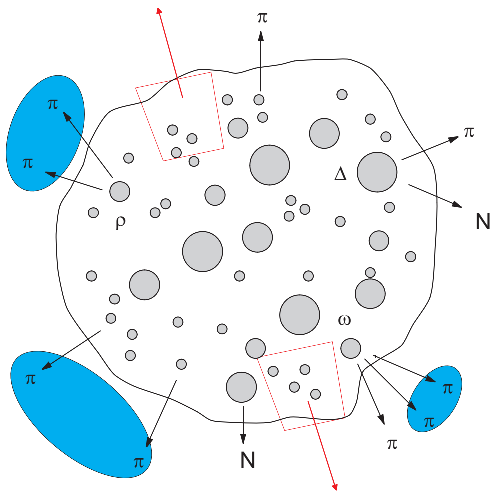
Production of resonances in a thermal model: invariant-mass spectra and balance functions

W. Broniowski [1], W. Florkowski [1,2]

[1] The Henryk Niewodniczański Institute of Nuclear Physics, Kraków
[2] Institute of Physics, Świętokrzyska Academy, Kielce

Quark Matter 2004, Oakland



Thermal model

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Bjorken, Gorenstein, Gaździcki, Heinz, Braun-Munzinger, Stachel, Redlich, Magestro, Csörgő, Becattini, Cleymans, Letessier,...

our variant

WB + WF, **PRL 87 (2001) 272302; PRC 65 (2002) 064905** ($p_\perp$ spectra of hadrons)
WB + WF + Brigitte Hiller, **PRC 68 (2003) 034911** (pion invariant-mass distributions)
Piotr Bożek + WB + WF, **nucl-th/0310062** (pion balance functions)

single freeze-out model

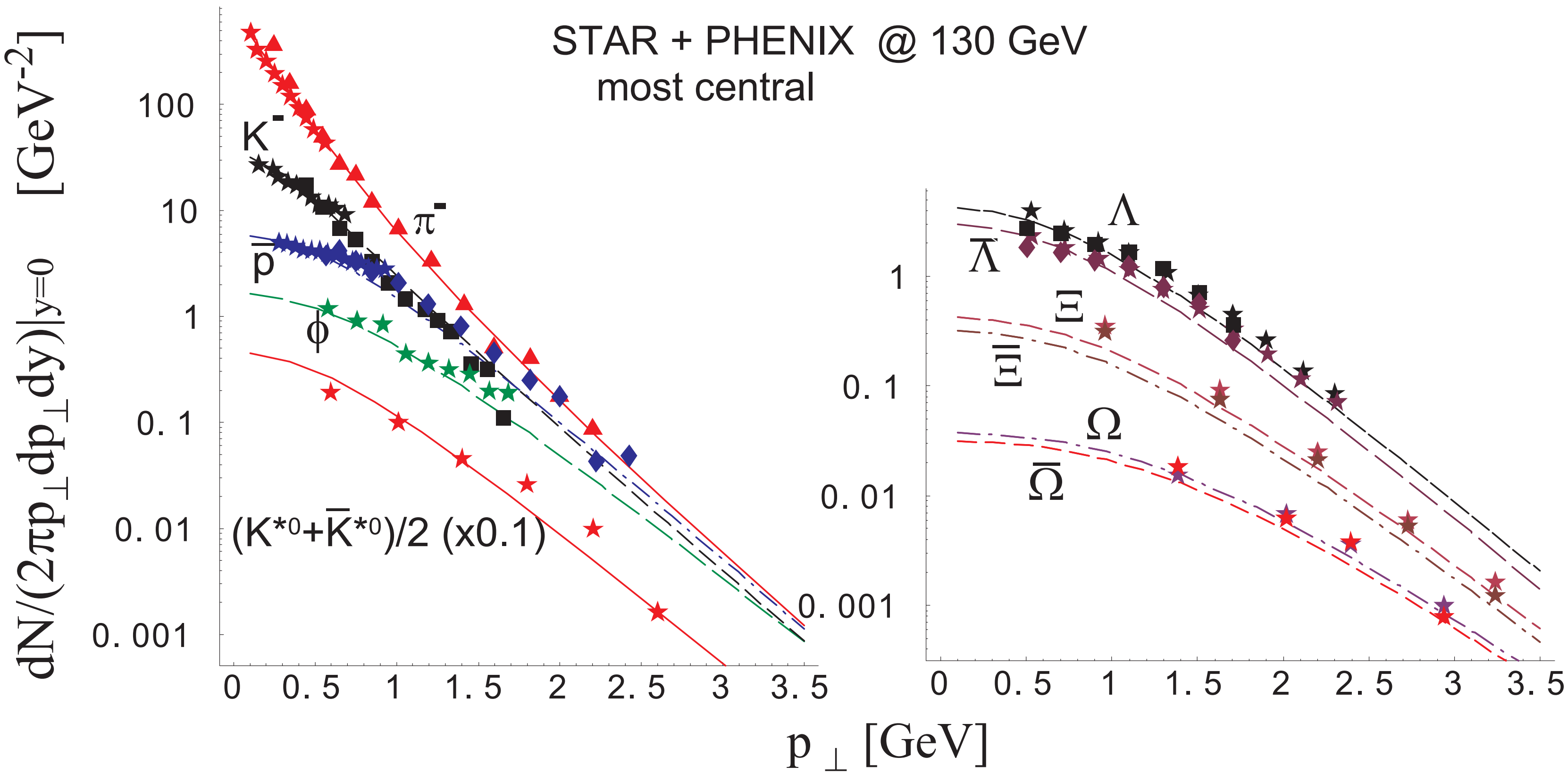
1. $T_{\rm chem} = T_{\rm kin} \equiv T$
2. Complete treatment of resonances
3. Special choice of the freeze-out hypersurface, $\tau = \sqrt{t^2 - x^2 - y^2 - z^2} = \text{const}$
4. Only 4 parameters: T,μ_B (fixed by the ratios of the particle abundances), invariant time at freeze-out τ (controls the overall normalization), transverse size $\rho_{\rm max}$ ($\rho_{\rm max}/\tau$ controls the slopes of the $p_\perp$ spectra)
5. Hubble-like flow, $u^\mu = \frac{x^\mu}{\tau} = \frac{t}{\tau}(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t})$

not unique! – see poster by Torrieri

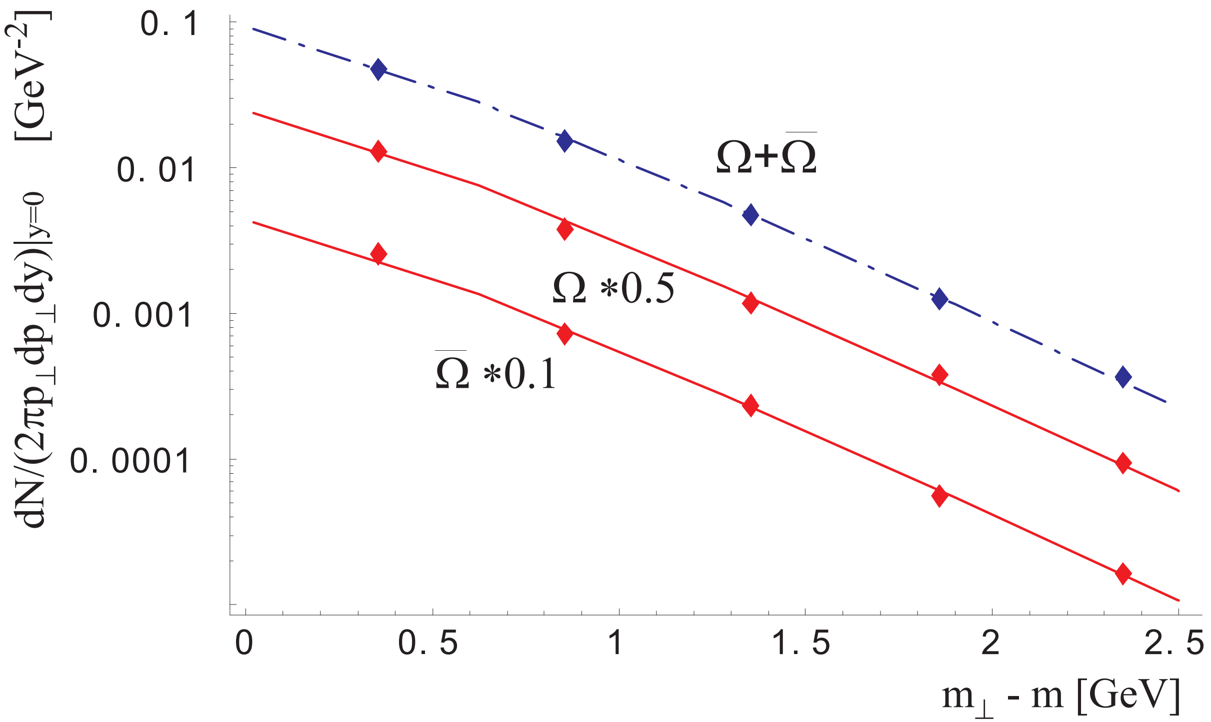
2 thermal parameters fitted from particle ratios

T [MeV] $= 165 \pm 7$, μ_B [MeV] $= 41 \pm 5$, @ 130 GeV
T [MeV] $= 166 \pm 5$, μ_B [MeV] $= 29 \pm 4$, @ 200 GeV

2 geometric parameters fitted from the spectra of $\pi^{\pm}, K^{\pm}, p$, and $\bar{p}$

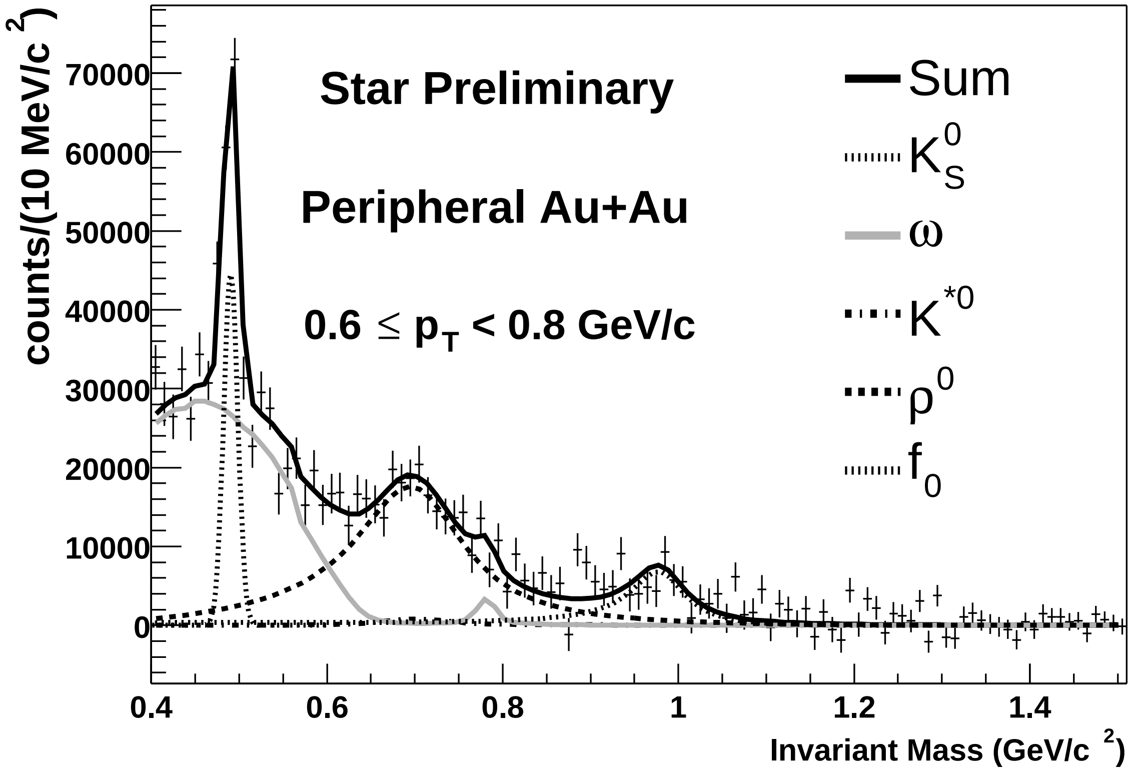


hyperon spectra from STAR @ 200 GeV, C. Suire, QM2002



besides particle ratios and transverse-momentum spectra we may try to calculate further observables, e.g., invariant-mass distributions, charge correlations, ...

$\pi^+\pi^-$ pairs from STAR – P. Fachini



Can we explain such resonance structure in the thermal model?

More importantly, can we explain the shift of the rho-meson peak?

recent papers by Shuryak and Brown, Kolb and Prakash, and Rapp indicate that the shift is a real dynamic effect

The phase-shift formula for the density of resonances

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman; **Weinhold, Friman, Nörenberg**; WB+WF+BH, PRC 68 (2003) 034911; Pratt, Bauer, nucl-th/0308087

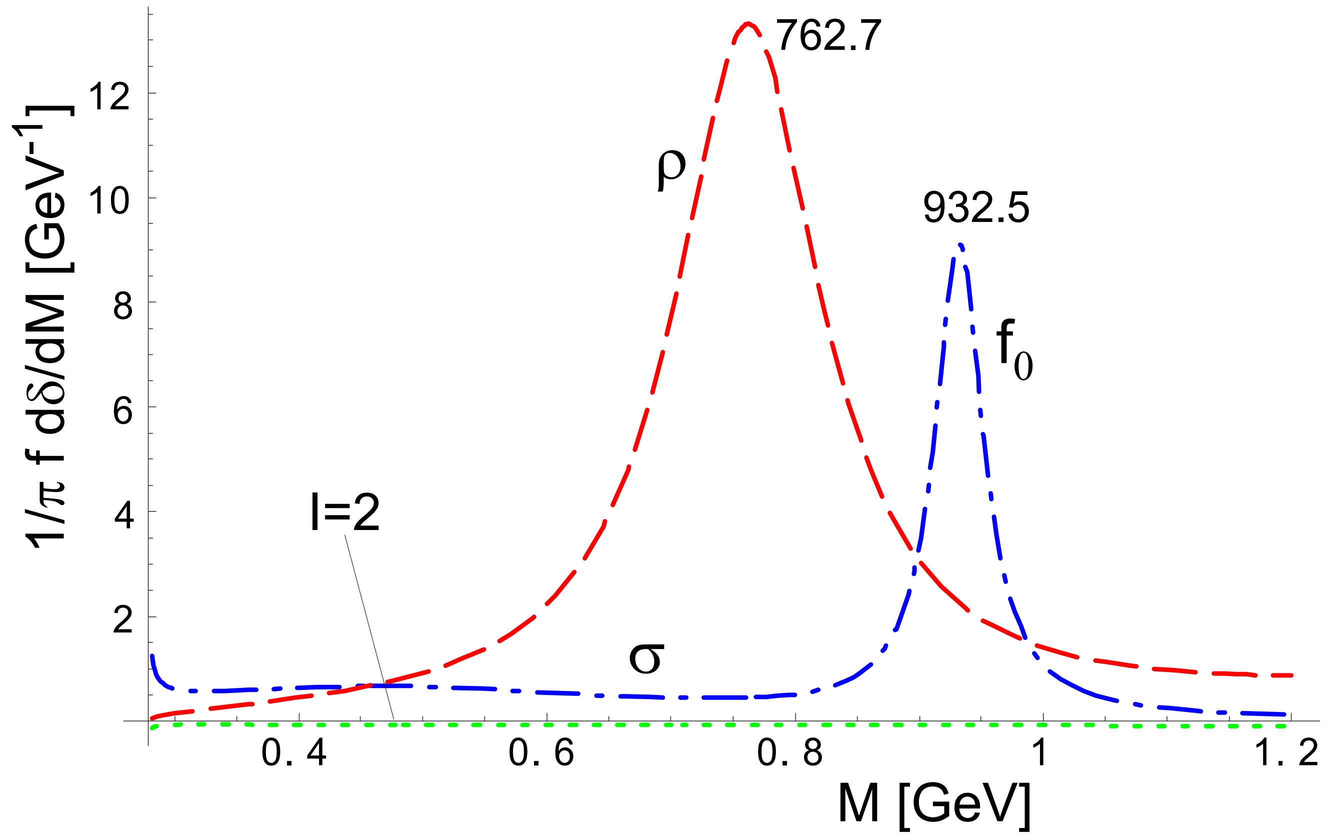
$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{12}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

In some works the spectral function of the resonance is used instead of the derivative of the phase shift. For narrow resonances this does not make a difference, since then $d\delta_{12}(M)/dM \simeq \pi\delta(M - m_R)$, and

$$n^{\mathrm{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\sqrt{m_R^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

For wide resonances, or for effects of tails, the difference between the correct formula and the one with the spectral function is significant

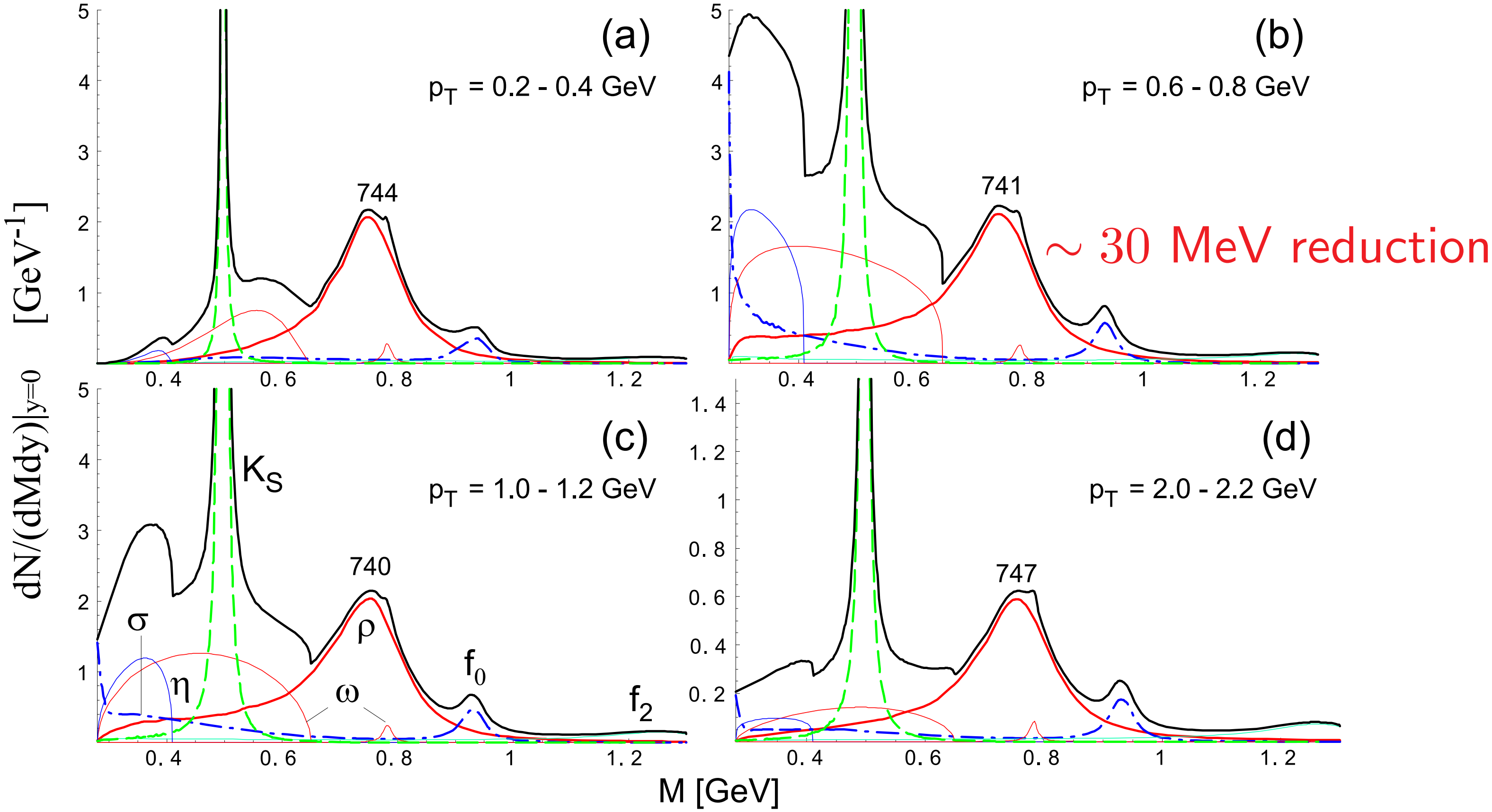
$d\delta_{\pi\pi}(M)/dM$



Small contribution from σ, negative and tiny contribution from $I = 2$

peaks slightly shifted to lower M

Cuts/flow in the single-freeze-out model



The invariant $\pi^+\pi^-$ mass spectra in the single-freeze-out model for four sample bins in the transverse momentum of the pair, p_T, plotted as a function of M. η indicates $\eta + \eta'$. All kinematic cuts of the STAR experiment are incorporated

Role of the resonance decays

The calculation leads to the following enhancement factors coming from the decays of higher resonances:

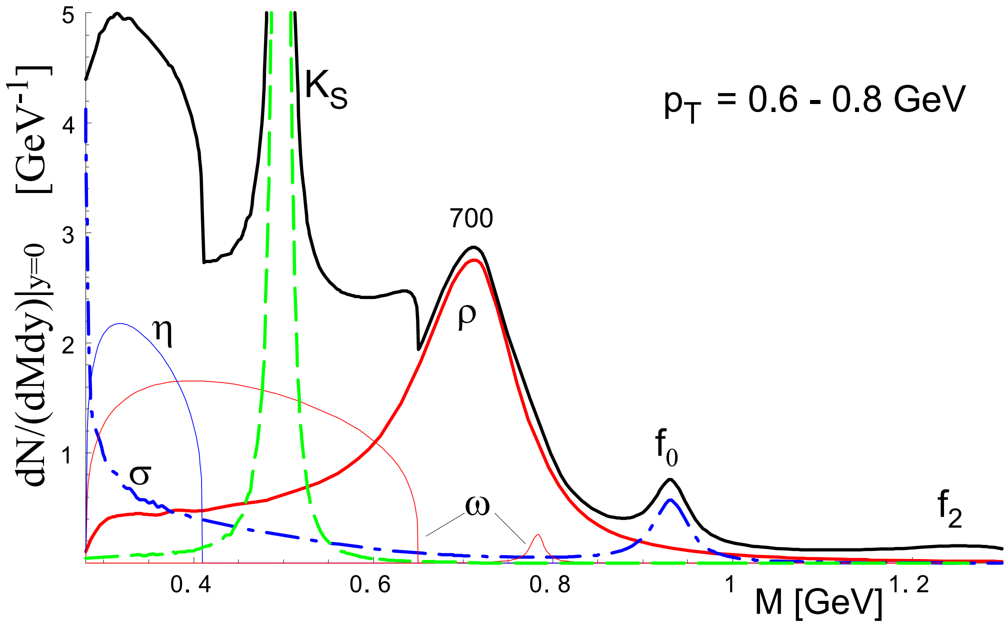
$d_{K_S} = 1.98$, $d_\eta = 1.74$, $d_\sigma = 1.13$, $d_\rho = 1.42$, $d_\omega = 1.43$, $d_{\eta'} = 1.08$, $d_{f_0} = 1.01$, and $d_{f_2} = 1.28$

The effects is strongest for light particles, K_S, η, ρ, and ω, while it is weaker for the heavier η' and scalar mesons

Full model, with feeding from higher resonances and flow/cuts at $T = 165$ MeV is similar to the naive model at $T = 110$ MeV !

Thermal efects may reduce the rho-meson mass only by 30 MeV – there is a place for other dynamic effects

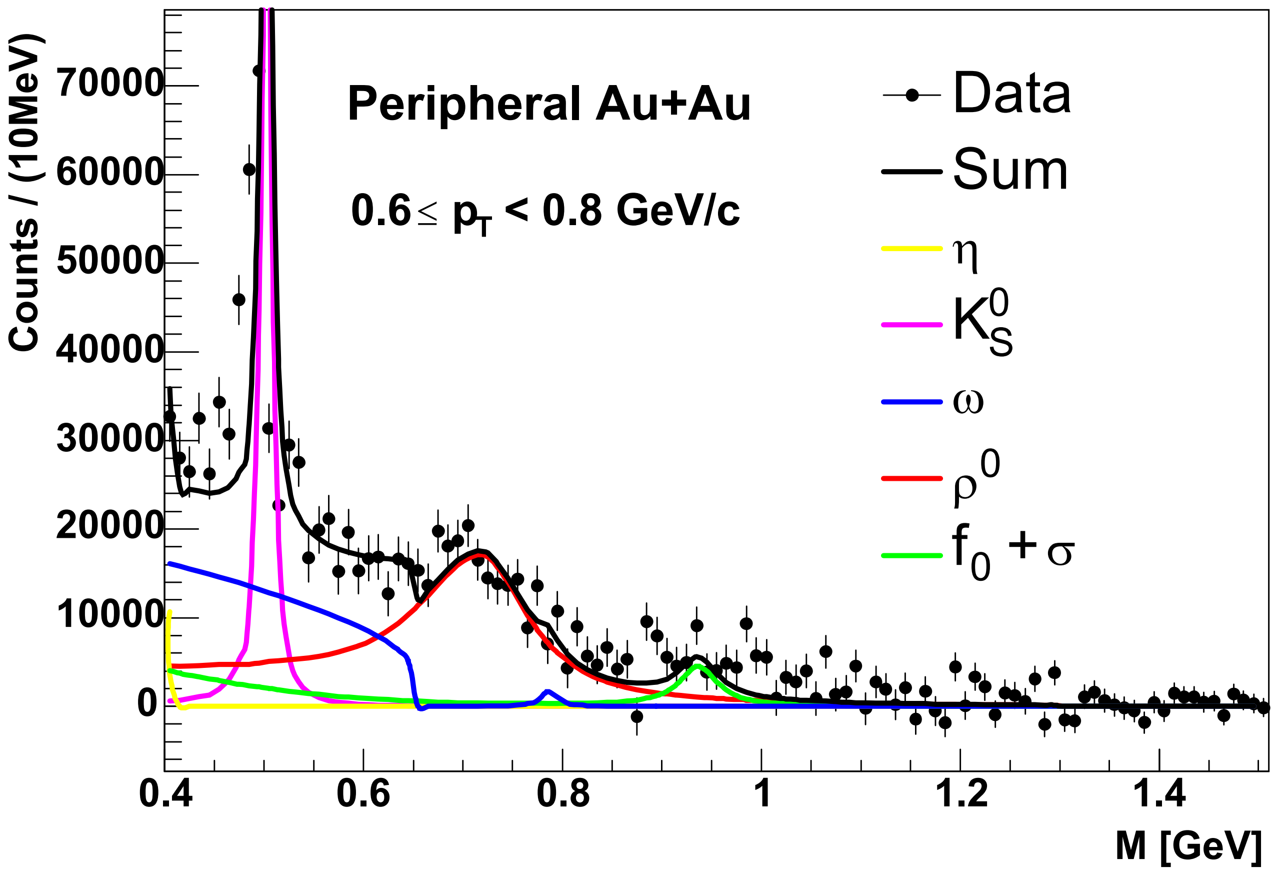
Medium effects?



Our model + position of ρ shifted down from the vacuum value by 9%

STAR vs. single freeze-out model

compiled by P. Fachini

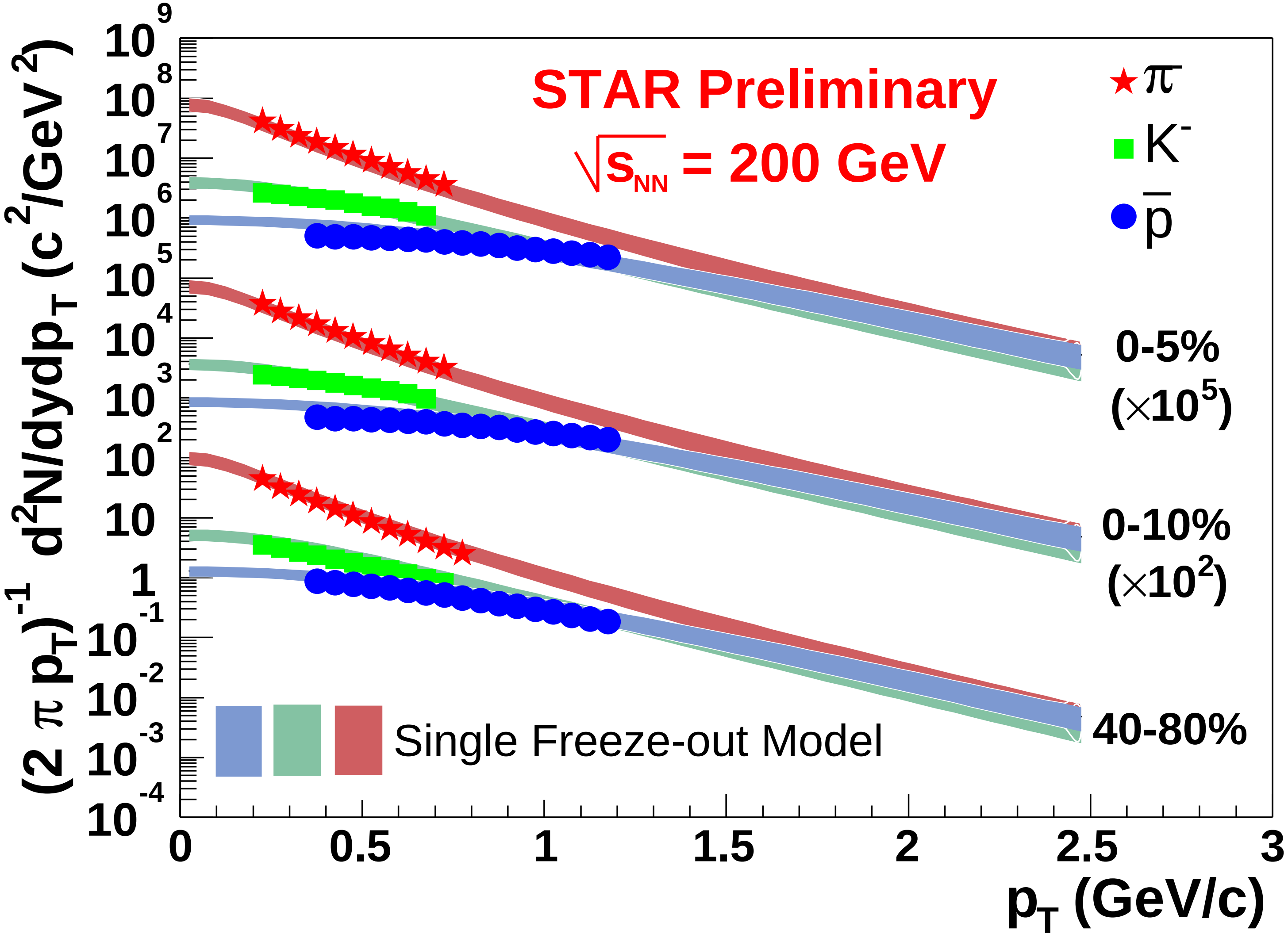


Ratios including resonances

	$m_\rho^* = 770$ MeV	$m_\rho^* = 700$ MeV	Experiment
T [MeV]	$T = 165.6 \pm 4.5$	$T = 167.6 \pm 4.6$	
μ_B [MeV]	$\mu_B = 28.5 \pm 3.7$	$\mu_B = 28.9 \pm 3.8$	
η/π^-	0.120 ± 0.001	0.112 ± 0.001	
ρ^0/π^-	0.114 ± 0.002	0.135 ± 0.001	0.183 ± 0.028 (40-80%)
ω/π^-	0.108 ± 0.002	0.102 ± 0.002	
$K^*(892)/\pi^-$	0.057 ± 0.002	0.054 ± 0.002	
ϕ/π^-	0.025 ± 0.001	0.024 ± 0.001	
η'/π^-	0.0121 ± 0.0004	0.0115 ± 0.0003	
$f_0(980)/\pi^-$	0.0102 ± 0.0003	0.0097 ± 0.0003	0.042 ± 0.021 (40-80%)
$K^*(892)/K^-$	0.33 ± 0.01	0.33 ± 0.01	0.205 ± 0.033 (0-10%) 0.219 ± 0.040 (10-30%) 0.255 ± 0.046 (30-50%) 0.269 ± 0.047 (50-80%)
$\Lambda(1520)/\Lambda$	0.061 ± 0.002	0.062 ± 0.002	0.022 ± 0.010 (0-7%) 0.025 ± 0.021 (40-60%) 0.062 ± 0.027 (60-80%)
$\Sigma(1385)/\Sigma$	0.484 ± 0.004	0.485 ± 0.004	

STAR spectra vs. single freeze-out model

compiled by P. Fachini



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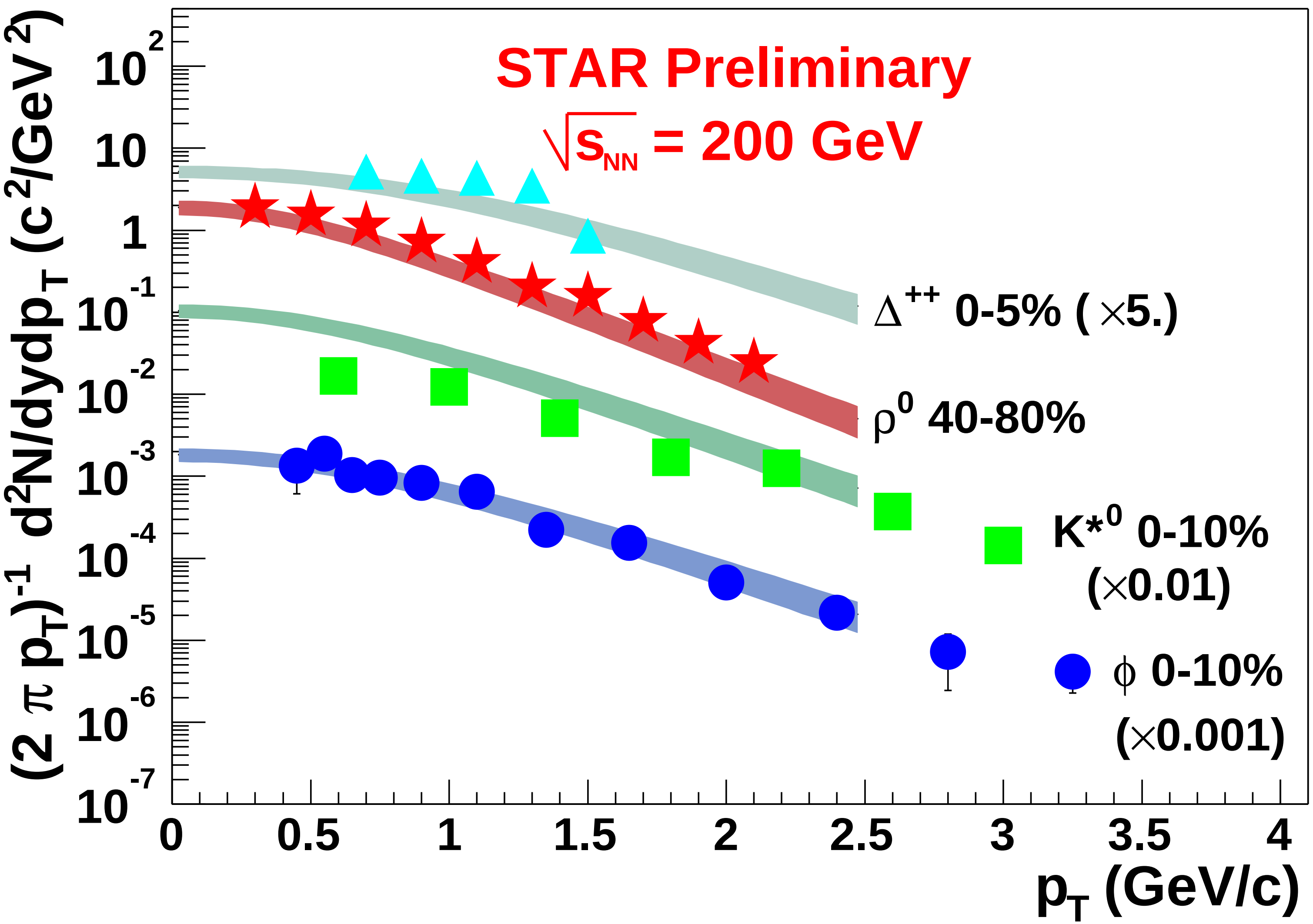


Concept of the balance functions

S. Bass, P. Danielewicz, and S. Pratt, PRL 85 (2000) 2689

$$B(\delta, Y) = \frac{1}{2}\left\{ \frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_{+}\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_{-}\rangle} \right\}$$

N_{+-} and N_{-+} numbers of the unlike-sign pairs

N_{++} and N_{--} numbers of the like-sign pairs

two members of a pair fall into the rapidity window Y, their relative rapidity is

$$\delta = \Delta y = |y_2 - y_1|$$

N_{+} (N_{-}) number of positive (negative) particles in the interval Y

Two contributions for the $\pi^+\pi^-$ balance function

1) RESONANCE CONTRIBUTION (R) is determined by the decays of neutral hadronic resonances which have a $\pi^+\pi^-$ pair in the final state

$$K_S,\ \eta,\ \eta',\ \rho^0,\ \omega,\ \sigma,\ f_0$$

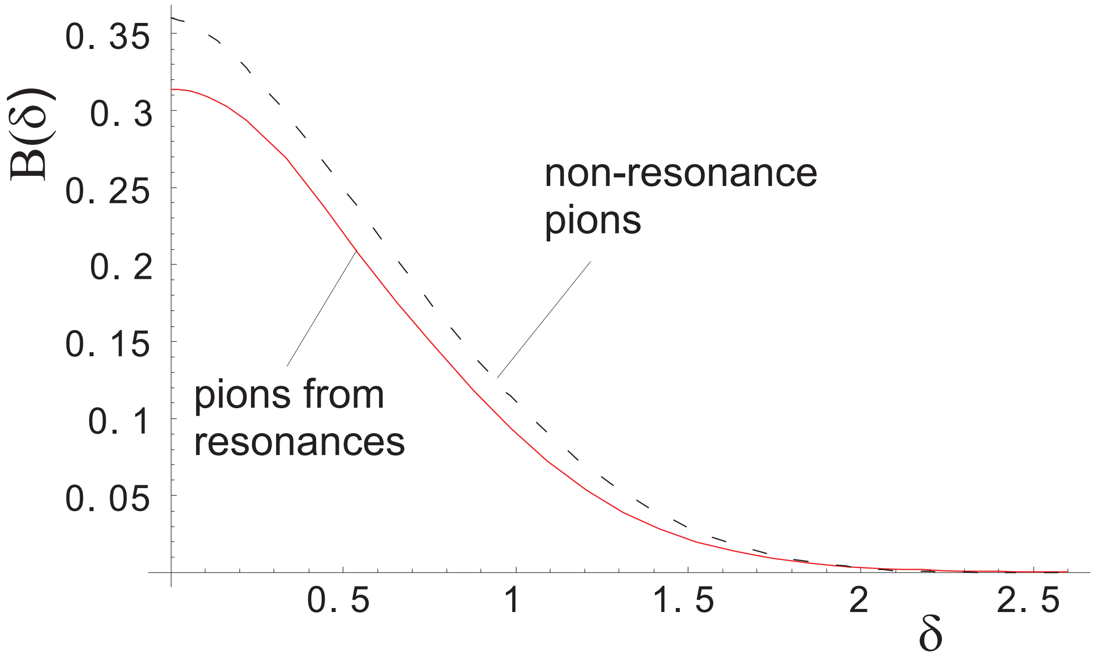
2) NON-RESONANCE CONTRIBUTION (NR) other possible correlations among the charged pions

in our approach the non-resonance two-particle distribution is determined by the local relative thermal momenta of particles

The pion balance function is constructed as a sum of the two terms

$$B(\delta, Y) = B_{\rm R}(\delta, Y) + B_{\rm NR}(\delta, Y)$$

Results



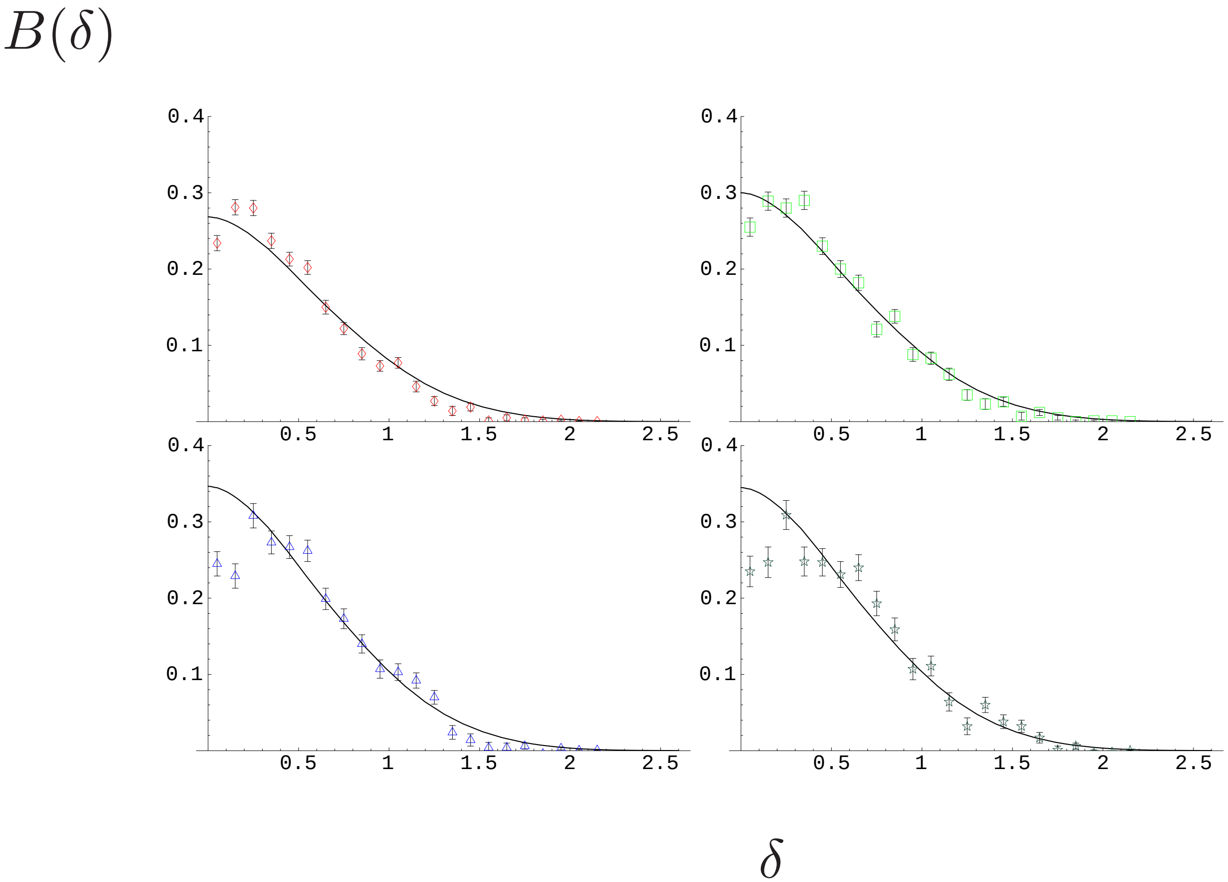
$\rho_{\max}/\tau = 0.9 \rightarrow \langle \beta_\perp \rangle = 0.5$

$\langle \delta \rangle \equiv \int_{0.2}^{2.4} \delta\, B(\delta)\, d\delta$

$\langle \delta \rangle_{NR} = 0.67,\ \langle \delta \rangle_{R} = 0.65,\ \langle \delta \rangle_{R+NR} = 0.66$

STAR measurement: $\langle \delta \rangle = 0.59$ for central, $\langle \delta \rangle = 0.66$ for peripheral

Fit to the STAR data



four different centralities: 0-10%, 10-40%, 40-70%, 70-96%

rescaling factors: 0.40, 0.44, 0.51, 0.51 (χ^2 fits)

poor man's way of taking into account the detector efficiency

Summary

1. Single freeze-out model works well for the pion invariant-mass distributions and the pion balance functions; it gives similar results at T=165 MeV to the naive calculation at T=110 MeV, 4 parameters fitted long before by the ratios and spectra

2. Derivative of phase shift is used as weight, not the spectral density
3. Kinematic cuts and flow important, resonance decays important
4. Not possible to place the ρ peak at the experimental value (medium or other effects?)

5. The resonance contribution to the pion balance function is determined in the unique way, the form of the non-resonance contribution should be assumed
6. The two calculated contributions have similar δ-dependence, the width of the sum is larger (12%) than the width measured by STAR for central events, however the shape is quite right except for the very small values of δ where the Bose-Einstein correlations are important
7. The overall normalization must be fitted in order to take into account the effect of the efficiency of the detector, this brings a relatively large factor ~ 0.50

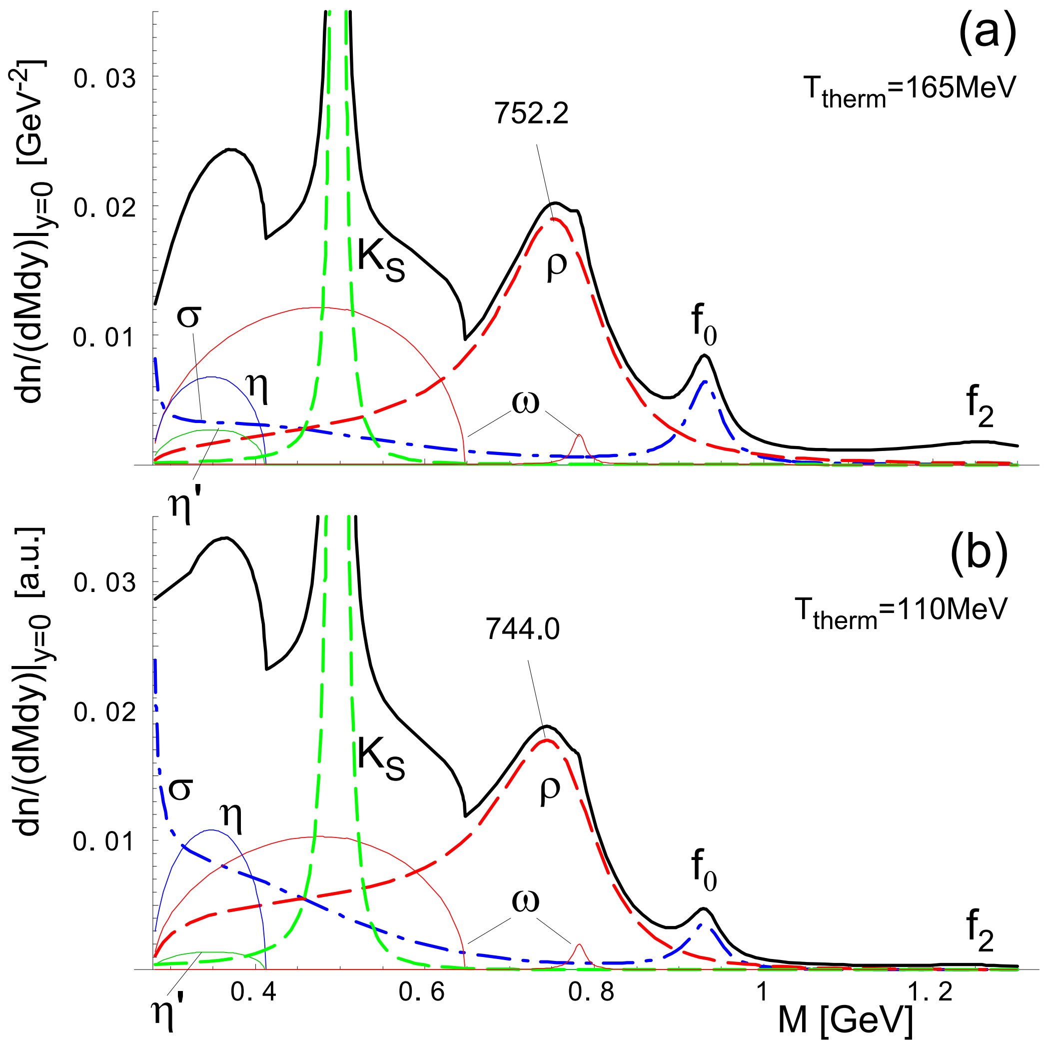
single freeze-out model describes well: ratios, spectra, $R_{\rm out}/R_{\rm side} \approx 1$, v_2 (with two extra parameters), invariant masses, balance functions,....

Back-up slides

Warm-up calculation - static source

We compute the spectra at mid-rapidity, hence

$$\left.\frac{dn}{dMdy}\right|_{y=0} = \sum_i f_i \int_{0.2\mathrm{GeV}}^{2.2\mathrm{GeV}} \frac{p_\perp dp_\perp}{(2\pi)^2} \frac{d\delta_i(M)}{\pi dM} \frac{\sqrt{M^2+p_\perp^2}}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}_\perp{}^2}}{T}\right)-1}$$



flat

steep

The STAR cuts

The cuts in the STAR analysis of the $\pi^+\pi^-$ invariant-mass spectra have the following form (Fachini):

$$\begin{aligned} &|y_\pi| \le 1, \\ &|\eta_\pi| \le 0.8, \\ &0.2 \text{ GeV} \le p_\pi^\perp \le 2.2 \text{ GeV}, \end{aligned} \tag{1}$$

while the bins in $p_T \equiv |\boldsymbol{p}_\pi^\perp + \boldsymbol{p}_\pi^\perp|$ start from the range $0.2 - 0.4$ GeV, and step up by 0.2 GeV until $2 - 2.4$ GeV.

For two-body decays, the relevant formula for the number of pairs of particles 1 and 2 has the form

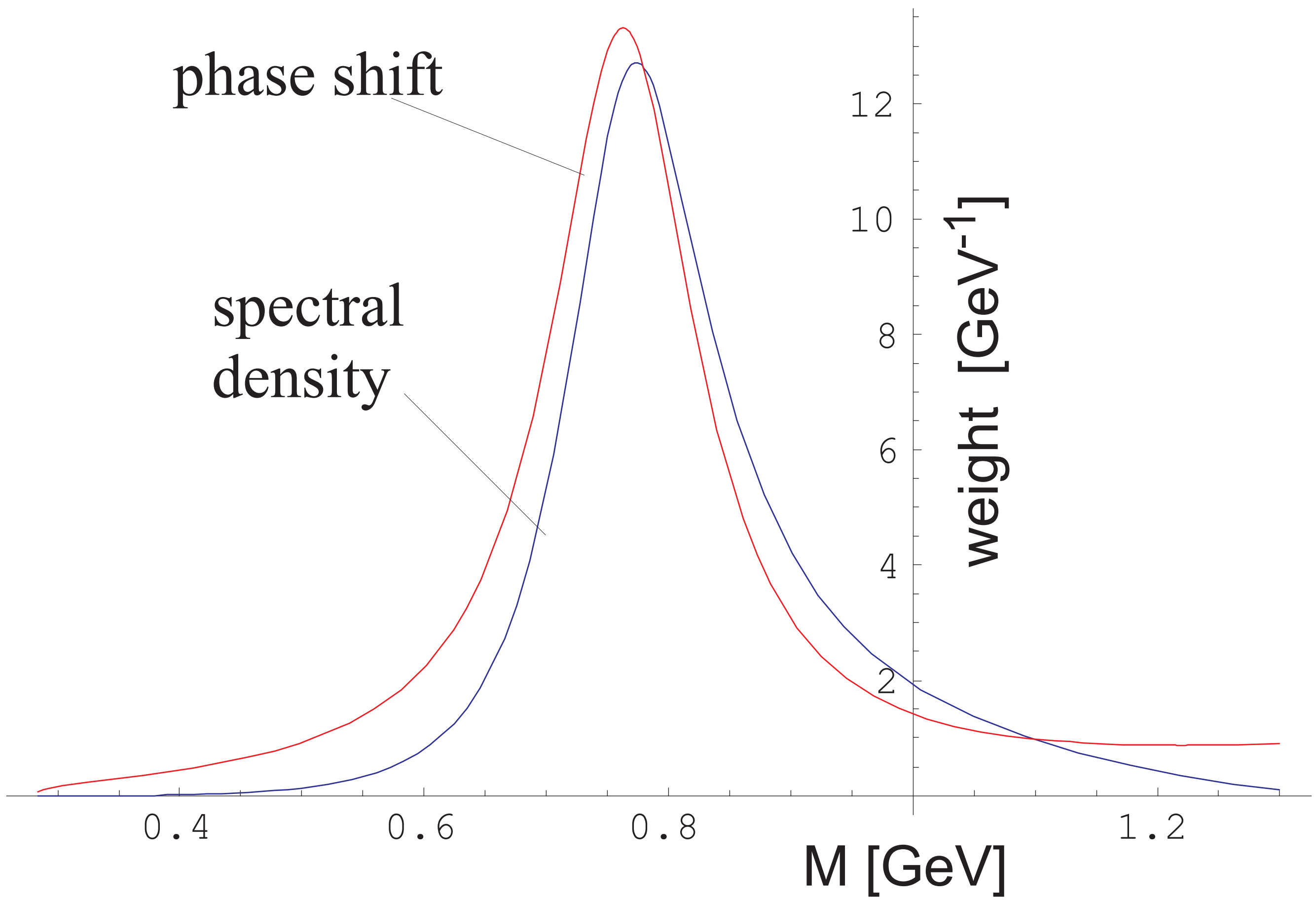
$$\begin{aligned} \frac{dN_{12}}{dM} \quad &= \quad \frac{d\delta_{12}}{dM}\frac{bm}{p_1^*}\int_{p^\perp_{1,\mathrm{low}}}^{p^\perp_{1,\mathrm{high}}} dp_1^\perp \int_{y_{1,\mathrm{low}}}^{y_{1,\mathrm{high}}} dy_1 \int_{p^\perp_{\mathrm{low}}}^{p^\perp_{\mathrm{high}}} dp^\perp \int_{y_{\mathrm{low}}}^{y_{\mathrm{high}}} dy \\ &\times C_2^0 C_1^\eta C_2^\eta \frac{\theta(1-\cos^2\gamma_0)}{|\sin\gamma_0|} S(p^\perp), \end{aligned} \tag{2}$$

Lowering the ρ mass

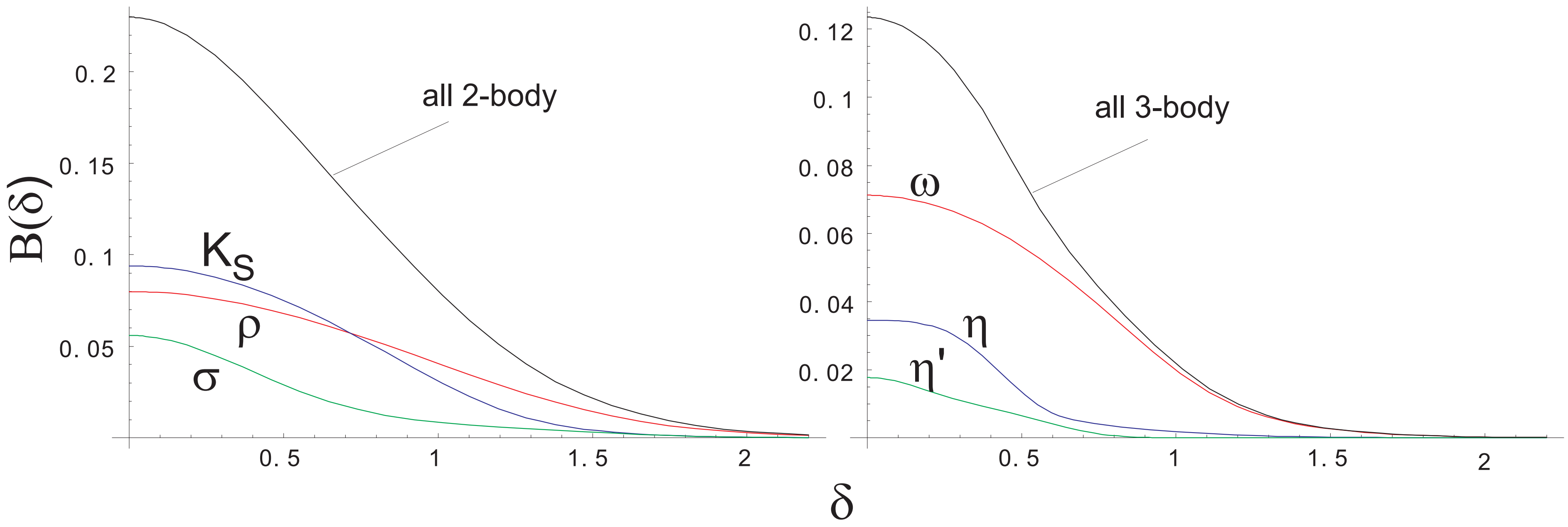
In order to show how the medium modifications will show up in the $\pi^+\pi^-$ spectrum, we have scaled the $\pi\pi$ phase shift in the ρ channel, according to the simple law

$$\delta_1^1(M)_{\mathrm{scaled}} = \delta_1^1(s^{-1}M)_{\mathrm{vacuum}}, \tag{3}$$

Phase shift vs. spectral density



Anatomy of the resonance contribution



heavier resonance – wider

two-body wider than three-body