Diffraction in the Dipole Cascade Picture

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Work done with
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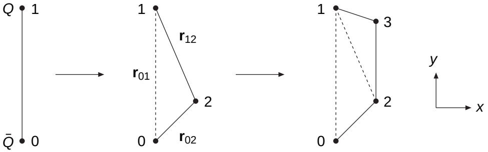
Outline

Mueller Dipole model

- Evolution in rapidity of dipoles in transverse coordinate space.



- Emission probability

$$\frac{d\mathcal{P}}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\boldsymbol{r}_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}$$

- Eg. $\gamma^\star\gamma^\star$, each $\gamma^\star$ splits into $q\bar{q}$ dipoles which evolve through dipole splitting $\Leftrightarrow$ gluon emission.
- Interaction through dipole–dipole scattering probability

$$f = \frac{\alpha_s^2}{2}\left\{\log\left[\frac{|\boldsymbol{x}_1 - \boldsymbol{x}_2| \cdot |\boldsymbol{y}_1 - \boldsymbol{y}_2|}{|\boldsymbol{x}_1 - \boldsymbol{y}_2| \cdot |\boldsymbol{y}_1 - \boldsymbol{x}_2|}\right]\right\}^2$$

- Equivalent to LO BFKL
- Easy to include multiple dipole–dipole scatterings with scattering amplitude

$$T = 1 - e^{-\sum_{ij} f_{ij}}$$

- Less easy to include saturation in the evolution

MC implementation of Mueller Dipoles

- First done by Salam: OEDIPUS
- Dipole splitting is divergent for small dipole sizes
- Final result independent of cutoff because small dipoles has small interaction probability
- Problem for MC implementation — too many dipoles

The [*insert name here*] MC implementation

- Small size dipoles correspond to high $p_\perp \propto 1/r$ gluons
- We may have infinitely many small *virtual* dipoles but there is not enough energy for all of them to interact.
- Our MC tracks each parton: y, $\boldsymbol{x}$, $p_\perp$
- Require each emission to be ordered in p_+ and p_-
- Take into account recoils
- Neighboring dipoles are correlated

- A right-moving dipole with negative p_- must collide with left-moving dipole with enough positive p_- so that all partons can be put on-shell (+vv.)
- Energy-momentum conservation gives a dynamical cutoff for small dipoles in the evolution
- Also non-interacting (virtual) side chains takes energy, energy conservation effects are somewhat over estimated.
- Hopefully, the MC can be used to also study final-state properties

Modeling the proton

- The virtual photon wave functions are well known
- How do we describe the initial dipole state of a proton?
- We have at three valence quarks $\Rightarrow$ three dipole ends.
- Three independent dipole didn't work very well
- Three (correlated) dipoles in Δ-geometry worked very well.
- Just one parameter, R, Gaussian width for the position of the "valence gluons"

The Dipole Swing

- Each dipole carries a colour index
- Two dipoles with the same index are allowed to reconnect $(\boldsymbol{x}_1, \boldsymbol{y}_1), (\boldsymbol{x}_2, \boldsymbol{y}_2) \to (\boldsymbol{x}_1, \boldsymbol{y}_2), (\boldsymbol{x}_2, \boldsymbol{y}_1)$ with probability

$$\propto \frac{(\boldsymbol{x}_1 - \boldsymbol{y}_1)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_2)^2}{(\boldsymbol{x}_1 - \boldsymbol{y}_2)^2 (\boldsymbol{x}_2 - \boldsymbol{y}_1)^2}$$

- Coefficient adjusted so that the swing saturates
- Can be interpreted as gluon exchange, but also as modeling the quadrupole as two combinations of dipoles.
- This gives saturation in the evolution (the number of dipoles are not decreased, but smaller dipoles are preferred).

- The size of the proton, R.
- Λ_{QCD} in the running coupling
- The strength of the dipole swing
- The quark masses for the photon wave function

hep-ph/0503181, hep-ph/0610157, hep-ph/0702087

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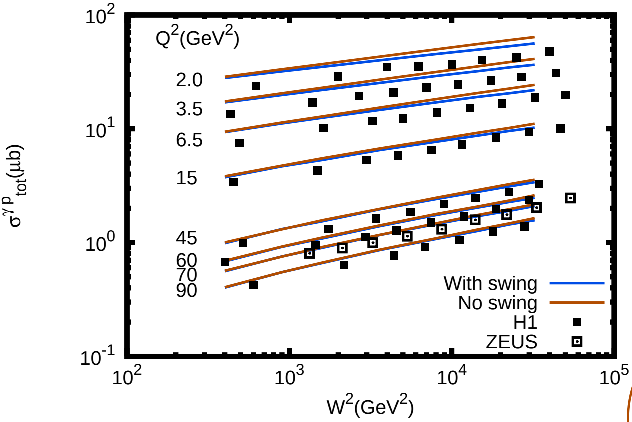
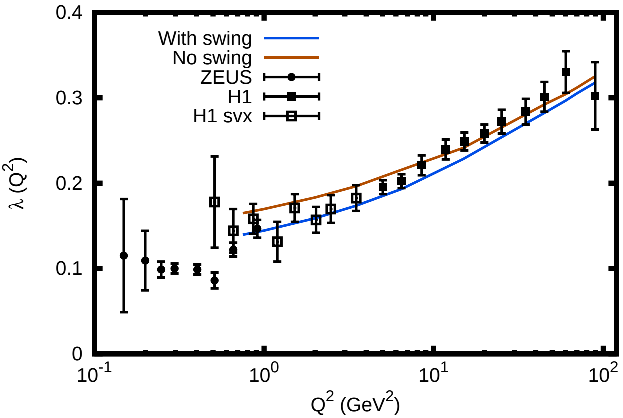
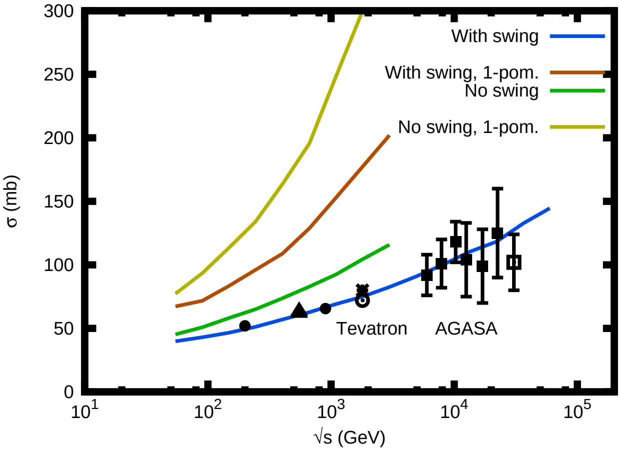

10^2
10^1
10^0
10^{-1}
$\sigma^{\gamma^* p}_{tot}(\mu b)$
$Q^2(GeV^2)$
2.0
3.5
6.5
15
45
60
70
90
With swing
No swing
H1
ZEUS
10^2
10^3
10^4
10^5
$W^2(GeV^2)$


With swing
No swing
ZEUS
H1
H1 svx
0.4
0.3
0.2
0.1
0
λ (Q²)
10⁻¹
10⁰
10¹
10²
Q² (GeV²)


300
250
200
150
100
50
0
σ (mb)
10^1
10^2
10^3
10^4
10^5
√s (GeV)
With swing
With swing, 1-pom.
No swing
No swing, 1-pom.
Tevatron
AGASA

- The total cross section: $\frac{d\sigma_{\rm tot}}{d^2b} = 2\langle 1 - e^{-F}\rangle_{RL}$ where $F = \sum_{ij} f_{ij}$ is b-dependent
- Should be independent of Lorentz frame
- In the end we want to describe final states. Let's start with some simple semi-inclusive observables.
- We use the Good & Walker picture of diffraction, using the dipole states as eigenstates of the diffraction.
- Elastic cross section: $\frac{d\sigma_{\rm el}}{d^2b} = \langle 1 - e^{-F}\rangle_{RL}^2$
- Should also be frame-independent

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- Diffractive cross sections

$$\frac{d\sigma_{SD}^{R}}{d^2b} = \langle\langle 1-e^{-F}\rangle_L^2\rangle_R - \langle 1-e^{-F}\rangle_{R,L}^2$$
$$\frac{d\sigma_{SD}^{L}}{d^2b} = \langle\langle 1-e^{-F}\rangle_R^2\rangle_L - \langle 1-e^{-F}\rangle_{R,L}^2$$
$$\frac{d\sigma_{DD}}{d^2b} = \langle(1-e^{-F})^2\rangle_{R,L} - \langle\langle 1-e^{-F}\rangle_L^2\rangle_R - \langle\langle 1-e^{-F}\rangle_R^2\rangle_L + \langle 1-e^{-F}\rangle_{R,L}^2.$$

- Not frame-independent. And it shouldn't be.

- Assuming a total rapidity interval $Y = \log s$, evolving left-moving dipole system to y and right-moving to $Y - y$
- $\sigma_{diff}/\sigma_{tot}$ only gives the probability to have a gap at y, not how large the gap is.
- Looking at hard diffraction in DIS we want to measure $\sigma_{diff}(m_X)/\sigma_{tot}$, where m_X is the mass of the diffracted photon.
- We evolve the proton to $y_p = \ln W^2/m_X^2$
- We evolve the photon to $y_{\gamma^\star} = \ln \max(z, 1-z) m_X^2/p_\perp$
-
$$\sigma_{diff}(m_X) = \int^{\ln m_X^2} \frac{d\sigma_{diff}}{d \ln m_X^2} d \ln m_X^2$$

- But first we must make sure σ_{tot} is frame-independent
- Not a big problem for *pp*. Difficult for $\gamma^{\star}p$
- Important to treat gluon emission and dipole–dipole scattering in the same way.
- The swing is necessary but not a big effect for $\gamma^{\star}p$
- Energy momentum conservation important
- Running α_S is important
- But most of all. . .

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Confinement effects

- We have confinement effects in the initial proton
- We also had a naive suppression of large dipoles in the evolution, but nothing in the dipole–dipole interaction
- Taking away confinement in the evolution did not work
- Need to include confinement effect everywhere

$$f(\boldsymbol{x}_i,\boldsymbol{y}_i|\boldsymbol{x}_j,\boldsymbol{y}_j)=\frac{g^4}{8}(\Delta(\boldsymbol{x}_i-\boldsymbol{x}_j)-\Delta(\boldsymbol{x}_i-\boldsymbol{y}_j)-\Delta(\boldsymbol{y}_i-\boldsymbol{x}_j)+\Delta(\boldsymbol{y}_i-\boldsymbol{y}_j))^2$$

where $\Delta(\boldsymbol{r})$ is the Green's function given by

$$\Delta(\boldsymbol{r})=\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\boldsymbol{k}^2}.$$

This assumes a Coulomb potential.
Let's instead use a Yukawa potential with $M=1/R$

$$\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\boldsymbol{k}^2+M^2}=\frac{1}{2\pi}K_0(rM)$$

$$f_{ij}\to\frac{\alpha_s^2}{2}\Big(K_0(|\boldsymbol{x}_i-\boldsymbol{y}_j|/R)-K_0(|\boldsymbol{x}_i-\boldsymbol{x}_j|/R)-$$
$$-K_0(|\boldsymbol{y}_i-\boldsymbol{y}_j|/R)+K_0(|\boldsymbol{x}_j-\boldsymbol{y}_i|/R)\Big)^2$$

$$\frac{d\mathcal{P}}{dY} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{z} \left(\frac{\mathbf{x}-\mathbf{z}}{(\mathbf{x}-\mathbf{z})^2} - \frac{\mathbf{y}-\mathbf{z}}{(\mathbf{y}-\mathbf{z})^2} \right)^2.$$

The two terms comes from the integration

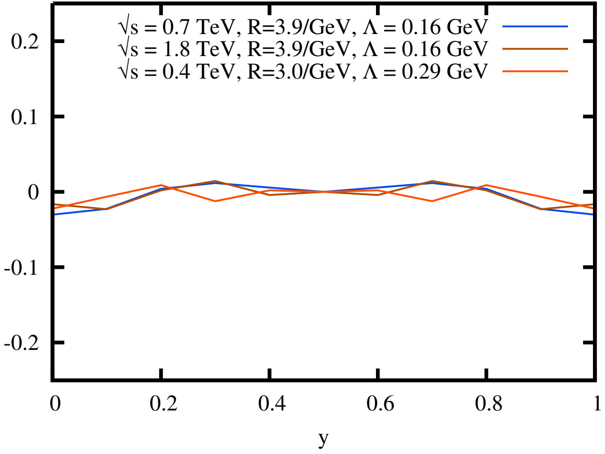
$$\int \frac{d^2\mathbf{k}}{(2\pi)^2 i} \frac{\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2} = -\nabla \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2}.$$

Again changing to a Yukawa potential

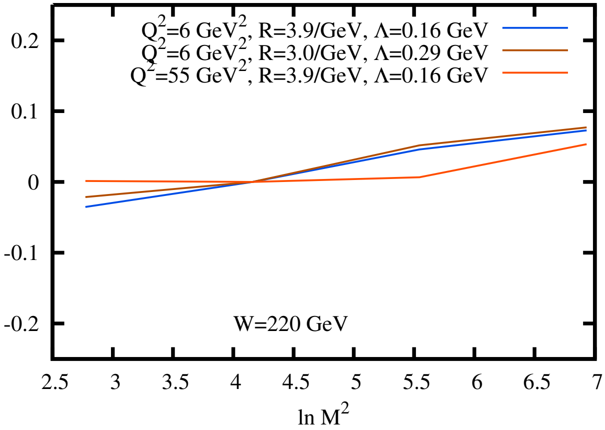
$$\frac{d\mathcal{P}}{dY} \to \frac{\bar{\alpha}}{2\pi} d^2\mathbf{z} \left(\frac{1}{R} \frac{\mathbf{x}-\mathbf{z}}{|\mathbf{x}-\mathbf{z}|} K_1\left(\frac{|\mathbf{x}-\mathbf{z}|}{R}\right) - \frac{1}{R} \frac{\mathbf{y}-\mathbf{z}}{|\mathbf{y}-\mathbf{z}|} K_1\left(\frac{|\mathbf{y}-\mathbf{z}|}{R}\right) \right)^2.$$

Same as before for $r \ll R$. Exponentially damped for $r \gg R$

Frame-independence of σ_{tot}^{pp}



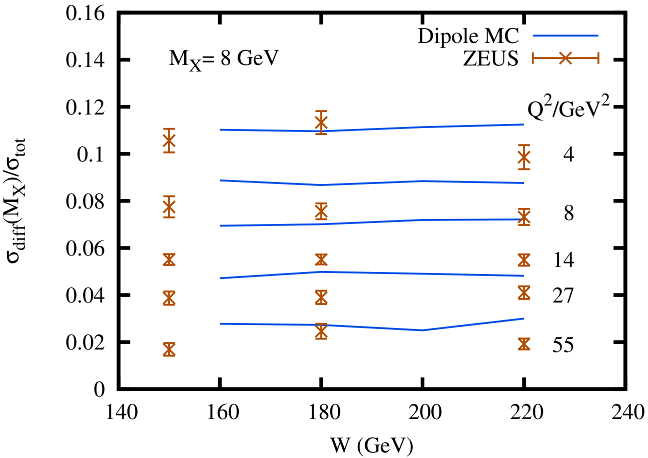
Frame-independence of $\sigma_{\mathrm{tot}}^{\gamma^{\star}p}$



Results

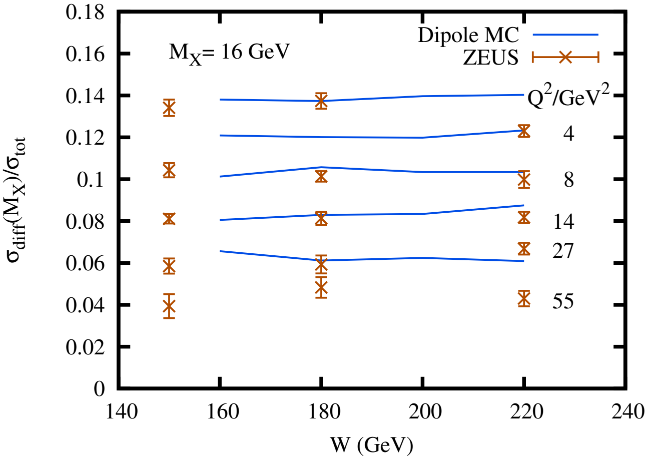
- For the Tevatron we get $\sigma_{\rm el}/\sigma_{\rm tot} \approx \spadesuit.\heartsuit\diamondsuit$
- Increases to $\spadesuit.\clubsuit\diamondsuit$ at the LHC
- Tevatron: $\spadesuit.\heartsuit\clubsuit$
- Results for single/double diffraction in *pp* still to come

Diffractive excitation of $\gamma^\star$ at HERA



Data has $m_N < 2.3$ GeV

Diffractive excitation of $\gamma^\star$ at HERA



Summary

- We have a Monte Carlo implementation of Mueller Dipoles
- Key ingredients:
  - Energy-momentum conservation
  - Dipole swing
  - Simple proton model
  - Confinement effects
  - . . . possibility to study final states
- Reasonable description of data:
  - Total cross sections for *pp* and $\gamma^\star p$
  - Diffraction at HERA
  - Elastic scattering in *pp*
  - . . . more to come

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