Decoherence of colour in QCD vacuum

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The model of QCD stochastic vacuum

- The model of QCD stochastic vacuum is one of the popular phenomenological models which explains quark confinement [*Savvidy'77*, *Ambjorn'80*, *Simonov'96*, *Dosch'02*, *Simonov'04*].
- It is based on the assumption that one can calculate vacuum expectation values of gauge-invariant quantities as expectation values with respect to some well-behaved stochastic gauge field.
- It is known that such vacuum provides confining properties, giving rise to QCD strings with constant tension at large distances.

Our model

- Most frequently the model of QCD stochastic vacuum is used to calculate Wilson loops, string tensions and field configurations around static charges [*Simonov'96*, *Dosch'02*].
- In this paper we will consider the colour states of quarks themselves.
- Usually white wave functions of hadrons are constructed as gauge-invariant superpositions of quark colour states.
- Here we will show that white objects can be also obtained as white mixtures of states described by the density matrix.

Our model

- Suppose that we have some quantum system which interacts with the environment.
- Interactions with the environment can be effectively represented by additional stochastic terms in the hamiltonian of the system
- The density matrix of the system in this case is obtained by averaging with respect to these stochastic terms [*Haken'72*, *Reineker'82*, *Haake'91*, *Peres,95*].
- Instead of considering complicated nonperturbative dynamics of Yang-Mills fields one introduces external stochastic field and average over its implementations [*Savvidy'77*, *Ambjorn'80*, *Simonov'96*, *Dosch'02*, *Simonov'04*].
- Interactions with the environment result in decoherence and relaxation of quantum superpositions [*Haake'91*, *Peres,95*].
- QCD stochastic vacuum can be considered as the environment in quantum-optical language.
- Information on the initial state of the quantum system is lost after sufficiently large time. Here the analogy between QCD vacuum and the environment can be continued: information on colour states is also lost in QCD vacuum due to confinement phenomenon.

In order to demonstrate the emergence of white states which is caused by decoherence processes let us consider the following example. Consider propagation of heavy spinless quark along some fixed path γ from the point x to the point y. The amplitude of such process in Feynman-Schwinger representation and in quenched approximation factorizes into the kinetic part and the parallel transport operator, hence in semiclassical approximation the colour state vector of the quark in the point y is obtained by parallel transport from the point x.

$$|\phi(y)\rangle = \mathcal{P} \exp \left(i \int\limits_{\gamma} dx^{\mu} \hat{A}_{\mu} \right) |\phi(x)\rangle$$

where by kets we denote colour state vectors (unit vectors in N_c-dimensional complex space), $\mathcal{P}$ is the path-ordering operator and $\hat{A}_{\mu}$ is the gauge field vector. Equivalently we can describe evolution of colour state vectors by parallel transport equation $\partial_{\mu}|\phi\rangle = i\hat{A}_{\mu}|\phi\rangle$. In order to consider mixed states we introduce the colour density matrix $\hat{\rho} = \sum\limits_{k} w_k |\phi_k\rangle\langle\phi_k|$, where w_k is the probability to find the system in the state $|\phi_k\rangle$. Density matrix obeys the constraint $\mathrm{Tr}\,\hat{\rho} = 1$.

We first obtain the colour density matrix of the quark which propagates in a fixed external gauge field, which is some particular implementation of QCD stochastic vacuum. We will denote this solution by $\hat{\rho}_1(\gamma)$. The colour density matrix $\hat{\rho}_1$ is parallel transported according to the following equation:

$$\partial_\mu \hat{\rho}_1 = i\left[\hat{A}_\mu, \hat{\rho}_1\right]$$

In order to find the solution of this equation we decompose the colour density matrix into the pieces which transform under trivial and adjoint representations of the gauge group:

$$\hat{\rho}_1 = N_c^{-1}\hat{I} + \rho_1^a \hat{T}_a$$

$$\partial_\mu \rho_1^a = A^a_{b\,\mu} \rho_1^b$$

$$\rho_1^a(y) = \mathcal{P}\exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right)^a_b \rho_0^b$$

$$\hat{\rho}_1(\gamma) = N_c^{-1}\hat{I} + \hat{T}_a \mathcal{P} \exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right)^a_b \rho_0^b$$

According to the definition of the density matrix we should finally average this result over all implementations of stochastic gauge field. Here some remarks should be made. First, in the model of QCD stochastic vacuum only expectation values of path-ordered exponents over closed paths are defined. This means that while working in the framework of this model we can consider only closed paths γ with $x = y$. Physical reason for it is that if $x \neq y$ than the colour state in the point y can be transformed into any other state by some gauge transformation. Averaging procedure in this case is not applicable, since in the model of stochastic vacuum only expectation values of gauge-invariant quantities are defined. Closed path corresponds to a process in which the particle-antiparticle pair is created, propagate and finally annihilate. Thus we will consider only closed paths γ. Second remark concerns the expectation value of the path-ordered exponent. Due to the Schur's lemma in colour-neutral stochastic vacuum it is proportional to the identity, therefore we can write it as follows:

$$\langle\langle \mathcal{P}\exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right)^a_b\rangle\rangle =$$

$$= \left(N_c^2-1\right)^{-1}\delta^a_b\langle\langle \mathcal{P}\exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right)^d_d\rangle\rangle = \delta^a_b W_{adj}\left(\gamma\right)$$

where by $\langle\langle\ldots\rangle\rangle$ we denote averaging over implementations of stochastic vacuum and $W_{adj}\left(\gamma\right)$ is the Wilson loop in the adjoint representation. Finally after averaging over implementations of stochastic vacuum we obtain for the colour density matrix of the colour charge which was parallel transported along the loop γ:

$$\hat{\rho}\left(\gamma\right) = \langle\langle\hat{\rho}_1\left(\gamma\right)\rangle\rangle = N_c^{-1}\hat{I} + \left(\hat{\rho}_0 - N_c^{-1}\hat{I}\right) W_{adj}\left(\gamma\right)$$

This expression shows that if the Wilson loop in the adjoint representation decays, the colour density matrix obtained as a result of parallel transport along the loop γ tends to white colourless mixture with $\hat{\rho} = N_c^{-1}\hat{I}$, where all colour states are mixed with equal probabilities and all information on the initial colour state is lost.

In the model of QCD stochastic vacuum decay rates of Wilson loops in different representations of the gauge group are proportional to each other (Casimir scaling), therefore if the Wilson loop in the adjoint representation decays, the Wilson loop in the fundamental representation also decays. But Wilson loop decay points at confinement of colour charges, therefore the stronger are the colour charges confined, the quicker their states transform into white mixtures. It is important that the path γ is closed, which means that actually one observes particle and antiparticle.

As the Wilson area law typically holds for the Wilson loop, we can obtain an explicit expression for the density matrix. Here it is convenient to choose the rectangular loop $\gamma_{R\times T}$ which stretches time T and distance R:

$$\hat{\rho}\left(\gamma_{R\times T}\right) = N_c^{-1}\hat{I} + \left(\hat{\rho}_0 - N_c^{-1}\hat{I}\right)\exp\left(-\sigma_{adj}RT\right)$$

where $\sigma_{adj} = \sigma_{fund} C_{adj} C_{fund}^{-1}$ is the string tension between charges in the adjoint representation, σ_{fund} is the string tension between charges in the fundamental representation and C_{adj}, C_{fund} are the eigenvalues of quadratic Casimir operators.

Here we have used the Casimir scaling [*Simonov'96*, *Dosch'02*, *Simonov'04*].

- Finally we can obtain the decoherence rate, which is introduced using the concept of purity $p = \mathrm{Tr}\,\hat{\rho}^2$. For pure states the purity is equal to one. For our colour density matrix the purity is:

$$p\left(T,R\right) = N_c^{-1} + \left(1 - N_c^{-1}\right) \exp\left(-2\sigma_{fund} C_{adj} C_{fund}^{-1} RT\right)$$

- Purity decay rate is proportional to the string tension and the distance R.
- Purity decay rate is proportional to the string tension and the distance R. It can be inferred from this expression that the stronger is the quark-antiquark pair coupled by QCD string or the larger is the distance between quark and antiquark, the quicker information about colour states is lost as a result of interactions with the stochastic vacuum.

Conclusions

- We show that in QCD stochastic vacuum white states of colour charges in the fundamental representation of $SU(N_c)$ gauge group can be obtained as a result of decoherence of pure colour state into a mixed state.
- Decoherence rate is found to be proportional to the tension of QCD string and the distance between colour charges.
- The purity of colour states is calculated.

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