J/ψ and η_c in the Deconfined Plasma from Lattice QCD

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QM2004, Oakland CA, U.S.A. January 16, 2004

PLAN

- Spectral Function
- Necessity of MEM *(Maximum Entropy Method)*
  - MEM Outline
  - Importance of Error Analysis
- Finite Temperature Results for J/ψ and η_c
  - Error Analysis
    - Statistical
    - Systematic

Spectral Function

Definition of Spectral Function

$$\frac{A_{\eta\eta'}(k_0,\vec{k})}{(2\pi)^3} \equiv \sum_{n,m} \frac{e^{-E_n/T}}{Z} \langle n | J_\eta(0) | m \rangle \langle m | J^\dagger_{\eta'}(0) | n \rangle (1 \mp e^{-P^0_{mn}/T}) \delta^4(k^\mu - P^\mu_{mn})$$

$-(+)$: Boson(Fermion)

$J_\eta(0)$: A Heisenberg Operator with some quantum #

$|n\rangle$: Eigenstate with 4-momentum P^μ_n

$P^\mu_{mn} = P^\mu_m - P^\mu_n$

- Pretty important function to understand QCD

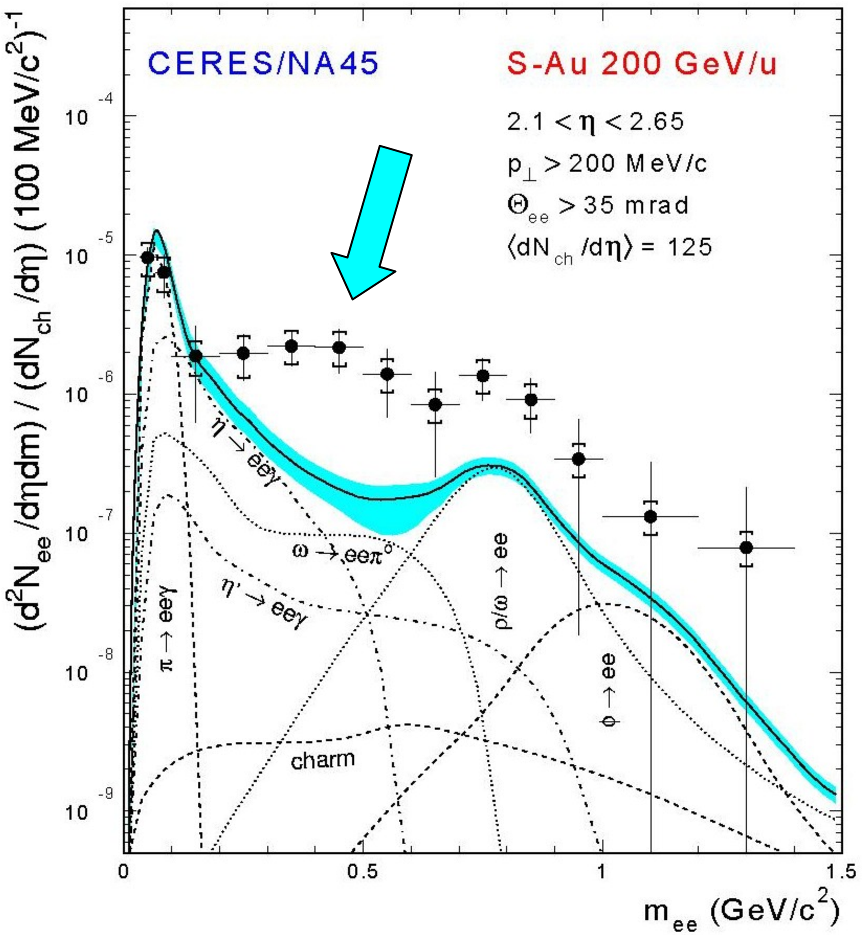
Dilepton production rate, Real Photon production rate, ...etc.

$$\frac{dN(e^+e^- \text{ production at } T)}{d^4x d^4k} = -\frac{\alpha^2}{3\pi^2 k^2} \frac{\boxed{A^\mu_\mu(k_0,\vec{k})}}{e^{k_0/T}-1}$$

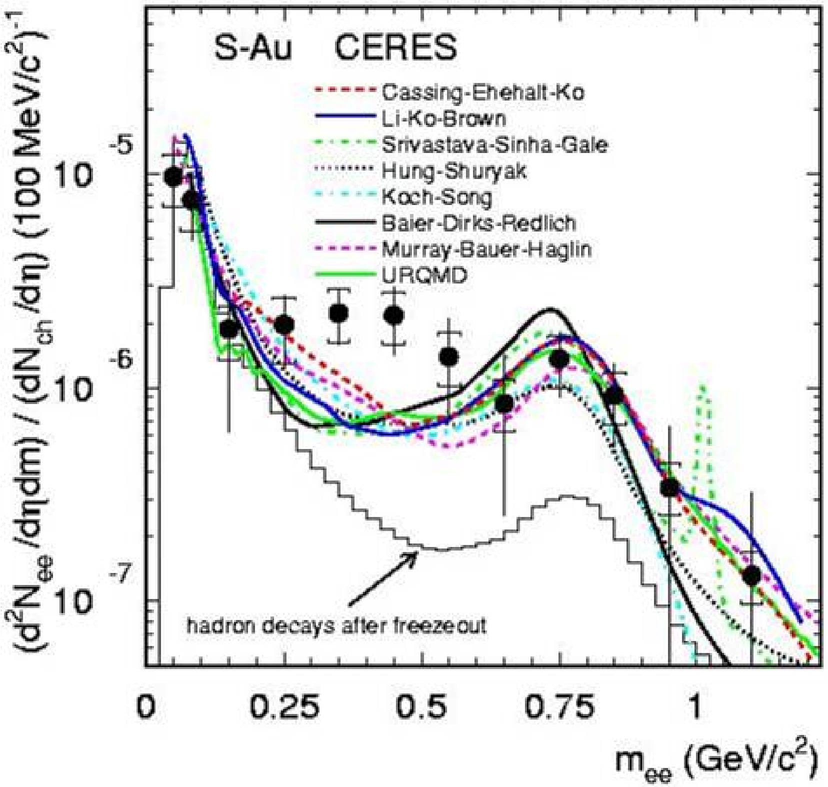
holds regardless of states, either in Hadron phase or QGP

Hadron Modification in HI Collisions?

Experimental Data



Comparison with Theory
(with no hadron modification)



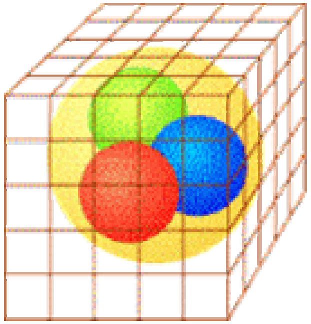
Mass Shift ? Broadening ? or Both ?
or More Complex Structure ?

M. Asakawa (Kyoto University)

Lattice? But there was difficulty ...

- What's measured on Lattice is Correlation Function $D(\tau)$

$$D(\tau) = \int \left\langle O(\tau, \vec{x}) O'^{\dagger}(0, \vec{0}) \right\rangle d^3x$$



$D(\tau)$ and $A(\omega) \equiv A(\omega, \vec{0})$ are related by

$$D(\tau) = \int_0^\infty K(\tau, \omega) A(\omega) d\omega$$

However,

χ^2-fitting : inconclusive !

- Measured in Imaginary Time
- Measured at a Finite Number of discrete points
- Noisy Data ⟸ Monte Carlo Method

Difficulty on Lattice

Thus, what we have is

Inversion Problem

$$D(\tau) = \int_0^\infty K(\tau,\omega) A(\omega)\, d\omega$$

$$D(\tau) \Rightarrow A(\omega)$$

d i s c r e t e

continuous

noisy

Difficulty on Lattice

Thus, what we have is

Inversion Problem

$$D(\tau) = \int_0^\infty K(\tau,\omega)A(\omega)\,d\omega$$

$$D(\tau) \Rightarrow A(\omega)$$

Typical ill-posed problem

Problem since Lattice QCD was born

Principle of MEM

- MEM

a method to infer the most statistically probable image $(= A(\omega))$ given data

In MEM, Statistical Error can be put to the Obtained Image

- Theoretical Basis: Bayes' Theorem

$$P[X|Y] = \frac{P[Y|X]P[X]}{P[Y]}$$

$P[X|Y]$: Probability of X given Y

- In Lattice QCD

D: Lattice Data (Average, Variance, Correlation…etc.)

H: All definitions and *prior knowledge* such as $A(\omega) \geq 0$

Bayes Theorem $\Rightarrow$ $P[A|DH] \propto P[D|AH]P[A|H]$

In MEM, basically Most Probable Spectral Function is calculated

Ingredients of MEM

- $P[D|AH]$

  $= \chi^2$-likelihood function
  $P[D|AH] = \exp(-L)/Z_L$

- $P[A|H]$

  given by Shannon-Jaynes Entropy

$$P[A|H\alpha m] = \frac{\exp(\alpha S)}{Z_S}$$

$$S = \int\left[A(\omega) - m(\omega) - A(\omega)\log\left(\frac{A(\omega)}{m(\omega)}\right)\right]d\omega$$

max at $A(\omega) = m(\omega)$

$$Z_S = \int e^{\alpha S}[dA], \quad \alpha \in \mathbf{R}$$

Default Model $m(\omega) \in \mathbf{R}$: Prior knowledge about $A(\omega)$

such as semi-positivity,
perturbative asymptotic value, …etc.

For further details,
Y. Nakahara, and T. Hatsuda, and M. A., Prog. Part. Nucl. Phys. 46 (2001) 459

Error Analysis in MEM (Statistical)

- MEM is based on Bayesian Probability Theory
  - In MEM, *Errors can be and must be assigned*
  - This procedure is *essential* in MEM Analysis
- For example, Error Bars can be put to

Average of Spectral Function in $I=[\omega_1,\omega_2]$, $\langle A_\alpha \rangle_I = \dfrac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2} A_\alpha(\omega)d\omega$

$$\left\langle (\delta A_\alpha)^2 \right\rangle_I = \frac{1}{(\omega_2-\omega_1)^2}\int [dA]\int_{I\times I} d\omega d\omega' \delta A(\omega)\delta A(\omega') P[A\,|\,DH\alpha m]$$

$$\simeq -\frac{1}{(\omega_2-\omega_1)^2}\int_{I\times I} d\omega d\omega' \left(\frac{\delta^2 Q(A)}{\delta A(\omega)\delta A(\omega')}\right)^{-1}_{A=A_\alpha}$$

$$\delta A(\omega) = A(\omega) - A_\alpha(\omega)$$

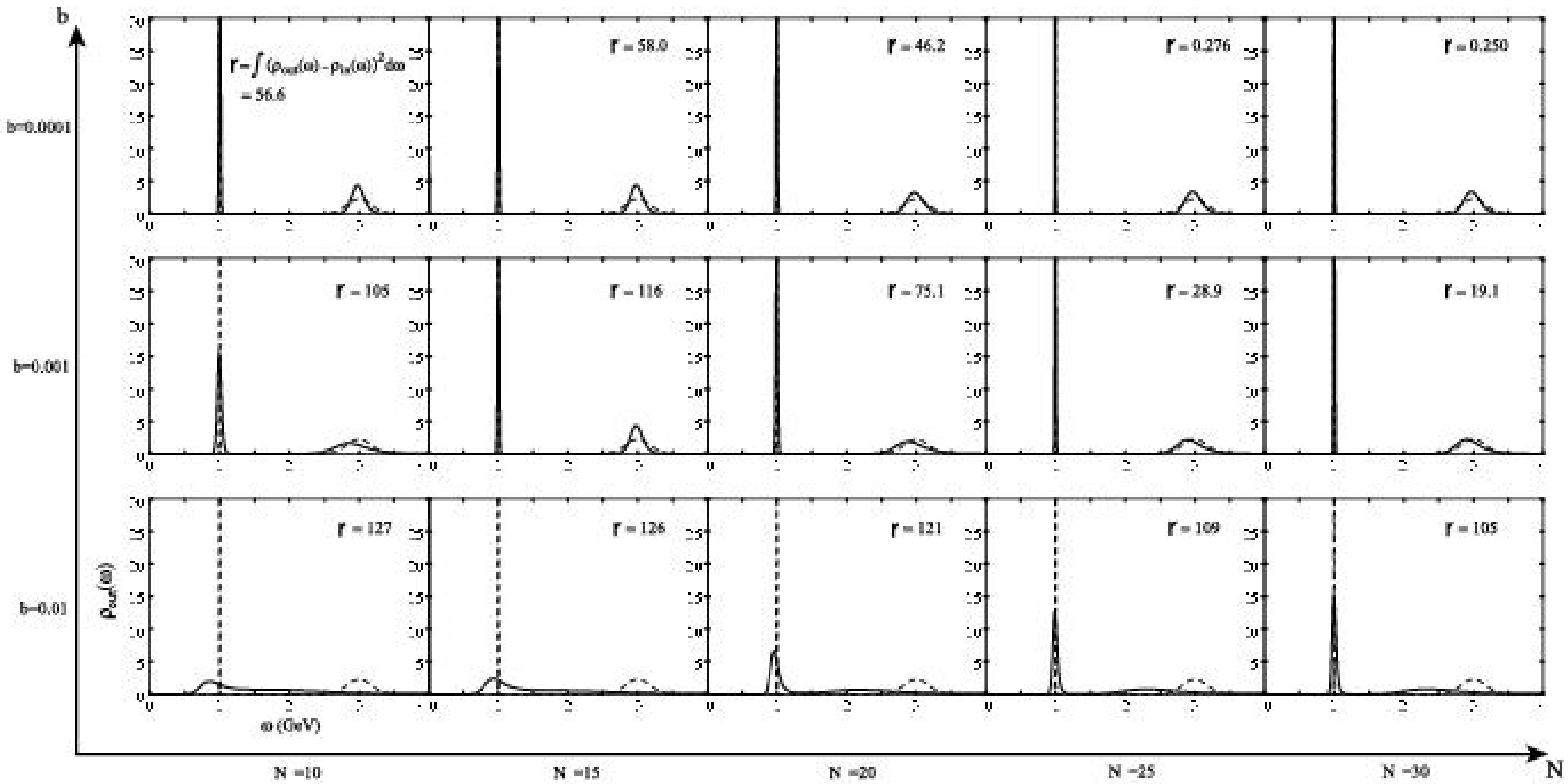
$$Q(A) = \alpha S - L$$

$$[dA] = \prod_{l=1}^{N_\omega} \frac{dA_l}{\sqrt{A_l}}$$

Gaussian approximation

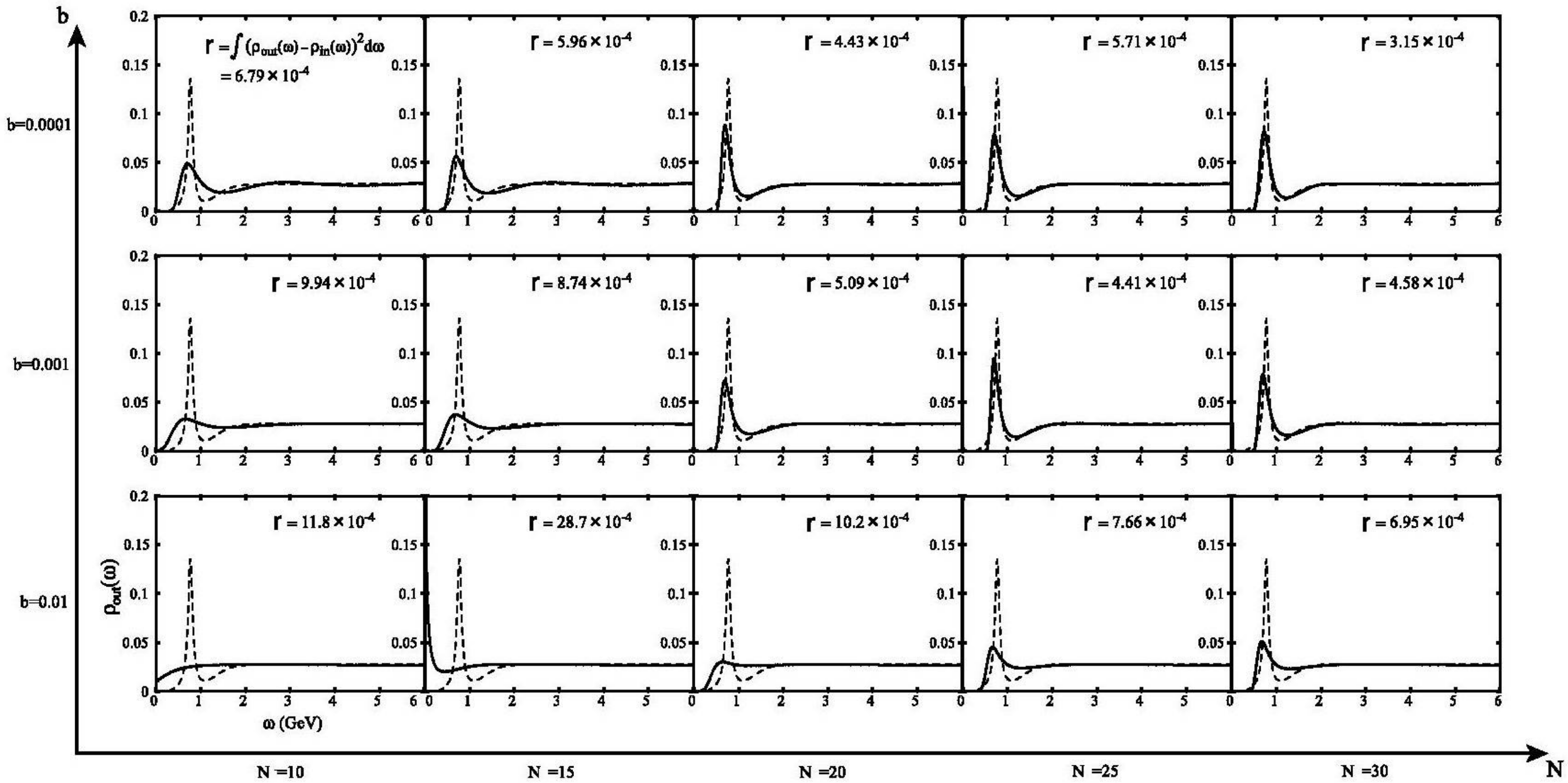
Result of Mock Data Analysis (1)

N(# of data points)-b(noise level) dependence



Result of Mock Data Analysis (2)

N(# of data points)-b(noise level) dependence



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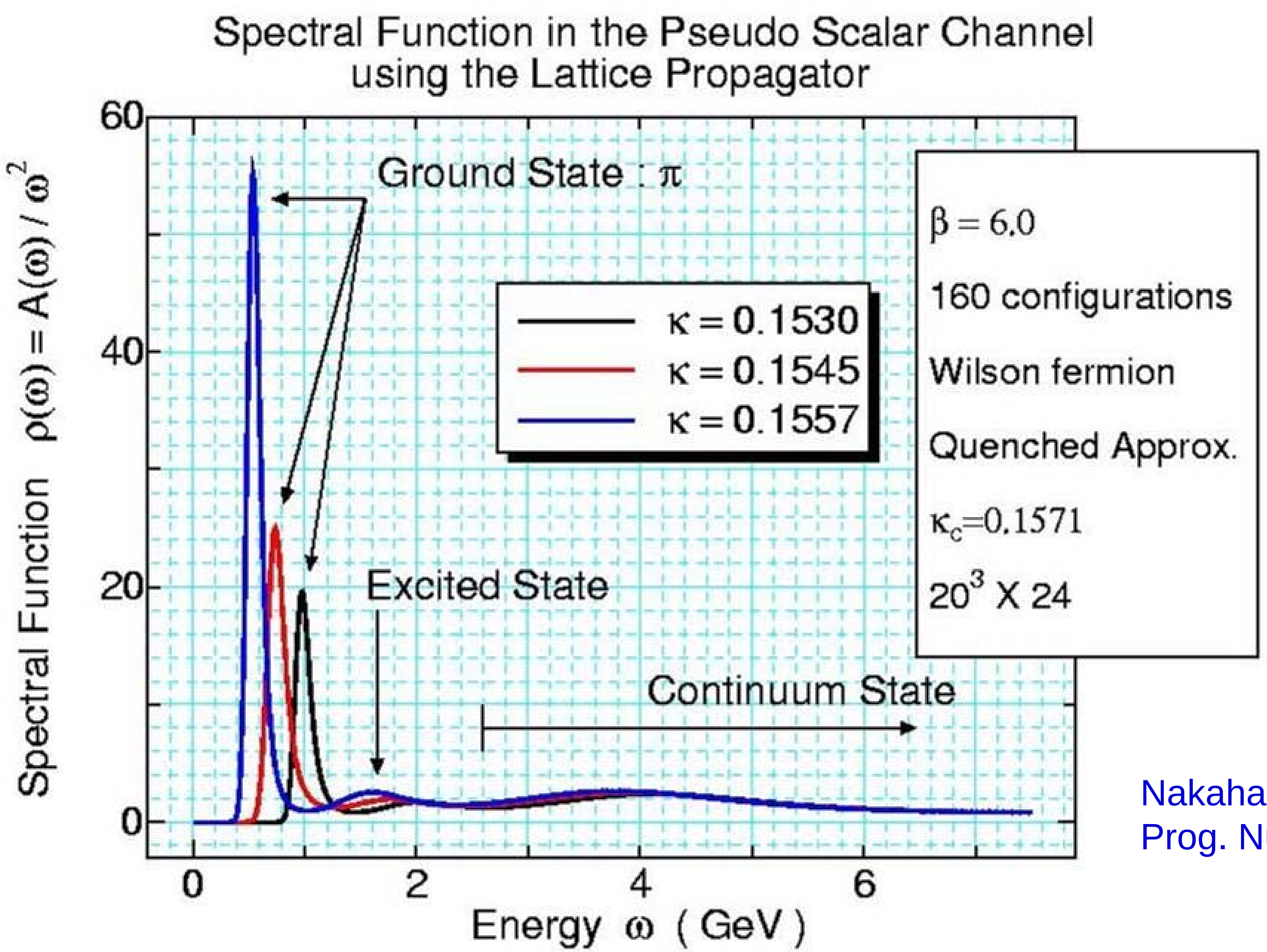
Application of MEM to Lattice Data (T=0)



Nakahara, Hatsuda, and M.A.
Prog. Nucl. Theor. Phys., 2001

Resonance Physics has become possible on Lattice

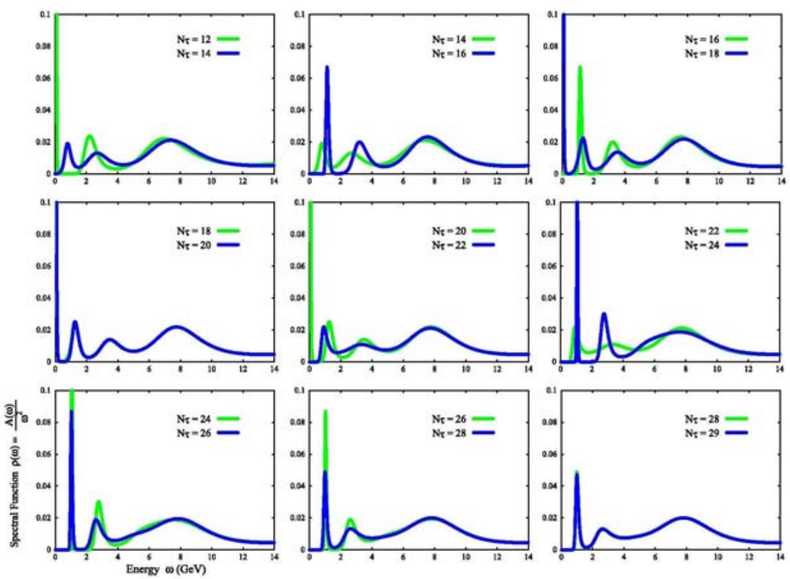
What Result of Mock Data Analysis tells us

- General Tendency (always statistical fluctuation exists)

The *Larger the Number of Data Points* and the *Lower the Noise Level*

The result is closer to the original image

How many data points are needed ?



$40^3 * 30$ lattice
$\beta = 6.47$, 150 confs.
isotropic lattice ($T<T_c$)

may depend on statistics, β, K, ξ,...etc.

$N_{\tau,\, min} \cong 30$ or larger

Parameters on Lattice

1. Lattice Sizes
   $32^3 * 32$ ($T = 2.33T_c$)
   40 ($T = 1.87T_c$)
   42 ($T = 1.78T_c$)
   44 ($T = 1.70T_c$)
   46 ($T = 1.62T_c$)
   54 ($T = 1.38T_c$)
   72 ($T = 1.04T_c$)
   80 ($T = 0.93T_c$)
   96 ($T = 0.78T_c$)
2. $\beta = 7.0$, $\xi_0 = 3.5$
   $\xi = a_\sigma / a_\tau = 4.0$ (anisotropic)
3. $a_\tau = 9.75 * 10^{-3}$ fm
   $L_\sigma = 1.25$ fm
4. Standard Plaquette Action
5. Wilson Fermion
6. Heatbath : Overrelaxation
   = 1 : 4

   1000 sweeps between measurements
7. Quenched Approximation
8. Gauge Unfixed
9. $\mathbf{p} = \mathbf{0}$ Projection
10. Machine: CP-PACS

Parameters in MEM analysis

- Default Models used in the Analysis

channel	PS	V
$m(\omega)/\omega^2$	1.15	0.40

With Renormalization of Each Composite Operator on Lattice
The m-dependence of the result is weak

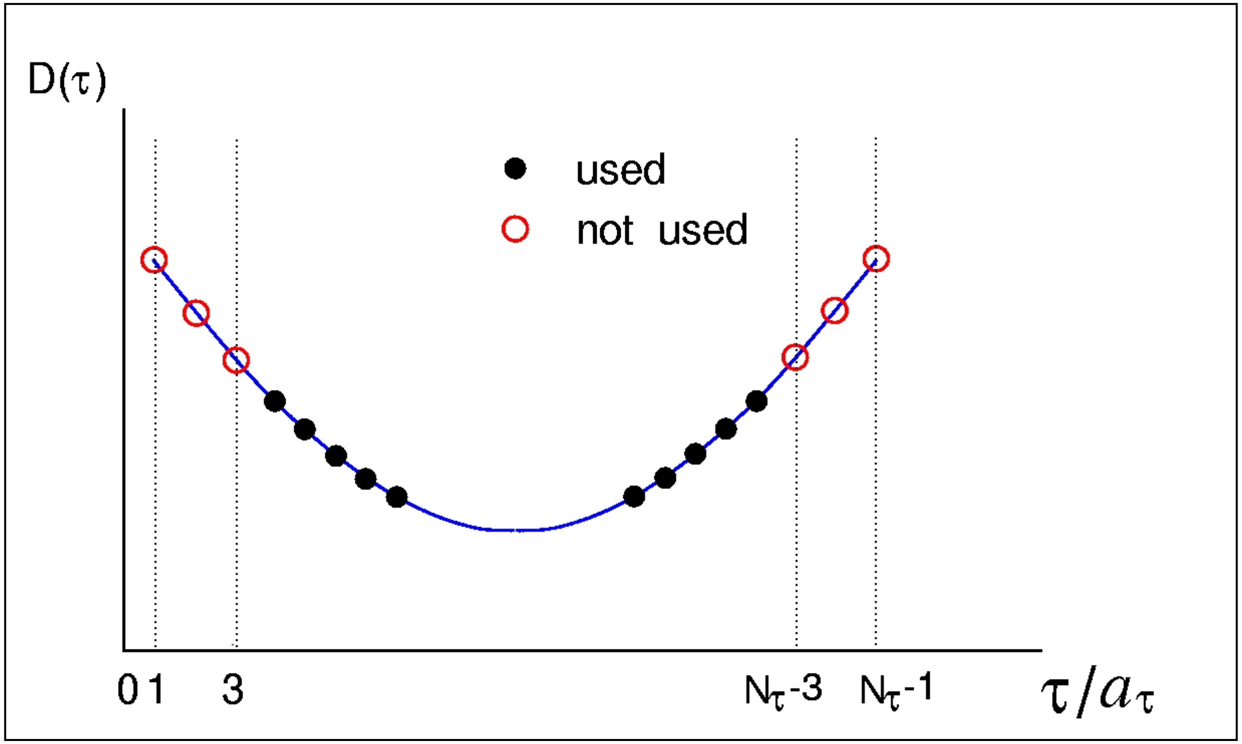
- Continuum Kernel ⟵ Small Enough Temporal Lattice Spacing

- Data Points at $\tau / a_\tau = 0, \cdots, 3,\ N_\tau - 3, \cdots, N_\tau - 1$ are not used

⟵ $|\vec{p}|, \omega \leq \pi / a_\sigma$ and $a_\sigma / a_\tau = 4$
Information at $\omega \geq \pi / a_\sigma$: not physical

Data at these points can be dominated by such *unphysical* noise

Parameters in MEM Analysis (cont'd)

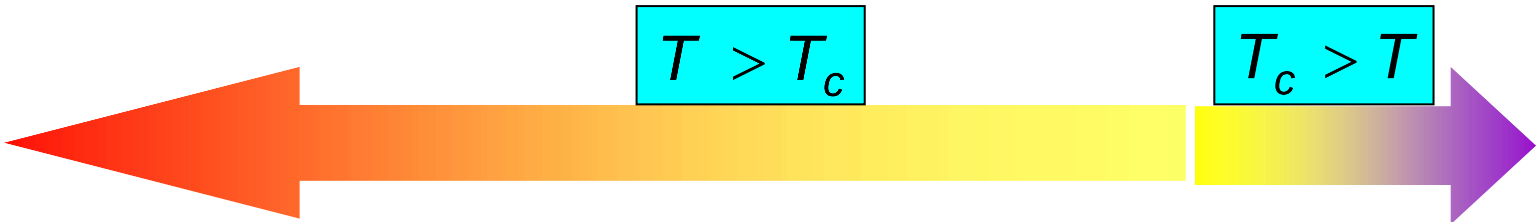


- Furthermore, in order to fix resolution, *a fixed number of data points* (default value = 33 or 34) are used for each case
- Dependence on the Number of Data Points is also studied (systematic error estimate)

Number of Configurations

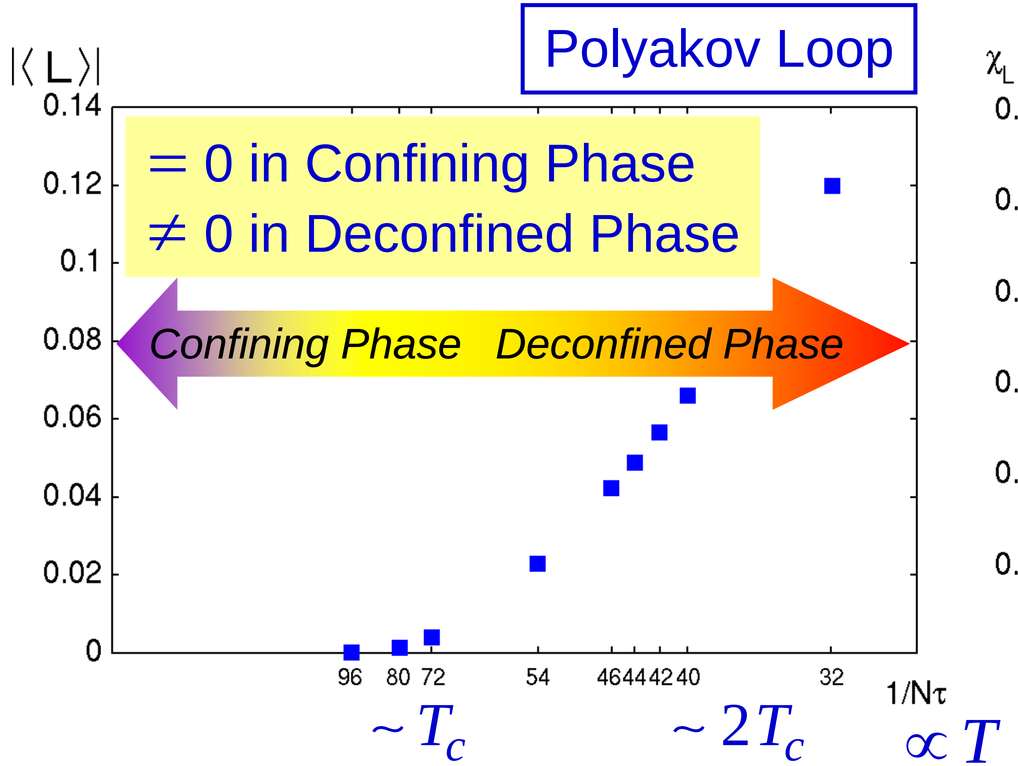
$N_\sigma = 32, \quad \beta = 7.0, \quad \xi = 4.0$

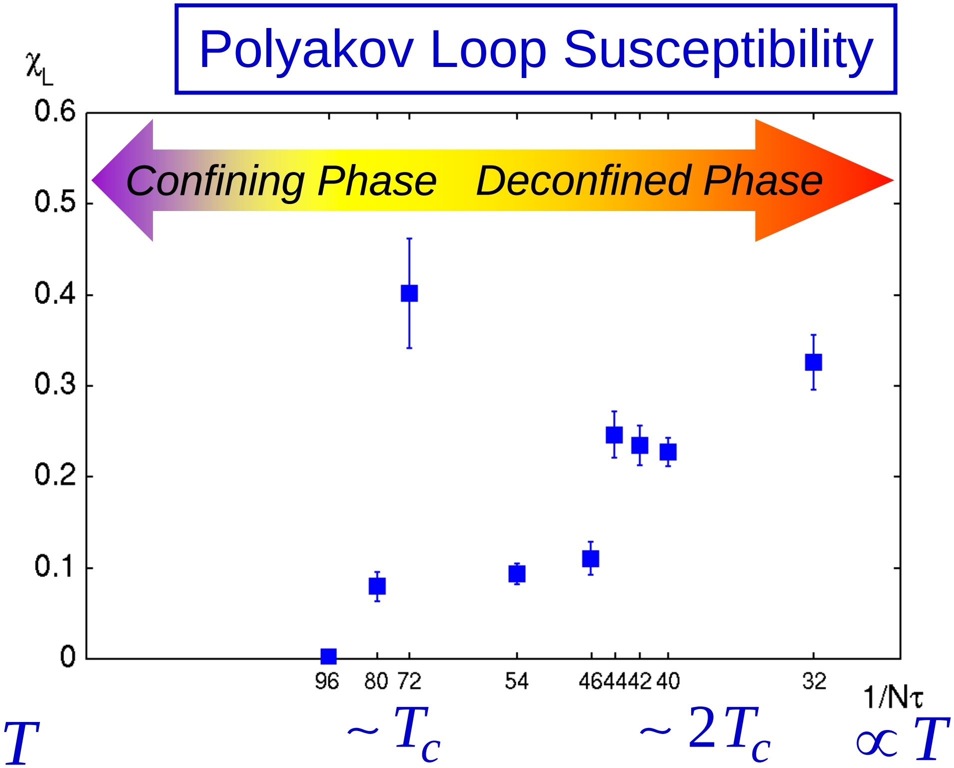
As of January 16, 2004



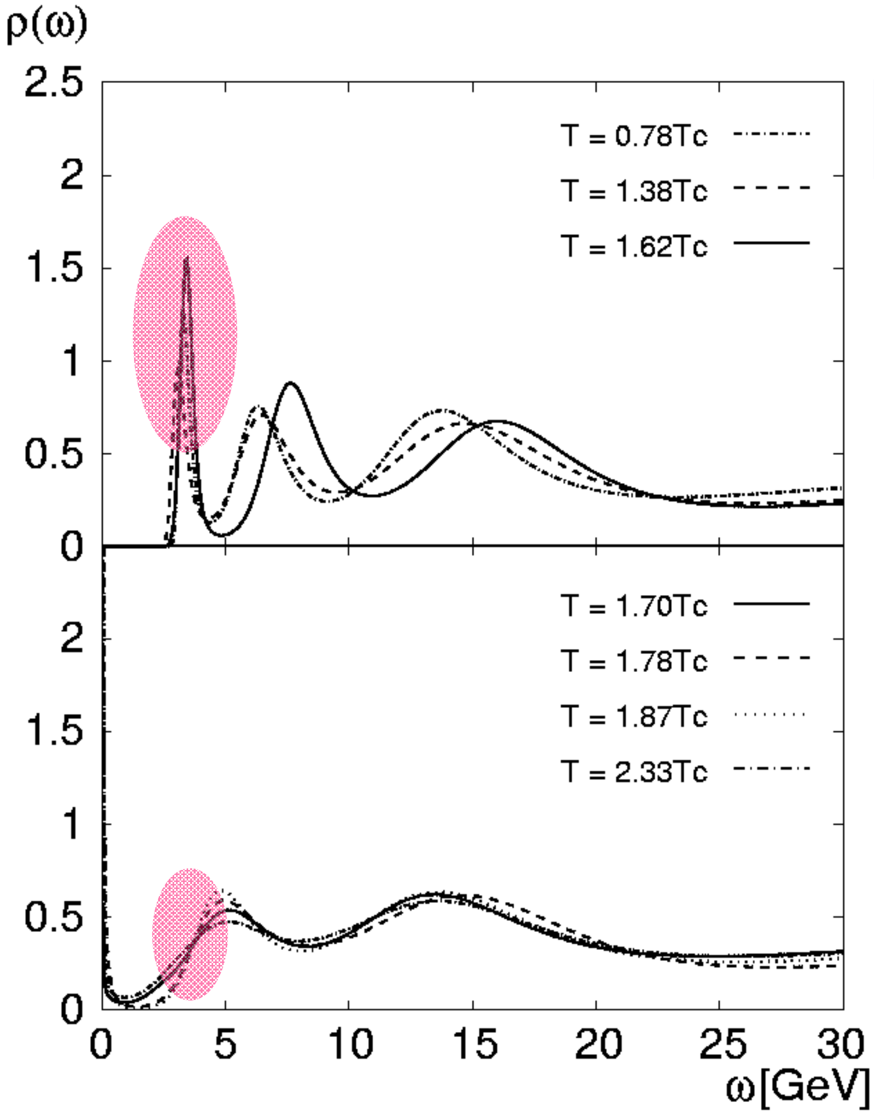
N_τ	32	40	42	44	46	54	72	80	96
T / T_c	2.33	1.87	1.78	1.70	1.62	1.38	1.04	0.93	0.78
# of Config.	141	181	180	180	182	150	150	110	194

Polyakov Loop and PL Susceptibility





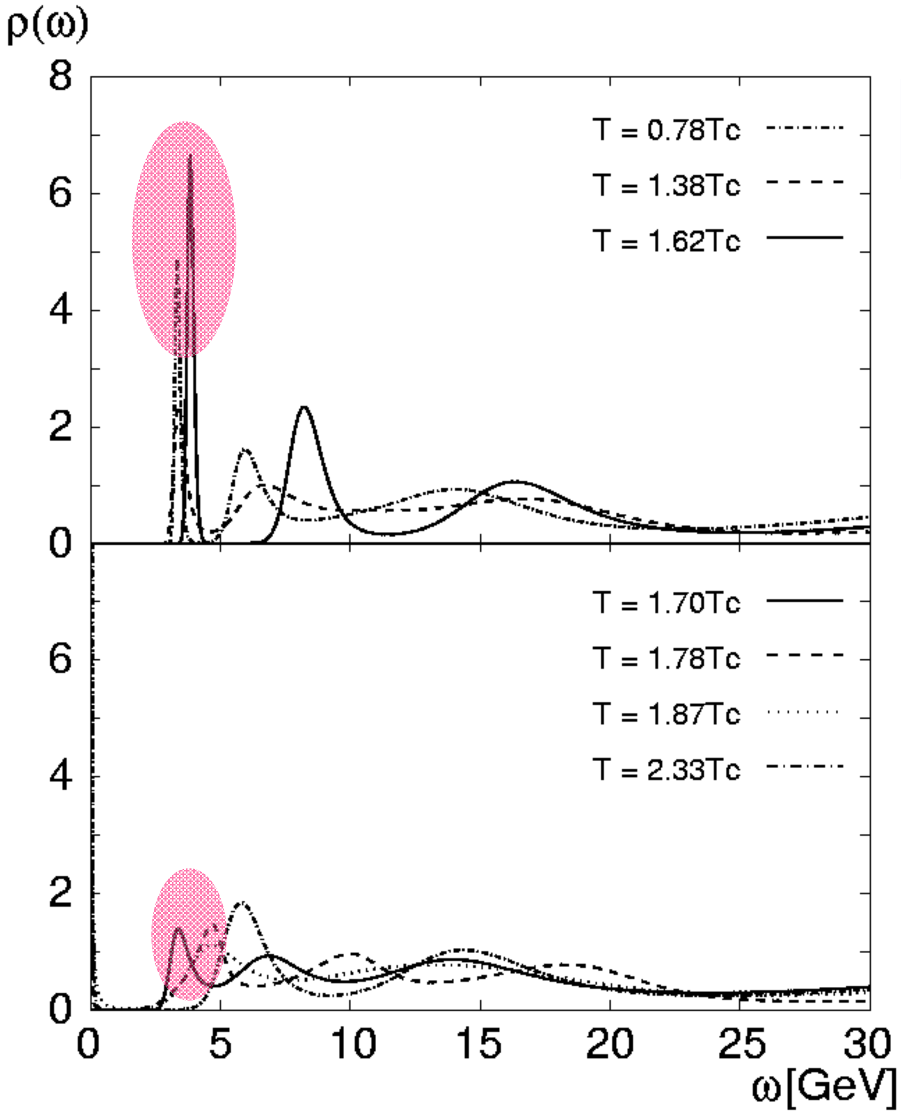
Result for V channel (J/ψ)



$$A(\omega) = \omega^2 \rho(\omega)$$

$J/\psi\,(\mathbf{p}=\mathbf{0})$ disappears between $1.62T_c$ and $1.70T_c$

Result for PS channel (η_c)

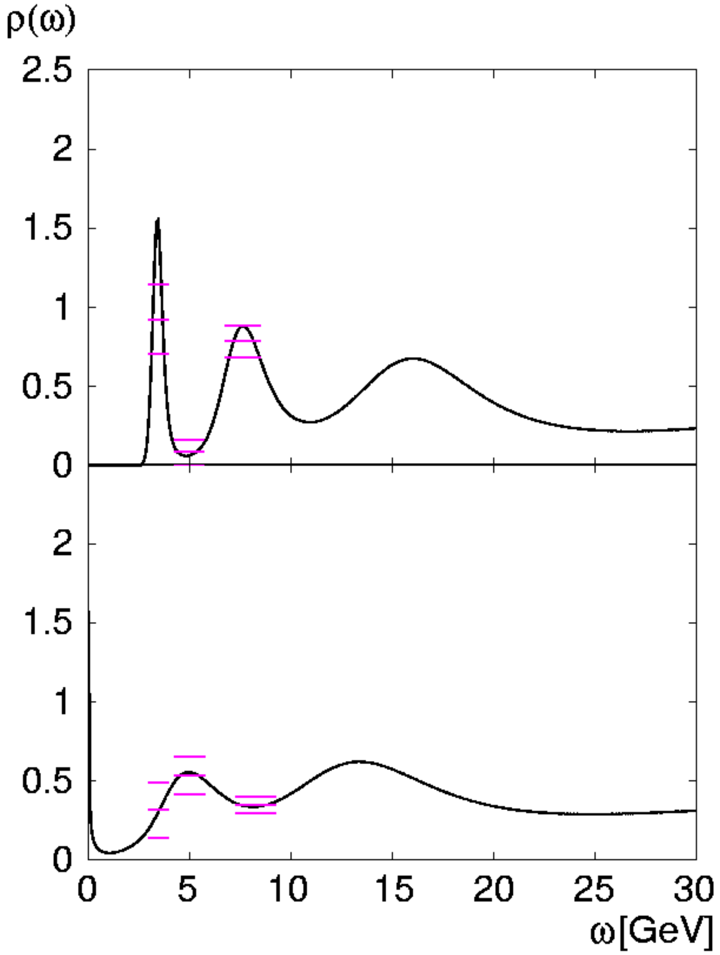


$$A(\omega) = \omega^2 \rho(\omega)$$

η_c ($\mathbf{p} = \mathbf{0}$) also disappears between $1.62T_c$ and $1.70T_c$

Statistical Significance Analysis for J/ψ

Statistical Significance Analysis = Statistical Error Putting

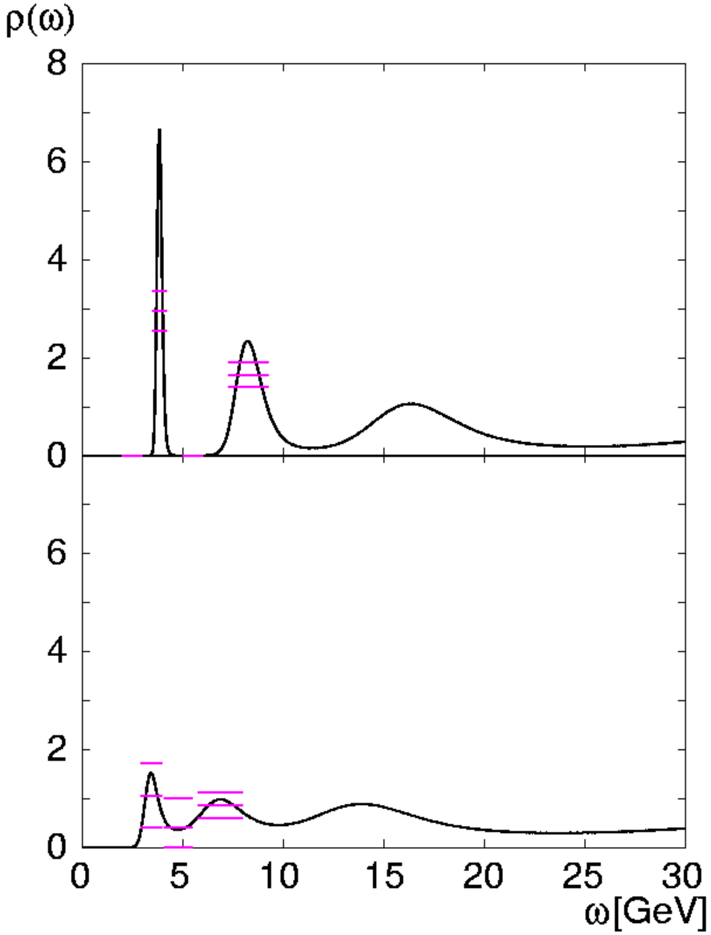


$T = 1.62T_\mathrm{c}$

$T = 1.70T_c$

Statistical Significance Analysis for η_c

Statistical Significance Analysis = Statistical Error Putting

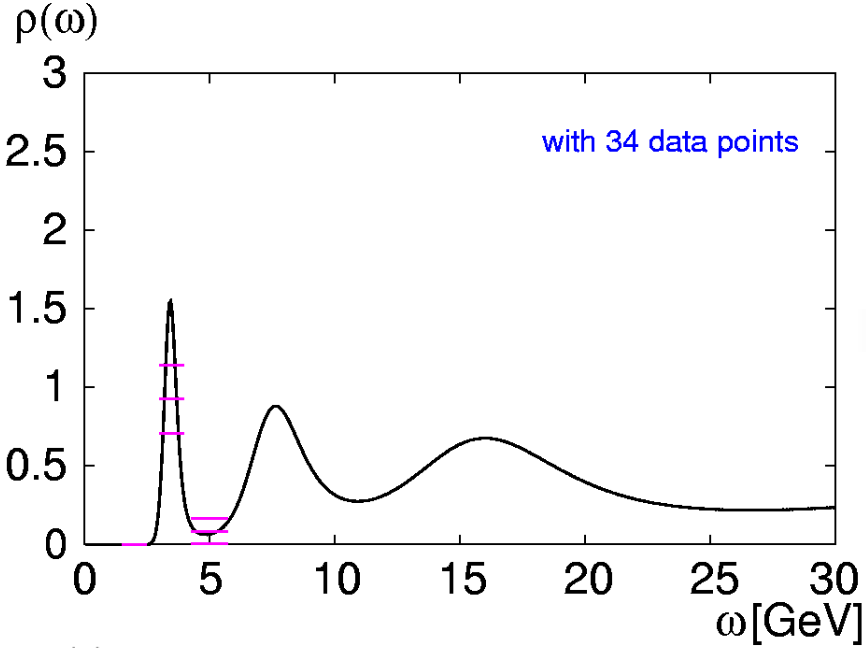


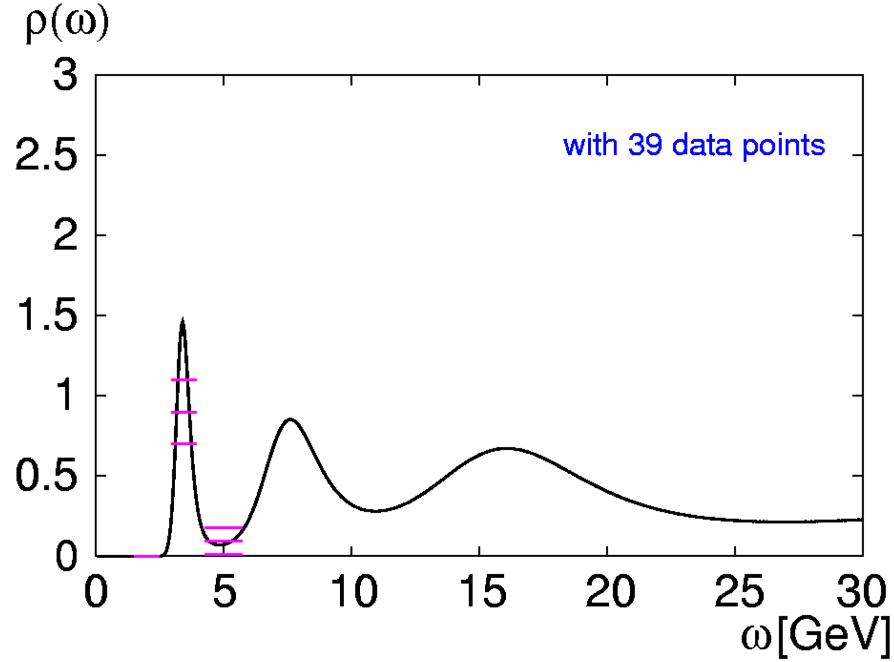
$T = 1.62T_c$

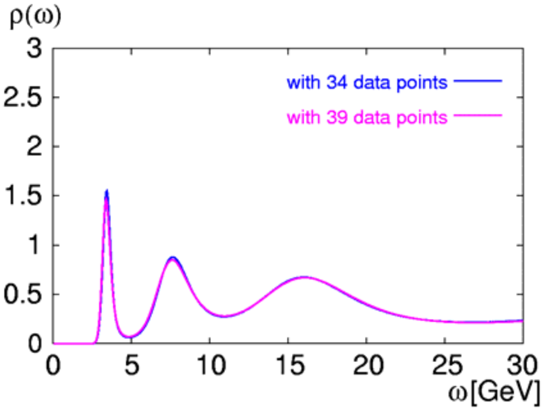
$T = 1.70T_c$

Dependence on Data Point Number (1)

Data Point # Dependence Analysis = Systematic Error Estimate



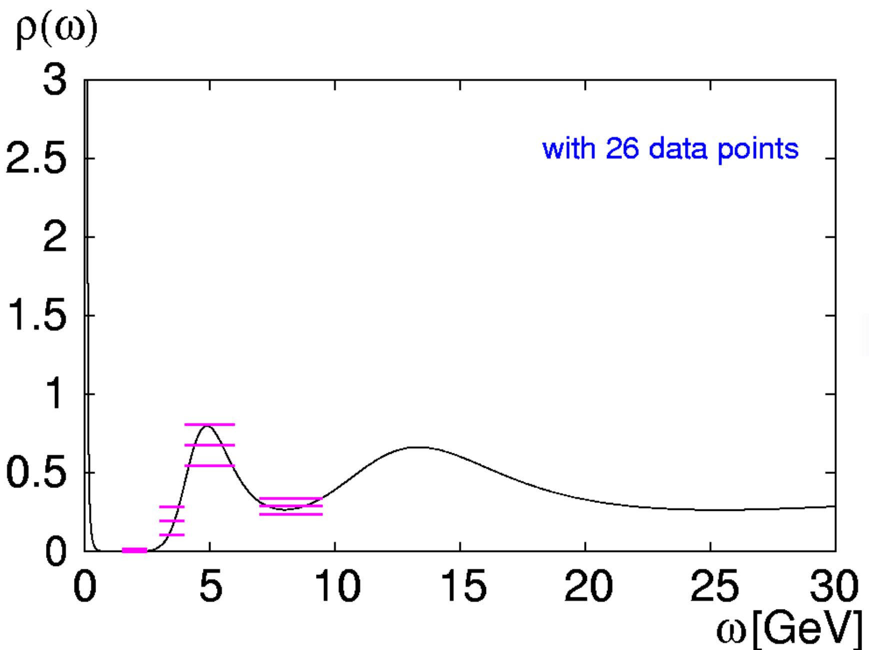


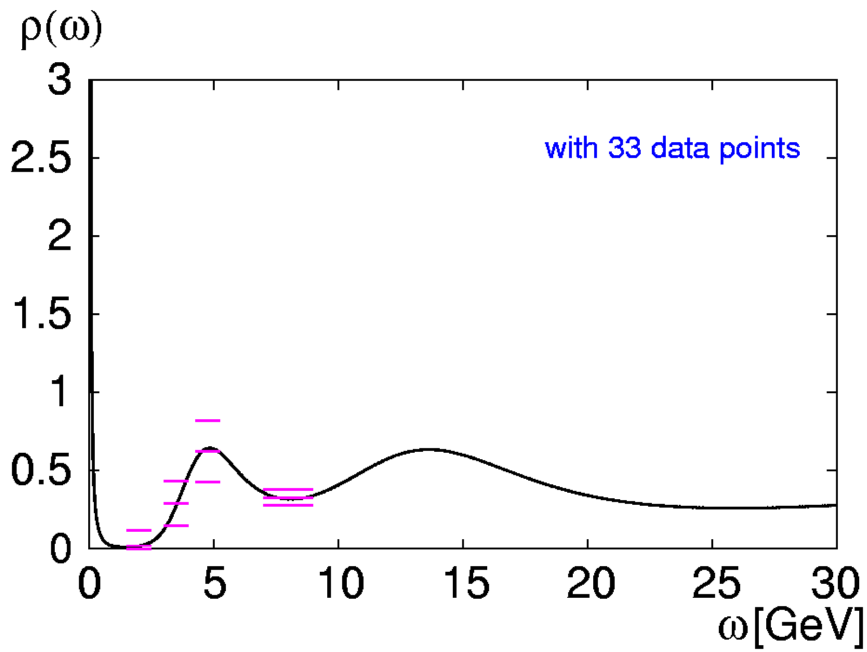


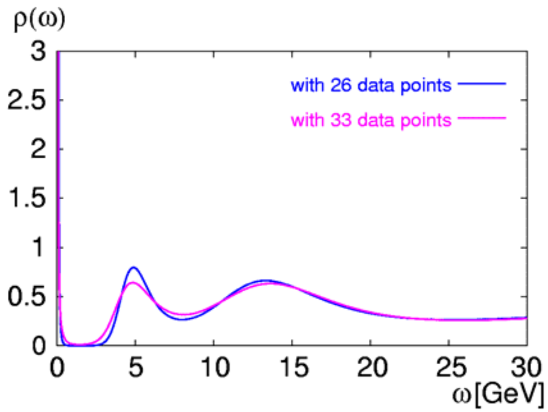
$N_\tau = 46$ ($T = 1.62T_c$)
V channel (J/ψ)

Dependence on Data Point Number (2)

Data Point # Dependence Analysis = Systematic Error Estimate







$N_\tau = 40$ ($T = 1.87T_c$)
V channel (J/ψ)

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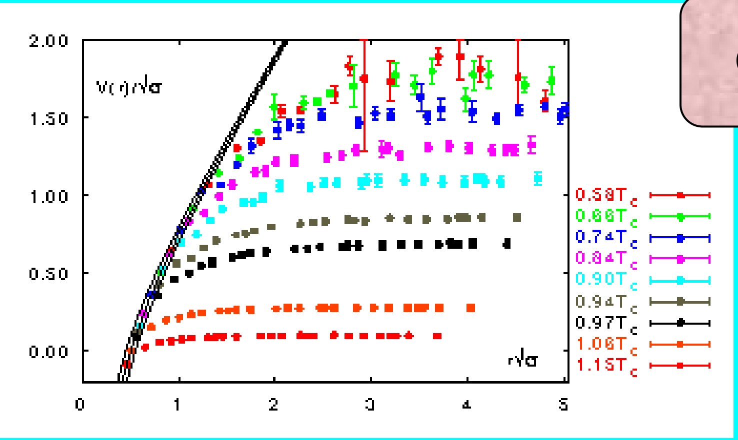
Debye Screening in QGP

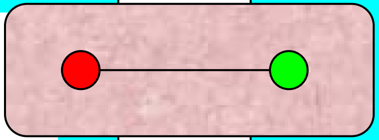
- Original Idea of *J*/ψ Suppression
  as a signature of QGP Formation: Debye Screening (Matsui & Satz, 1986)

Debye Screening of Potential between $c\bar{c}$

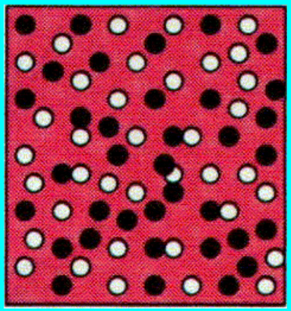


$$N_f = 3, \quad \frac{N_{PS}}{N_V} \sim 0.7$$

Karsch et al. (2000)



J/ψ Melting at $T \gtrsim 1.1 T_c$



Need to start over asking a question
“ What is QGP ? ” ?

Summary and Perspectives

- Spectral Functions in QGP Phase were obtained
  for heavy quark systems at $\mathbf{p} = \mathbf{0}$ on *large* lattices at several T
- Both *Statistical and Systematic Error Estimates*
  have been carefully carried out

- It seems J/ψ and η_c $(\mathbf{p} = \mathbf{0})$ remain in *QGP up to ~1.6*T_c
- *Sudden Qualitative Change between 1.62*T_c *and 1.70*T_c
- ~34 Data Points look sufficient to carry out MEM analysis
  on the present Lattice and with the current Statistics
  (This is Lattice and Statistics dependent)
- Physics behind is still unknown

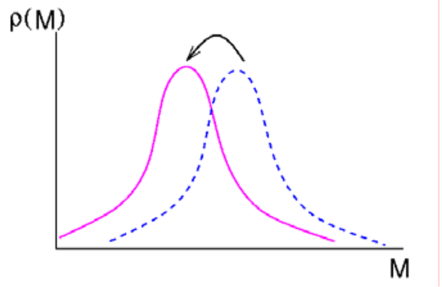
Further study needed
for better understanding of QGP
and Hadronic Modes in QGP !

Back Up Slides

Why Theoretically Unsettled

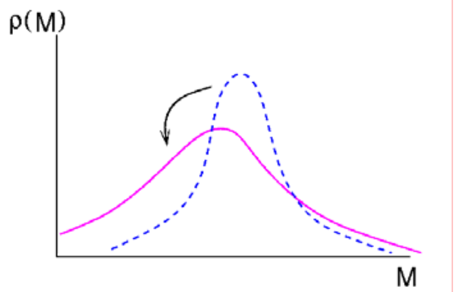
Mass Shift
(Partial Chiral Symmetry Restoration)

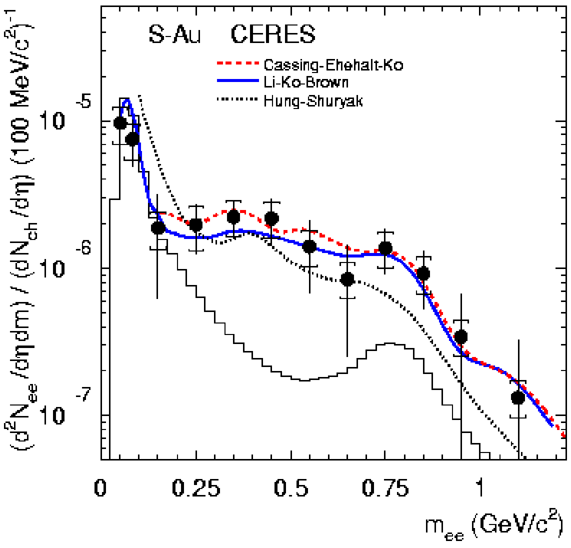
Spectrum Broadening
(Collisional Broadening)

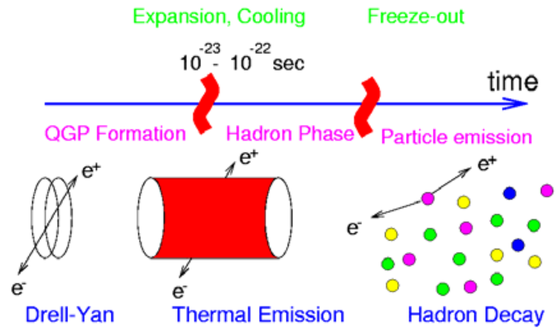


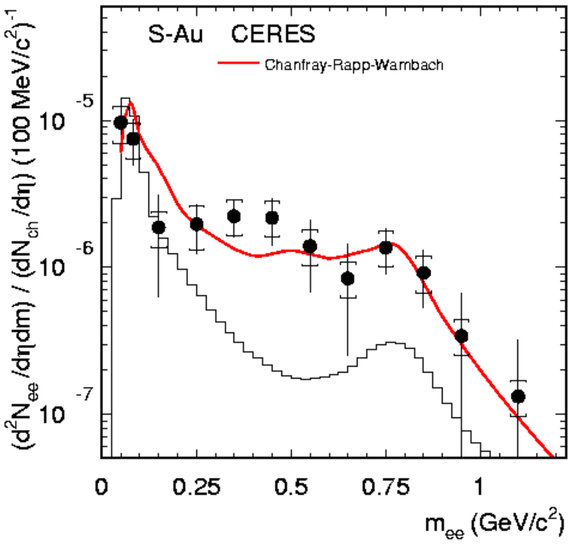
Observed Dileptons

Sum of All Contributions
(Hot and Cooler Phases)









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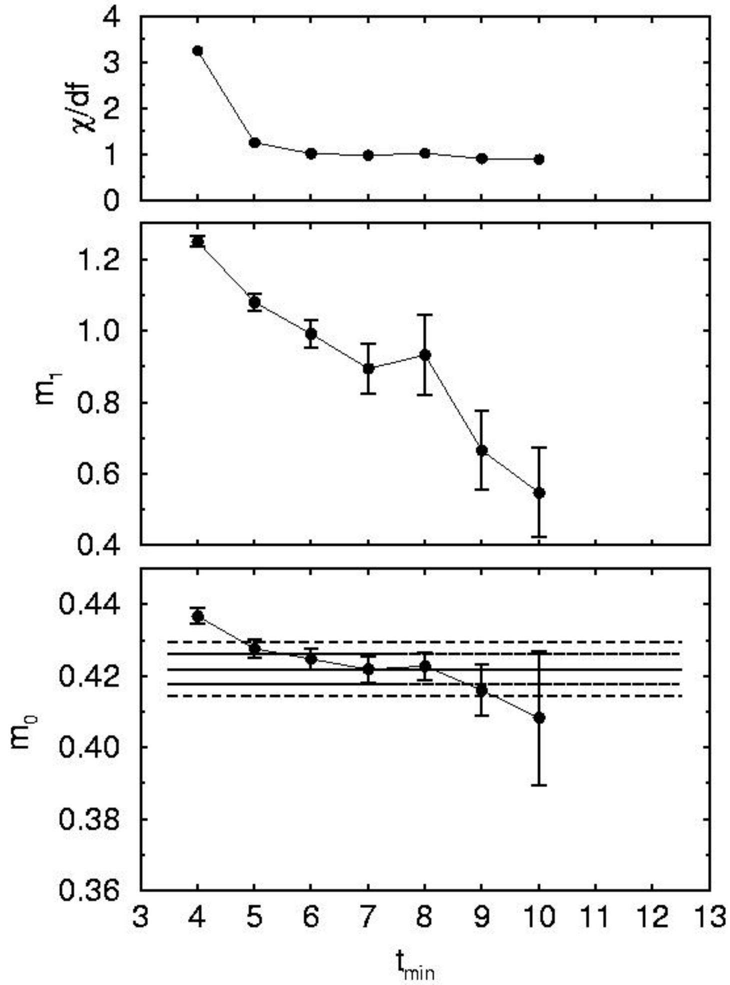
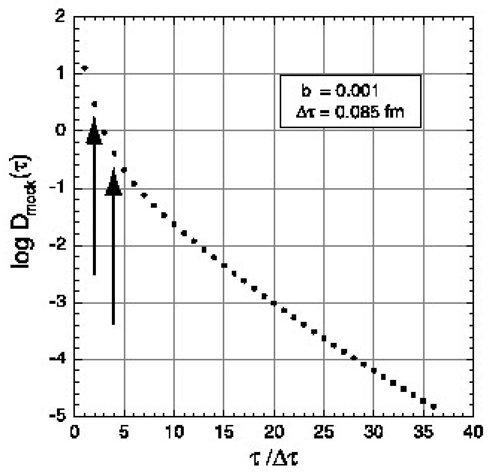
Way out ?

- Example of χ^2-fitting failure

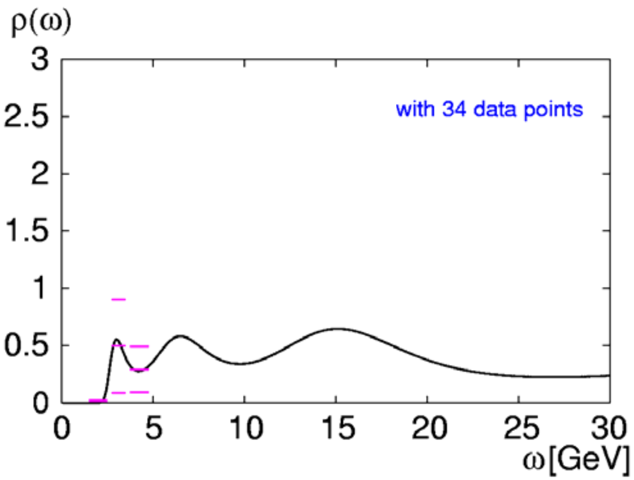


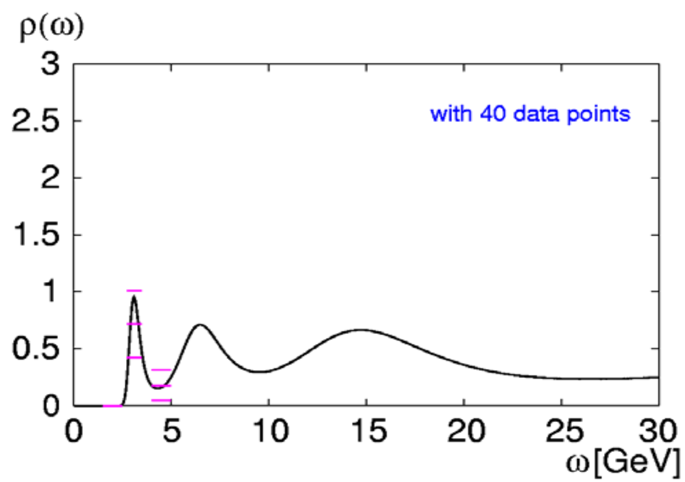
2 pole fit
by QCDPAX (1995)

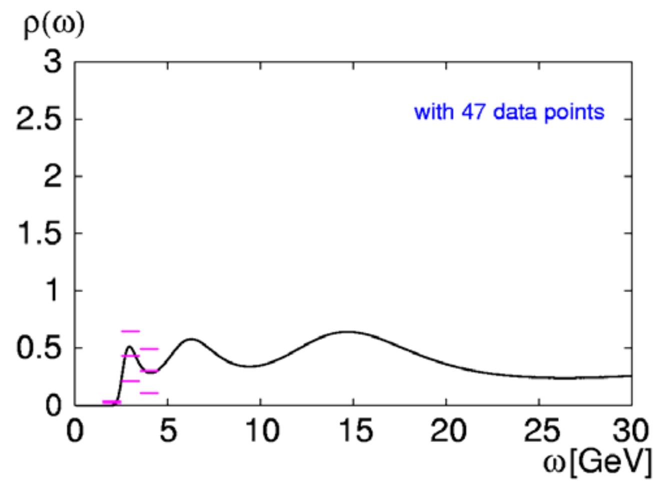
$\beta = 6.0$
$24^3 \times 54$ lattice



Dependence on Data Point Number



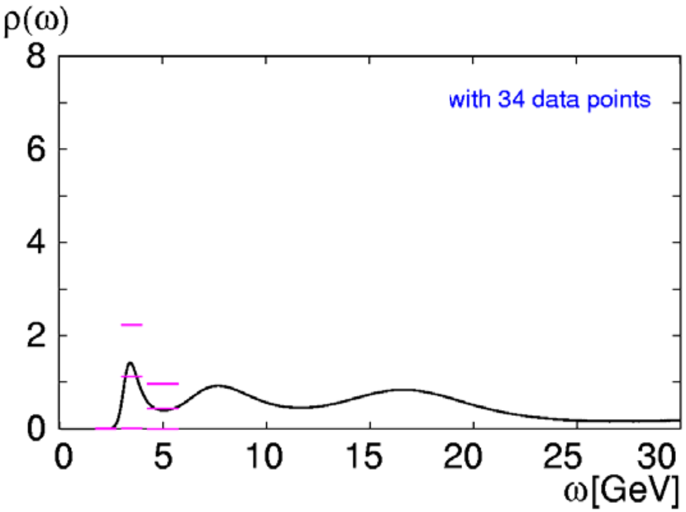


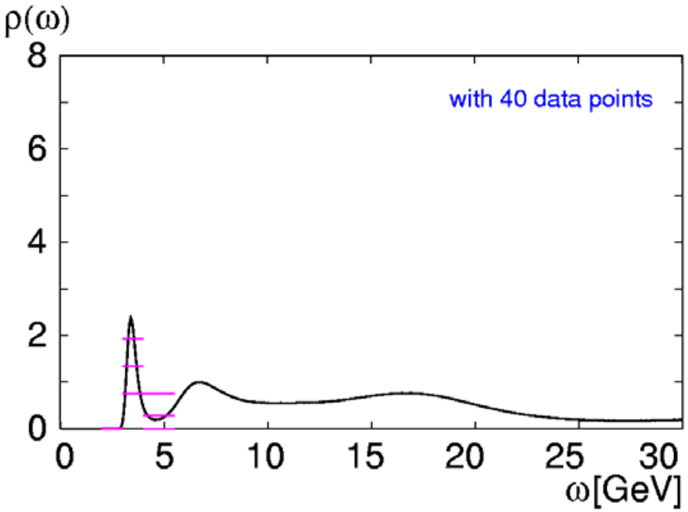


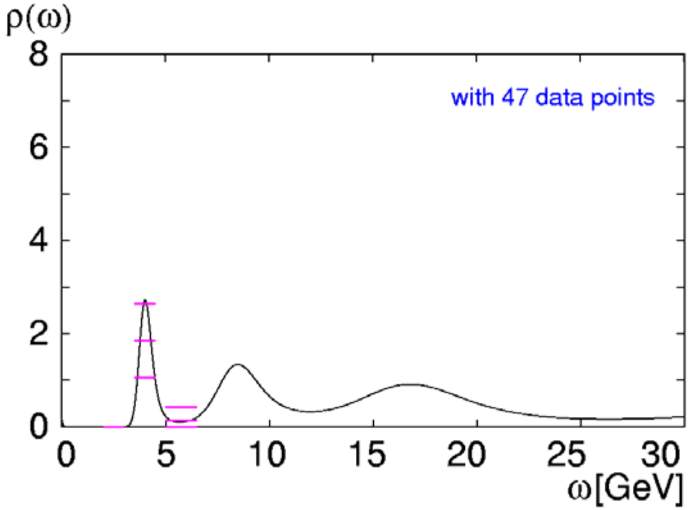
$N_\tau = 54$ ($T = 1.38 T_c$)
V channel (J/ψ)

M. Asakawa (Kyoto University)

Dependence on Data Point Number







$N_\tau = 54$ ($T = 1.38T_c$)
PS channel (η_c)

Dependence on Data Point Number

$N_\tau = 46$ ($T = 1.62T_c$)
PS channel (η_c)