The Instanton Molecule Liquid

G. E. B., C.-H. Lee, Manuque Rho
& Edward Shuryak

the perfect liquid in which the mfp = distance between particles ($\eta/s \sim 1/4\pi$).

Just the opposite of the QGP.

We will work at $T \gtrsim T_c$.
In order to make a smooth ~ 2nd-order phase transition
m_π & m_σ should be zero just above T_c

But LGS (quenched) give* $m_q \sim 1$ GeV. ∴ $q\bar{q}$ pair must be very small $x \sim .1$ fm in order to be bound by 2 GeV.

* quarks are not the thermodynamic variables.

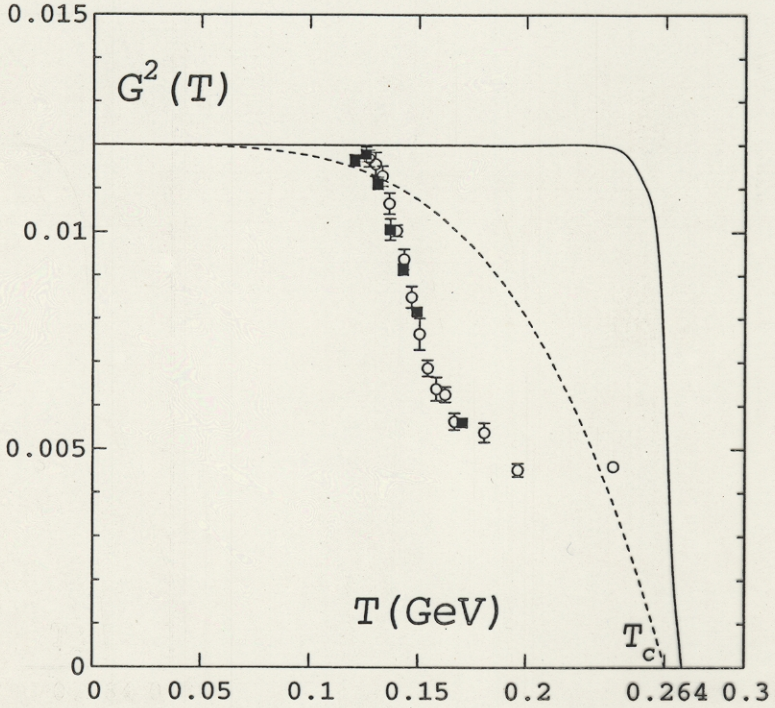


Fig. 2. Gluon condensates taken from Miller(12). The lines show the trace anomaly for SU(3) (solid) and the ideal gluon gas (broken) in comparison with that of the light dynamical quarks denoted by the open circles and the heavier ones by filled circles. The T_c marked in the figure is that for quenched QCD, whereas we deal with unquenched QCD in this note.

Use Nambu - Jona - Lasinio

Binding energy of the glue

$$B.E. = -E_{vac} = -\left\{4\int_0^{\Lambda}\frac{d^3k}{(2\pi)^3}\left\{\sqrt{k^2+m_N^2}-k\right\}\right.$$

$\Lambda \sim 700$ MeV gives $T_c \sim 170$ MeV when constituent quarks, are used. Field energy left out.

$$E_{glue} = -E_{vac} = (.012\ \text{GeV})^4$$

Now at $T \sim 125$ MeV (see figure) nucleons loosen into constituent quarks

$$B.E. = -\left\{12\int_0^{\Lambda}\frac{d^3k}{(2\pi)^3}\left\{\sqrt{k^2+m_Q^2}-k\right\}\right.$$

$$E_{soft\ glue} = \sim \frac{1}{2}(.012\ \text{GeV})^4$$

Adami, Hatsuda, Zahed
Phys. Rev. B43 (1991) 921

Remaining $\gtrsim \frac{1}{2}$ trace anomaly is used up in rearranging random instanton vacuum into instanton molecules.

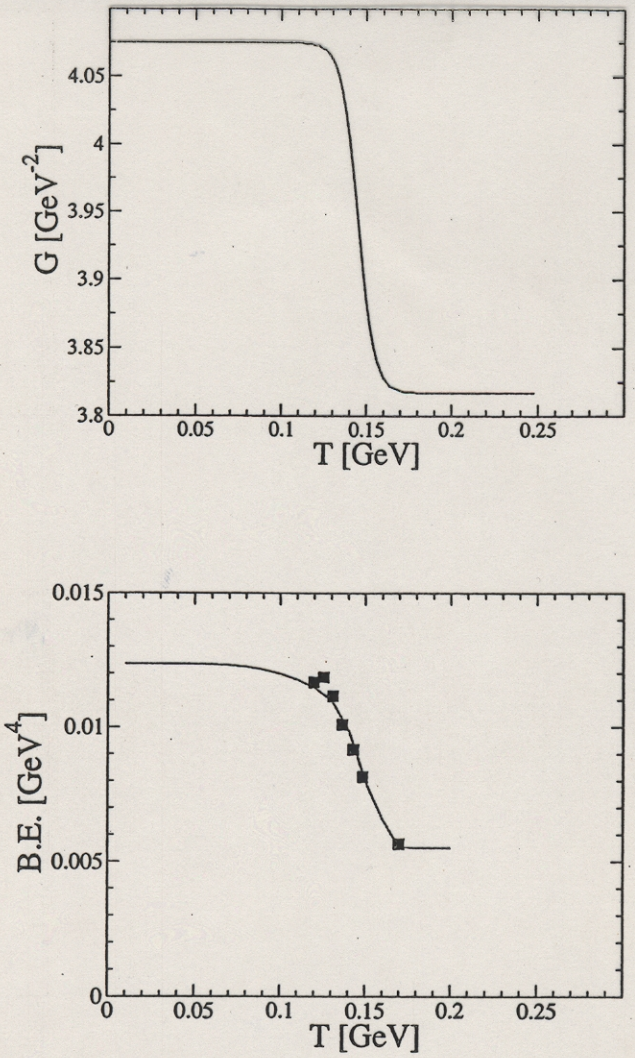


Fig. 1. (a) Temperature dependence of the coupling G and (b) $B.E.(soft\ glue)$ with varying G. In this calculation $\Lambda = 695$ MeV and $m_{Q,current} = 0$ are used.

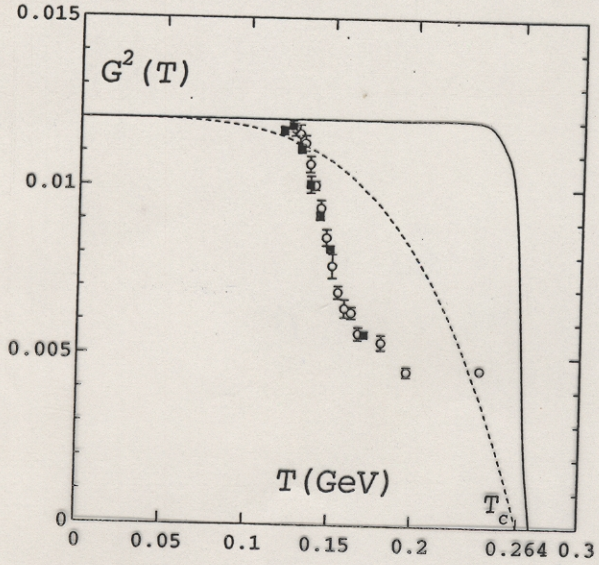


Fig. 2. Gluon condensates taken from Miller[15]. The lines show the trace anomaly for SU(3) (solid) and the ideal gluon gas (broken) in comparison with that of the light dynamical quarks denoted by the open circles and the heavier ones by filled circles. The T_c marked in the figure is that for quenched QCD, whereas we deal with unquenched QCD in this note.

G in NJL carries the information about the Glue. G decreases from 4.075 to 3.825 as the soft glue is melted. Only 6%.

1. Chiral Symmetry Breaking is very fragile!

2. We have the

$$G = 3.825 \text{ GeV}^{-2}$$

to use in our new chirally restored NJL with instanton molecules above T_c.

Instanton Molecule Interactions

$$\mathcal{L}_{IML} = G \Big\{ (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}\tau^a\gamma_5\psi)^2$$

$$+ \frac{1}{4}\Big[(\bar{\psi}\tau^a\gamma_\mu\psi)^2 - (\bar{\psi}\tau^a\gamma_\mu\gamma_5)^2 \Big]$$

$$- (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \Big\}$$

$$\tau^a = (\tau, 1) \qquad (4\text{-vector})$$

32 degrees of freedom (as compared with 37 effective boson d.o.f. in quarks & gluons)

$G = 3.825\ \mathrm{GeV}^{-2}$, only 6% smaller than G for the chirally broken NJL.

$m_q \sim 1\ \mathrm{GeV}$ both at $T = \frac{3}{2}T_c$ (quenched)

and $3T_c$ (quenched) from LGS.

The IML interaction is $\propto$ to $G|\psi^2(0)|$; as T increases, free charge & the IM size increases so $|\psi^2(0)|$ decreases.

Color Coulomb + Velocity - Velocity (G.E.B. 1958)

$$\frac{e^2}{r}(1 - \underline{\alpha}_1 \cdot \underline{\alpha}_2) \qquad (T = T_c)$$

$$\Rightarrow \frac{2\alpha_s}{r} \quad \text{if } \underline{\alpha}_1 \cdot \underline{\alpha}_2 = -1, \quad 0 \text{ if } \underline{\alpha}_1 \cdot \underline{\alpha}_2 = 1$$

$\therefore \alpha_s = 0.5$ for charmonium $\rightarrow$
(need to bind charmonium up to $\gtrsim 1.6 T_c$)
A & H.

$(\alpha_s)_{eff} = 1$ if the quark & antiquark go around in opposite directions.

$$T_c < T \lesssim 2T_c$$

Dynamical confinement into color singlets. Hard glue (epoxy) is clamped into IMs. LGS show that epoxy is melted by $\sim 2T_c$.

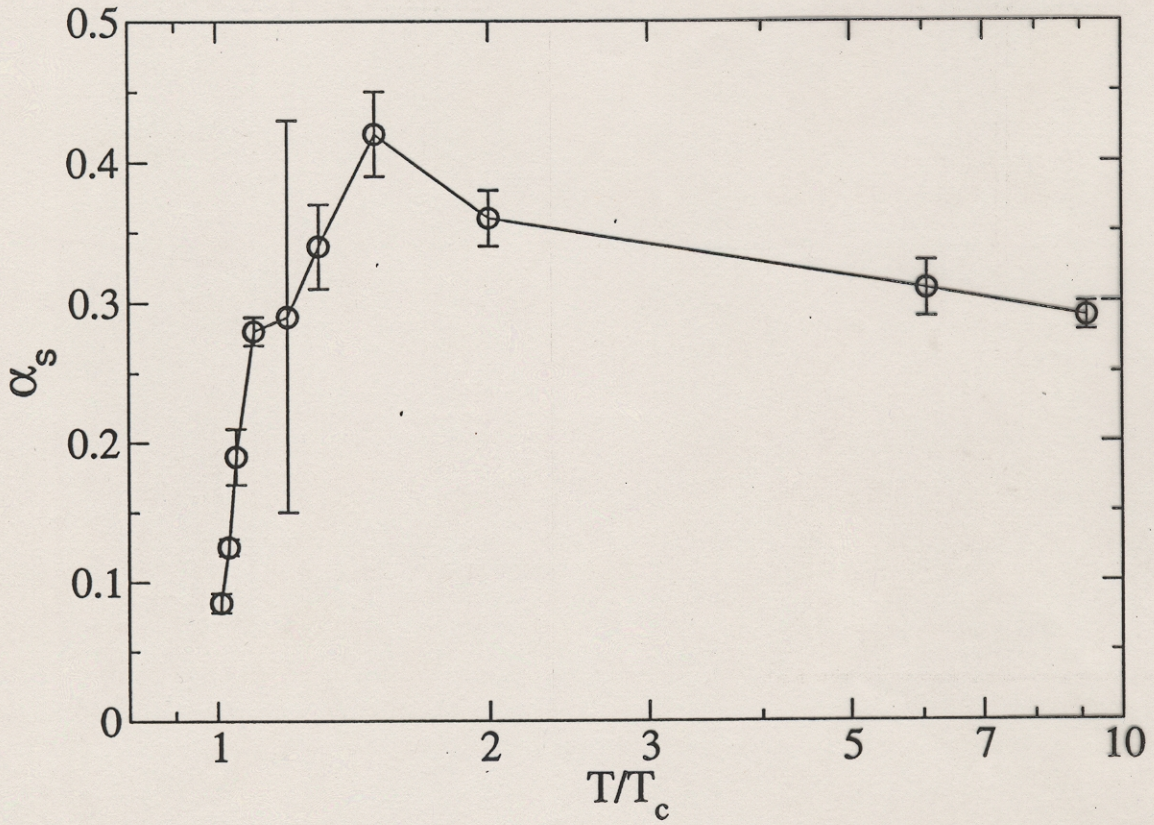


Fig. 1. Lattice Gauge calculation of α_s by Santo Fortunato (private communication).

their hydro calculations, where there is complete equilibration in the material (at $\tau \sim 2/3$ fm/c). Quenched calculations should be adequate for the above determinations, because the loop effects which bring the π and σ masses down to zero at T_c, will not be appreciable for the relatively large, ~ 1 fm radius, $\bar{q}q$ bound states in the temperature region $\sim T_{zb}$.

As the temperature decreases from T_{zb} to T_c, the chirally restored mesons become much smaller in extent, down to rms radii $\sim 1/3$ fm for the π and σ at T_c [5]. We now discuss their interactions just above T_c.

BLRS[5] obtained the 8 π's and σ's as massless, the four vectors (ρ and ω) polarized towards the time direction at a mass of 560 MeV, and the four axial vectors polarized in this direction at 1440 MeV. If all 16 of these d.o.f. were massless, their density at $T_c = 175 MeV$ would be

$$n = 1 \text{ fm}^{-3}, \tag{3}$$

Results: for $T_c \sim 170$ MeV

Color Coulomb

$-\frac{1}{2} m_q$

$|M| - \frac{3}{2} m_q$

$\sqrt{\langle r^2 \rangle} = 0.3$ fm

no screening

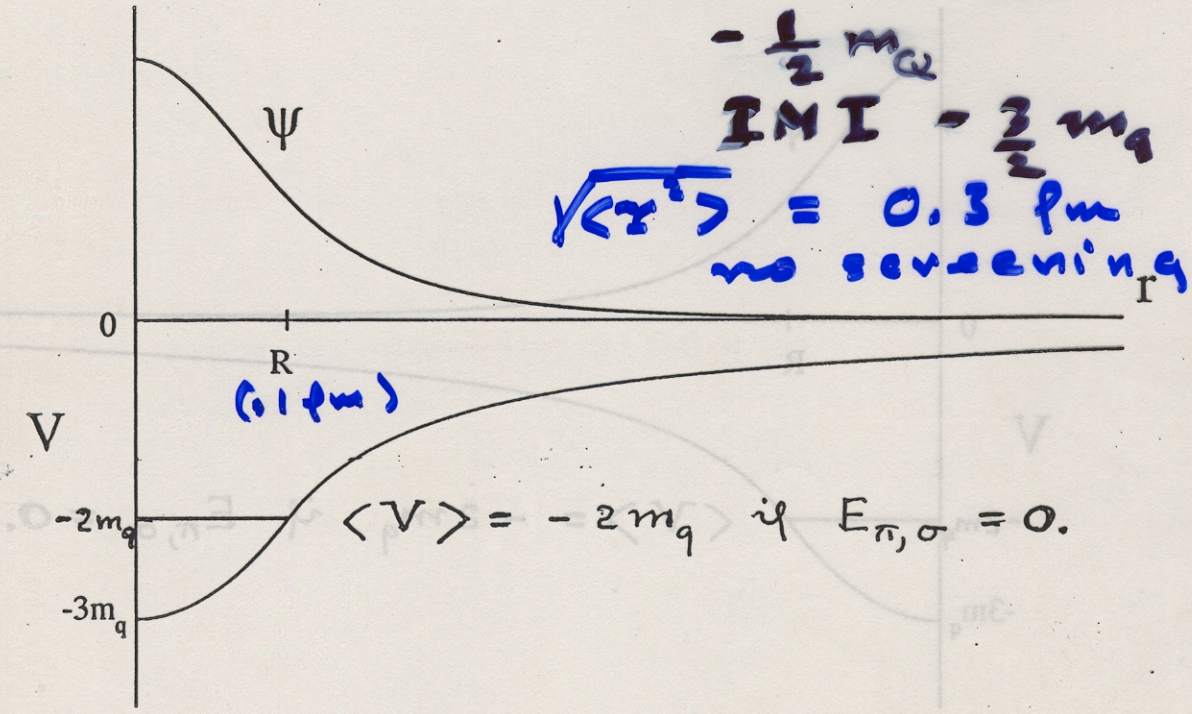
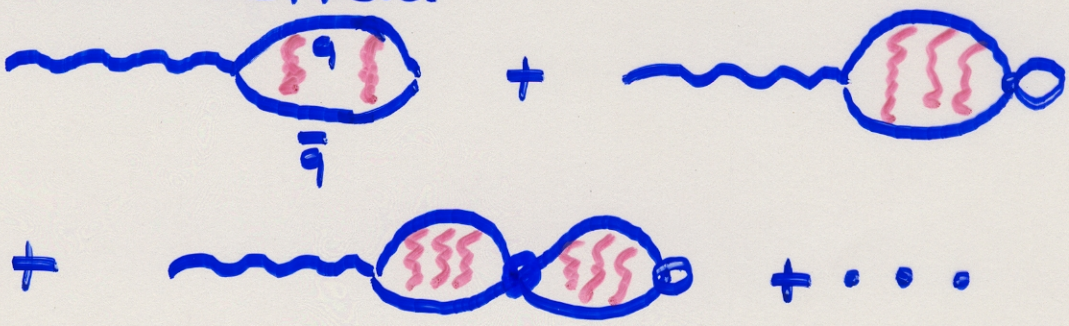


Fig. 1. The color Coulomb potential V and the corresponding wave function ψ for relativistic Klein-Gordon case. The interaction eq. (8) with $R = \hbar/2m_q$ was used. The ground state energy with $\alpha_s = 1$ corresponds to $E_{\text{ground}} = 0.645\ m_q$. The minimum of the potential at the origin is at $-3m_q$ here.

Source Current



ξ = Coulomb

$\alpha_s = 0.5$ $\qquad 2m_q = 2$ GeV (arbitrary)

Coulomb Binding $\sim -.5$ GeV

IMInt -1.5 GeV

Finite size correction $0.75 = F$

$$\Delta E = -0.5\,\text{GeV} - \frac{1\,\text{GeV}}{1 - \frac{1\,\text{GeV}\,F^2}{1.5\,\text{GeV}}}$$

By $1.5(T_c)_{\text{quenched}} \approx 2(T_c)_{\text{unquenched}}$

the free charge $\longrightarrow$ Debye screening

Molecules get bigger & break up.

(Hard Glue is melted) LGS.

As shown in Sec.2, this is necessary for reliable MEM analysis.

In Fig. 4, we show the results at $N_\tau = 40$, i.e., $T \simeq 1.9T_c$. There is an apparent peak around $\omega = 0$. A possible explanation of the peak is the effect of Landau damping [8]. However, this peak is, at the moment, not statistically significant yet. The peaks around $\omega \simeq 4.5$ GeV are significantly broader than those around 2 GeV at $T \simeq 1.9T_c$.

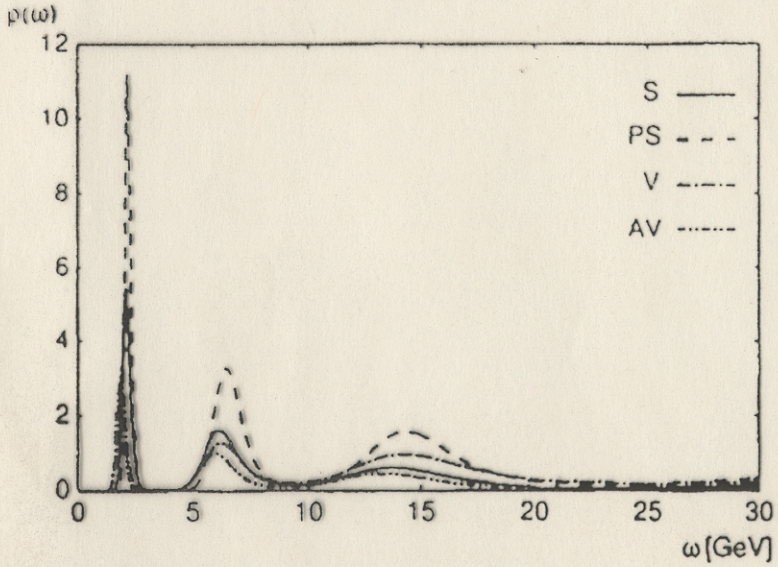


Figure 3. Spectral Functions for $N_\tau = 54$ ($T \simeq 1.4T_c$)

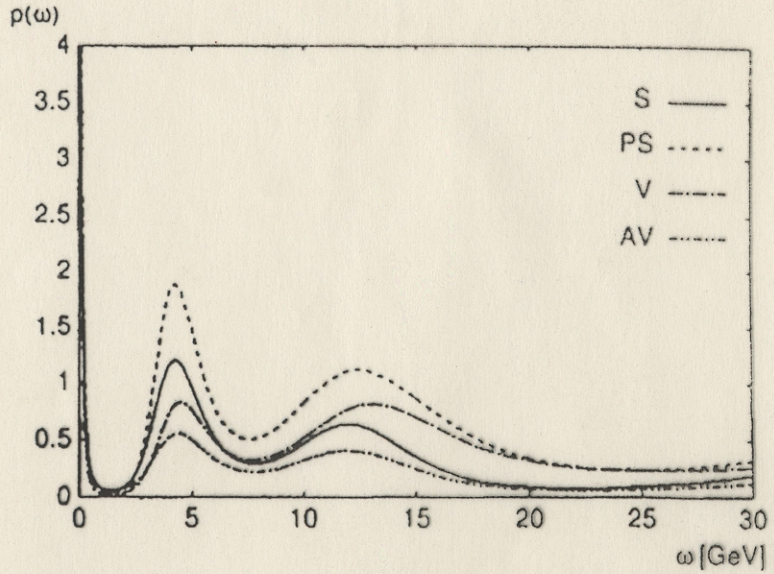


Figure 4. Spectral Functions for $N_\tau = 40$ ($T \simeq 1.9T_c$)

Asakawa & Hatsuda Resonances.

1.5 T_c = 405 MeV RHIC Initial T

2 GeV peak

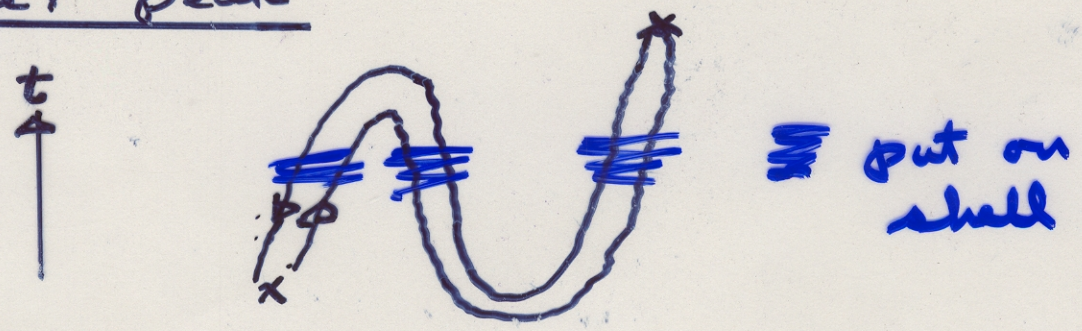
All mesons degenerate S, P, V, AV

Test the temperature where mesons unbind : zero Binding energy, $a = \infty$ ∴ create quark & antiquark costs

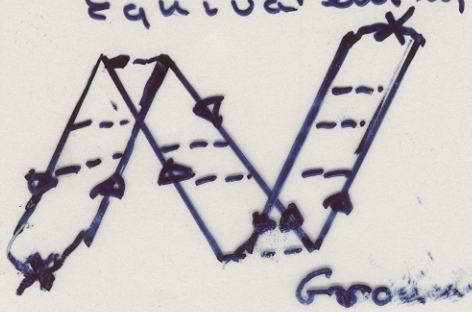
$$m_q + m_{\bar{q}} \cong 2\ \text{GeV}.$$

no temperature smearing bec. start out with zero B.E. Fermi sol

6 GeV peak



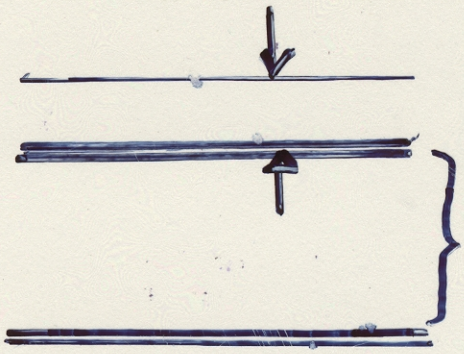
Equivalently



Ground State Correlation.

Representation

Color Coulomb = 0.5 GeV



$2 m_q \sim 2$ GeV

$q\bar{q}$ bound state at 1.5 GeV

So the unperturbed states are all at 1.5 GeV at $T = T_c$.

But we want the (chirally restored π and ρ to have $m = 0$ in order to make a 2nd-order phase transition).

The 4-point (δ-function) Instanton Molecule Interaction has constant matrix elements. Secular equation:

$$1.5\text{ GeV}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \text{etc.} & \\ 0 & 0 & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}\psi - V\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}\psi$$

$$= E\psi$$

(Brown & Bolsterli 1959)

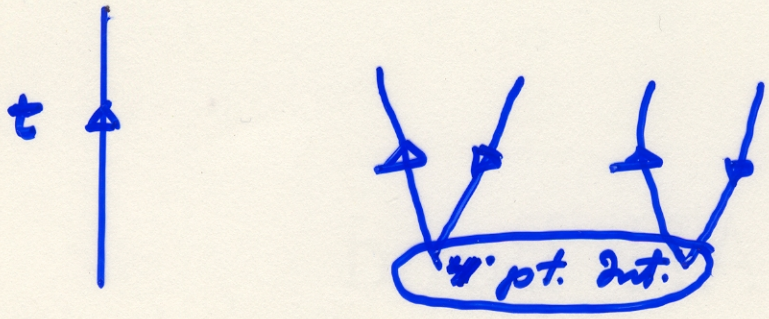
Principle of insufficient reason.

$$\psi = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$E_{coll} = 1.5 \text{ GeV} - NV$$

$$= .75 \text{ GeV}$$

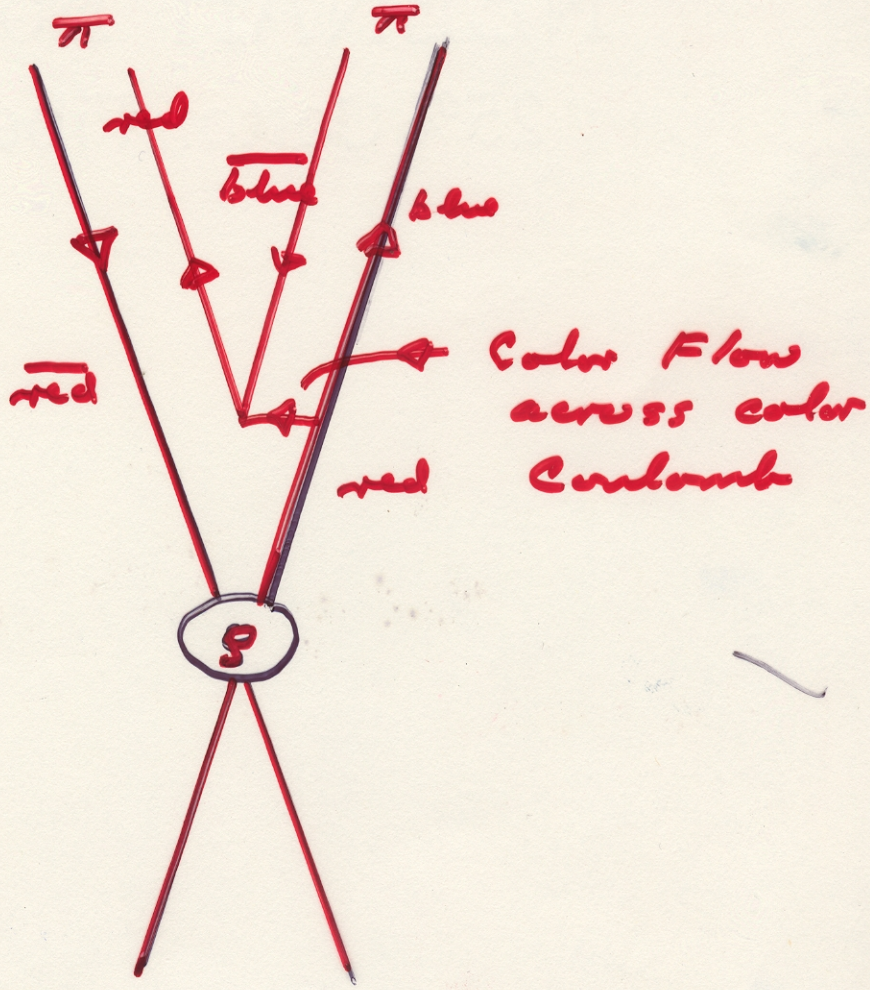
If the bubbles going forward in time bring the energy down half way, then ground-state correlations (backward going graphs) will bring it down all the way to zero.



So the $2 \times 1.5N$ GeV binding energy in the vibrations & in the lowering of the vacuum energy by the two-body correlations is available for thermal energy.*

Vibrational modes will be mixed through nonlinearities

e.g. $\rho \to 2\pi$



* also the Coulomb binding.

Rep 3

Compare the matrix element in 2nd-order perturbation theory (Fermi Golden Rule) with that in the chirally broken (usual ρ) case

Color Coulomb Interaction

$$\delta H = \frac{e^2}{r_{\pi\rho}} \cong \frac{e^2}{r_{\pi\rho_{cm}}} + \frac{e^2 r_\rho \cos\theta_\rho}{r^2_{\pi\rho_{cm}}}$$

Because of ρ-spin, keep only $\cos\theta_\rho$

$$R = \frac{4 e^2 (r_1 / r^2_{\pi\rho_{cm}})}{g_V m_\rho} \sqrt{\frac{1}{3}} \sim \frac{2}{3}$$

Remember $\Gamma_\rho \sim 150$ MeV chirally broken.

But the 2/3 is for the $\bar{q}q$ bound state; no collectivity yet.

Now add the instanton molecule interaction.

From earlier: The color Coulomb interaction brings the $\bar{q}q$ state down from 2 GeV to 1.5 GeV, 1/4 of the way to zero.

The Instanton Molecule interaction brings the collective interactions to zero. ∴ it is 3 times larger. Putting it on each vertex could increase the $\frac{2}{3}$ to $\frac{2}{3} \times 4 \times 4 \times 4$ & then must square. But as soon as $\Gamma_{\rho \to 2\pi} = m_\rho$, should stop (complete nonlinearity.)

Peter, Johanna & Wettenich want "the scattering of collective excitations . . . expressed in the form of multihadron scattering just in the region of T_c . . . & this gives it to them.

$$\Gamma_\rho = 0 \quad T \to T_c \text{ from below}$$

(Harada & Yamawaki). Phys. Rept. 381 (2003) 1. g_V, $g_\sigma \to 0$. Particles equilibrated just above T_c fly freely out.