Fragmentation of the Fireball and Event-by-Event Fluctuations of Rapidity Distributions

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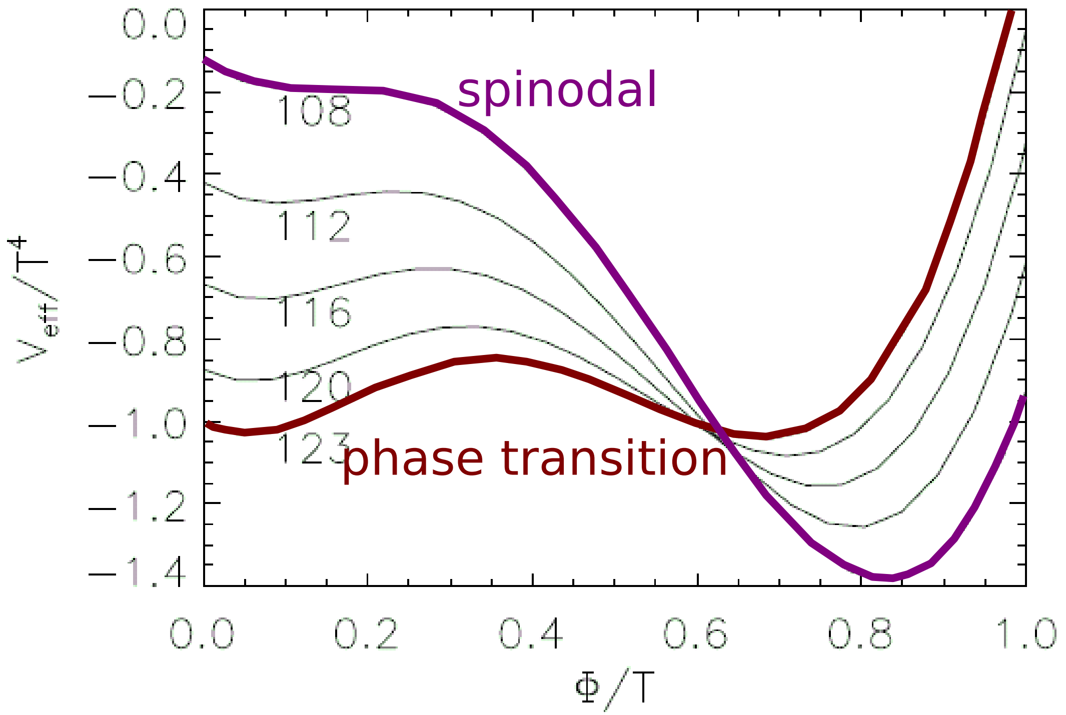
Fragmentation: where and how to see it

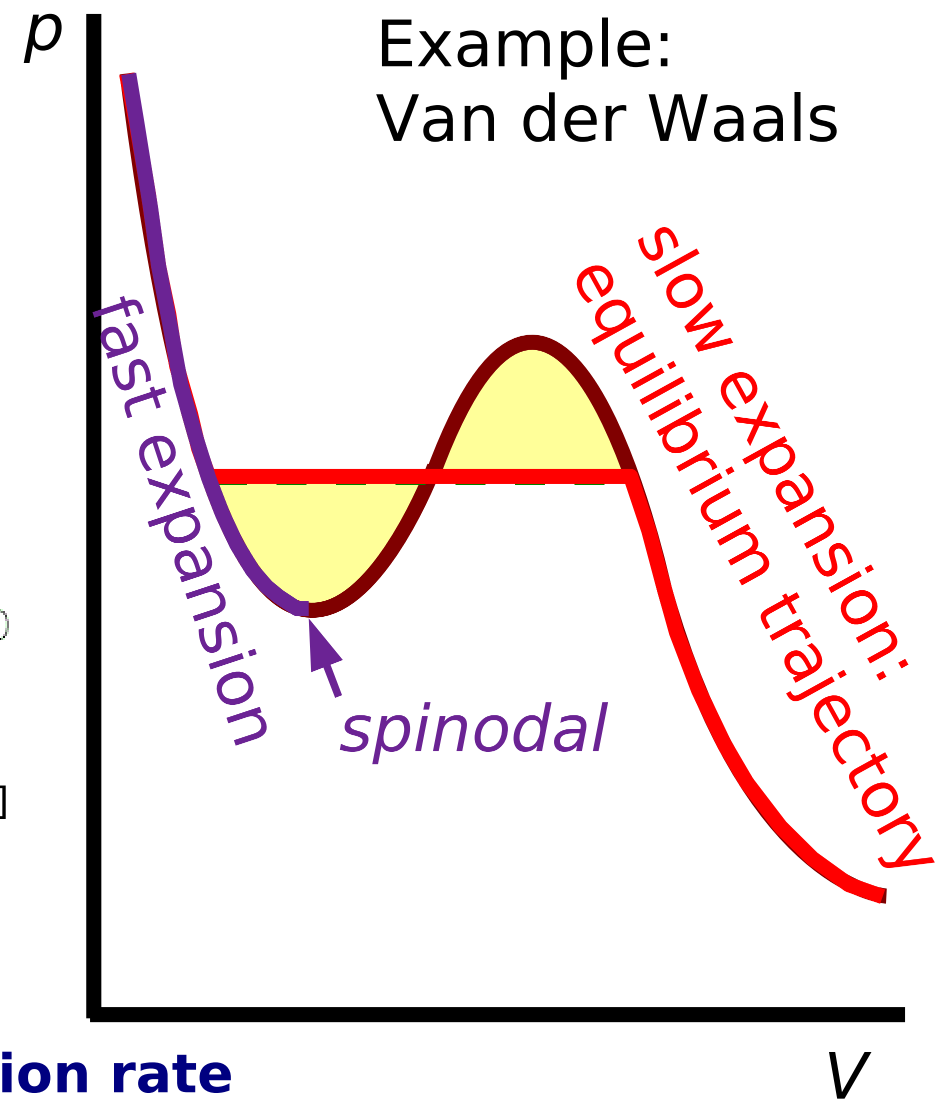
- Fragmentation
  - at 1st order PT: spinodal decomposition
  - at crossover: due to bulk viscosity

- How to see it?
  - Look for differing rapidity distributions
  - Look at rapidity correlations
  - ...

Spinodal fragmentation at phase transition



Example: linear sigma model coupled to quarks
[O. Scavenius *et al.*, Phys. Rev. D **63** (2001) 116003]



What is fast?

bubble nucleation rate < expansion rate

Bubble nucleation rate

eg. [L. Csernai, J. Kapusta: Phys. Rev. Lett. **69** (1992) 737]

nucleation rate:

$$\Gamma = \Gamma_0 \exp\left(-\frac{\Delta F_*}{T}\right)$$

difference in free energy:

$$\Delta F = \frac{4\pi}{3}\left[p_q(T) - p_h(T)\right] R^3 + 4\pi R^2 \sigma$$

critical size bubble:

$$R_*(T) = \frac{2\sigma}{p_h(T) - p_q(T)}$$

at phase transition

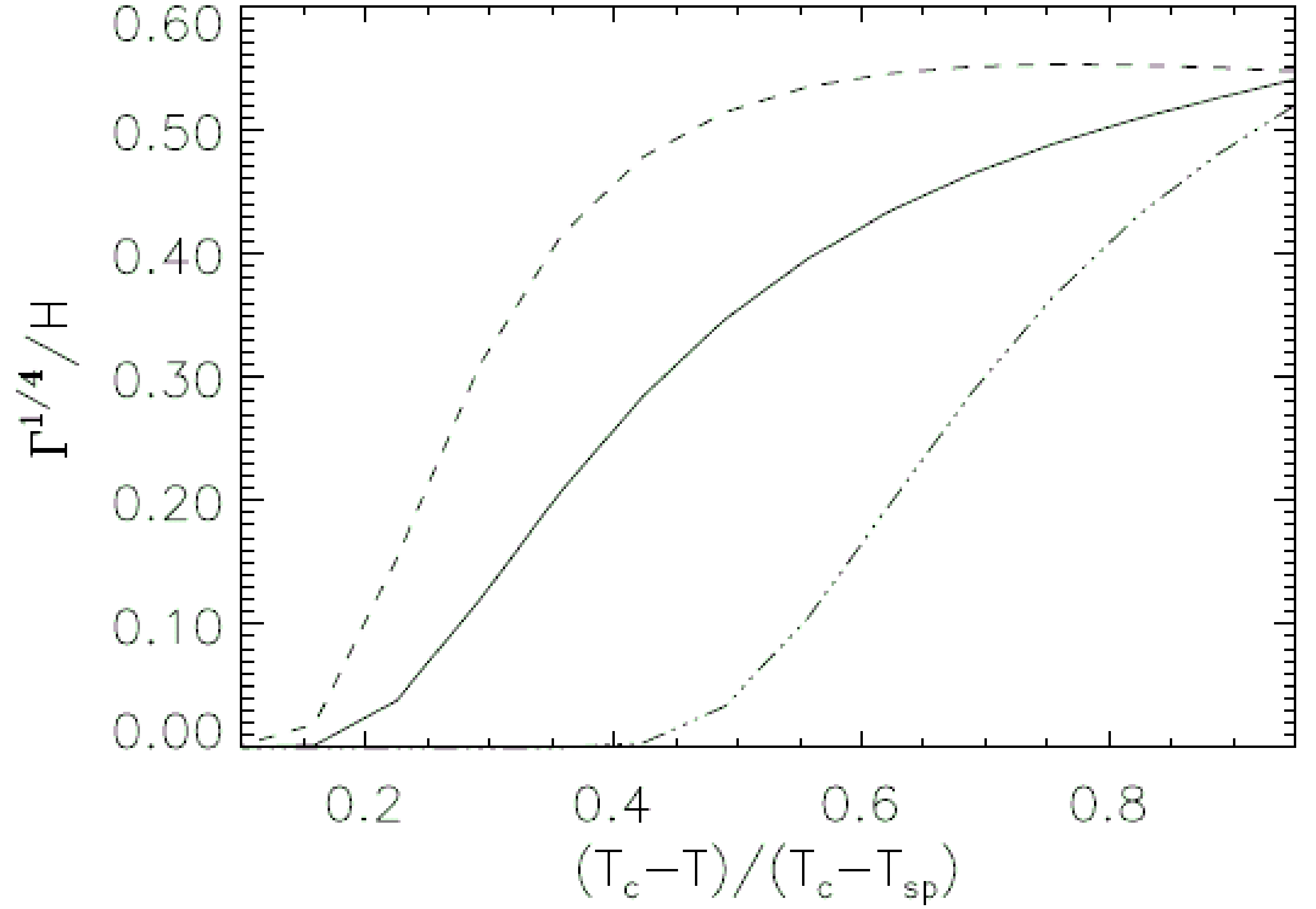
$$R_*(T_c) \to \infty$$

=> must supercool

Supercooling down to spinodal

O. Scavenius *et al.*, Phys. Rev. D **63** (2001) 116003:

compare nucleation rate with Hubble constant (1D)



likely to reach spinodal

The size of droplets

[I.N. Mishustin, Phys. Rev. Lett. **82** (1998), 4779; Nucl. Phys. A**681** (2001), 56c]

Minimize thermodynamic potential per volume

$$\frac{\Delta\Omega}{V} = \frac{\Delta\Omega_{\rm bulk} + \Delta\Omega_{\rm surf} + \Delta\Omega_{\rm kin}}{V}$$

where

$$\begin{aligned} \Delta\Omega_{\rm bulk} &= -\frac{4\pi}{3} R^3 \left(p_q - p_h\right) \\ \Delta\Omega_{\rm surf} &= 4\pi R^2 \sigma \\ \Delta\Omega_{\rm kin} &= \frac{2\pi}{5} R^5 \Delta\mathcal{E}\,\mathcal{H}^2 \end{aligned}$$

this gives

$$R_* = \left(\frac{5\sigma}{\Delta\mathcal{E}\,\mathcal{H}^2}\right)^{1/3}$$

also [D.E. Grady, J.Appl.Phys. **53** (1982) 322]

Bulk viscosity near T_c

Bulk viscosity increases near T_c:

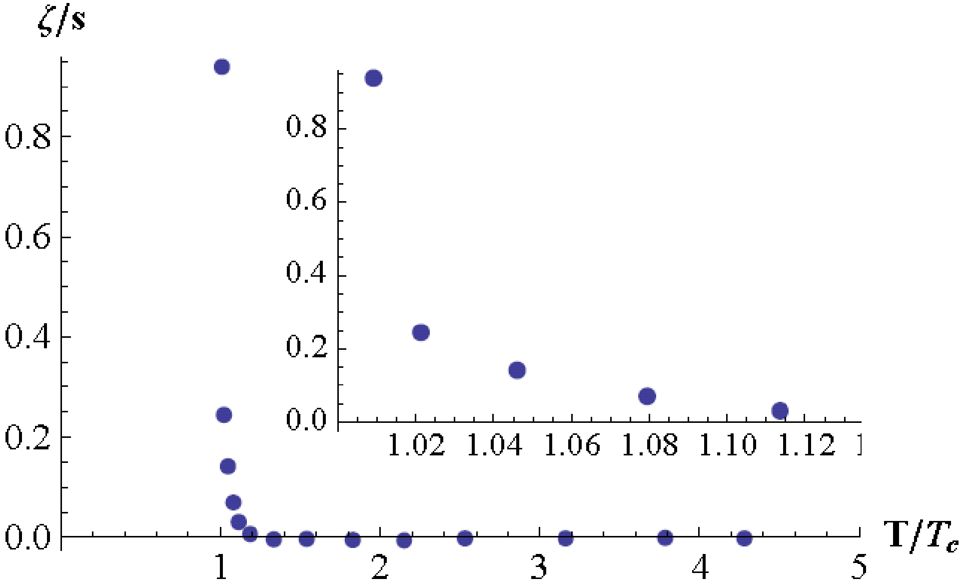
[K. Paech, S. Pratt, Phys. Rev. C **74** (2006) 014901,
D. Kharzeev, K. Tuchin, arxiv:0705.4280 [hep-ph]]

Kubo formula for bulk viscosity:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3r \, e^{i\omega t} \langle [\theta^\mu_\mu(x), \theta^\mu_\mu(0)] \rangle$$

From lattice QCD,
using low energy theorems:

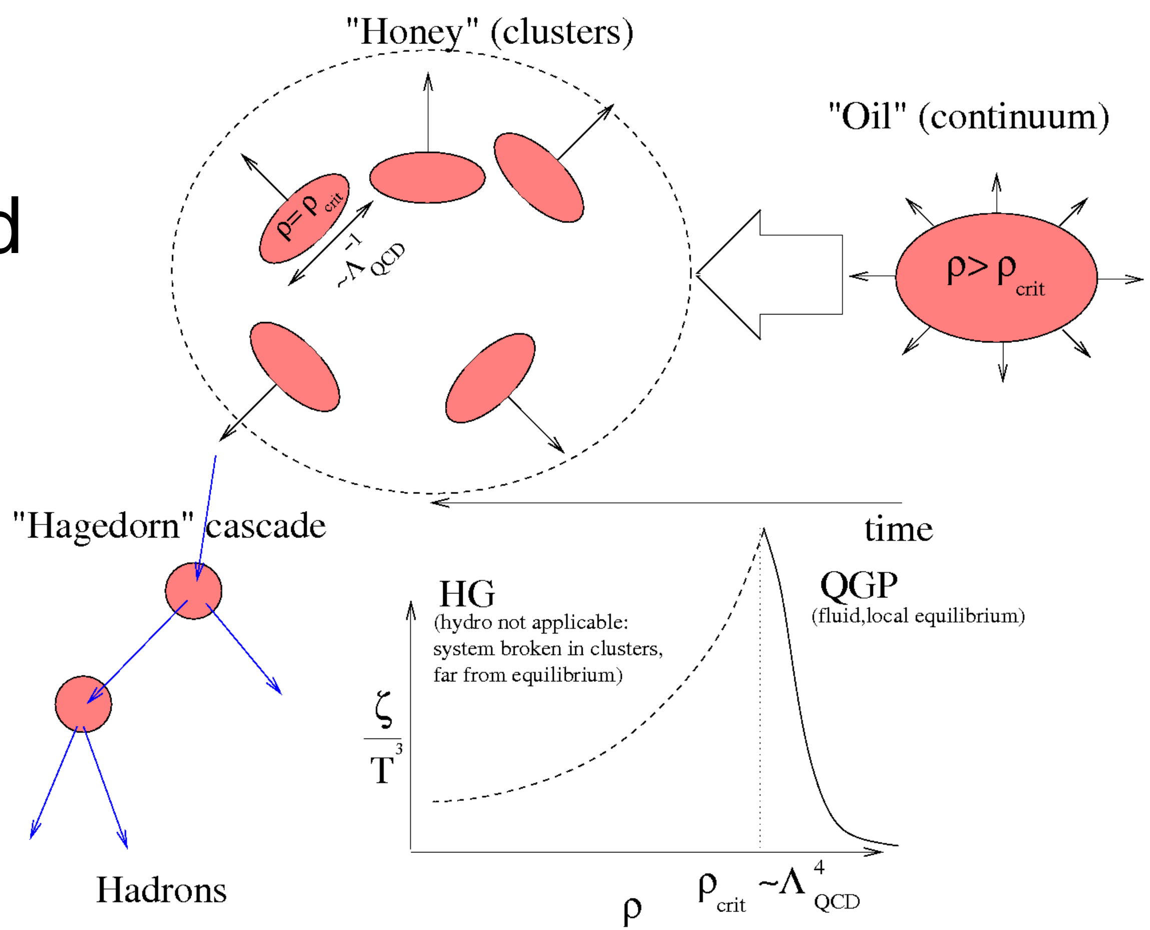
$$\zeta = \frac{1}{9\,\omega_0} \left\{ T^5 \frac{\partial}{\partial T} \frac{(\mathcal{E} - 3P)_{\rm LAT}}{T^4} + 16 |\epsilon_v| \right\}$$



Bulk-viscosity-driven fragmentation

... and freeze-out

1. (s)QGP expands easily
2. Bulk viscosity singular at critical temperature
3. System becomes rigid
4. Inertia may win and fireball will fragment
5. Fragments evaporate hadrons



“Sudden viscosity” => fragmentation

Energy-momentum tensor

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \zeta\, \partial_\rho u^\rho \left(g^{\mu\nu} - u^\mu u^\nu\right)$$

energy density decrease rate

$$\frac{1}{\varepsilon} u^\mu \partial_\mu \varepsilon = \frac{\varepsilon + p - \zeta \partial_\rho u^\rho}{\varepsilon} \partial_\mu u^\mu$$

fragment size:

kinetic energy = dissipated energy

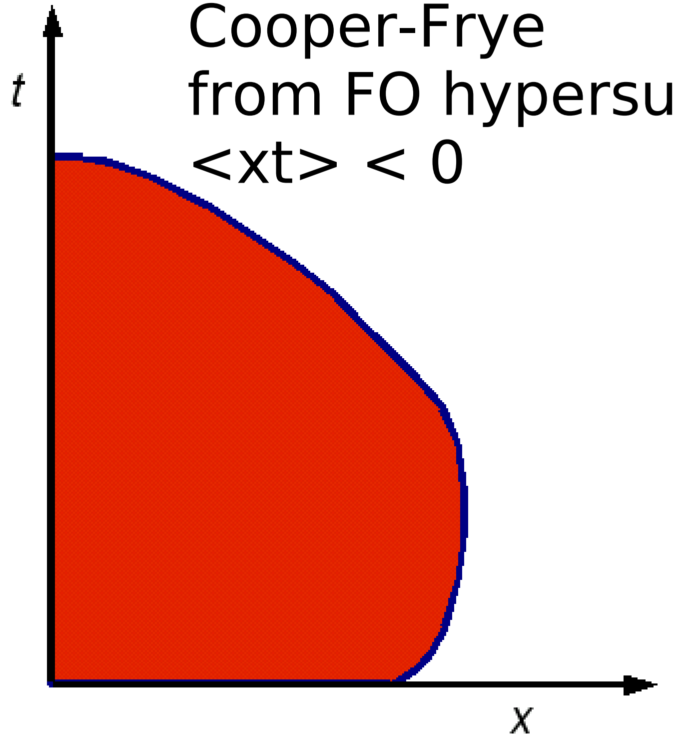
$$L^2 = \frac{24 \zeta_c \tau_c}{\varepsilon_c}$$

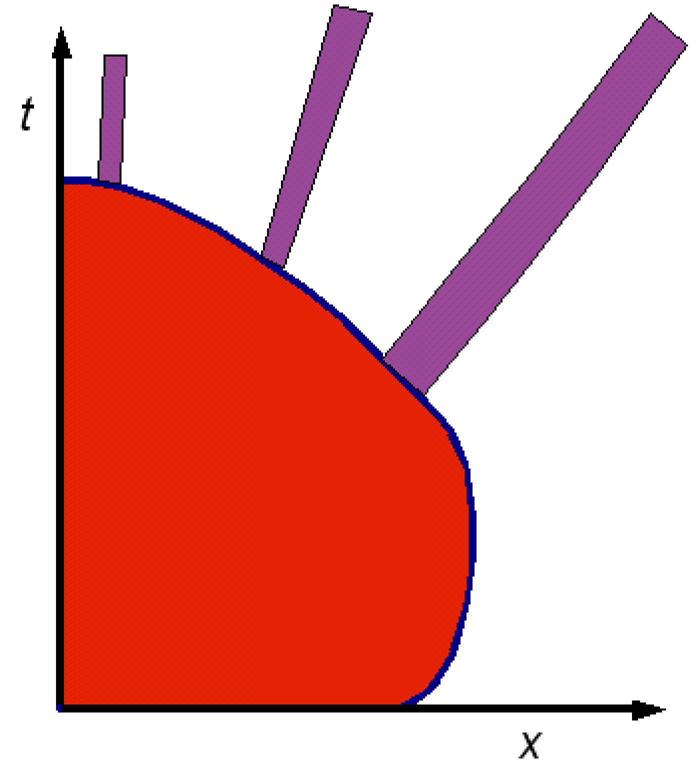
where $\zeta(\tau) = \zeta_c\, \tau_c\, \delta(\tau - \tau_c)$

Almost universal size!

Fragmentation and freeze-out: HBT puzzle

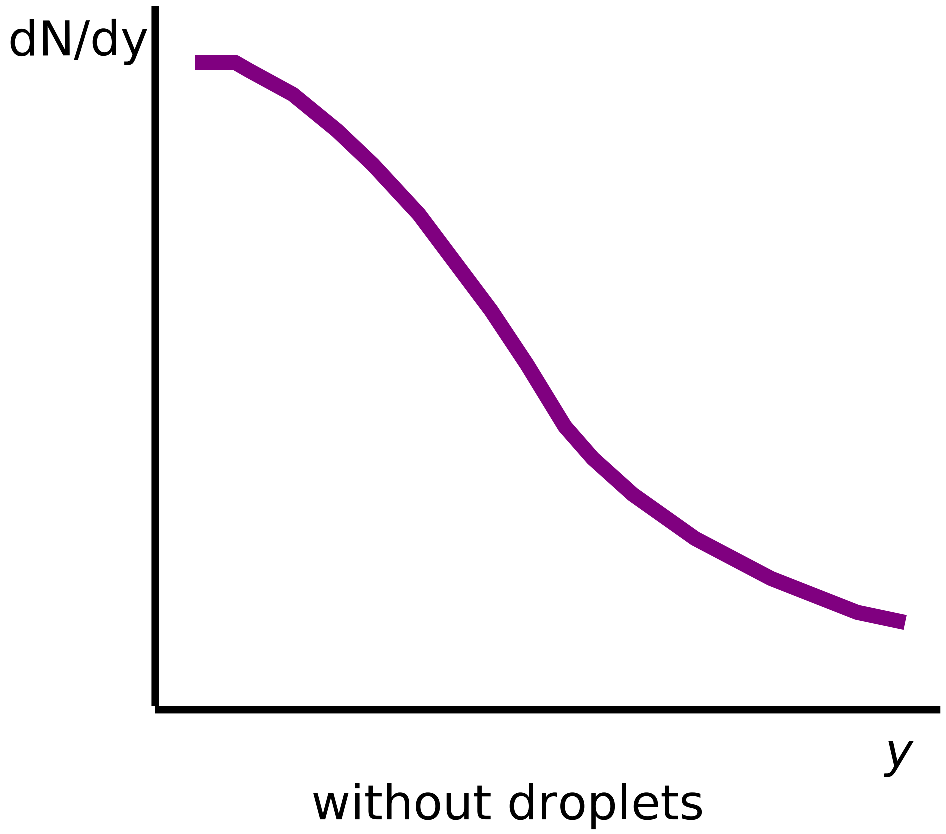
$$R_o^2 = \langle \tilde{x}^2 \rangle - 2\beta_t \langle \tilde{x}\tilde{t} \rangle + \beta_t^2 \langle \tilde{t}^2 \rangle$$
$$R_s^2 = \langle \tilde{y}^2 \rangle$$

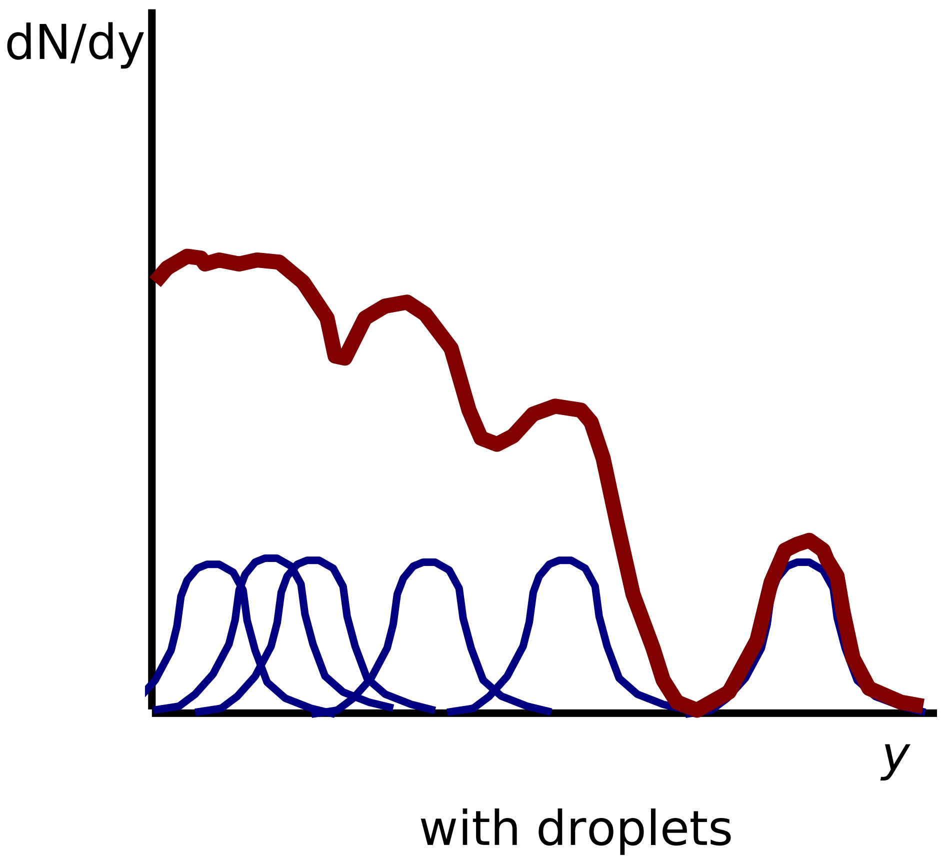




Droplets and rapidity distributions

rapidity distribution in a single event

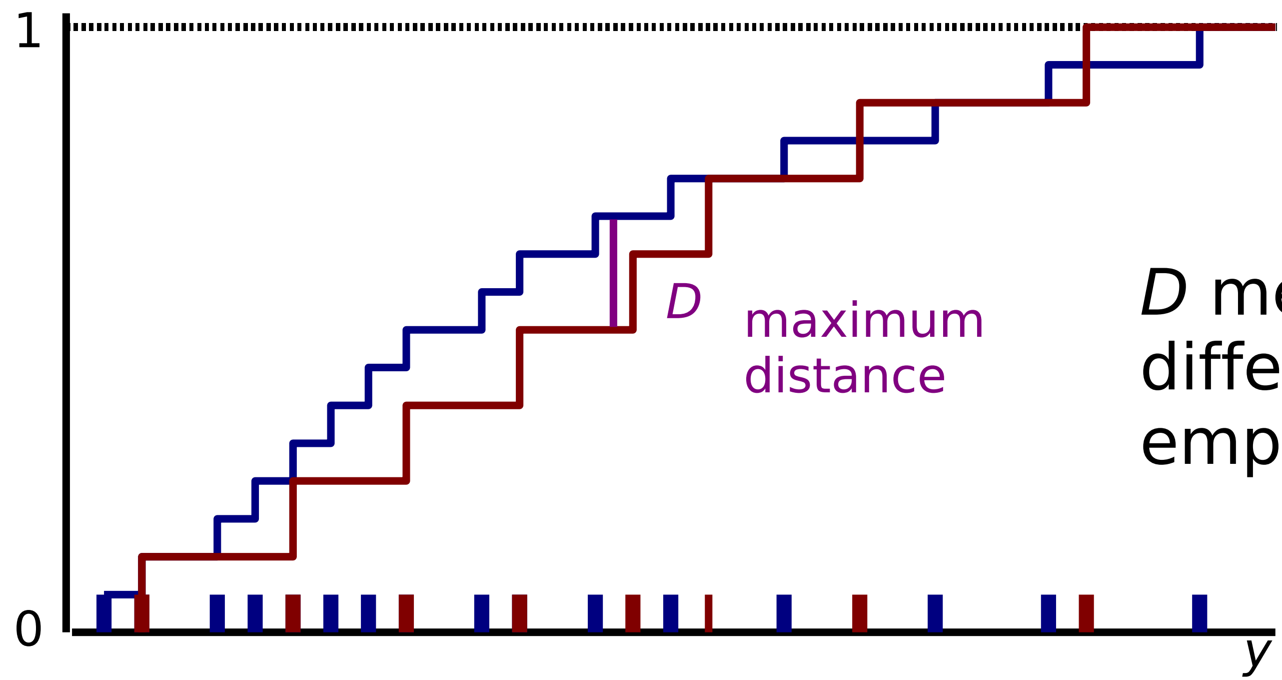




If we have droplets, each event will look differently

The measure of difference between events

- Kolmogorov–Smirnov test:
- Are two empirical distributions generated from *the same* underlying probability distribution?



D measures the difference of two empirical distributions

Kolmogorov-Smirnov: theorems

How are the *D*'s distributed?

Smirnov (1944):
If we have two sets of data generated from the same underlying distribution, then D's are distributed according to

$$\lim_{n_1,n_2\to\infty} P(\sqrt{n}D < t) = \sum_{k=-\infty}^{\infty} (-1)^k \exp\left(-2k^2t^2\right) \quad \textbf{with} \quad n = \frac{n_1 n_2}{n_1+n_2}$$

This is independent from the underlying distribution!

For each $t{=}D$ we can calculate

$$Q(\sqrt{n}D) = 1 - \sum_{k=-\infty}^{\infty} (-1)^k \exp\left(-2k^2\, n\, D^2\right)$$

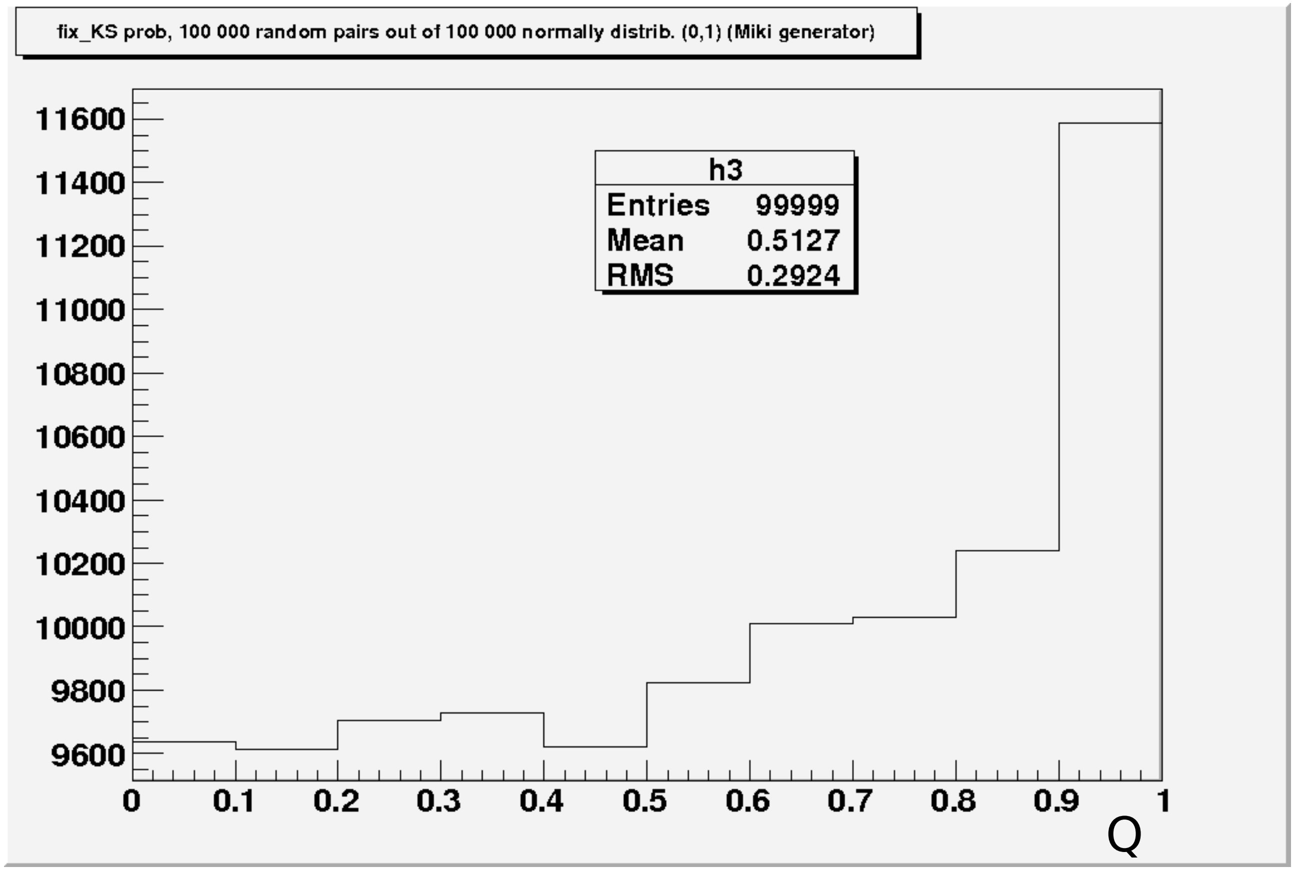
For events generated from the same distribution, Q's will be distributed **uniformly**.

KS-test on generated distributions

- random data generated according to half-normal distribution

- peak close to Q = 1 (“too similar” events)

WHY?



- we use asymptotic formula valid for very large *n*

results improve with larger *n* and/or correction terms

- we have limited number of significant figures

Does it matter in other e-by-e studies?

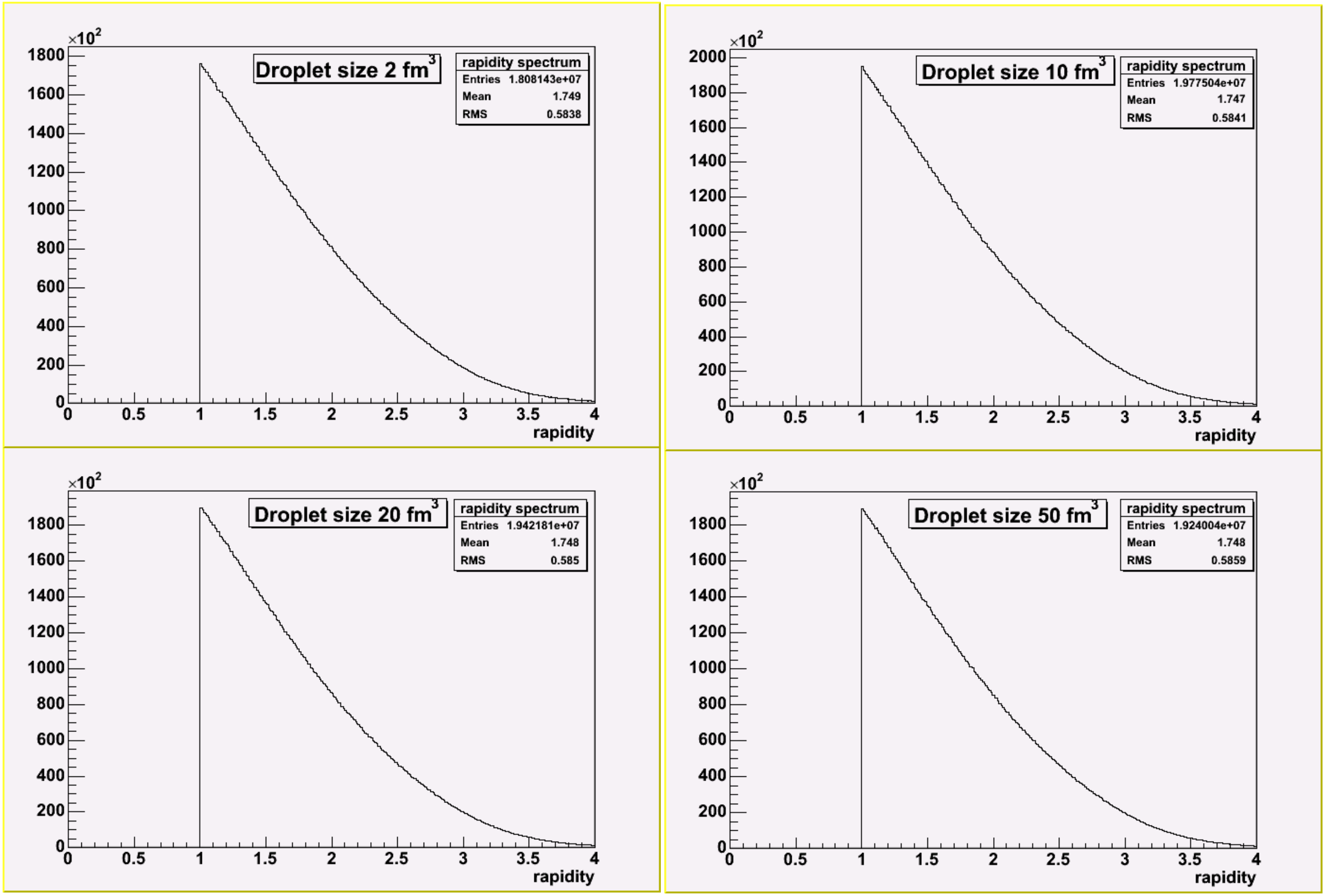
Droplet generator

MC generator of (momenta and positions of) particles

- some particles are emitted from droplets (clusters)
- droplets are generated from a blast-wave source (tunable parameters)
- droplets decay exponentially in time (tunable time, T)
- tunable size of droplets: Gamma-distributed or fixed
- no overlap of droplets
- also directly emitted particles (tunable amount)
- chemical composition: equilibrium with tunable params.
- rapidity distribution: uniform or Gaussian
- possible OSCAR output

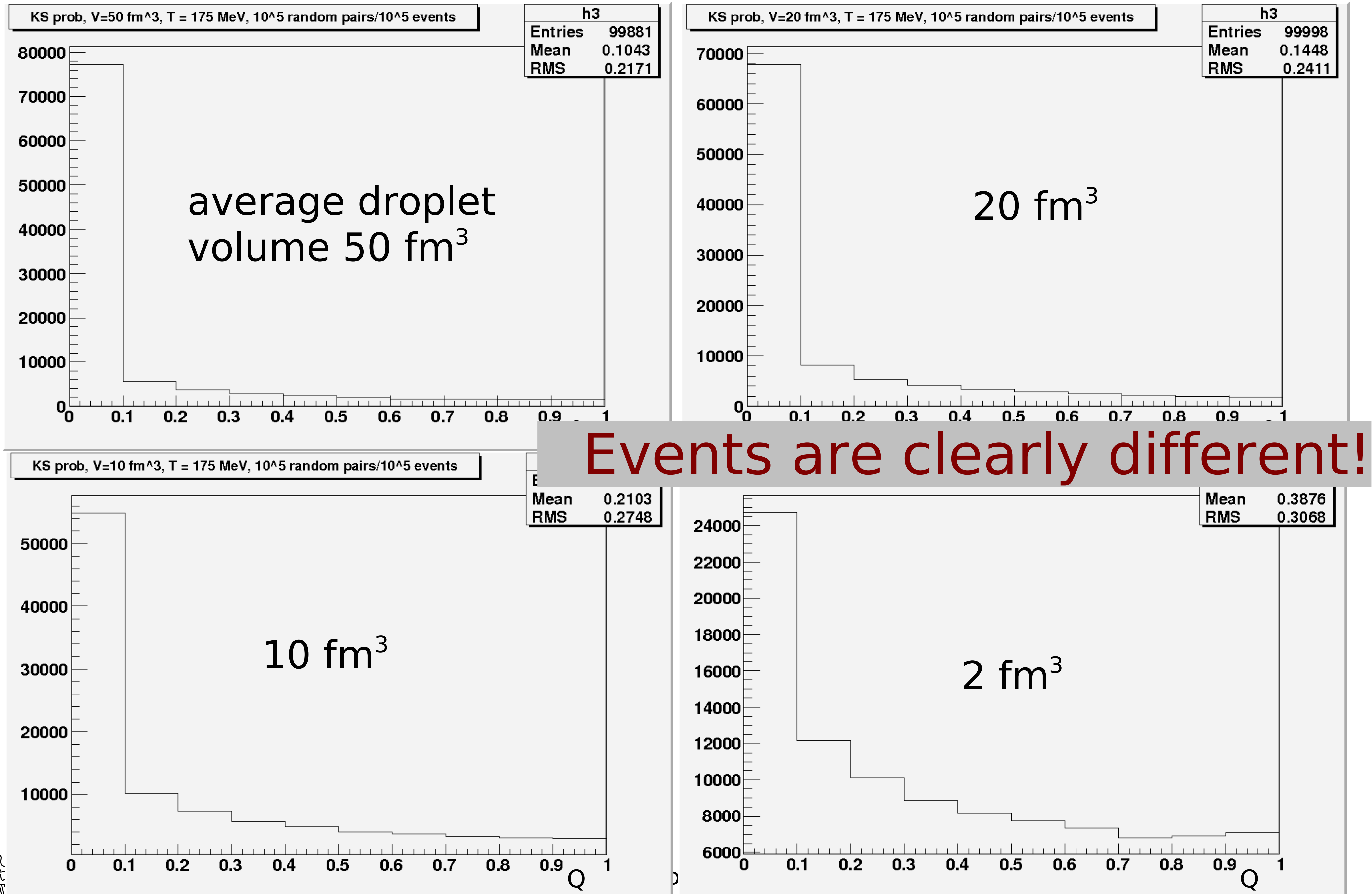
Droplet generator: rapidity spectra

T = 175 MeV, various droplet sizes, all particles from droplets

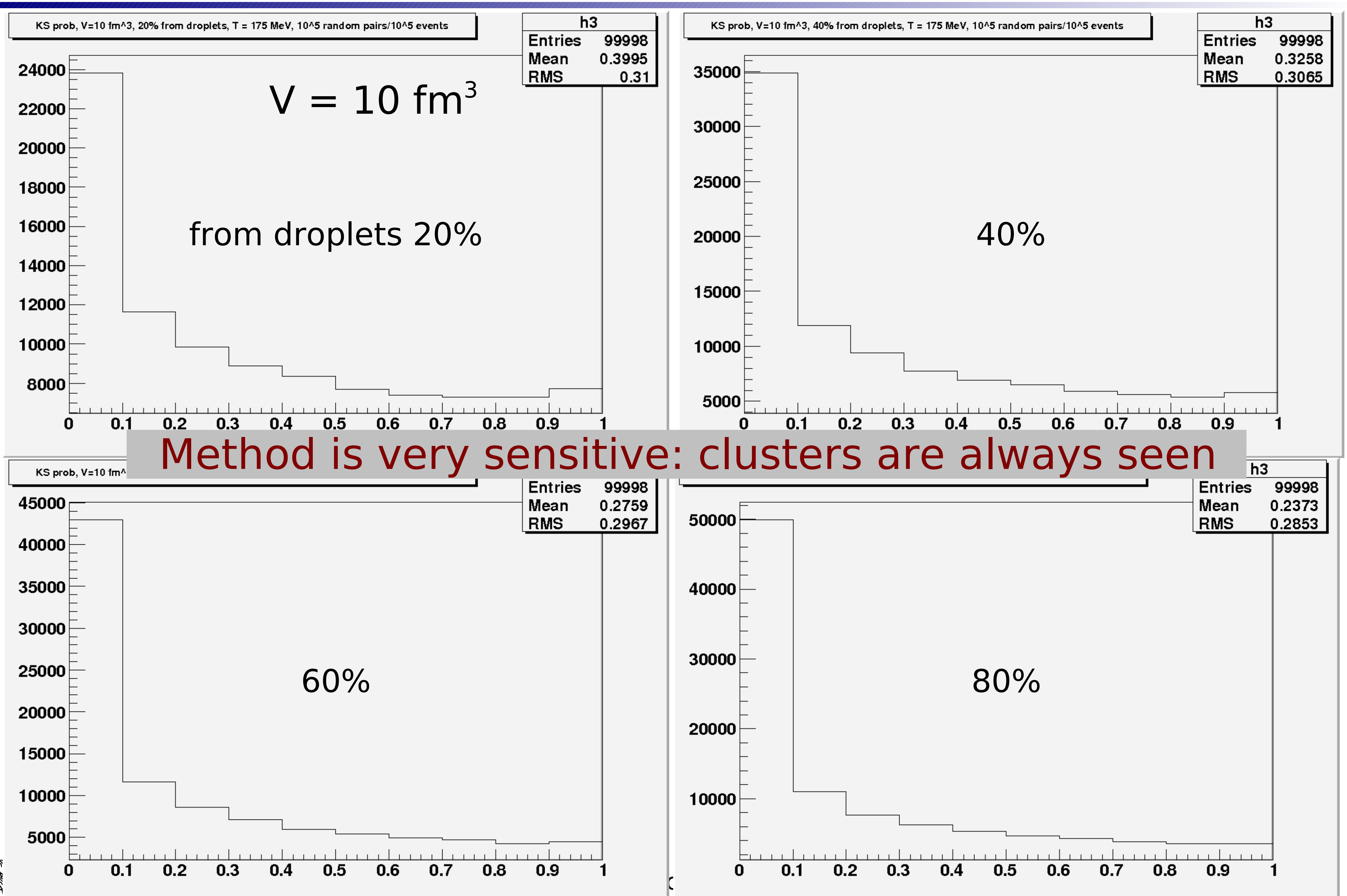


No difference in distributions summed over all events!

Droplet generator: all from droplets

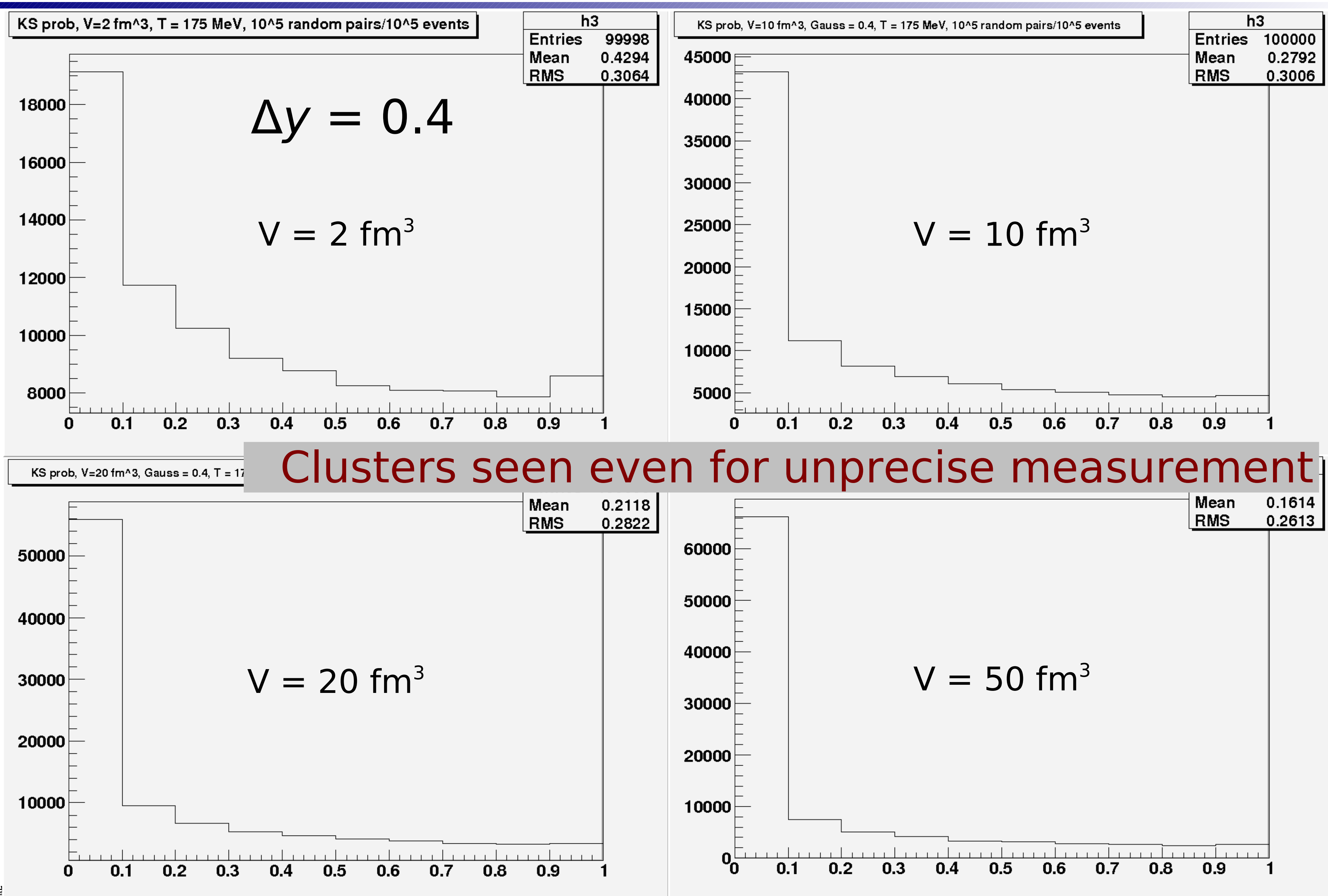


Droplet generator: part from droplets



Method is very sensitive: clusters are always seen

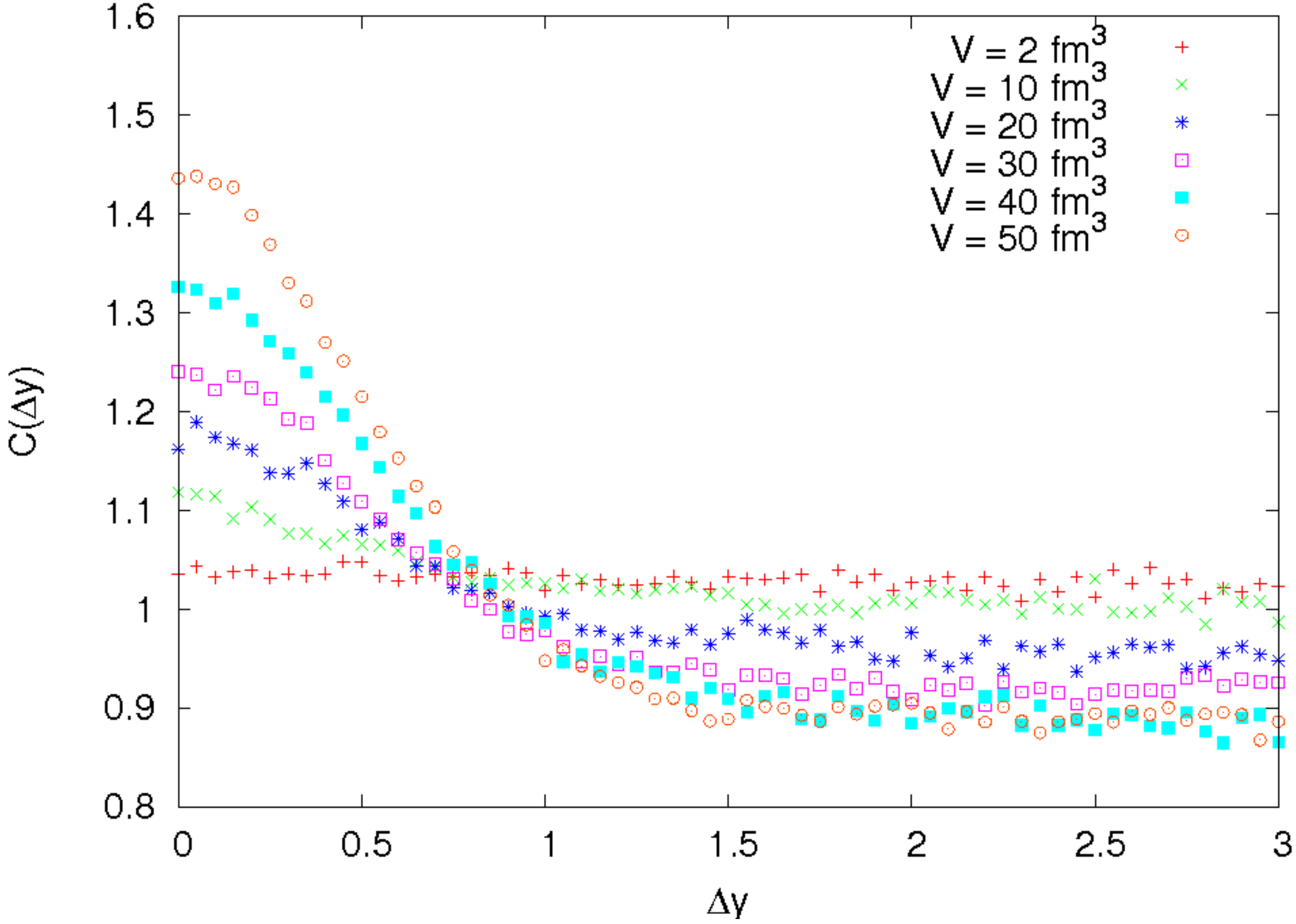
Droplet generator: smeared rapidities



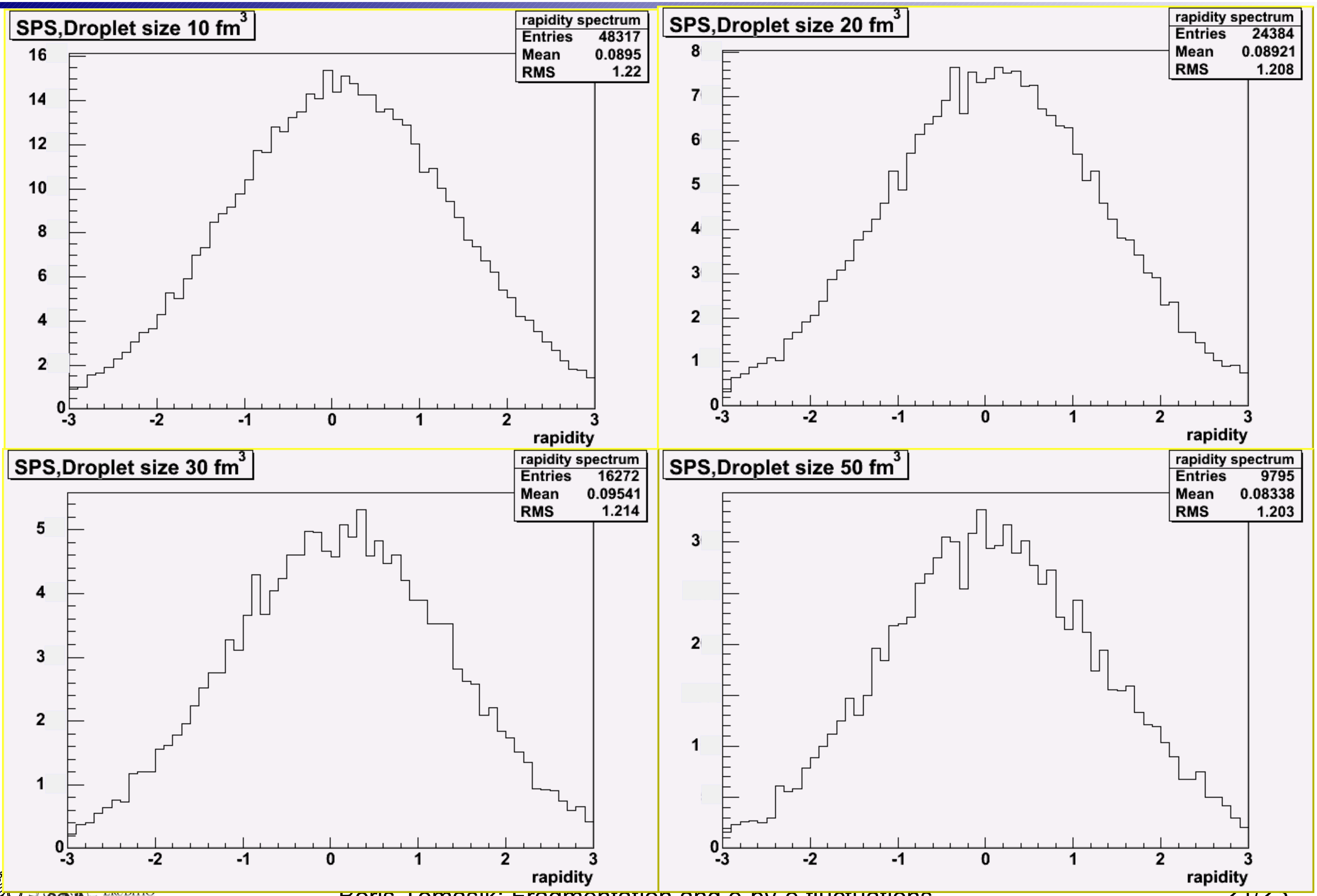
Proton-proton rapidity correlations

Motivation:
S. Pratt, PRC **49** (1994) 2722; J. Randrup, nucl-th/0406031

- Only correlations due to droplets included
- Not normalised
- Gaussian rapidity profile
- 100% protons from droplets

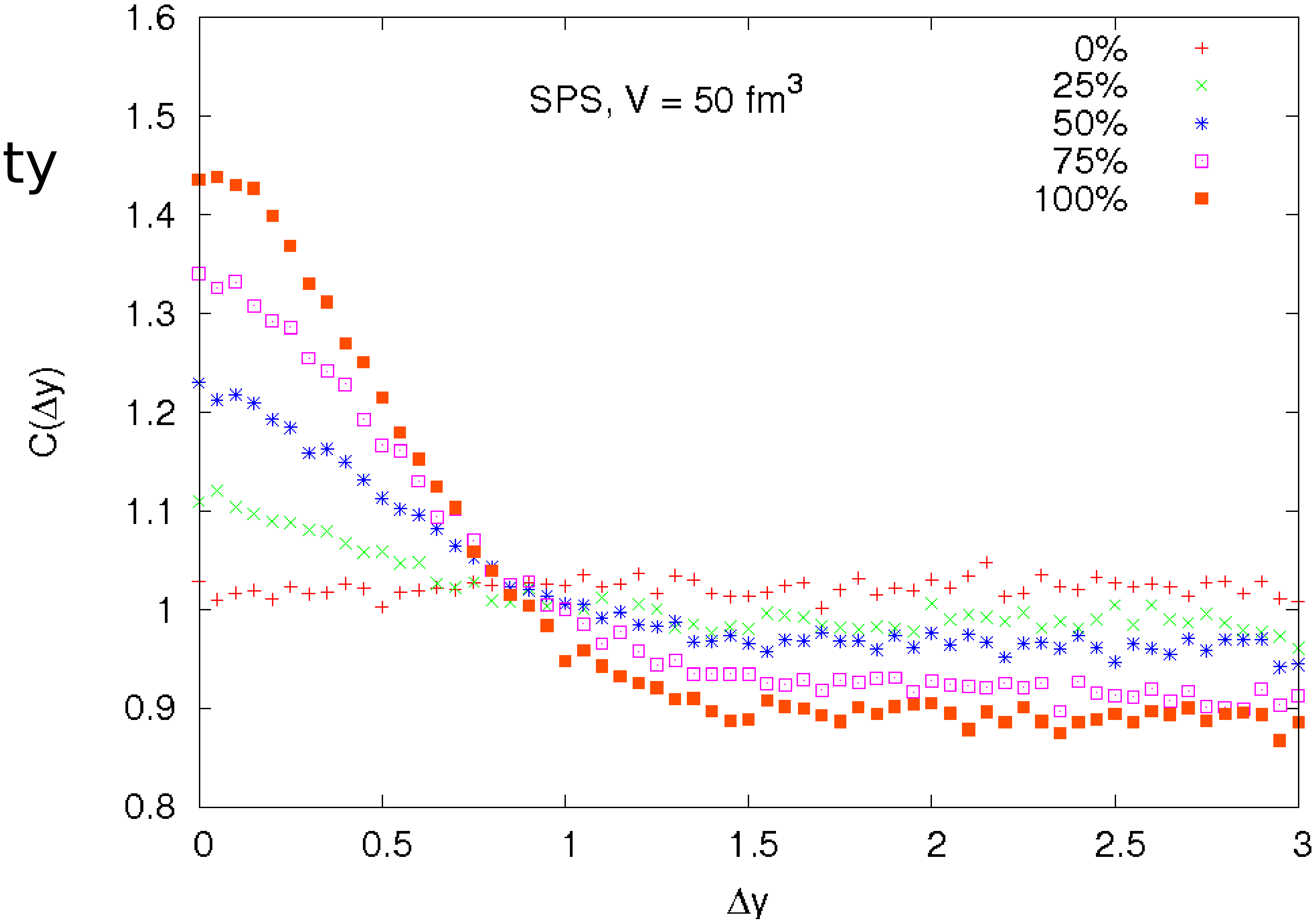


Rapidity density of droplets



Include also directly produced protons

- Only correlations due to droplets included
- Not normalised
- Gaussian rapidity profile



Conclusions

- Fragmentation of the fireball:
  - due to spinodal decomposition (lower energies)
  - bulk-viscosity driven

Fragments can be seen:
- KS method – different rapidity distributions
- Femtoscopy
- Rapidity correlations
- Fluctuations
- ...