y,i}^2}{\langle p_y^2 \rangle} + \frac{p_{z,i}^2}{\langle p_z^2 \rangle} + \frac{\left(E_i - \langle E \rangle\right)^2}{\langle E^2 \rangle - \langle E \rangle^2}\right)\right)$$

We will return to this….

in this case, the index i is only keeping track of particle type, really

k-particle correlation function

$$C(p_1,\ldots,p_k) \equiv \frac{\tilde{f}_c(p_1,\ldots,p_k)}{\tilde{f}_c(p_1)\ldots\tilde{f}_c(p_k)}$$

$$= \frac{\left(\frac{N}{N-k}\right)^2}{\left(\frac{N}{N-1}\right)^{2k}} \frac{\exp\left(-\frac{1}{2(N-k)}\sum_{i=1}^{k}\left(\frac{\left(\sum_{i=1}^{k} p_{x,i}\right)^2}{\langle p_x^2\rangle} + \frac{\left(\sum_{i=1}^{k} p_{y,i}\right)^2}{\langle p_y^2\rangle} + \frac{\left(\sum_{i=1}^{k} p_{z,i}\right)^2}{\langle p_z^2\rangle} + \frac{\left(\sum_{i=1}^{k}(E_i - \langle E\rangle)\right)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)}{\exp\left(-\frac{1}{2(N-1)}\sum_{i=1}^{k}\left(\frac{p_{x,i}^2}{\langle p_x^2\rangle} + \frac{p_{y,i}^2}{\langle p_y^2\rangle} + \frac{p_{z,i}^2}{\langle p_z^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)}$$

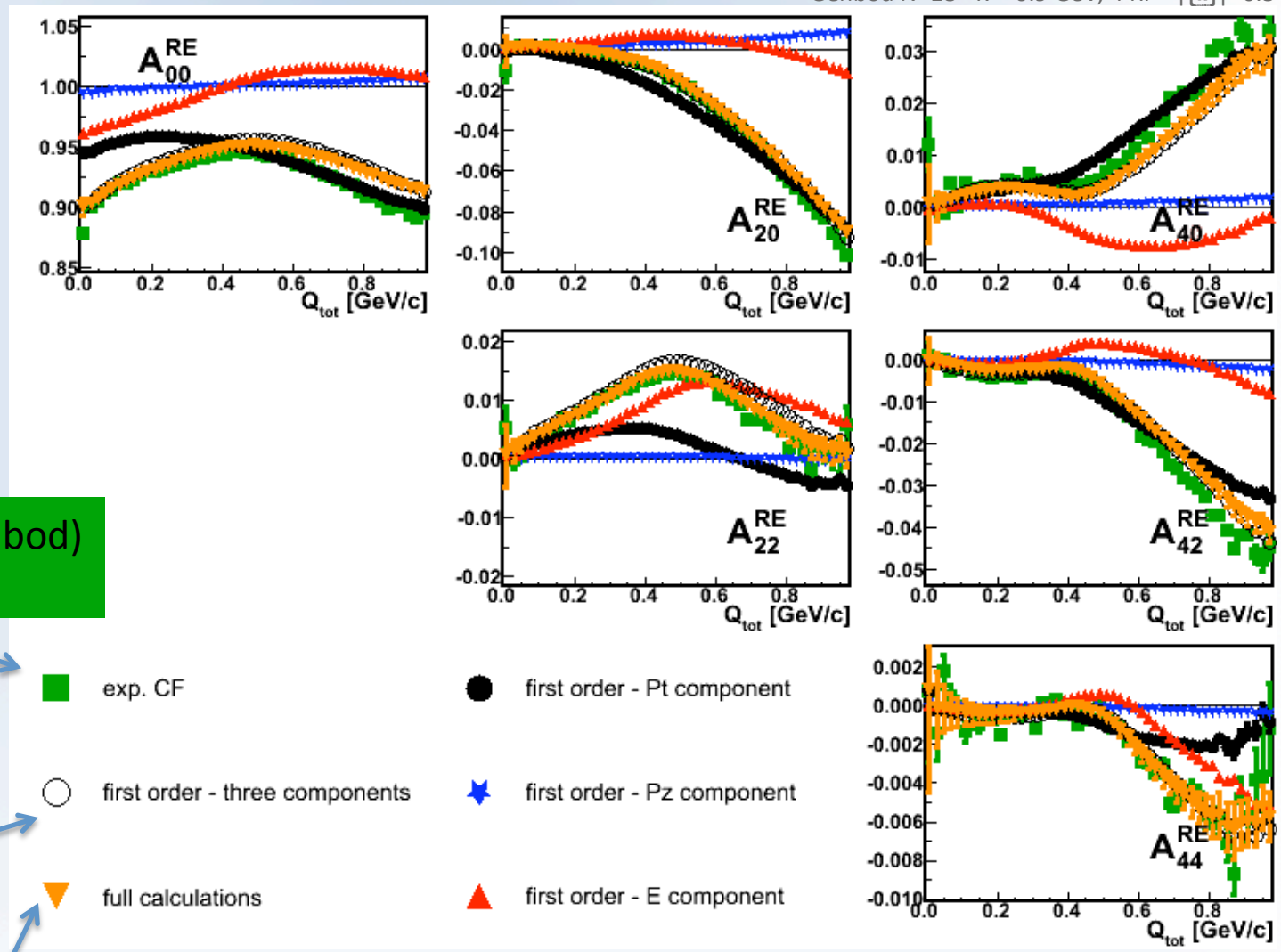
Dependence on "parent" distrib f vanishes,
except for energy/momentum means and RMS

2-particle correlation function (1st term in 1/N expansion)

$$C(p_1,p_2) \cong 1 - \frac{1}{N}\left(2\frac{\vec{p}_{T,1}\cdot\vec{p}_{T,2}}{\langle p_T^2\rangle} + \frac{p_{z,1}\cdot p_{z,2}}{\langle p_z^2\rangle} + \frac{(E_1 - \langle E\rangle)\cdot(E_2 - \langle E\rangle)}{\langle E^2\rangle - \langle E\rangle^2}\right)$$

How do EMCICs look ? – nontrivial !

Genbod N=18 <K>=0.9 GeV; PRF - |[illegible]|<0.5



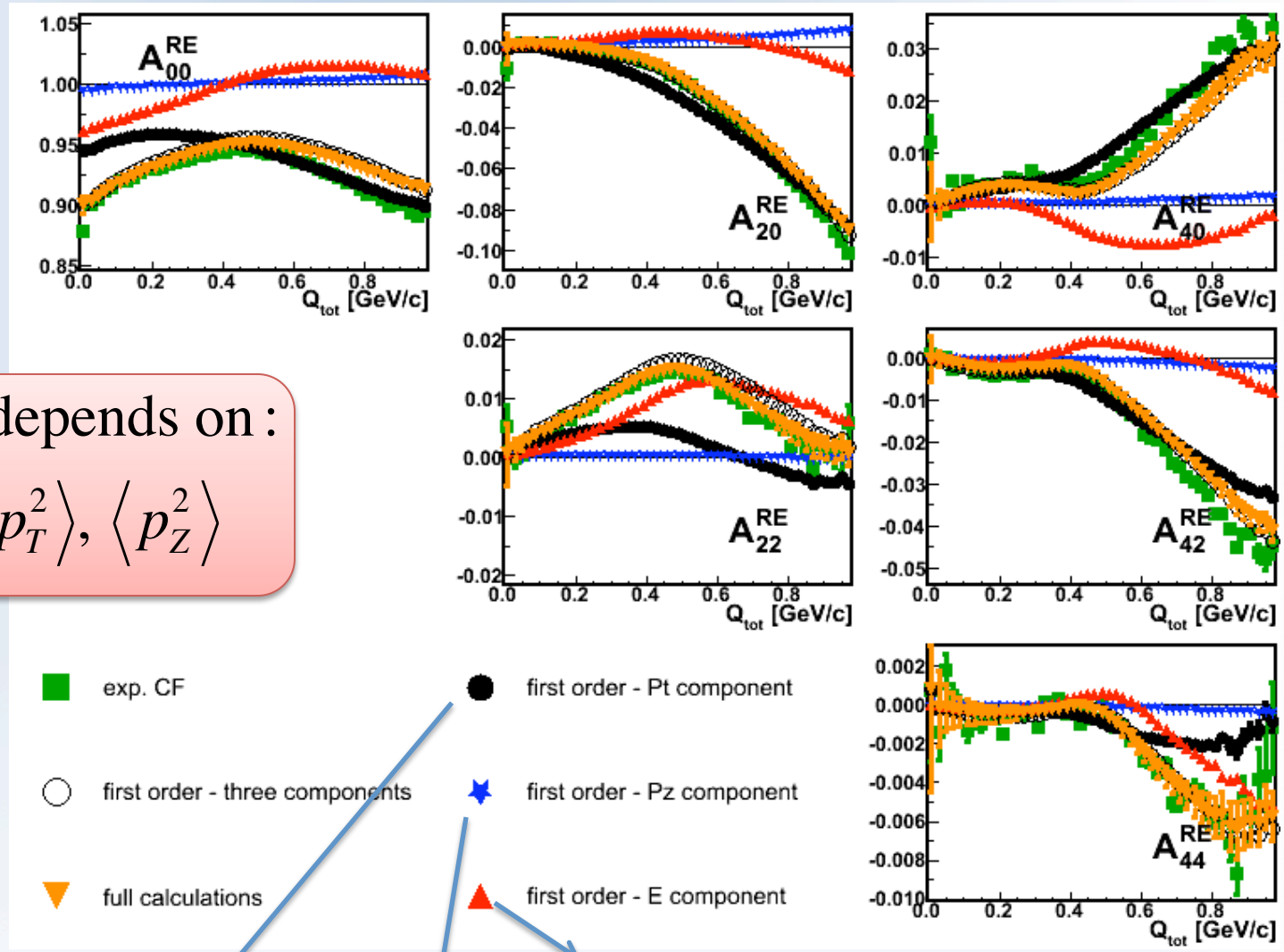
event generator (genbod) with only EMCICs

O(1/N) term in CLT approximation

CLT approximation

- **structure <u>not</u> confined to large Q**
- **kinematic cuts have strong effect**

How do EMCICs look ? – nontrivial !



Detailed shape depends on:

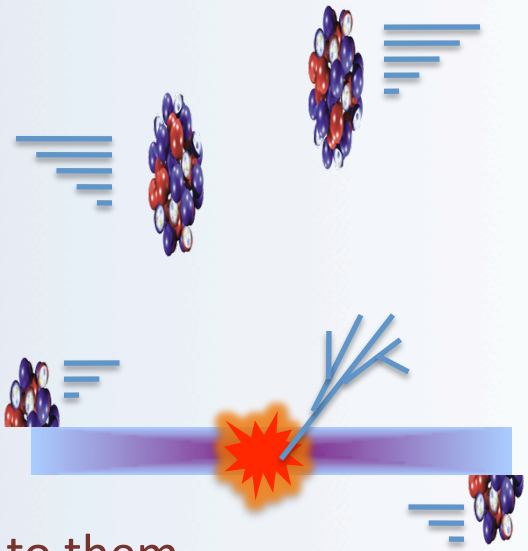
$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

$$C(p_1,p_2) \cong 1 - \frac{1}{N}\left(2\frac{\vec{p}_{T,1}\cdot\vec{p}_{T,2}}{\langle p_T^2\rangle} + \frac{p_{z,1}\cdot p_{z,2}}{\langle p_z^2\rangle} + \frac{(E_1-\langle E\rangle)\cdot(E_2-\langle E\rangle)}{\langle E^2\rangle - \langle E\rangle^2}\right)$$

"the system" ... a nontrivial concept

$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared



- Not known a priori
- should *track* measured quantities, but not be identical to them

1. *N* includes all primary particles (including unmeasured γ's etc)
2. secondary decay (resonances, fragmentation) smears connection b/t $\langle E^2 \rangle$ and measured one
3. $\langle E^2 \rangle$ etc: averages of the *parent* distribution

$$\langle p_\mu^2 \rangle \equiv \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}(p)}_{\text{unmeasured parent distrib}} \neq \int d^3p \cdot p_\mu^2 \cdot \underbrace{\tilde{f}_c(p)}_{\text{measured}}$$

4. "relevant system" almost certainly not the "whole" (4π) system
   - e.g. beam fragmentation probably not relevant to system emitting at midrapidity
     - characteristic physical processes (strings etc): Δy ~ 1÷2
   - jets: "of the system" ??
     - or just stealing energy *from* "the system?"
   - if "relevant system" ≠ "whole system", then total energy-momentum ***will fluctuate*** e-by-e

The underlying event appears indep of jet – CDF

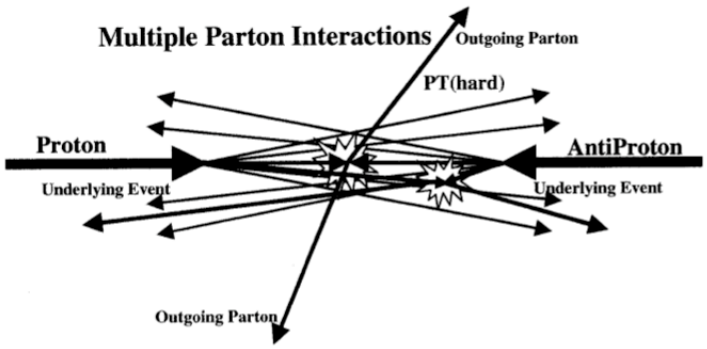
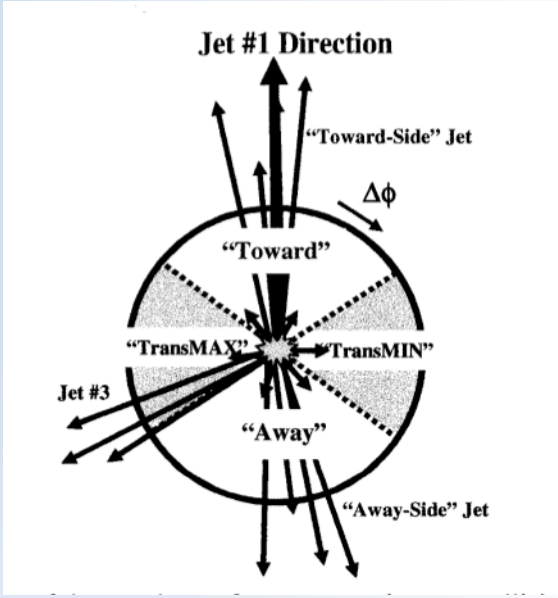
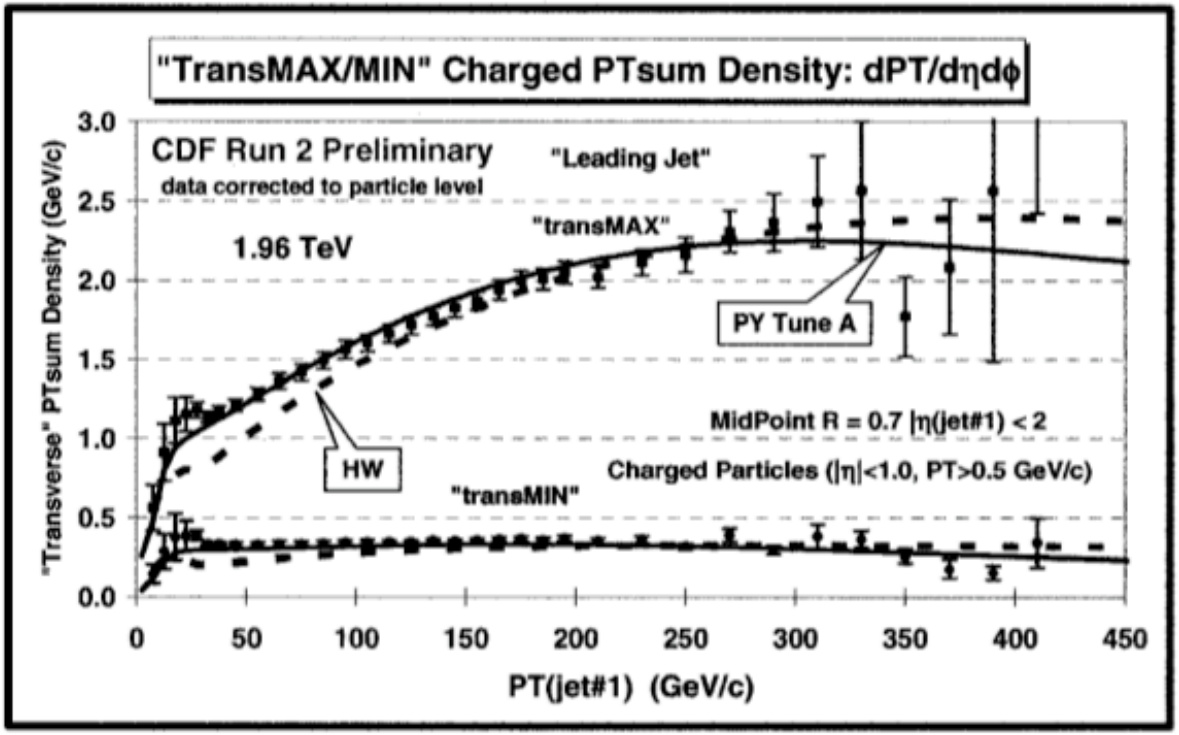


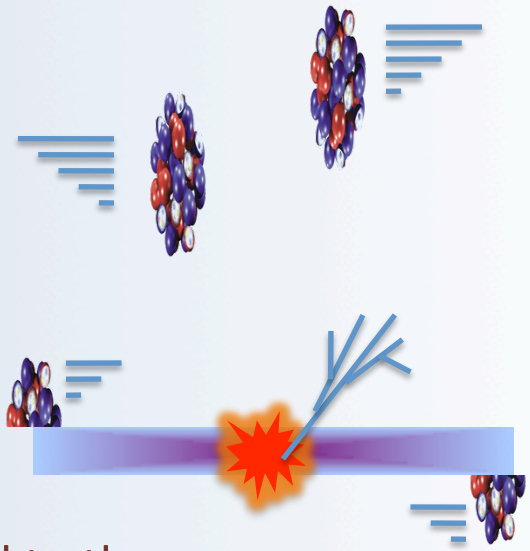
Fig. 1-5. Illustration of the way PYTHIA models the "underlying event" in proton-antiproton collision by including multiple parton interactions. In addition to the hard 2-to-2 parton-parton scattering with transverse momentum, P_T(hard), there is a second "semi-hard" 2-to-2 parton-parton scattering that contributes particles to the "underlying event".





L.A. Cruz, PhD Thesis, U. Florida
"Underlying event at Tevatron" (2005)

"the system"... a nontrivial concept



$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared

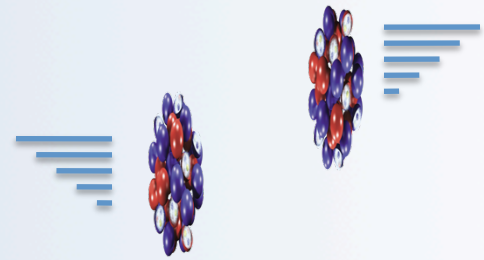
- Not known a priori
- should *track* measured quantities, but not be identical to them

- We will treat them as parameters: what to expect?

Maxwell - Boltzmann parent $\dfrac{d^3N}{d^3p} \sim e^{-E/T}$

	non - rel	ultra - rel	if $T = .15 \div .35$
$\langle p_T^2 \rangle$	$2mT$	$8T^2$	$0.045 \div 0.98\ (\mathrm{GeV/c})^2$
$\langle E^2 \rangle$	$\frac{15}{4}T^2 + m^2$	$12T^2$	$0.10 \div 1.5\ \mathrm{GeV}^2$
$\langle E \rangle$	$\frac{3}{2}T + m$	$3T$	$0.36 - 1\ \mathrm{GeV}$

"the system" ... a nontrivial concept



$$N, \langle E \rangle, \langle E^2 \rangle, \langle p_T^2 \rangle, \langle p_Z^2 \rangle$$

Characteristic scales of relevant system in which limited energy-momentum is share

- Not known a priori
- should *track* measured quantities, b
- What to expect?

Blastwave, $T = 100$ MeV $\rho_0 = 0.9$

$\langle p_T^2 \rangle_\pi = 0.240$ GeV^2 $\left(\langle p_T \rangle_\pi = 0.405 \text{ GeV}\right)$

$\langle m_T \rangle_\pi = 0.435$ GeV

$\langle m_T^2 \rangle_\pi = 0.259$ GeV^2

Maxwell - Boltzmann parent $\frac{d^3N}{d^3p} \sim e^{-E/}$

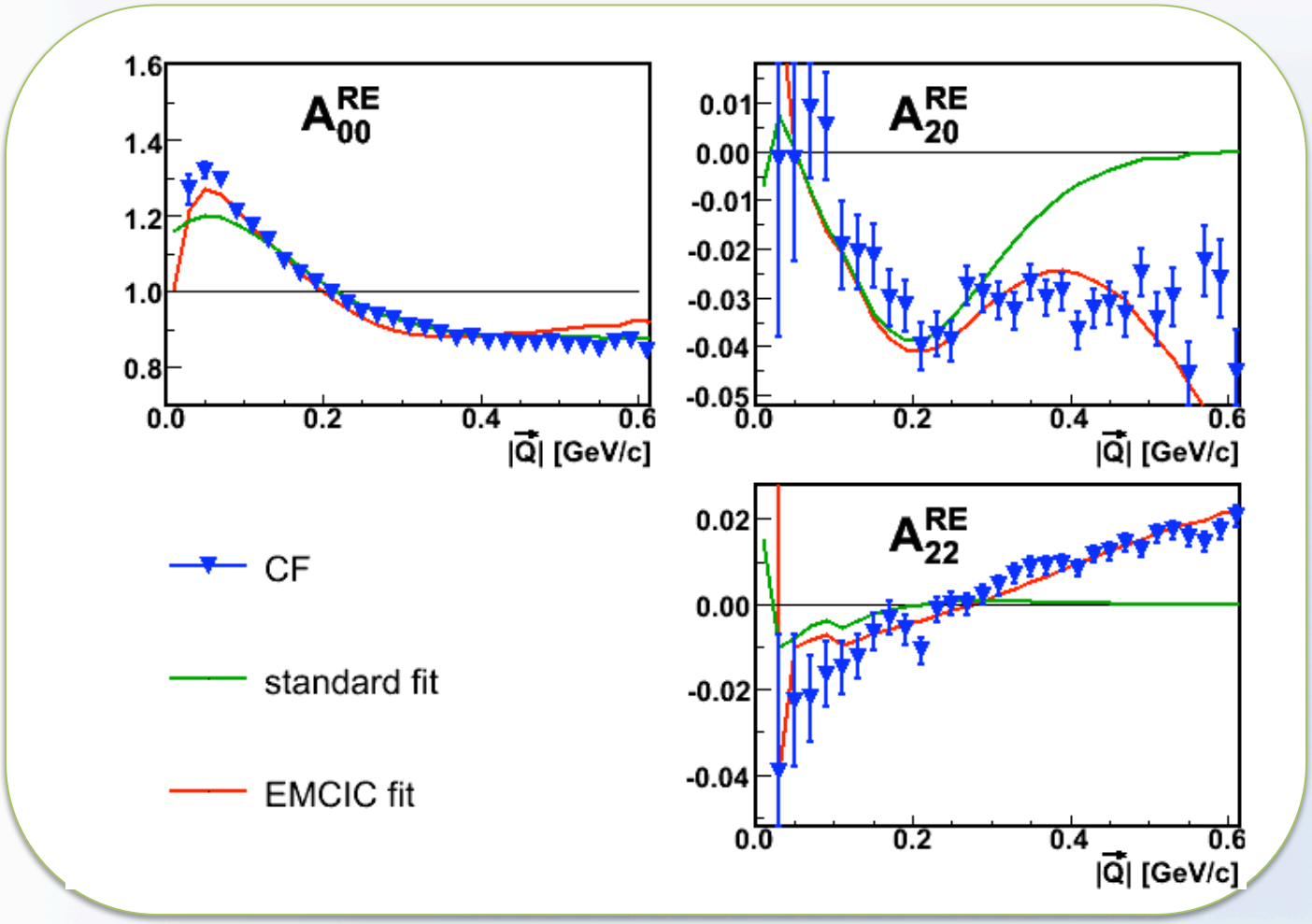
	non - rel	ultra - rel	if $T = .15$ -
$\langle p_T^2 \rangle$	$2mT$	$8T^2$	$0.045 \div 0.$
$\langle E^2 \rangle$	$\frac{15}{4}T^2 + m^2$	$12T^2$	$0.10 \div 1.5$
$\langle E \rangle$	$\frac{3}{2}T + m$	$3T$	$0.36 - 1$ G

η_{max}	$\langle N \rangle$	$\langle p_T^2 \rangle_c$	$\langle p_z^2 \rangle_c$	$\langle E^2 \rangle_c$	$\langle E \rangle_c$
1.0	16	0.20	0.11	0.40	0.44
2.0	29	0.21	0.76	1.05	0.68
3.0	39	0.21	3.5	3.8	1.2
4.0	47	0.21	24	25	2.2
5.0	51	0.22	88	89	3.7

TABLE I: For a given selection on pseudorapidity $|\eta| < \eta_{max}$, the number and kinematic variables for primary particles from a PYTHIA simulation of $p+p$ collisons at $\sqrt{s_{NN}} = 200$ GeV are given. Units are GeV/c or (GeV/c)^2, as appropriate.

p+p minbias

Z. Chajecki, WPCF 2008

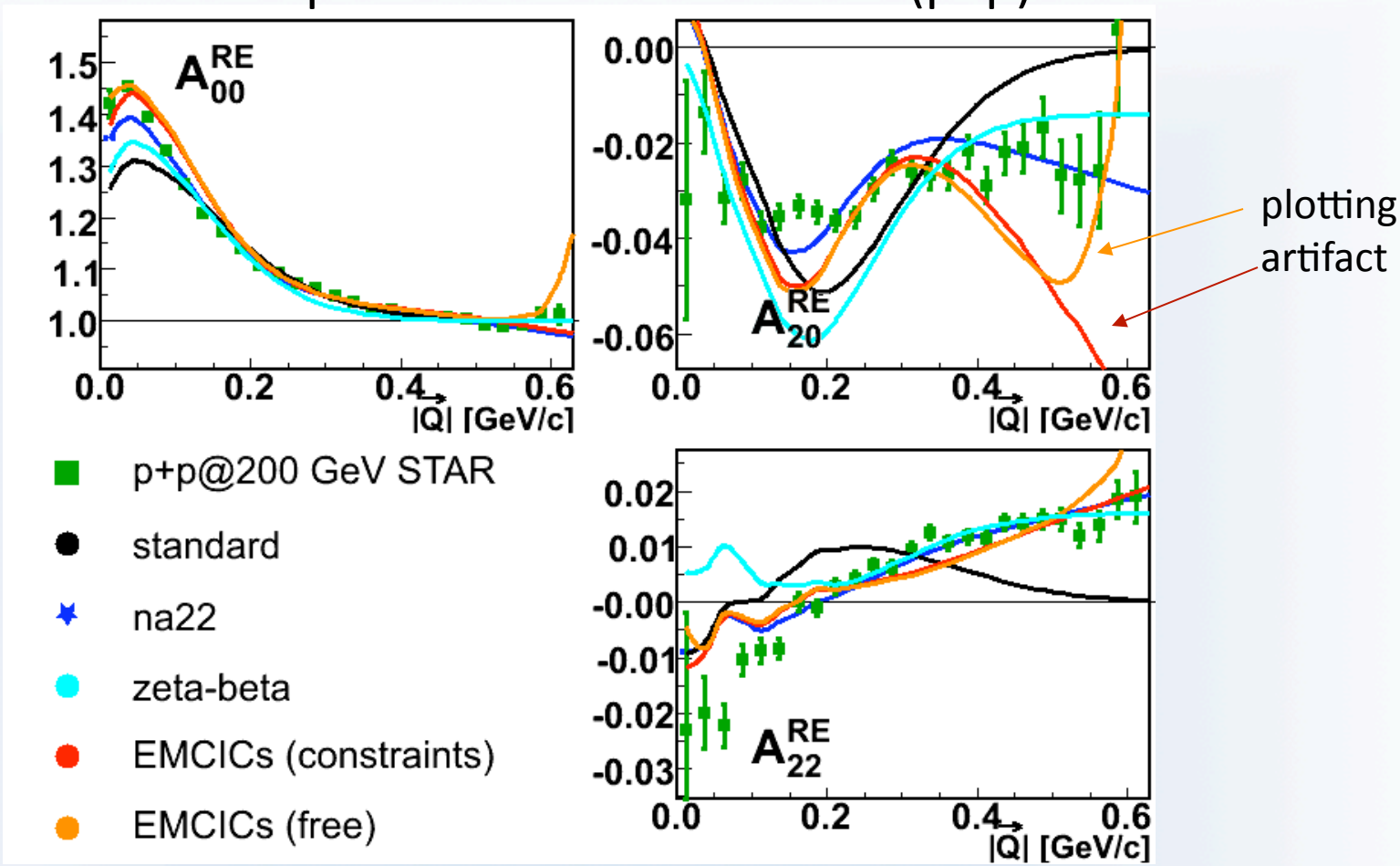


λ = 0.38 ± 0.01
R_o = 0.81 ± 0.01 fm
R_s = 0.84 ± 0.02 fm
R_l = 1.29 ± 0.02 fm

λ = 0.69 ± 0.01
R_o = 0.96 ± 0.04 fm
R_s = 0.98 ± 0.03 fm
R_l = 1.26 ± 0.02 fm

N = 13.6
$\langle E \rangle$ = 0.68 GeV
$\langle E^2 \rangle$ = 0.54 GeV^2
$\langle p_T^2 \rangle$ = 0.17 GeV^2
$\langle p_z^2 \rangle$ = 0.33 GeV^2

Various fits to the pion correlation function (p+p)



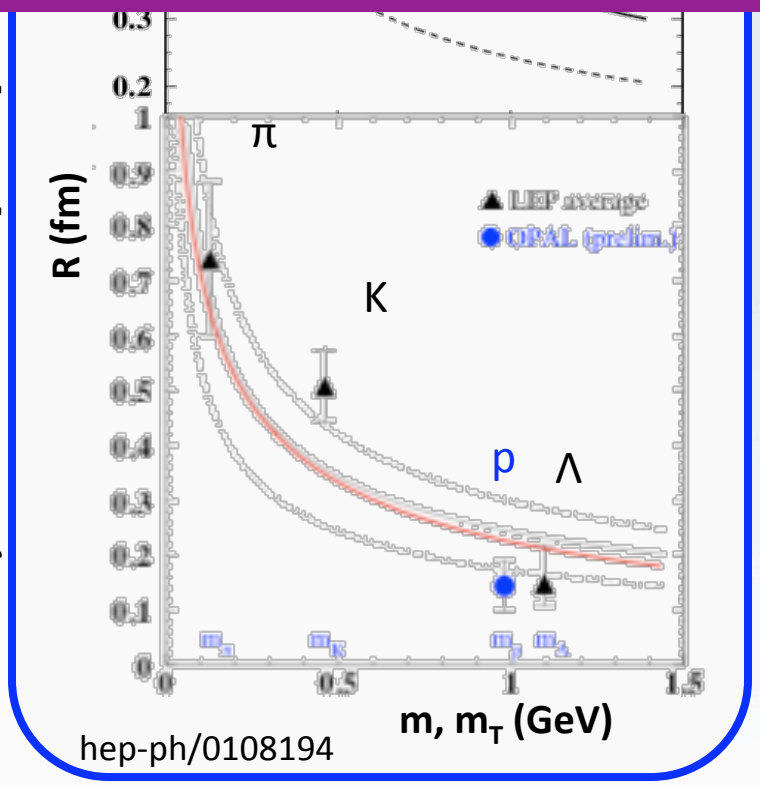
fit method	R_{out} [fm]	R_{side} [fm]	R_{long} [fm]
standard	0.65 +/- 0.01	0.85 +/- 0.01	1.42 +/- 0.02
"NA22"	1.18 +/- 0.02	1.05 +/- 0.02	1.75 +/- 0.03
"zeta-beta"	1.01 +/- 0.03	0.79 +/- 0.03	1.52 +/- 0.05
EMCICs (constr.)	1.05 +/- 0.02	1.06 +/- 0.02	1.66 +/- 0.03
EMCICs(free)	1.06 +/- 0.02	1.08 +/- 0.02	1.69 +/- 0.03

Zbigniew Chajecki QM05

femtoscopy in p+p @ STAR

1. **Heisenberg uncertainty?**
2. **String fragmentation? (Lund)**
3. **Resonance effects?**
4. **Flow???**

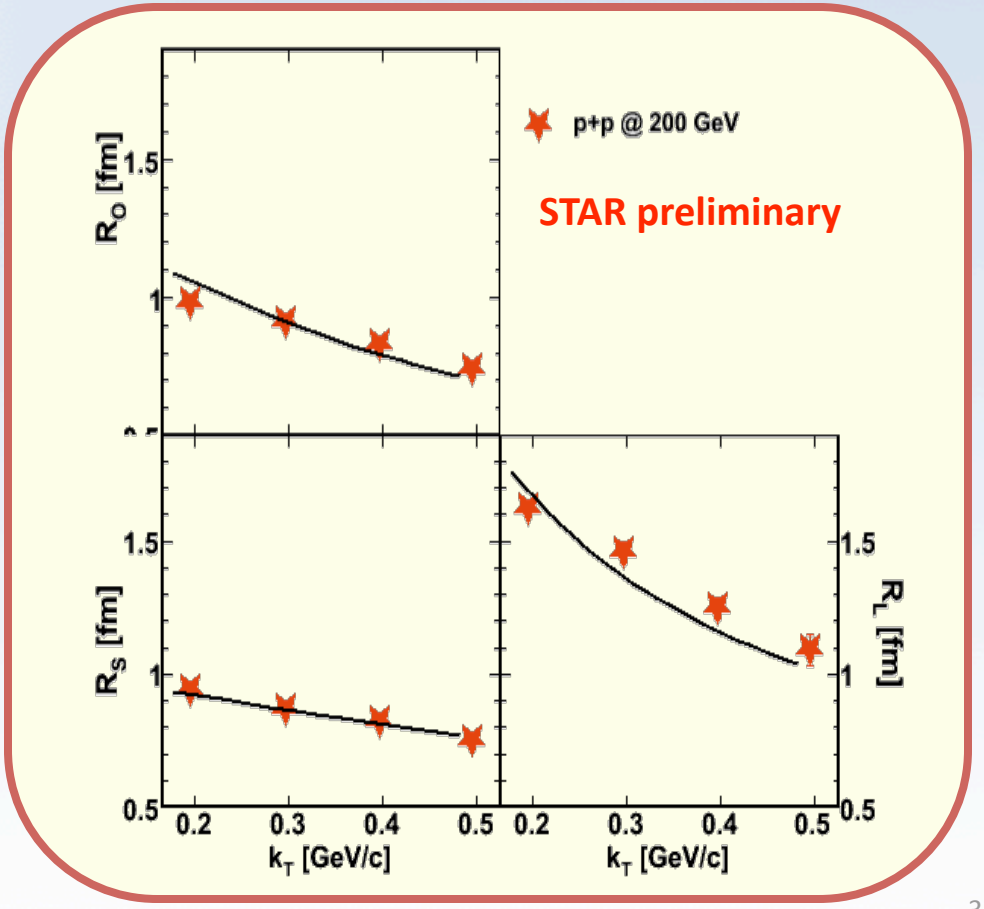
- Increasingly suggested in HEP experiments



c.f. Z. Chajecki arXiv:0901.4078 [nucl-ex]

p+p and A+A measured in *same* experiment, *same* acceptance, *same* techniques

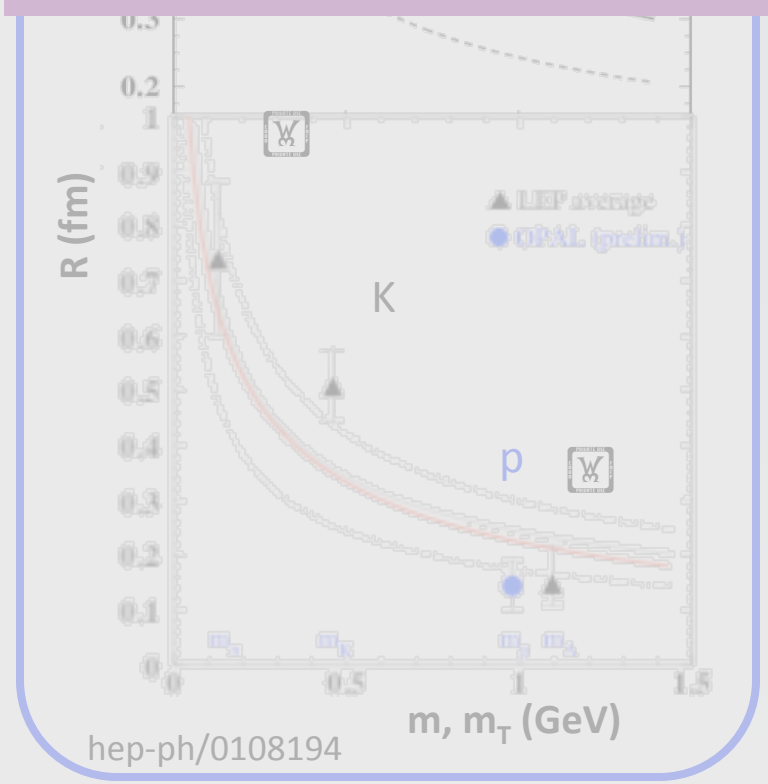
- unique opportunity to compare physics
- what causes p_T-dependence in p+p?
- same cause as in A+A?



femtoscopy in p+p @ STAR

1. Heisenberg uncertainty?
2. String fragmentation? (Lund)
3. Resonance effects?
4. Flow???

- Increasingly suggested in recent experiments



hep-ph/0108194

p+p and A+A measured in *same* experiment, *same* acceptance, *same* techniques

- unique opportunity to compare physics
- what causes p_T-dependence in p+p?

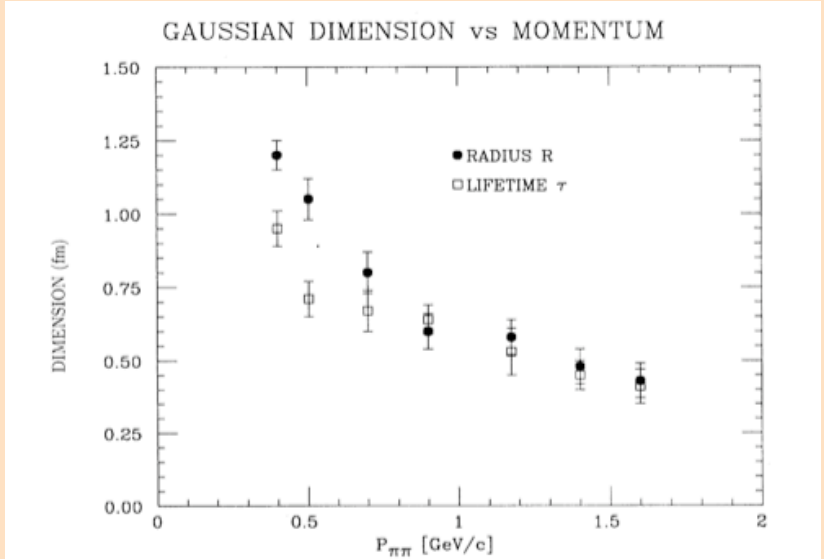


FIG. 9. Interaction "radius" and lifetime as a function of the total momentum $P_{\pi\pi}$ of the pion pair. R_G is primarily a source dimension along the beam direction. τ might possibly be interpreted as a source dimension transverse to the beam. Data are from Table III.

E735 Collaboration, PRD**48** 1931 (1993)
also PLB 2002
consistent with an expanding shell model.

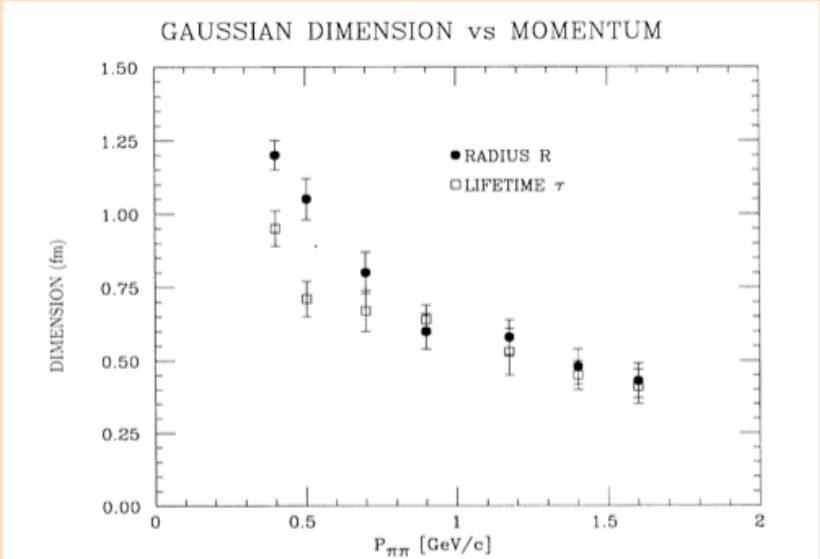
femtoscopy in p+p @ STAR

1. Heisenberg uncertainty?
2. String fragmentation? (Lund)
3. Resonance effects?
4. Flow???

- Increasingly suggested in recent experiments

p+p and A+A measured in *same* experiment, *same* acceptance, *same* techniques

- unique opportunity to compare physics
- what causes p_T-dependence in p+p?



ime as a function of the
$_G$ is primarily a source
ight possibly be inter-
to the beam. Data are

W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

... and suggests the existence of an important "collective flow", even in the system of particles produced in e+e− annihilation!

A 1/√m T scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a "transverse flow" even there!

8 1931 (1993)

ling shell model.

1. Heisenberg uncertainty?

2. String fragmentation? (Lund)

3. Resonance effects?

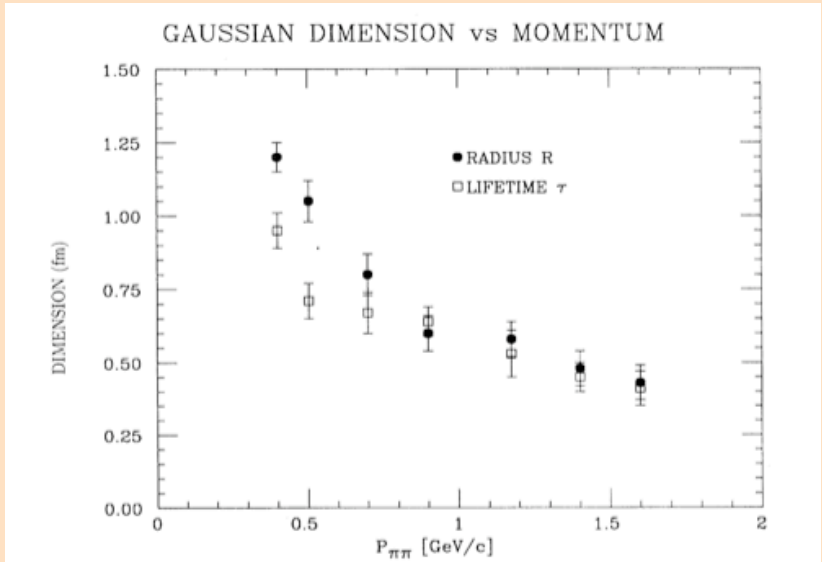
4. Flow???

- Increasingly suggested in recent experiments

NA22 Collaboration Z. Phys. C 71, 405–414 (1996) (hadron-hadron collisions)

[based on shape of C(q)…]

Our data do not confirm the expectation from the string type model… A good description of our data is, however, achieved in the framework of the hydrodynamical expanding source model.



ime as a function of the
$_G$ is primarily a source
ight possibly be inter-
to the beam. Data are

8 1931 (1993)

ling shell model.

W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

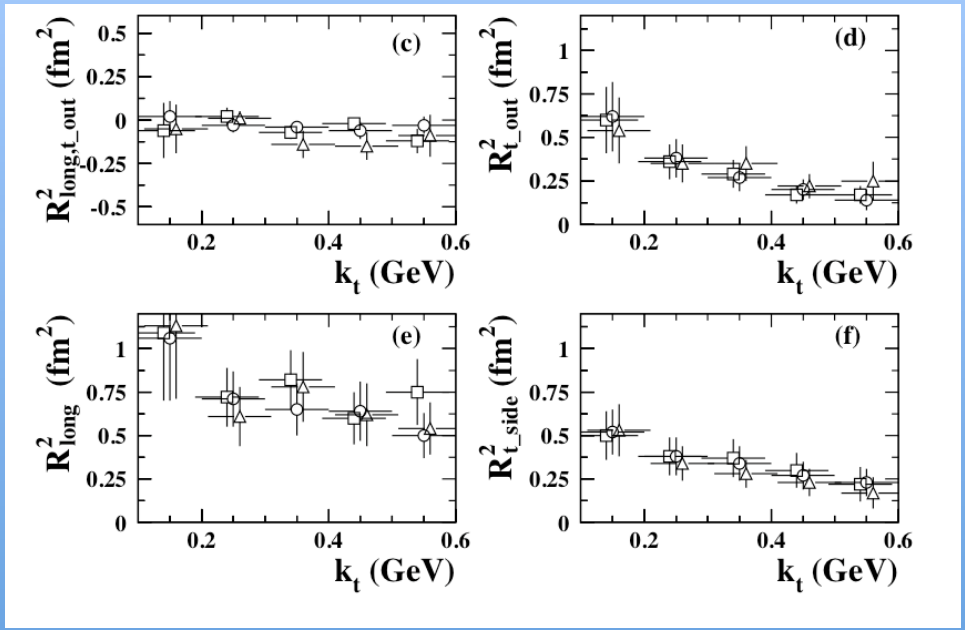
… and suggests the existence of an important "collective flow", even in the system of particles produced in e+e− annihilation!

A 1/√m T scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a "transverse flow" even there!

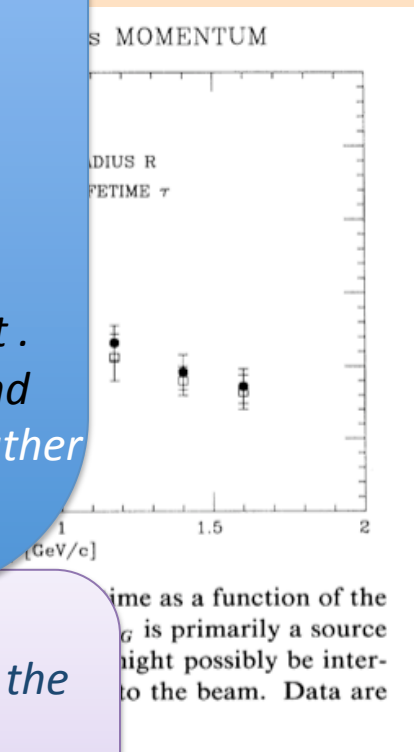
C 71, 405–414 (1996)

pectation from
description of
in the framework
ng source model.

OPAL Collaboration, Eur.Phys.J.C52:787-803,2007; arXiv:0708.1122 [hep-ex]



R2tside , R2tout and, less markedly, R2long decrease with increasing kt . The presence of correlations between the particle production points and their momenta is an indication that the pion source is not static, but rather expands during the particle emission process.



W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

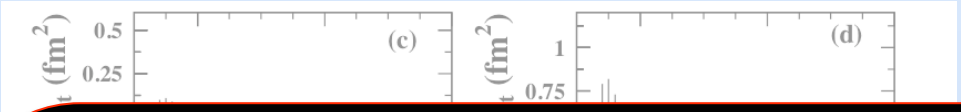
… and suggests the existence of an important "collective flow", even in the system of particles produced in e+e− annihilation!

A 1/√m T scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a "transverse flow" even there!

8 1931 (1993)

ling shell model.

OPAL Collaboration, Eur.Phys.J.C52:787-803,2007; arXiv:0708.1122 [hep-ex]



C 71, 405–414 (1996)

R2ts
The p
their
expands

RHIC: "comparison machine"

Vary size. All else fixed. [acceptance, technique…]

- spectra
- femtoscopy

compare with a system we "know" is flowing

W. Kittel Acta Phys.Polon. B32 (2001) 3927 [Review article]

… and suggests the existence of an important "collective flow", even in the system of particles produced in e+e− annihilation!

A 1/√m T scaling first observed in heavy-ion collisions is now also observed in Z fragmentation and may suggest a "transverse flow" even there!

8 1931 (1993)

ling shell model.

OPAL Collaboration, Eur.Phys.J.C52:787-803

RHIC: "comparison m

Vary size. All else fixe

- spectra
- femtoscopy

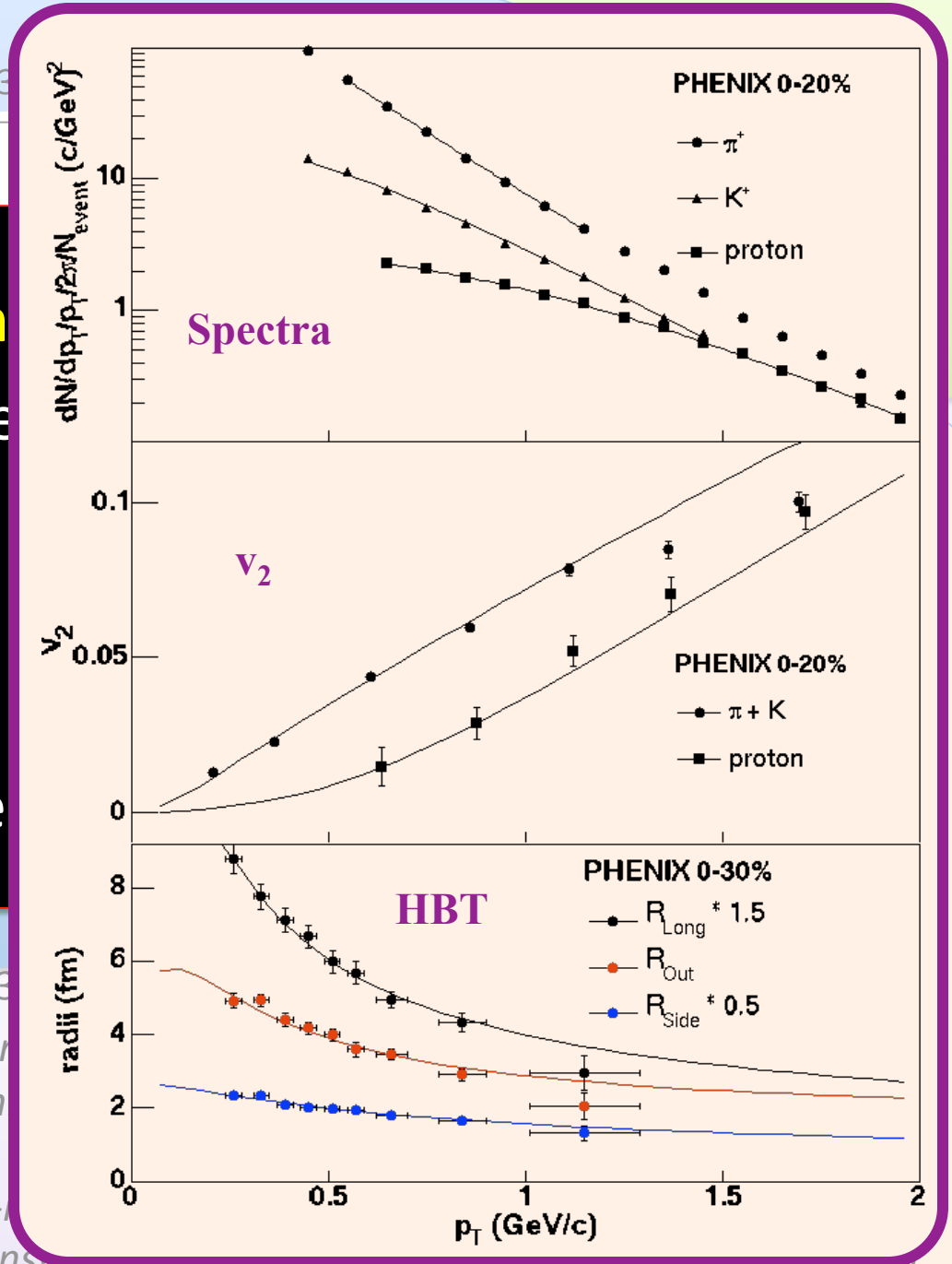
compare with a syste

R2ts
The p
their
expands

W. Kittel Acta Phys.Polon. B32 (2001) 3

... and suggests the existence of an impor
system of particles produced in e+e− ann

A 1/√m T scaling first observed in heavy-
Z fragmentation and may suggest a "transverse flow" even there.



Apples:apples comparison…

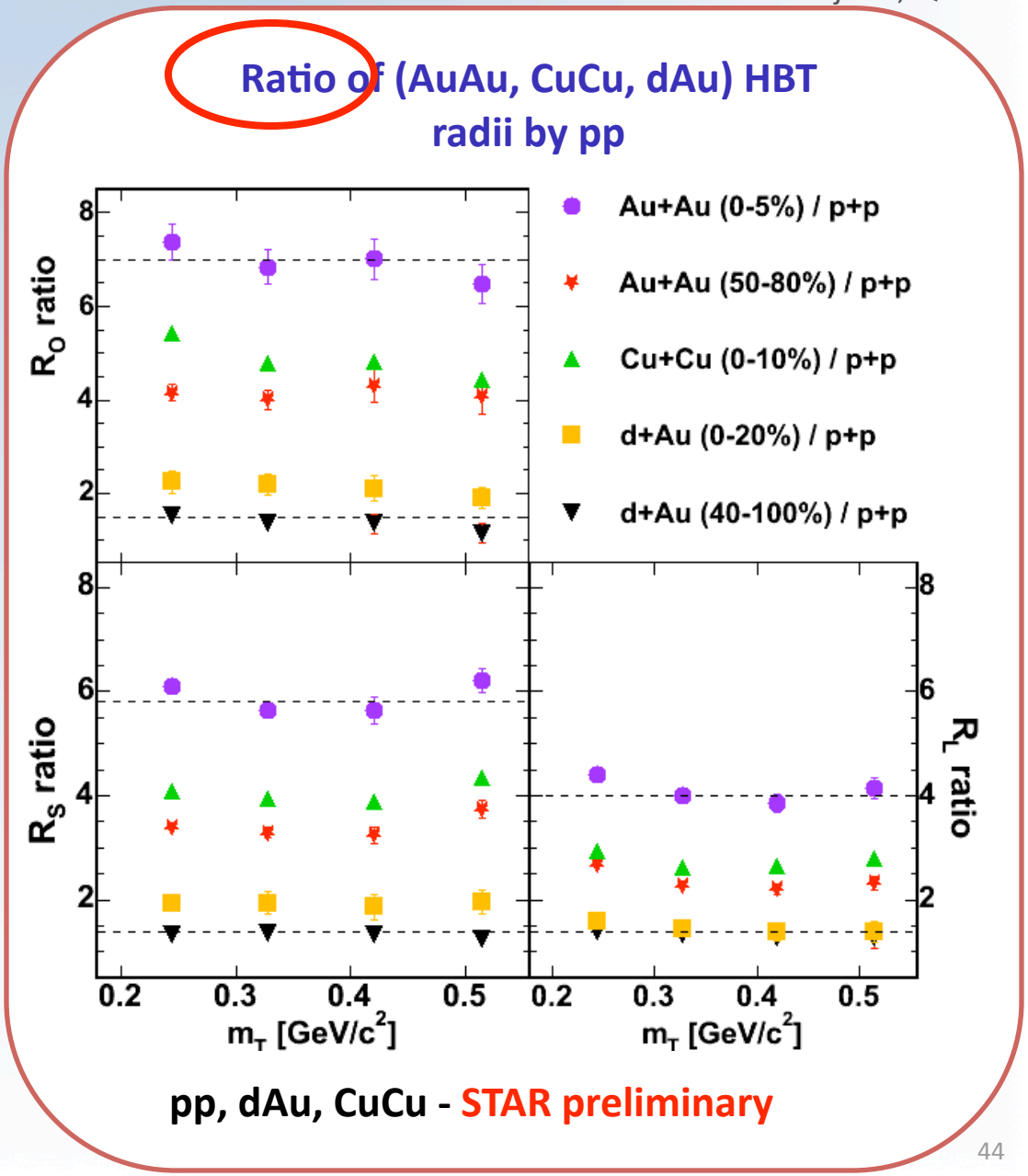
Z. Chajecki, QM05

$R(p_T)$ taken as strong space-time evidence of flow in Au+Au

- clear, quantitative consistency predictions of BlastWave

"Identical" signal seen in p+p

- cannot be of "identical" origin? (other than we "know it cannot"…)

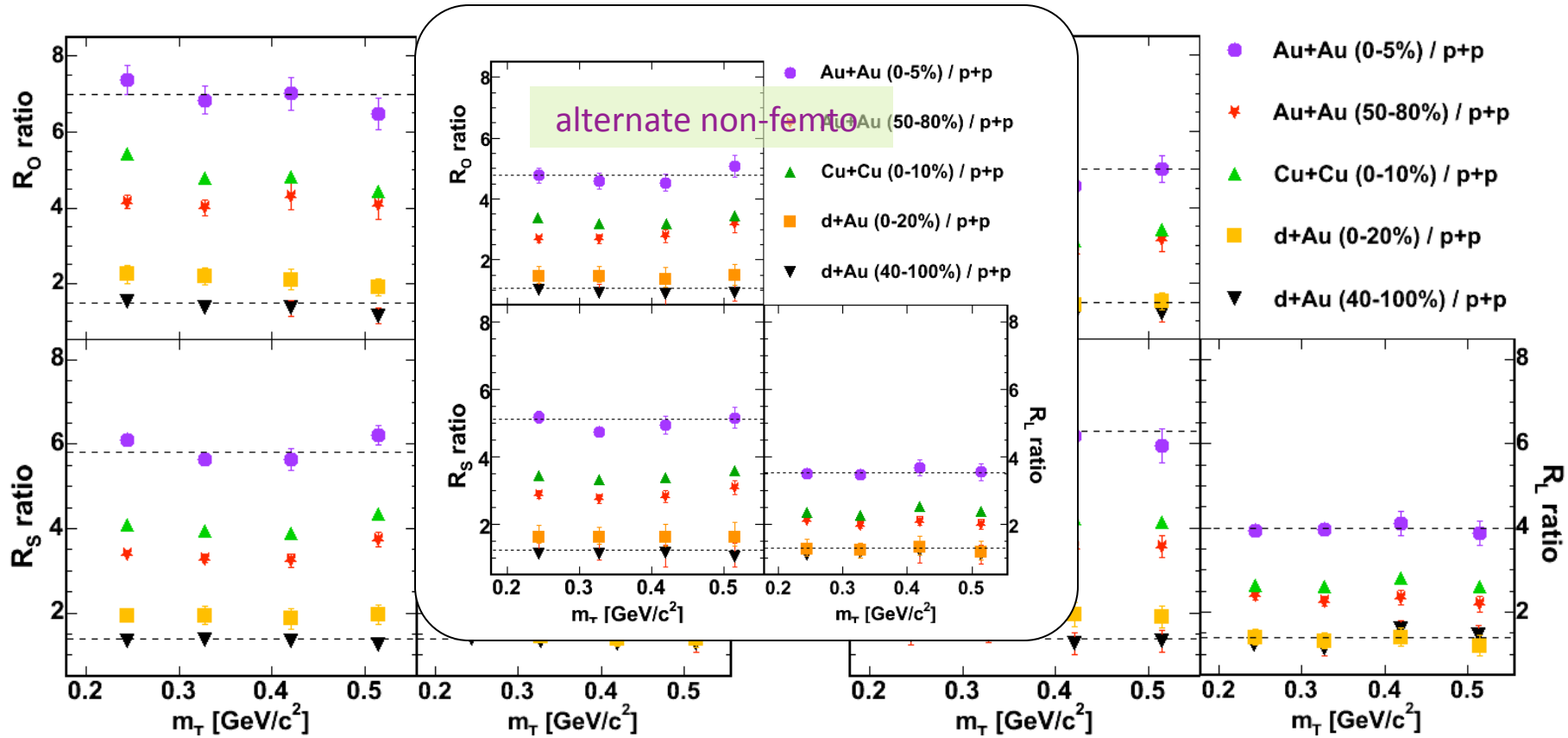


Significant non-femto correlations, but little effect on "message"

STAR preliminary

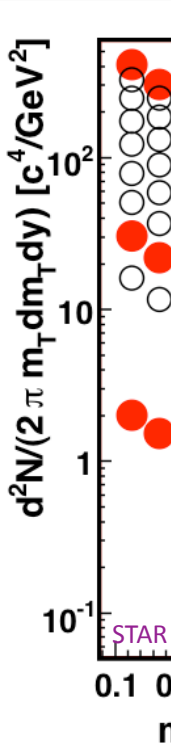
rather, "suggestion": explosive flow in p+p?

radii by pp



Fit w/o baseline parameterization

NEW fit w/ baseline parameterization



NO…..?

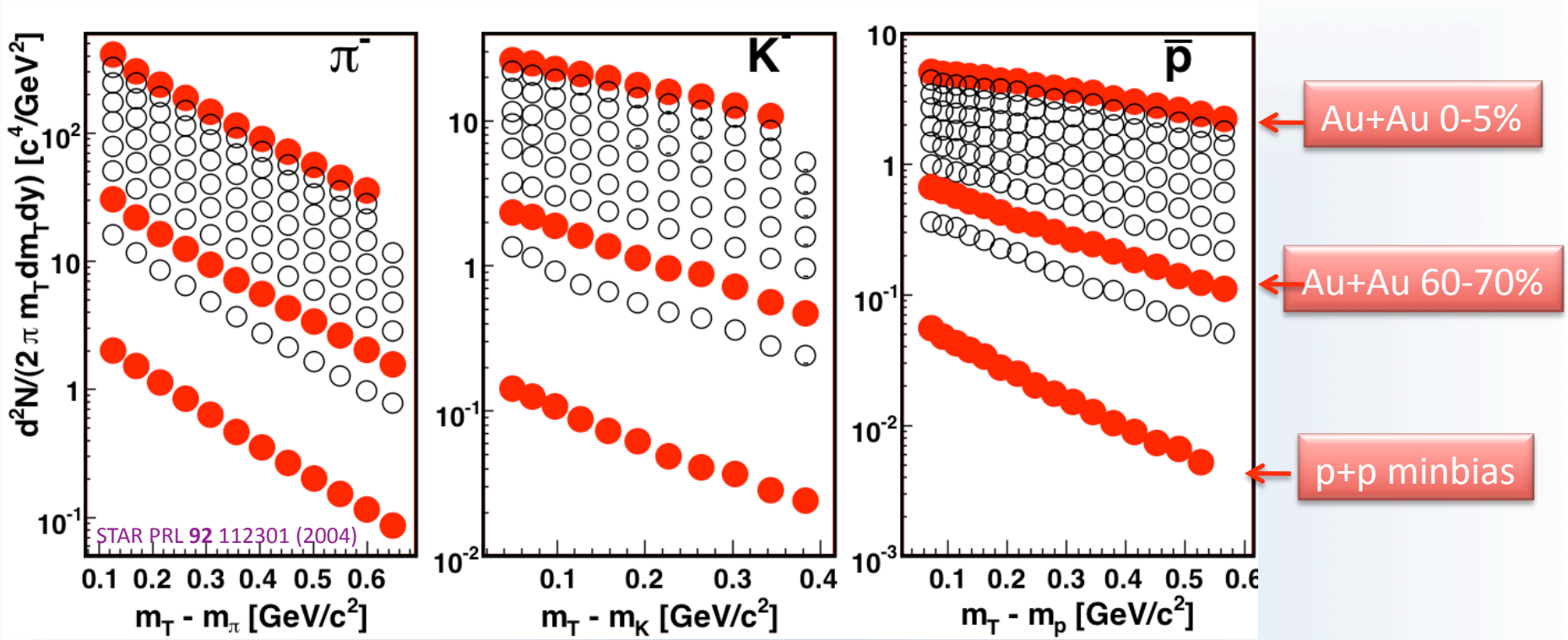
Au+Au 0-5%

Au+Au 60-70%

p+p minbias

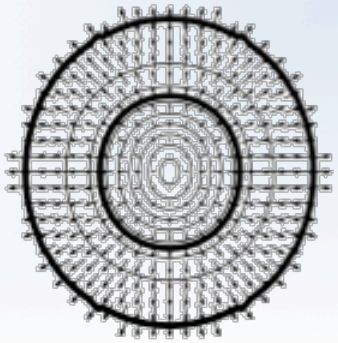
Blast-wave

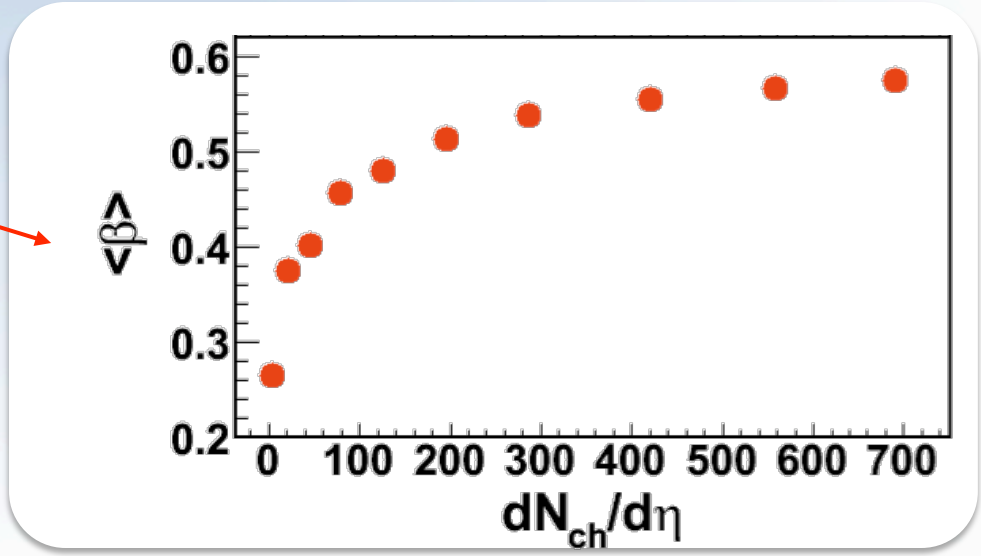
- much les



Blast-wave fit to spectra:

- much less explosive flow in p+p collisions





Don't forget - EMCICs even for k=1

measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i)\underbrace{\left(\frac{N}{N-1}\right)^2 \exp\left(-\frac{1}{2(N-1)}\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{p_{z,i}^2}{\langle p_z^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)}_{\text{“distortion” of single-particle spectra}}$$

“matrix element”

$$N, \langle E\rangle, \langle E^2\rangle, \langle p_T^2\rangle, \langle p_Z^2\rangle$$

Characteristic scales of relevant system in which limited energy-momentum is shared

EMCICs even for k=1

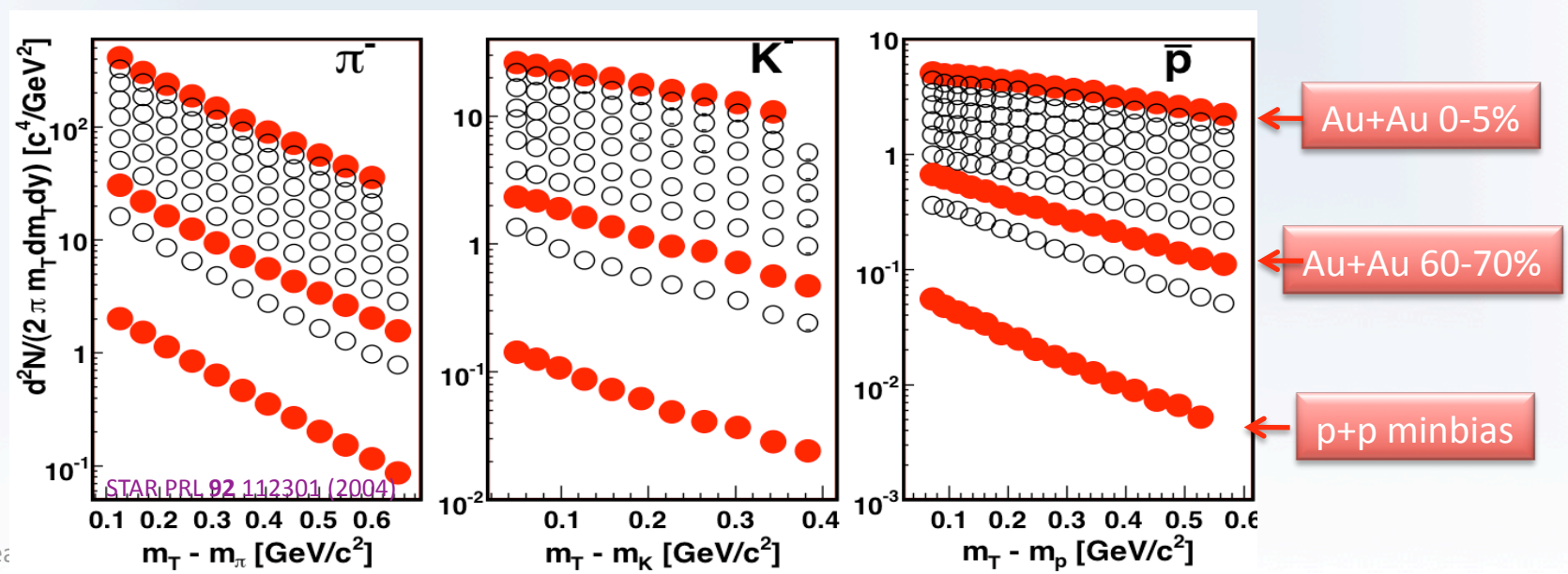
measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i)\underbrace{\left(\frac{N}{N-1}\right)^2 \exp\left(-\frac{1}{2(N-1)}\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{p_{z,i}^2}{\langle p_z^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)}_{\text{“distortion” of single-particle spectra}}$$

“matrix element”

What if the only difference between p+p and A+A collisions was *N*?

$$\text{same } \tilde{f}(p)\ ,\ \langle p_T^2\rangle\ ,\ \langle E\rangle\ ,\ \langle E^2\rangle$$



EMCICs even for k=1

measured

$$\tilde{f}_c(p_i) = \tilde{f}(p_i)\underbrace{\left(\frac{N}{N-1}\right)^2 \exp\left(-\frac{1}{2(N-1)}\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{p_{z,i}^2}{\langle p_z^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)}_{\text{“distortion” of single-particle spectra}}$$

“matrix element”

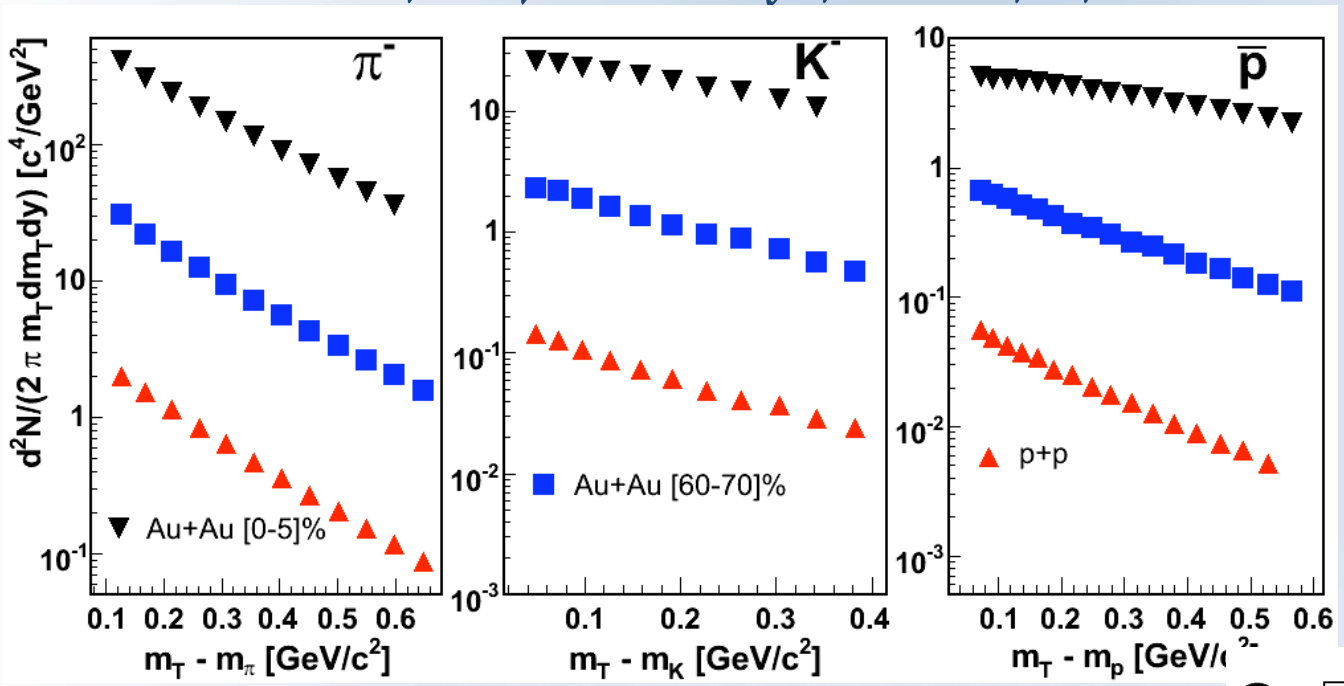
What if the only difference between p+p and A+A collisions was *N*?

$$\text{same } \tilde{f}(p)\ ,\ \langle p_T^2\rangle\ ,\ \langle E\rangle\ ,\ \langle E^2\rangle$$

Then we would measure:

$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} = \left(\frac{(N_{AA}-1)N_{pp}}{(N_{pp}-1)N_{AA}}\right)^2 \exp\left(\left(\frac{1}{2(N_{AA}-1)} - \frac{1}{2(N_{pp}-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)$$

Multiplicity evolution of spectra - p+p to A+A (soft sector)

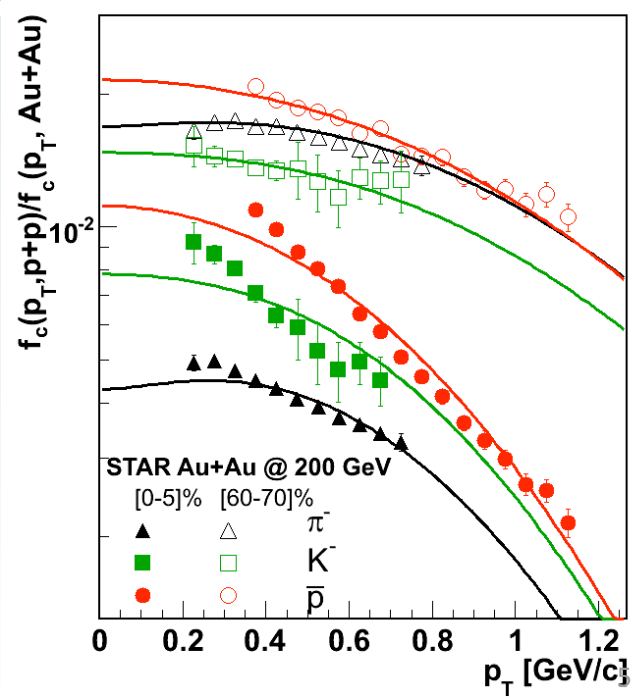


$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} \propto \exp\left(\left(\frac{1}{2(N_{AA}-1)} - \frac{1}{2(N_{pp}-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)$$

N evolution of spectra dominated by PS "distortion"

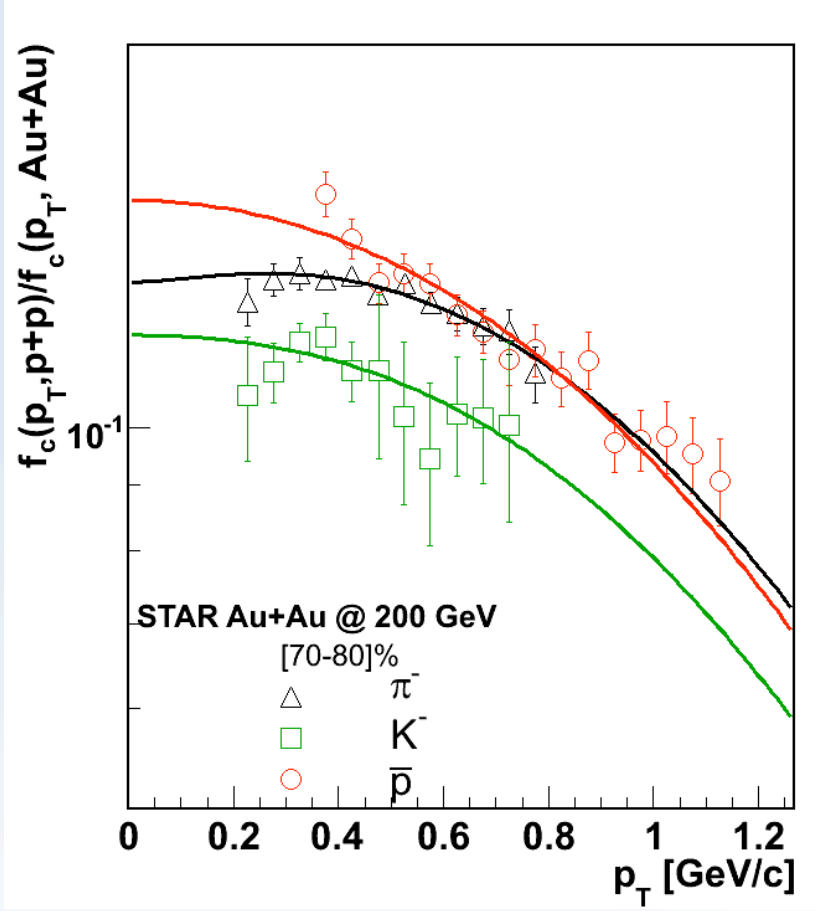
p+p system samples *same* parent distribution, but under stronger PS constraints

K ~ unity. driven by conservation of discrete quantum #s (strangeness, etc)



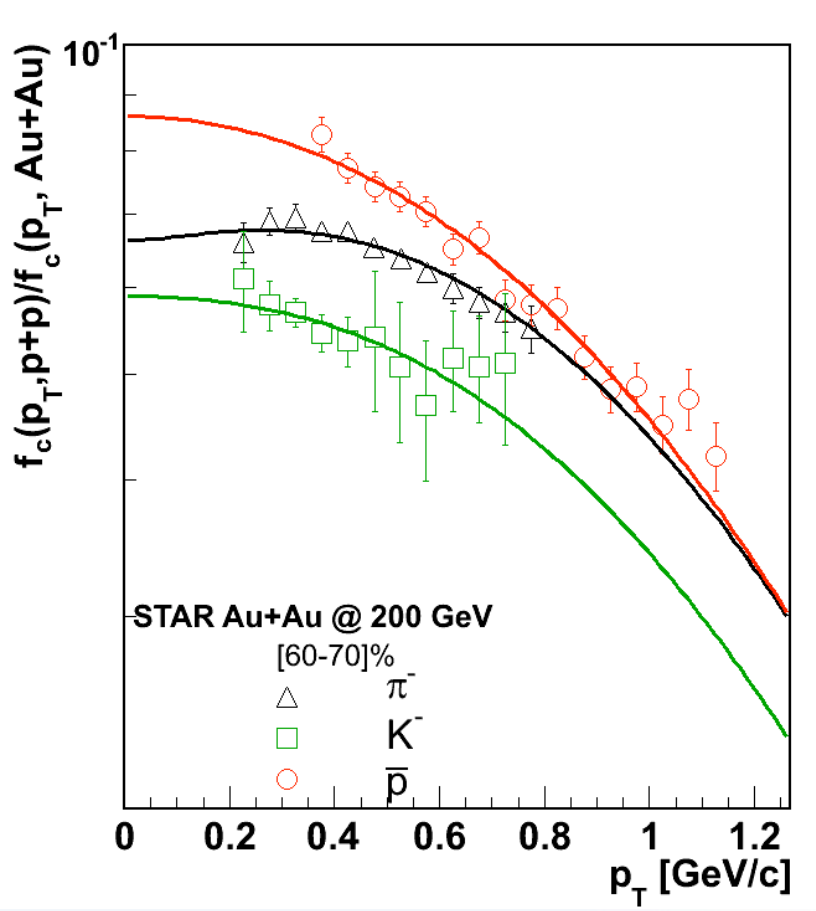
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"



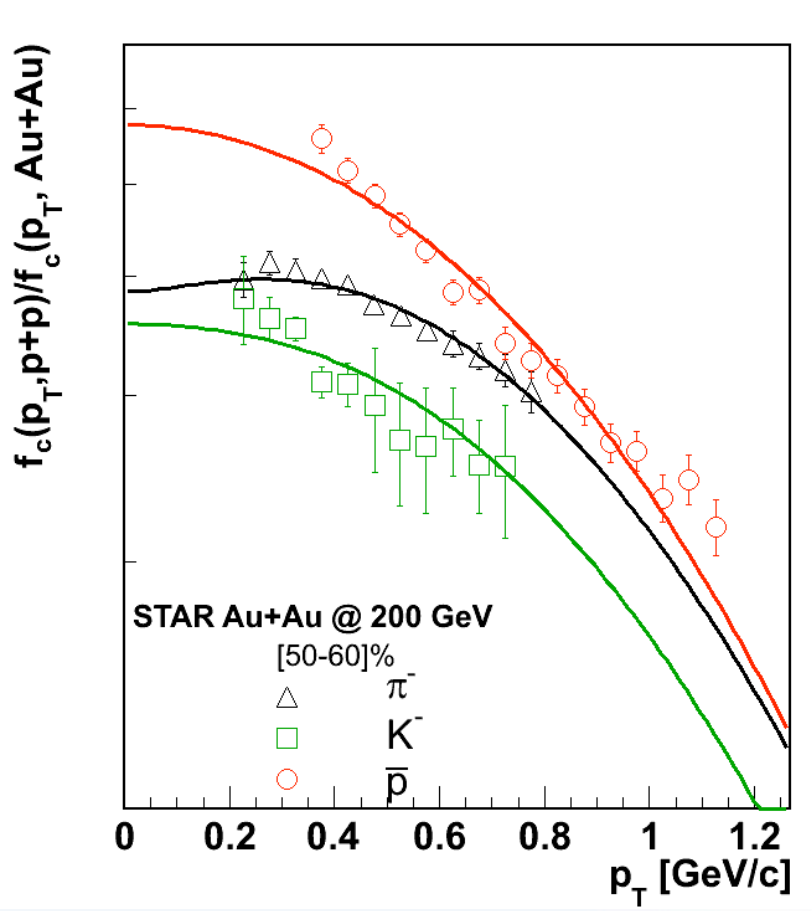
TMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"



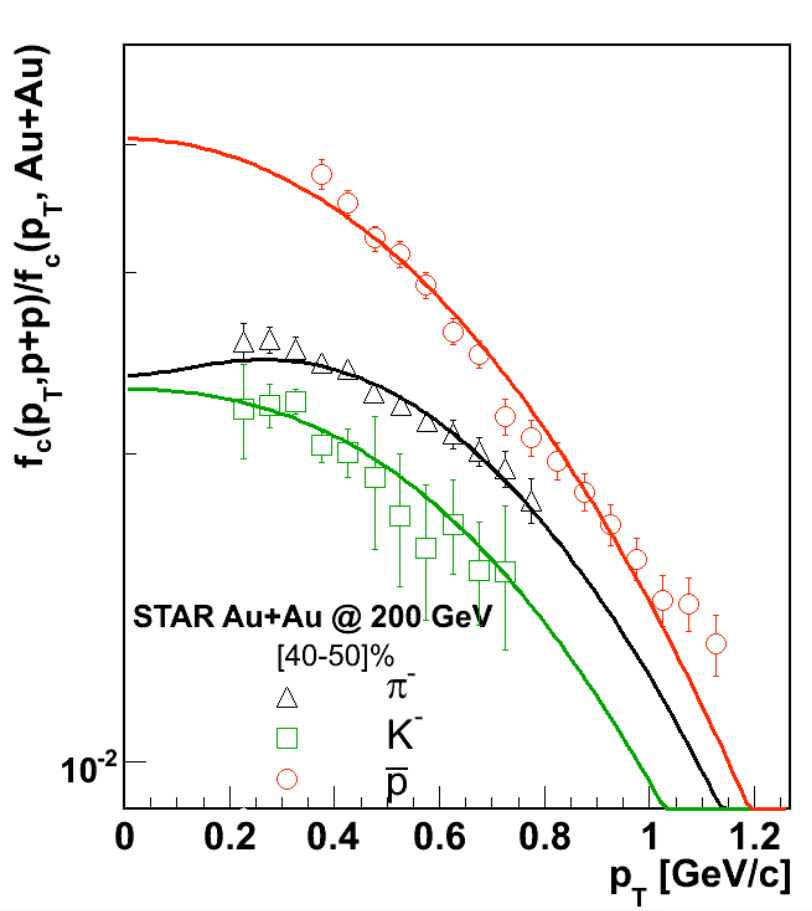
TMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"



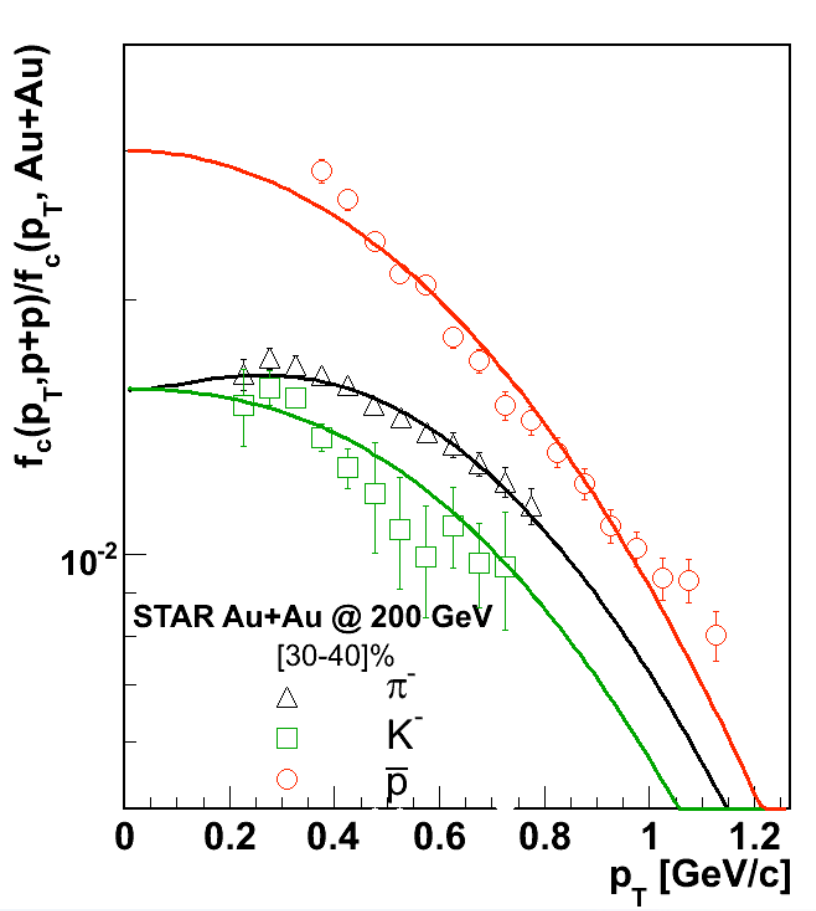
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"



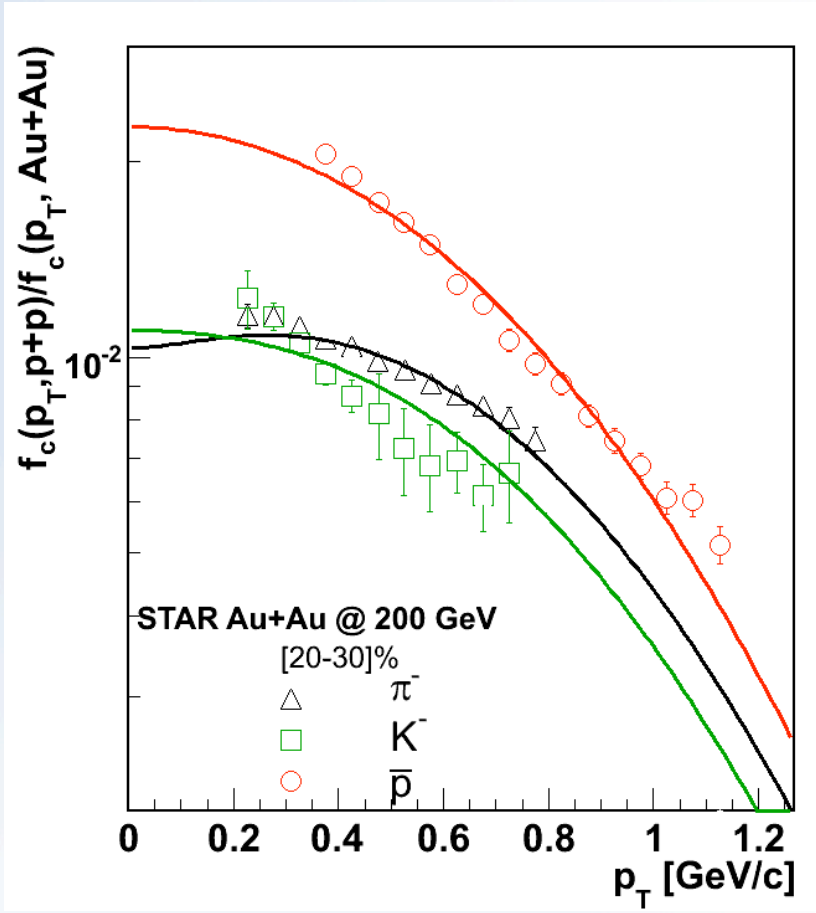
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"



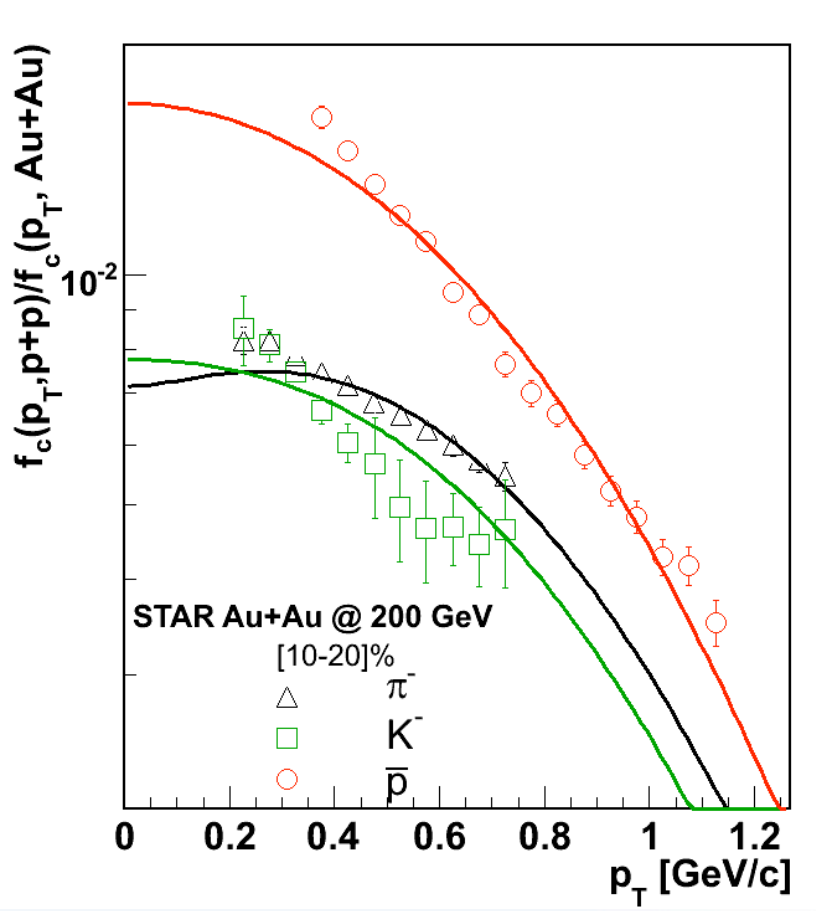
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"
$Au+Au$ 20-30%	219	"	"	"



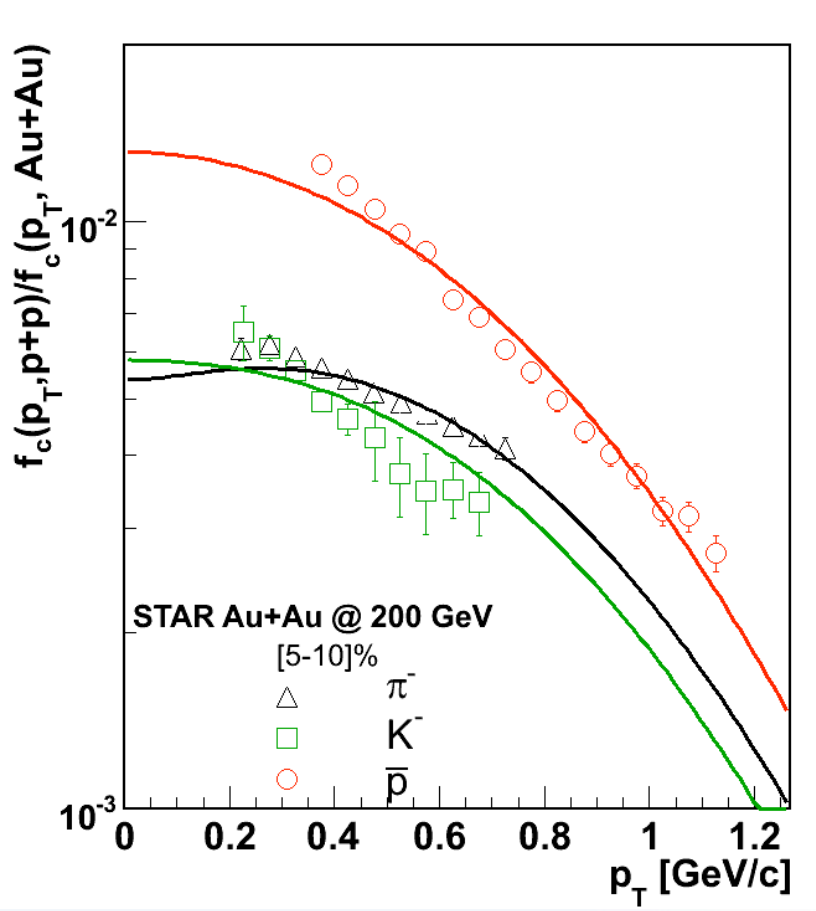
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"
$Au+Au$ 20-30%	219	"	"	"
$Au+Au$ 10-20%	> 300	"	"	"



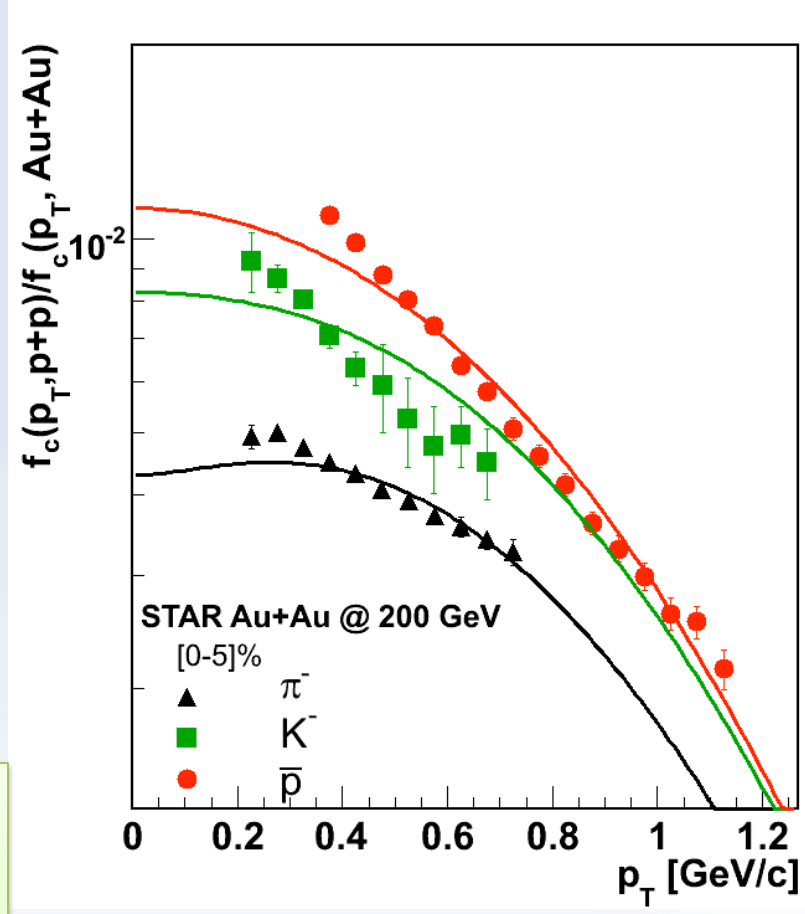
IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"
$Au+Au$ 20-30%	219	"	"	"
$Au+Au$ 10-20%	> 300	"	"	"
$Au+Au$ 5-10%	> 300	"	"	"



IMPT: What changes with multiplicity…? multiplicity does !!

Event selection	N	$\langle p_T^2 \rangle$ [(GeV/c)^2]	$\langle E^2 \rangle$ [GeV^2]	$\langle E \rangle$ [GeV]
$p+p$ min-bias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"
$Au+Au$ 20-30%	219	"	"	"
$Au+Au$ 10-20%	> 300	"	"	"
$Au+Au$ 5-10%	> 300	"	"	"
$Au+Au$ 0-5%	> 300	"	"	"



postulate of *same* parent consistent with *all* spectra

- magnitude
- pT dependence (shape)
- mass dependence

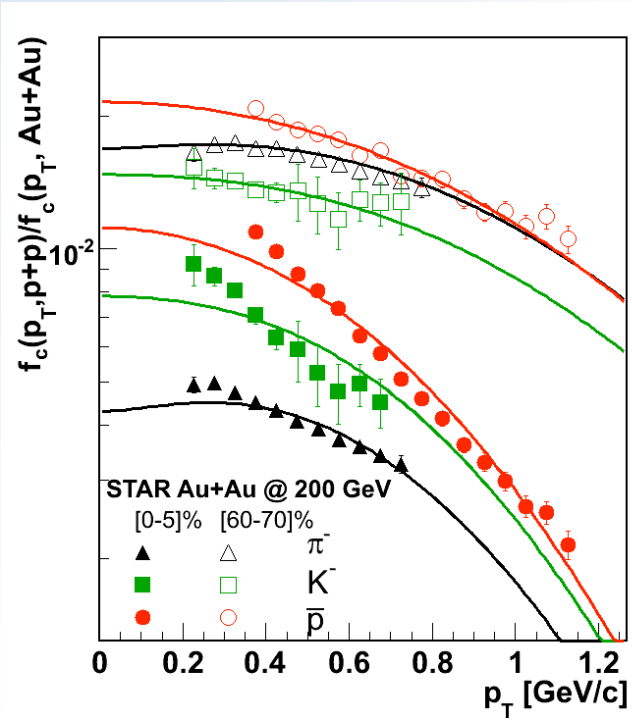
Kinematic scales of "the system"

$$\frac{\tilde{f}_c^{E_1}(p_{T,i})}{\tilde{f}_c^{E_2}(p_{T,i})} = \left(\frac{(N_2-1)N_1}{(N_1-1)N_2}\right)^2 \exp\left(\left(\frac{1}{2(N_2-1)} - \frac{1}{2(N_1-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{(E_i-\langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)$$

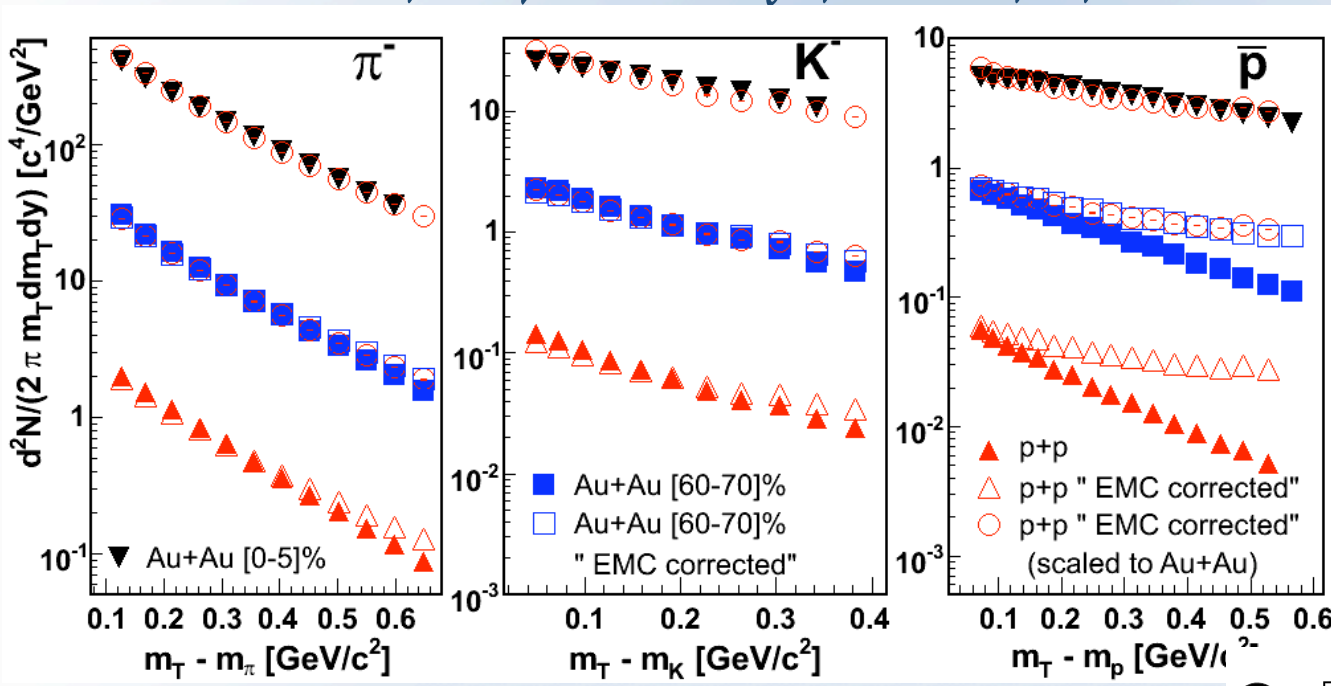
	non - rel	ultra - rel	if $T = .15 \div .35$	What we find
$\langle p_T^2\rangle$	$2mT$	$8T^2$	$0.045 \div 0.98\ (\mathrm{GeV/c})^2$	$0.12\ (\mathrm{GeV/c})^2$
$\langle E^2\rangle$	$\frac{15}{4}T^2 + m^2$	$12T^2$	$0.10 \div 1.5\ \mathrm{GeV}^2$	$0.43\ \mathrm{GeV}^2$
$\langle E\rangle$	$\frac{3}{2}T + m$	$3T$	$0.36 - 1\ \mathrm{GeV}$	$0.61\ \mathrm{GeV}$

Event selection	N	$\langle p_T^2\rangle$ [(GeV/c)^2]	$\langle E^2\rangle$ [GeV^2]	$\langle E\rangle$ [GeV]
$p+p$ minbias	10.3	0.12	0.43	0.61
$Au+Au$ 70-80%	15.2	"	"	"
$Au+Au$ 60-70%	18.3	"	"	"
$Au+Au$ 50-60%	27.3	"	"	"
$Au+Au$ 40-50%	38.7	"	"	"
$Au+Au$ 30-40%	67.6	"	"	"
$Au+Au$ 20-30%	219	"	"	"
$Au+Au$ 10-20%	> 300	"	"	"
$Au+Au$ 5-10%	> 300	"	"	"
$Au+Au$ 0-5%	> 300	"	"	"

TABLE II: Multiplicity and parent-distribution kinematic parameters which give a reasonable description of the spectrum ratios for identified particles in the soft sector. See text for details. Note that the multiplicity changes with event class; the parent distribution is assumed identical.



Multiplicity evolution of spectra - p+p to A+A (soft sector)

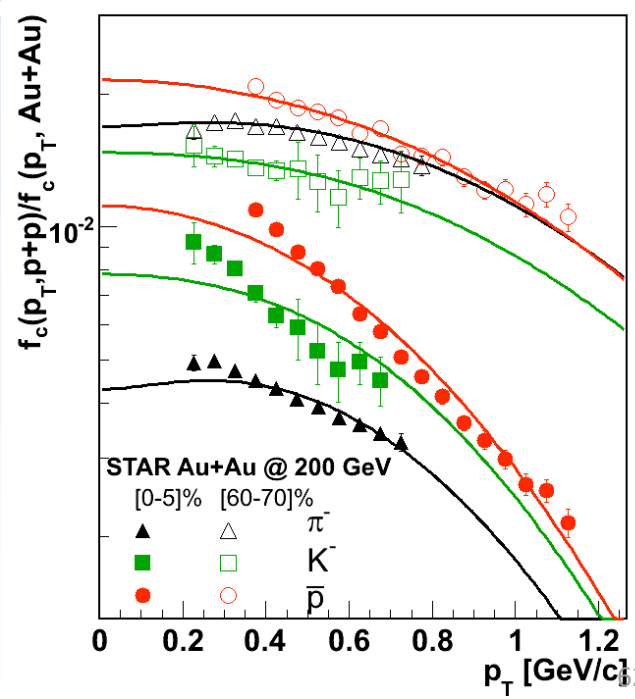


$$\frac{\tilde{f}_c^{pp}(p_{T,i})}{\tilde{f}_c^{AA}(p_{T,i})} \propto \exp\left(\left(\frac{1}{2(N_{AA}-1)} - \frac{1}{2(N_{pp}-1)}\right)\left(\frac{2p_{T,i}^2}{\langle p_T^2\rangle} + \frac{(E_i - \langle E\rangle)^2}{\langle E^2\rangle - \langle E\rangle^2}\right)\right)$$

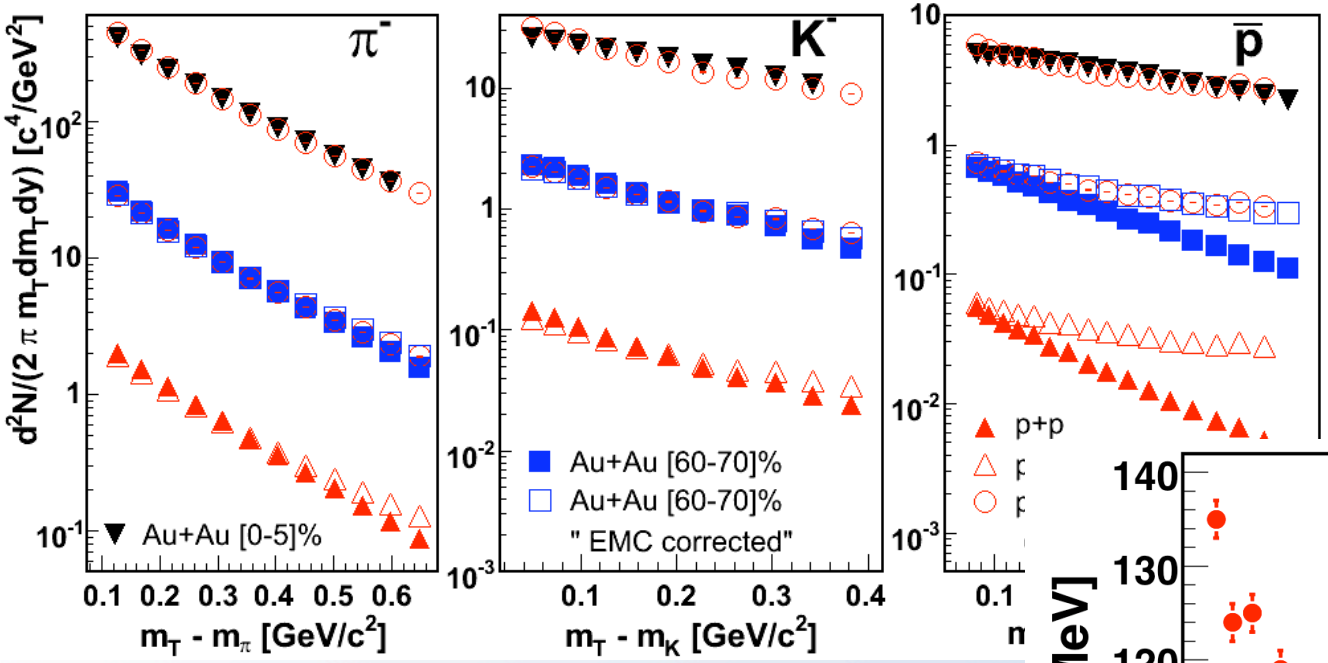
N evolution of spectra dominated by PS "distortion"

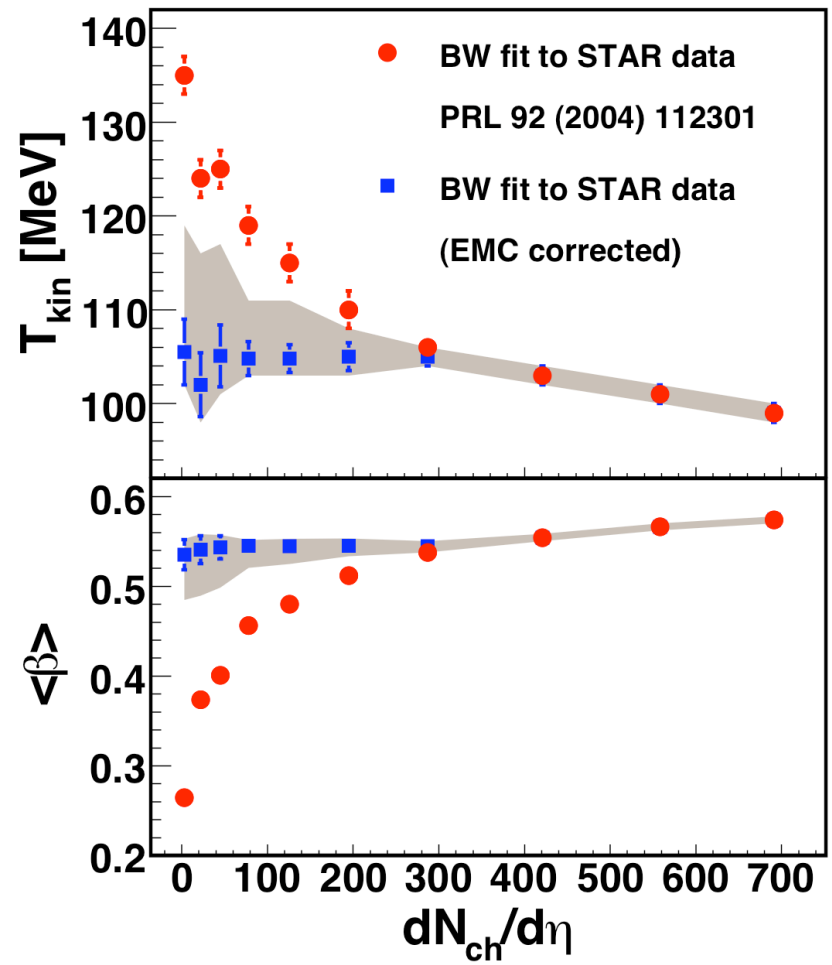
p+p system samples *same* parent distribution, but under stronger PS constraints

K ~ unity. driven by conservation of discrete quantum #s (strangeness, etc)



By popular demand





Almost universal "flow" & "temperature" parameters in a BlastWave fit

Apparent changes in β, T with dN/dη caused by EMCICs*

* EMCIC = Energy & Momentum Conservation Induced *Constraint*

Blast-wave in p+p: simultaneous description of spectra, HBT

$T = 105.5$ MeV

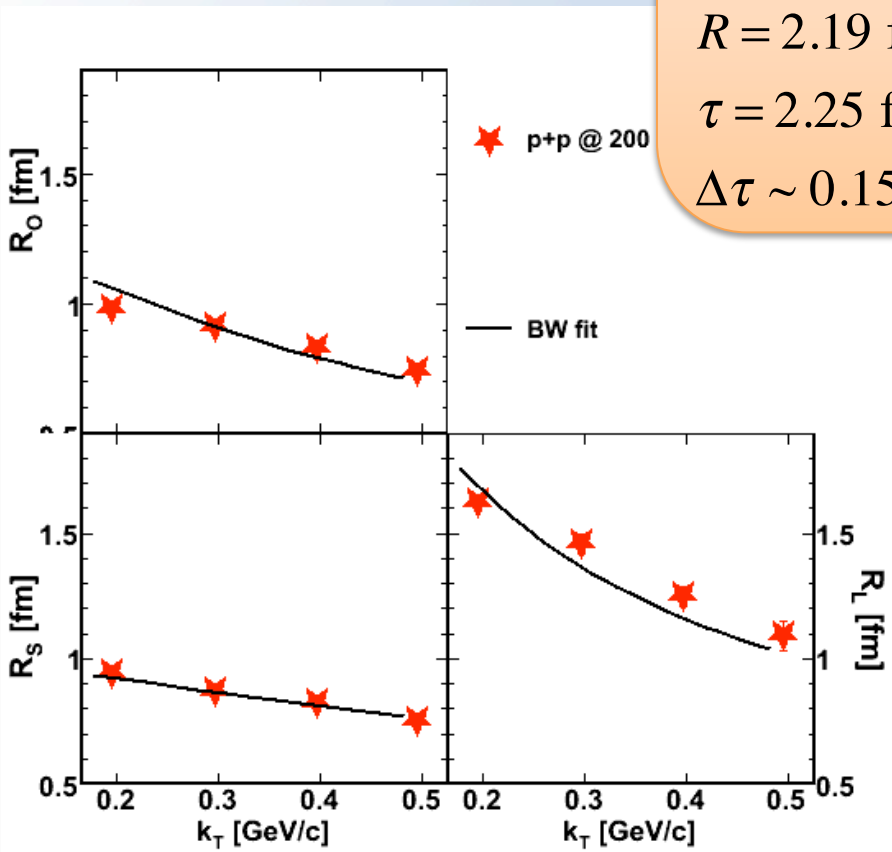
$\rho_0 = 0.934 \quad (\langle\beta\rangle = 0.535)$

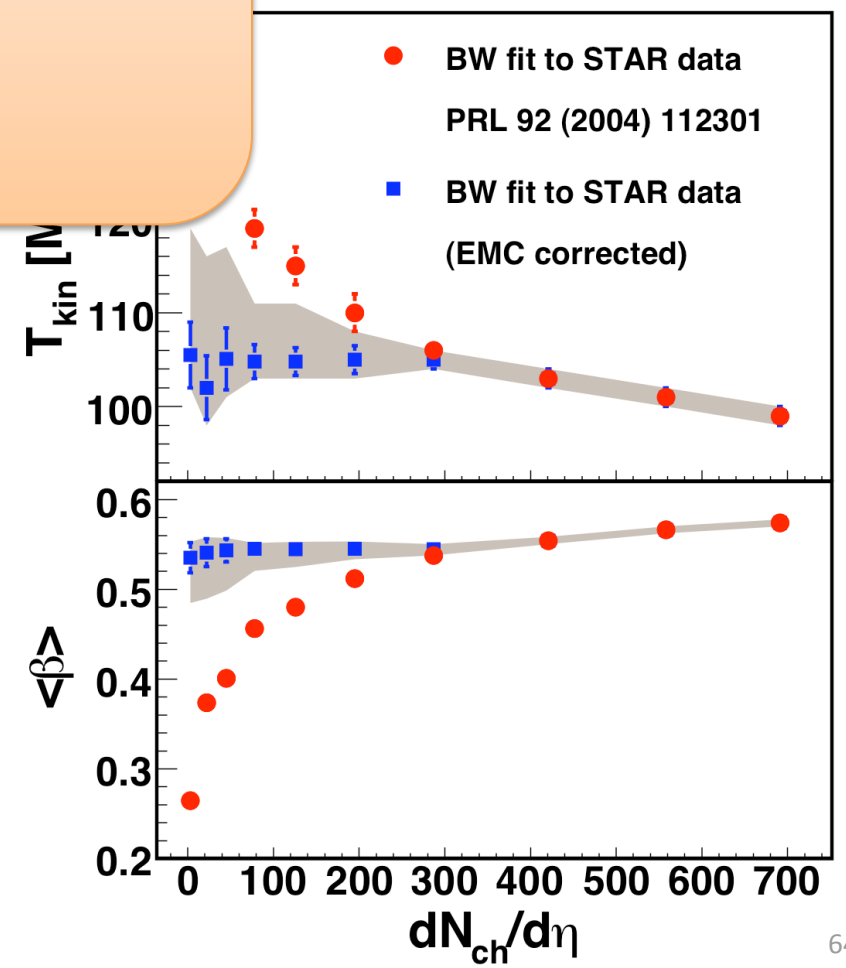
determined entirely by spectra

$R = 2.19$ fm

$\tau = 2.25$ fm/c

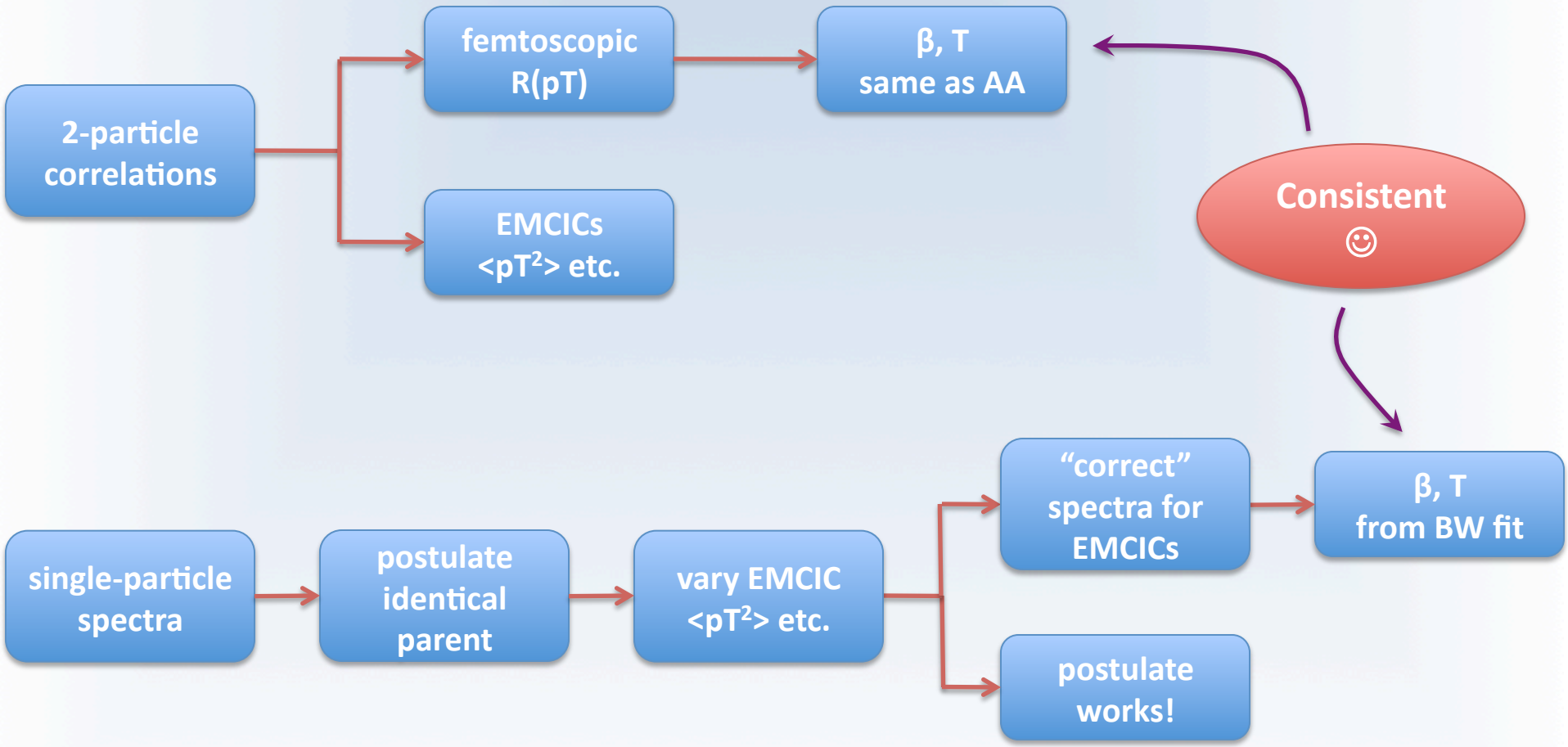
$\Delta\tau \sim 0.15$





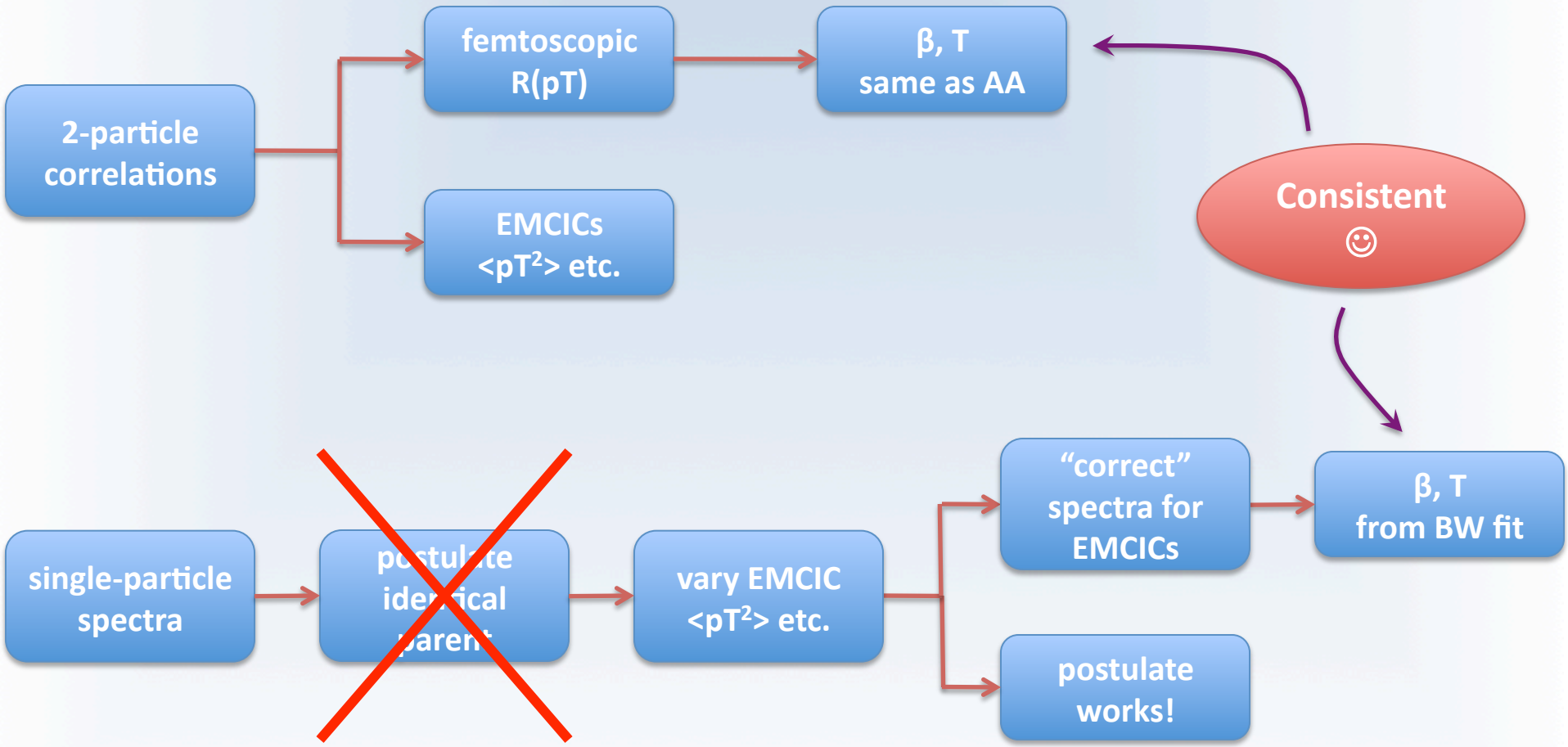
Can't follow the game without a program…

PHASE I



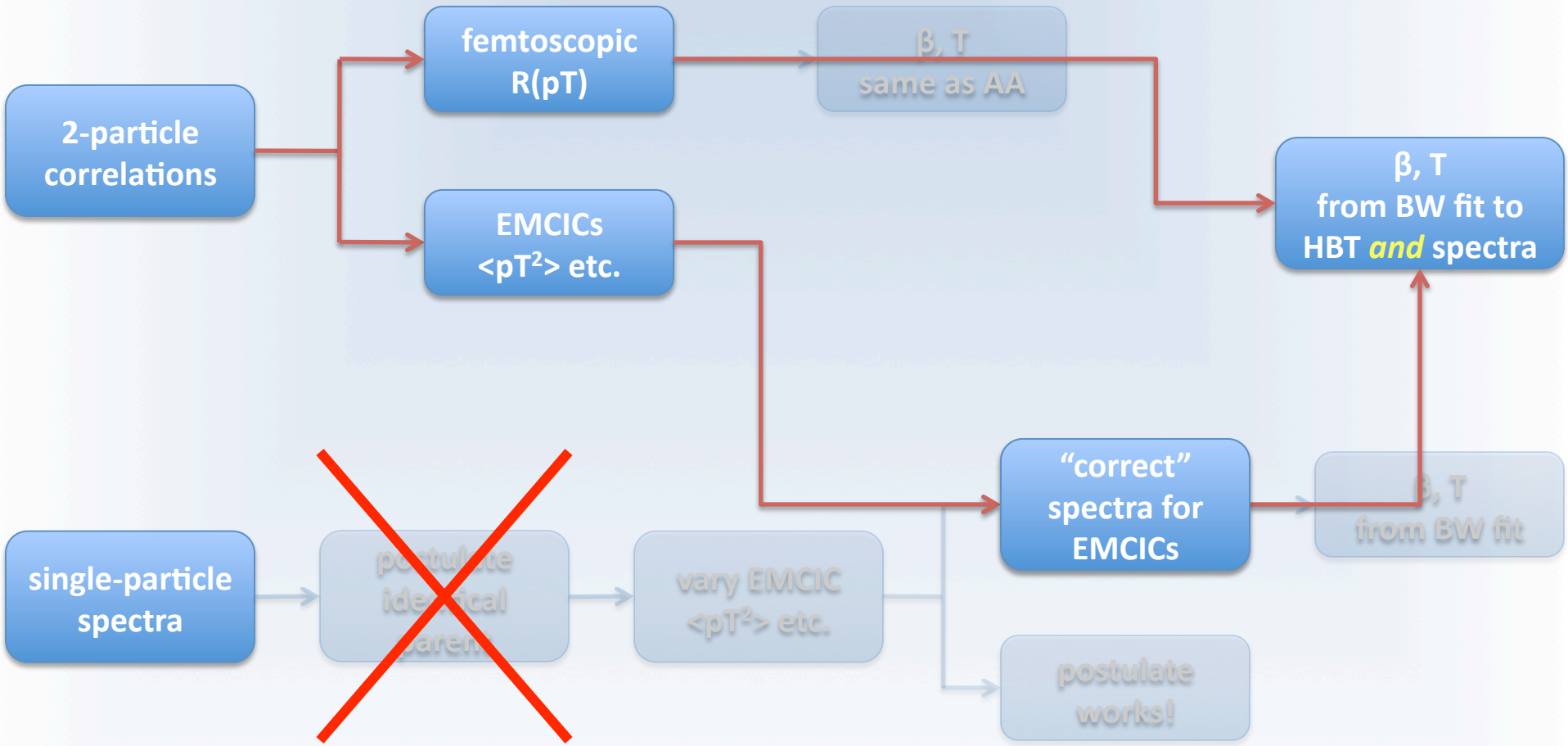
Can't follow the game without a program…

PHASE I



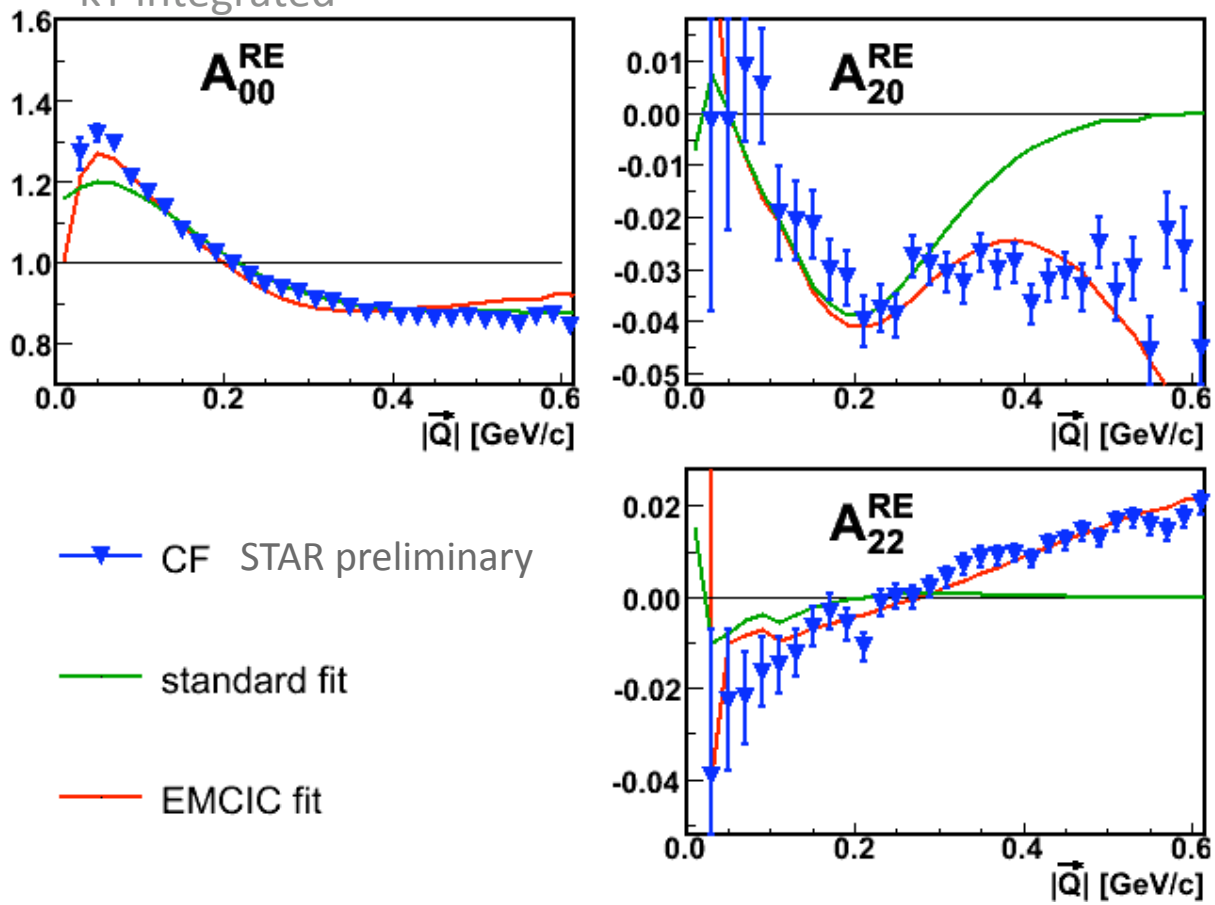
Can't follow the game without a program…

PHASE II



Femto and "system" parameters from 2-particle correlations

kT integrated

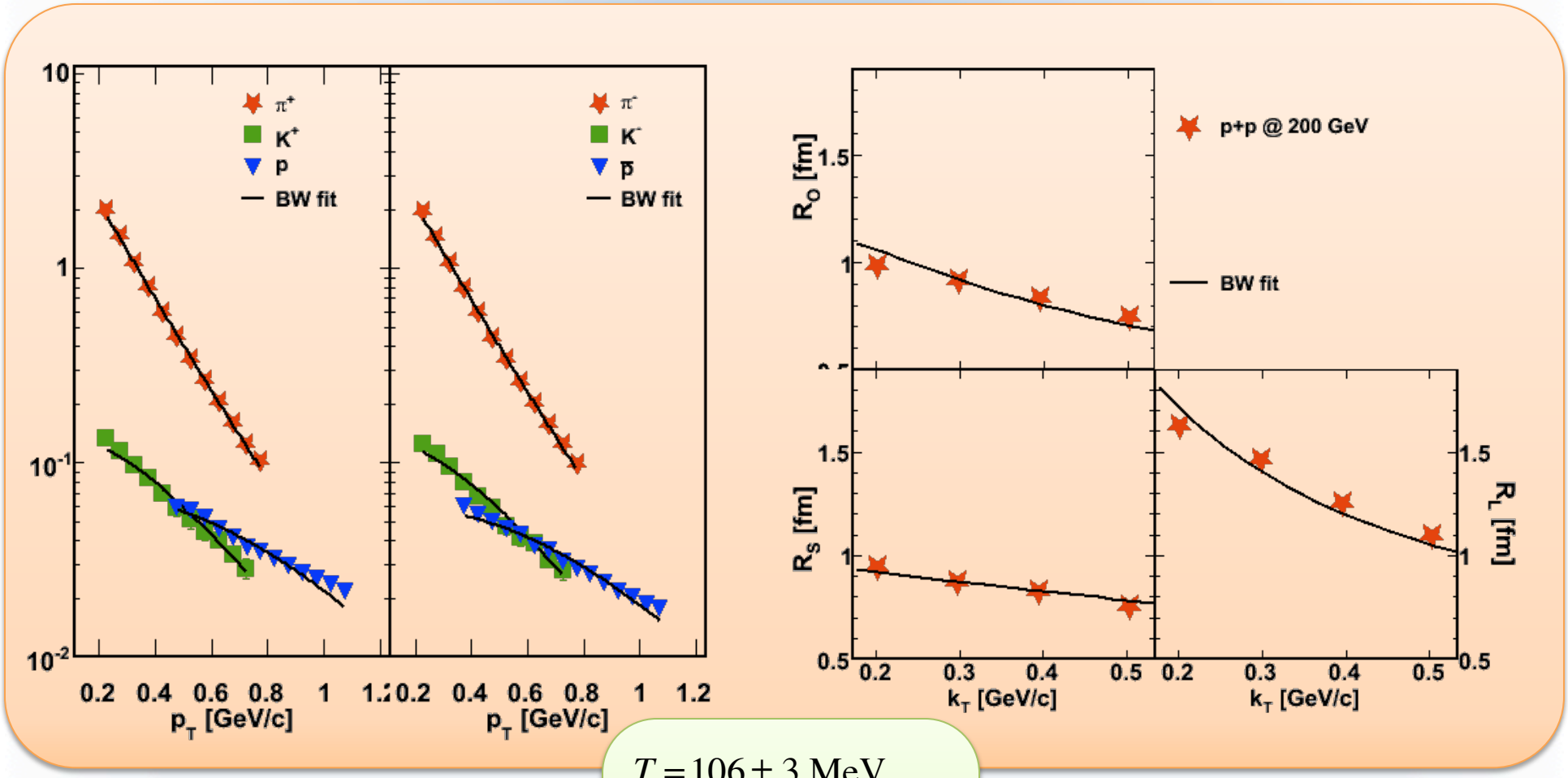


λ = 0.38 ± 0.01
R_o = 0.81 ± 0.01 fm
R_s = 0.84 ± 0.02 fm
R_l = 1.29 ± 0.02 fm

λ = 0.69 ± 0.01
R_o = 0.96 ± 0.04 fm
R_s = 0.98 ± 0.03 fm
R_l = 1.26 ± 0.02 fm

N = 13.6
$\langle E \rangle$ = 0.68 GeV
$\langle E^2 \rangle$ = 0.54 GeV^2
$\langle p_T^2 \rangle$ = 0.17 GeV^2
$\langle p_z^2 \rangle$ = 0.33 GeV^2

Combined fit: consistent flow-based description



$$T = 106 \pm 3 \text{ MeV}$$
$$\langle \beta \rangle = 0.48 \pm 0.03$$
$$R = 2.09 \pm 0.04 \text{ fm}$$
$$\tau_0 = 2.25 \pm 0.05 \text{ fm/c}$$
$$\Delta\tau = 0.1 \pm 0.2 \text{ fm/c}$$

Combined fit: consistent flow-based description

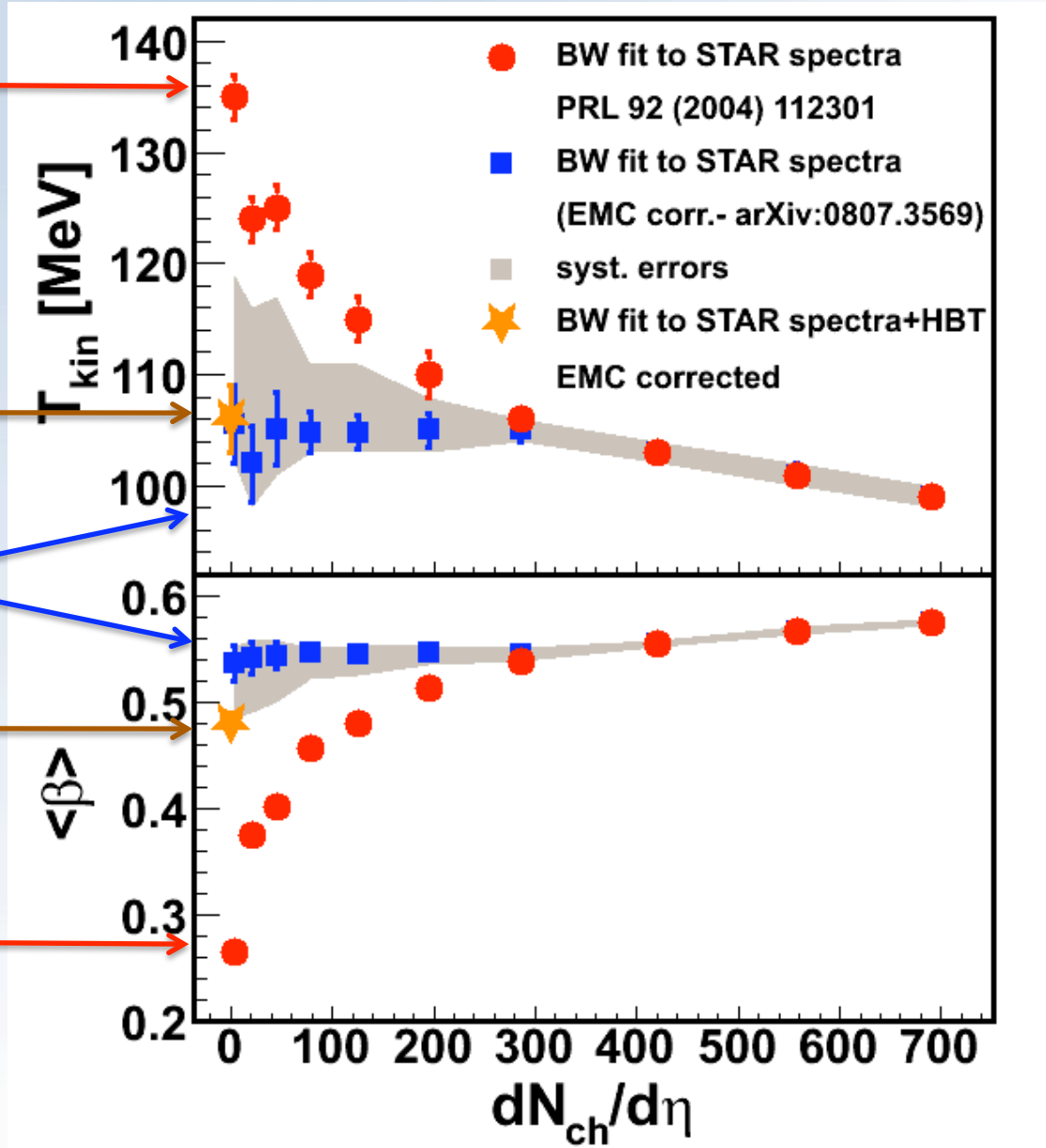
"raw" (ignoring EMCICs)

EMCICs **fixed by correlations**
Joint spectra/HBT BW fit

EMCICs **free adjusted**
to spectra & fit to spectra

EMCICs **fixed by correlations**
Joint spectra/HBT BW fit

"raw" (ignoring EMCICs)



Combined fit: consistent flow-based description

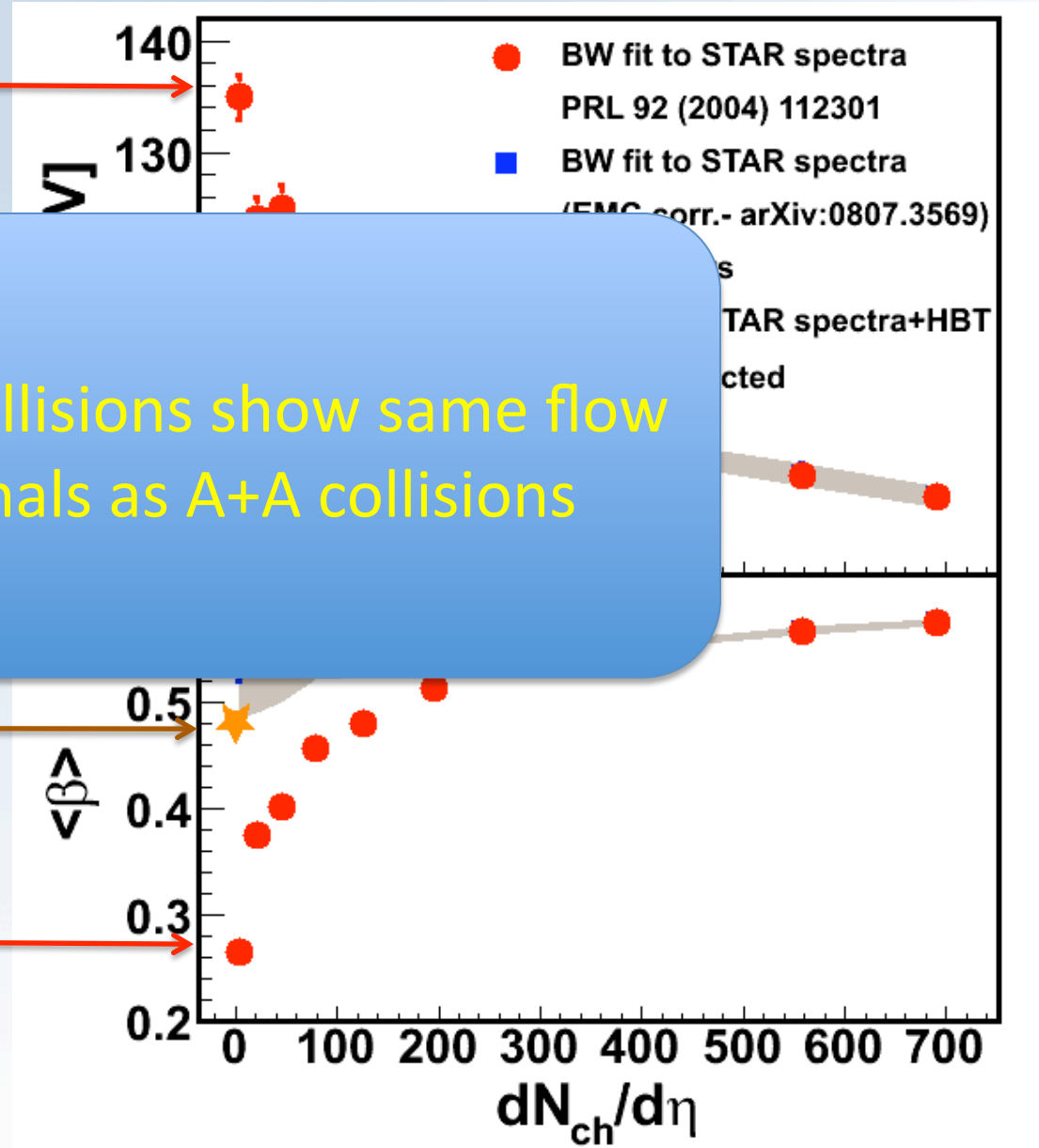
"raw" (ignoring EMCICs)

EMCICs fixed by cor
Joint spectra/HBT B

EMCICs free adjuste
to spectra & fit to sp

EMCICs fixed by correlations
Joint spectra/HBT BW fit

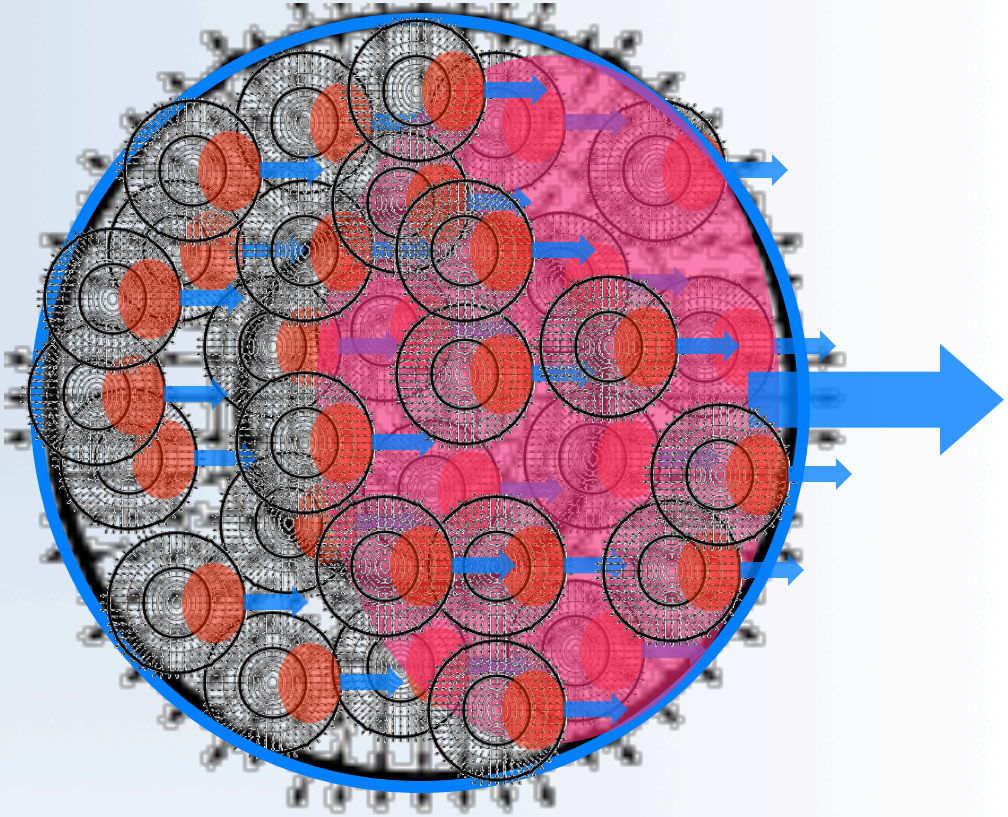
"raw" (ignoring EMCICs)



p+p collisions show same flow signals as A+A collisions

Implication: A+A is just a collection of flowing p+p?

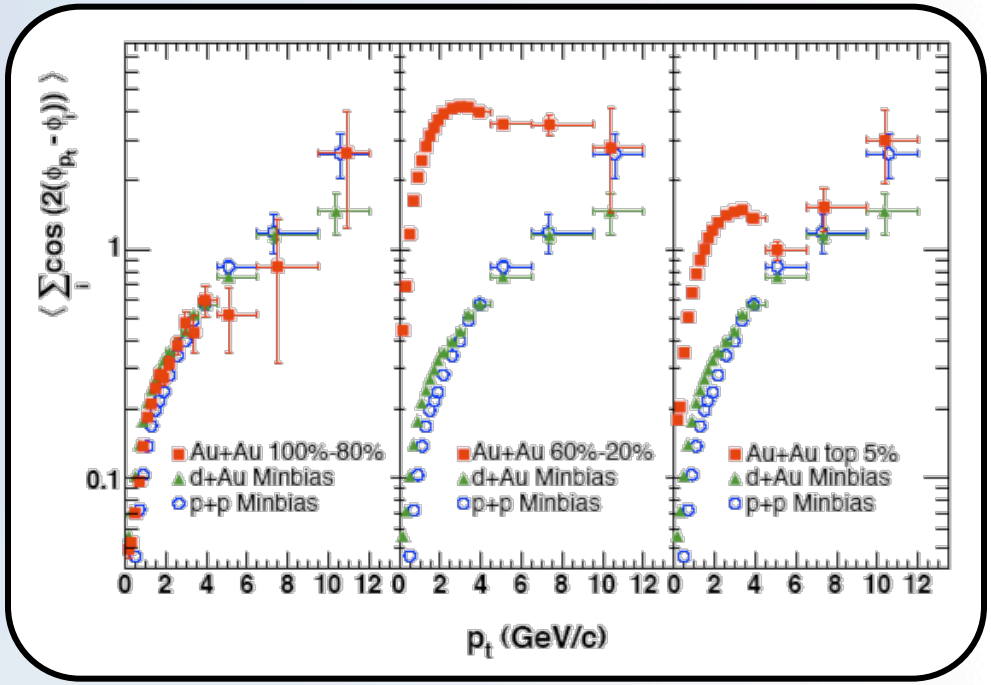
- No! Quite the opposite.
  - **femtoscopically**
    - A+A looks like a big BlastWave
    - *not* superposition of small BlastWaves
    - A+A has thermalized globally
  - **spectra**
    - superposition of spectra from p+p has same shape as a spectrum from p+p!
    - relaxation of P.S. constraints indicates A+A has thermalized globally



- rather, p+p looks like a "little A+A"

Implication: A+A is just a collection of flowing p+p?

- No! Quite the opposite.
  - **femtoscopically**
    - A+A looks like a big BlastWave
    - *not* superposition of small BlastWaves
    - A+A has thermalized globally
  - **spectra**
    - superposition of spectra from p+p has same shape as a spectrum from p+p!
    - relaxation of P.S. constraints indicates A+A has thermalized globally
- rather, p+p looks like a "little A+A"

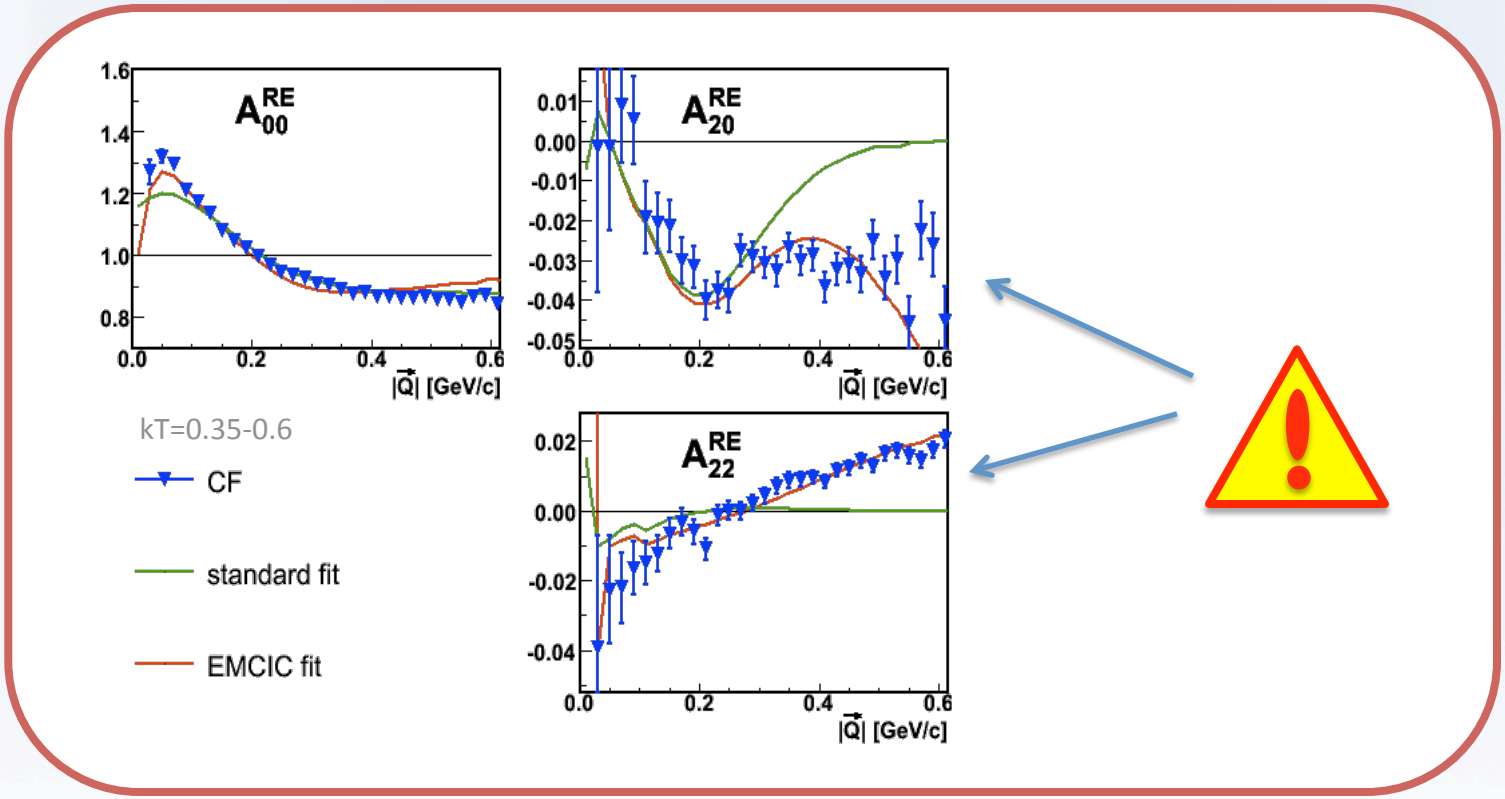


anisotropic flow

- A+A shows increased signal over superposition of p+p
- is the p+p signal "flow" ??

Summary

- E&M conservation induces phasespace constraints w/ explicit N dependence
  - should not be ignored in (crucial!) N-dependent comparisons
  - significant effect on 2- (and 3-) particle correlations [c.f. Ollitrault, Borghini, Voloshin…]
  - …*and* single-particle spectra (often neglected because no "red flags")



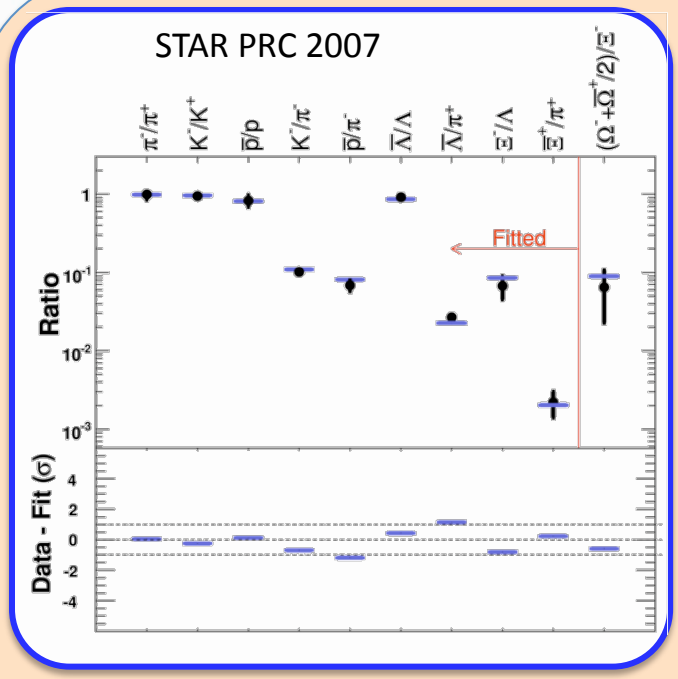
Summary

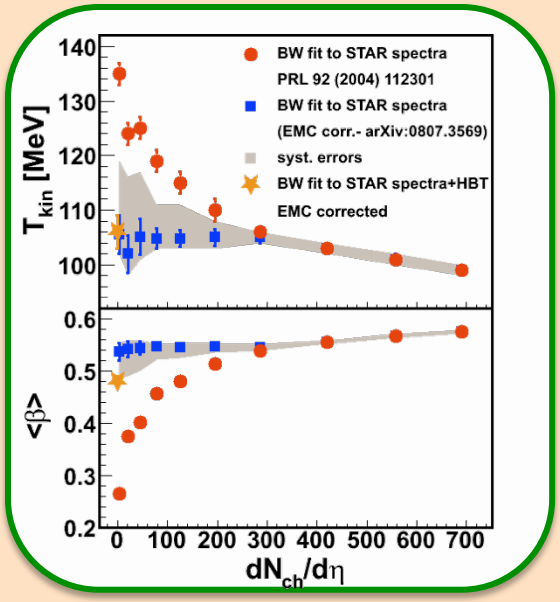
- E&M conservation induces phasespace constraints w/ explicit N dependence
  - should not be ignored in (crucial!) N-dependent comparisons
  - significant effect on 2- (and 3-) particle correlations [c.f. Ollitrault, Borghini, Voloshin…]
  - …*and* single-particle spectra (often neglected because no "red flags")

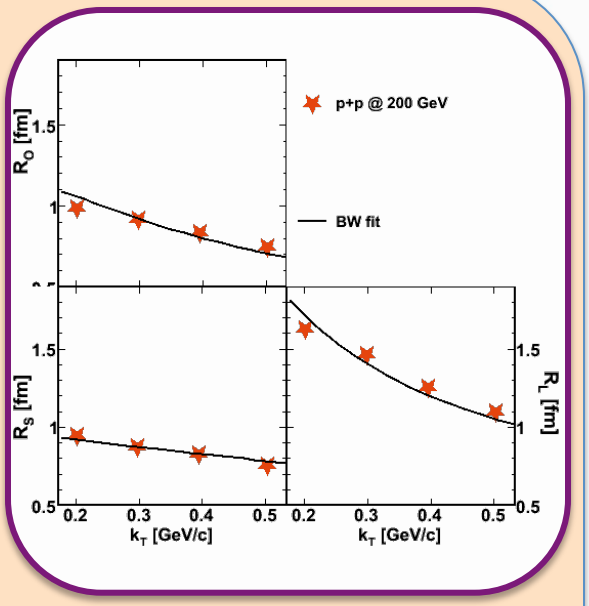
- Femtoscopy & Spectra
  - in H.I.C., well understood, detailed fingerprint of flow
  - RHIC – first opportunity for direct comparison with p+p
  - accounting for EMCICs, identical flow signals in p+p

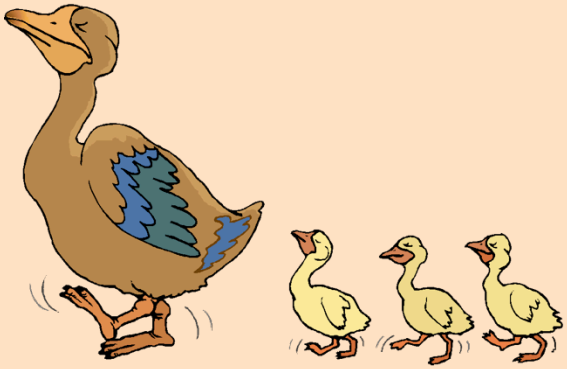
- is pp/AA physics very similar, or are measurements insensitive to diff physics?
- **Has AA become the reference system for pp in non-perturbative sector???**
- Thermalization, hadronization, very early color dynamics…

Summary









"It is a capital mistake to theorize before one has data."
– Sir Arthur Conan Doyle *[Debasish Das @ WWND09]*

"It is even worse, when one *has* data, to insist upon two orthogonal theoretical interpretations for the same systematics."
– Prof. Mike Lisa