Traces of Thermalization at RHIC

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onset of thermal equilibration – common centrality dependence of $\langle p_t \rangle$, p_t and charge dynamic fluctuations

I. Fluctuations from nonequilibrium two-body correlations

- multiplicity and p_t fluctuation observables
- approach to equilibrium
- initial and near-equilibrium fluctuations

II. Experiments

- PHENIX and STAR mean p_t and fluctuations
- energy dependence, net charge fluctuations

nucl-th/0308067

Dynamic Fluctuations

variance minus thermal contribution

Pruneau, Voloshin & S.G.

multiplicity N

$$R_{AA} = \frac{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle}{\langle N \rangle^2}$$

mean p_t

$$\boxed{\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle}$$

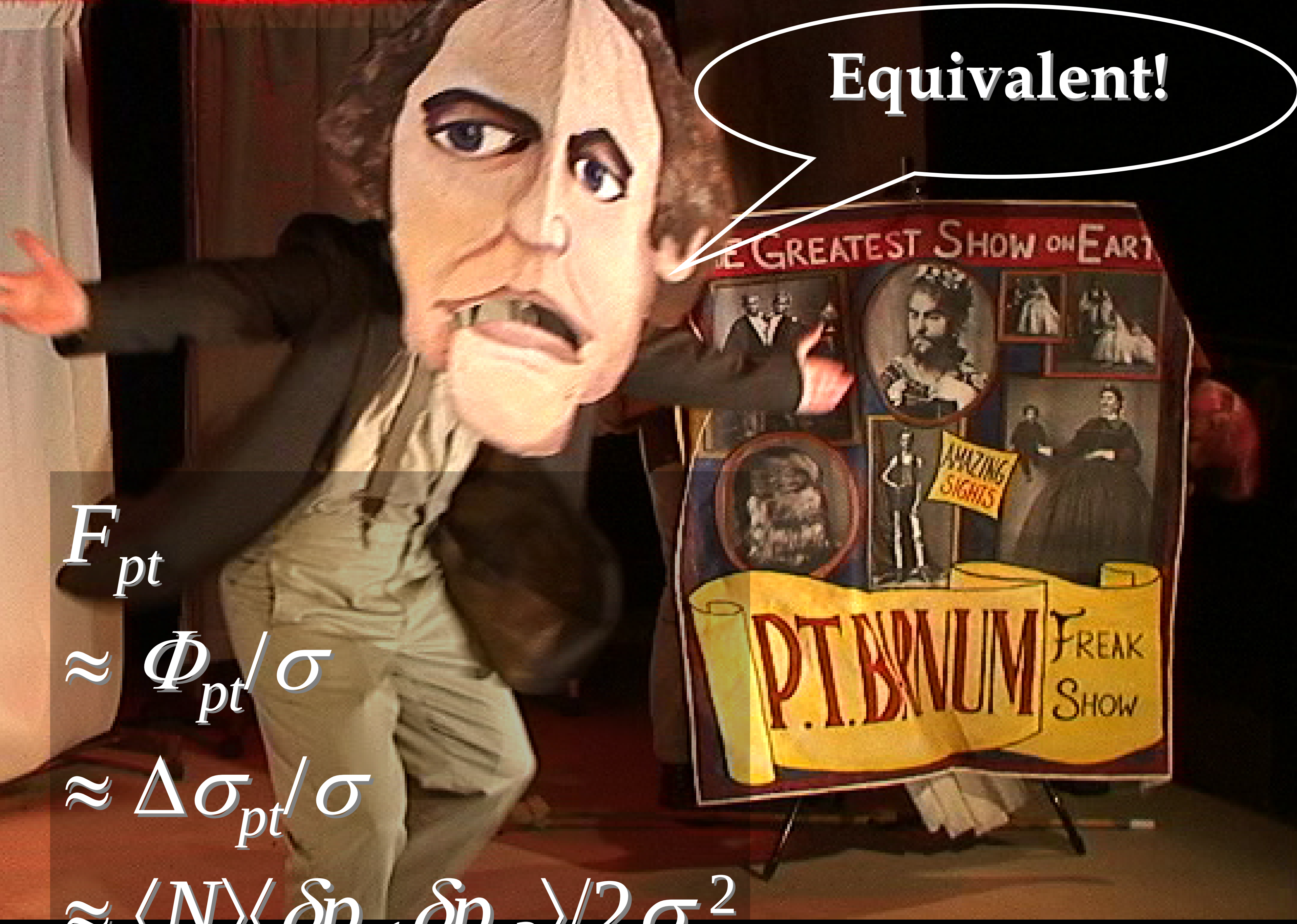
$$\delta p_t \equiv p_t - \langle p_t \rangle$$

probe two-body correlation function:

$$r(p_1, p_2) = \frac{dN}{dp_1 dp_2} - \frac{dN}{dp_1} \frac{dN}{dp_2}$$

$$R_{AA} \propto \iint r(p_1, p_2)$$

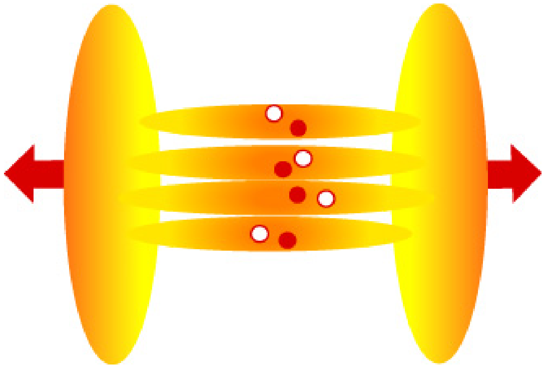
$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \iint \delta p_{t1} \delta p_{t2}\, r(p_1, p_2)$$



$$F_{pt} \approx \Phi_{pt}/\sigma \approx \Delta\sigma_{pt}/\sigma \approx \langle N\rangle\langle \delta p_{t1}\delta p_{t2}\rangle/2\sigma^2$$

Time Scales

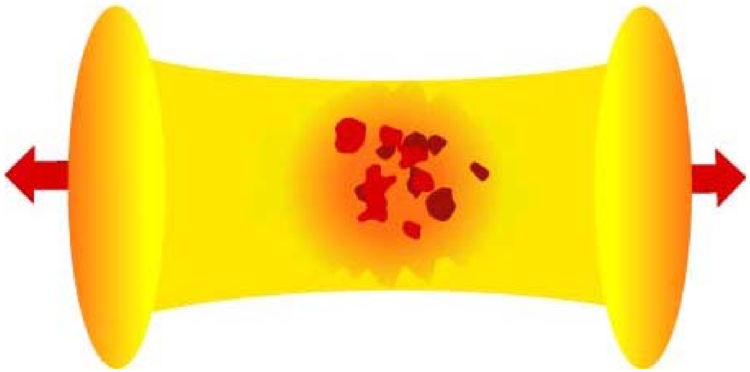
initially – string fragmentation



later – clumps, size ~ ξ

- local thermalization; time ~ ν^{-1}

  scattering rate $\nu = \langle\sigma\, v_{rel}\rangle n$

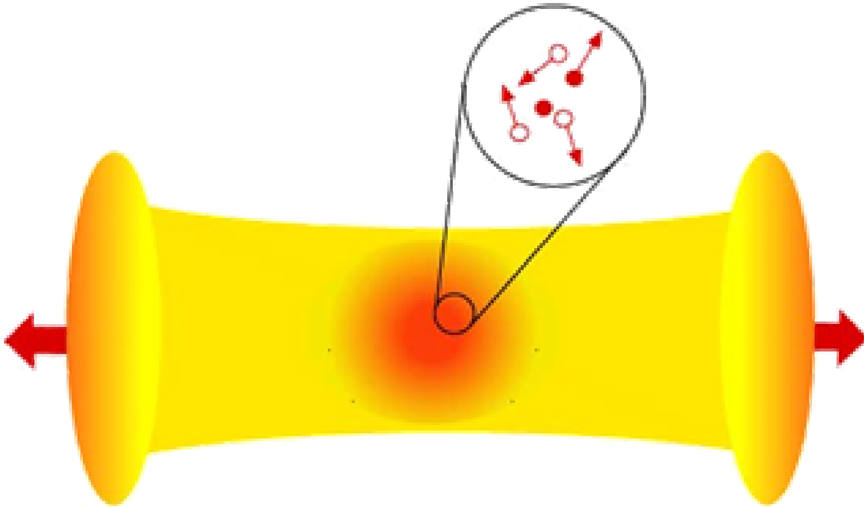


much later (if ever)

- flow between clumps →

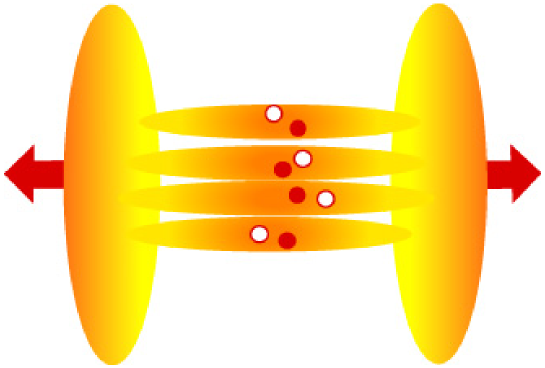
  homogeneity

- diffusion time $t_{diff} \sim \nu\xi^2$



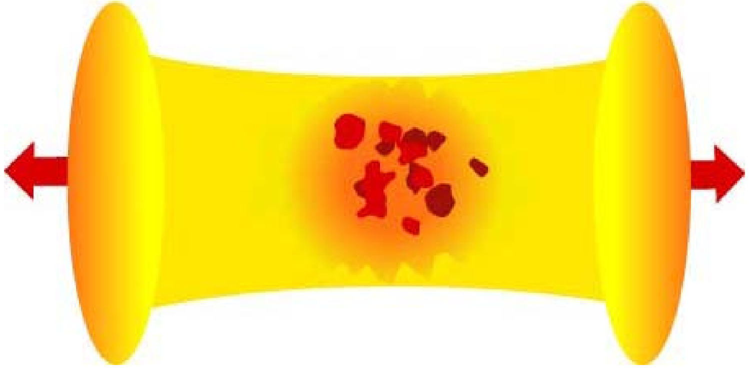
Fluctuation Sources

dynamic fluctutations ~ (strings)$^{-1}$



dynamic fluctutations ~ (clumps)$^{-1}$

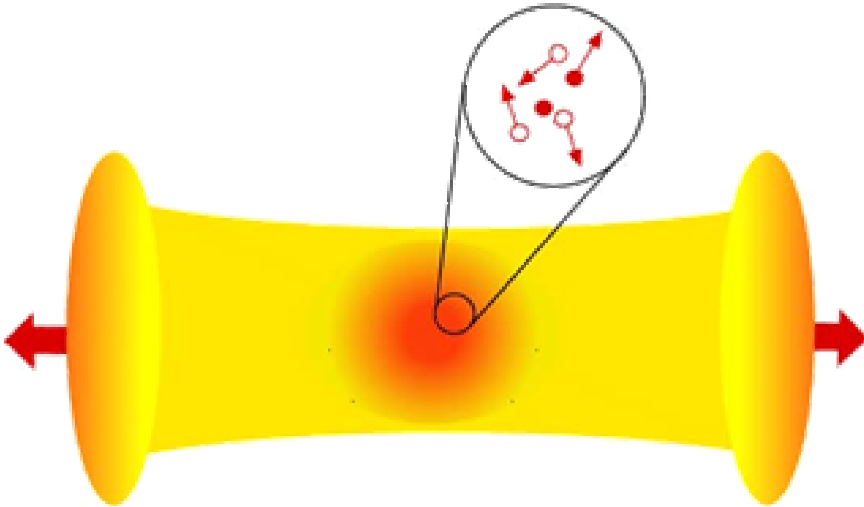
-- larger? depends on clump size



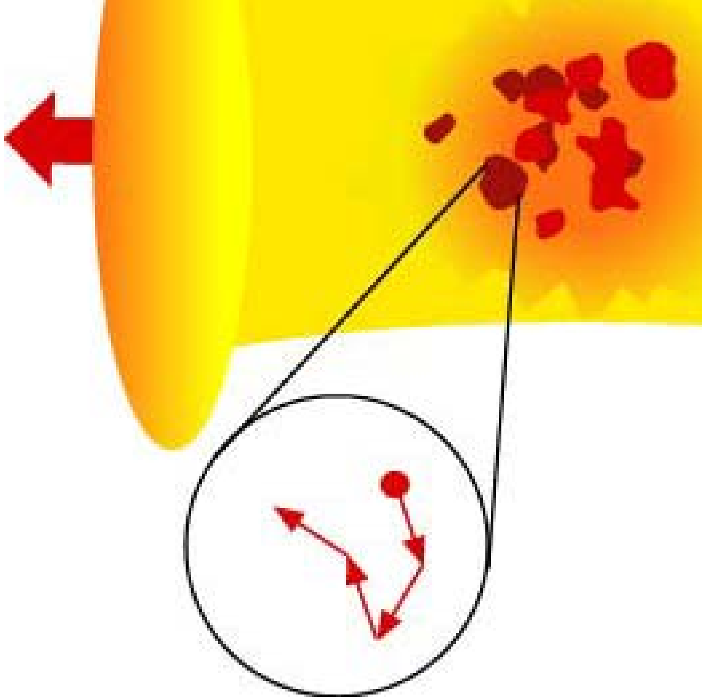
Gazdzicki & Mrowczynski

statistical fluctuations ~ (particles)$^{-1}$

dynamic fluctuations **vanish**



Partial Thermalization



random walk – n collisions

$$\langle p_t^{\,2} \rangle = \langle p_t^{\,2} \rangle_0 + \kappa (n-1)$$

collision: many walkers bouncing off each other – energy conservation limits p_t increase

Boltzmann equation $\Rightarrow$ $$\langle p_t^{\,2} \rangle = \langle p_t^{\,2} \rangle_0 S + \langle p_t^{\,2} \rangle_e (1-S)$$

survival probability: $$S = e^{-\int \nu \, d\tau} \approx (\tau_0 / \tau_F)^\alpha$$

few collisions $\langle p_t^{\,2} \rangle_e - \langle p_t^{\,2} \rangle_0 \sim \kappa$ many collisions – saturates at $\langle p_t^{\,2} \rangle_e$

Nonequilibrium $\langle p_t \rangle$

central collisions: more participants N

longer lifetime $\Rightarrow$ smaller survival probability S

$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1-S)$$

Boltzmann equation +
longitudinal expansion

- survival probability

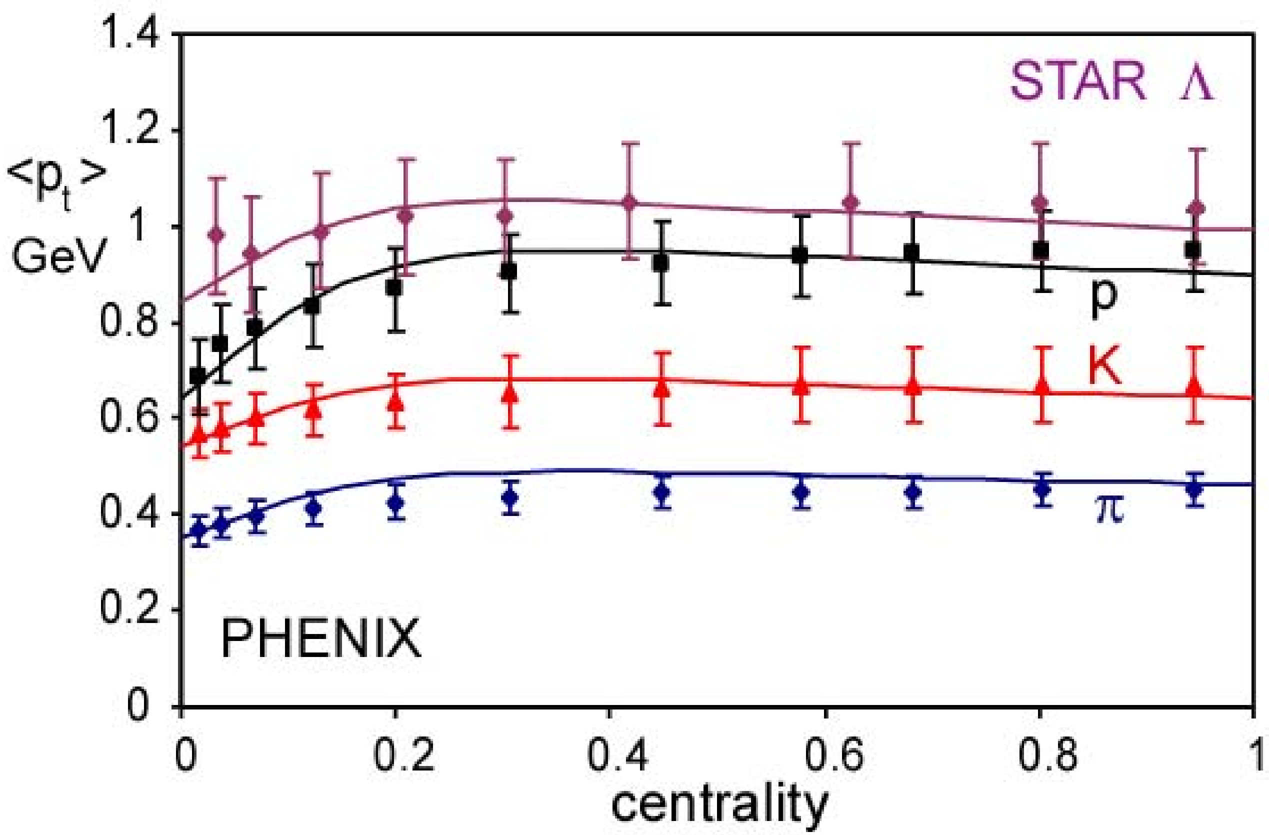
  $S = (t_0 / t_F)^{\alpha}$

- equilibrium cooling

  $\langle p_t \rangle_e \propto (t_0 / t_F)^{\gamma}$

- centrality dependence

  $t_F \propto N^x, \alpha \propto N^y$



Nonequilibrium Dynamic Fluctuations

Boltzmann equation with Langevin noise $\Rightarrow$ phase-space correlations $\Rightarrow$ dynamic fluctuations

simple limits:

independent initial and near-local equilibrium correlations

$$\langle \delta p_{t1}\delta p_{t2}\rangle = \langle \delta p_{t1}\delta p_{t2}\rangle_0 S^2 + \langle \delta p_{t1}\delta p_{t2}\rangle_{le}(1-S)^2$$

initial correlations equilibrium-like

$$\langle \delta p_{t1}\delta p_{t2}\rangle = \langle \delta p_{t1}\delta p_{t2}\rangle_0 S^2 + \langle \delta p_{t1}\delta p_{t2}\rangle_{le}\left(1-S^2\right)$$

same centrality dependence of S

Initial and Near-Local Equilibrium Fluctuations

initial fluctuations:

M participant nucleons

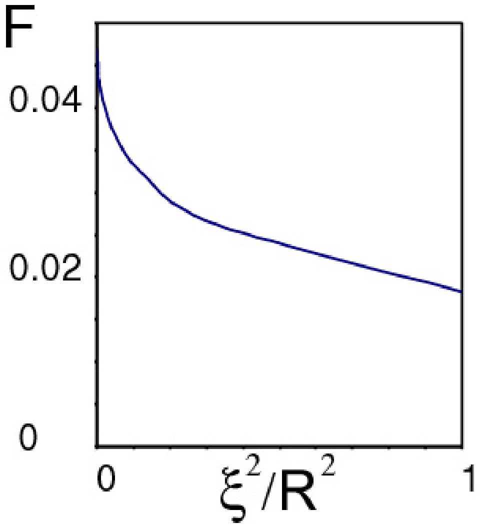
$\rightarrow$ independent strings

$$\langle \delta p_{t1} \delta p_{t2} \rangle \approx \frac{2\langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{M} \left(\frac{1+R_{pp}}{1+R_{AA}} \right)$$

near equilibrium: correlations from clumpiness – more likely to find particles near "hot spots" $\rightarrow$ spatial correlation function $r(x_1,x_2)$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \iint \delta \overline{p}_{t1}(x_1) \delta \overline{p}_{t2}(x_2)\, r(x_1, x_2)\, dx_1 dx_2$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle_e \approx \frac{\langle p_t \rangle^2 R_{AA}}{1+R_{AA}} F(\xi / R_t)$$



transverse size R_t, correlation length ξ

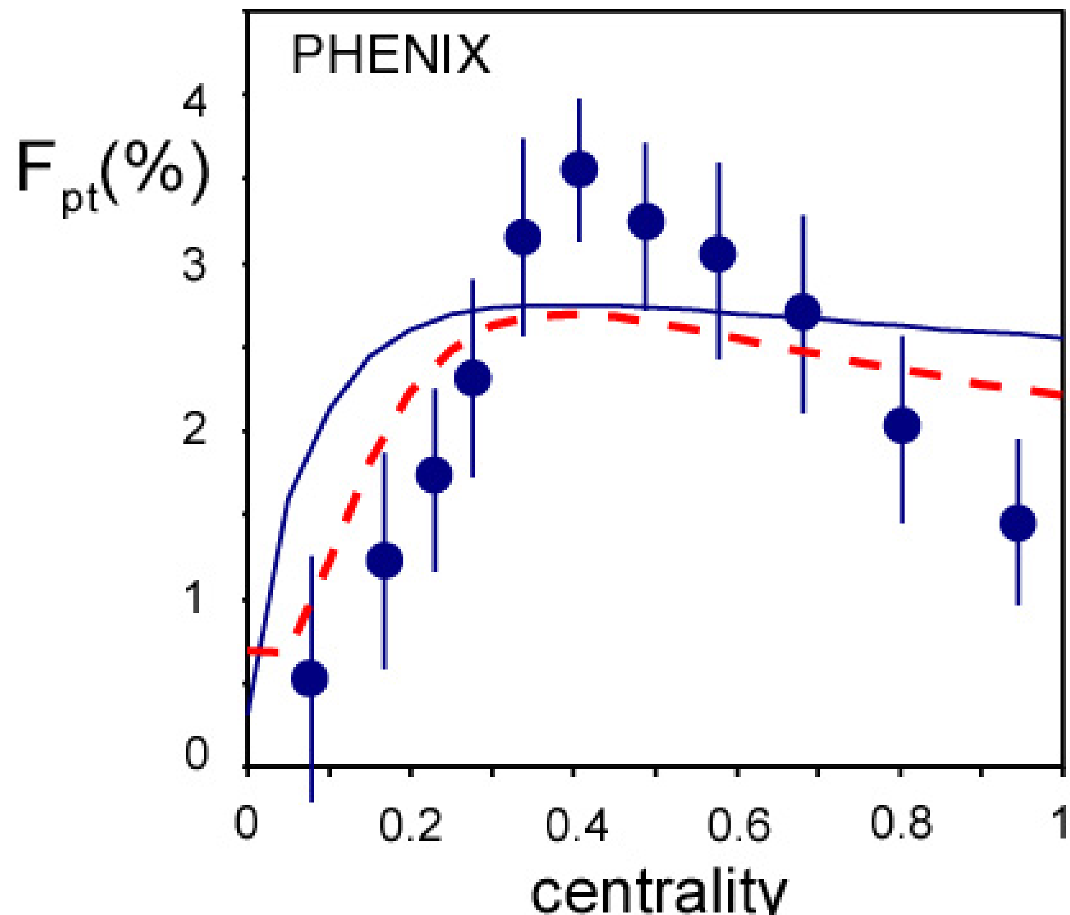
Nonequilibrium Fluctuations

$$\left\langle \delta p_{t1} \delta p_{t2} \right\rangle = \left\langle \delta p_{t1} \delta p_{t2} \right\rangle_0 S^2 + \left\langle \delta p_{t1} \delta p_{t2} \right\rangle_{le} \left(1 - S^2\right)$$

initial fluctuations –
wounded nucleons

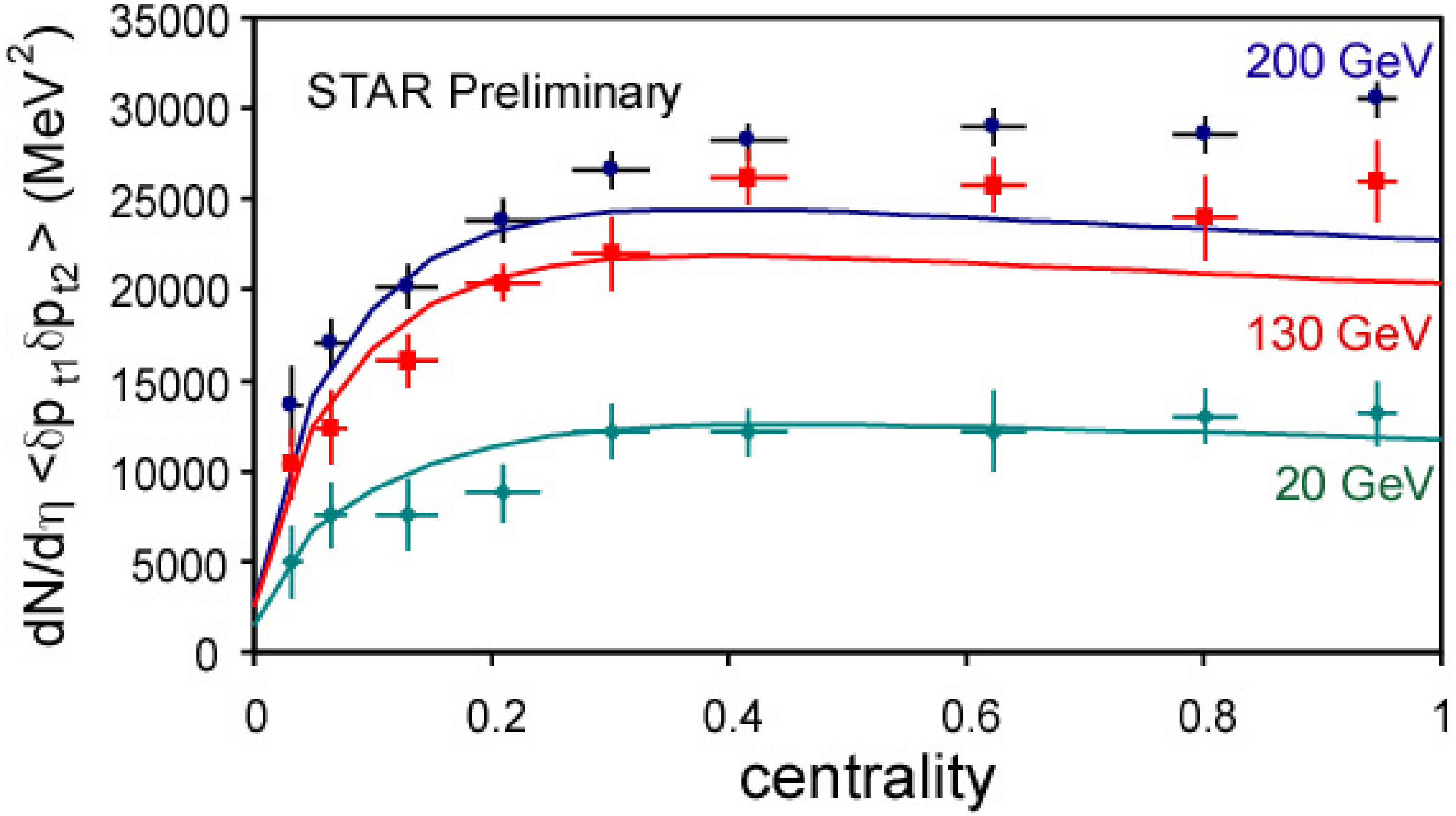
near equilibrium
correlation length
$\xi \sim 1$ fm; $R \propto N^{1/2}$

survival probability
centrality dependence
from $\langle p_t \rangle$ data



Energy Dependence

multiplicity, scattering rate $\alpha \propto dN/d\eta$, $R_{AA} \propto (dN/d\eta)^{-1}$



FIND: partial thermalization describes trends.

Deviation in central collisions at highest energies – jets?

Similar Charge Fluctuations

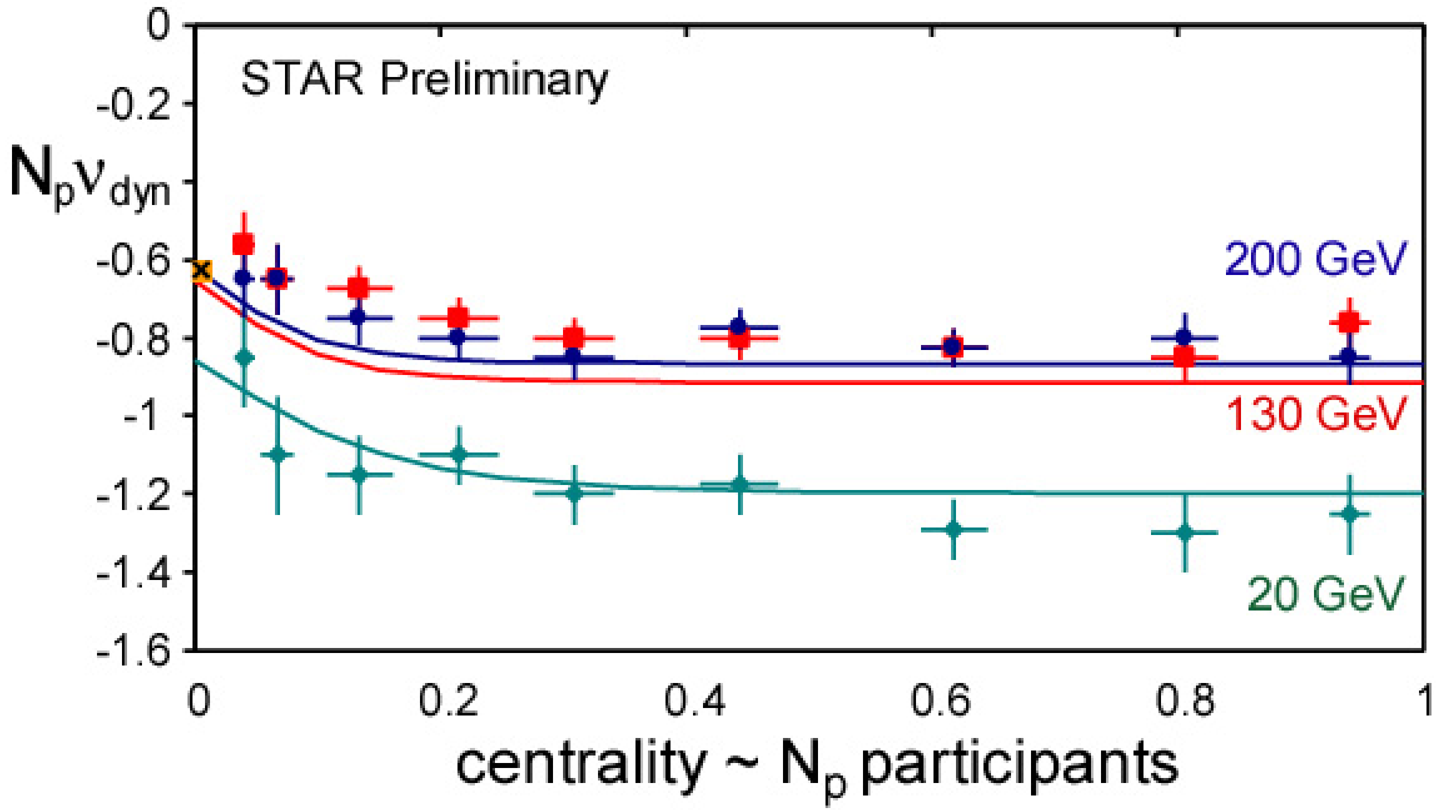
expect same charge fluctuations → correlations $r(p_1,p_2)$

$$\nu_{dyn} = R_{++} + R_{--} - 2R_{+-} \longrightarrow \nu_0 S^2 + \nu_{le}\left(1 - S^2\right)$$

rapidity correlation length

- $\Delta\eta_{rms} \sim 0.5$ in *AA*
- $\Delta\eta_{rms} \sim 1$ in *pp*

consistent with balance function measurements



Fluctuations: not just for Exotic Phenomena!

$\langle p_t \rangle$, p_t and charge fluctuations

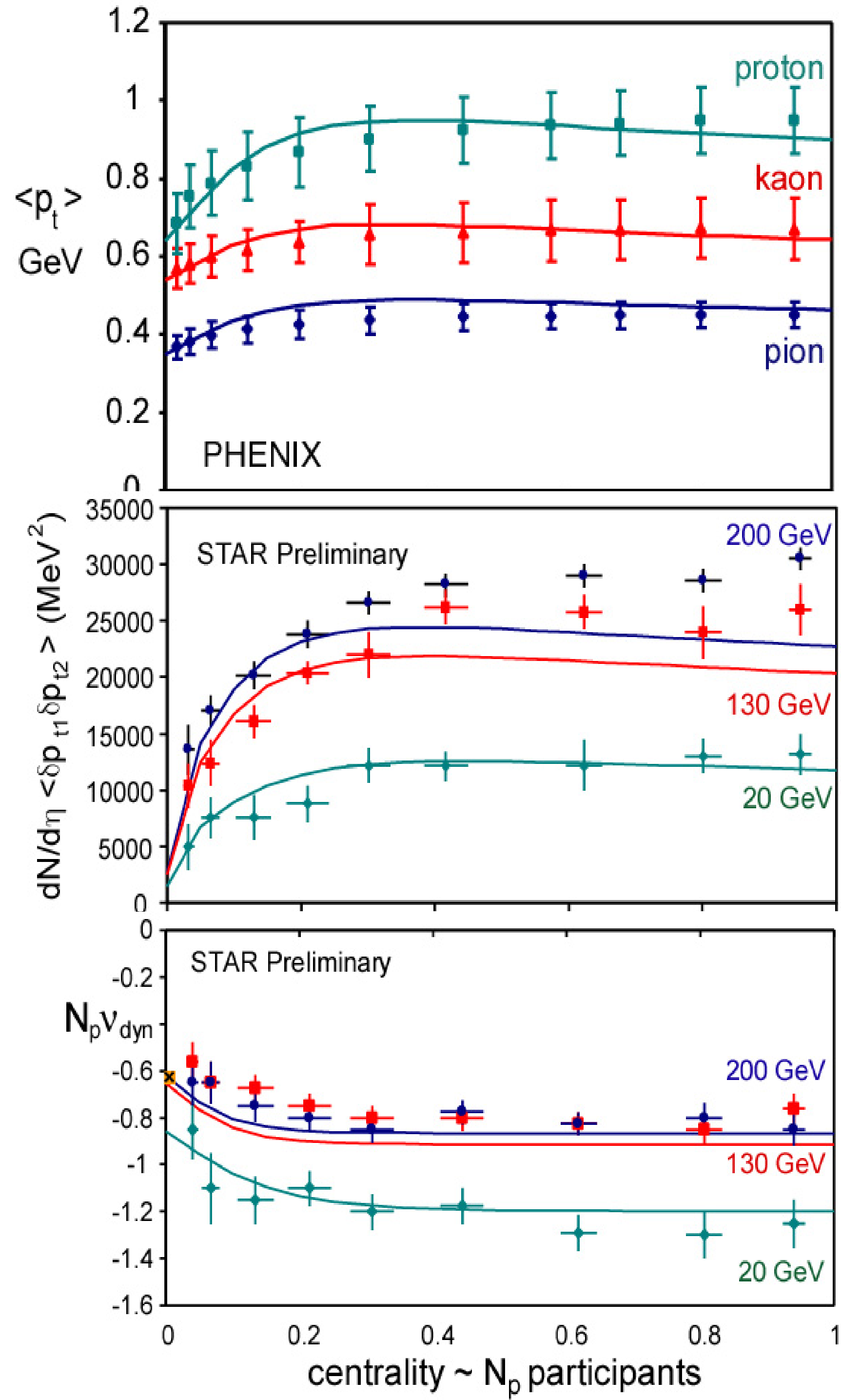
- common behavior – peripheral
- but: alternative indivdual explanations

central collisions?

- cooling? PHENIX: yes, STAR: no
- jet contribution to fluctuations?

partons or hadrons?

- species independence for $\langle p_t \rangle$
- **need particle-identified fluctuations**



Summary: Partial Thermalization

$\langle p_t \rangle$, p_t and charge fluctuations

- common behavior – peripheral
- but: alternative indivdual explanations

central collisions?

- cooling? PHENIX: yes, STAR: no
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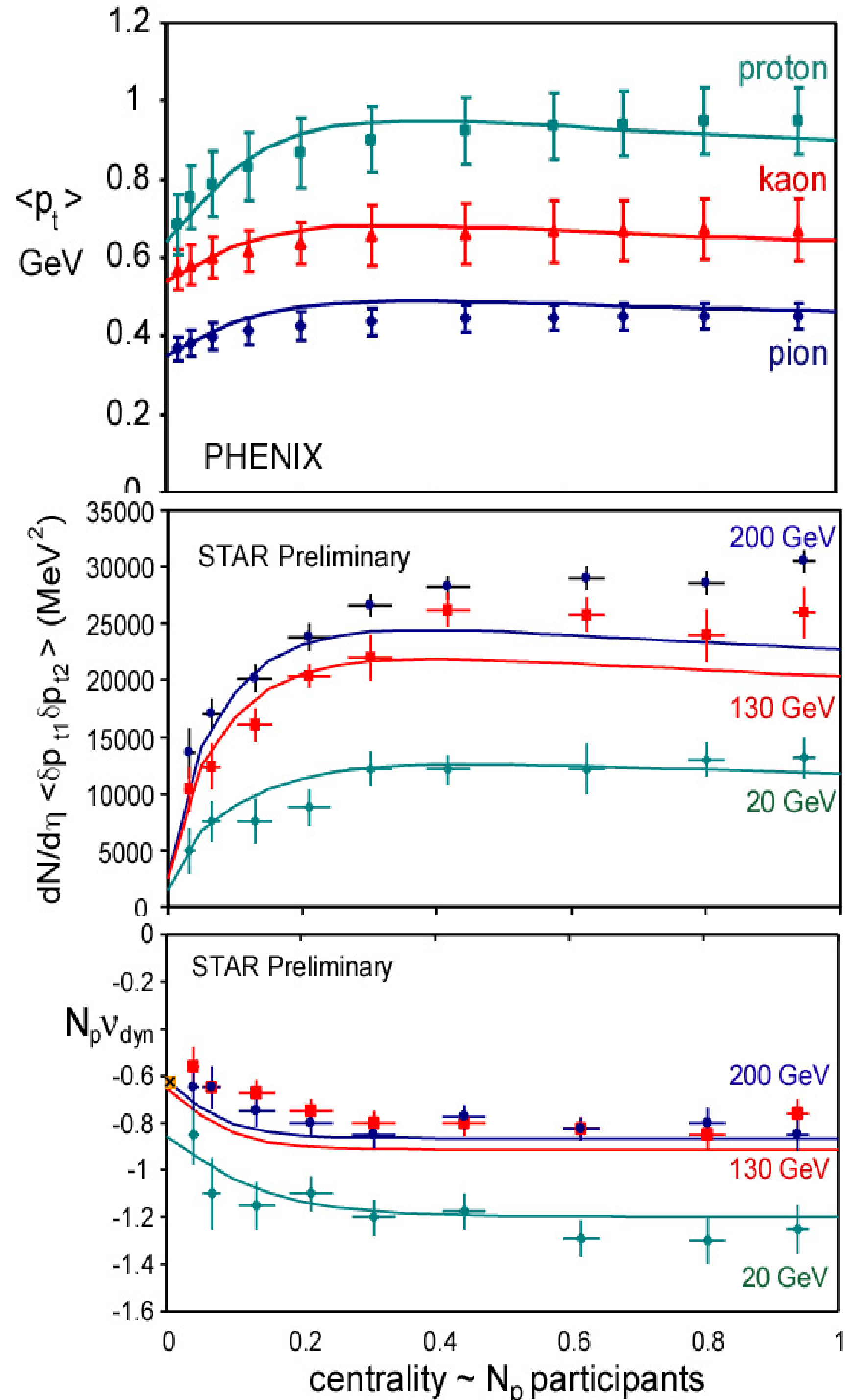
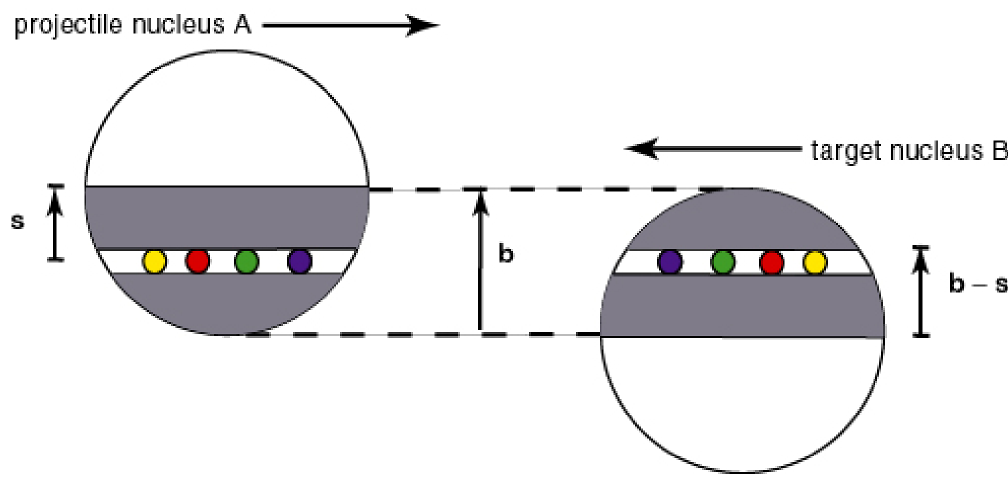


Initial Fluctuations

AA collision:

M participant nucleons

$\rightarrow$ independent strings



multiplicity $\propto M$

variance

$R_{AA} \propto M^{-1}$

dynamic p_t fluctuations

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

$$\langle N(N-1) \rangle = \langle N \rangle^2 (1 + R_{AA})$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \approx \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{M} \left(\frac{1 + R_{pp}}{1 + R_{AA}} \right)$$

Fluctuations Near Equilibrium

correlations from non-uniformity – more likely to find particles near "hot spots" $\rightarrow$ spatial correlation function

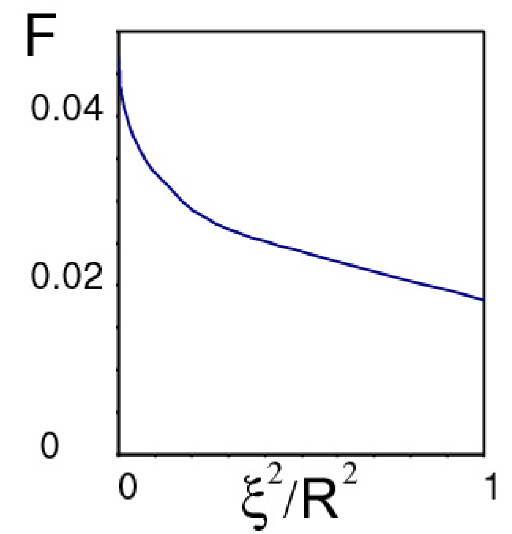
$$\left\langle \delta p_{t1} \delta p_{t2} \right\rangle \propto \iint \delta \overline{p}_{t1}(x_1) \delta \overline{p}_{t2}(x_2)\, r\left(x_1, x_2\right) dx_1 dx_2$$

assume:

- longitudinal Bjorken expansion
- uncorrelated longitudinal and transverse d.o.f.'s
- $\delta p_t \propto \delta T(x)$ independent of rapidity
- gaussian densities, correlation function

obtain:

$$\left\langle \delta p_{t1} \delta p_{t2} \right\rangle_e \approx \frac{\left\langle p_t \right\rangle^2 R_{AA}}{1 + R_{AA}} F(\xi / R_t)$$



transverse size R_t, correlation length ξ

Approach to Equilibrium

Boltzmann Equation for phase space density $f(x,p)$

$$\frac{\partial f}{\partial t} + \vec{v}_{\vec{p}} \cdot \vec{\nabla} f = -\nu\left(f - f^{e}\right)$$

approximation: relaxation time ν^{-1}

Bjorken scaling: $$f = f_0(p_z', p_t, s)S(t,t_0) + \int_{t_0}^{t} f^{e}(p_z', p_t, s)S(t,s)\nu\, ds$$

Baym; Gavin; B. Zhang & Gyulassy

where $p_z' = p_z(t/t_0)$

survival probability –
chance of no scattering

$$S(t,t_0) = e^{-\int_{t_0}^{t} \nu\, ds}$$

$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1 - S)$$

Nonequilibrium Fluctuations

Introduce fluctuations:

- Boltzmann Equation plus Langevin noise
- Langevin noise for f^e

relaxation equation for correlation functions

$$\mathrm{P}(x_1, p_1, x_2, p_2) = \langle f_1 f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle - \delta_{12} \langle f_1 \rangle$$

obtain:

$$\mathrm{C}(x_1, p_1, x_2, p_2) = \langle f_1 f_2{}^e \rangle - \langle f_1 \rangle \langle f_2{}^e \rangle$$

$$\frac{d\mathrm{P}_{12}}{dt} = -2\nu \mathrm{P}_{12} + \nu(\mathrm{C}_{12} + \mathrm{C}_{21}); \qquad \frac{d\mathrm{C}_{12}}{dt} = -\nu(\mathrm{C}_{12} - \mathrm{P}_{21}{}^e)$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v_{p_1} \cdot \nabla_1 + v_{p_2} \cdot \nabla_2$$

Bjorken flow:

$$\mathrm{P}_{12} = \mathrm{P}_{12}{}^0 S^2 + 2\mathrm{C}_{12}{}^0 S(1-S) + \mathrm{P}_{12}{}^e (1-S)^2$$

Near Local Equilibrium Correlations

single particle:

$$f(p) = e^{-(E - \vec{p}\cdot\vec{v})/T}, \qquad \vec{v} << 1$$

particle correlations

$$\mathrm{P}^{e}(x_1, p_1, x_2, p_2) = f(p_1) f(p_2) A$$

density correlations

$$A(x_1, p_1, x_2, p_2) = r(x_1, x_2)$$
$$+ s(x_1, x_2)\left(\vec{v}(x_1)\cdot\vec{p}_1\right)\left(\vec{v}(x_1)\cdot\vec{p}_1\right)$$
$$+ t(x_1, x_2) E_1 E_2$$

Near Local Equilibrium Correlations

single particle:

$$f(p) = e^{-(E-\vec{p}\cdot\vec{v})/T}, \qquad \vec{v} << 1$$

particle correlations

$$\mathrm{P}^{e}(x_1, p_1, x_2, p_2) = f(p_1)f(p_2)A$$

$$A(x_1, p_1, x_2, p_2) = r(x_1, x_2) + s(x_1, x_2)\left(\vec{v}(x_1)\cdot\vec{p}_1\right)\left(\vec{v}(x_1)\cdot\vec{p}_1\right) + t(x_1, x_2)E_1E_2$$

momentum correlations

Near Local Equilibrium Correlations

single particle:

$$f(p) = e^{-(E-\vec{p}\cdot\vec{v})/T}, \qquad \vec{v} << 1$$

particle correlations

$$\mathrm{P}^{e}(x_1, p_1, x_2, p_2) = f(p_1) f(p_2) A$$

$$\begin{aligned} A(x_1, p_1, x_2, p_2) = r(x_1, x_2) \\ + s(x_1, x_2)\big(\vec{v}(x_1)\cdot\vec{p}_1\big)\big(\vec{v}(x_1)\cdot\vec{p}_1\big) \\ + t(x_1, x_2) E_1 E_2 \end{aligned}$$

temperature correlations

Near Local Equilibrium Correlations

single particle:

$$f(p) = e^{-(E - \vec{p}\cdot\vec{v})/T}, \qquad \vec{v} << 1$$

particle correlations

$$P^e(x_1, p_1, x_2, p_2) = f(p_1) f(p_2) A$$

$$\begin{aligned} A(x_1, p_1, x_2, p_2) = {} & r(x_1, x_2) \\ & + s(x_1, x_2)\big(\vec{v}(x_1)\cdot\vec{p}_1\big)\big(\vec{v}(x_1)\cdot\vec{p}_1\big) \\ & + t(x_1, x_2) E_1 E_2 \end{aligned}$$