GAPLESS

COLOR-FLAVOR LOCKED

QUARK MATTER

KRISHNA RAJAGOPAL

(MIT)

with: MARK ALFORD

(Washington University)

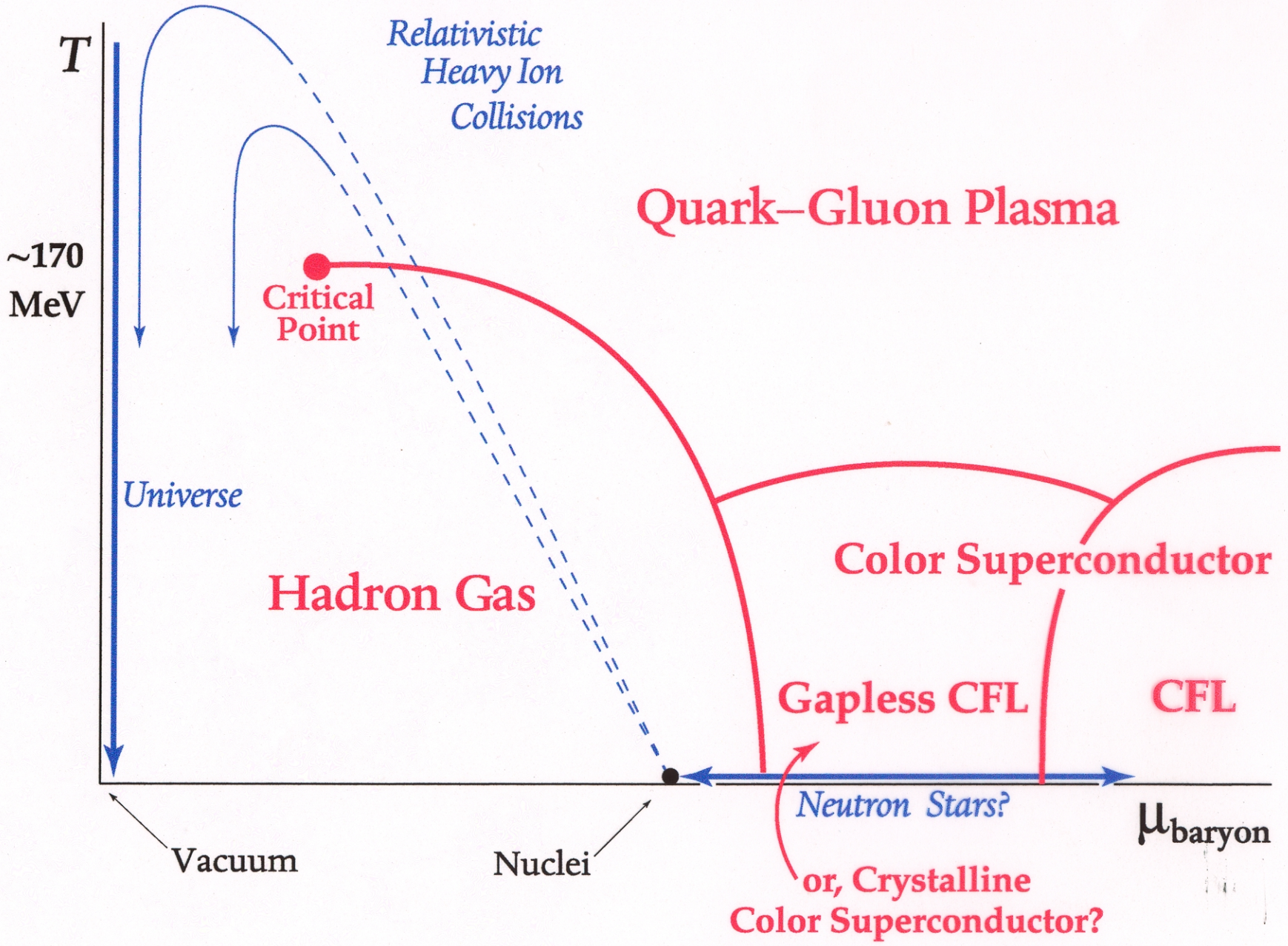
CHRIS KOUVARIS

(MIT)

QUARK MATTER 2004

OAKLAND, CA 1/15/2003

EXPLORING *the* PHASES *of* QCD



CFL

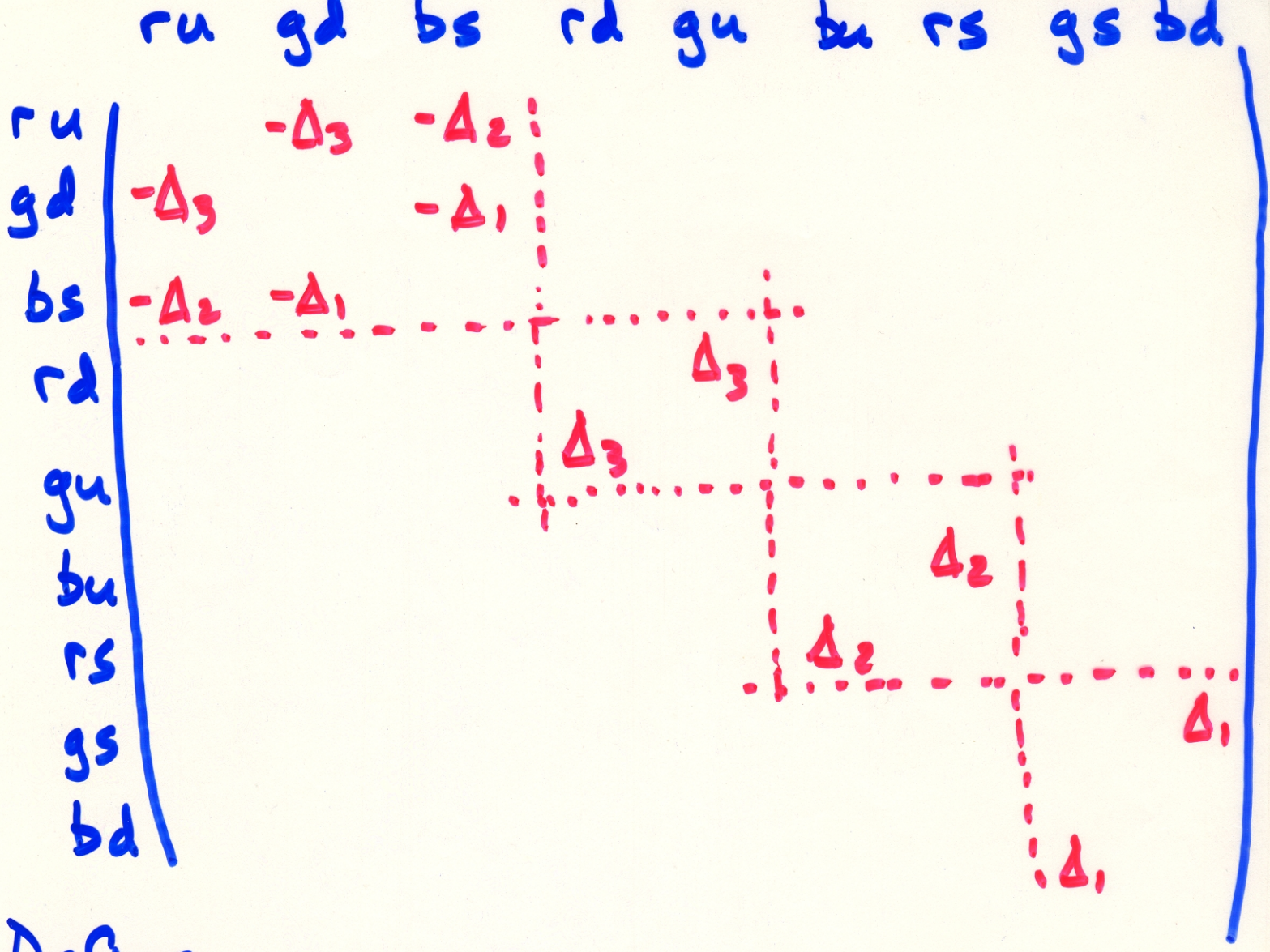
In cold quark matter, quarks near their Fermi surfaces pair → color superconductivity

Pattern of pairing:

$$\langle \psi_a^\alpha C\gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}$$

(α, β: color; a, b: flavor; $C\gamma_5$: Lorentz scalar)

- antisymmetry in color & Dirac indices energetically favored; flavor antisym. forced by Pauli
- If density great enough that M_s can be neglected, $\Delta_1 = \Delta_2 \simeq \Delta_3$
- All 9 quarks pair, maximizing condensation energy; leaves largest symmetry unbroken
- Demonstrated rigorously at asymptotic density.
- Unbroken symmetries all are color+flavor



Define

$$\tilde{Q} = \begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix} \begin{matrix} u \\ \text{for } d \\ s \end{matrix} + \begin{pmatrix} -2/3 \\ 1/3 \\ 1/3 \end{pmatrix} \begin{matrix} r \\ \text{for } g \\ b \end{matrix}$$

and check $\tilde{Q} = 0$ for every pair in the condensate.

⇒ One linear combination of photon & gluon does not get "Meissnered."

$$U(1)_{EM} \times SU(3)_{color} \to U(1)_{\tilde{Q}}$$

A TRANSPARENT INSULATOR

What can the $\tilde{Q}$-photon scatter off? No electrons.... And,...

- the CFL condensate itself is $\tilde{Q}$-neutral.
- once you include nonzero quark masses, all excitations with $\tilde{Q} \neq 0$ are massive.
- $\therefore$ for $T \ll M_{\text{lightest excitation with } \tilde{Q} \neq 0}$ (likely a kaon),

the CFL phase is transparent to the $\tilde{Q}$-photon. It is a $\tilde{Q}$-insulator, with some index of refraction $n_{CFL} \neq 1$.

NEUTRALITY

We study bulk matter in weak eqbm.
Must be electrically and color neutral.

Define μ_e, μ_3, μ_8: (color)electrostatic potentials coupled to electric charge and $T_3 = (\frac{1}{2}\ -\frac{1}{2}\ 0)$ and $T_8 = \mathrm{diag}(\frac{1}{3}\ \frac{1}{3}\ -\frac{2}{3})$ color charges.

These are zeroth components of gauge fields, and the gauge field dynamics enforce neutrality:

$$\frac{\partial \Omega}{\partial \mu_e} = \frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = 0$$

In the CFL phase, this implies

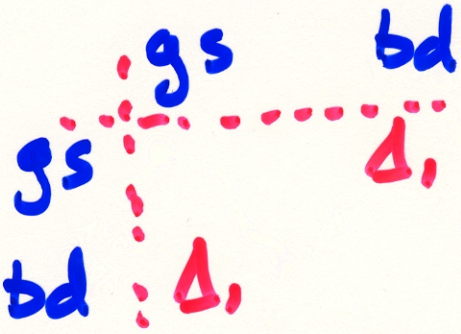
$$\mu_e = \mu_3 = 0 \qquad \mu_8 = -\frac{M_s^2}{2\mu}$$

to leading order in M_s^2/μ^2.
(Model independent result.)

Steiner Reddy Prakash; Alford KR

BREAKDOWN OF CFL

Focus on gs-bd "block":



$$\mu_{bd}^{eff} = \mu - \frac{2}{3}\mu_8$$

$$\mu_{gs}^{eff} = \mu + \frac{1}{3}\mu_8 - \frac{M_s^2}{2\mu}$$

↑ leading effect of M_s

In unpaired quark matter, $\mu_8 = 0$

and $P_{F_{bd}} - P_{F_{gs}} = M_s^2/2\mu$

In CFL, $P_{F_{bd}} = P_{F_{gs}}$ even though the μ's "want" them split, because this maximizes pairing energy.

So ....

Breaking a gs-bd pair

costs $2\Delta_1$,

benefits $\mu_{bd}^{eff} - \mu_{gs}^{eff} = \frac{M_s^2}{2\mu} - \mu_8 = \frac{M_s^2}{\mu}$

∴ CFL phase is UNSTABLE with respect to some phase which has more bd quarks than gs quarks if

$$\frac{M_s^2}{\mu} > 2\Delta_1$$

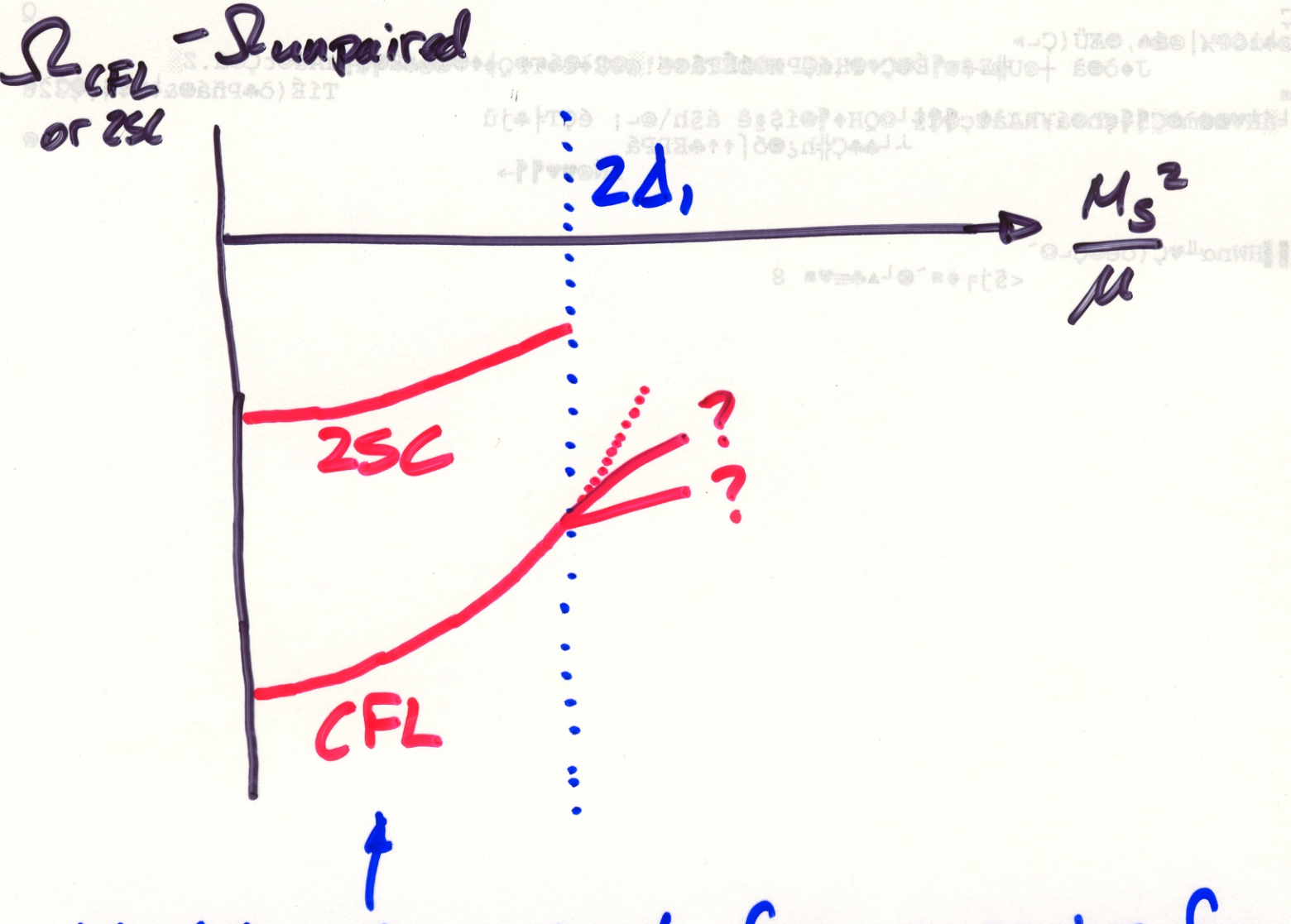
The new phase cannot be

unpaired $\Delta_1 = \Delta_2 = \Delta_3 = 0$

or 2SC $\Delta_1 = \Delta_2 = 0$; $\Delta_3 \neq 0$

quark matter. (See figure.)

So far, argument model-independent.



Model independent free energies for CFL & 2SC phases, to leading order in M_s^2, Δ^2. Alford, Rajagopal

$\Rightarrow$ whatever the new phase is that replaces CFL at $\frac{M_s^2}{\mu} = 2\Delta_1$, it is not 2SC or unpaired.

A MODEL

At the densities of interest, neither lattice QCD nor weak coupling methods are available.

We have studied the outcome of the instability in a model.

NJL model, whose 4 fermion interaction has the quantum numbers of one gluon exchange.

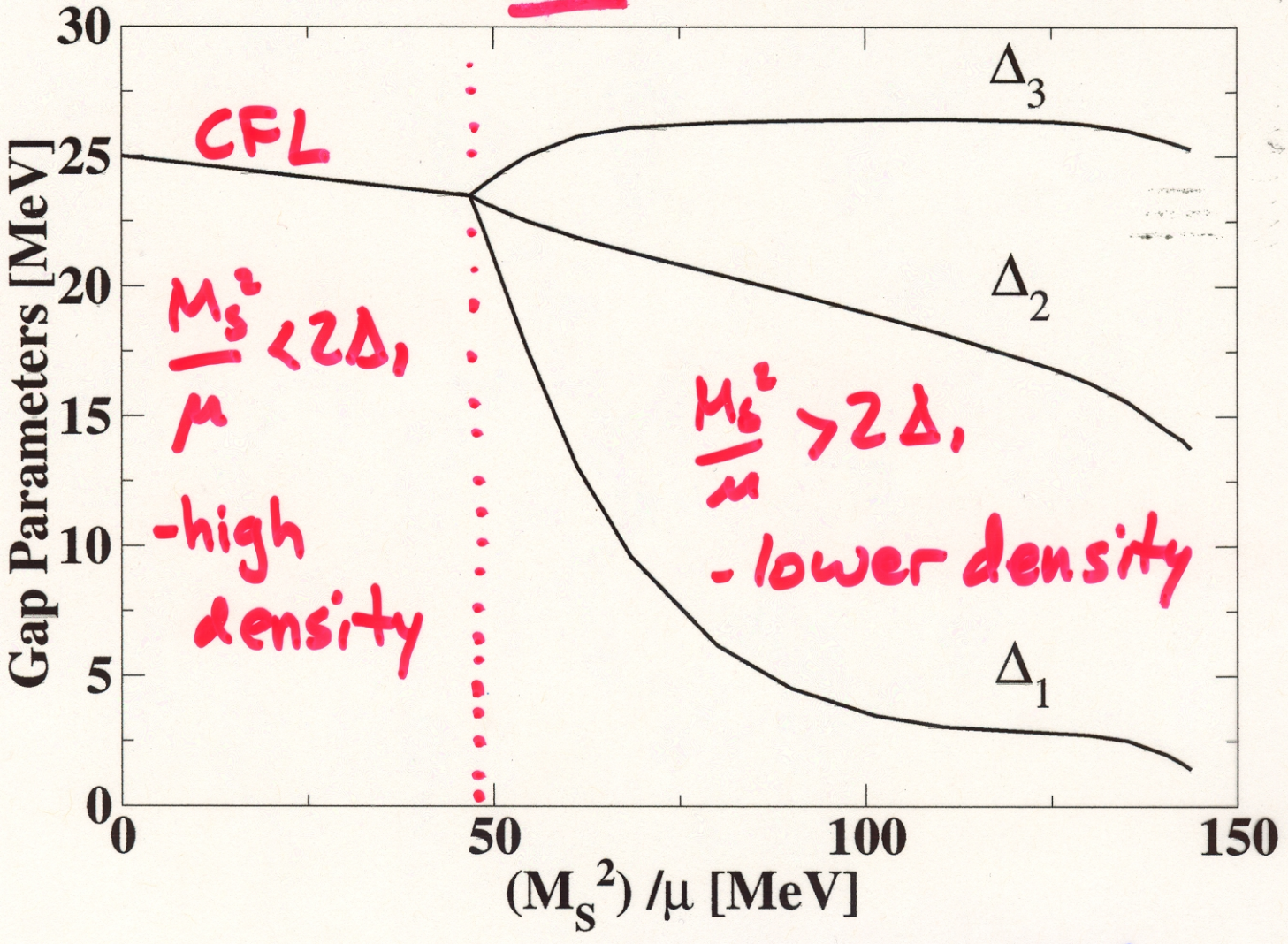
Introduce μ_e, μ_3, μ_8 by hand, as there are no gauge fields.

Solve six coupled integral equations (3 gap eq^s; 3 neutrality eq^s) to obtain $\Delta_1\ \Delta_2\ \Delta_3\ \mu_e\ \mu_3\ \mu_8$.

Figures to come all have

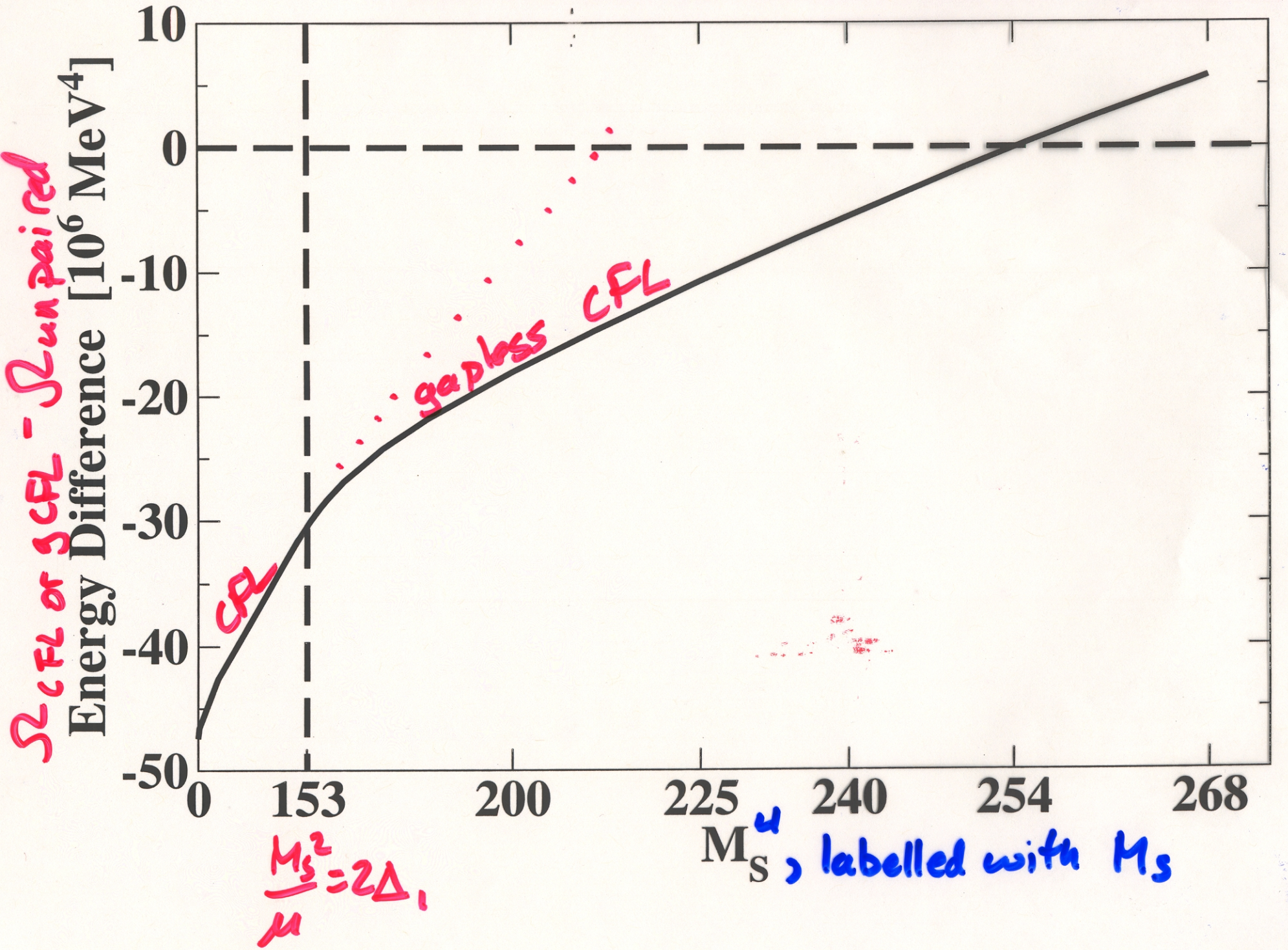
$\mu = 500$ MeV Δ's $= 25$ MeV @ $M_s = 0$ (fixes coupling)

Δ's

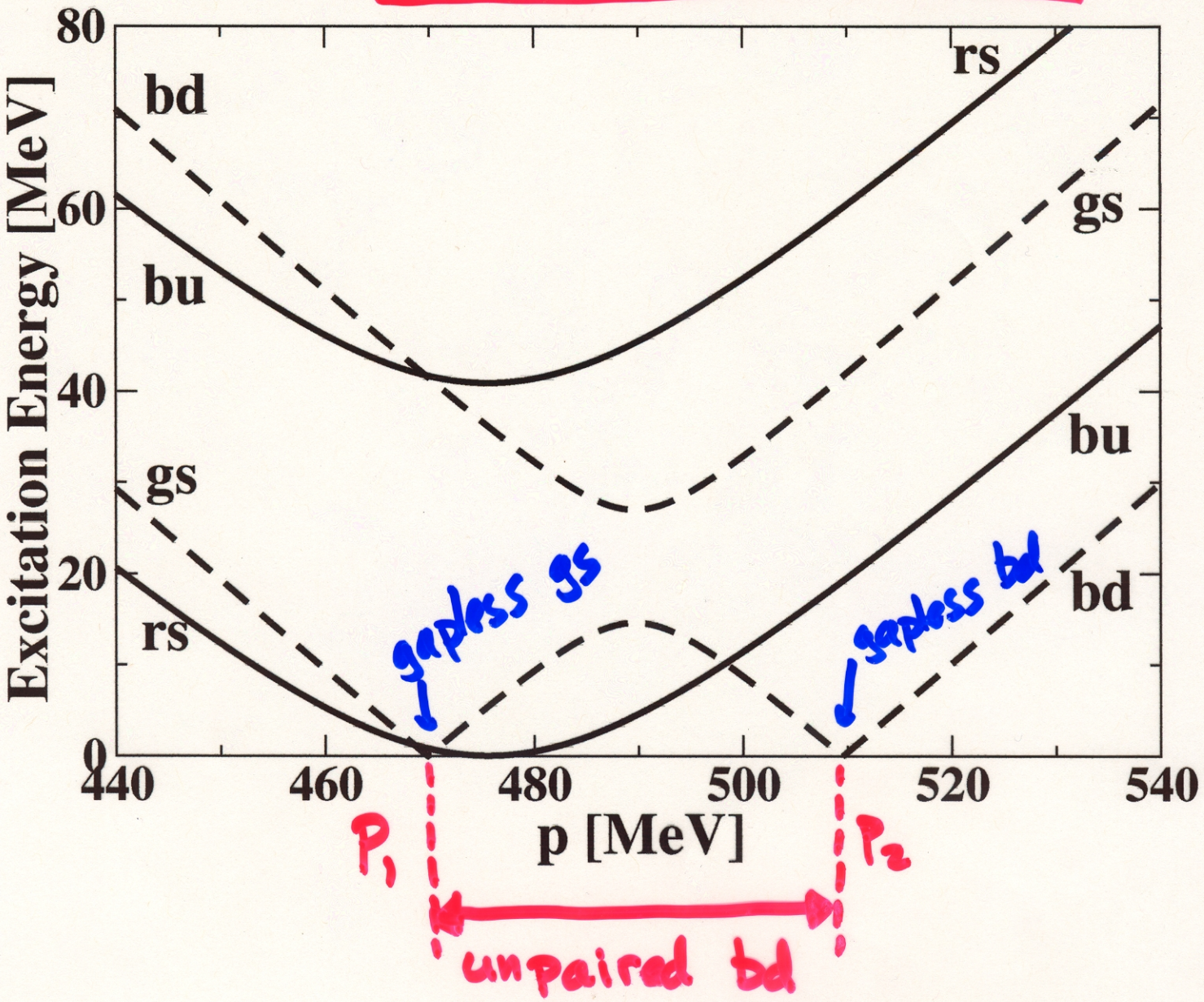


We shall see that this phase is gapless. Clearly, it still locks color and flavor: same unbroken U(1) symmetries, including $U(1)_{\tilde{Q}}$, as CFL

SECOND ORDER TRANSITION (AT T=0 !)



DISPERSION RELATIONS



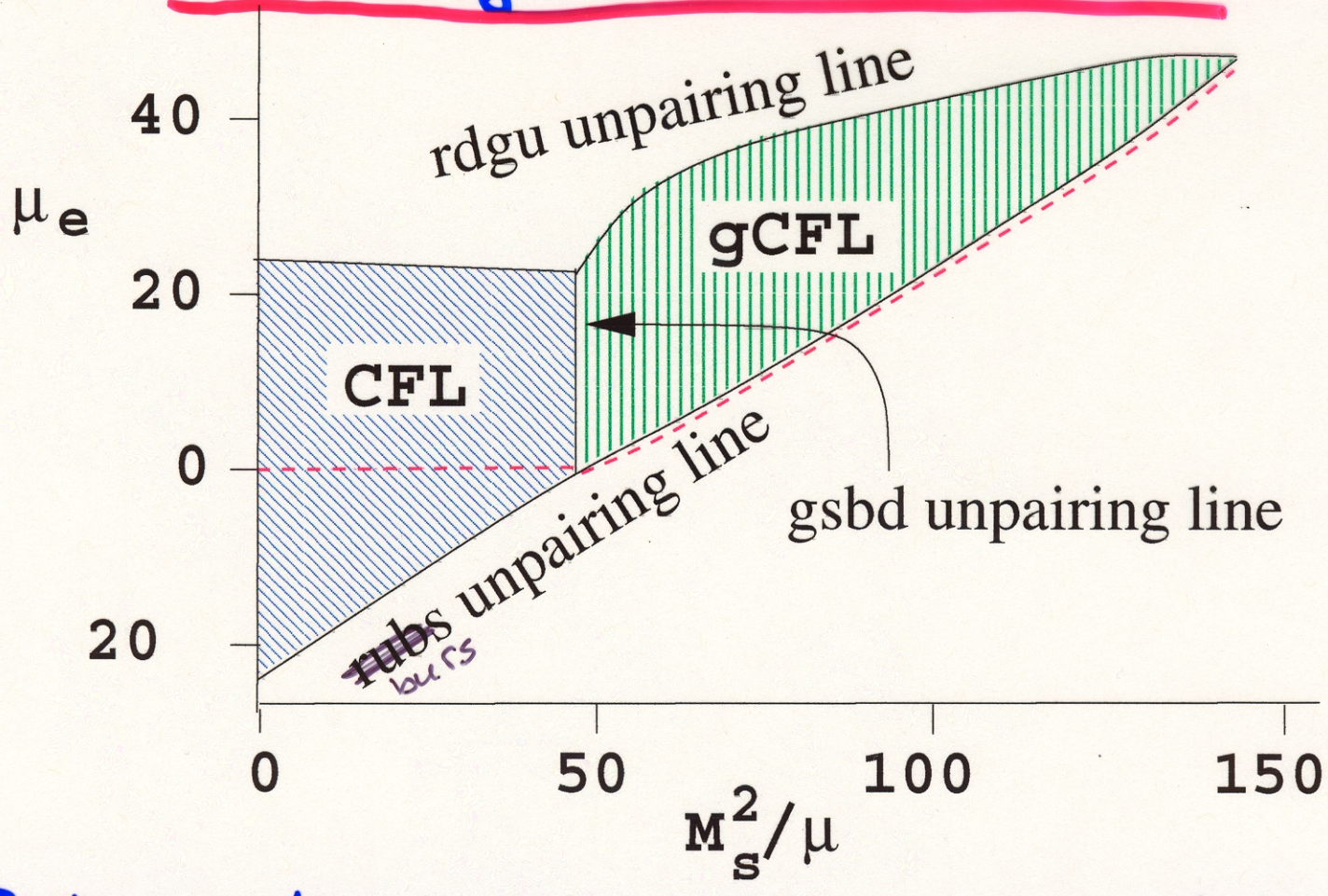
$$p_2 - p_1 \sim \left(\frac{M_s^2}{\mu} - 2\Delta_1\right)^{1/2} \Delta_1^{1/2}$$

Plot is for $M_s^2/\mu = 80$ MeV.

NB: gs & bd have $\tilde{Q} = 0$. Even with unpaired bd quarks, $\tilde{Q}$-insulator.

NB: what is happening with bu & rs ???

CFL & gCFL "BAND GAP"



Between the curves, if we ignore electrons we find (g)CFL solutions, and quark matter that is a $\tilde{Q}$-insulator:

Ω independent of $\mu_{\tilde{Q}} = -\frac{1}{q}(\mu_e + \mu_3 + \mu_8)$

between "breakdown voltages" at which either bu-rs or rd-gu unpair.

With electrons: - - - -

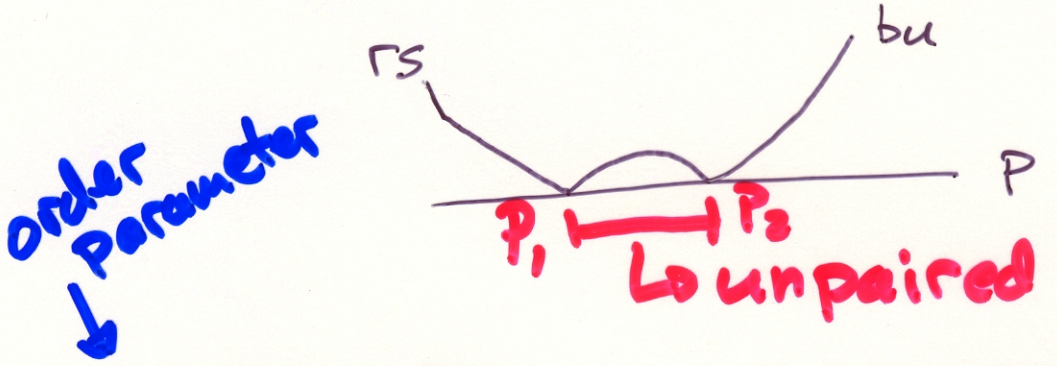
$\mu_e = 0$, no electrons, in CFL

$\mu_e \neq 0$, just outside band gap, in gCFL

NEUTRALITY WITH ELECTRONS

$\mu_e \neq 0 \Rightarrow$ electrons (with nonzero $Q + \tilde{Q}$ charge)

Must be cancelled by unpaired bu quarks, that have $\tilde{Q} = +1$.



$n_e \sim \mu_e^3$ $\quad n_{bu} \sim (P_2^{bu} - P_1^{bu})\mu^2$

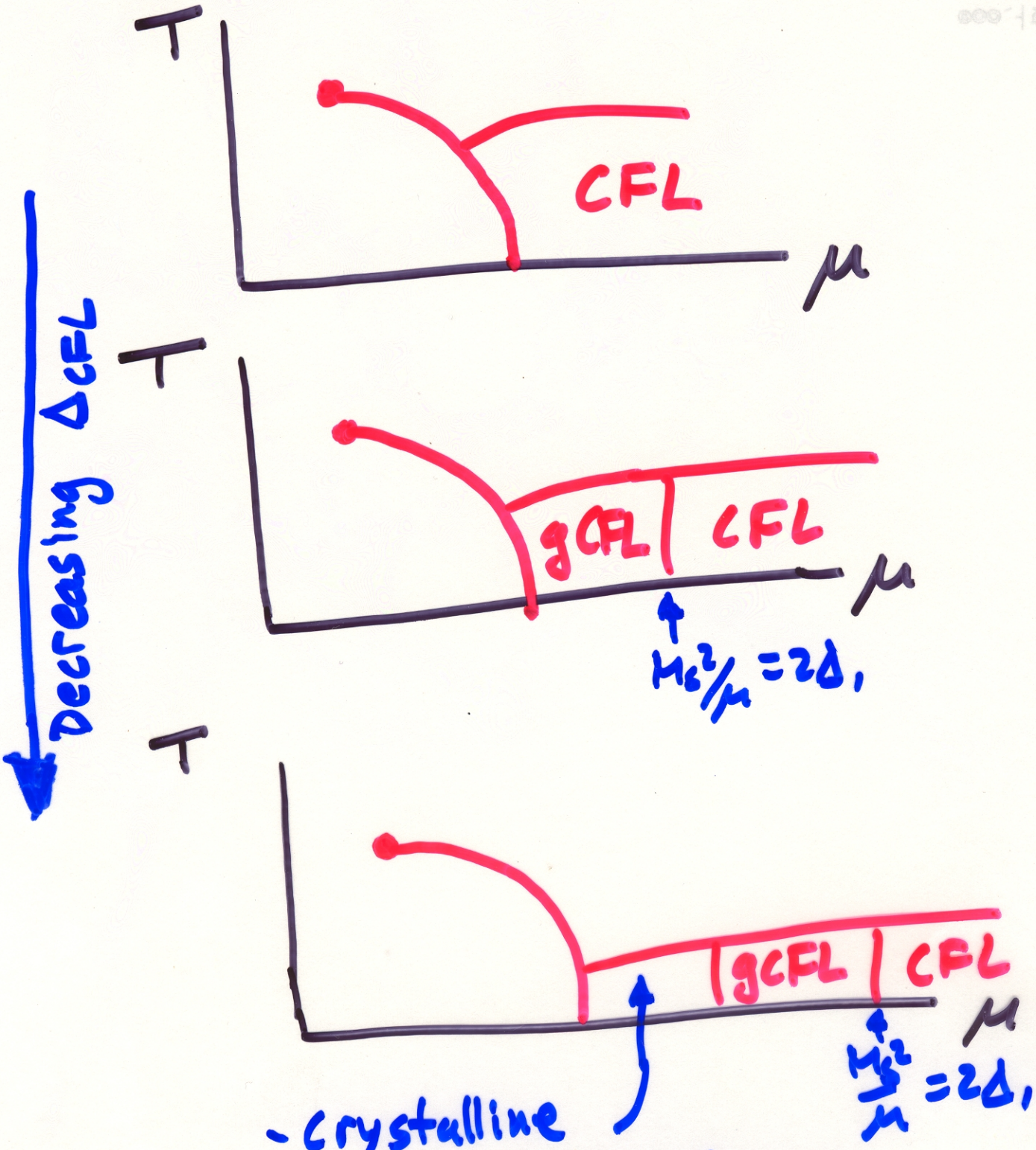
$\Rightarrow P_2^{bu} - P_1^{bu} \sim \mu_e^3/\mu^2$

$= 0.0046$ MeV in previous figure!!

Whereas gs-bd dispersion relation is "grazing" or "quadratic" only where M_s^2/μ tuned to critical point, bu-rs dispersion relation forced to be quadratic throughout gCFL region.

CFL	gCFL
$U(1)_{\tilde{Q}}$ unbroken	same
$U(1)_B$ broken, $\therefore$ superfluid	same
$\tilde{Q}$ insulator	$\tilde{Q}$ conductor
$n_e = 0$	$n_e \neq 0$
Chiral symmetry broken $\rightarrow$ Goldstone bosons	Same, but with additional contributions to GB mass.
$C_V \sim T^3$ at low T ($\ll$ that of nuclear matter)	$C_V \sim T^{1/2} \mu^2 \Delta_2^{1/2} \gg$ n.m. (dominated by gapless mode with quadratic dispersion relation)
Neutrino emissivity at low T exponentially suppressed	$\sim T^{5.5}$, ditto, faster than in any other phase.
Irrelevant to cooling of neutron stars	Controls cooling of neutron stars, if present

POSSIBLE PHASE DIAGRAMS



- crystalline color superconductor?
- gapless 2SC?
- quark matter with only very small uu, dd, ss pairing?

CONCLUSIONS

- CFL (possibly augmented by K^0-cond.) is stable for $Ms^2/\mu < 2\Delta_1$
- CFL unstable for $Ms^2/\mu > 2\Delta_1$, and is <u>not</u> replaced by 2SC
- CFL replaced by gapless CFL at a 2nd order "metal-insulator" quantum critical point. Order parameter is n_e.
- gCFL has gapless quasiparticles at three momenta:
  - two with standard linear dispersion relations
  - one with (very close to) quadratic
- $C_V \sim T^{1/2}$, ν Emissivity $\sim T^{5.5}$
- If present, gCFL dominates neutron star cooling, and star cools fast. Quantitative calculation in progress.