Imaging Overview

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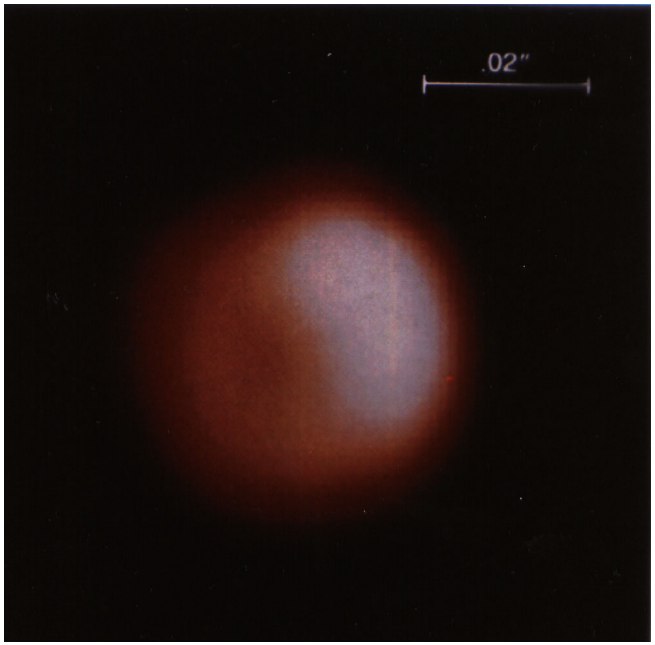
Outline

1. Introduction
2. Accessing Sources
   - Imaging Relation and Its Inversion
   - pp Imaging
3. 3D Structure
   - Cartesian Harmonics
   - Sample Source Decomposition
4. Practice of HE Restoration
   - Tests of π-π Source Restoration
   - PHENIX Analysis
5. Conclusions

Astronomy

Intensity/phase interferometry first used to assess sizes of astronomical objects. Astronomers have since moved to details:



red giant Betelguese

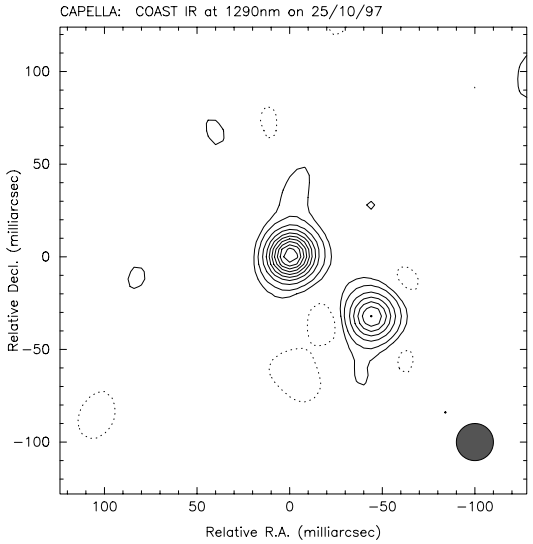


Figure 5.7: Reconstructed image of Capella, from data taken on 25 October 1997 at a wavelength of 1.3 µm. The contours are at -4, 4, 10, 20, 30, …, 90% of the peak ux. The map has been restored with a circular beam for clarity.

binary star Capella, Monnier Rep Prog Phy 66(03)789

Can we do comparably well?

Imaging

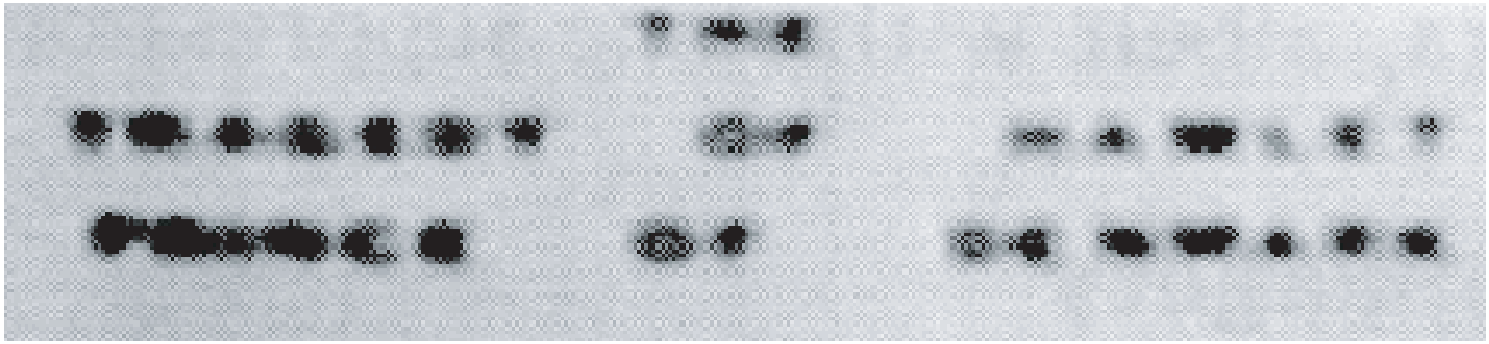
Source f/correlation: case of imaging. General task:

$$C(q) = \int dr\, K(q,r)\, S(r)$$

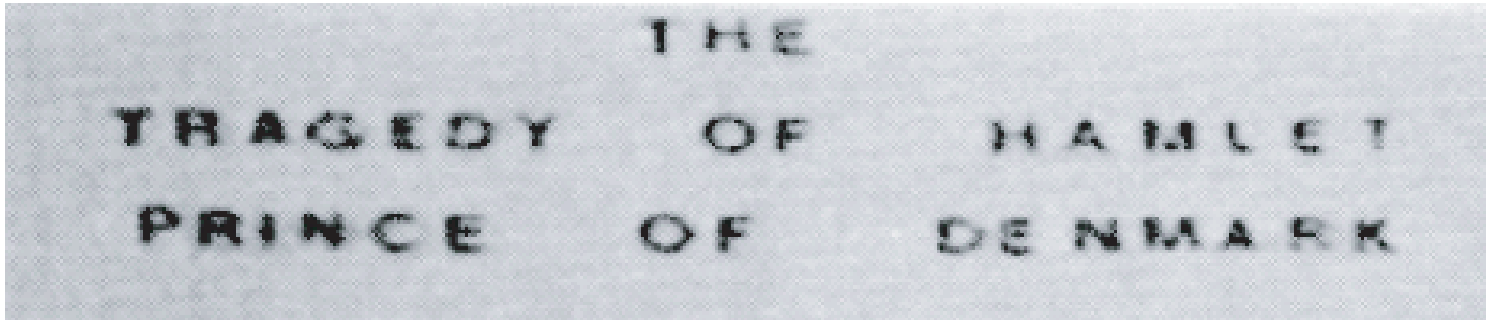
From data w/ errors, $C(q)$, determine the source $S(r)$.
Requires inversion of the kernel K.

Optical recognition: K - blurring function, max entropy method

C:

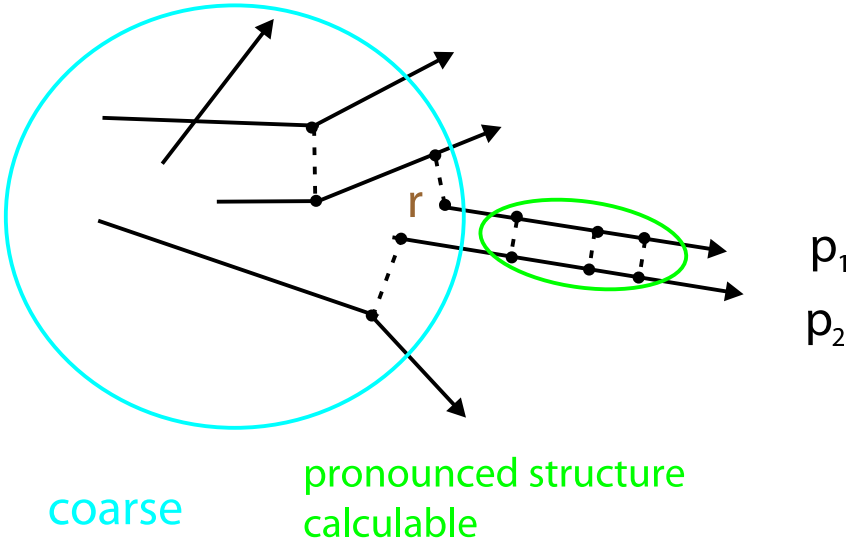


S:



Electron scattering: $d\sigma/dq \to \rho(r)$

Factorization of Final-State Amplitude in Reactions



2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\underbrace{\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}_{\text{data}} = \int \mathrm{d}\mathbf{r}\, \underbrace{S'_{\mathbf{P}}(\mathbf{r})}_{\text{source}}\, \underbrace{|\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2}_{\text{2-ptcle wf}}$$

S': distribution of emission points in 2-ptcle CM

Normalizing with 1-ptcle cross sections yields correlation f:

$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_2}} = \int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

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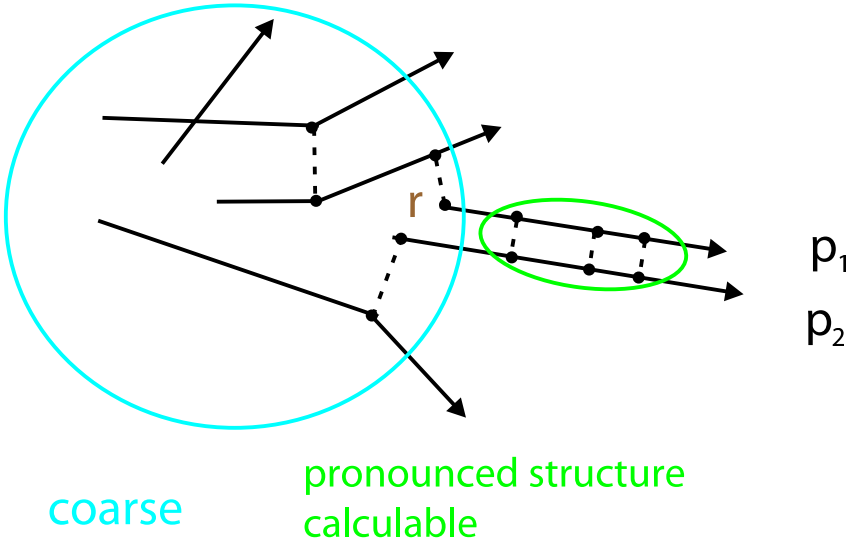

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Integral Relation

Of interest the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int d\mathbf{r} \left(|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int d\mathbf{r}\, K(\mathbf{q}, \mathbf{r})\, S(\mathbf{r})$$

Learning about S possible when scat wf $|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair. For some ptcle pairs, it may easier to learn about S than for others. Ease of learning about shape anisotropy may depend on a pair.

For pure interference, π^0's or γ's, $\Phi_{\mathbf{q}}^{(-)}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}} \right)$, kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$:

$$K(\mathbf{q}, \mathbf{r}) = |\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 = \cos(2\mathbf{q}\mathbf{r})$$

$$\Rightarrow \quad \mathcal{R}(\mathbf{q}) = \int d\mathbf{r} \cos(2\mathbf{q}\mathbf{r}) S(\mathbf{r})$$

3D cosine-transform source-correlation relation.
Brown&PD, PLB398(97)252

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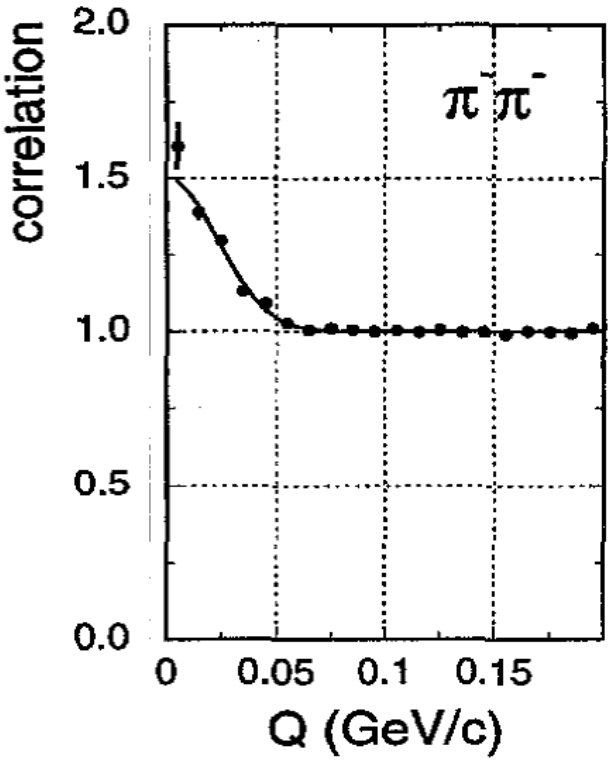
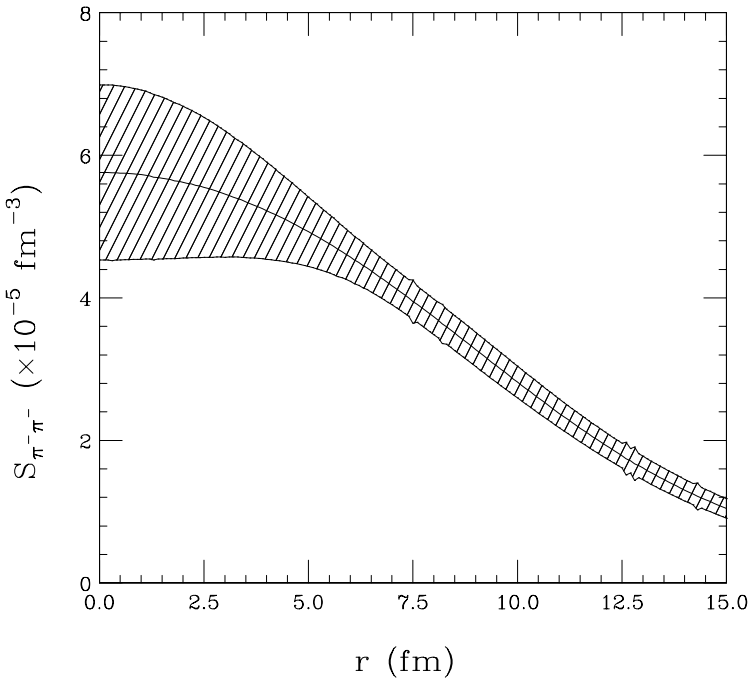
Fourier-Transform of Angle-Averaged Correlation



Coulomb-corrected 1D π^--π^- correlation-function (Miskowiec *et al.*)



restored source: relative dstr of π^--π^- emission pts in central Au+Au at 10.8 GeV/c (E877)

Brown, PD PLB398(97)252

$S(r \to 0)$: entropy, freeze-out density (Brown, PD, Panitkin ...)

$S(0) \searrow \Leftrightarrow$ entropy $\nearrow$

Multipole Decomposition

$$\boxed{\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\mathbf{r}\, K(\mathbf{q},\mathbf{r})\, S(\mathbf{r})}$$

source-correlation relation

but a spin-averaged kernel depends on the relative angle only

$$K(\mathbf{q},\mathbf{r}) = K(q, r, \cos\theta_{\mathbf{qr}}) = \sum_{\ell} (2\ell+1)\, K_\ell(q,r)\, P^\ell(\cos\theta_{\mathbf{qr}})$$

With

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \quad S(\mathbf{r}) = \sqrt{4\pi} \sum S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D relations for different deformation coefficients:

$$\boxed{\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S^{\ell m}(r)}$$

Different deformation coefficients for the source and correlation functions are directly related to each other.

For pure interference $K_\ell(q,r) = (-1)^{\ell/2} j_\ell(2qr)$, for even ℓ only, where j_ℓ is spherical Bessel function.

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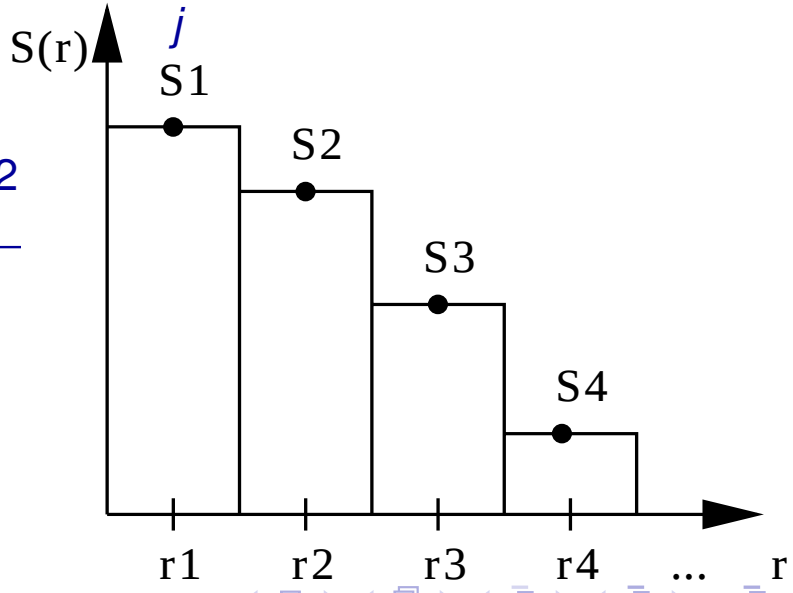
1D Source Imaging

The $\ell = 0$ 1D relation connects angle-averaged source $S(r)$ and correlation $\mathcal{R}(q)$ functions, in terms of angle-averaged kernel K_0:

$$\mathcal{R}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_0(q, r)\, S(r)$$

Source discretization strategy works for any ptcle pair & ℓ

1. Discretize integral
$$\mathcal{R}_i = \sum_j 4\pi\, \Delta r\, r_j^2\, K_0(q_i, r_j)\, S(r_j) \equiv \sum_j K_{ij}\, S_j$$
2. Vary $S(r_j)$ to minimize χ^2:
$$\chi^2 = \sum_i \frac{(\sum_j K_{ij}\, S_j - \mathcal{R}_i^{exp})^2}{\sigma_i^2}$$
3. Mtx result from minimization:
$$S = (K^\top K)^{-1}\, K^\top\, \mathcal{R}^{exp}$$

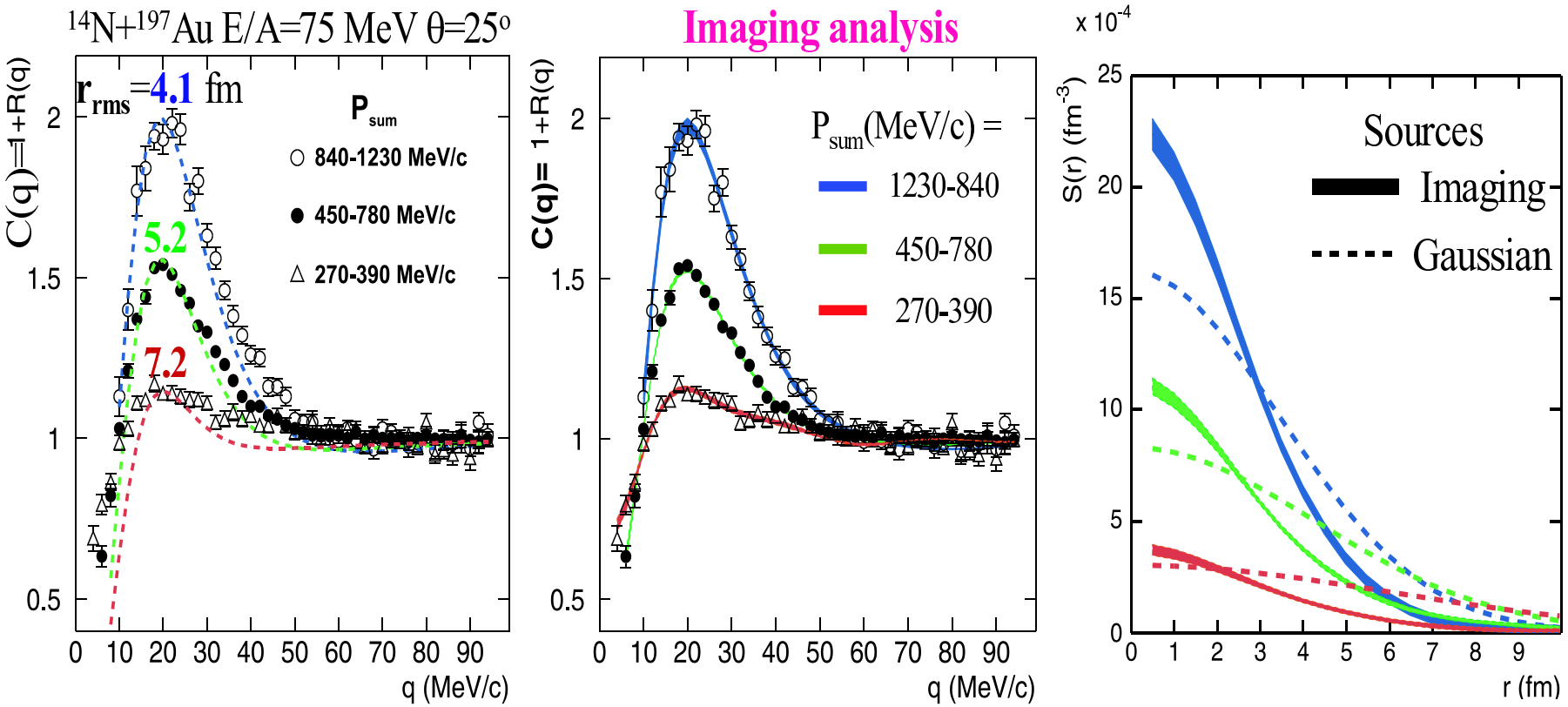


Brown&PD, PRC57(98)2474

Understanding 1D pp Correlations

Altered interpretation of C_{pp}: Gaussian fit vs imaging

Verde PRC65(02)054609

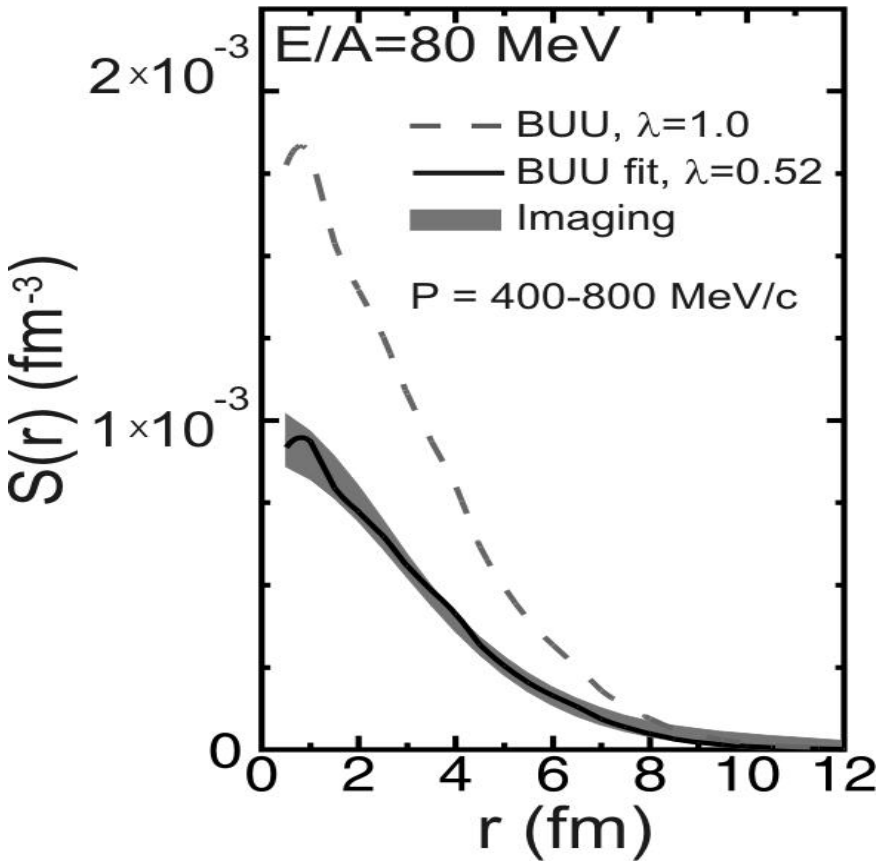


Gauss par: quickly changing radii. Imaging: quickly changing preequilibrium fraction, non-Gaussian source shapes!

Preequilibrium fraction: $\int^{r<R} d\mathbf{r}\, S \equiv \lambda$

Imaged pp Source Compared to Transport

Verde PRC67(03)034606: Ar+Sc central collisions at 80 MeV/u, fast $400 < P_{tot} < 800$ MeV/c pairs



Nucleon-based transport reproduces correctly the *shape* of the preequilibrium source.

The transport *cannot* describe correctly the preequilibrium pair fraction.

Towards Anisotropy: Basis in Spherical Angle

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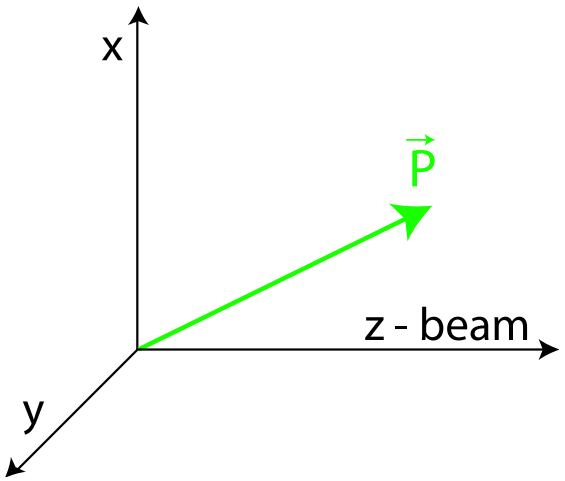

Problem: Why giving up real quantities, R & S, for imaginary, $R^{\ell m}$ & $S^{\ell m}$?

Another basis??

Take the direction vector: $\hat{n}_\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Rank-ℓ tensor product:

$$(\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \equiv \hat{n}_{\alpha_1}\, \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell} = \sum_{\ell' \le \ell, m} c_{\ell' m}\, \mathrm{Y}^{\ell' m}$$

$\mathcal{P}^{(\ell,\ell)}$ projection operator that, within the space of rank-ℓ cartesian tensors, removes $\mathrm{Y}^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{P}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} = \sum_m c_{\ell m}\, \mathrm{Y}^{\ell m}$$

The components $\mathcal{P}\hat{n}^\ell$ are real and can be used to replace $\mathrm{Y}^{\ell m}$

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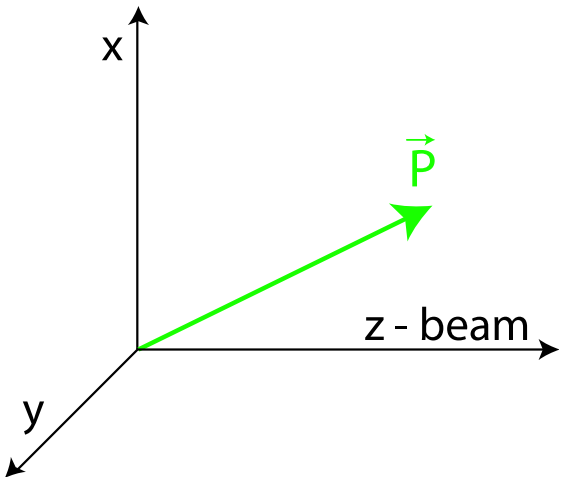

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Low-ℓ Cartesian Harmonics

$$
\begin{aligned}
\mathcal{P}\hat{n}^0 &= 1 \\
(\mathcal{P}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\
(\mathcal{P}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\
&\vdots
\end{aligned}
$$

Applequist Theor. Chem. Acc 107(02)103

PD&Pratt PLB618(05)60

$\mathcal{P}$ can be called a detracing operator as

$$\sum_\alpha (\mathcal{P}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0$$

Completeness relation ($\mathcal{P} = \mathcal{P}^\top = \mathcal{P}^2$):

$$
\begin{aligned}
\delta(\Omega' - \Omega) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n'}^\ell)_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \\
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Consequences

- $$\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\Omega'\, \delta(\Omega' - \Omega)\, \mathcal{R}(\mathbf{q}') = \sum_{\ell} \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q)\, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

  where $$\mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{\mathrm{d}\Omega_{\mathbf{q}}}{4\pi}\, \mathcal{R}(\mathbf{q})\, (\mathcal{P}\hat{q}^{\ell})_{\alpha_1 \ldots \alpha_\ell}$$

- Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

$$\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)$$

- For weak anisotropies, only lowest-ℓ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q)\, \hat{q}_\alpha + \sum_{\alpha_1\, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q)\, \hat{q}_{\alpha_1}\, \hat{q}_{\alpha_2} + \ldots$$

  monopole, dipole, quadrupole...

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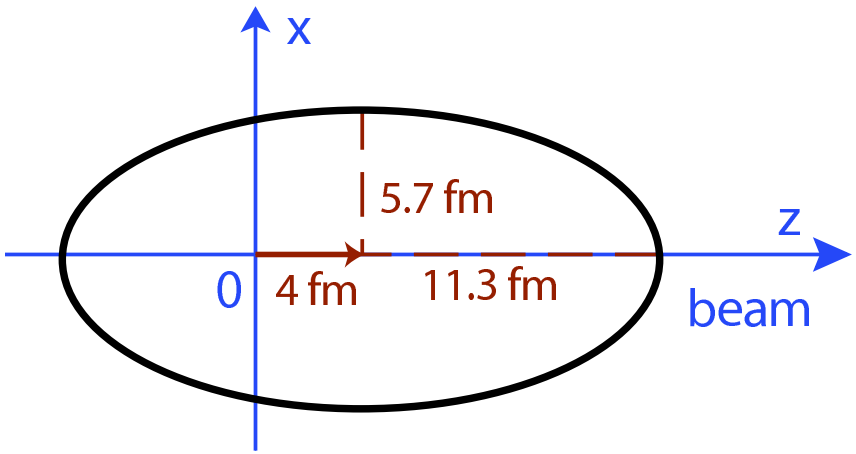
- For weak anisotropies, only lowest-ℓ matter:

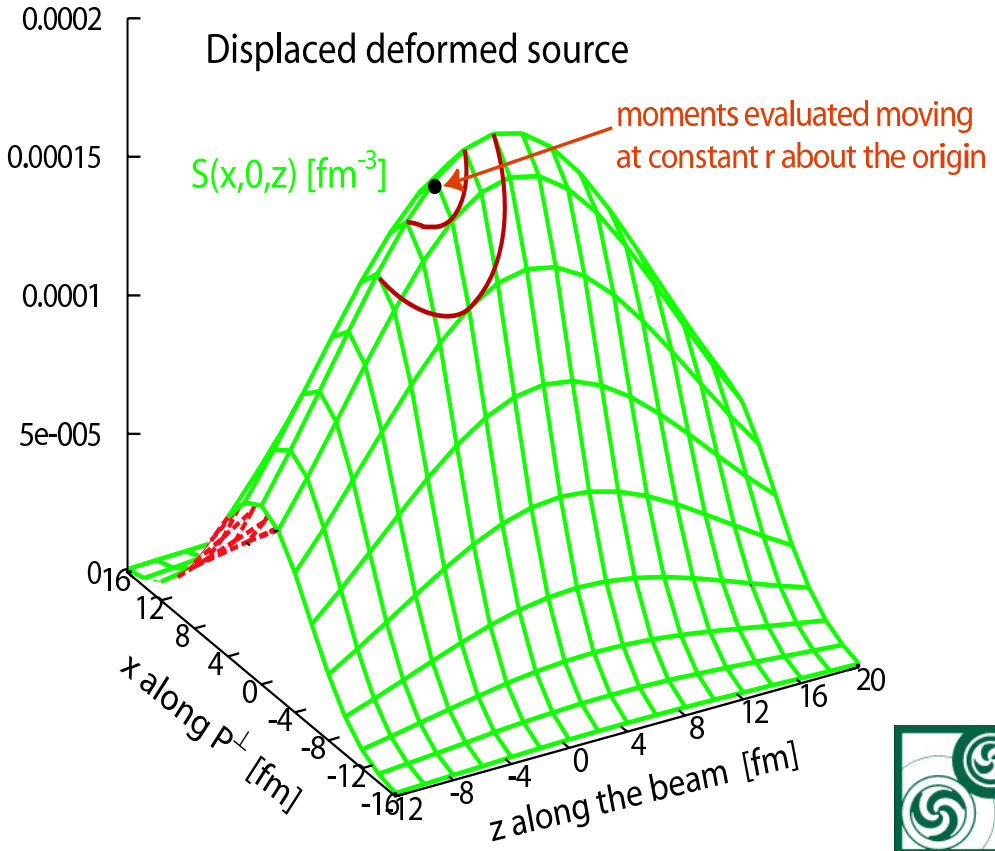
  $$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q)\, \hat{q}_\alpha + \sum_{\alpha_1\, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q)\, \hat{q}_{\alpha_1}\, \hat{q}_{\alpha_2} + \ldots$$

  monopole, dipole, quadrupole...

Sample Gaussian Source

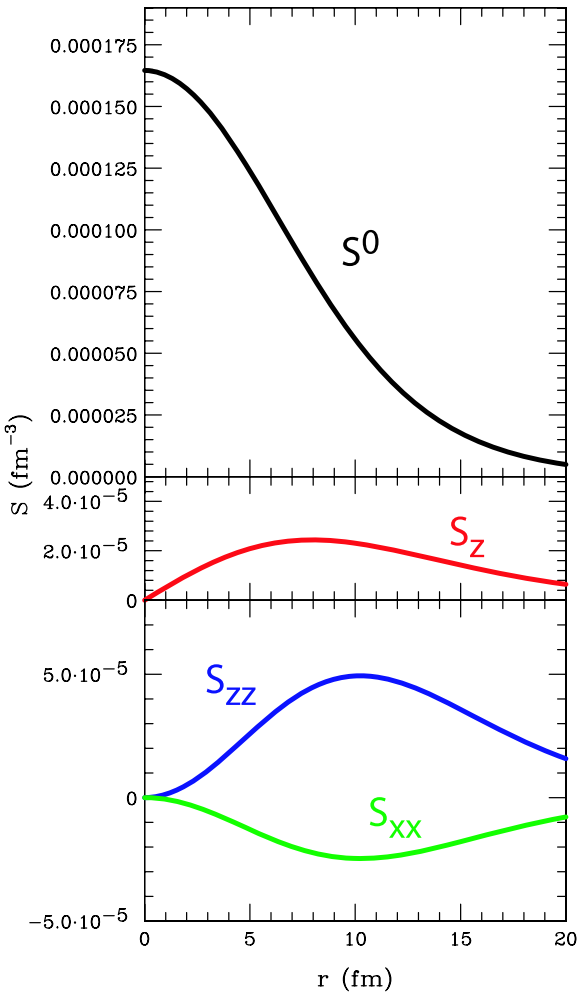
Anisotropic Gaussian, elongated and displaced along the beam axis



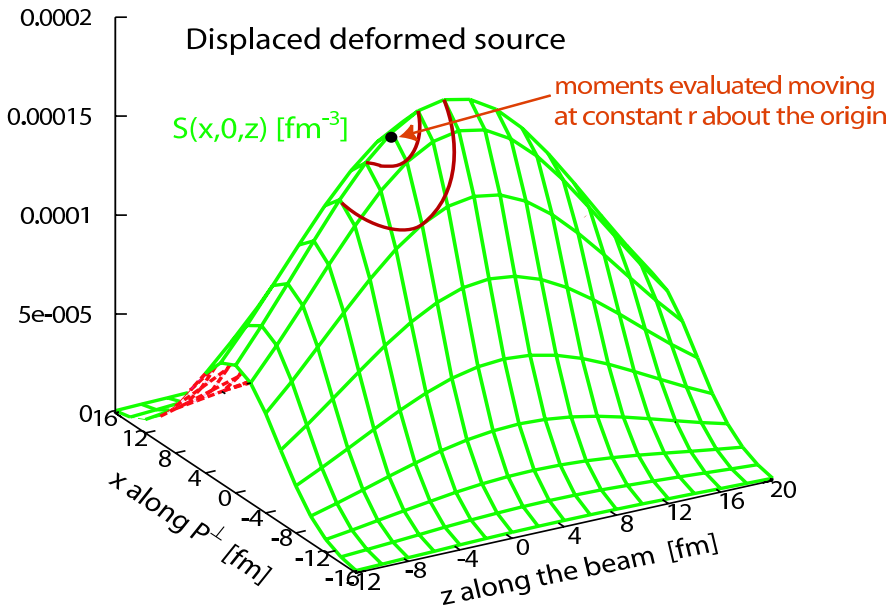


Low-ℓ Characteristics

3D info in terms of 1D plots



$S^{(\ell)} \propto r^\ell$



Along z-axis:

$$S(r) = S^0(r) + S_z(r) + S_{zz}(r) + \ldots$$

In xz plane, at 45° to z-axis:

$$S(r) = S^0(r) + \frac{1}{\sqrt{2}} S_z(r) + \frac{1}{2}\left(S_{xx}(r) + S_{zz}(r)\right)$$

$$S_{xx} + S_{yy} + S_{zz} = 0$$

Source & Correlation Symmetries

Midrapidity identical-π sources in A+A collisions, e.g. NA49 Pb+Pb at SPS or PHENIX/STAR Au+Au at RHIC:

- Identical ptcles: $\vec{r} \to -\vec{r} \Leftrightarrow$ even-ℓ only
- Midrapidity: $z \to -z$ (beam dir) $\Leftrightarrow$ even-z moments only
- Reaction-plane averaging: $y \to -y$ (sideways) $\Leftrightarrow$ even-y moments only
- In the end, also: $x \to -x$ (outward) $\Leftrightarrow$ even-x moments only

- $\ell = 0$
- $\ell = 2$: x^2, y^2, z^2 (only 2 independent)
- $\ell = 4$: $x^4, y^4, z^4, \; x^2 y^2, x^2 z^2, y^2 z^2$ (only 3 independent)

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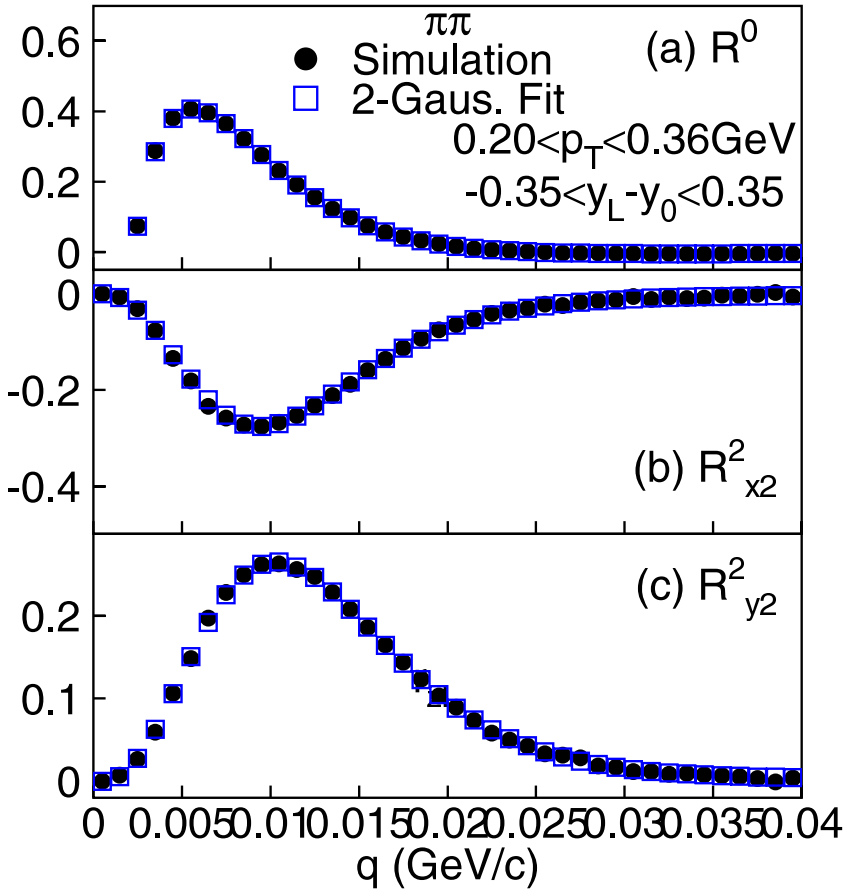
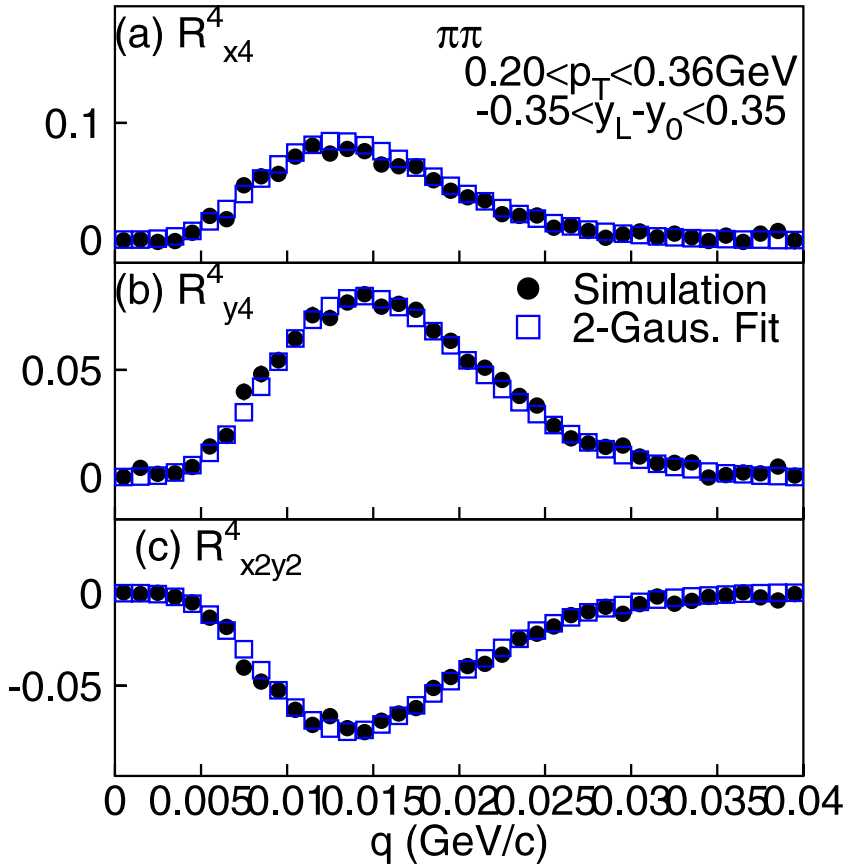
$\ell = 0$-4 Correlation Moments

Emission simulated from Gaussian source characterized by $R_x = 7$ fm, $R_y = 5$ fm, $R_z = 10$ fm and $\tau = 5$ fm/c.

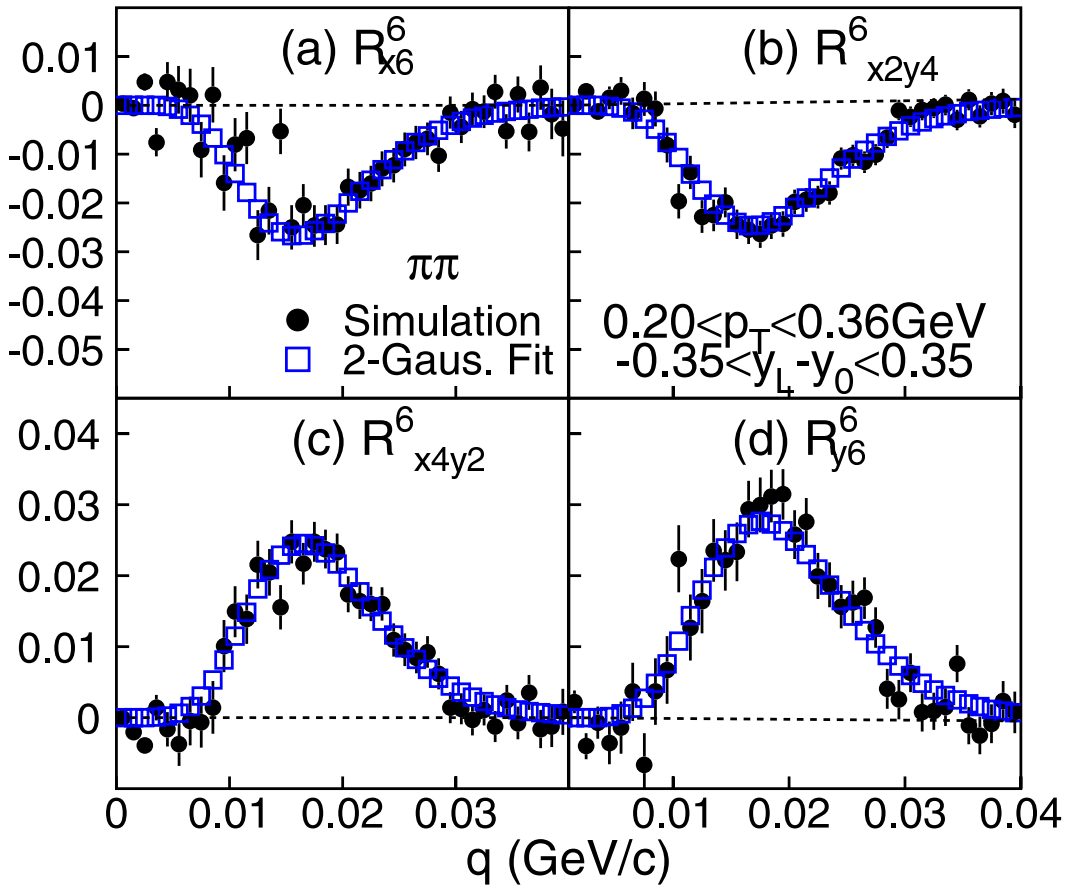




Simulation + fit to simulated correlation

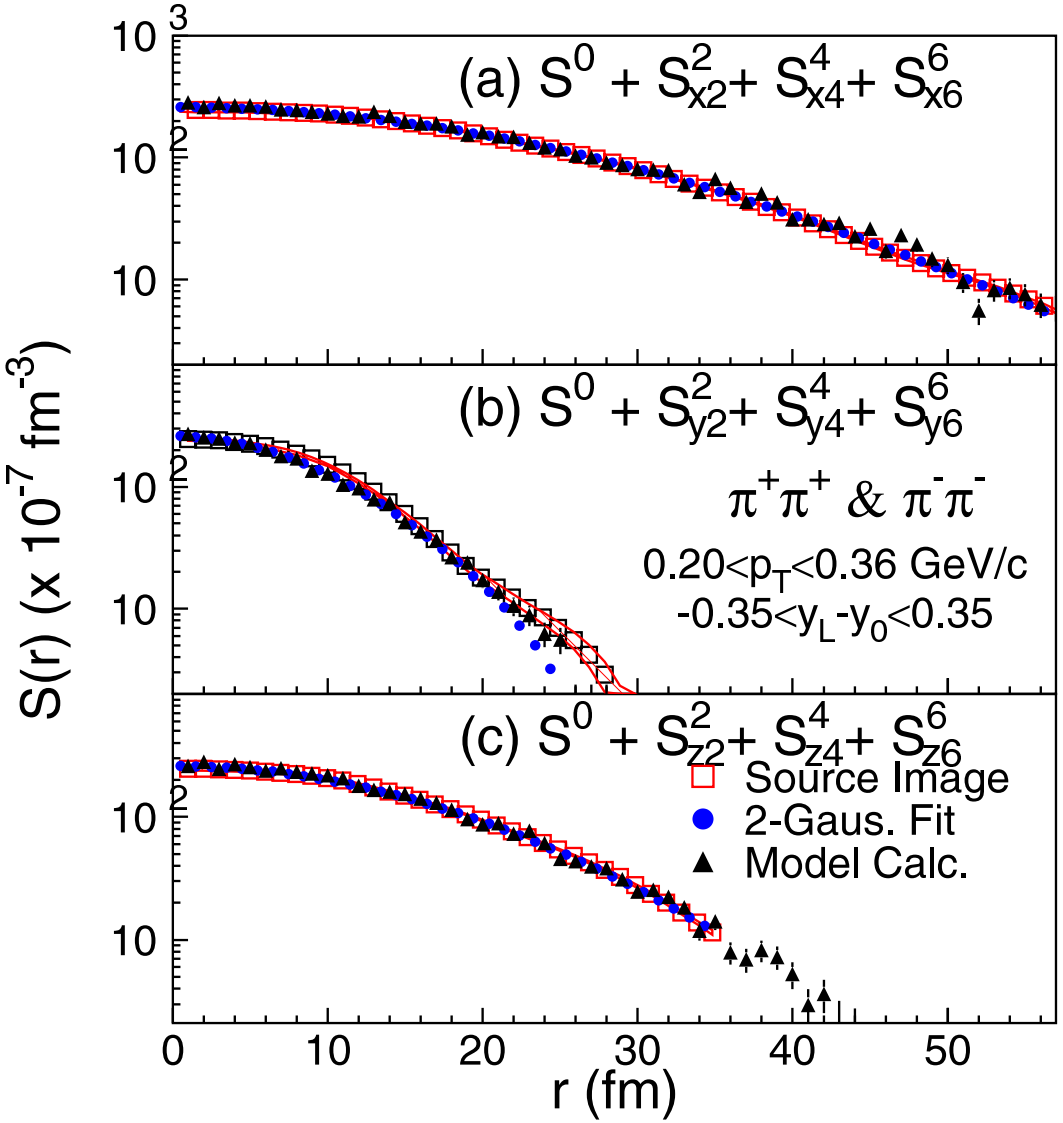
$\ell = 6$ Correlation Moments

Gaussian w/$R_x = 7$ fm, $R_y = 5$ fm, $R_z = 10$ fm and $\tau = 5$ fm/c.



Coulomb repulsion is fully accounted for and contributes to correlation anisotropy.

Test Source Restoration



From moments, function fit & imaging

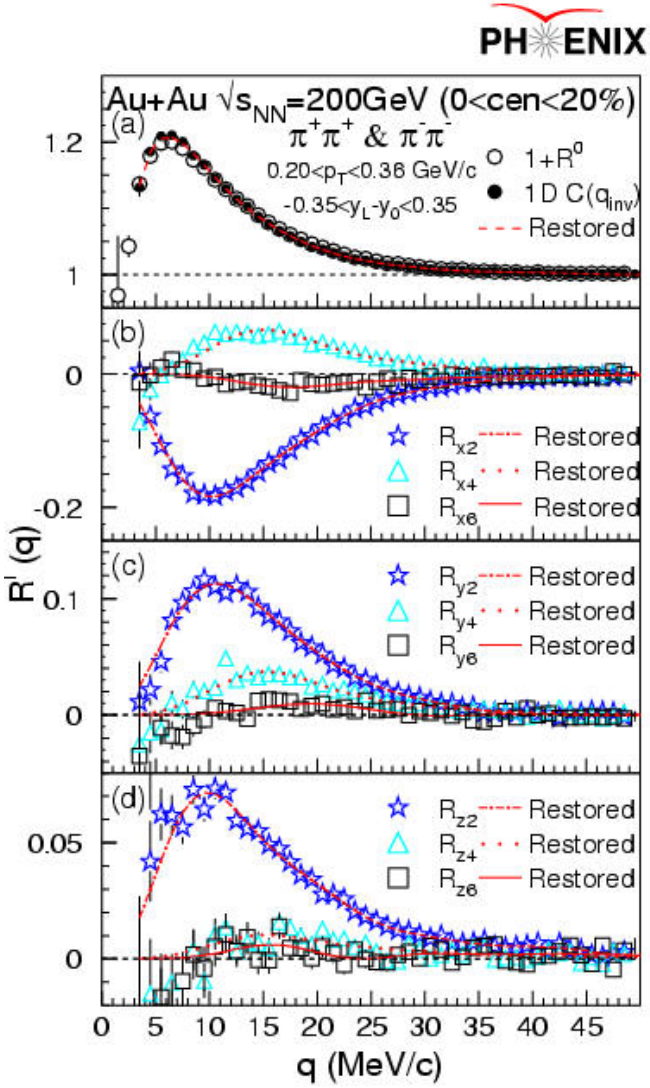
$R_x = 7$ fm, $R_y = 5$ fm, $R_z = 10$ fm and $\tau = 5$ fm/c.

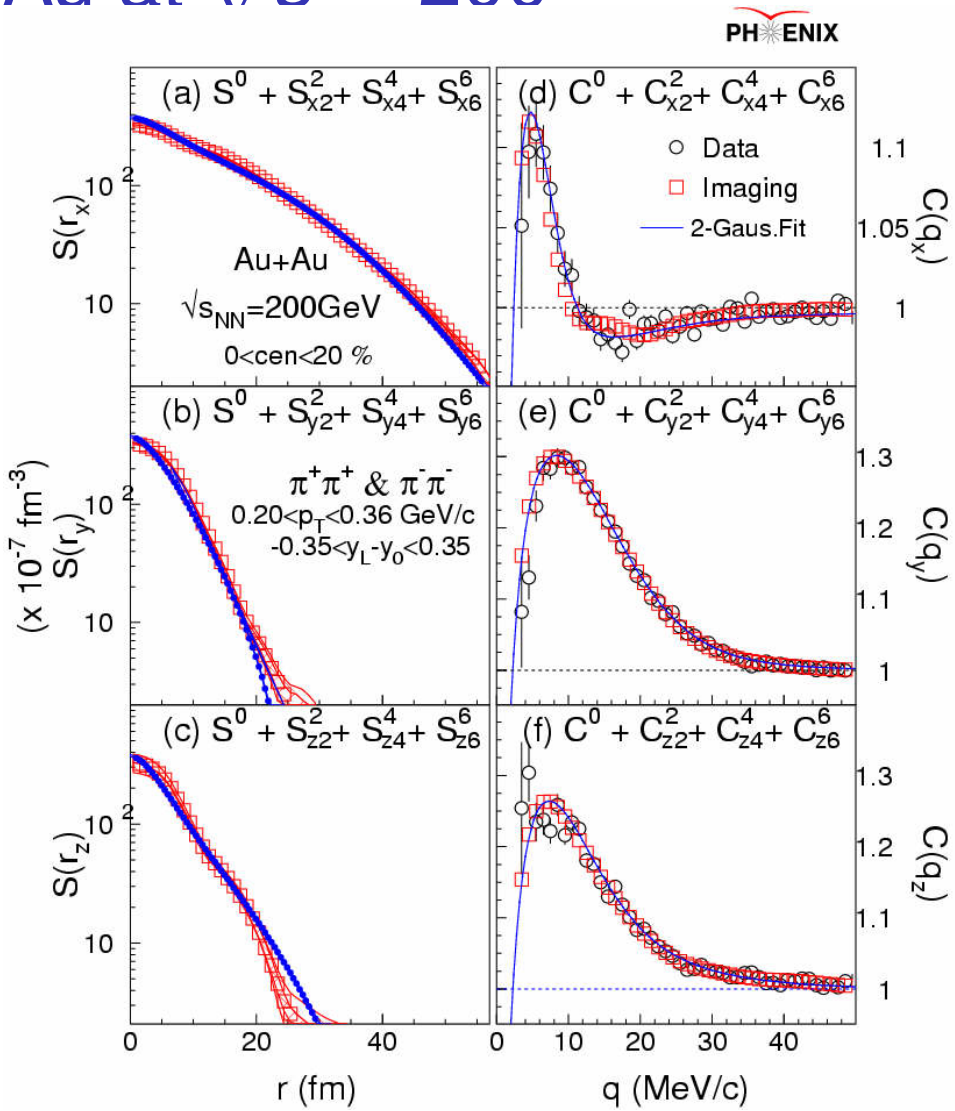
z - longitudinal, along beam axis

x - outward, along pair mo

y - perpendicular to x and y

$\pi\pi$ in Au+Au at $\sqrt{s} = 200$





Data/restoration

Reach up to $r \sim 50$ fm

Conclusions

- Correlations at low relative velocities yield access to source spatial characteristics in the pair CM! Temporal information in the pair CM is not directly accessible.
- Different final-state effects can provide information on source asymmetry, including identity interference, Coulomb and strong interactions.
- Cartesian harmonics provide easy means f/representing & manipulating info in functions dependent on spherical angle, such as correlation and source functions.
- Given data with high resolution, restoration can yield access to sources at far-away distances, up to ~ 50 fm.

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