Running coupling in small x evolution

Heribert Weigert
The Ohio State University

ISMD, Berkeley 2007

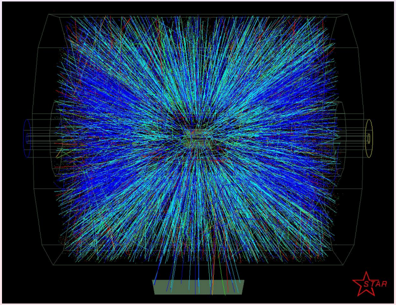
hep-ph/0609087
Gardi, Kuokkanen, Rummukainen, Weigert

hep-ph/0609090,hep-ph/0612071
Kovchegov, Weigert

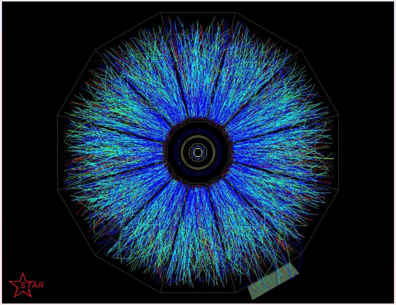
Outline

Outline

RHIC: searching for the Quark Gluon Plasma



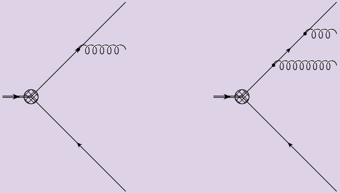
side view



front view

From photons to gluons

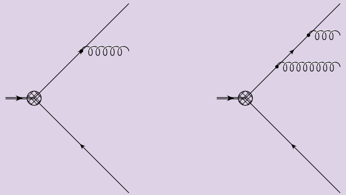
photon-like contributions



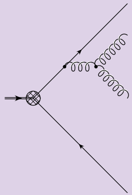
- enhanced by phase space integrals $\frac{dE}{E}\frac{d\theta}{\theta}$ → $\alpha_s \ln E \ln \theta$
- all orders calculation needed $\sum_{n=0}^{\infty}(\alpha_s \ln E)^n \ldots$

From photons to gluons

photon-like contributions

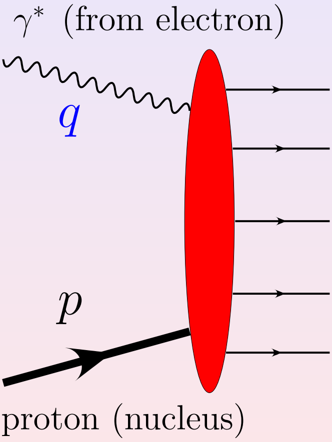


QCD: charged gluons



- enhanced by phase space integrals $\frac{dE}{E}\frac{d\theta}{\theta} \longrightarrow \alpha_s \ln E \ln\theta$
- all orders calculation needed $\sum_{n=0}^{\infty}(\alpha_s \ln E)^n \ldots$
- gluons charged $\longrightarrow$ radiation nonlinear in QCD

Kinematic variables: transverse resolution vs energy

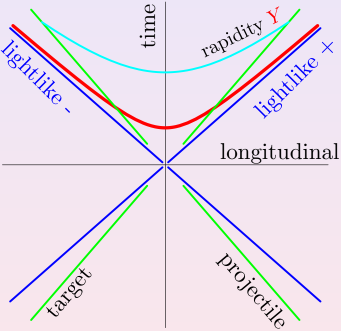


- $Q^2 := -q^2 \gg 0$

  spacelike!
  transverse resolution
  $\Delta \boldsymbol{r} \sim \frac{1}{Q}$

- $x = x_{\mathrm{Bj}} := \frac{Q^2}{2p.q} = \frac{Q^2}{2m\, E_{\mathrm{rest}}}$

Kinematic variables: transverse resolution vs energy



- $Q^2 := -q^2 \gg 0$

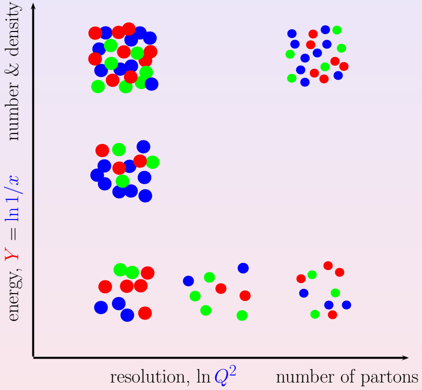
  spacelike!
  transverse resolution
  $\Delta \boldsymbol{r} \sim \frac{1}{Q}$

- $x = x_{\rm Bj} := \frac{Q^2}{2p.q} = \frac{Q^2}{2m\, E_{\rm rest}}$

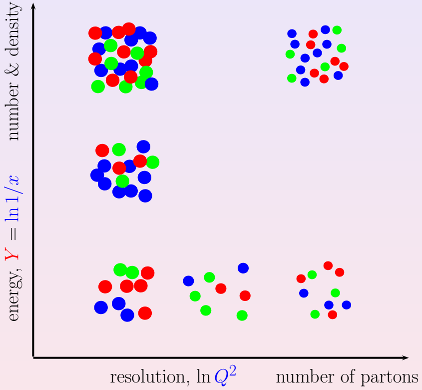
- $Y = \ln\frac{1}{x} \propto \ln E_{\rm rest}$

  all used synonymously

Large energies mean large densities

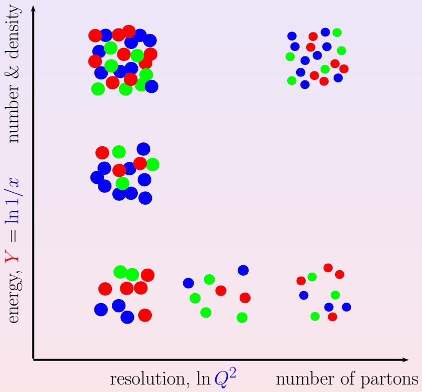


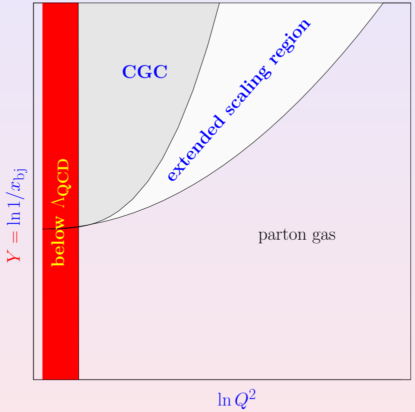
Large energies mean large densities



- density $\longrightarrow$ finite correlation length R_s

Large energies mean large densities



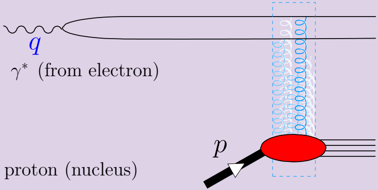


- density → finite correlation length R_s

Outline

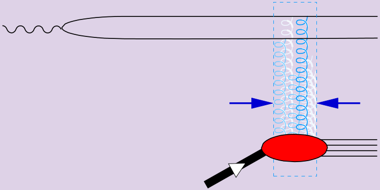
Gluon production at increasing energy

the photon splits



Gluon production at increasing energy

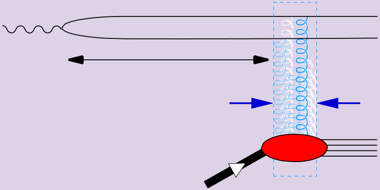
the photon splits



- target gluons Lorentz contracted to $\delta(x^-)$

Gluon production at increasing energy

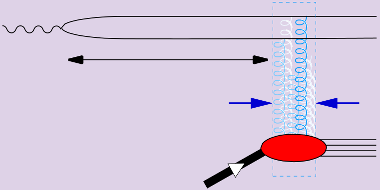
the photon splits



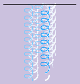
- target gluons Lorentz contracted to $\delta(x^-)$
- creation time dilated $\sim \frac{1}{x}$

Gluon production at increasing energy

the photon splits

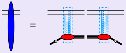


- target gluons Lorentz contracted to $\delta(x^-)$
- creation time dilated $\sim \frac{1}{x}$

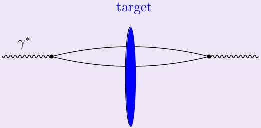
$\sum_{\text{gluons}}$  $= \mathrm{P}\exp -ig\int dz_\mu A^\mu(z)$

$= U_{\boldsymbol{x}}$

Cross section



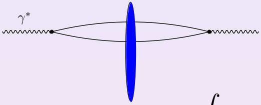
$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$

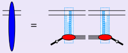


Cross section

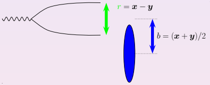
energy, $\ln 1/x$

$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$





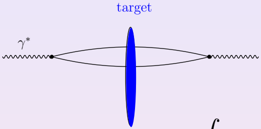
$$= \int d^2r \; |\psi^2|(r^2Q^2) \underbrace{\int d^2b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y)}_{\sigma_{\mathrm{dipole}}}$$

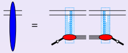


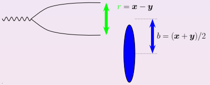
Cross section

energy, $\ln 1/x$

$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$







$$= \int d^2 r \; |\psi^2|(r^2 Q^2) \left(\int d^2 b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y) \right)$$

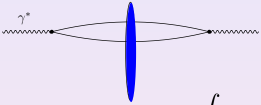
σ_{dipole}

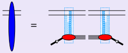
- σ_{dipole} contains $U_{\boldsymbol{x}}$

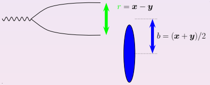
Cross section

energy, $\ln 1/x$

$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$







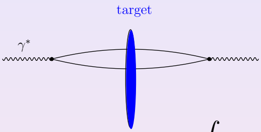
$$= \int d^2 r \; |\psi^2|(r^2 Q^2) \underbrace{\int d^2 b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})}{N_c} \rangle (Y)}_{\sigma_{\text{dipole}}}$$

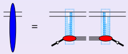
- σ_{dipole} contains $U_{\boldsymbol{x}}$
- $\langle \ldots \rangle (Y)$ hard!
  target wavefunction: non-perturbative

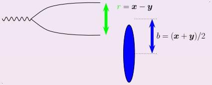
Cross section

energy, $\ln 1/x$

$$\sigma_{\mathrm{DIS}}(Y, Q^2) = \mathrm{Im}$$



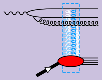




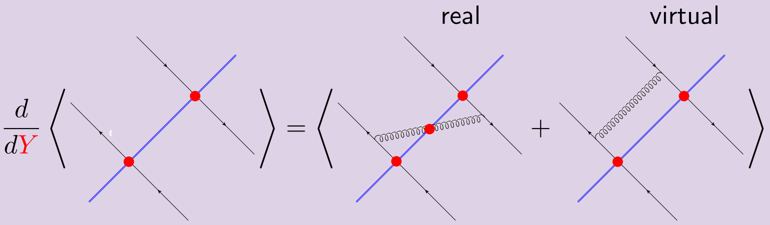
$$= \int d^2 r \; |\psi^2|(r^2 Q^2) \underbrace{\int d^2 b \; \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)}{N_c} \rangle (Y)}_{\sigma_{\text{dipole}}}$$

- σ_{dipole} contains $U_{\boldsymbol{x}}$
- $\langle \ldots \rangle (Y)$ hard!
  target wavefunction: non-perturbative

$$\frac{d}{dY} \langle \ldots \rangle (Y)$$



Diagrammatic input to LO-JIMWLK



$$\frac{d}{dY}\langle\ U_{\boldsymbol{x}} \otimes U_{\boldsymbol{y}}^\dagger\ \rangle = \langle\ U_{\boldsymbol{z}}^{ab}\ t^a U_{\boldsymbol{x}} \otimes t^b U_{\boldsymbol{y}}^\dagger + U_{\boldsymbol{x}} t^a \otimes t^a U_{\boldsymbol{y}}^\dagger\ \rangle$$

LO-JIMWLK-Kernel

$$\frac{\alpha_s(\mu^2)}{\pi}\mathcal{K}_{\boldsymbol{xzy}} \propto \frac{\alpha_s(\mu^2)}{\pi}\int \frac{d^2q}{(2\pi)^2}\frac{d^2q'}{(2\pi)^2}\ e^{-i\boldsymbol{q}\cdot(\boldsymbol{z}-\boldsymbol{x})+i\boldsymbol{q}'\cdot(\boldsymbol{z}-\boldsymbol{y})}\ \frac{\boldsymbol{q}\cdot\boldsymbol{q}'}{\boldsymbol{q}^2\boldsymbol{q}'^2}$$

fixed coupling!

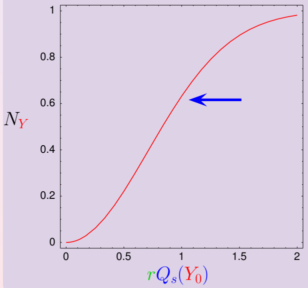
Saturation scale and cross section

$$\langle \ldots \rangle(Y) \quad \Longrightarrow \quad \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U_{\boldsymbol{0}}^\dagger)}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Saturation scale and cross section

$$\langle \ldots \rangle(Y) \quad \Rightarrow \quad \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U^\dagger_{\boldsymbol{0}})}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Correlation length shrinks



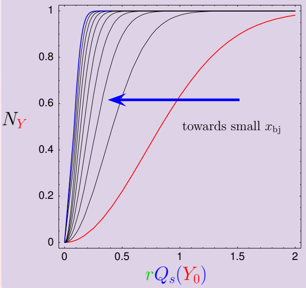
- $$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

$R_s(Y) \equiv$ correlation length
$Q_s(Y) \equiv$ saturation scale

Saturation scale and cross section

$$\langle \ldots \rangle(Y) \rightarrow \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U^\dagger_{\boldsymbol{0}})}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Correlation length shrinks



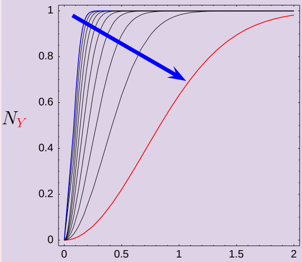
- $R_s(Y) \sim \frac{1}{Q_s(Y)}$

$R_s(Y) \equiv$ correlation length
$Q_s(Y) \equiv$ saturation scale

Saturation scale and cross section

$$\langle \ldots \rangle(Y) \Rightarrow \int d^2b \, \langle \frac{\mathrm{tr}(1 - U_{\boldsymbol{r}} U_{\boldsymbol{0}}^\dagger)}{N_c} \rangle(Y) =: N_Y(\boldsymbol{r})$$

Correlation length shrinks



change units: $rQ_s(Y_0) \to rQ_s(Y)$

- $$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

  $R_s(Y) \equiv$ correlation length
  $Q_s(Y) \equiv$ saturation scale
- scaling solutions
  initial conditions erased
  "attractor"

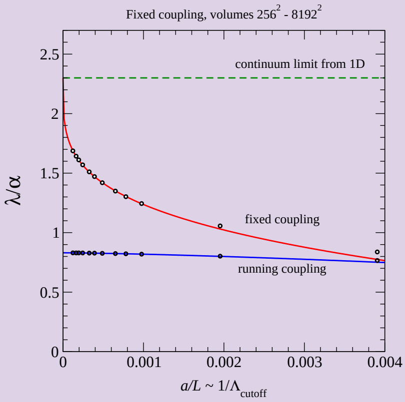
Outline

1. Motivation: gluons form the CGC
   - Current and planned collider experiments
   - Enhanced gluon production at high energies
2. JIMWLK evolution: properties of the CGC
   - Gluons in observables
   - The evolution equation
   - The saturation scale
3. **Running coupling**
   - Running coupling is essential
   - Fermion bubble diagrams
   - Running coupling at all orders
4. Consequences
   - Generic slowdown
   - Applications
5. Wrapping up

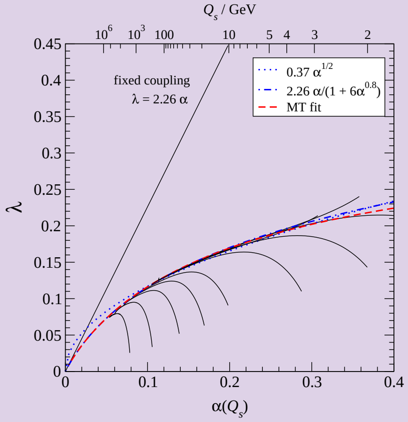
Running coupling is essential:

assume parent dipole running

phase space reduction



$\lambda(Y) := \partial_Y \ln Q_s(Y)$



with Kari Rummukainen, Nucl.Phys.A739:183-226,2004 [hep-ph/0309306]

The large N_f limit: running coupling on the cheap

- Trace running coupling corrections by N_f contributions

recalculate JIMWLK with fermion bubble chain insertions

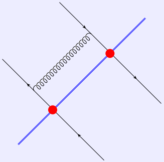
$$\frac{d\alpha_s(\mu^2)/\pi}{d\ln\mu^2} = -\beta_0\ \left(\alpha_s(\mu^2)/\pi\right)^2 - \beta_1\ \left(\alpha_s(\mu^2)/\pi\right)^3 + \cdots$$

$$\beta_0 = \frac{11}{12}C_A - \frac{1}{6}N_f$$

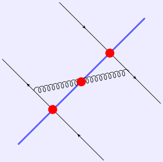
- recover QCD running coupling by subst:

$$N_f \longrightarrow -6\beta_0$$

Diagrams

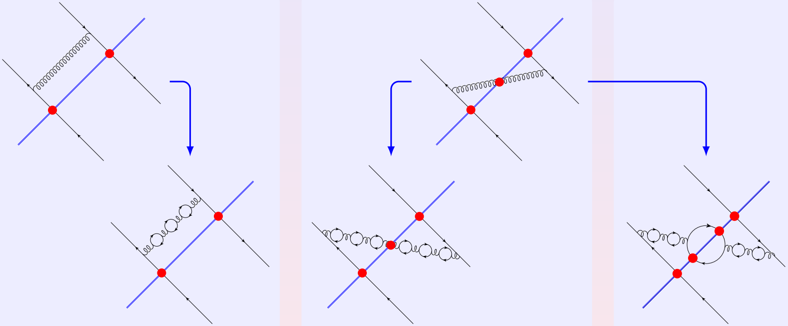


$$t^a U_{\boldsymbol{x}} \otimes U^\dagger_{\boldsymbol{y}} t^a$$



$$U^{ab}_{\boldsymbol{z}}\ t^a U_{\boldsymbol{x}} \otimes t^b U^\dagger_{\boldsymbol{y}}$$

Diagrams

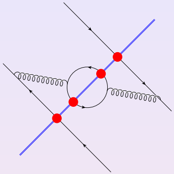


$$t^a U_{\boldsymbol{x}} \otimes U_{\boldsymbol{y}}^\dagger t^a$$

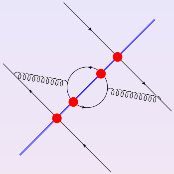
$$U_{\boldsymbol{z}}^{ab}\ t^a U_{\boldsymbol{x}} \otimes t^b U_{\boldsymbol{y}}^\dagger$$

$$2\mathrm{tr}(t^a U_{\boldsymbol{z}_1} t^b U_{\boldsymbol{z}_2}^\dagger) \times t^a U_{\boldsymbol{x}} \otimes t^b U_{\boldsymbol{y}}^\dagger$$

New channels contain running coupling corrections

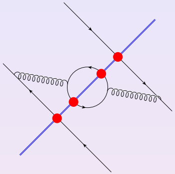


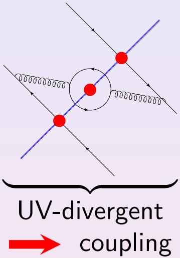
New channels contain running coupling corrections



$$\lim_{z_1, z_2 \to z} 2\mathrm{tr}(t^a U_{z_1} t^b U^\dagger_{z_2}) = U^{ab}_{z}$$

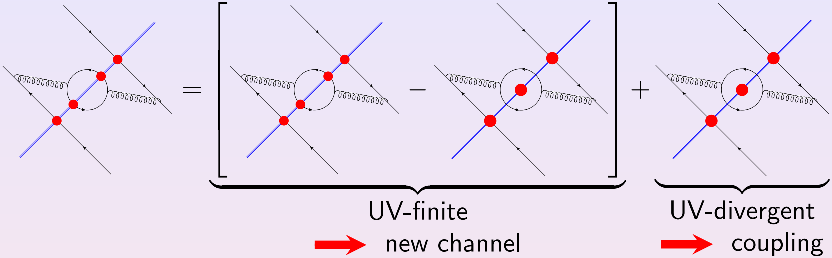
New channels contain running coupling corrections





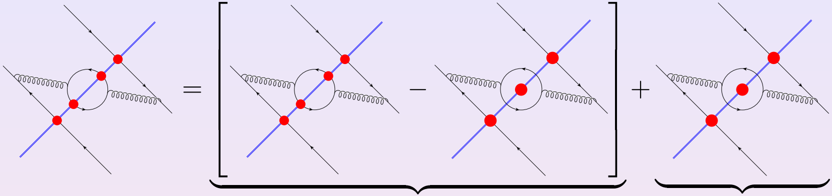
$$\lim_{z_1, z_2 \to z} 2\mathrm{tr}(t^a U_{z_1} t^b U^\dagger_{z_2}) = U^{ab}_{z}$$

New channels contain running coupling corrections



$$\lim_{\boldsymbol{z}_1,\boldsymbol{z}_2\to\boldsymbol{z}} 2\mathrm{tr}(t^a U_{\boldsymbol{z}_1} t^b U^\dagger_{\boldsymbol{z}_2}) = U^{ab}_{\boldsymbol{z}}$$

New channels contain running coupling corrections



UV-finite → new channel

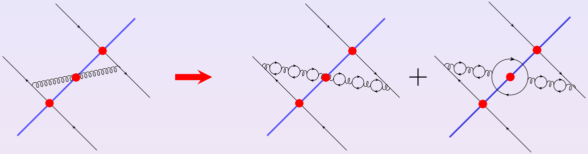
UV-divergent → coupling

$$\lim_{\boldsymbol{z}_1,\boldsymbol{z}_2\to\boldsymbol{z}} 2\mathrm{tr}(t^a U_{\boldsymbol{z}_1} t^b U^\dagger_{\boldsymbol{z}_2}) = U^{ab}_{\boldsymbol{z}}$$

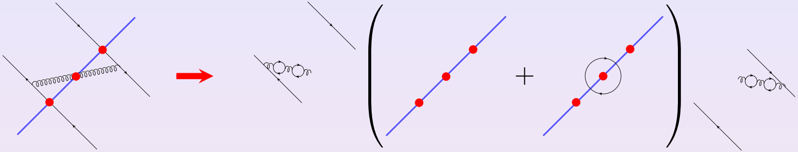
choice of z → separation schemes

- $U^{ab}_{\boldsymbol{z}}$ @ gluon pos: hep-ph/0609087 hep-ph/0609090
- $U^{ab}_{\boldsymbol{z}}$ @ quark pos: hep-ph/0609105 (Balitsky)
- sum unaffected

Running coupling to all orders: triumvirates

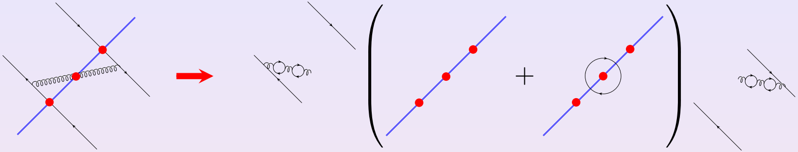


Running coupling to all orders: triumvirates



$$\alpha_s\left(\boldsymbol{q}^2\right) \qquad \frac{1}{\alpha_s\left(\tilde{Q}^2\right)} \qquad \alpha_s\left(\boldsymbol{q}'^2\right)$$

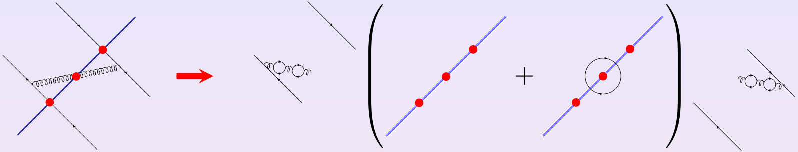
Running coupling to all orders: triumvirates



$$\alpha_s(\mu^2) \quad \rightarrow \quad \frac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)} \qquad \sim \mathcal{O}(\alpha_s)$$

triumvirate of couplings

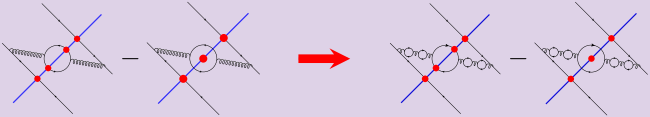
Running coupling to all orders: triumvirates



$$\alpha_s(\mu^2) \quad \longrightarrow \quad \frac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)} \qquad \sim \mathcal{O}(\alpha_s)$$

triumvirate of couplings

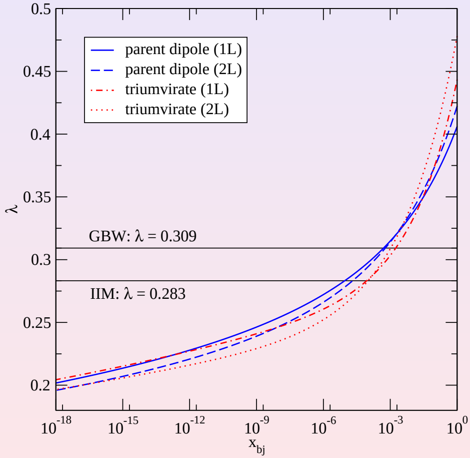
New channels: also dressed, formally higher order, e.g:



$\sim \mathcal{O}(\alpha_s^2)$

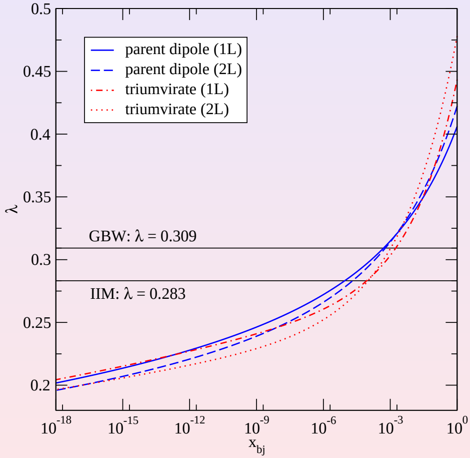
Outline

Main effect: Evolution slows down



- $\lambda = \frac{d}{dY} \ln Q_s(Y)$

Main effect: Evolution slows down

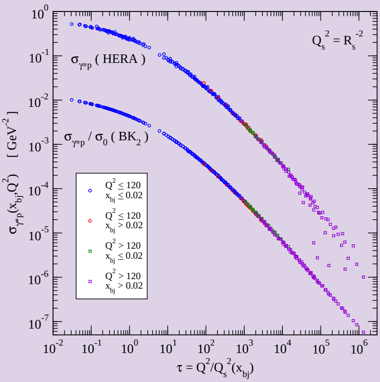


- $\lambda = \frac{d}{dY} \ln Q_s(Y)$

- triumvirates slower (despite "optimal" scheme for parent dipole)
- 1 loop → 2 loop: moderate correction $10^{-6} < x_{\text{bj}} < 10^{-2}$
- new channels?

Successful fit to HERA data

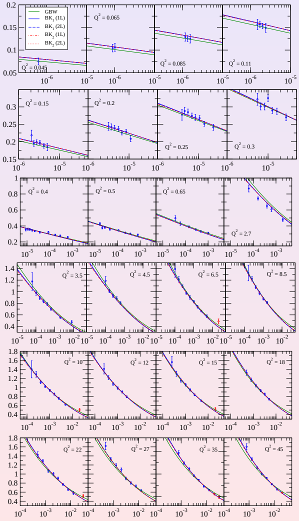
Kuokkanan, Rummukainen, Weigert, in prep.

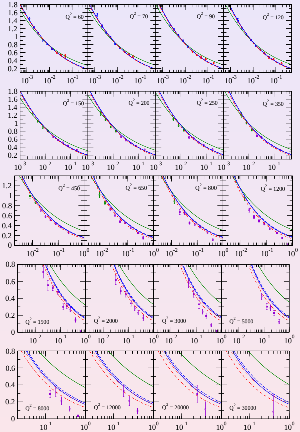


- input:
  - triumvirate running
  - energy conservation corr. (DGLAP)
- excellent global fit
- in pseudo-scaling region (no remnants of initial conditions!)

Successful fit to HERA data

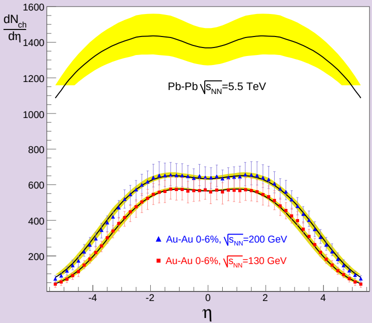
Kuokkanan, Rummukainen, Weigert, in prep.





Pseudorapidity distributions: from RHIC to LHC

J. L. Albacete, arXiv:0707.2545 [hep-ph]

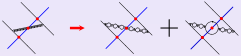


- input
  - k_t factorization: eA-dipoles → AA'-multiplicities
  - running + $q\bar{q}$-channel
- shape @ fixed s
- growth with s
- RHIC not in scaling region: origin of saturation: large A, not s (consistent with Cronin observation)

Outline

Summary and outlook

- $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

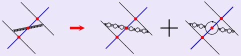


triumvirate coupling

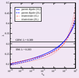
Summary and outlook

- $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

  triumvirate coupling



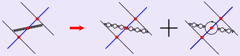
- Evolution slows down



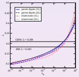
Summary and outlook

- $\alpha_s(\mu^2) \longrightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

  triumvirate coupling

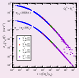


- Evolution slows down

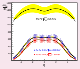


- need additional DGLAP input to match Hera data

  (Kuokkanen, Rummukainen, Weigert)



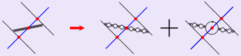
- pseudorapidity distributions: from RHIC to LHC (Albacete)



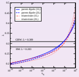
Summary and outlook

- $\alpha_s(\mu^2) \rightarrow \dfrac{\alpha_s\left(\boldsymbol{q}^2\right)\ \alpha_s\left(\boldsymbol{q}'^2\right)}{\alpha_s\left(\tilde{Q}^2\right)}$

  triumvirate coupling

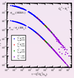


- Evolution slows down

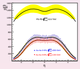


- need additional DGLAP input to match Hera data

  (Kuokkanen, Rummukainen, Weigert)



- pseudorapidity distributions: from RHIC to LHC (Albacete)



- Now that we have stopped modelling running coupling corrections: stop modelling new physics channels! → next talk