QCD Pomeron from Gauge/String Duality

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Aug. 6, 2007, ISMD 2007
LBL, Berkeley, CA

* First principle derivation of Pomeron as a Regge pole --
- - Graviton propagating in AdS space - -

* Conformal invariance beyond perturbative QCD

- work based on papers by R. Brower, J. Polchinski, M. Strassler, and C-I Tan, hep-th/0603115, hep-th/0707.2408, and more (in preparation)

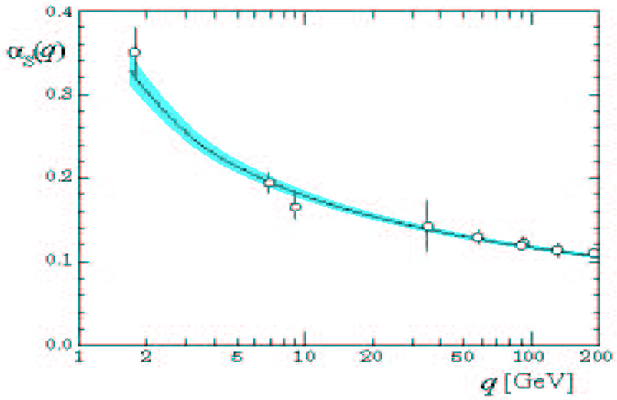
Outline

- Scales in QCD
  - Hard Pomeron (BFKL) -- Scale Invariance
  - Soft Pomeron (Glueballs) -- Confinement scale
- QCD Pomeron as “metric fluctuations” in AdS space
  - Unified Pomeron is Regge Pole in AdS: (Conformal Invariance)
  - Pomeron as a Reggeized Massive Graviton: (Confinement)
- Pomeron Kernel in Transverse Space: AdS_3
- Other developments: Unitarization, Froissart Bound, Confinement, etc.

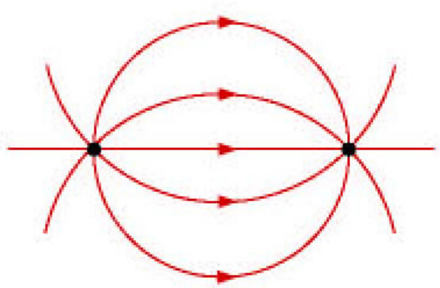
I. Scalar Dependence of QCD and History of Diffractive Dcattering at High Energies

Asymptotic Freedom

perturbative



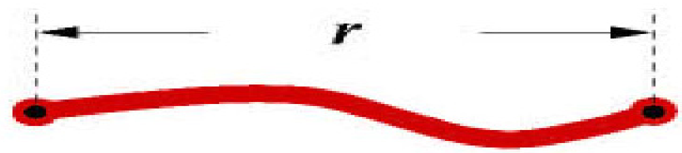
$$\alpha_s(q) \equiv \frac{\bar{g}(q)^2}{4\pi} = \frac{c}{\ln(q/\Lambda)} + \ldots$$



r < 0.1 fm

Confinement

non-perturbative



r >> 1 fm

Force at Long Distance--Constant Tension/Linear Potential, Coupling increasing, Quarks and Gluons strongly bound <==> "Stringy Behavior"

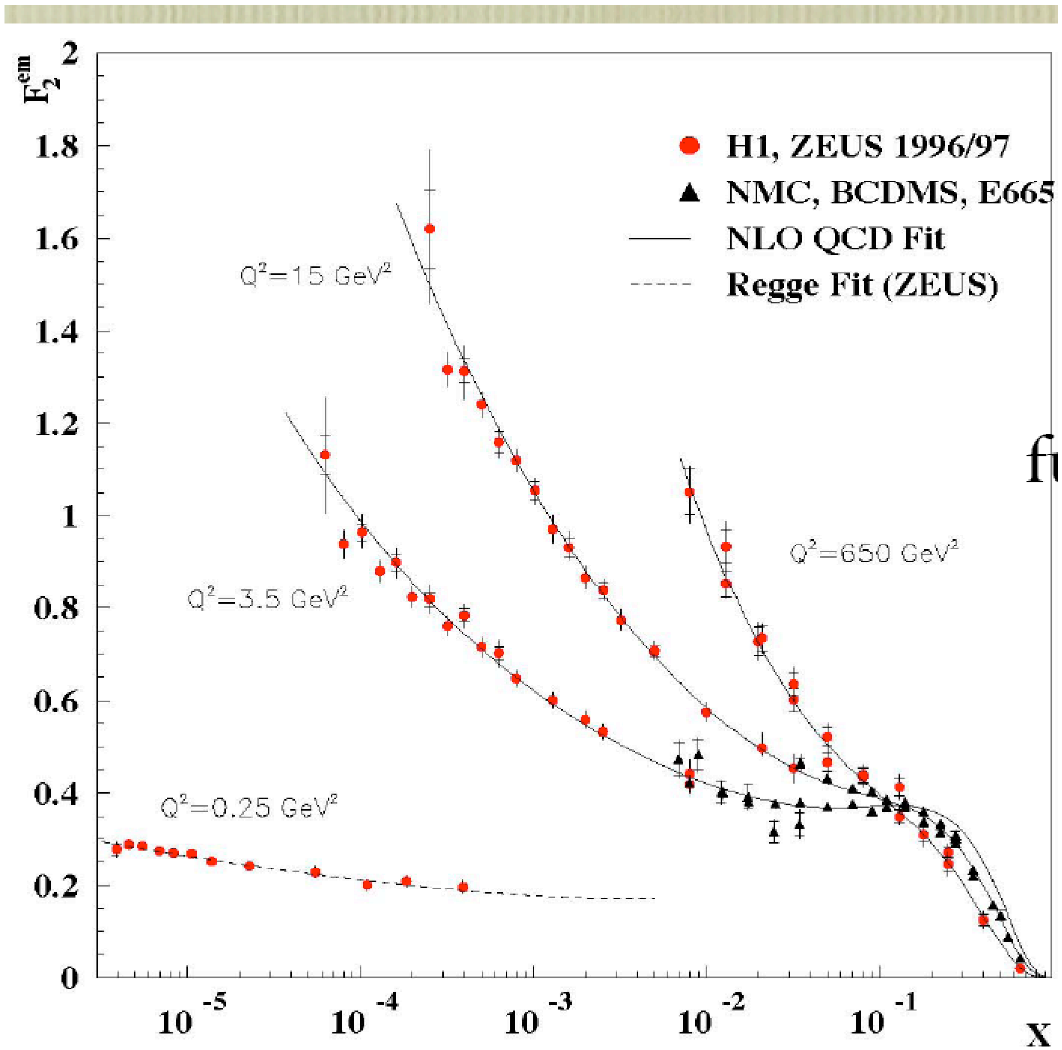
Test of Perturbative QCD-- Deep Inelastic Scattering (DIS)

Anomalous Dimension of

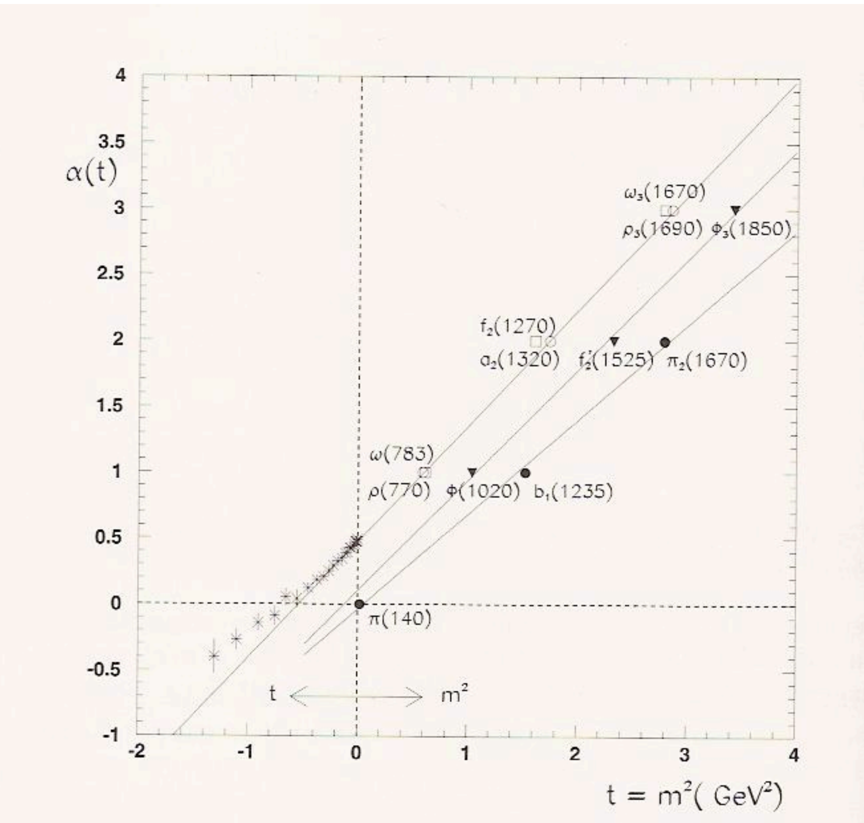
Leading twist operator

DGLAP evolution

$$\mathrm{tr}(F_{+\mu} D_+^{j-2} F_+{}^{\mu})$$

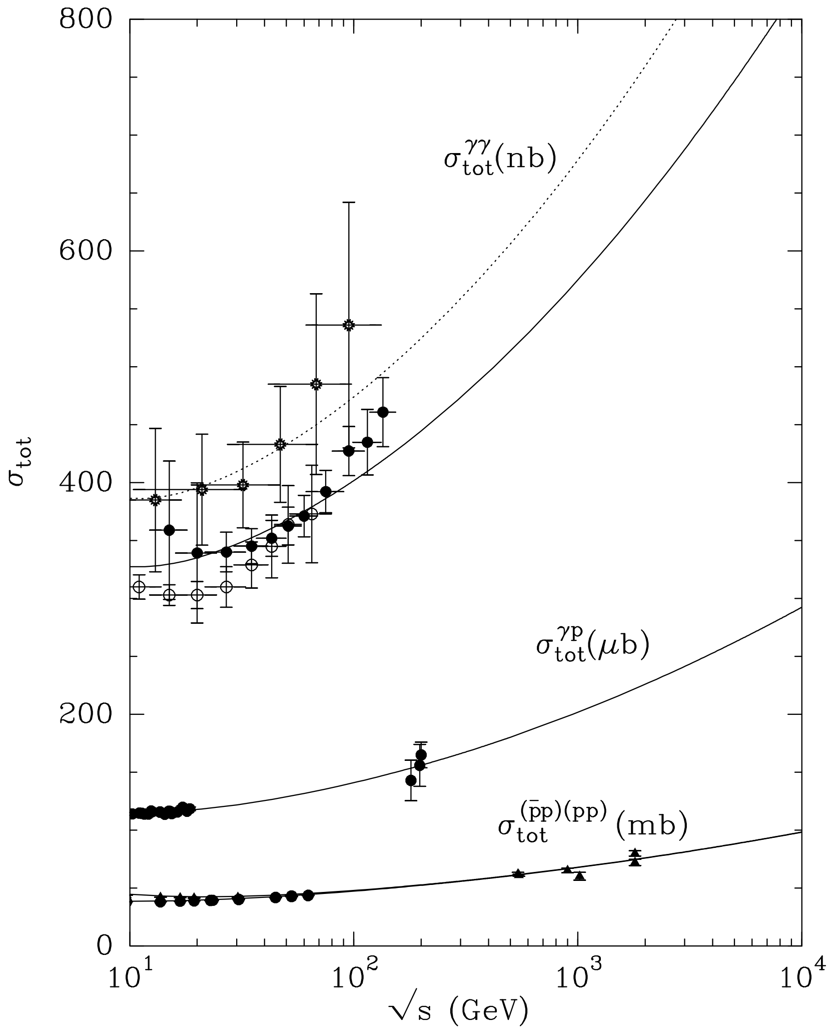


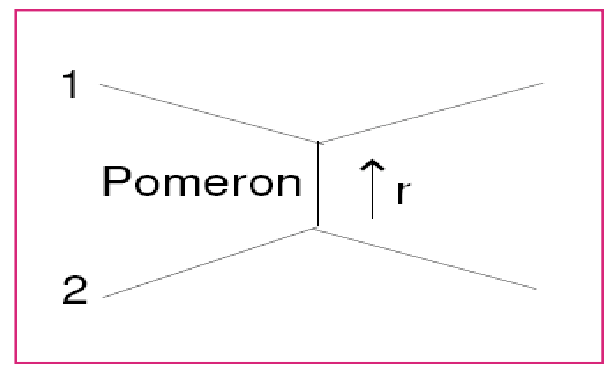
Regge Behavior and Regge Trajectory



$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

Total Cross Sections





$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

$$\sigma_{total} \sim \mathcal{A}(s,0)/s \sim S^{J(0)-1} \sim s^{\alpha(0)-1}$$

$$\alpha(0) > 1$$

IR (Soft) Pomeron ??

BFKL (Balitsky-Lipatov-Fadin-Kuraev)

BFKL Summation: Scale Invariance

- Weak perturbation theory: 1st order in α_s and all orders $(\alpha_s \log s)^n$
- Implies "planar" diagrams (e.g. $N_c = \infty$) and conformal scaling
- BFKL is essentially a large N_c CFT results!

$$A(s,t=0) \simeq \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s;k_\perp,k'_\perp) \Phi_2(k'_\perp)$$

$$K(s,k_\perp,k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi \ln s}} e^{-\left[(\ln k'_\perp - \ln k_\perp)^2 / 4\mathcal{D} \ln s\right]}$$

Diffusion in "virtuality" $k_\perp$

Weak Coupling:

$$\alpha(0) = 1 + \ln(2) g^2 N/\pi^2$$

$$\mathcal{D} = \frac{14\zeta(3)}{\pi} g^2 N/4\pi^2.$$

BFKL vs Soft Pomeron

- Perturbative QCD
- Short-Distance
- α_{BFKL} (0) ~ 1.4
- Increasing Virtuality
- No Shrinkage of elastic peak
- Fixed-cut in t
- Diffusion in Virtuality
-

- Non-Perturbative
- Long-distance: Confinement
- $\alpha_P(0)$ ~ 1.08
- Fixed trans. Momenta
- Shrinkage of Elastic Peak: <|t|> ~1/ log s
- $\alpha'(0)$ ~ 0.3 Gev^{-2}
- Diffusion in impact space

UV Pomeron (BFKL): Scale Invariance

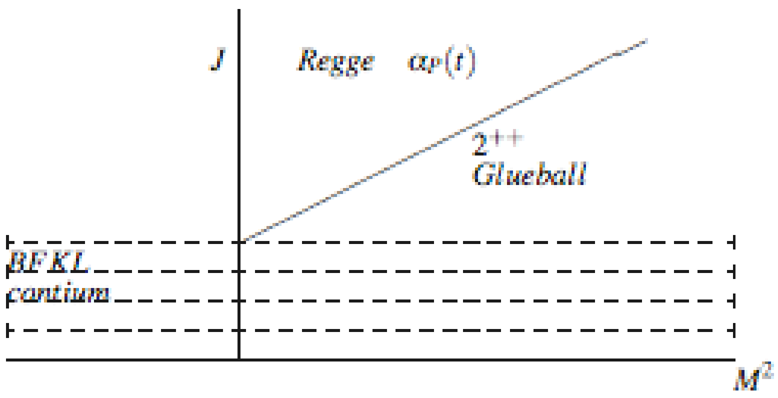
IR Pomeron (Soft Pomeron): Confinement

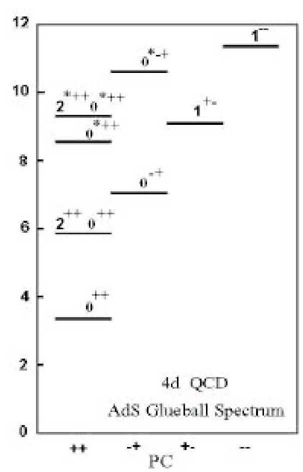
II: Gauge/String Duality

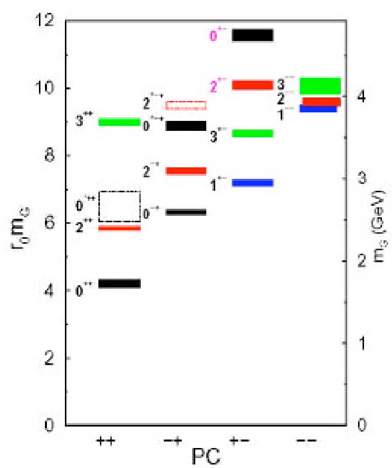
QCD Pomeron as "metric fluctuations" in AdS

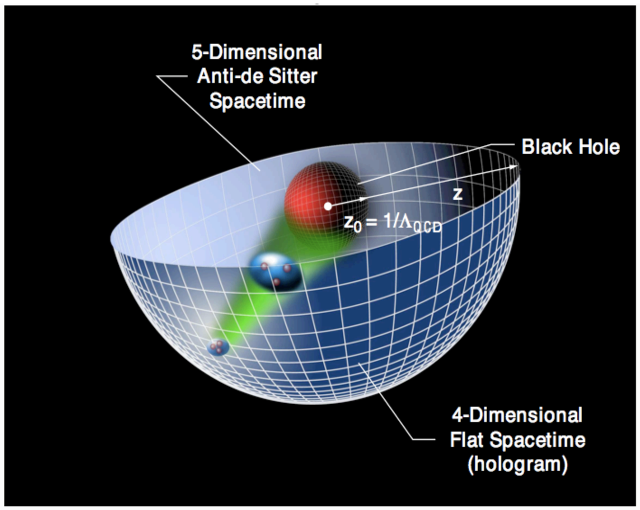
- Strong <==> Weak duality
- Scale Invariance:
- Confinement:
- Pomeron as Reggeized Massive Graviton

QCD Pomeron <====> Graviton (metric) in AdS









The QCD Pomeron

We show that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

Pomeron can be associated with a Reggeized Massive Graviton.

Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

II-a. Gauge/String Duality

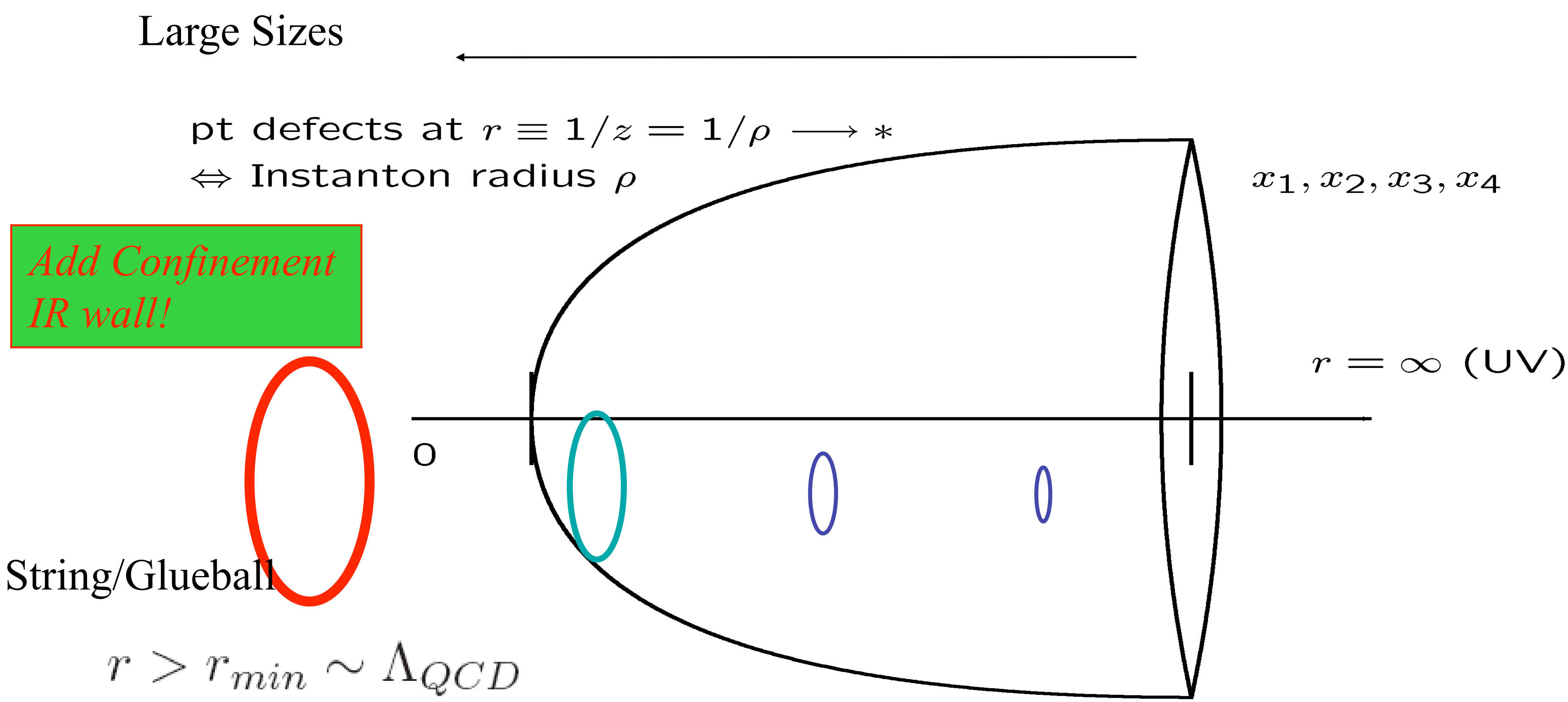
Scale Invariance and Cutoff AdS$_5$

** Maldacena:
at large r (UV), AdS-5

$$ds^2 \simeq -r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dr^2}{r^2} + ds_X^2$$

** Scale Invariance of QCD in UV:

$$x \to \zeta x; \quad r \to \frac{r}{\zeta}$$



Gauge/String Dual: Confinement Deformation

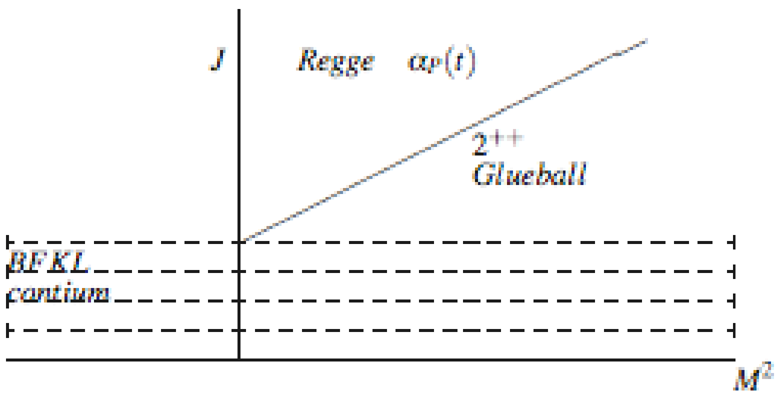
- Use models to provide concrete mathematical realization of Gauge/String for QCD

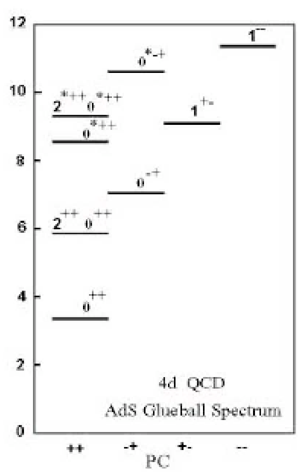
- For simplicity, mostly use Hard-Wall Model

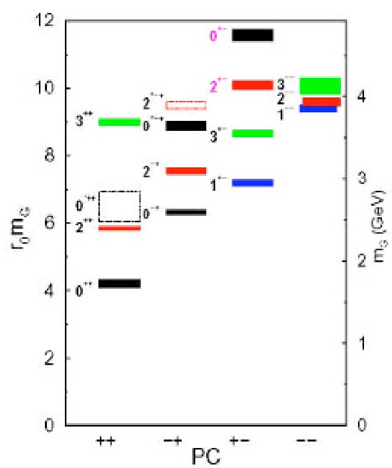
Identify model independent features.

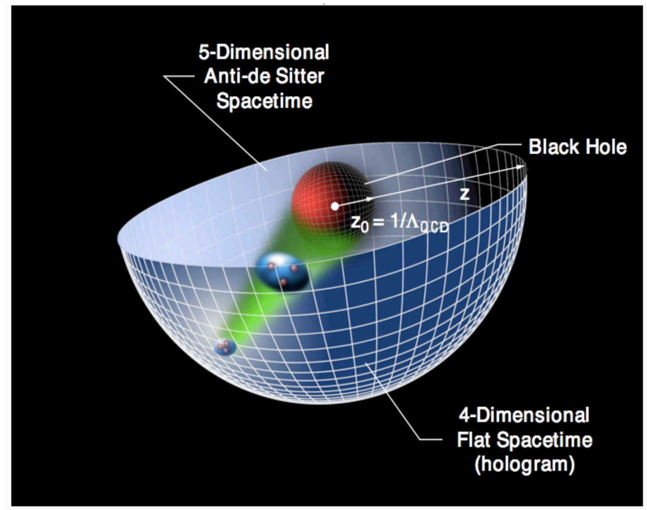
II-6. Spectrum at strong coupling

QCD Pomeron <===> Graviton (metric) in AdS









4-Dim Massive Graviton

5-Dim Massless Mode:

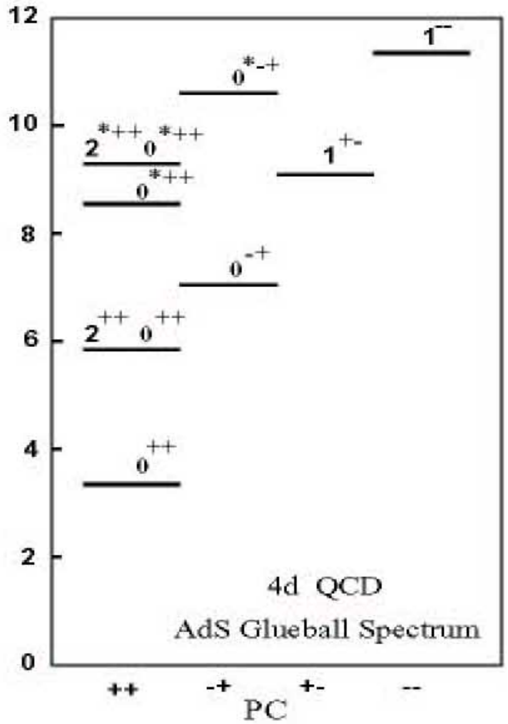
$$0 = E^2 - (p_1{}^2 + p_2{}^2 + p_3{}^2 + p_r{}^2)$$

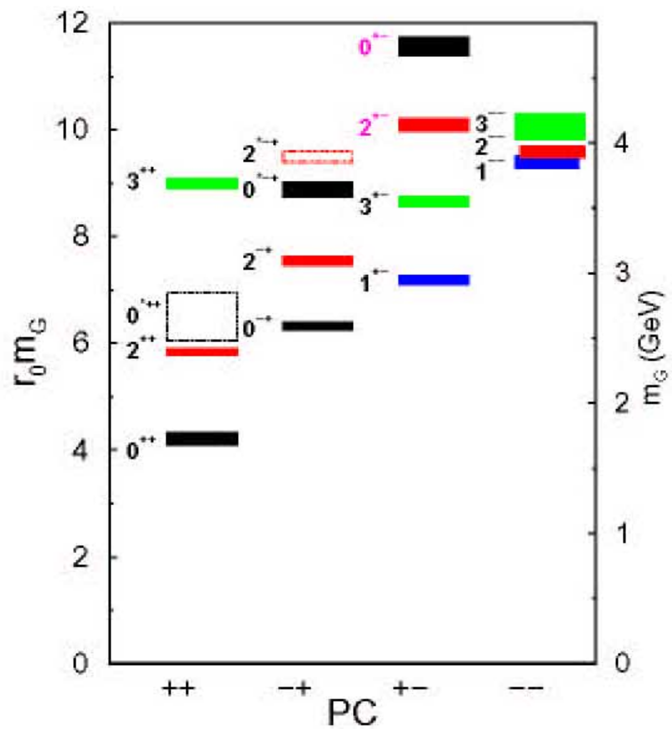
If, due to Curvature in fifth-dim, $p_r{}^2 \neq 0$,

Four-Dimensional Mass:

$$E^2 = (p_1{}^2 + p_2{}^2 + p_3{}^2) + M^2$$

Glueball Spectrum

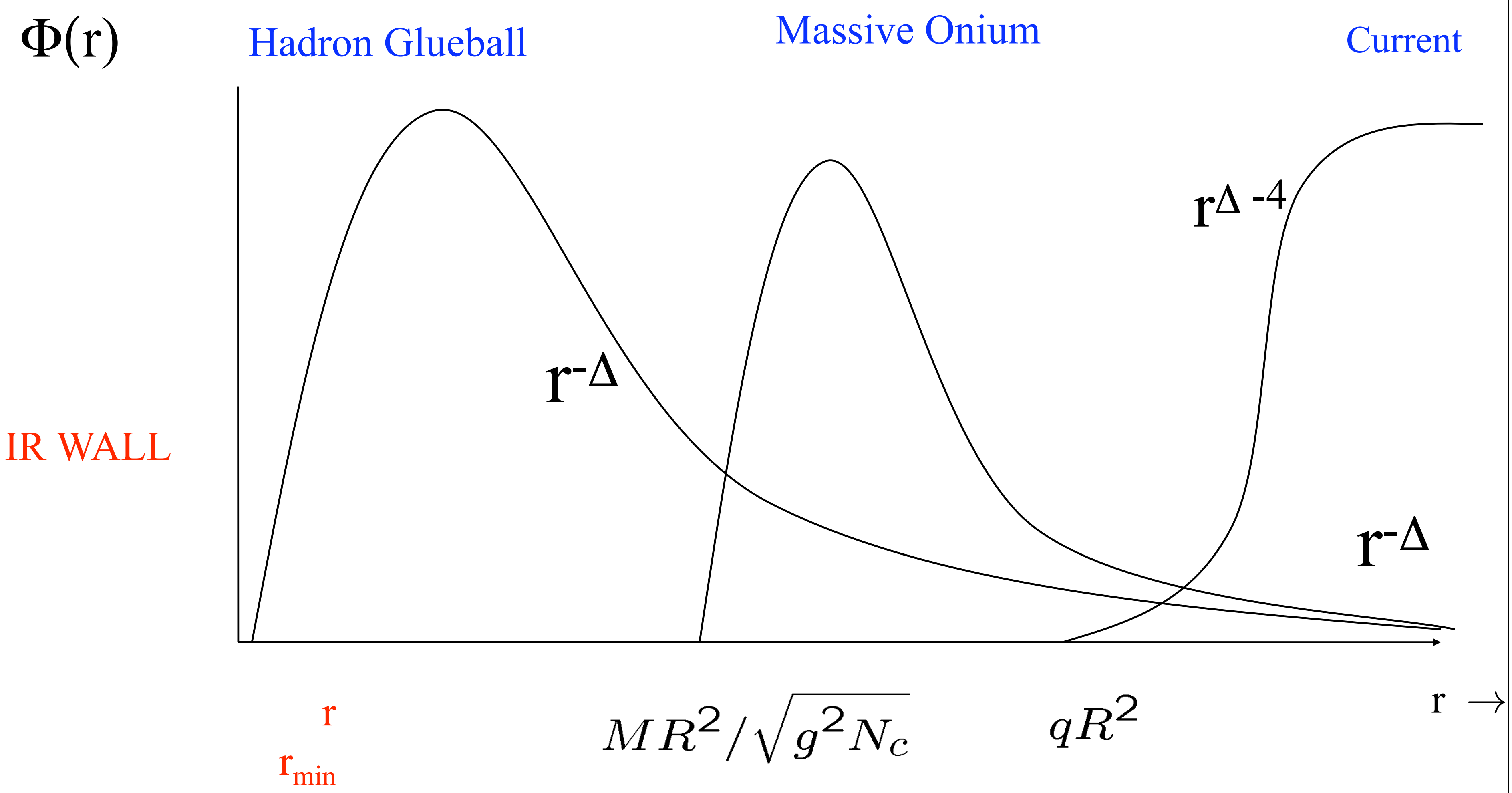




The AdS^7 glueball spectrum for QCD_4 in strong coupling (left) compared with the Morningstar/Peardon lattice spectrum for pure SU(3) QCD (right) with $1/r_0 = 410$ Mev.

R. Brower, S. Mathur, and C-I Tan, hep-th/0003115, "Glueball Spectrum of QCD from AdS Supergravity Duality".

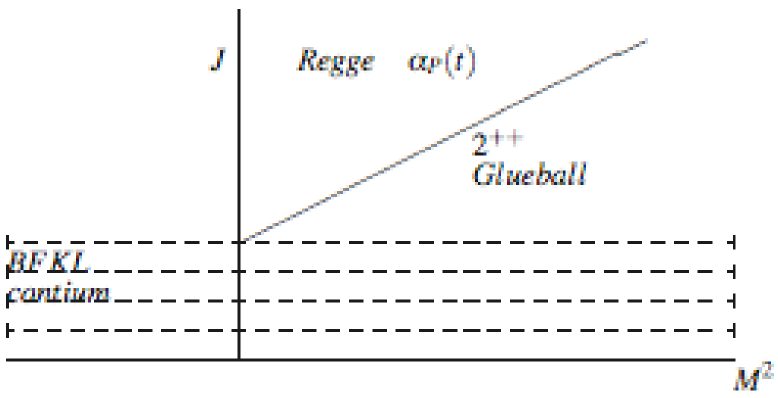
Approx. Scale Invariance and the 5th dimension

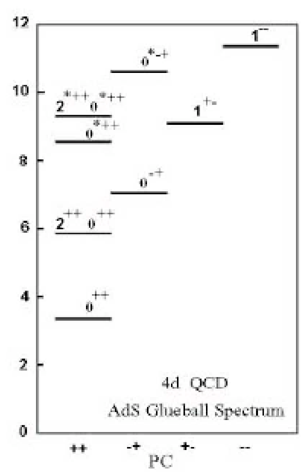


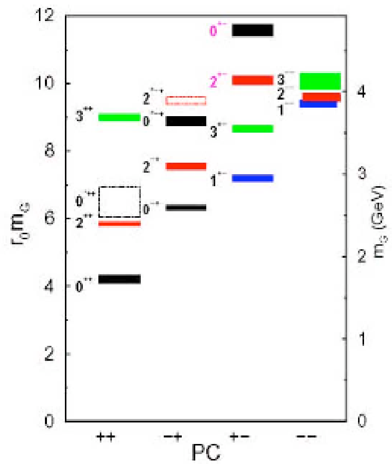
==> Hard Scattering (Polchinski-Strassler)

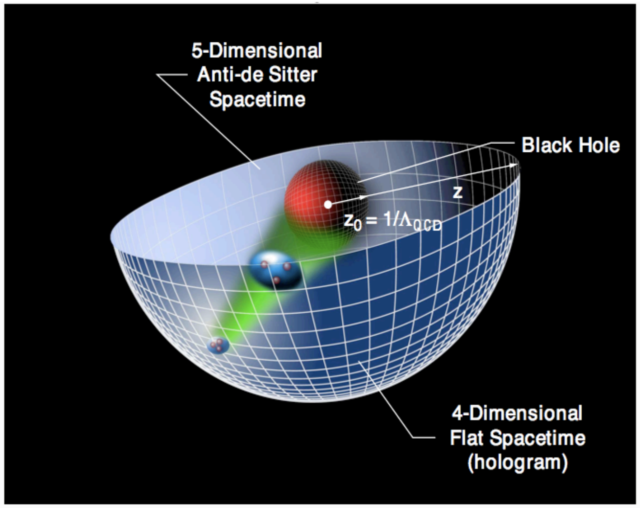
II-c: Various Facets of the AdS Pomeron

QCD Pomeron <===> Graviton (metric) in AdS









Regge Behavior in AdS_5

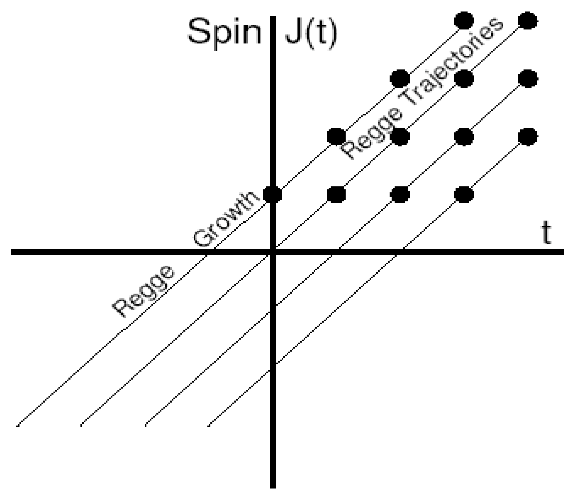
$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

$$t \leftrightarrow -\nabla^2$$

Regge in Flat Space

String amplitudes $\longrightarrow$ Regge behavior $\mathcal{A} \sim \sum_i s^{J_i(t)}$

$[J(t) = \alpha(t) = \alpha_0 + \alpha' t]$



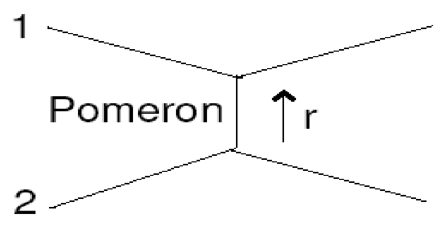
- t negative; Fourier transform momentum space $\longrightarrow$ position space

$$J(t) \sim \alpha_0 + \alpha' t \Rightarrow \mathcal{A} \sim s^{\alpha_0} \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

Strings grow: $\langle |\vec{x}|^2 \rangle \sim \ln s$

(random-walk diffusion, with $\tau \sim \ln s$)

Regge in Curved Space



$$\mathcal{A} \sim s^{J(t)} = s^{2+\alpha' t/2} \quad \text{(flat space)}$$
$$\rightarrow s^{2+\alpha' \nabla^2/2} \quad \text{(curved space)}$$
$$= s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau}$$

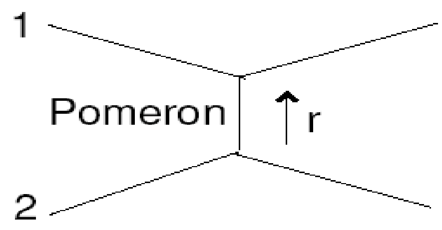
where $\tau \propto \ln s$ is again a diffusion time, and

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla^2_{\mathbf{r}} + 0 = -\partial_u^2 + (4 - e^{-2u} t/t_0)$$

where $u = \ln r$

A Schrödinger operator with potential $V(u;t) = 4 - e^{-2u} t/t_0$

Diffusion in u=log r: (Effective Hamiltonian at t=0)



$$\mathcal{A} \sim s^{J(t)} = s^{2+\alpha' t/2} \quad \text{(flat space)}$$
$$\to s^{2+\alpha' \nabla^2/2} \quad \text{(curved space)}$$
$$= s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau}$$

where $\tau \propto \ln s$ is again a diffusion time, and for $t = 0$,

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla_{\mathbf{r}}^{\mathbf{2}} + 0 = -\partial_u^2 + 4$$

where $u = \ln r$

A Schrödinger operator with potential $V(u;t) = 4$

$$\mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Diffusion in AdS: $u = \log r$ (continued)

$$\mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{\lambda}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Sandwiching this differential operator between the two scattering hadrons, writing the kernel explicitly, and recalling $\tau \propto \ln s$, $u = \ln r$,

$$\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \, \Phi_1(r) \;\; s^{j_0} \frac{e^{-\left[(\ln[r'/r])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \;\; \Phi_2(u')$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Same form as the BFKL kernel for $t = 0$:

Comparison of Diffusion in AdS and BFKL

$$\mathcal{A} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) \quad s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \quad \Phi_2(k'_\perp)$$

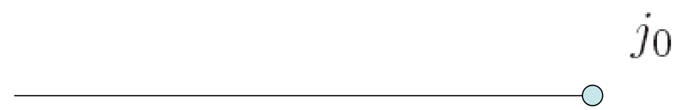
$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \ , \quad \mathcal{D} = \frac{7\zeta(3)}{\pi}\alpha N \ .$$

$$\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \ \Phi_1(r) \quad s^{j_0} \frac{e^{-\left[(\ln[r'/r])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \quad \Phi_2(u')$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}} \ , \ \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

Conformal Pomeron

- Fixed branch point in J-plane:



- Weak coupling:

$$j_0 = 1 + \frac{4 \ln 2}{\pi} \alpha N$$

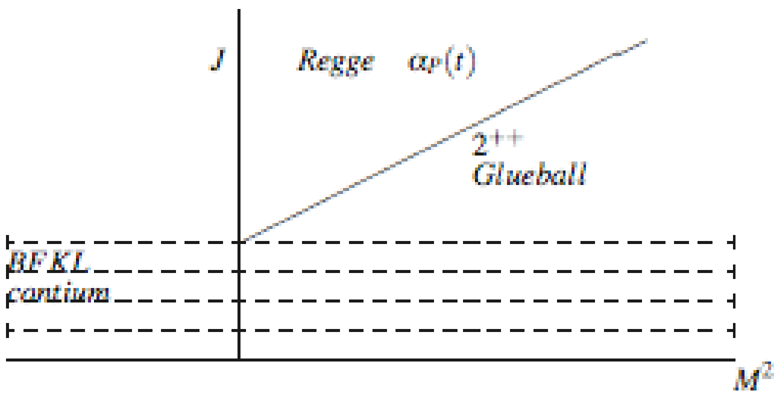
- Strong coupling:

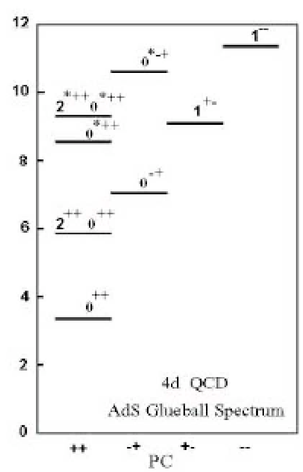
$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

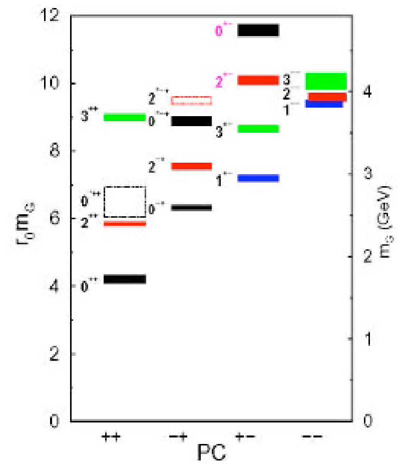
II-d. Synthesis of Conformal (BFKL) and Confined (Soft)Pomeron

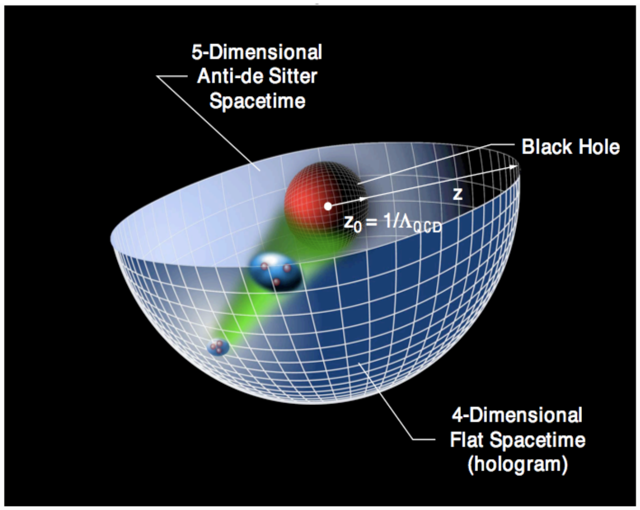
"heterotic Pomeron": Levin and Tan, (hep-ph/9302308)

QCD Pomeron <====> Graviton (metric) in AdS









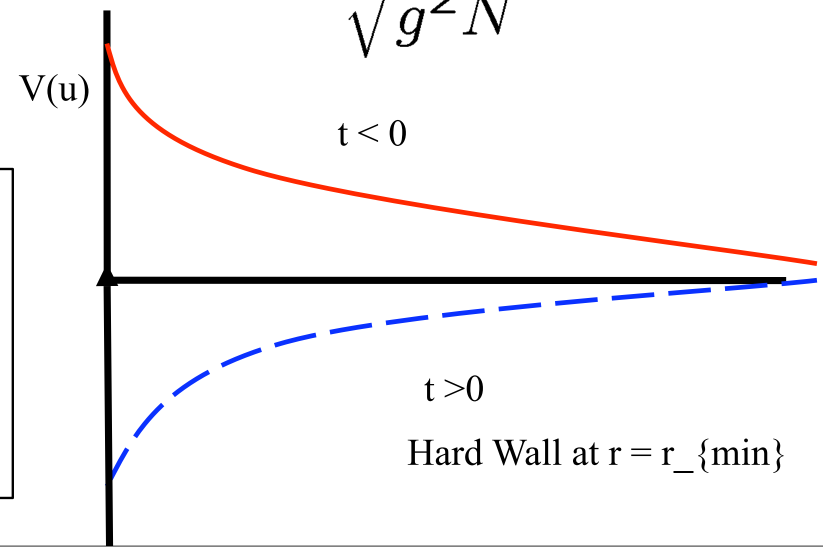
Pomeron in Confined AdS Deformation

Confinement----> Regge trajectory, Resonances, etc.

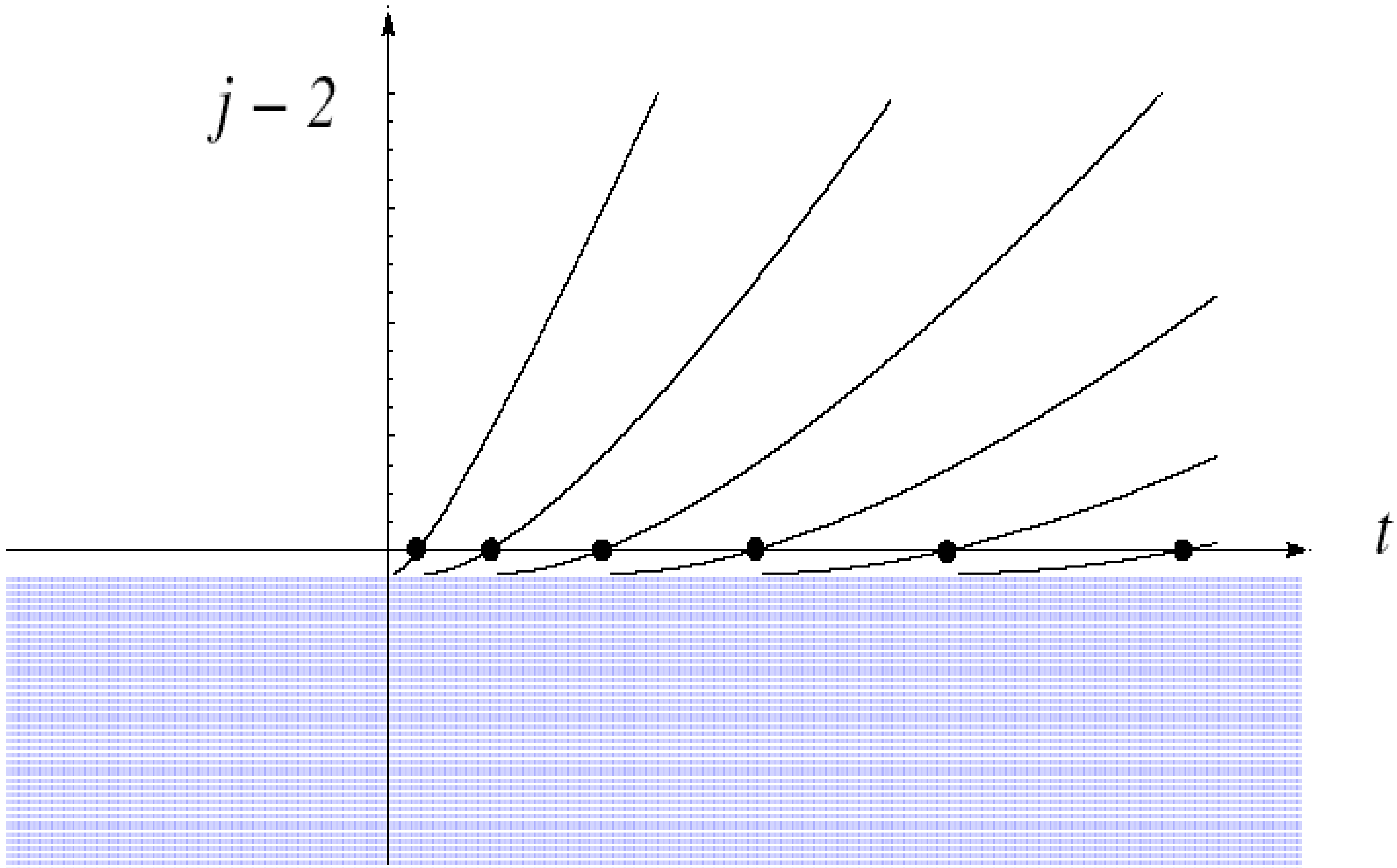
Simplest Model: Hard-Wall

$$\frac{1}{2\sqrt{g^2N}}\left[\,-\frac{d}{du^2}-te^{-2u}\,\right]\Psi(u,J)=\left(2-J-\frac{2}{\sqrt{g^2N}}\right)\Psi(u,J)$$

- $V(u) = -t\, e^{-u}$, $0 < u < \infty$
- Attractive for $t > 0$, Regge Pole + BFKL cut
- $t < 0$ only scattering state for BFKL

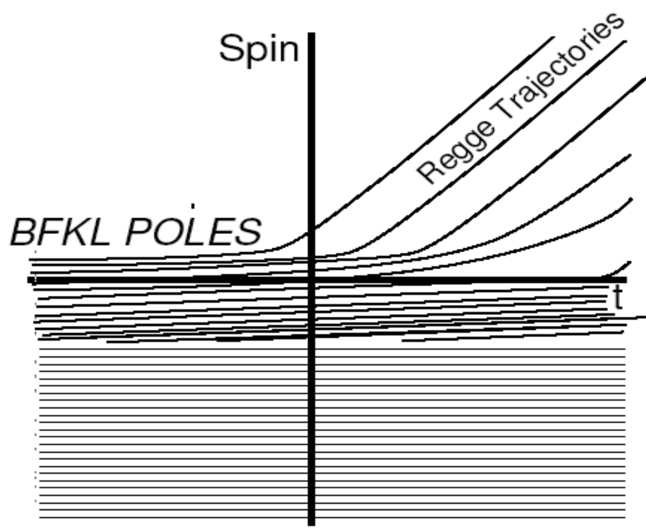


Hardwall Regge Spectrum and Cut



Pomeron in QCD

Running UV, Confining IR (large N)



The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

The QCD Pomeron

Have shown that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

Pomeron can be identified as Reggeized Massive Graviton.

Both the IR Pomeron and the UV Pomeron are dealt in a unified single step.

Both conceptual and practical advantages.

III. Pomeron Kernel in Impact Space

* Reduction to AdS_3

* New Realization of Conformal Invariance

  - Conformal limit:

  - Confinement:

AdS Graviton Exchange at High Energies

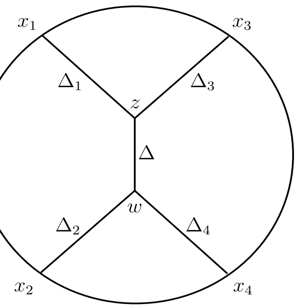
$$ds^2 \simeq \frac{R^2}{z^2}(-\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds_X^2$$

$$z = R^2/r$$

One Graviton in Momentum Representation at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde\Phi_\Delta(p_1^2,z)\tilde\Phi_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde\Phi_\Delta(p_2^2,z')\tilde\Phi_\Delta(p_4^2,z')$$

$$p_1 + p_2 \rightarrow p_3 + p_4$$



$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

Reduction to AdS-3 at High Energy for Near Forward Scattering

* momentum transfer q is transverse:

$$(zz')G^{(3)}_{\Delta=3}(x^{\perp},z,z') = \int \frac{dq^{\perp}}{(2\pi)^2} e^{ix^{\perp}q^{\perp}} G^{(5)}_{\Delta=4}(q^{\pm}=0,q^{\perp},z,z')$$

* AdS-3 Propagator:

$$\mathcal{K}(s,x^{\perp},z,z') = (zz's)^2 (zz')G^{(3)}_3(x^{\perp},z,z')$$

* Isometry of Euclidean AdS-3 is SL(2C) ---
the same symmetry group as BFKL kernel

$$\{-\partial_z z^{-1}\partial_z - z^{-1}\partial^2_{x^{\perp}} + 3z^{-3}\}G^{(3)}_3(x_{\perp},x'_{\perp},z,z') \;\;=\;\; \delta(z-z')\delta^{(2)}(x_{\perp}-x'_{\perp})$$

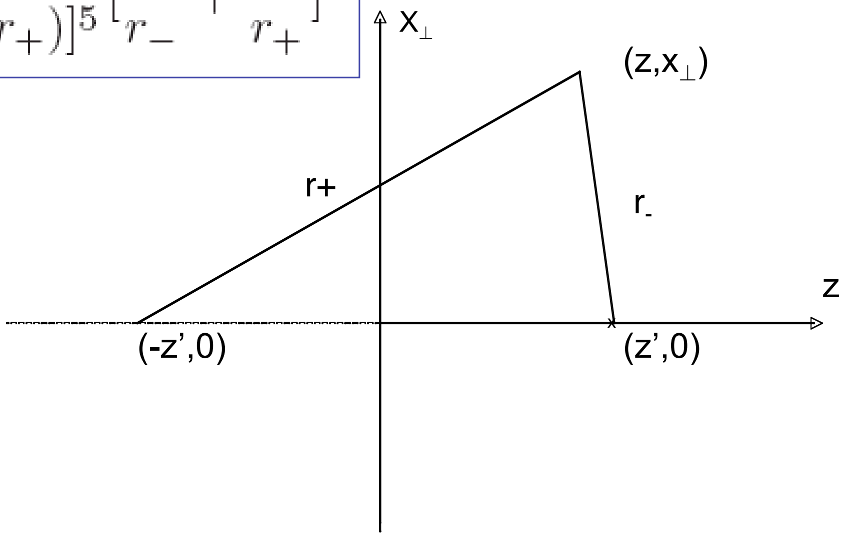
AdS-3 Propagator in Conformal Limit

$$G_3^{(3)}(x_\perp, z, z') \;=\; \frac{1}{4\pi}\frac{1}{[y+\sqrt{(y^2-1)}]^2\sqrt{y^2-1}}$$

$$y = \frac{z^2 + z'^2 + (x-x')^2}{2zz'}$$

$$r_\pm = \sqrt{(z \pm z')^2 + x_\perp^2} = \sqrt{(2zz')(y \pm 1)}$$

$$G_3^{(3)}(x_\perp, z, z') = \frac{(2zz')^3}{\pi}\frac{1}{[(r_- + r_+)]^5}[\frac{1}{r_-} + \frac{1}{r_+}]$$



Randall-Sundrum with vanishing
Direchlet bdry condition at z=0

Finite Strong Coupling Pomeron Propagator--Conformal Limit

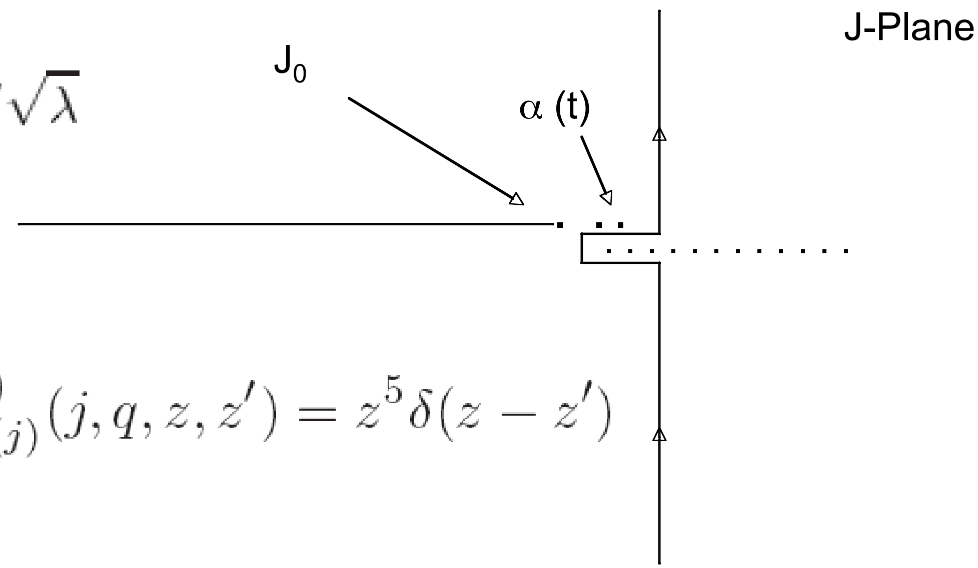
- Spin 2 --------> J by Using Complex angular momentum representation
- Reduction to AdS-3

Complex j-Plane:

$$\mathcal{T}^{(1)}(p_i, z, z') = \int \frac{dj}{2\pi i}\, \frac{(1 + e^{-i\pi j})}{\sin \pi j}\, (\tilde{s})^j\, G^{(5)}(j, q, z, z')$$

Integration Contour for Mellin Transform

$$j_0 = 2 - 2/\sqrt{\lambda}$$



$$\{2\sqrt{\lambda}(j-2) - z^5\partial_z z^{-3}\partial_z - z^2 t\} G^{(5)}_{\Delta(j)}(j, q, z, z') = z^5 \delta(z - z')$$

Reduction to AdS-3:

$$G^{(5)}_{\Delta}(j, q^{\pm} = 0, q^{\perp}, z, z') \to (zz') G^{(3)}_{(\Delta-1)}(j, q_{\perp}, z, z')$$

Strong Coupling Pomeron Propagator--

Conformal Limit

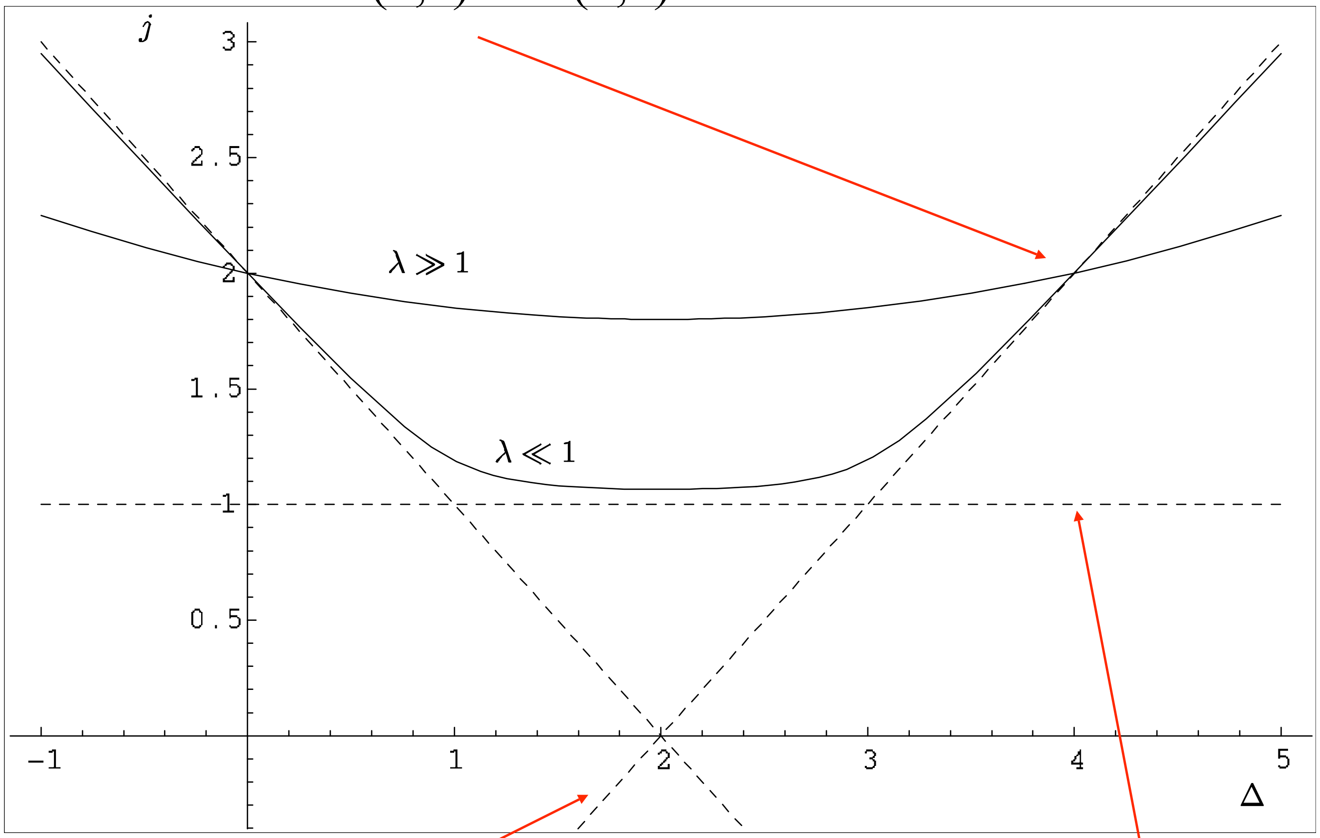
- Use J-dependent Dimension

$$\Delta : \quad 4 \rightarrow \Delta(J) = 2 + [2\sqrt{\lambda}(J - J_0)]^{1/2} = 2 + \sqrt{\tilde{j}}$$

- BFKL-cut: $J_0 = 2 - \frac{2}{\sqrt{\lambda}}$

Spin-Dimension Curve

(4,2) and (0,2) have zero anomalous dimension



λ = 0 Anomalous
Dim=0

λ = 0, BFKL

inversion symmetry: Δ ➔ 4 - Δ

Strong Coupling Pomeron Propagator-- Conformal Limit and Comparison with BFKL

- AdS-3 propagator:

$$\mathcal{K}(j, x_\perp - x'_\perp, z, z') = \frac{1}{4\pi z z'} \frac{\left[y + \sqrt{y^2 - 1}\right]^{(2-\Delta_+(j))}}{\sqrt{y^2 - 1}} ,$$

$$y \pm 1 = \frac{(z \mp z')^2 + (x_\perp - x'_\perp)^2}{2zz'}$$

- BFKL kernel:

$$\Phi_{n,\nu}(b_1 - b_0, b_2 - b_0) = \left[\frac{b_1 - b_2}{(b_1 - b_0)(b_2 - b_0)}\right]^{i\nu+(1+n)/2} \left[\frac{\bar{b}_1 - \bar{b}_2}{(\bar{b}_1 - \bar{b}_0)(\bar{b}_2 - \bar{b}_0)}\right]^{i\nu+(1-n)/2}$$

IV. Further Developments:

- "Eikonalization for AdS-Graviton," (hep-th/0707:2408, R Brower, M. Strassler, C-I Tan)
- "Froissart Bound and Confinement," (in preparation, R Brower, M. Strassler, C-I Tan)
- Nonlinear Gravity effects: e.g., fan diagrams,
- Loops: e.g., AdS-3 Pomeron-Field Theory,
- Odderon, Diffractive Jets, Higgs', etc. at LHC,
- High Density Phenomena, etc.

V. Summary

- Provide meaning for Pomeron Pole non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences.