Low x and Diffraction

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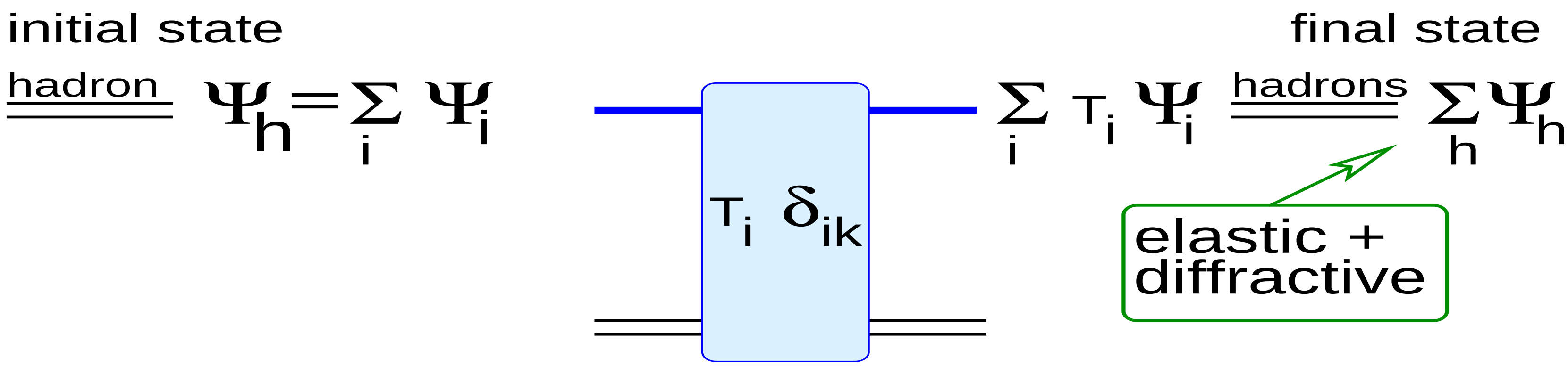
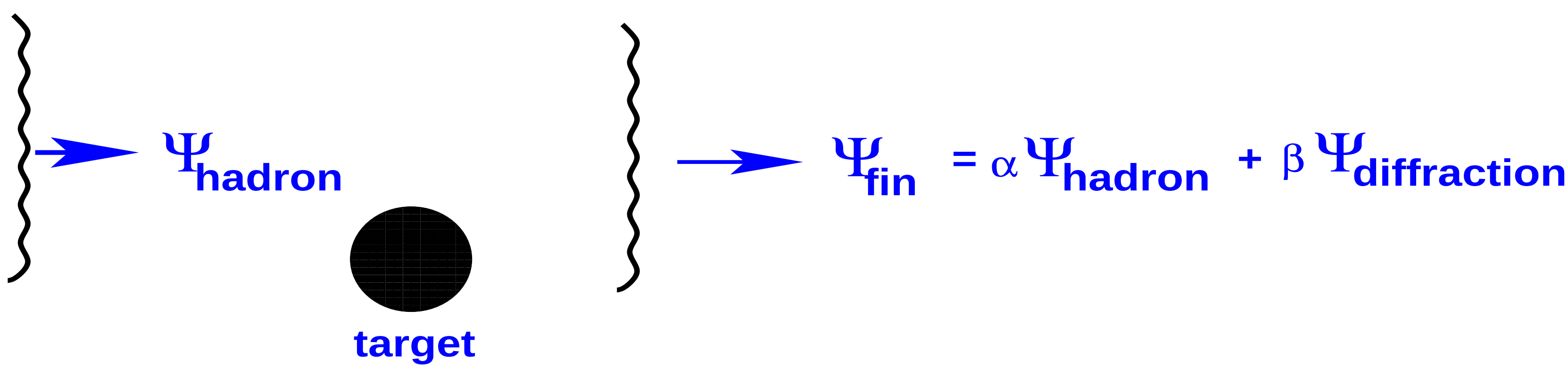
Questions for answer:

- **What are main ideas from QCD ?**
- **Key words: correct d.o.f., saturation, CGC and ...**
- **What have been seen experimentally ?**
- **Key words: black disc (?), geometrical scaling, hot spots and ...**
- **What are theoretical achivements and problems ?**
- **Key words: summing Pomeron loops, diffusion scale and ...**

Ideas from QCD

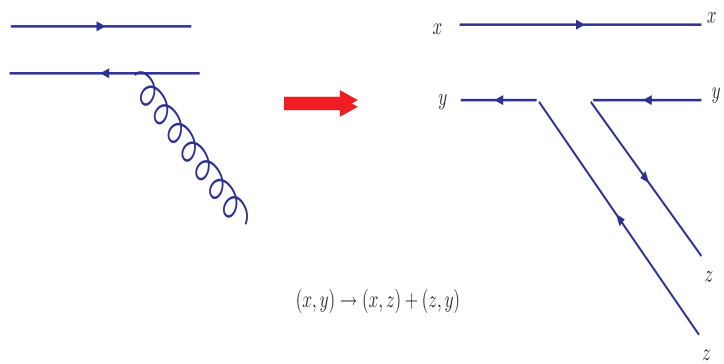
- **Degrees of Freedom and Diffraction**



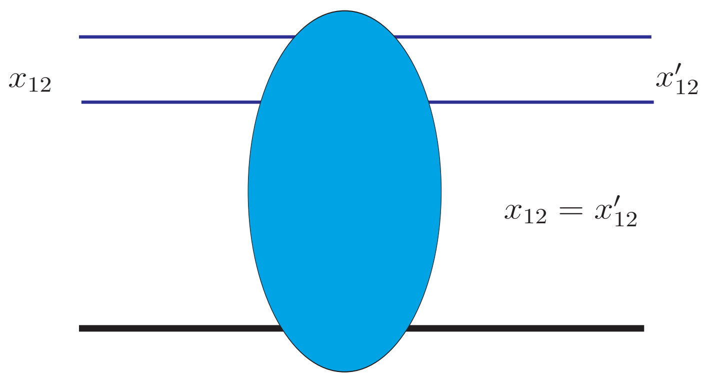


- **Correct d.o.f. at high energy: colourless dipoles** (Mueller (1994)).

  Large diffraction production $\longrightarrow$ **hadrons are not correct d.o.f.**

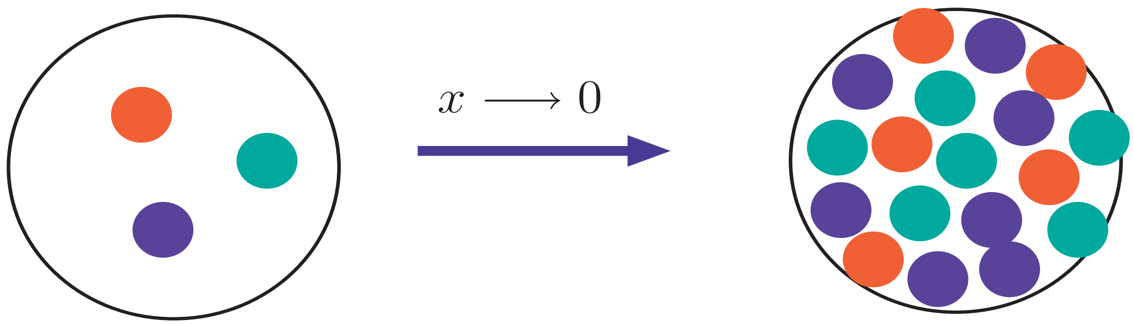


Interaction: dipole $\longrightarrow$ **two dipoles decay**



Interaction: dipole does not change the size during scattering

Saturation



Packing factor of partons $\left(\kappa(Q^2, x_{Bj})\right) \longrightarrow$ 1

$Q^2 \propto 1/r^2$ **where r is the size of dipole**

- **Saturation scale (momentum) $Q_s(x)$:**

Q_s is the solution of the equation

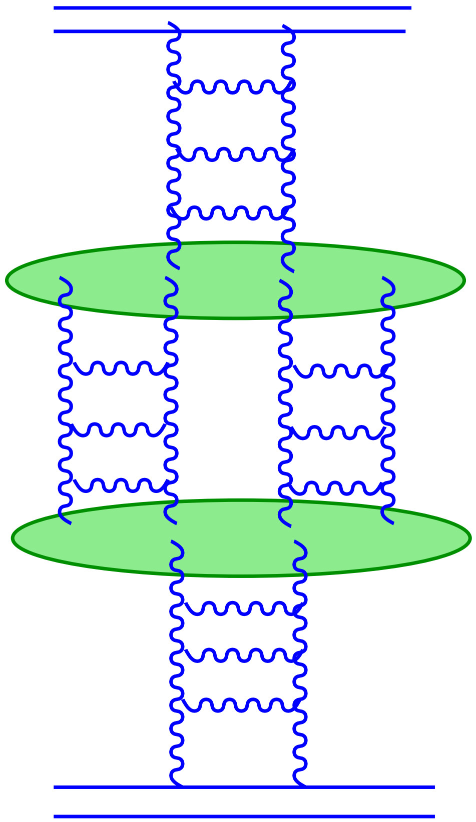
$$\kappa\left(Q_s^2, x_{Bj}\right) = 1$$

where

$$\kappa(r^2, x) = \frac{3\pi^2\,\alpha_S^2\,r^2\,xG\left(x, Q^2 = 4/r^2\right)}{4\,\pi R^2}$$

$$\boxed{Q_s^2 \propto x^{-\lambda}}$$

- **BFKL Pomeron Calculus = BFKL Pomerons and their interactions**



- $A\,(1P) \;=\; \bar{\alpha}_S^2\, s^{\Delta}$;
$\Delta \;=\; C_1 \bar{\alpha}_S \;+\; C_2 \bar{\alpha}_S^2$;

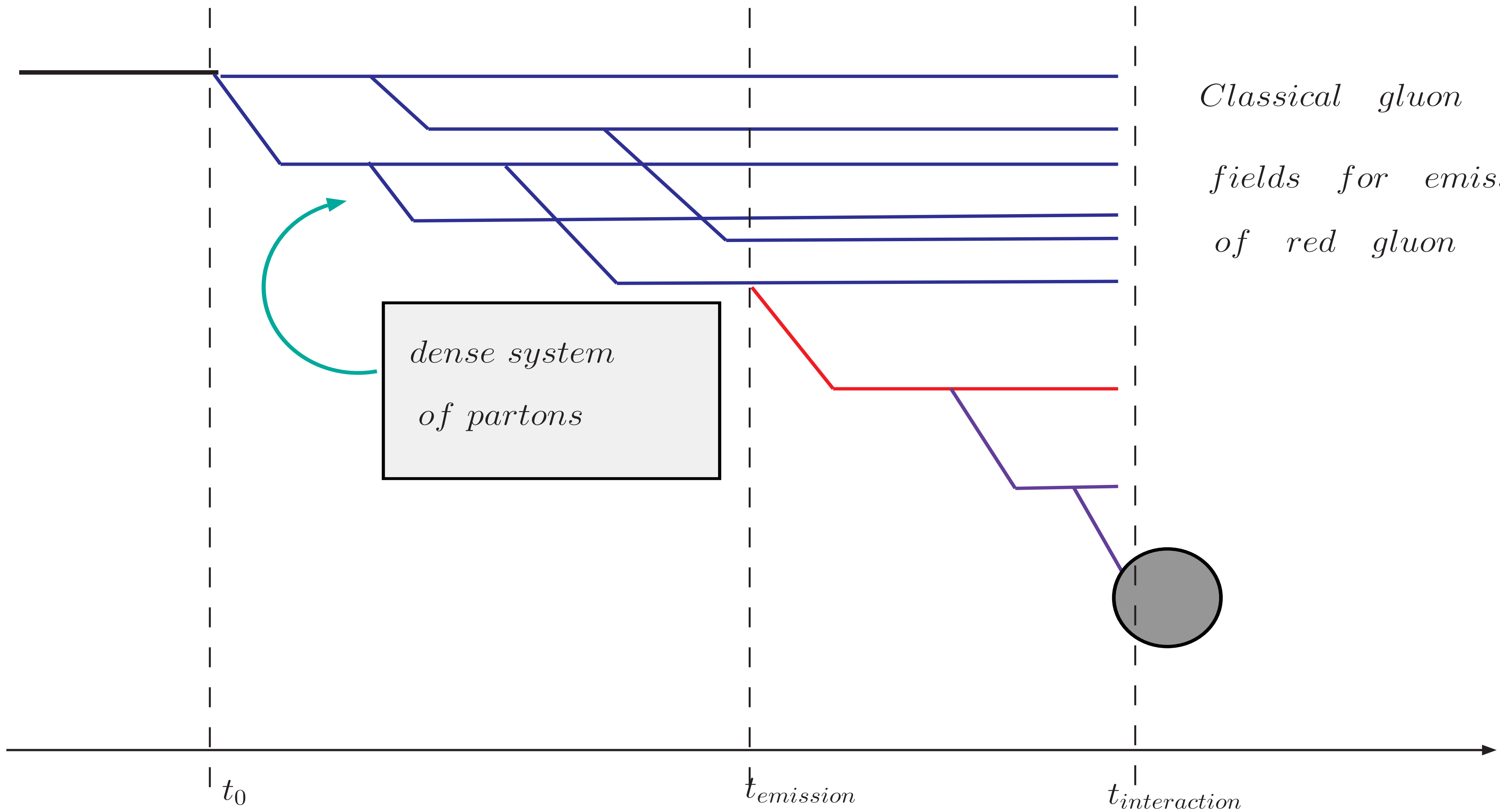
- $A\,(2P) \;=\; \bar{\alpha}_S^4\, s^{2\,\Delta}$; $Y \;=\; \ln\, s$

- **BFKL Pomeron :** $\boxed{1 \ll \;\bar{\alpha}_S\, Y}$;

- **Pomeron interaction:**
$\boxed{\ln(1/\bar{\alpha}_S^2) \ll \bar{\alpha}_S\, Y \ll 1/\bar{\alpha}_S}$;

- **Pomeron interaction + NLO**
BFKL + ... : $\boxed{1/\bar{\alpha}_S \ll \bar{\alpha}_S Y}$;

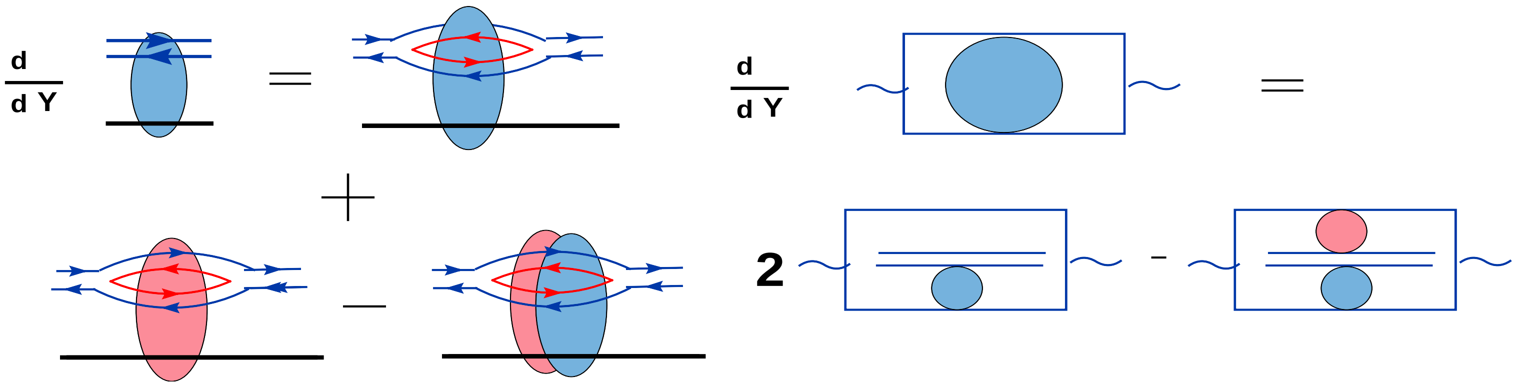
• Strong (classical) gluon fields (CGC) (McLerran & Venogapalan)



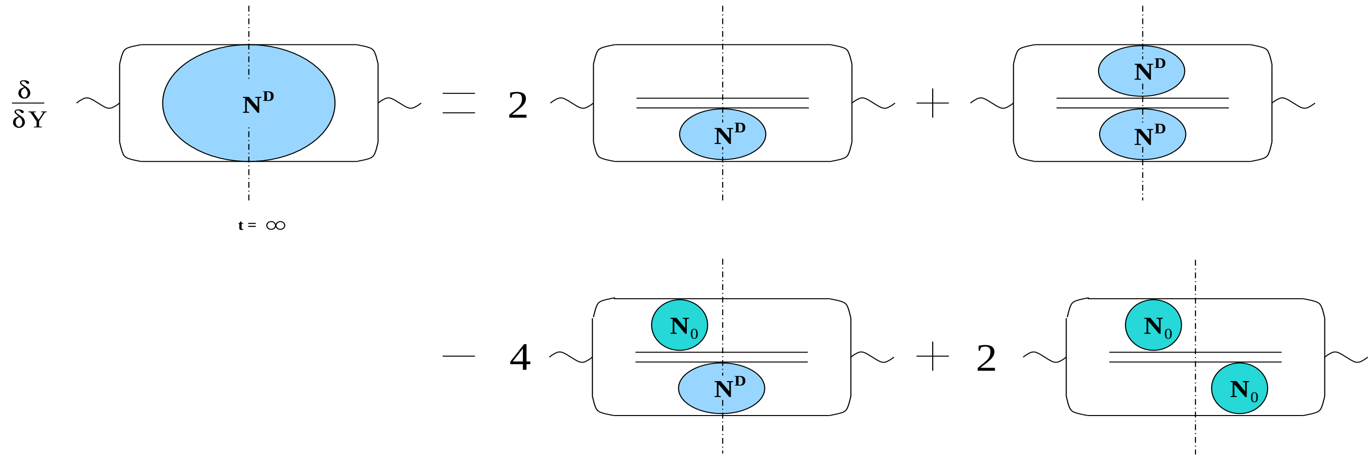
$$t_0 - t_{emission} << t_{emission} - t_{intereaction}$$

● **Equation for diffractive production** (Kovchegov & Levin (1999)) :

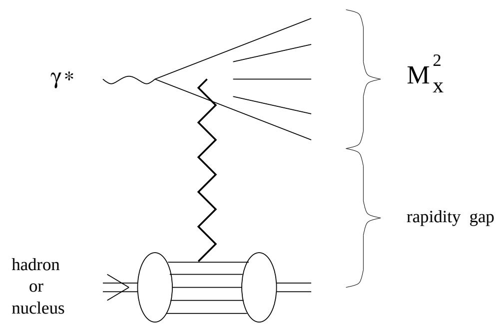
♠ **Balitsky - Kovchegov equation for N_{el}:**

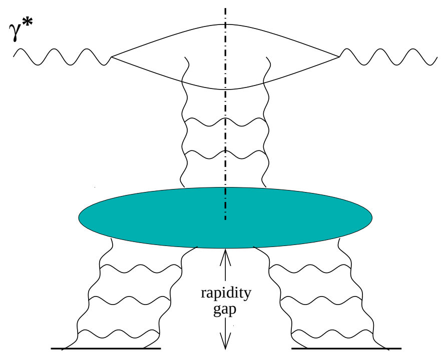


♠ **Kovchegov - Levin equation for diffractive production in MFA:**



Geometrical scaling behaviour.





- Single diffractive dissociation process. The bunch of produced particles is absorbed in cut Pomeron.

For total and total single diffraction cross sections

$\sigma_{tot}(\gamma^* p)$ and $\sigma_{sd}(\gamma^* p \to M^2 p,$ integrated over M)

$$\Longrightarrow \frac{1}{Q_s^2(x)} F\left(Q^2/Q_s^2\right)$$

$\sigma_{tot} \longrightarrow$ GLR; Mueller & Qiu; Bartels & E.L..; McLerran &Venogapalan; Kwiecinski, Stasto &Golec-Biernat; Iancu, Itakura &McLerran,

$\sigma_{sd} \longrightarrow$ E.L & Lublinsky; Kharzeev, E.L. & McLerran, Mueller

- **Typical distances in diffractive production in MFA.**

$$\sigma_{sd} \propto \int_{1/R^2}^{\infty} \frac{dk^2}{k^4}\, xG\left(\beta, Q^2/k^2\right)\ \left(xG\left(x_P, k^2\, R^2\right)\right)^2$$

$R^2 \rightarrow$ size of the target; $\beta = \frac{Q^2}{Q^2+M^2}$; $x_P = \frac{M^2}{W^2}$;

$Q^2 > Q_s^2(k^2;\beta)$ and $k^2\, R^2 > Q_s^2(x_P)$

$$\sigma_{sd} \propto \int_{1/R^2}^{\infty} \frac{dk^2}{k^4}\,(Q^2/k^2)^{\gamma}\,(k^2\, R^2)^{2\,\gamma_1} \propto (R^2)^{1+\gamma-2\gamma_1}$$

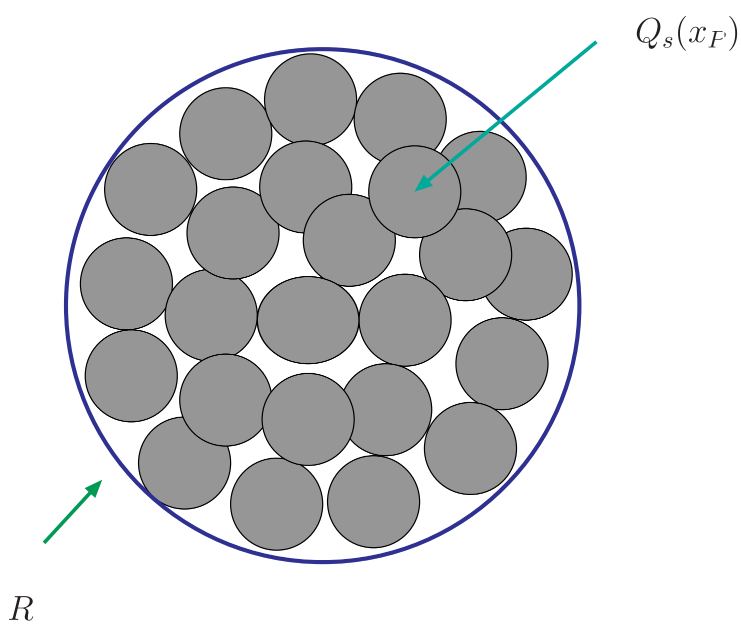
$Q^2 > Q_s^2(k^2;\beta)$ and $k^2\, R^2 < Q_s^2(x_P)$

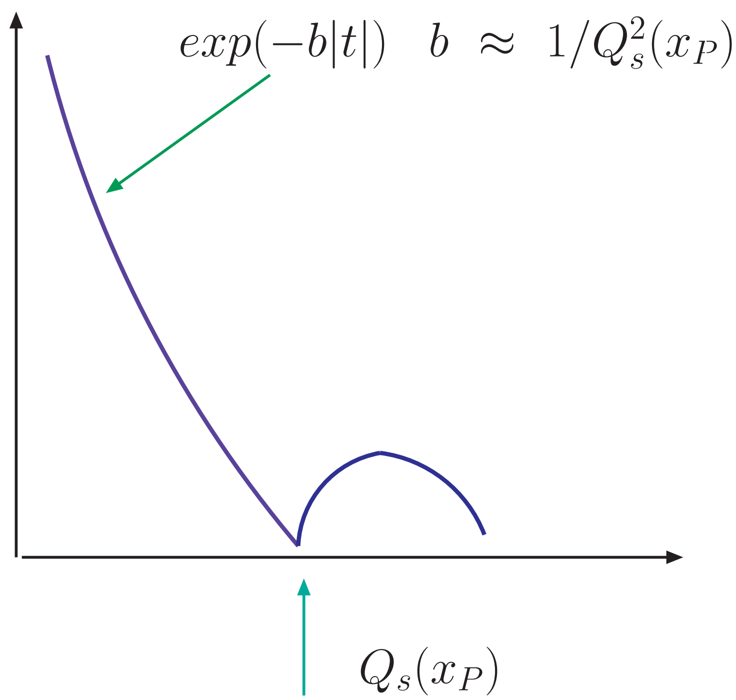
$$\sigma_{sd} \propto \int_{1/R^2}^{\infty} \frac{dk^2}{k^4}\,(Q^2/k^2)^{\gamma}\,(Q_s^2(x_P)\, R^2)^2 \propto (Q_s^2(x_P))^{1-\gamma}$$

Therefore

$$r^2 \approx \frac{1}{Q_s^2(x_P)}$$

- **'Hot spots'** **(Mueller,1992)**





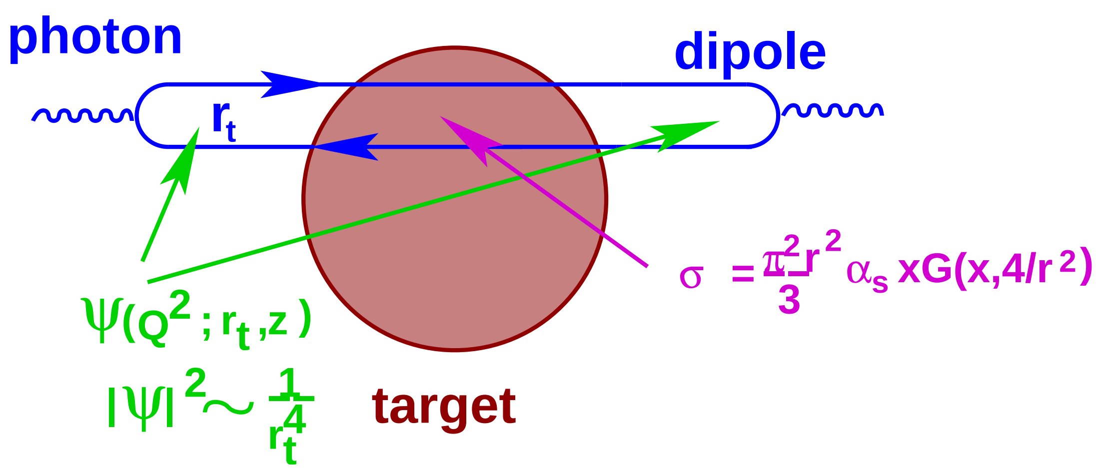
For σ_{tot} the size of hot spot $\propto 1/Q_s(x_{Bj})$

For σ_{sd} the size of hot spot $\propto 1/Q_s(x_P)$

- **The main ideas for soft processes**

> **There is no soft Pomeron but the parton system goes through the stage of parton saturation.**

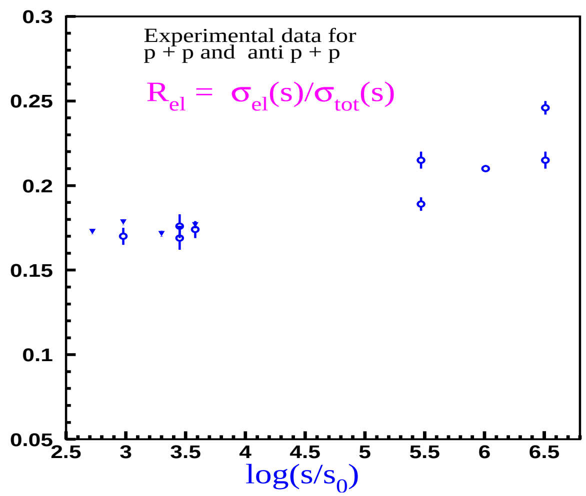
- **Saturation models** (Golec-Biernat & Wusthoff)

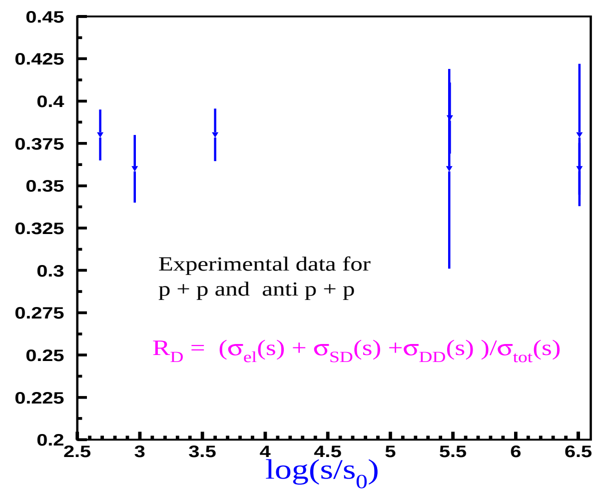


- $\sigma_{dipole}(r_t,x) = \sigma_0\left(1 - e^{-\frac{r^2 Q^2_{sat}(x)}{4}}\right)$
- $Q_s^2 = \frac{4\pi^2\alpha_S}{3} xG(x,4/r^2)$

Ideas versus experimental data

- **Soft diffraction**



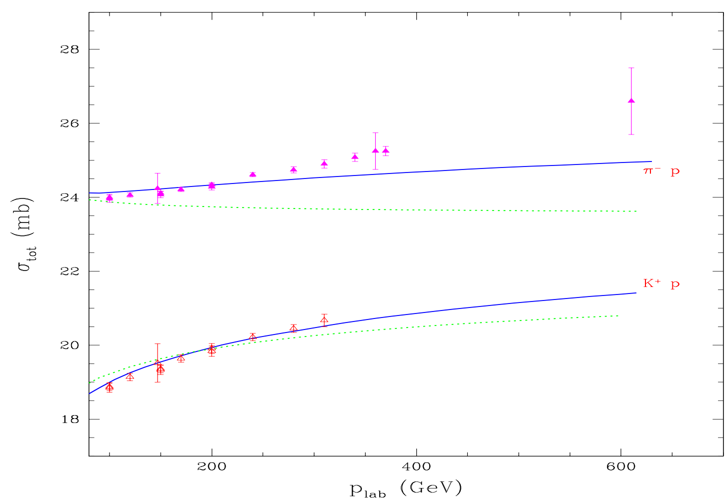


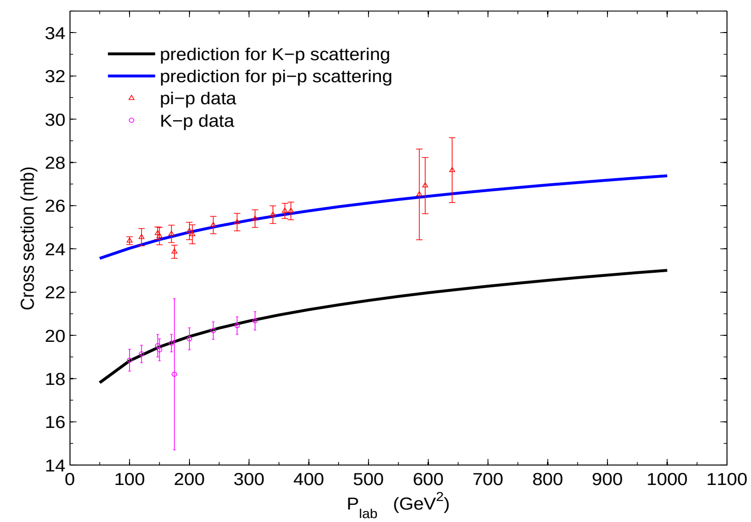
1. Hadron are **not** correct d.o.f.

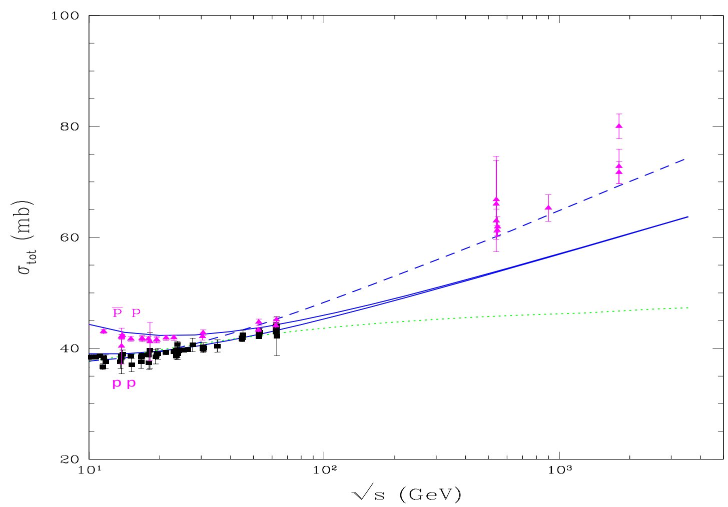
2. New d.o.f.: constituent quarks, dipoles, ...

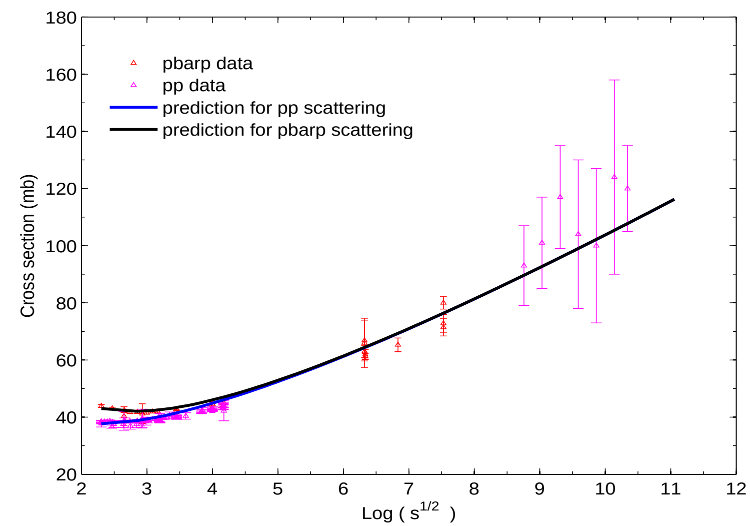
3. At least one d.o.f. scatters as a black disc

No need for a soft Pomeron



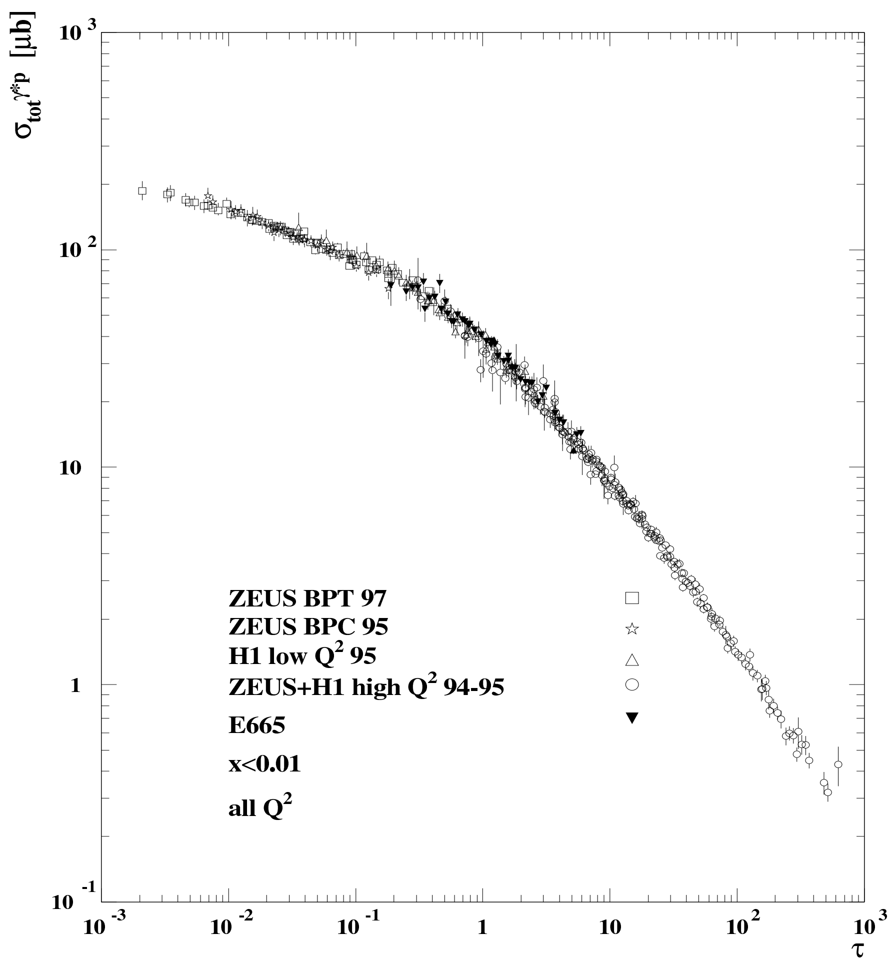






- **Bartels, Gotsman, E.L., Lublinsky & Maor**
- **Kormilitzin**

Geometrical scaling

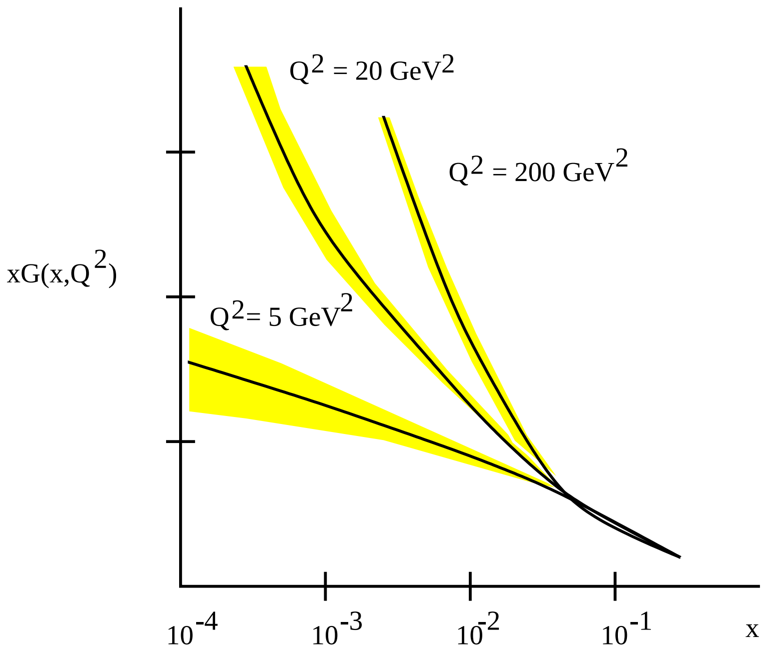


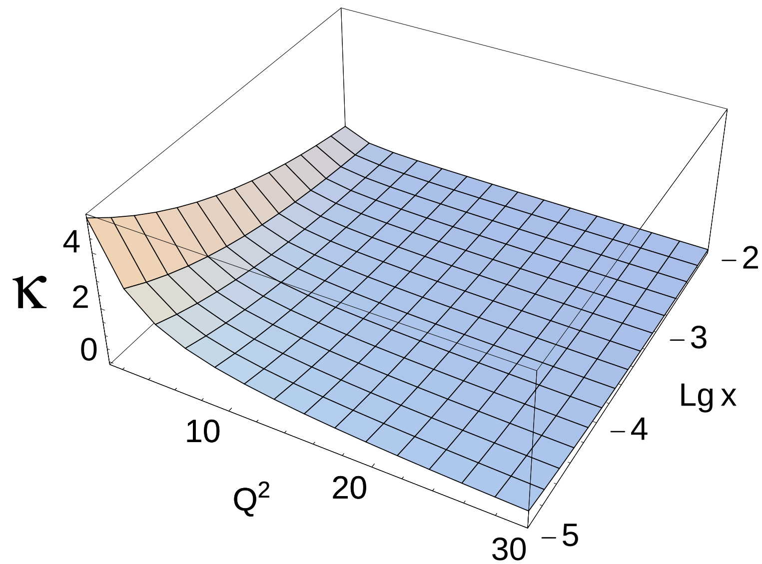
- For $Q^2 < Q_s^2$ Bartels & E.L.

- For $Q^2 > Q_s^2$ Iancu, Itakura & McLerran

- Kwiecinski, Golec-Biernat and Stasto

- **Packing factor (κ)**



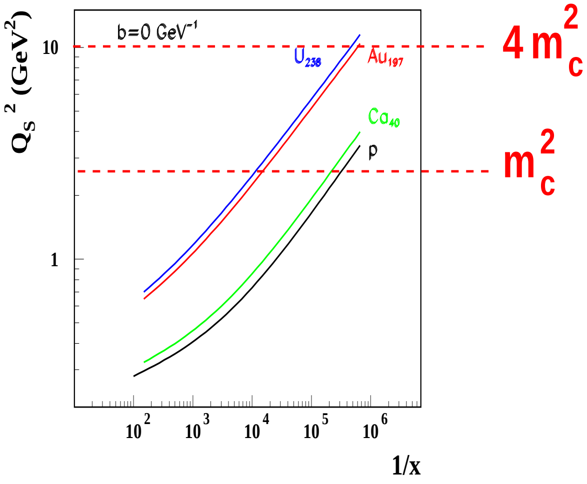


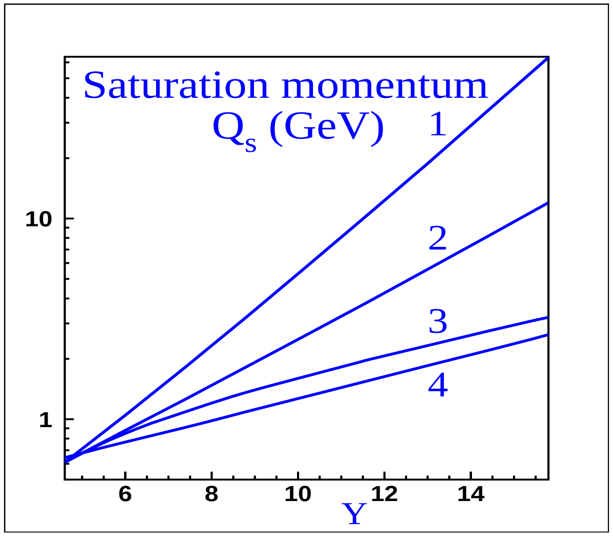
- **Large number of gluons xG at $x \to 0$;**

- **Large packing factor ;**

However

- **The value of Q_s is not large at HERA ($Q_s \approx 1\,GeV$) ;**
- **It depends on saturation model;**

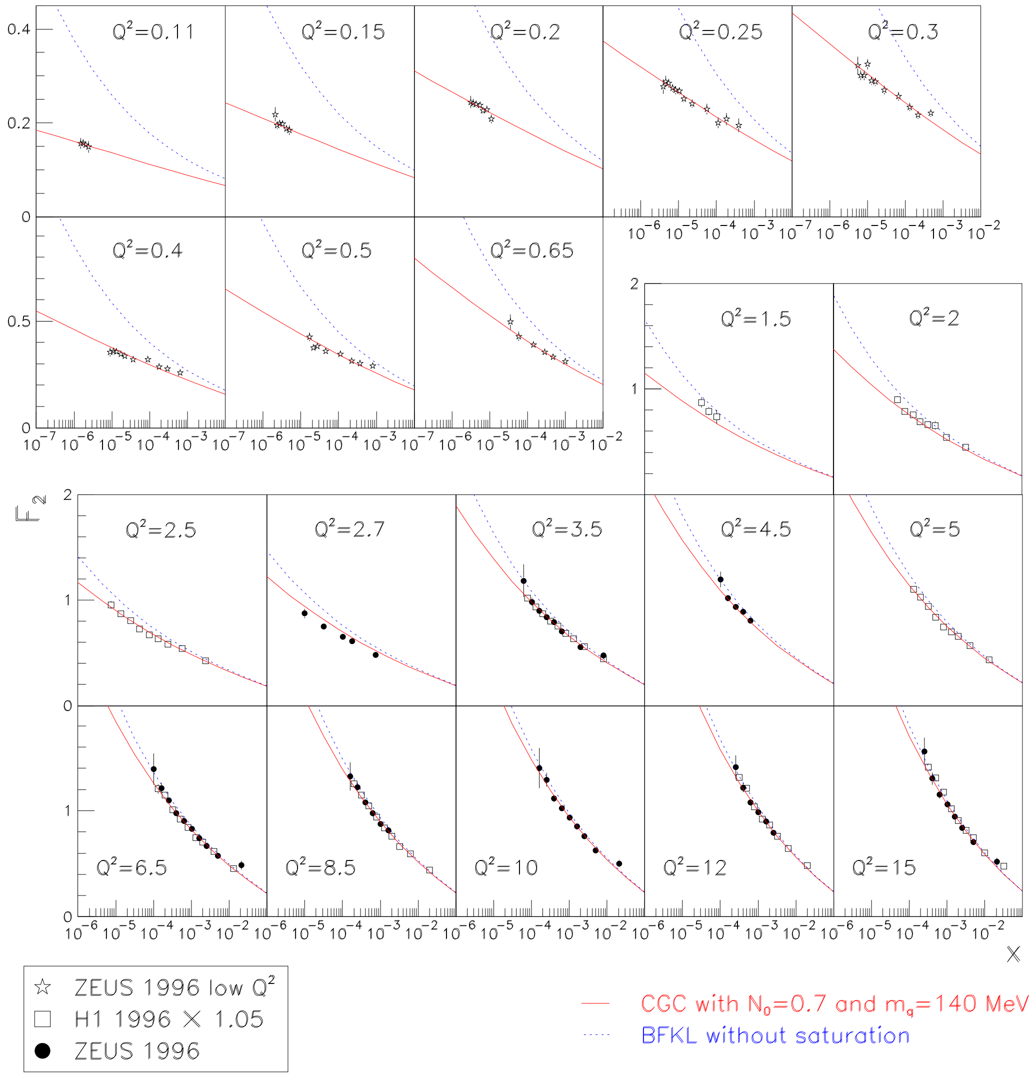




1. **LO BFKL ;**
2. **Modified MFA;**
3. **NLO BFKL (Durham);**
4. **BGW model;**

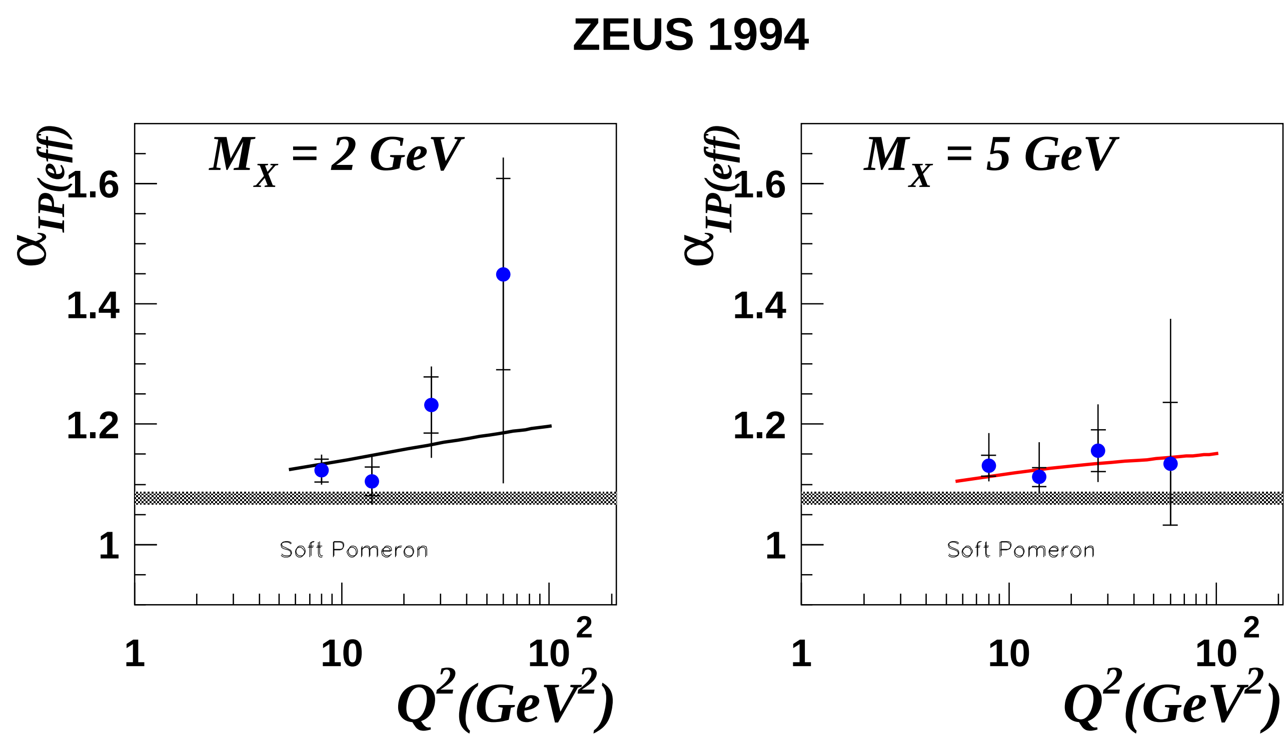
Good desription of F_2 in MFA (B-K equation)

(Lublinsky + TAU (02); Iancu, Itakura & Munier (03))



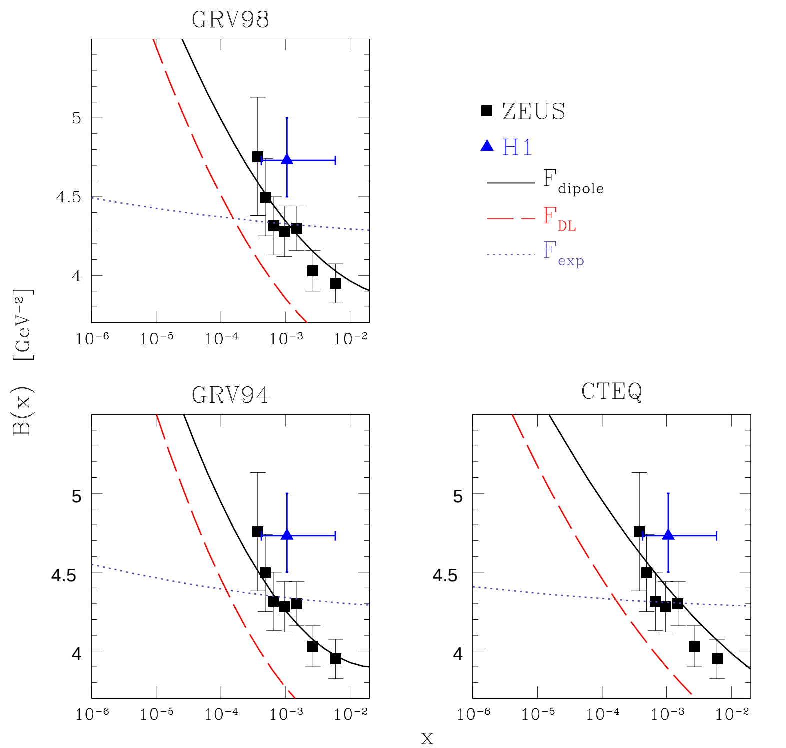
Diffractive production: typical dustances

$$\sigma_{sd} \propto \frac{1}{x^{2\alpha_{eff}}}$$



The fact that $\alpha_{P,eff} > \alpha_{P,soft}$ means that short distances contribute to the diffracttion production

• Diffractive production: shrinkage of diffraction peak

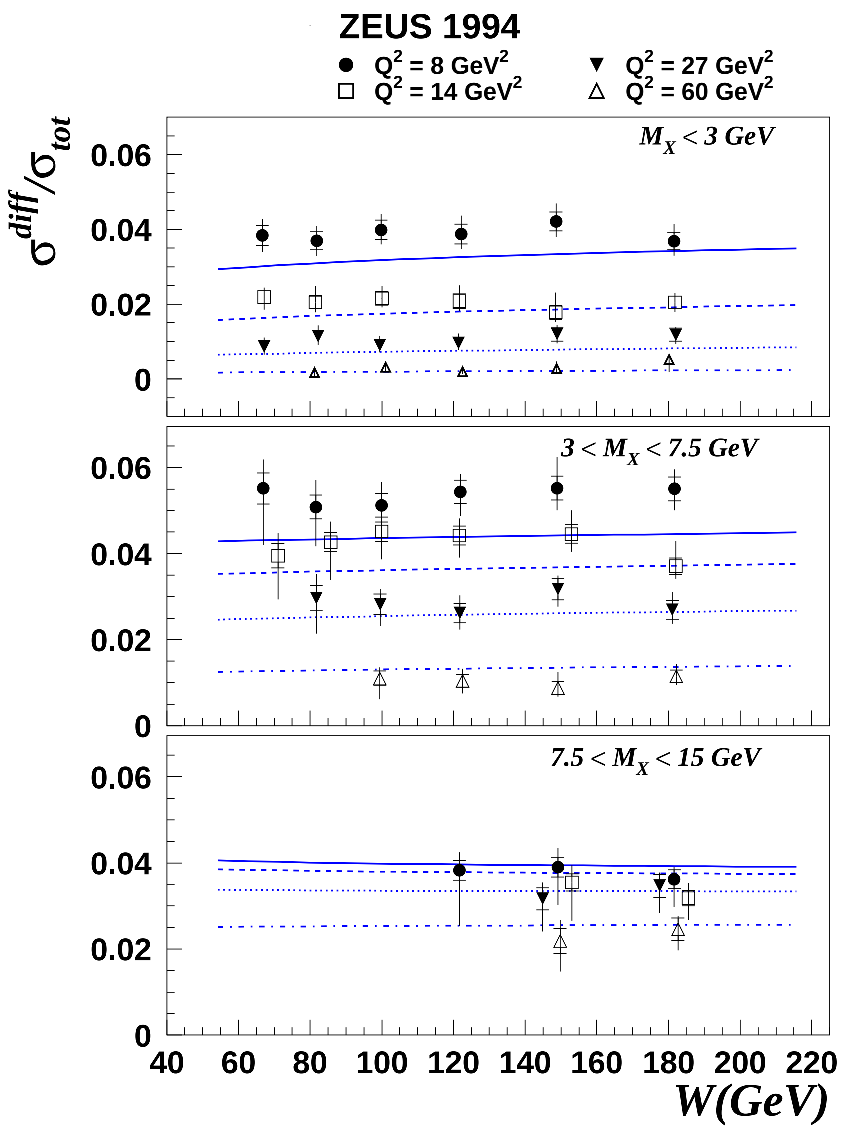


- For soft processes $B = B_0 + 2\alpha'_P \, ln(s/s_0)$ where $\alpha'_P = Const$(DL);
- For hard processes (without SC) $B = B_0$;
- For hard processes (with SC) $B = B_0 + 2\alpha'_P \, ln(s/s_0)$ but α'_P increases with energy;

Hot spots: $\sigma(\gamma^* p \to J/\Psi + p) \to \frac{1}{Q_s^2} F\left(r^2 Q_s^2(x,b)\right)$

$Q_s^2(x,b) = Q_s^2(x)\exp(-\mu b)$ therefore, $\boxed{b \,\propto\, (1/\mu)\ln Q_s^2}$

- **Diffractive production: $\sigma_{diff}(M^2)/\sigma_{tot}$**



- **As far as I know there is no other explanation why this ratio is constant;**

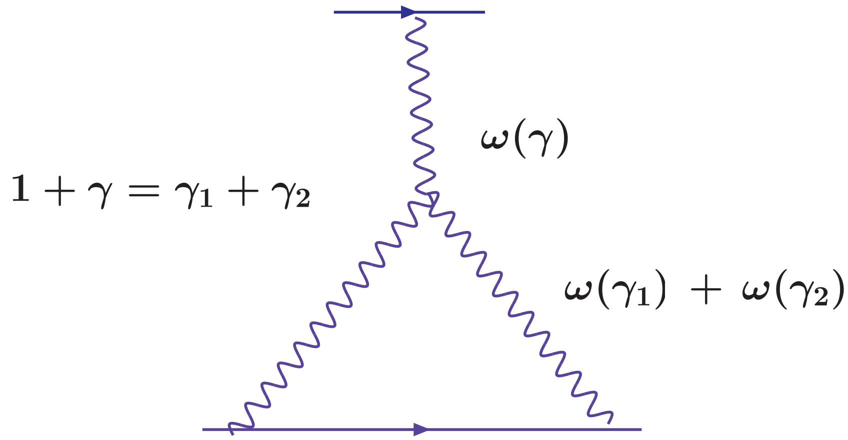
Theory development

- **The BFKL Pomeron calculus**

  Status:

  - **Everything that has been done during the past three years is nothing more than understanding of the BFKL Pomeron calculus (Kozlov,E.L. & Prygarin; Bondarenko);**

  - **The news: The Pomeron interaction generates a new state with the intercept larger than intercept of two BFKL Pomerons (Hatta & Mueller; E.L, Miller & Prygarin);**



- $A \propto \frac{1}{2\pi i}\int_{\epsilon - i\infty}^{\epsilon + i\infty} d\omega$ $e^{\omega\, Y}\,\frac{1}{\omega - \omega(\gamma)}\ \frac{1}{\omega - \omega(\gamma_1) - \omega(\gamma_2)}$

- $\omega(2\gamma_0\ -\ 1) = 2\ \omega(\gamma_0)$
- $\omega(\gamma_0)\ > \omega(\gamma = 1/2)$
- $A \propto\ Y\, e^{2\ \omega(\gamma_0)\, Y}$

The sad truth: we have to start from the very beginning not only in summing Pomeron loops but also in MFA ? !

- **Statistical approach: its beauty and problems**

We know from the old good days of Reggeon Field Theory (Pomeron calculus) that this theory is the field theory for directed percolation

(Grassberger & Sudermeyer (1978), Obukhov (1980), Cardy & Sugar (1980))

but because of the advent of QCD we did not investigate this idea in the full strength.

TIME HAS COME

- **People:** **Blaizot, Brunet, Derrida, Enberg, Golec-Biernat, Hatta, Iancu, Itakura, E.L., Lublinsky, McLerran, Marquet, Mueller, Munier, Peshanski, Shoshi, Soyez, Triantafyllopoulos + ... nearly everybody**

Langevin equation:

- $$\frac{\partial \Phi}{\partial Y} = \frac{\bar{\alpha}_S}{2\pi} K \bigotimes \Phi - \frac{2\pi \bar{\alpha}_S^2}{N_c} \Phi^2 + \zeta$$
- $$< |\zeta| > = 0; \qquad < |\zeta\,\zeta| > \neq 0$$

Langevin equation for Einstein diffusion:

- $$\frac{d\vec{v}}{dt} = -\lambda \vec{v} + \zeta$$

The main prediction:

(Iancu, Mueller & Munier (2004))

- violation of the geometrical scaling behaviour ;
- appearance of new saturation scale (diffusion scale);

- $$A(z,Y) \;=\; A\left(\frac{\ln(r^2\,Q^2_{new,s})}{\sigma}\right) \;=\; \frac{1}{\sigma\sqrt{2\,\pi}}\int dz\,T\,(z)\;\;e^{-\frac{(z-<z>)^2}{2\,\sigma^2}}$$
- $\sigma^2 \;\propto\; Y$; $\quad z \;=\; \ln(r^2\,Q^2_s)$ where r is the dipole size;
- $< z > \;=\; \ln(r^2\,Q^2_{new,s})$; $Q_{new,s}$ =new saturation (diffusion) scale
- $T\,(z) \;=$ solution in the MFA;

The BFKL Pomeron calculus leads to a very complicated form of the noise term and simplification that one has to make kills the main idea: to calculate Pomeron loops

- **Summing Pomeron loops in the BFKL Pomeron calculus**

  (E.L. , Miller & Prygarin)

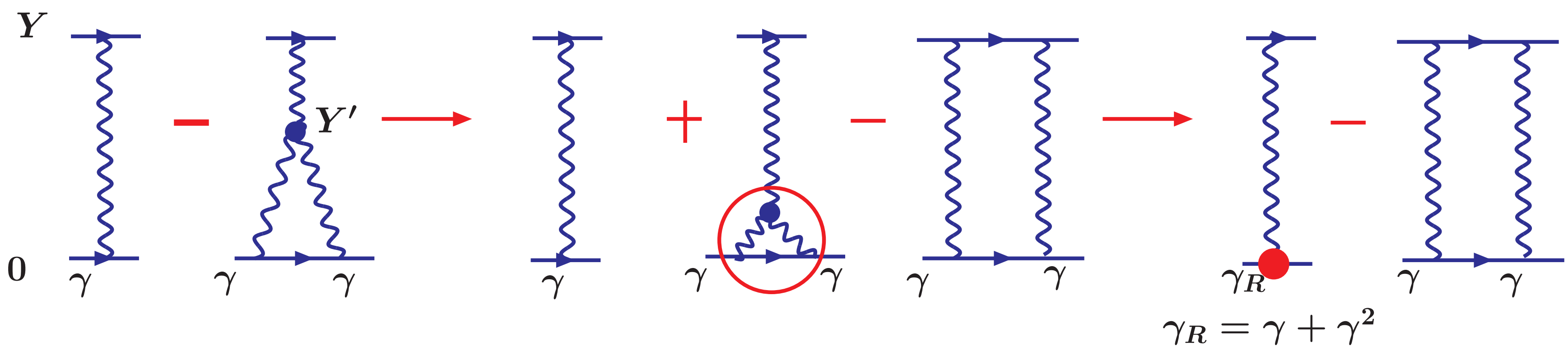
$$1 \approx \bar{\alpha}_S Y \,\leq\, \ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S$$

- $1 \approx \bar{\alpha}_S Y \,\leq\, \ln 1/\bar{\alpha}_S^2 \longrightarrow$ **LO BFKL Pomeron**
- $\ln 1/\bar{\alpha}_S^2 \leq \bar{\alpha}_S Y \leq 1/\bar{\alpha}_S \longrightarrow$ **BFKL Pomeron calculus**
- $1/\bar{\alpha}_S \leq \bar{\alpha}_S Y \longrightarrow$ **NLO BFKL Pomeron and non-linear QCD**

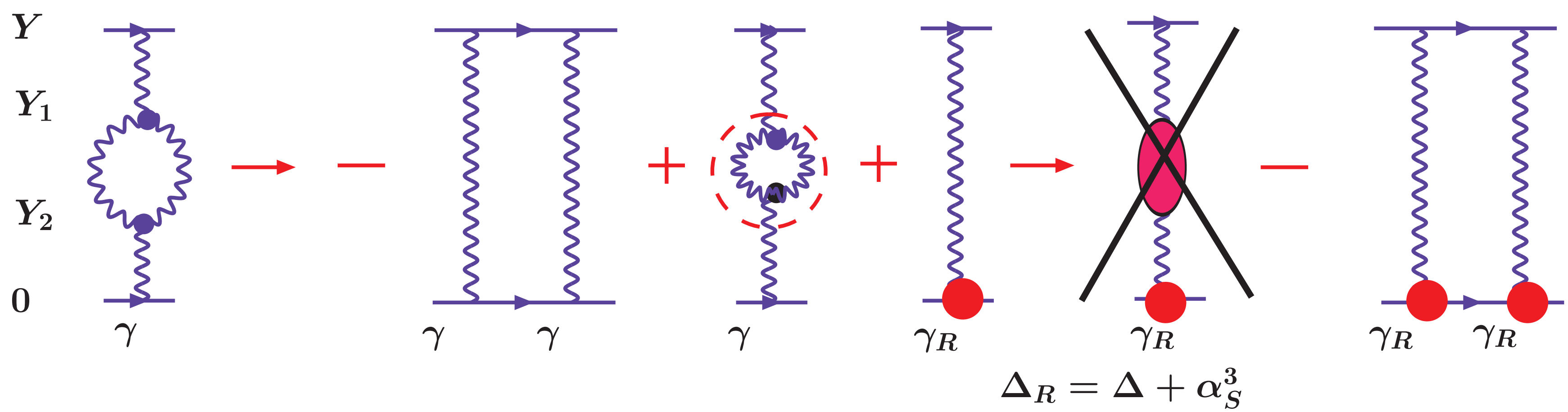
The main results:

- We can neglect the overlapping singularities;
- We are dealing with the system of the non-interacting BFKL Pomerons;
- For summing Pomeron loops we can use the Mueller-Patel -Salam -Iancu approximation, improved by the renormalization of the scattering amplitude at low energies;

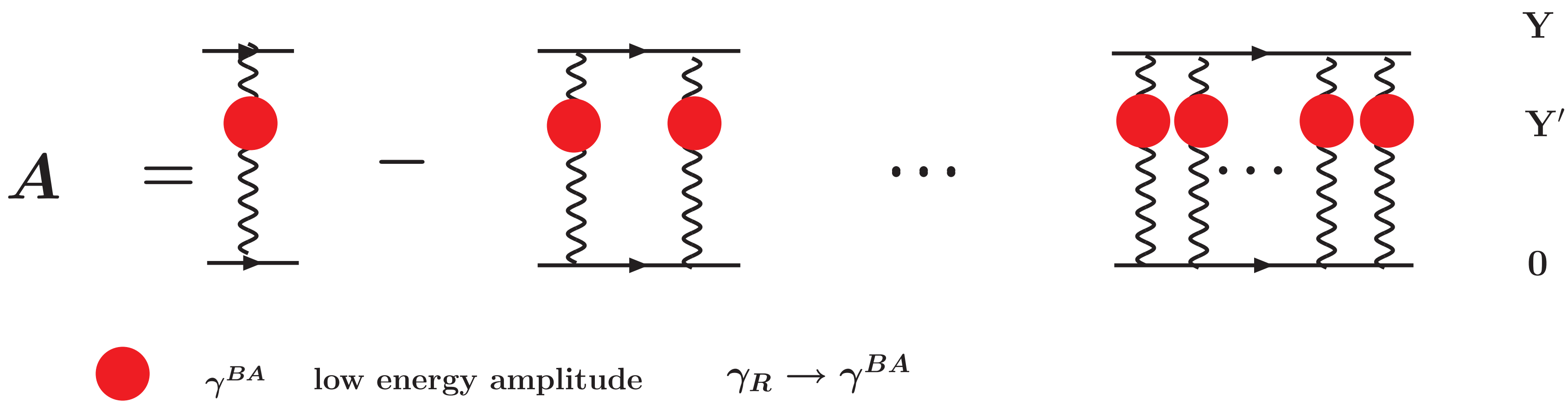
An example:



Y
Y'
0
γ
γ_R
$\gamma_R = \gamma + \gamma^2$



Y
Y_1
Y_2
0
γ
γ_R
$\Delta_R = \Delta + \alpha_S^3$



Solution:

For model BFKL kernel

$$\omega(\gamma) \;=\; \bar{\alpha}_S \begin{cases} \dfrac{1}{\gamma} & \text{for } r^2 Q_s^2 \ll 1 - \text{ summing } \left(\bar{\alpha}_S \ln(1/(r^2 Q_s^2))\right)^n; \\[2ex] \dfrac{1}{1-\gamma} & \text{for } r^2 Q_s^2 \gg 1 - \text{ summing } \left(\bar{\alpha}_S \ln(r^2 Q_s^2)\right)^n; \end{cases}$$

we obtain:

- **geometrical scaling behaviour;**

- **rather slow approaching the asymptotic value, namely $1 - N \propto \exp(-z)$ where $z = \ln(r^2 Q_s^2)$;**

Conclusions

" Once you eliminate the impossible what remains is the solution - no matter how improbable it may seem"

I hope that during this conference we will eliminate a couple of possibilities approaching to the solution