Pion Phase Space Density from STAR HBT Analysis

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Abstract

The phase space density[1] ⟨f⟩ of pions produced in ultra-relativistic heavy ion collisions is of considerable interest because it can be compared to thermal models[2] to extract the freeze-out temperature and can be used to predict the importance of multiparticle Bose-Einstein effects such as "true" multiparticle correlations[3,4]. We have used results from the initial HBT analysis of STAR data from 130 GeV/nucleon gold-gold collisions at RHIC, along with the preliminary STAR pion momentum spectrum[5] to estimate the pion phase space density. We find phase space densities that show evidence of strong transverse flow effects[6] and that are very similar to those previously observed in lead-lead collisions at the CERN SPS[2].

Introduction

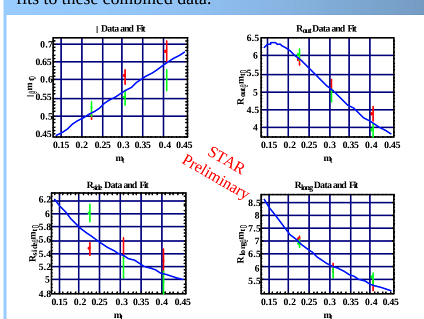
The 6-dimensional position-momentum space occupied by pions produced in RHIC collisions can be divided into "phase-space cells" having a six dimensional volume h^3. The ***phase space density***, i.e., the average number of pions per phase space cell, is of great interest because it determines the importance of multiparticle HBT correlation effects and because it can be directly compared with thermal models to provide estimates of dynamic collision characteristics such as freezout temperature and average flow velocity.

The phase space density ⟨f⟩ can be calculated from the observed the pion momentum spectrum $\mathbf{d^2N/dy\,dm_T}$ and the HBT observables λ, $\mathbf{R_O}$, $\mathbf{R_S}$, and $\mathbf{R_L}$. At midrapidity the phase space density as a function of transverse mass $\mathbf{m_T}$, with $\mathbf{E_\pi}$, the total pion energy, is given by the relation:

$$\langle f\rangle(\mathrm{m_T}) = \frac{(\hbar c)^3}{2\,\mathrm{E}_\pi\,\mathrm{m_T}}\,\frac{d^2\mathrm{N}}{dy\,d\mathrm{m_T}}\,\frac{(\lambda\,\pi)^{1/2}}{\mathrm{R_O\,R_S\,R_L}}$$

Fits to HBT Parameters

Before calculating the phase space density, we would like to fit the HBT observables as functions of the transverse mass $\mathbf{m_T}$ so that they can be extrapolated downward to eliminate flow effects. To do this, we assume that the chaoticity parameter λ depends linearly on $\mathbf{m_T}$, i.e., $\boldsymbol{\lambda = A + B(m_T - m_\pi)}$, while the radii follow power-law scaling[7], i. e., $\mathbf{R_X(m_T) = A\,(m_T/m_\pi)^B}$, so that $\boldsymbol{\lambda}$=A and $\mathbf{R_X}$=A when $\mathbf{p_T=0}$. However, the fits must satisfy the constraint that $\mathbf{R_S=R_O}$ at $\mathbf{p_T=0}$. With this constraint we use the same **A** parameter derived from the power-law fit to $\mathbf{R_S}$ in fitting $\mathbf{R_O}$, using the modified power law expression $\mathbf{R_O(m_T) = A\,(m_T/m_\pi)^{B+C/m_T}}$. The results of these fits, along with the fit parameters, are shown in the figure. The red • and green • points are HBT observables from $\pi^-\pi^-$ and $\pi^+\pi^+$ analyses, respectively, and the blue lines — are the least-squares fits to these combined data.



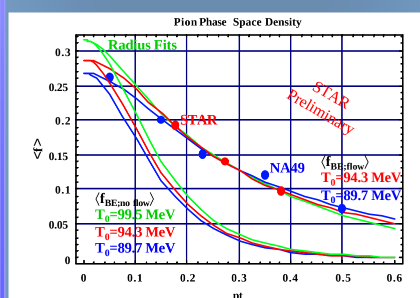
The fit parameters produced by these fits are given in the table below. Note that the fitted **B** parameter for $\mathbf{R_L}$ within its error limits, agrees with the value of -½ expected from Sinyukov scaling[7].

	A	B	C
λ	0.445±0.031	0.756±0.190	
R_S	6.205±0.376	-0.185±0.081	
R_O	6.205±0.376	-0.731±0.088	0.143±0.027
R_L	8.633±0.366	-0.452±0.058	

Phase Space Density

We use these fits, evaluated at three $\mathbf{p_T}$ values, to plot the phase space density ⟨f⟩ calculated from the measured pion spectra[5] and HBT observables.

The blue points • are for experiment **NA49** at the CERN SPS[2] and the red points • are for the **STAR** experiment at RHIC at $\mathbf{s^{1/2}}$ = 130 GeV/nucleon. We find the ⟨f⟩ values to be remarkably similar.



The green curve through the red points is the result of using the functions fitted to the STAR HBT observables, as just described. The lower curves are thermal Bose-Einstein phase space densities neglecting flow effects, as calculated from the relation:

$$\left\langle \mathrm{f}_{\mathrm{BE};no\,flow}\right\rangle(\mathrm{m_T}) = \left\{Exp[\mathrm{m_T}\;/\,\mathrm{T_0}] - 1\right\}^{-1}$$

where $\mathbf{T_0}$ is the intrinsic freezout temperature.

The upper red and blue curves are fits to the STAR and NA49 points, respectively, using a linearized version of the thermal Bose- Einstein phase space density including transverse and longitudinal flow, as derived by Tomasik[6]. His relation is:

$$\left\langle \mathrm{f}_{\mathrm{BE;flow}}\right\rangle(\mathrm{m_T}) = \left\{Exp[(\mathrm{m_T/T_0})\,Cosh(\eta_T)\,Cosh(\eta_L) - (\mathrm{p_T/T_0})\,Sinh(\eta_T)] - 1\right\}^{-1}$$

where $\boldsymbol{\eta_T}$ is the maximum transverse flow rapidity at a particular transverse momentum and $\boldsymbol{\eta_L}$ is the maximum longitudinal flow rapidity at a particular rapidity. Since the data are at midrapidity, on the average $\boldsymbol{\eta_L}$ = 0 and $\mathbf{Cosh(\boldsymbol{\eta_L}) = 1}$.

However, $\boldsymbol{\eta_T}$ is a model-dependent function of $\mathbf{p_T}$. To evaluate $\langle \mathbf{f_{BE;flow}}\rangle$, we use a simple linear model that assumes $\boldsymbol{\eta_T}$ = [illegible]$\mathbf{_T}$ $\mathbf{ArcTanh(p_T/m_T)}$, where [illegible]$\mathbf{_T}$ is an unknown constant related to the transverse flow velocity. In other words, we make the simple assumption that $\boldsymbol{\eta_T}$ scales linearly with transverse pion rapidity. This simple model appears to reproduce the effects on ⟨**f**⟩ of transverse flow. The temperatures derived from the HBT parameter extrapolation and from the model fits are given in the following table.

Temperatures from Extrapolations and Linearized Tomasik fits to STAR and NA49 ⟨f⟩ values.

	To (MeV)	[illegible]T
STAR Extp	99.50±0.50	
STAR Fit	94.30±0.58	0.372±0.007
NA49 Fit	89.72±1.22	0.427±0.019

Freezout Temperature and Flow Velocity

Extrapolating the HBT parameters to $\mathbf{p_T}$=0, where flow effects are suppressed, provides an estimate of the pion freezout temperature of $\mathbf{T_0}$ = 99.5 ±5 MeV, with the error arising primarily from the large systematic uncertainties inherent in the extrapolation. An alternate procedure with better control of systematic errors is the fit to the STAR ⟨**f**⟩ points using the linearized Tomasik relation. This provides our best estimate of the pion freezout temperature:

$$\mathbf{T_0 = 94.3 \pm 0.6\ MeV.}$$

This temperature can be combined with the effective temperature $\mathbf{T_{eff}}$=185±5 MeV obtained from the pion spectrum to calculate the average transverse flow velocity with the relation $\mathbf{b_T = (T_{eff}^2 - T_0^2)/(T_{eff}^2 + T_0^2)}$. This gives an average transverse flow velocity of:

$$\mathbf{b_T = 0.587 \pm 0.018.}$$

This velocity is very close to the velocity of sound of $\mathbf{b_{Sound} = 1/\sqrt{3}}$ = 0.577 predicted by relativistic hydrodynamics.

Conclusions

We have found that the pion phase space densities are remarkably similar for A-A collisions at the SPS and RHIC. The universal pion phase space density at freeze-out, previously pointed out in Reference 2 for AGS and SPS data, appears to be valid also at RHIC energies. The low $\mathbf{p_T}$ behavior of the STAR and NA49 data also provides very similar estimates of the pion temperatures at freezout. Further, the striking similarity of the flow-dependent phase space densities for STAR and NA49 suggest that both systems are constrained by the speed of sound in the systems, with the transverse flow velocity hard against the sonic limit.

We note also that the observed phase space densities, while well below 1.0, are well above static thermal density. We attribute this to phase space "squeezing" (i.e., position-momentum correlations at the source) arising from transverse flow. This suggests that significant multiparticle Bose-Einstein effects may be present, particularly away from midrapidity where longitudinal flow should produce additional phase space squeezing.

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