"QGP" at Strong Coupling

e.g $N=4$, QCD

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ph/0307267

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th/0310031

outline

- $N=4$ + AdS
- Coulomb's law
- $N=4$ $T \neq 0$
- QCD $T \simeq T_c$
- Conclusion

$N=4$ SYM

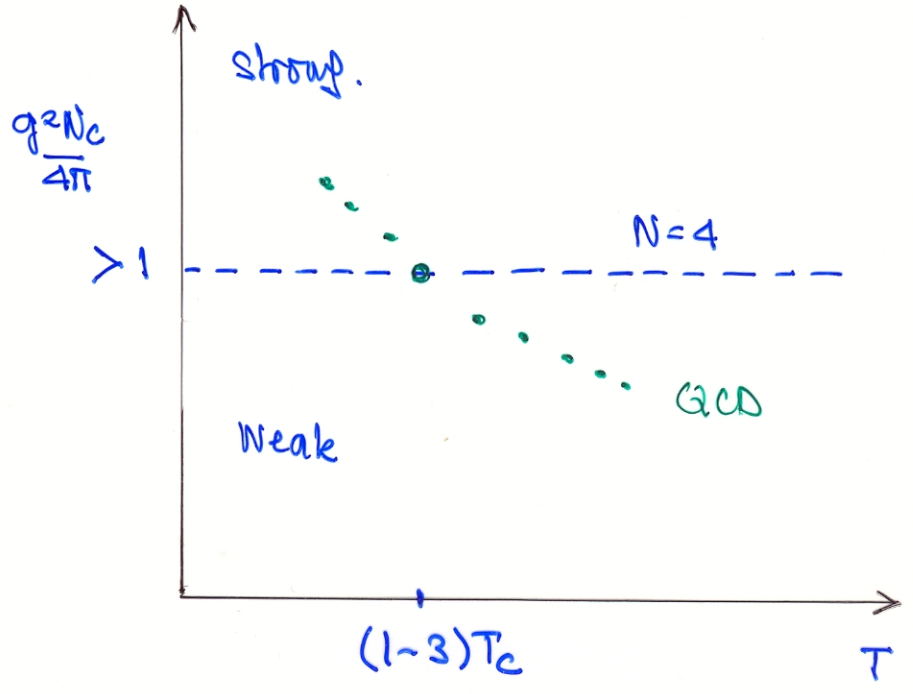
$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^{a\,2} + \frac{1}{2} \nabla_\mu \phi_i^{a\,2} + \frac{ig}{2} \bar{\psi}^a \Gamma^\mu \nabla_\mu \psi^a$$
$$+ \frac{g}{2} f^{abc} \bar{\psi}^a \Gamma^i \phi_i^b \psi^c + \frac{g^2}{2} f^{abc} f^{ade} \sum_{ij} \phi_i^b \phi_j^d \phi_i^c \phi_j^e$$
$$+ \text{q.t.}$$

A_μ^a :	$4\,(N_c^2-1)$	$S=1$	B
ϕ_i^a :	$6\,(N_c^2-1)$	$S=0$	B
ψ_j^a :	$4\,(N_c^2-1)$	$S=1/2$	F
$b_i c^i$:	$2\,(N_c^2-1)$	$S=0$	F

't Hooft $\quad N_c \to \infty \quad \lambda = g^2 N_c \ll 1$

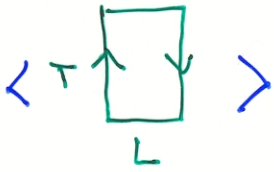
Maldacena $\quad N_c \to \infty \quad \lambda = g^2 N_c \gg 1$ ✓

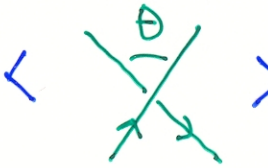
- $N=4$ in QCD



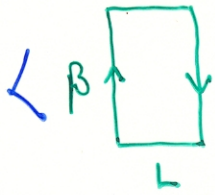
AdS T=0

$\sim e^{\sqrt{\lambda}}$ Gross + Strassler 99'

 $\sim e^{-\# \frac{\sqrt{\lambda}}{L} T}$ Maldacena 98'

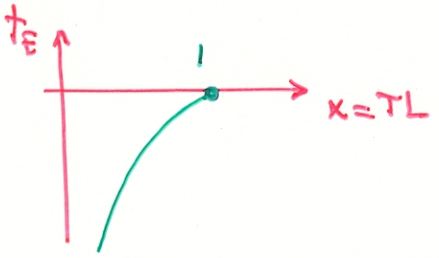
 $\sim T(s = e^{i\theta}, b_\perp)$ Jin+Rho+Zahed 99' Polshinski + Strassler 02'

AdS $T \neq 0$

- $V_E(L,T) \Leftarrow$  $_{\beta=\frac{1}{T}}$

Rey+Yee 99'

$$\simeq \frac{\sqrt{\lambda}}{L} t_E(TL)$$



- $V_M(L,T) = V_E(T,L)$

Shuryak+Zahed 03'

- $F(T,V_3) = -\frac{T}{V_3} \ln \langle \mathrm{tr}\, e^{-\beta H} \rangle$

$$= c_B N_c T^4 \left(\frac{3}{4} + \frac{\circ}{\lambda^{3/2}}\right)$$

Kim+Rey 00'

- $\Delta(t) = \langle P_\beta J(t_1) J(t_2) \rangle$

$$= \begin{pmatrix} \Delta_{++}(t) & \Delta_{+-}(t) \\ \Delta_{-+}(t) & \Delta_{--}(t) \end{pmatrix} \simeq \sum_{n_0}^{\infty} e^{-m_n t}$$

$$m_n = (C_\Delta + n\pi)(1-i)$$

e.g. $J_\pm(t) = \mathrm{Tr}\,(\phi_1 \pm i\phi_2)^\Delta(t)$

$$C_\Delta = \sqrt{\Delta(\Delta-4)}$$

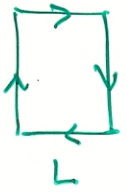
Son et al.
Shenker et al.
02'

- $L_{att} \simeq \frac{4}{3}\frac{2}{5T} = \frac{1}{3\pi T}$

$$\simeq \frac{1}{g^4 \ln\frac{1}{g} T} \qquad \lambda \ll 1$$

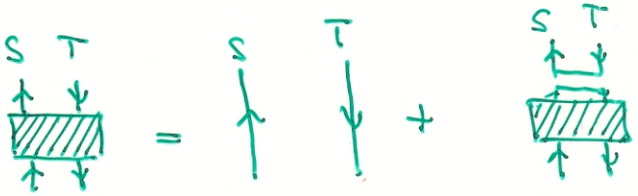
Son et al.
02'

T=0 Ladders

$$\Gamma_{TxL} = \langle$$  $$\rangle$$

$$V(L) = -\lim_{T\to\infty} \frac{1}{T} \ln \Gamma_{TxL}$$

planar + ladder + Feynman



$$\frac{\partial^2 \Omega}{\partial s\, \partial T} = \frac{\lambda/4\pi^2}{(s-T)^2 + L^2} \Gamma(s,T)$$

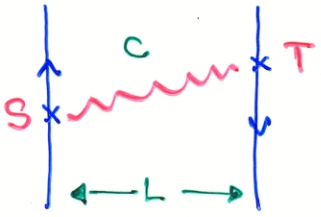
$$\Gamma(s,T) \simeq e^{-\frac{\sqrt{\lambda}}{4\pi}\left(\frac{s-T}{L}\right)^2} e^{\frac{\sqrt{\lambda}}{2\pi}\left(\frac{s+T}{L}\right)}$$

$$V_{Ladd} \simeq -\frac{\sqrt{\lambda}}{\pi L}$$

~

weak $-\frac{\lambda}{4\pi}\frac{1}{L}$!

Coulomb's law at strong coupling



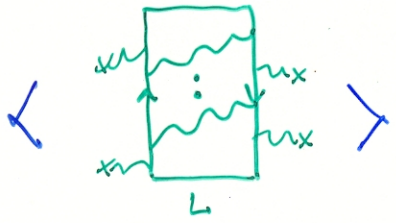
$S\text{-}T \simeq L/\lambda^{1/4}$

$\text{speed} \simeq c \simeq L/L/\lambda^{1/4} \simeq \lambda^{1/4} \gg 1$

$$V_E(L) \simeq -\frac{\lambda}{4\pi}\int_{L/\lambda^{1/2}} \frac{dt}{t^2+L^2} \simeq -\frac{\sqrt{\lambda}}{L}$$

$$V_E(L) \simeq -\frac{\lambda}{4\pi}\int_{\infty} \frac{dt}{t^2+L^2} \simeq -\frac{\lambda}{L}$$

Tfd Naïve



$$\frac{\partial^2 \Gamma_E}{\partial S \partial T} = \sum_{-\infty}^{+\infty} \frac{\lambda/4\pi^2}{(S-T+n\beta/L)^2+L^2} \Gamma_E(S,T)$$

$$\frac{\partial^2 \Gamma_M}{\partial S \partial T} = \sum_{-\infty}^{+\infty} \frac{\lambda/4\pi^2}{(n\beta)^2+(S-T)^2+L^2} \Gamma_M(S,T)$$

$$L V_{E,M}(T,L) = -\frac{\sqrt{\lambda}}{\pi}\left(1+2\sum_{n=1}^{\infty}\frac{1}{(\frac{n\beta}{L})^2+1}\right)$$

Screening at $\lambda \uparrow \infty$ *

$$\Pi_{00}(\omega,k) = -\omega_D^2 \left(1 - \frac{\omega}{2k} \ln\left|\frac{\omega+k}{\omega-k}\right|\right) \quad \overset{\#\lambda T^2}{\leq}$$

$$\Pi_{ii}(\omega,k) = \frac{\omega_D^2}{2} \left(\frac{\omega}{k} \ln\left|\frac{\omega+k}{\omega-k}\right|\right)$$

$\Pi \simeq$

$\lambda \downarrow 0$ or $\omega \downarrow 0$ $\quad \Pi_{00} = -\omega_D^2 \quad \Pi_{ii} = 0$

$\lambda \uparrow \infty$ or $\omega \uparrow \infty$ $\quad \Pi_{00} = -\omega_p^2 \frac{k^2}{\omega^2} \quad \Pi_T = \omega_D^2$

$\lambda \uparrow \infty$ or $\omega \uparrow \infty$ → short times → HTL Klimov, Pisarski, ...

Screened Scalar Ladders

$$\Delta(t,\vec{x}) = iT\sum_n \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\omega_n t + i\vec{k}\cdot\vec{x}}}{\omega_n^2 + k^2 + \Pi_{00}}$$

$$\simeq \frac{iT}{4\pi x}\left(e^{-m_D x} + 2\sum_{n=1}^{\infty} \cos\omega_n t\, \frac{\omega_n^2}{m_D^2}\, e^{-\omega_n^2 x/m_D}\right)$$

$$\simeq \frac{iT}{4\pi x}\left(e^{-\sqrt{\lambda}xT} + \frac{\cdot}{\lambda^{1/4}}\right)$$

$\downarrow 0$

Screened E-ladders (Feynman gauge)

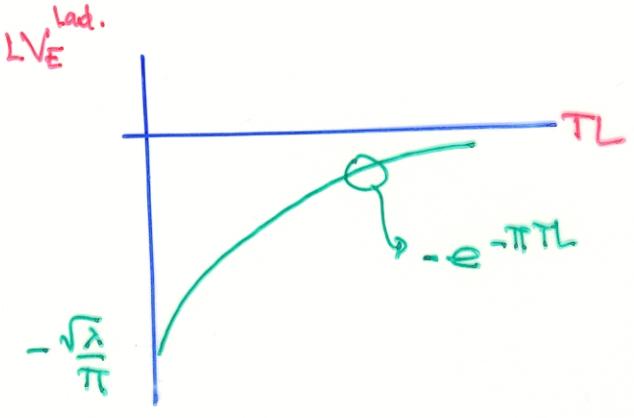
$$\Delta_\infty(t,\vec{x}) = iT \sum_{n=-\infty}^{+\infty} \int \frac{d\vec{k}}{(2\pi)^3} e^{-i\omega_n t + i\vec{k}\cdot\vec{x}}$$

$$* \frac{1}{k^4}\left(\frac{k^4}{k^2 - i\Pi_\infty} + \omega^2\right)$$

$$\Pi_\infty \sim \lambda T^2 \frac{k^2}{\omega^2}$$

$$\overset{t\neq 0}{\simeq} \frac{iT}{4}\left(\frac{e^{-m_D x}}{\pi x} + ix\frac{T^2}{8\zeta(3)\pi T x}\right)$$

$$V_E^{lad}(T,L) = -\frac{\sqrt{\lambda}}{\pi L}\,\frac{\pi TL}{|\sinh \pi TL|}$$

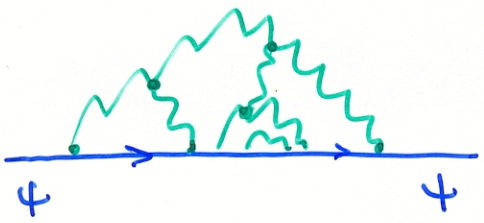


$$L_s \simeq \frac{1}{\pi T}$$

O.k. Aols

O.k. lattice

Heavy Excitations



$$\Sigma(E, \vec{k}\simeq 0) \simeq \gamma_4 \frac{M_F^2}{E} + \dots$$

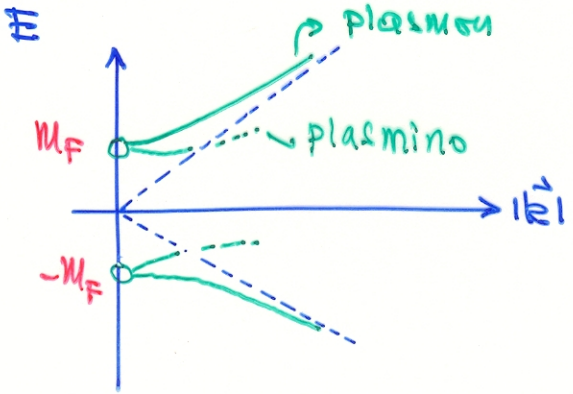
$M_F \simeq$ ($\lambda \gg 1$)

$=$

$= \frac{\sqrt{\lambda}}{L \simeq \frac{1}{T}} \simeq \sqrt{\lambda}\, T$

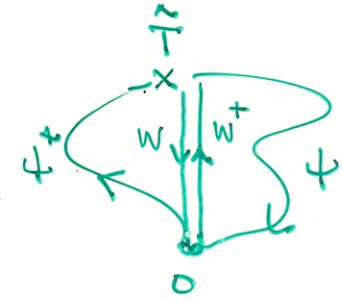
ok Ads

ok lattice



Correlators, e.g. White.

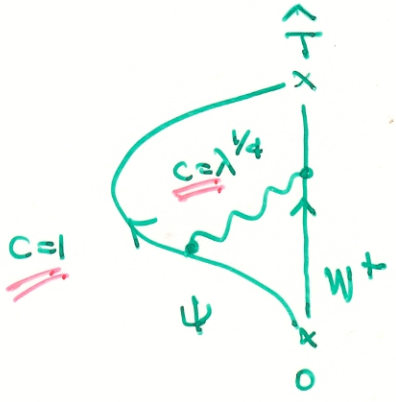
$$C(\hat{T}) \simeq \langle$$



$$\rangle$$

$$\simeq \langle \mathrm{Tr}\left(\psi^+(\tilde{T},\vec{0})\, W(\hat{T},\vec{0})\, \psi(0,\vec{0})\, W^+(\hat{T},\vec{0})\right) \rangle$$

$$\simeq e^{-E\hat{T}}$$



Potential problem even for Relativistic particle !

Light Excitations

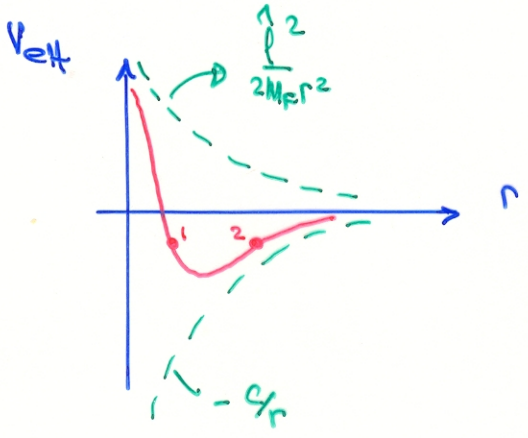
$$\left((E - V_E(\vec{x}))^2 - \left(-\vec{\nabla}^2 + m_F^2 \right) \right) \phi(\vec{x}) = 0$$

$$\phi(\vec{x}) = \frac{R_\ell(r)}{r} Y_{\ell m}(\hat{r})$$

$$-R_\ell'' = \left((E - V_E)^2 - m_F^2 - \frac{\hat{\ell}^2}{r^2} \right) R_\ell$$

WKB: $\hat{\ell} = \ell(\ell+1) \to (\ell+1/2)^2$

$V_E = -C/r$ 'no-screening'



$$p_r^2 = (E - V_E)^2 - m_F^2 - \frac{\hat{l}^2}{r^2}$$

$$p_r = 0 \rightarrow r_{1,2} = \frac{1}{E^2 - m_F^2}\left(EC \pm \sqrt{m_F^2 c^2 + \hat{l}^2 (E^2 - m_F^2)}\right)$$

≥ 0 bound!

i.e. $\hat{l} \simeq \sqrt{\lambda} + \frac{1}{\sqrt{\lambda}}$ for $C \simeq \sqrt{\lambda}$

$$\int_{r_1}^{r_2} p_r \, dr = \pi (n + 1/2)$$

$$E_{nl} = \pm m_F \left[1 + \left(\frac{C}{n + 1/2 + \sqrt{\hat{l}^2 - C^2}}\right)^2\right]^{-1/2}$$

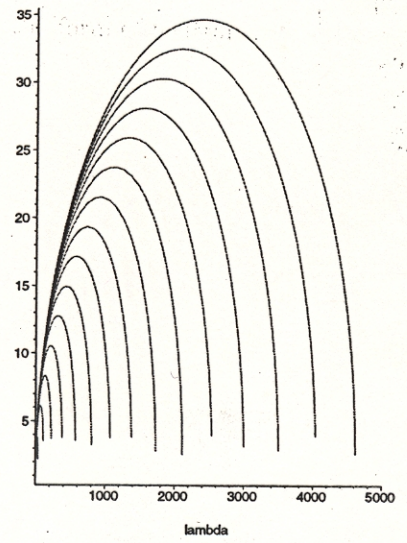
$C \simeq \lambda << 1$: $E_{nl} = \pm m_F \left(1 - \frac{C^2}{2(n+l+1)^2}\right)$ o.k.

$C \simeq \sqrt{\lambda} >> 1$: $E_{nl} = \pm \frac{m_F}{C}\left[(n + 1/2) + (\hat{l}^2 - C^2)^{1/2}\right]$

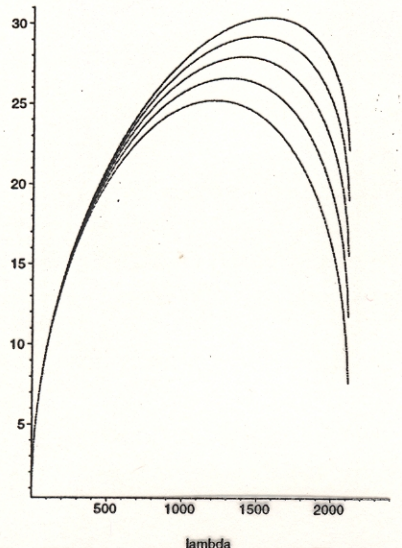
$\frac{m_F}{C} \rightarrow \pm T$; $(\hat{l}^2 - C^2)^{1/2} \rightarrow C_\Delta$

o.k. AdS

Size $\sim \frac{1}{\sqrt{\lambda} T}$ o.k. no-screening!



n=0



l=10

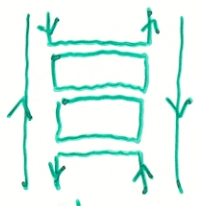
FIG. 5. The WKB spectrum versus 'tHooft coupling constant λ. (a) levels with fixed n=0 and $l = 1..15$, (b) levels with fixed $l = 10$ and $n = 0..5$.

Thermo with Composites

$N=4$, $T\neq 0$, $\lambda\uparrow\infty$: strongly coupled cb-phase.

ψ, A, ϕ

$E \simeq \sqrt{\lambda}\, T$, N_c^2 $\qquad$ $\underline{\underline{H}}$



A, ϕ, ψ^+ $\quad$ A, ϕ, ψ^+

$E = nT$, $\underline{\underline{N_c^2}}$ $\qquad$ $\underline{\underline{L}}$

$$e^{-\beta F} = \mathrm{Tr}\left(e^{-\beta H}\right) = \Big(\underbrace{\int_{min}^{max} d\ell^2}_{\mathbb{1}} \sum_{n=0}^{\infty} \underbrace{e^{-\frac{m_F}{cT}(n+1/2)}}_{2\,\mathrm{sh}\left(\frac{m_F}{2cT}\right)}\Big)^{N_c^2}$$

$$F = \underbrace{N_c^2\, T\, m}_{\text{ok. AdS.}} \left(2\,\mathrm{sh}\,\frac{m_F}{2cT}\right)$$

- Kinetic with light composites

Wigner's enhanced scattering:

$$\sigma(E) = \frac{2\pi}{m}\,\frac{1}{E+|E_b|}$$

$$\frac{g^2}{2}\,\frac{\lambda_T}{E_b} \simeq \frac{1}{T^2}$$

$$\lambda_{mfp} = \frac{1}{n\sigma} = \frac{T^2}{T^3} = \frac{1}{T} \qquad \text{'strong'}$$

$$= \frac{1}{g^4 m^{\frac{1}{2}}} \qquad \text{'weak'}$$

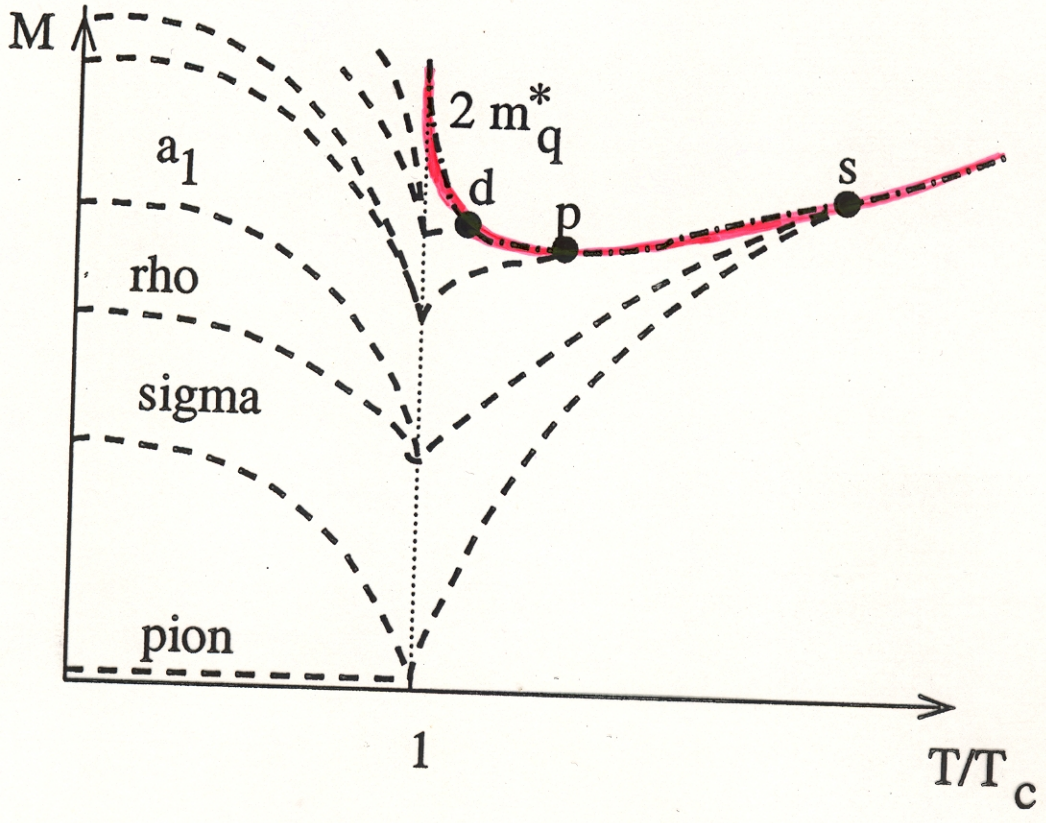
$$l_{sound} \simeq \frac{\eta}{s} \simeq \lambda_{mfp} \simeq \frac{1}{T}$$

$$\frac{\eta}{s} = \frac{s}{T} = \frac{T}{T}$$

ok AdS

Son et al. '02

'Running' QCD



$T_{\bar{q}q} \simeq 1.45\, T_c \simeq 250$ MeV

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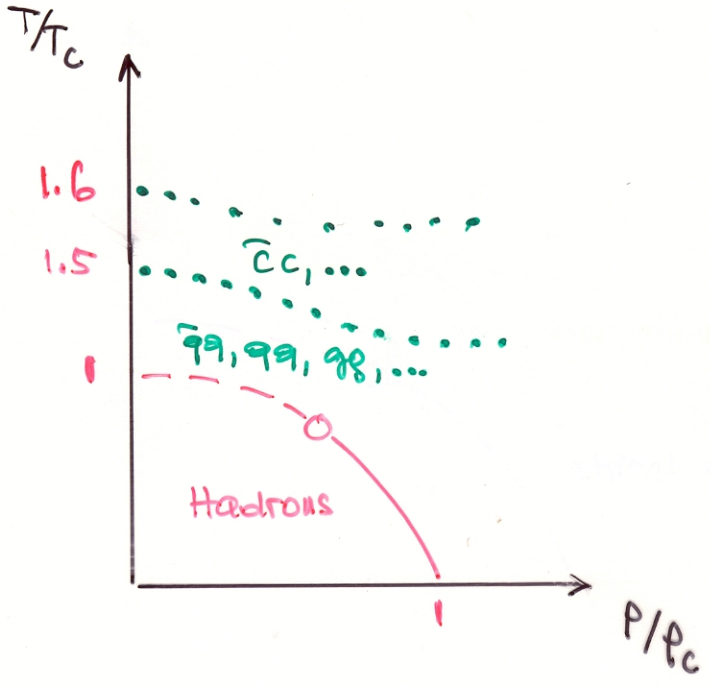
o.k. Haussou + Zahed 90'

Schafer + Shuryak 95'

...

Coulomb Phase Transitions in QCD



' Colored + neutral bound states exist above the chiral restoration line '

Conclusions

- QCD at $T \simeq (1\text{-}3)\, T_c$ strongly coupled.
- New Ionization lines above T_c
- Thermo near Black-Body
- Kinetic away from BB i.e. dense with λ_{mfp} small.