The Extended Model of Flux-Tube Phase Transitions & Deconfinement

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INPUT:

- Over the past few years the structure and the properties of deconfined state at moderate T above the critical one have received renewed interest.
- On the other hand, exp. evidence from RHIC indicates the formation of a "strongly interacting QCD" that manifests the saving or even occurrence of bound states in deconfinement phase.
- However, this does not allowed if one lives in the (classical) Wilson loop world.
- Concerning "Lattice": Lattice QCD indicates the formation of mesons (or resonances) as bound states. The support comes either from lattice-based potentials in K-G equation or even from phenomenological potentials within the two-body nonrelativistic equation.
- To better understand the phase transition $\rightarrow$ implemented Abelian Higgs model into the extended model of flux-tube.

Generalization of Color Confinement

- Unexpectedly successful way to understand strongly coupled gauge theories and estimate their low energy properties

Modelling NonPerturbative QCD

- Liner Rising Confinement
- Heavy Quark Potential Pattern
- Spectrum of Low-Lying Hadrons
- Radial Excitations
- ………….

QCD /Gauge Correspondence

Strongly Coupled Quark-Gluon Matter *(sQGP)*

↓ ↓

Phase Transition
Agrees with lattice

Almost Ideal Fluid
Fits RHIC Discoveries

- Time for prediction for

Higher Temperature Effects

RHIC

LHC

- QUARK CONFINEMENT IN NATURE? IT'S ORIGIN?

  Y-M? VACUUM?

  FOR CALCULATIONS $\Rightarrow$ DUAL VERSION OF Y-M THEORY

- VACUUM OF Y-M THEORY IS REALIZED BY A CONDENSATE OF MONOPOLE PAIRS

  Nambu 1974
  Mandelstam 1976
  Polyakov 1977
  't Hooft 1978

- INTERACTION FIELD IS SQUEEZED INTO TUBE WITH $E_{tube} \sim |\vec{x}_1 - \vec{x}_2|$

- NO MONOPOLES IN Y-M THEORY $\longrightarrow$ 't Hooft ABELIAN PROJECTION (AP) 1981

- AP REDUCES QCD INTO $[U(1)]^2$ INCLUDING MONOPOLES

- LATTICE QCD $\xrightarrow{MA}$ GIVES NUMERICAL EVIDENCE OF QCD-MONOPOLE

  CONDENSATION & ABELIAN DOMINANCE FOR New Physics PHENOMENA

  Lattice Community, 90'th-00'th
  G.K., 2002-2004

$$SU(N) \leftarrow \textbf{topology} \quad \rightarrow [U(1)]^{N-1}$$

different

Y-M MANIFOLD $\Downarrow$ ABELIAN SUBGROUP

NEW TOPOLOGICAL OBJECTS CAN APPEAR

$$A_\mu(x) \rightarrow A_\mu(x) + \frac{g}{4\pi}\partial_\mu \Omega'(x), \qquad D_\mu(x) = \partial_\mu + ieA_\mu(x)$$

$$F_{\mu\nu}(x) = \frac{ig}{2\pi}\left(\left[\partial_\mu, \partial_\nu\right] - \left[D_\mu(x), D_\nu(x)\right]\right), \qquad e \cdot g = 2\pi$$

$$F_{\mu\nu}(x) \rightarrow F^{\Omega}_{\mu\nu}(x) = \Omega(x) F_{\mu\nu}(x) \Omega^{-1}(x) =$$

$$= \partial_\mu A^{\Omega}_\nu(x) - \partial_\nu A^{\Omega}_\mu(x) + ie[A^{\Omega}_\mu(x), A^{\Omega}_\nu(x)] + \frac{ig}{2\pi}\Omega(x)[\partial_\mu, \partial_\nu]\Omega^{-1}(x)$$

$$\textbf{singularity} = \tilde{G}_{\mu\nu}(x)$$

$$AP: \quad F^{\Omega}_{\mu\nu}(x) \rightarrow F^{\alpha}_{\mu\nu}(x) = \partial_\mu A^{\alpha}_\nu(x) - \partial_\nu A^{\alpha}_\mu(x) + \frac{ig}{2\pi}\tilde{G}_{\mu\nu}(x)$$

$$\textit{Dirac current}: J^{m}_\mu(x) = (ig/8\pi)\varepsilon_{\mu\nu\rho\sigma}\partial_\nu \Omega(x)[\partial_\rho, \partial_\sigma]\Omega^{-1}(x) \textit{ in Abelian sector}$$

FLUX TUBE SOLUTION (T = 0 case)

PHASE TRANSITION IN DUAL MODELS

↓ ASSOCIATED

CHANGE IN SYMMETRY ⟶ SYMMETRY BREAKING

MODEL: SCALAR ORDER PARAMETER $\langle \hat{B}_i(x) \rangle = \hat{B}_{i_0} \sim O(0.1\,GeV), i = 1,\ldots, N_c(N_c - 1)/2$

DUAL GAUGE FIELD $\hat{C}_\mu(x)$

DUAL Y-M THEORY { $[U(1)]^2$ *Higgs* **model** $\rightarrow$ $SU(3)$ **gluodynamics**

SCALAR FIELDS $\hat{B}_i(x)$ - monopole degrees of freedom

$$L = Tr\left[-\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \left(D_\mu \hat{B}_i\right)^2\right] - W(\hat{B}_i), \qquad \hat{F}_{\mu\nu} = \partial_\mu \hat{C}_\nu - \partial_\nu \hat{C}_\mu - ig[\hat{C}_\mu, \hat{C}_\nu]$$ Baker et al. ‘ 96

$$D_\mu \hat{B}_i = \partial_\mu \hat{B}_i - ig[\hat{C}_\mu, \hat{B}_i]$$

V.E.V. $\hat{B}_{0_i}$ PRODUCE $J_\mu^{mon}(x) \sim \partial^\mu G_{\mu\nu}(x), \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + \tilde{G}_{\mu\nu}$

$(\bar{Q}Q)$ BOUND STATE: $\hat{C}_\mu \rightarrow \lambda^a C_\mu, \quad \hat{F}_{\mu\nu} \rightarrow \lambda^a G_{\mu\nu}, \quad \hat{\tilde{G}}_{\mu\nu} \rightarrow \lambda^a \tilde{G}_{\mu\nu}, \quad a = 3, 8$

GAUGE FIELD IN SCALAR (MONOPOLE) CONDENSATE

$$gC_\mu(x) = \frac{J_\mu^{mon}(x)}{4g(\chi + B_0)^2} + \partial_\mu f(x), \qquad B(x) \sim e^{if(x)}\left[\chi(x) + B_0\right]$$

$$\downarrow$$

$$C_\mu(x) = \frac{1}{3m^2}\partial^\nu \tilde{G}_{\mu\nu}(x) - \frac{4}{m}\partial_\mu \chi(x)$$

GK 2002

$$J_\mu^{mon}(\vec{x} \to \infty) \to 8m^2 C_\mu, \qquad C_\mu(\vec{x} \to \infty) \to 0$$

$\chi(x)$: ***subsidiary objects in massive dual theory***

$D_\mu^i B_i = 0$ *at* $R \to \infty \Rightarrow$ FINITE ENERGY CONFIGURATION

- MATTER FIELDS B_i $(i = 1, 2, 3)$ CORRESPOND TO MAGNETICALLY CHARGED PARTICLES
- EQUATION OF MOTION CARRYING A UNIT OF Z_3 FLUX CONFINED IN A NARROW TUBE ALONG Z AXIS (QUARK SOURCES AT $Z = \pm\infty$)

$$\Downarrow$$

DUAL ANALOGY TO ABRIKOSOV (1957) MAGNETIC VORTEX SOLUTION

FLUX TUBE IN $\underbrace{\textit{excited matter}}$ $(T \neq 0)$

CONDENSATE at HIGH Temperatures

OSCILLATIONS OF FLUX TUBE → VISIBLE

UP TO THE EXCITATION ENERGY $e_\beta = s / \beta$ AT CERTAIN ENTROPY DENSITY s

SCALAR FIELD T -EXTENSION (Fourier expansion)

$$\hat{B}(\tau, \vec{x}) = \hat{B}(\vec{x}) + \sum_{n=1} \hat{B}_n(\vec{x}) \exp(2\pi i n \tau / \beta)$$

$\uparrow$ $\uparrow$

$T = 0$ **mode** ***heavy (high T)*** **modes**

PHYSICAL PATTERNS

- CLASSICAL MECHANICS

STOCHASTIC PROCESSES IN DYNAMIC SYSTEM

↓

"LARGE" SYSTEM ($\Rightarrow$ WEAK ACTION) "SMALL" SYSTEM — Krylov, Bogolyubov 1939

IN EQUILIBRIUM STATE (THERMOSTAT)

- DUAL QUANTUM PICTURE

"LARGE" SYSTEM ($\Rightarrow$ WEAK ACTION) "SMALL" SYSTEM — G.K. 2002-2006

↓ $\left|(\partial_\nu - i\, g C_\nu)\phi\right|^2$ ↓

HIGGS- LIKE CONDENSATE
AT $T \neq 0$ (EQUILIBRIUM STATE)

CONFINED CHARGES REGION
(FLUX TUBE)

- STATIONARY STOCHASTIC PROCESSES IN THE DECONFINED STATE ARE DISTORTED BY THE RANDOM SOURCE

  Dirac string $\tilde{G}_{\mu\nu}(x)$ in $G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\nu + \tilde{G}_{\mu\nu}$

DUAL CONFINEMENT

$$Z = \sum_{flux} \exp[-\beta E(R)] = \sum_{R} N(R) \exp[-\beta E(R)]$$

$$\downarrow$$

divergent if $T > T_0$

$$(\vec{Q}_{\alpha}^{2} / 16\pi) m^{2} R [a - b \ln(\tilde{\mu} R)]$$

G.K. 2002

SU(3) Algebra: $\vec{Q}_{\alpha} = e\vec{\rho}_{\alpha}$ Abelian color charge

$N(R)$: Discrete space of scalar Higgs-like condensate

Flux tube: along the links of 3D cubic lattice

Lattice size: $l \approx (\sqrt{2\lambda} B_0)^{-1} = \mu^{-1}$ - mass (inverse) of scalar (Higgs) field

B_0 - threshold energy to excite monopole (Higgs) in dual QCD vacuum

(Bogolyubov particle in superconductor)

STRING TENSION

Entropy density $s:$ $\qquad N(R) \to \tilde{N}(R) = \exp[s\,(R/l)], \quad l << R$

$\downarrow$

Number of configuration counting

$$\boldsymbol{Z \approx \frac{V}{l^3} \sum_R \exp[-\beta\, \sigma_{eff}(\beta)\, R]}$$

$$\sigma_{eff}(\beta) = \tilde{\sigma}_0 - \frac{s}{l\,\beta}$$

↵ *phase transition order parameter;* $\quad \sigma_{eff}(T_0) = 0$

$$\tilde{\sigma}_0 = \sigma_0 \left(1 - \frac{1}{4} \ln \frac{\tilde{\mu}^2}{m_R^2} \right), \qquad \sigma_0 = \frac{3}{4} \alpha(Q)\, m^2 \to 0.18\, GeV^2$$

$m = 0.85\ GeV, \quad \alpha = 0.37$ (heavy quark-antiquark states fitting)

Higher Temperatures

High $T \to$ flux tube(s) disappear: QCD loses confinement

$T \to T_0$: flux tube arbitrary long

$$m^2(\beta) = \frac{4}{3}\frac{\sigma_{eff}(\beta)}{\alpha(Q,\beta)} \qquad \begin{cases} m(\beta) \to m, & \beta \to \infty \\ m(\beta) \to 0, & \beta^{-1} \to T_0 \end{cases}$$

$$T_0 = \frac{3}{4}\frac{1}{s}\alpha(Q)\frac{m^2}{\mu}\left(1 - \frac{1}{4}\ln\frac{\tilde{\mu}^2}{m_R^2}\right), \quad s \neq 0 \quad \Leftarrow \quad \sigma_{eff}(T_0) = 0$$

Main model parameters: $T_0 \approx O(200\, MeV), \quad B_0 \approx 276\, MeV$

Phase transition limit, $m(\sim T_0) \to 0$, leads:

Dirac string tensor $\partial^{\nu}\tilde{G}_{\mu\nu} = \left(m^2\, C_{\mu} + 4\, m\, \partial_{\mu} b\right) \to 0$ as $T \to T_0$

On the other hand, Dirac current $\partial^{\mu}\tilde{G}_{\mu\nu}(x) = -g\int_{\Gamma} dz_{\mu}\ \delta^4(x-z) \quad \to 0$ as $g \to 0$

Extended Model

$$\sigma_{eff}(\beta) \;\to\; \sigma_{eff}(\beta) = \bar{\sigma}_{eff}(\beta) + \frac{1}{R}\bar{\varepsilon}(T)\;\theta(T - T_0)$$

$$\downarrow$$

$$\underbrace{\sigma_0}_{0.18\,GeV^2}\left(\underbrace{1 - \frac{1}{4}\ln\frac{\tilde{\mu}^2}{m_R^2}}_{\sim O(1)}\right) - \underbrace{s\,\mu}_{1\,\mathrm{GeV}}\,T\,, \quad \bar{\varepsilon}(T): \quad V(R) = -(4/3)\;R^{-1}e^{-m_D(T)R}$$

$$\bar{\varepsilon}(T) \simeq m_D^2(T)/m_Q \to g^2(T)\;(1 + N_f/6)\;T^2/m_Q\;\;\theta(T - T_0)$$

$\bar{\sigma}_{eff}(T_0) = 0 \;\Rightarrow\; T_0 \approx 180\,MeV$ Phase transition temperature

New result: “bound state” energy in deconfined phase

$$E(T > T_0) = \sigma_{eff}(\beta)\;R \to \;g^2(T)\;T^2/M$$

$\sim T^2$ increased behavior (!)

Strongly QCD effect

Recall:

$$m^2(\beta) \sim g^2 \delta^2(0)$$

$$\downarrow$$

$$\to \sigma^{-1}_{flux\,tube}, \quad \sigma \to \infty \;\; as \;\; m \to 0$$

$$\sigma_{eff}(\beta) - effective\ measure\ of\ the\ phase\ transition$$

$$\sigma_{eff}(\beta_0) = 0 \quad special\ case \quad (q \xleftarrow{R\to\infty} \to \bar{q})$$

$$at\ T = T_0 \mapsto s = E/T_0, \quad e^s \to e^{M/T}\ for \quad mass \quad spectrum$$

T-Dependent Strong Coupling

In hot environment: $\alpha(Q) \to \alpha(Q,T)$

$$\alpha^{-1}(Q,T) \approx b \ln[Q/\Lambda(T)] = b\{\ln(Q/\Lambda) - \ln[\Lambda(T)/\Lambda]\}$$

$$\ln[\Lambda(T)/\Lambda]: \quad z^{(1)}(T) = -b\ln[\Lambda(T)/\Lambda] \quad \textit{renormalization cons.}$$

$$\alpha^{-1}(Q,T) = \alpha^{-1}(Q) + z^{(1)}(T)$$

$$\alpha(Q,T) = [1 + \rho(T)]\alpha(Q)$$

$$z^{-1}(T) = 1 + \rho(T) = \{1 - b\ln[\Lambda(T)/\Lambda]\alpha(Q)\}^{-1}$$

$$\ln\frac{\Lambda(T)}{\Lambda} = -\frac{1}{b}\int_0^{\kappa = T/Q} y(x)\frac{dx}{x} \quad \textit{Nakkagawa et al } (1988)$$

$$\frac{\partial \alpha(Q,T)}{\partial \kappa} = \overline{\beta}_\kappa(\alpha,T) = -y(T)\alpha^2(Q)$$

THERMAL FLUCTUATIONS (gauge field case)

m_{el} ~ g T   screening effect

- INSTANTONS ← Abelian Projection Theory → MONOPOLES

SU(N):

$$\frac{1}{\alpha(Q,T)}=\frac{1}{\alpha(Q)}\left\{\left[1-\frac{\Pi^{00}(q^0=0,\vec{q}\to 0;\beta)}{\vec{Q}^2}\right]+\frac{\alpha(Q)}{6\pi}\left(\frac{11N}{2}-N_f\right)\ln\frac{\vec{Q}^2}{M^2}\right\}$$

$$-\Pi^{00}\left(q^0=0,\vec{q}\to 0;\beta\right)=g^2\ T^2\left[\frac{N}{3}+\frac{N_f}{\pi^2T^2}I_F\left(\beta,\bar{\mu},m_q\right)\right]=m_{el}^2\left(\beta\right)$$

$$I_F=\int_0^{\infty}\frac{dx\ \mathrm{x}^2}{\sqrt{x^2+m_q^{\ 2}}}\left[n_F\left(x^2\right)+\bar{n}_F\left(x^2\right)\right],\qquad \rho_B\sim\int\frac{d^3x}{(2\pi)^3}\left[n_F\left(x^2\right)-\bar{n}_F\left(x^2\right)\right]$$

$$n_F\left(x^2\right)=\frac{1}{\exp\left[\left(\sqrt{x^2+m_q^{\ 2}}-\bar{\mu}\right)\beta\right]+1},\qquad \bar{n}_F\left(x^2\right)=\frac{1}{\exp\left[\left(\sqrt{x^2+m_q^{\ 2}}+\bar{\mu}\right)\beta\right]+1}$$

FINALLY: $\alpha^{-1}(Q,T)$ EXPANSION

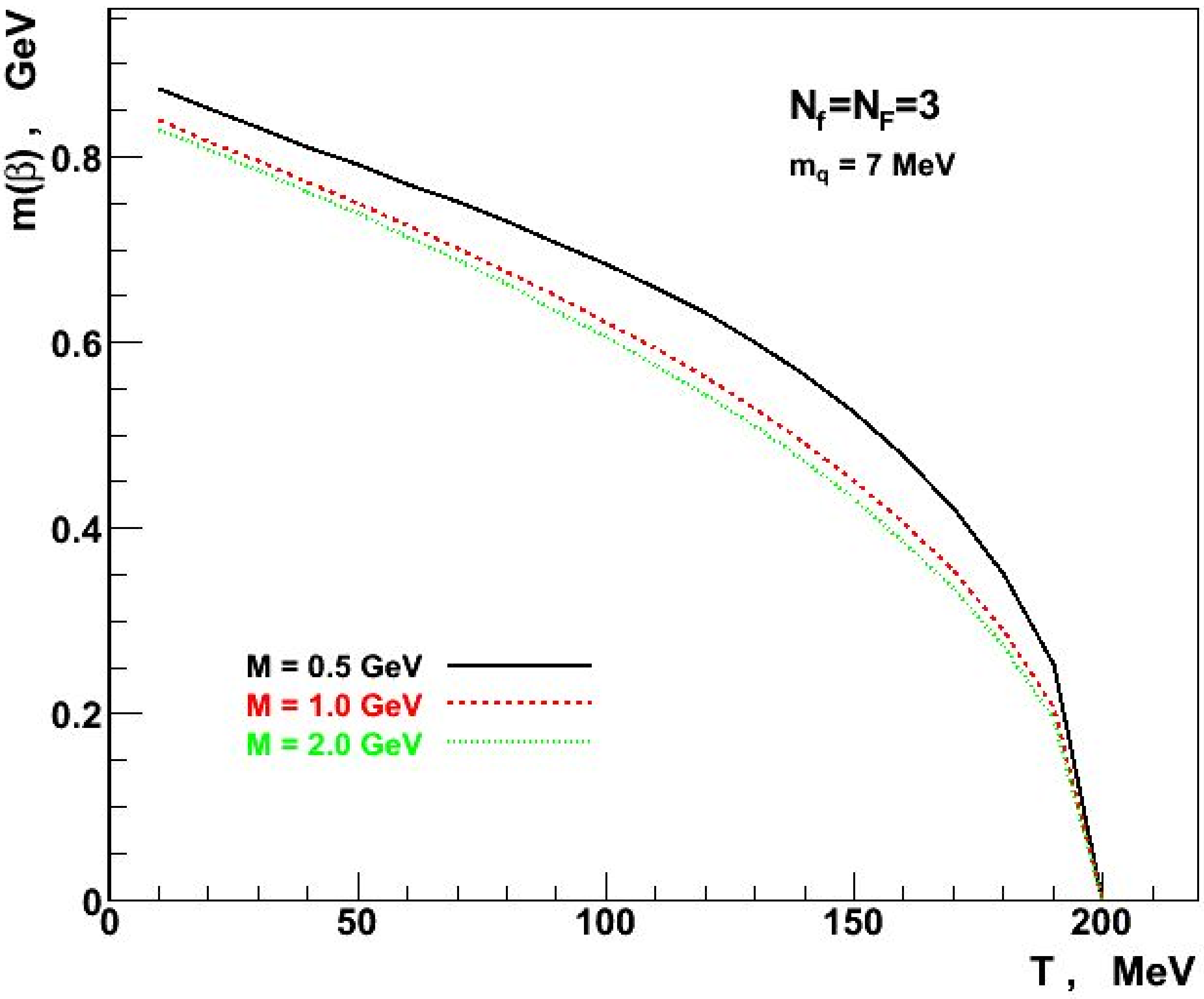
$$\frac{1}{\alpha(Q,T)} = \frac{1}{\alpha(Q)} + \frac{1}{6\pi}\left(\frac{11N}{2} - N_f\right)\ln\frac{\vec{Q}^2}{M^2} + 4\pi\frac{T^2}{\vec{Q}^2}\left[\frac{N}{3} + \frac{N_F}{\pi^2 T^2} I_F\left(\beta, \overline{\mu}, m_q\right)\right]$$

where $\alpha(Q,T) \to 0 \quad as \quad T \to \infty$

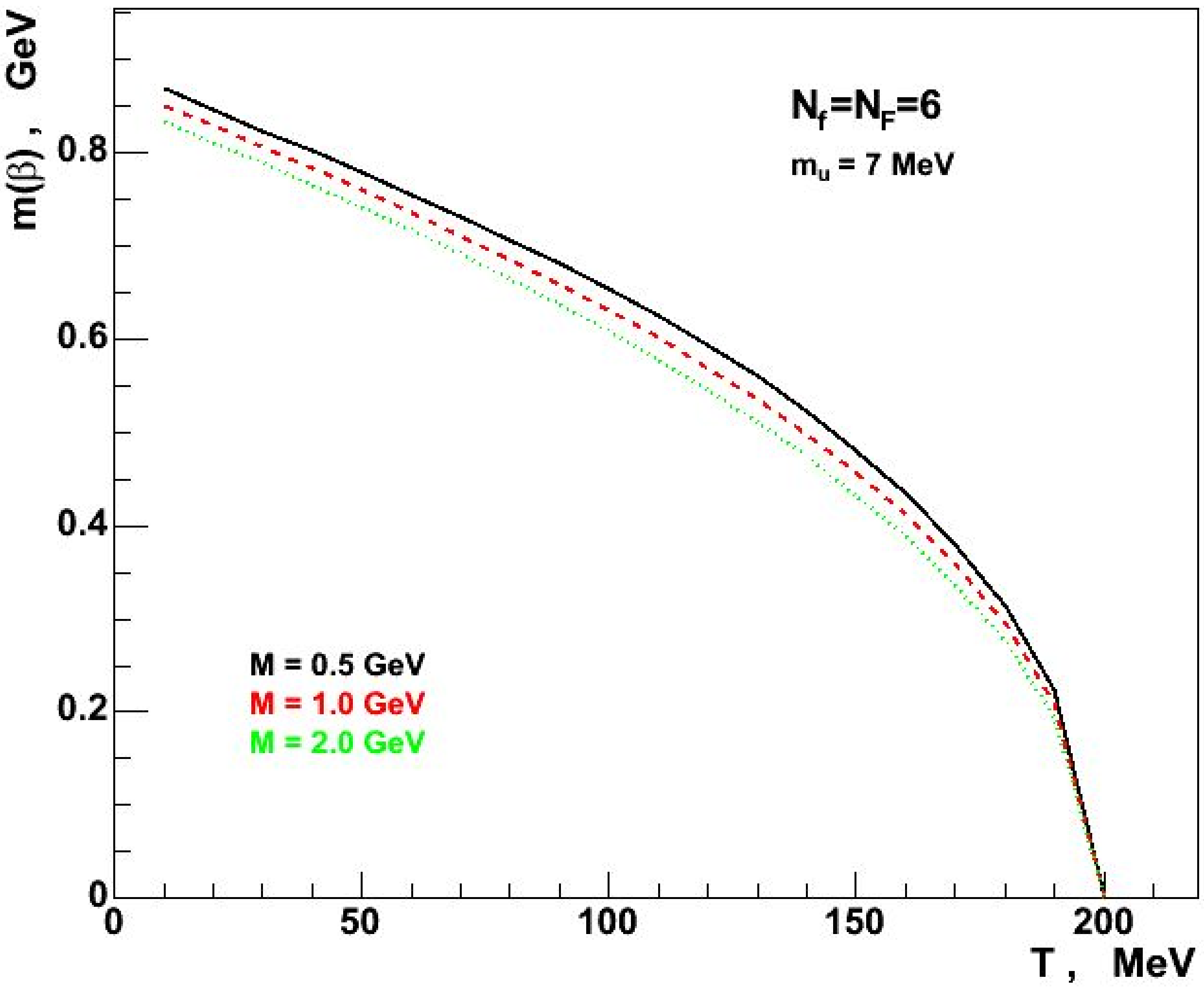
UPPER LIMIT ON T_0 (under the requirement $\frac{m}{\mu} < 1$, Type-II superconductor, magnetic Abrikosov-like vorteces)

$$T_0 < \frac{3}{4}\alpha(Q) m \left(1 - \frac{1}{4}\ln\frac{\tilde{\mu}^2}{m_R^2}\right)$$

$$T_0 < 222 MeV \quad at \quad B_0 = 276 MeV, \quad \alpha = 0.37, \quad m = 0.85 GeV, \quad s \sim O(1)$$



m(β) , GeV
0.8
0.6
0.4
0.2
0
0
50
100
150
200
T , MeV
N_f=N_F=3
m_q = 7 MeV
M = 0.5 GeV
M = 1.0 GeV
M = 2.0 GeV



m(β) , GeV
0.8
0.6
0.4
0.2
0
0
50
100
150
200
T , MeV
N_f=N_F=6
m_u = 7 MeV
M = 0.5 GeV
M = 1.0 GeV
M = 2.0 GeV

FLUX TUBE SOLUTIONS – PROFILE FUNCTIONS

- DUAL GAUGE FIELD along the Z-axis (cylindrical symmetry)

$$\tilde{C}(r,\beta)=\frac{4n^{\nearrow \text{topological charge}}}{7g(\beta)}-\sqrt{\frac{\pi m(\beta)r}{2\kappa}}\exp\left[-\kappa m(\beta)r\right]\left[1+\frac{3}{8\kappa m(\beta)}\right]$$

- COLOR-ELECTRIC FIELD E (rotation of the dual gauge field)

$$\vec{E}=\vec{\nabla}\times\vec{C}=\frac{1}{r}\frac{d\tilde{C}(r)}{dr}\vec{e}_z=E_z(r)\vec{e}_z$$

↓

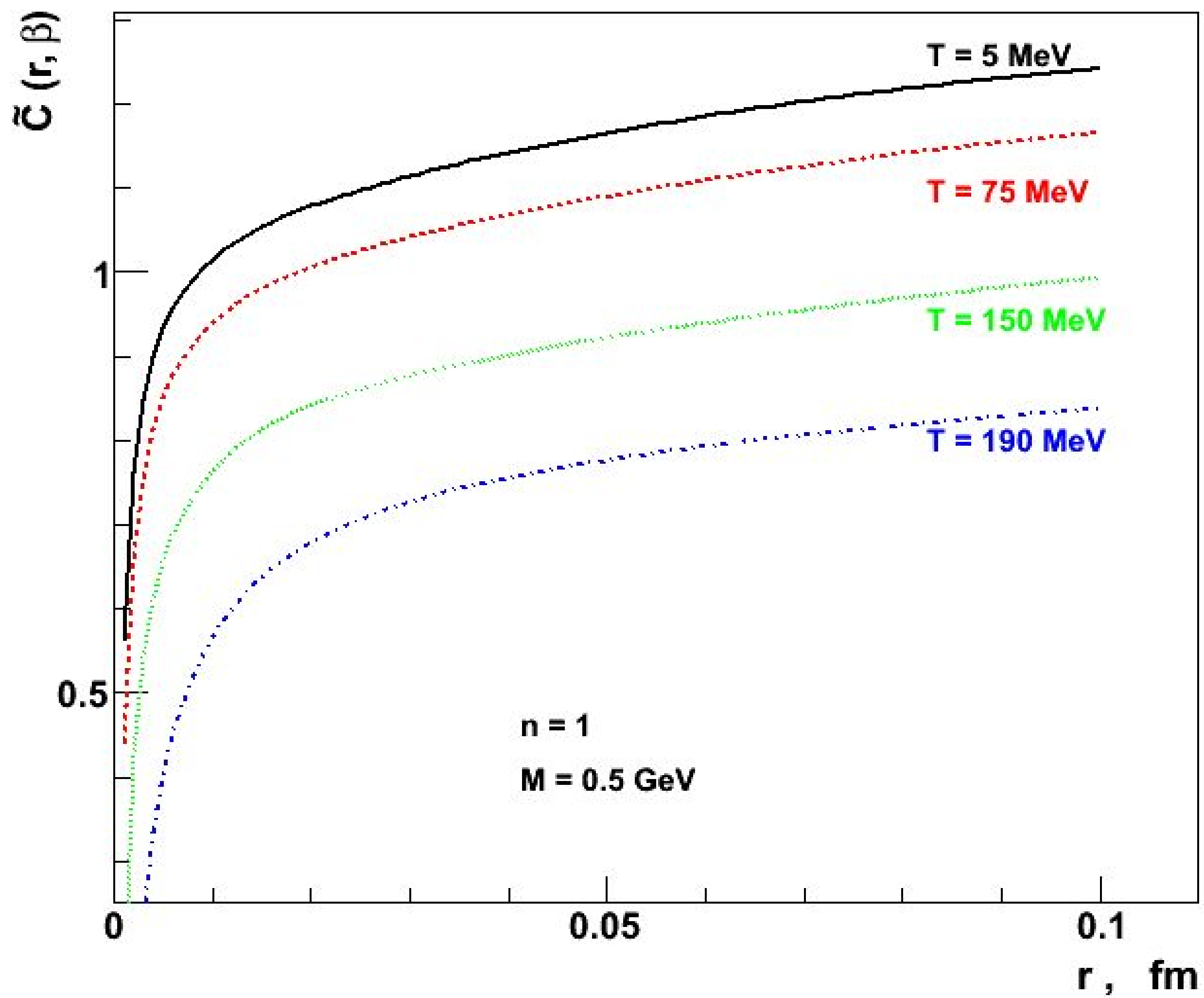
unit vector along the z-axis

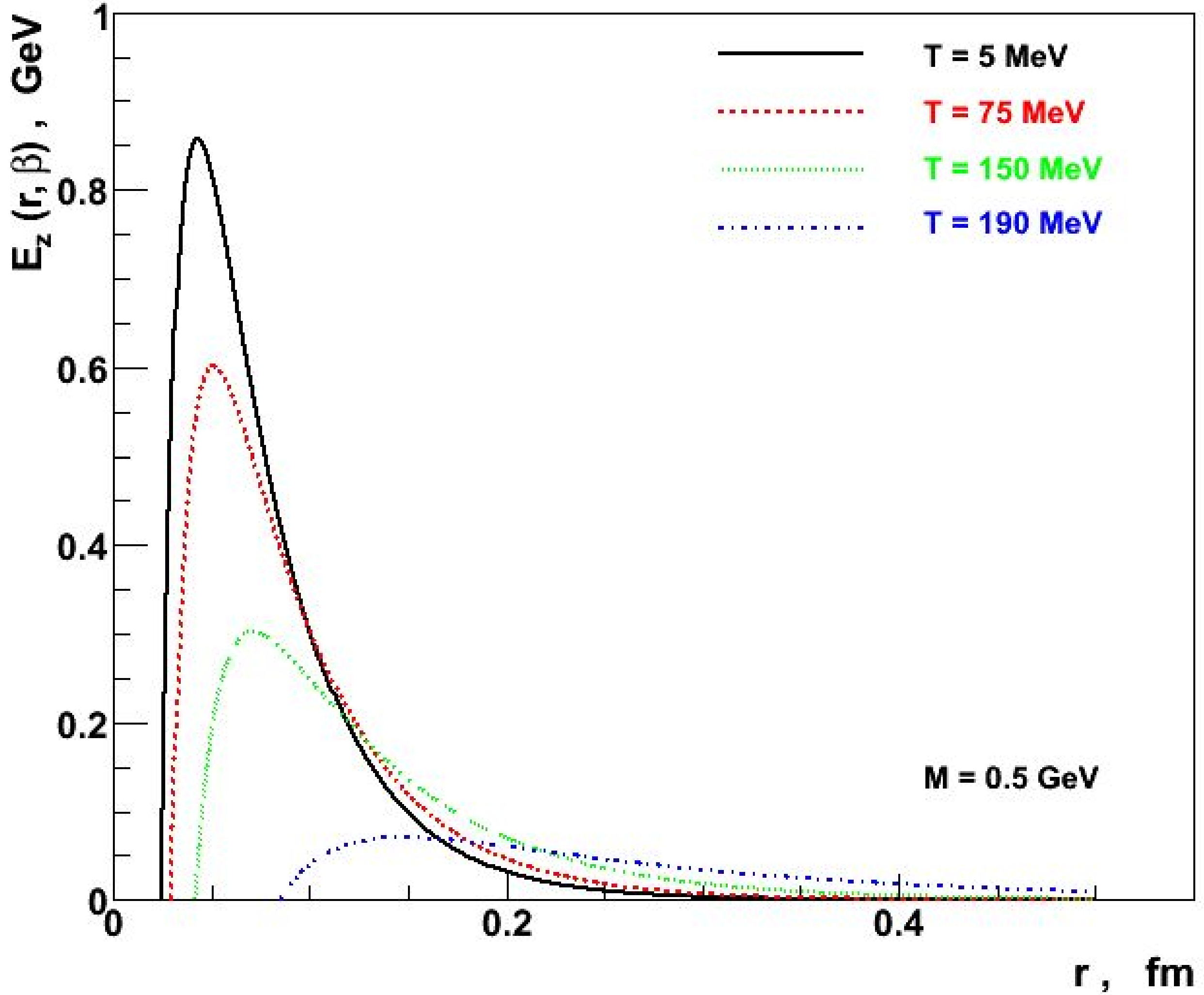
- T-dependent COLOR-ELECTRIC FIELD

$$E_z(r,\beta)=\sqrt{\frac{\pi m(\beta)}{2\kappa r}}\exp\left[-\kappa m(\beta)r\right]\left[\kappa m(\beta)-\frac{1}{2r}\right]$$

! Excluded from the vacuum and hence confined inside $\boldsymbol{r<m^{-1}(\beta)}$

We got: Flux-tube configuration ← vortex-type


$\tilde{C}$ (r, β)
T = 5 MeV
T = 75 MeV
T = 150 MeV
T = 190 MeV
1
0.5
n = 1
M = 0.5 GeV
0
0.05
0.1
r , fm



1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
$E_z(r, \beta)$, GeV
r , fm
T = 5 MeV
T = 75 MeV
T = 150 MeV
T = 190 MeV
M = 0.5 GeV

THE LOWER BOUND ON THE RADIAL COORDINATE

$$r_0 > \frac{1}{2\kappa m(\beta)} \to 0.03\ fm \quad as \quad T \to 0$$

DECONFINEMENT: $r_0 \to \infty \quad as \ \ m(\beta) \to 0 \quad at \ \ T \to T_0$

OBTAINED:

- Analytical form $\left\{ \begin{array}{c} monopole \\ dual\ gauge\ bosons \end{array} \right\}$ propagators
  $\Downarrow$
  leads
  Consistent perturbative expansion of Green's functions
- No dependence found on m_q (check for $m_q = 7,\ 10,\ 135\ MeV$)
- No essential dependence found for different N_f and N_F
- Phase transition T_0 essentially depends on $\alpha_s(Q,T)$ and dual gauge field m
- Bound states exist at $\boldsymbol{T_{cr} > T > T_0} \qquad \boldsymbol{T_{cr} - ?} \qquad \boldsymbol{T_{cr} \sim 2T_0,\ 3T_0, \ldots}$