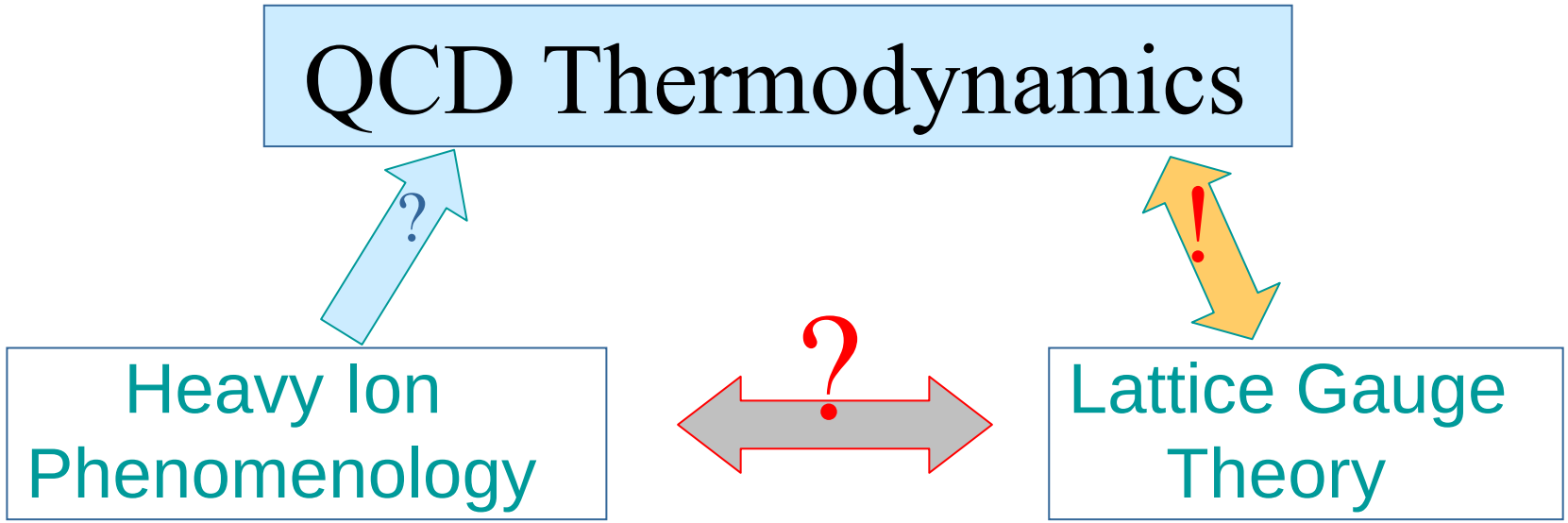
Heavy ion collisions and Lattice QCD at finite baryon density

Krzysztof Redlich

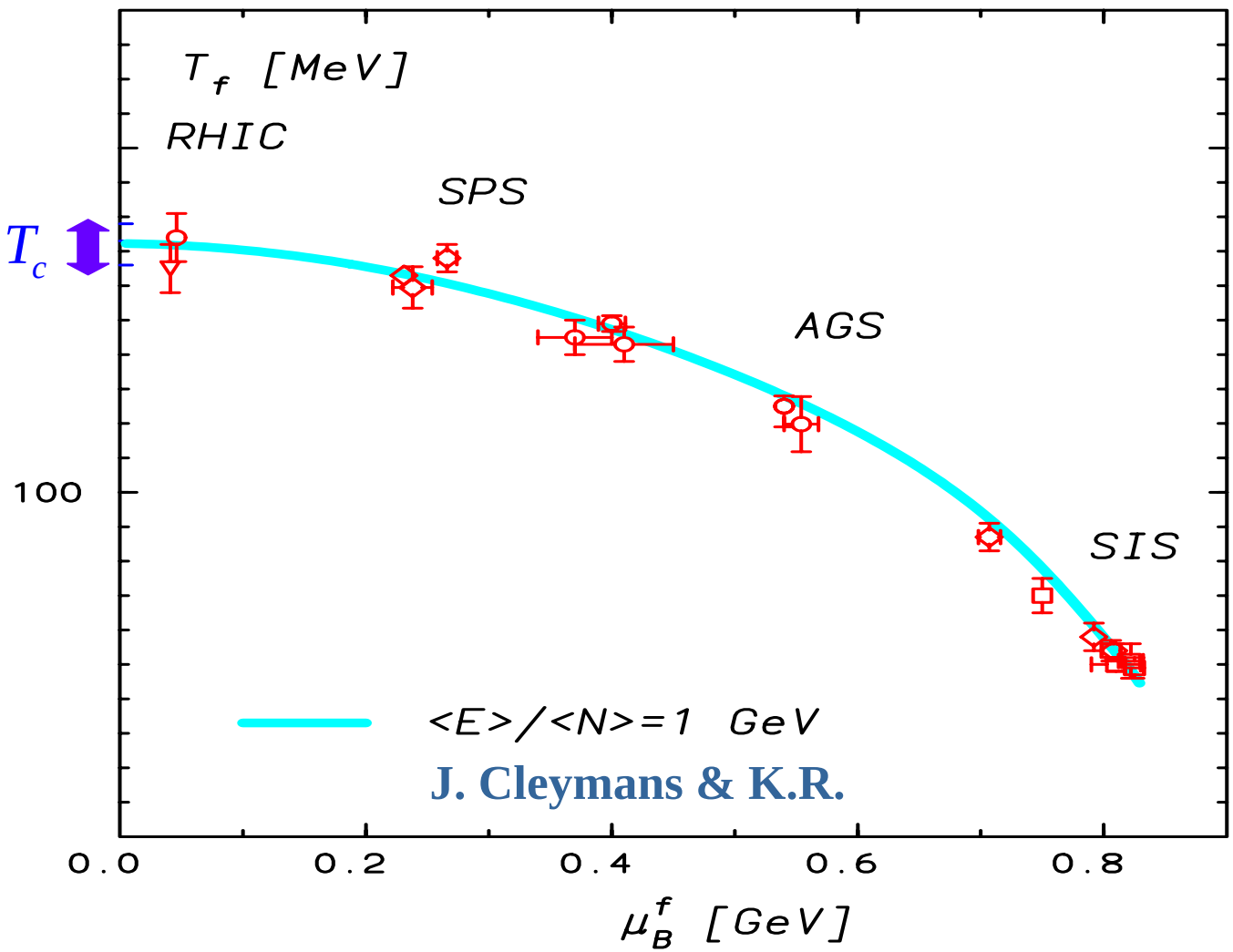


- **Equation of state at finite baryon density**
- **Freezeout and critical conditions**

Based on common work with:
Frithjof Karsch &
Abdel-Nasser Tawfik

Chemical freeze out curve from heavy ion data

$T_c^{\mu=0} \approx 173 \pm 8 [MeV]$ ⟺ **coincides with freezeout T at RHIC and SPC**



AGS, SPS, RHIC

F. Becattini, et al.
P. Braun-Munzinger, et al.
J. Cleymans, et al.
M. Kaneta & Nu Xu
J. Stachel, et al.

SIS

R. Averbeck, R. Holzmann,
V. Metag, R. S. Simon
H. Oeschler, et al..

Thermal Freezeout

⇩

see recent results of
CERES Collaboration
& Broniowski, Florkowski

QCD at non-vanishing chemical potential $\mu_q > 0$

Bielefeld-Swansea approach

$$Z(V,T,\mu) = \int DA \det M(\mu) e^{-S(V,T)}$$

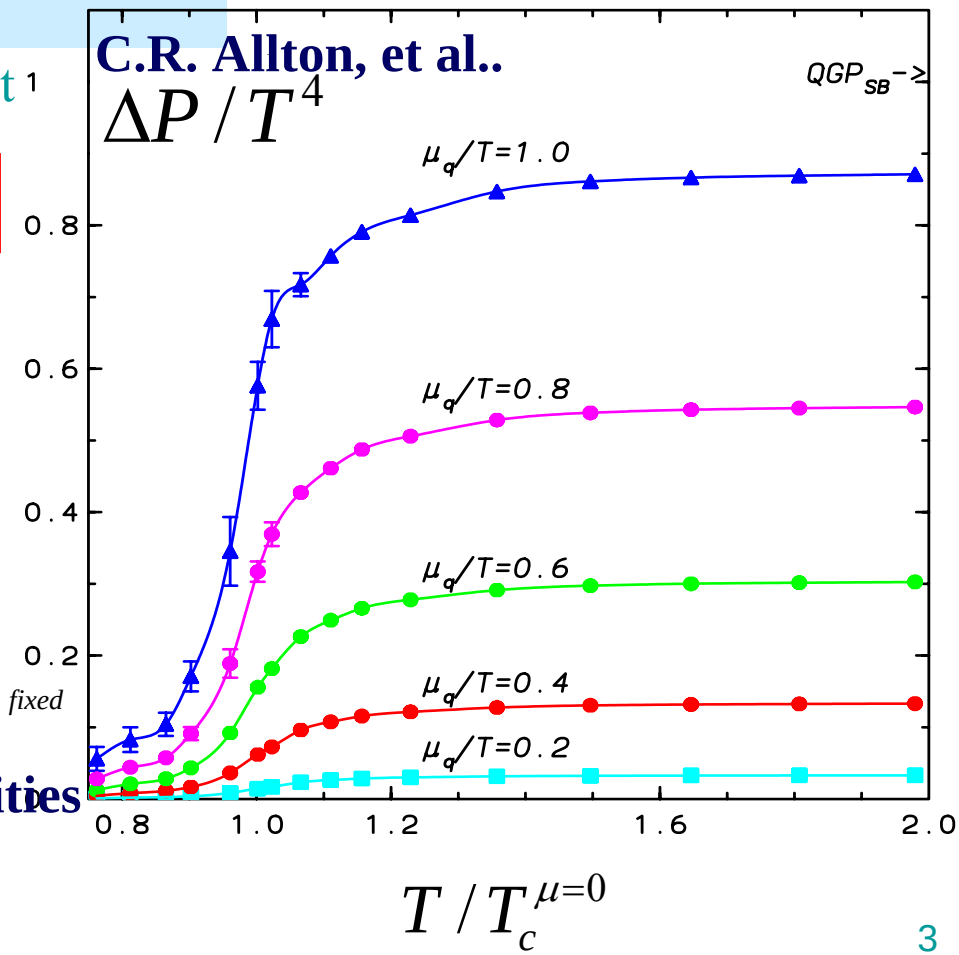
complex fermion determinant [1]

$$\Delta P = P(\mu) - P(0)$$

Taylor expansion of $P(\mu/T)$:

$$\frac{\Delta P(T,\mu)}{T^4} = \sum_{n=1}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

$$\frac{n_q}{T^3} = \left(\frac{\partial}{\partial(\mu/T)} \frac{P}{T^4}\right)_{T\ fixed}, \quad \frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu/T)^2} \frac{P}{T^4}\right)_{T\ fixed}$$



From μ dependence of chiral susceptibilities

$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \alpha(m_q) \left(\frac{\mu}{T_c(0)}\right)^2$$

Taylor expansion of resonance pressure

Factorization of the baryonic pressure

$$\frac{\Delta P_B}{T^4} \approx F(T)(\cosh(\frac{3\mu_q}{T}) - 1)$$

baryon mass spectrum

$$F(T) = \frac{1}{2\pi^2}\int dm\rho(m)(\frac{m}{T})^2 K_2(\frac{m}{T})$$

Compare wich LGT results:

$$\frac{\Delta P}{T^4} \approx F(T)\left[c_2(\frac{\mu_q}{T})^2 + c_4(\frac{\mu_q}{T})^4\right]$$

$$\frac{n_q}{T^3} \approx F(T)\left[2c_2(\frac{\mu_q}{T}) + 4c_4(\frac{\mu_q}{T})^3\right]$$

$$\frac{\chi_q}{T^2} \approx F(T)\left[2c_2 + 12c_4(\frac{\mu_q}{T})^2\right]$$

Consequences:

For fixed μ_q / T **any ratio of these observables is T-independent**

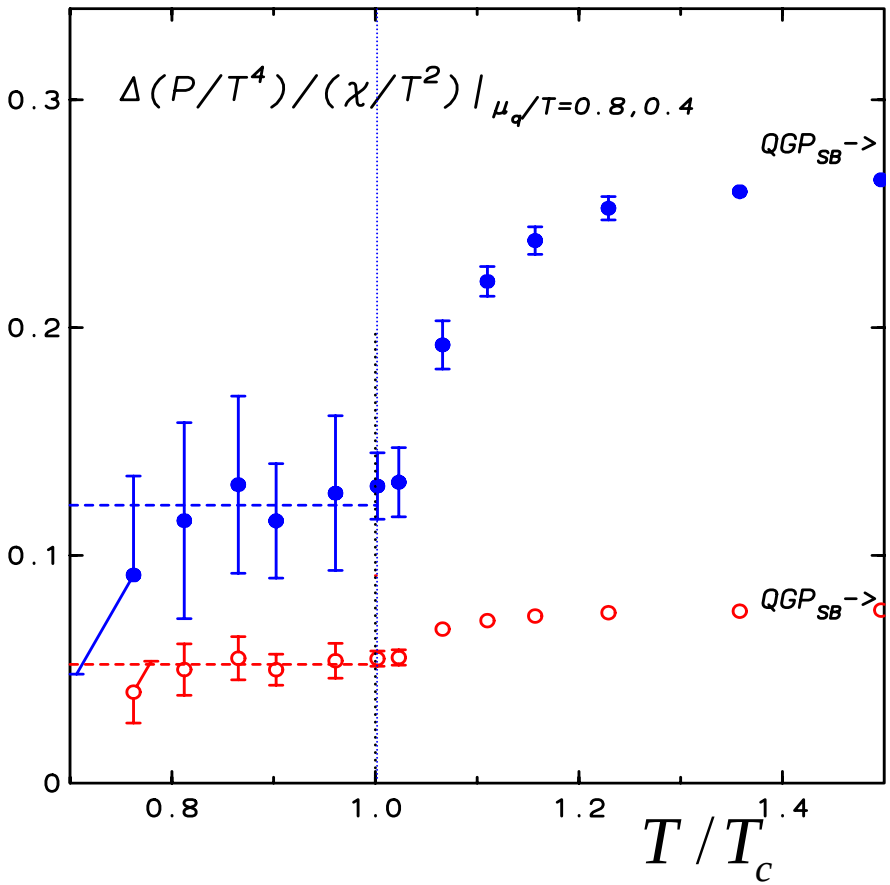
the ratio of the O(2) and O(4) coefficients: $\frac{c_4}{c_2} = \frac{3}{4}$

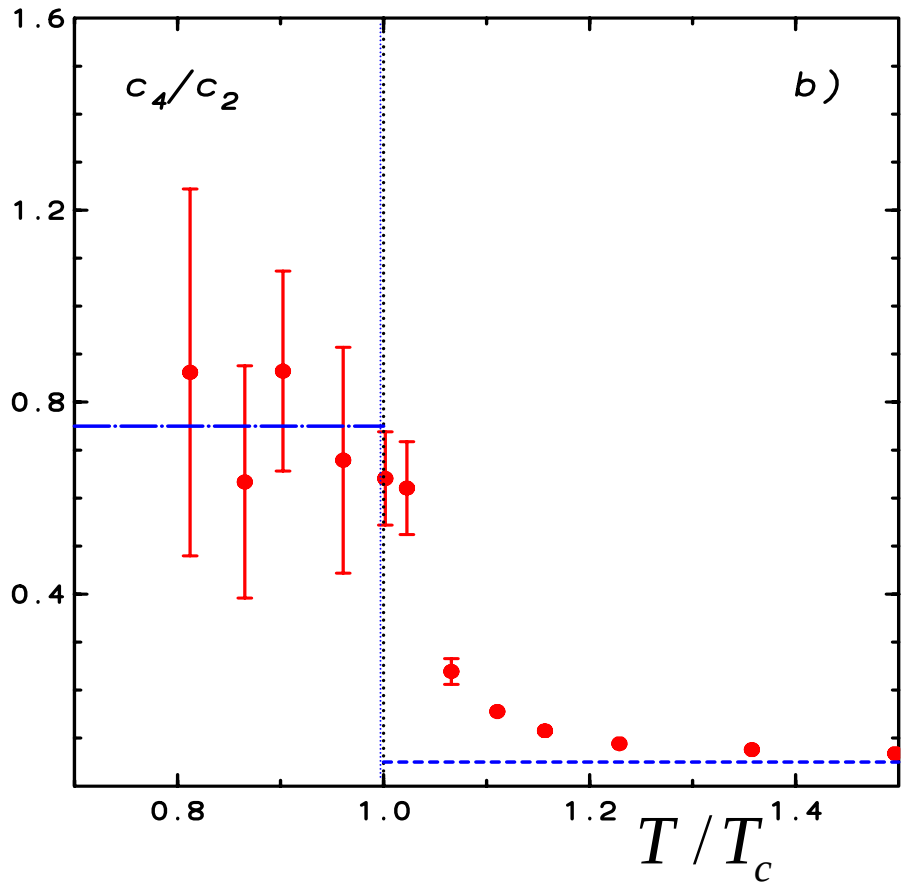
QCD partition fuction from LGT and Phenomenology

F. Karsch, A. Tawfik, K.R.

μ/T — **factorization**

Taylor cooficients of cosh(x)





$$\frac{P_B}{T^4} \approx F(T)\cosh\left(\frac{3\mu_q}{T}\right)$$

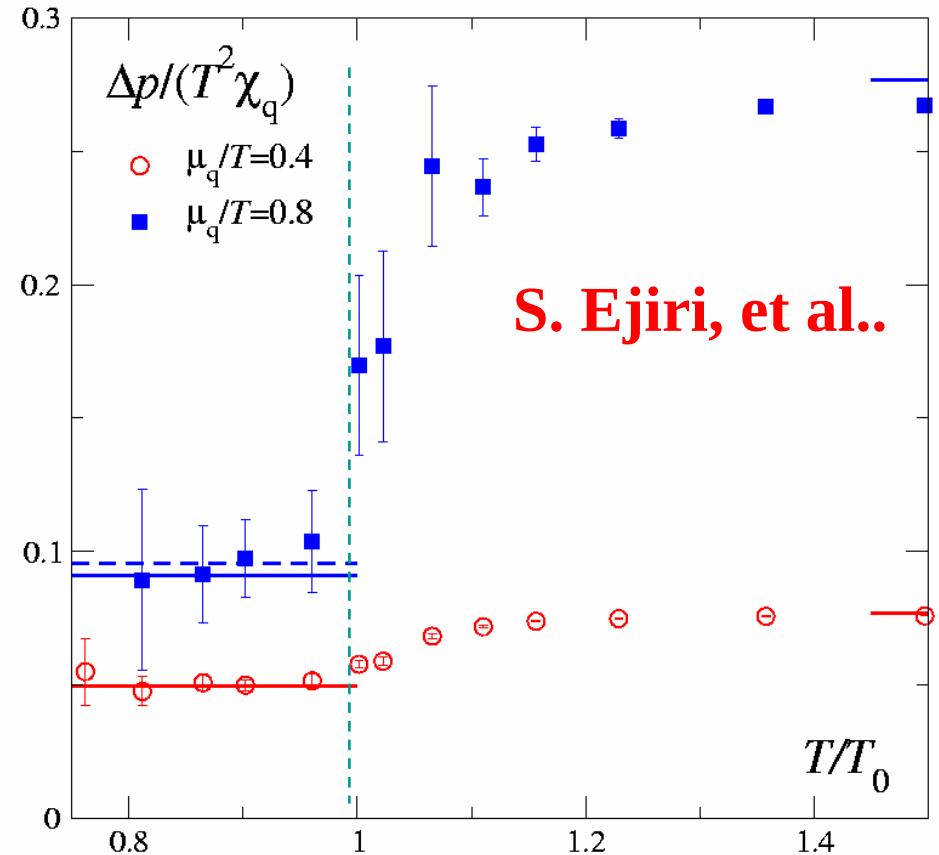
check T-dependence in F(T)

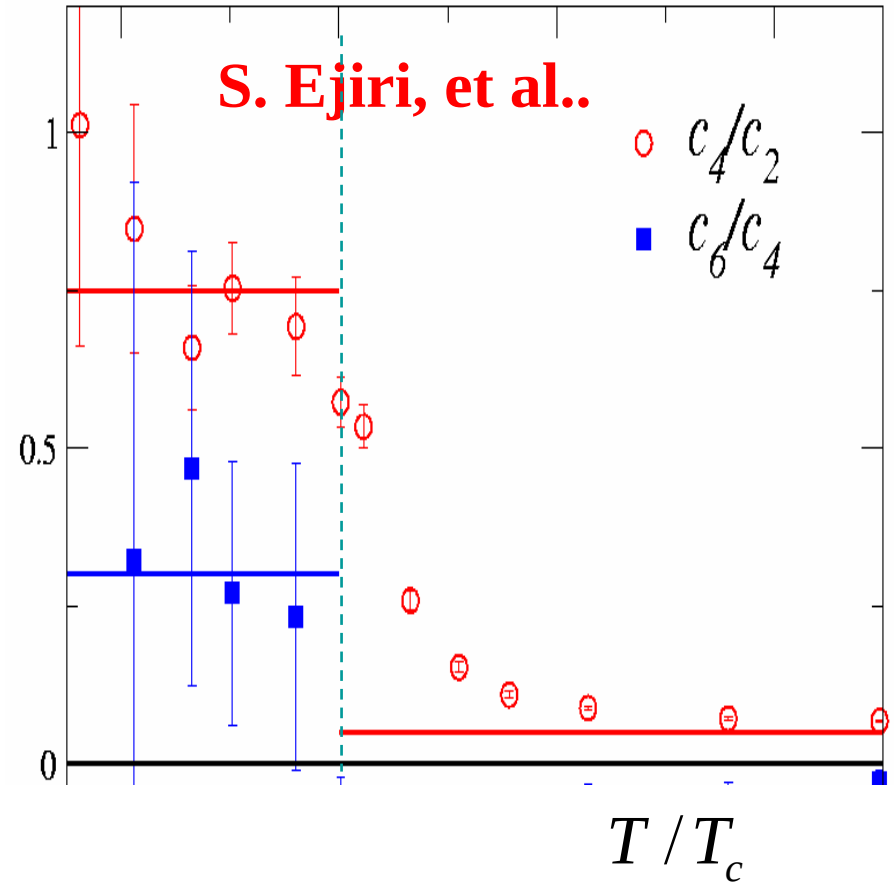
required: $m_{hadron} = f(m_{quark})$

QCD partition fuction from LGT and Phenomenology

μ/T — factorization

Taylor cooficients of cosh(x)





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Hadron Mass Spectrum versus quark mass

chiral limit $m_q \to 0$

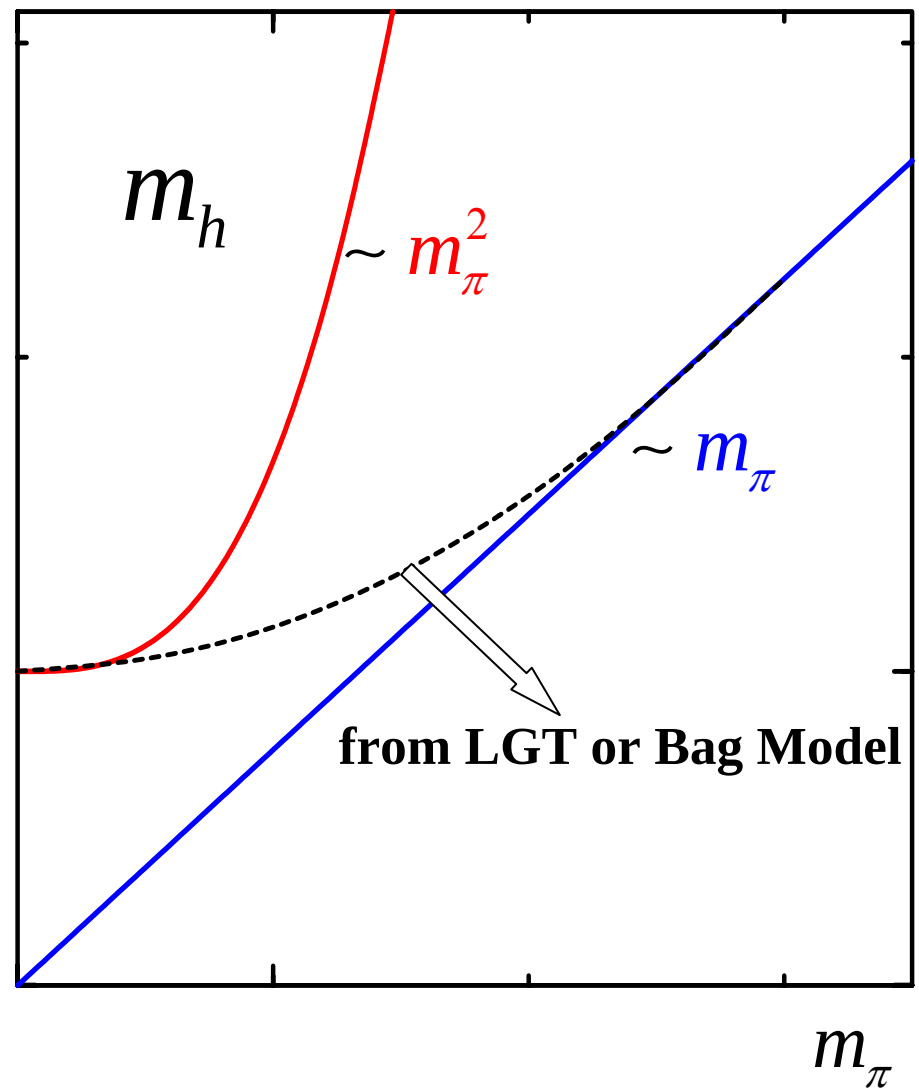
$$m_\pi^2 \simeq m_q \left[1 - c m_q \ln\left(\frac{A}{m_q} \right) \right]$$

$$m_h \simeq m_h^{phys.} + a m_\pi^2$$

quenched limit $m_q \to \infty$

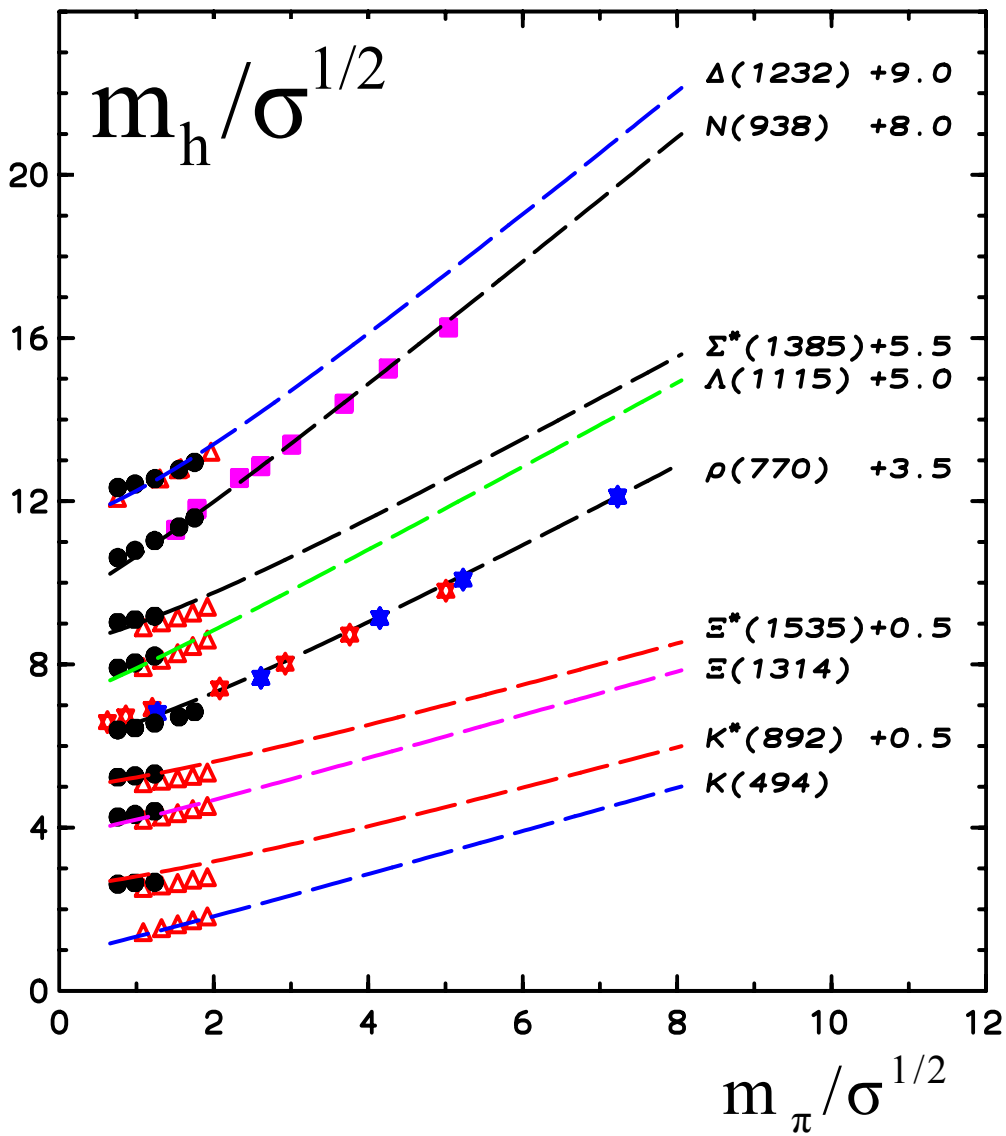
$$m_\pi \simeq 2 m_q$$

$$m_h \simeq \frac{N_f}{2} m_\pi$$



Hadron Mass Spectrum – LGT and Bag model results

F. Karsch, A. Tawfik, K.R.



LGT results for pion mass dependence of N, Δ and their parity partners

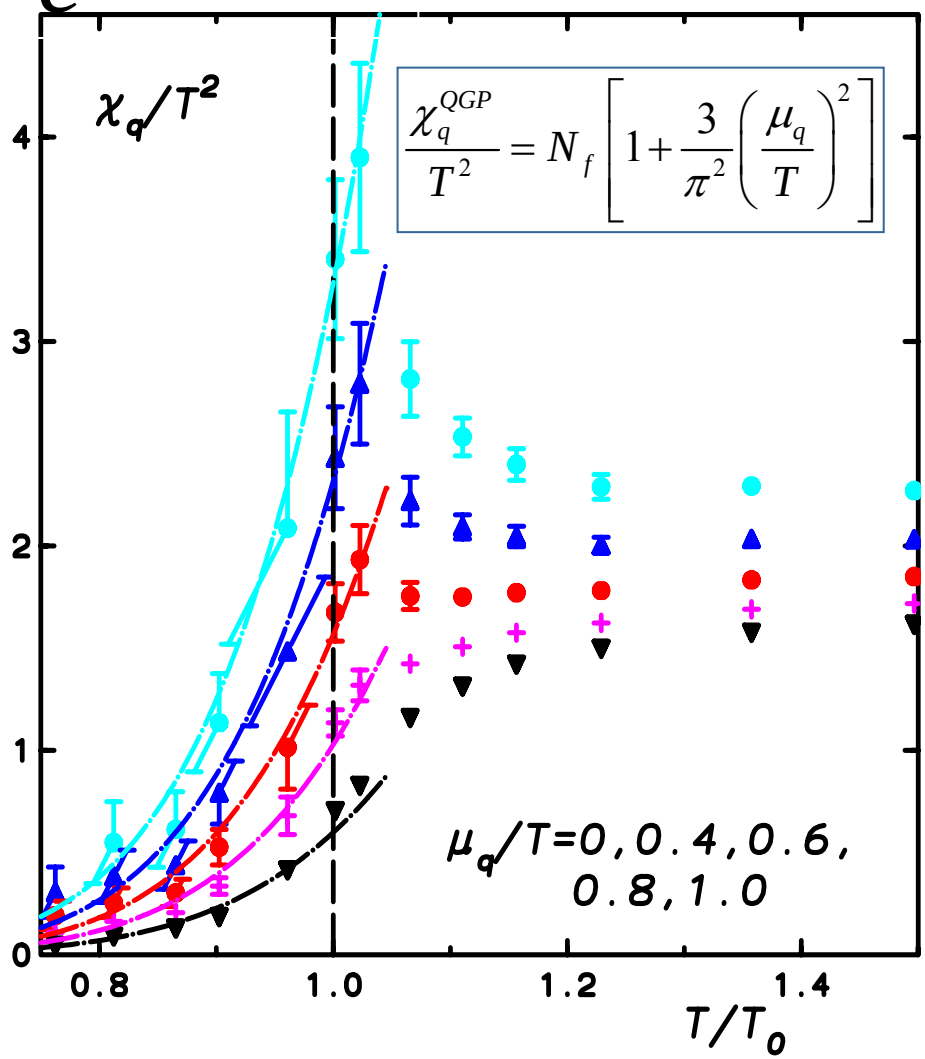
$$\frac{m_h(m_\pi)}{m_h^{phys.}} \approx 1 + A\left(\frac{m_\pi}{m_h^{phys.}}\right)^2$$

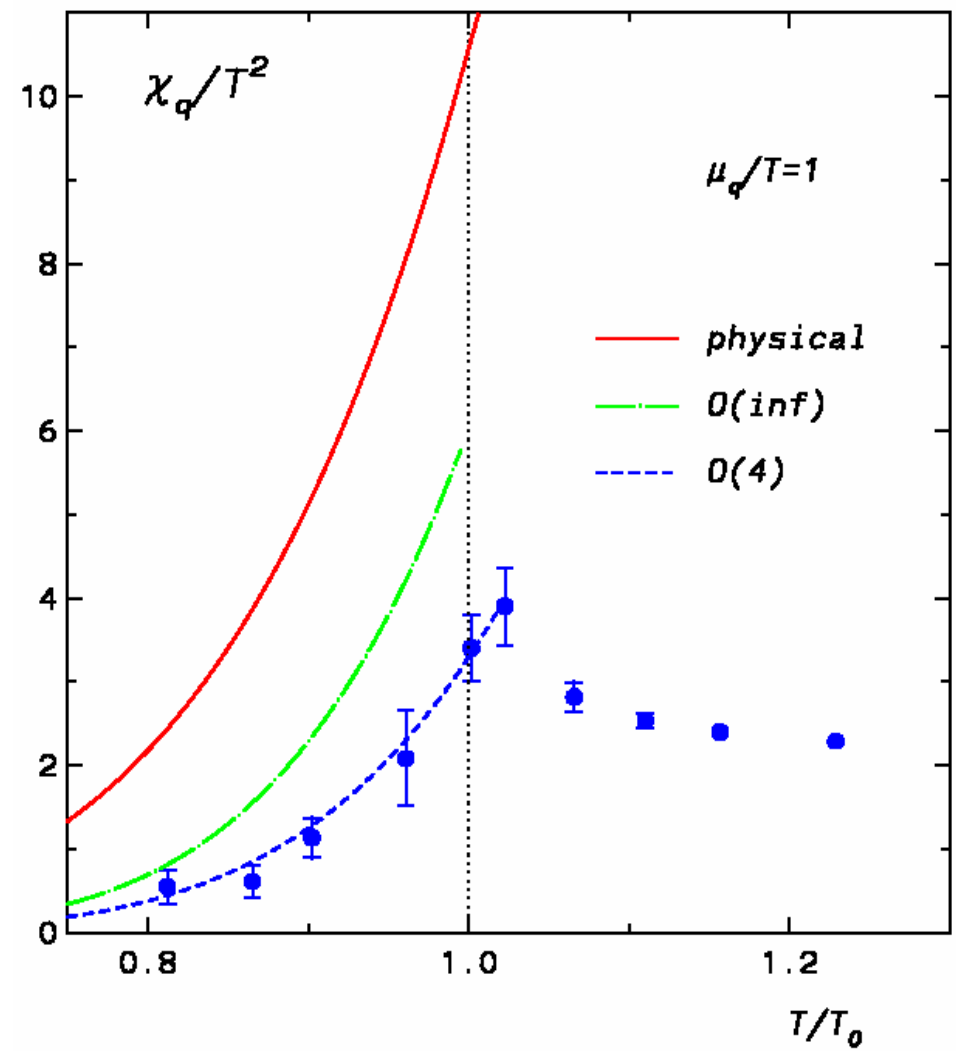
$$A = 1 \pm 0.1$$

QCDSF Coll., M. Göckeler, et al..

Hadron resonance gas model and LGT thermodynamics

$\sim e^{(\mu - m_N)/T}$





Deconfinement is density driven - (percolation)

LGT result shows:

strong dependence of T_c

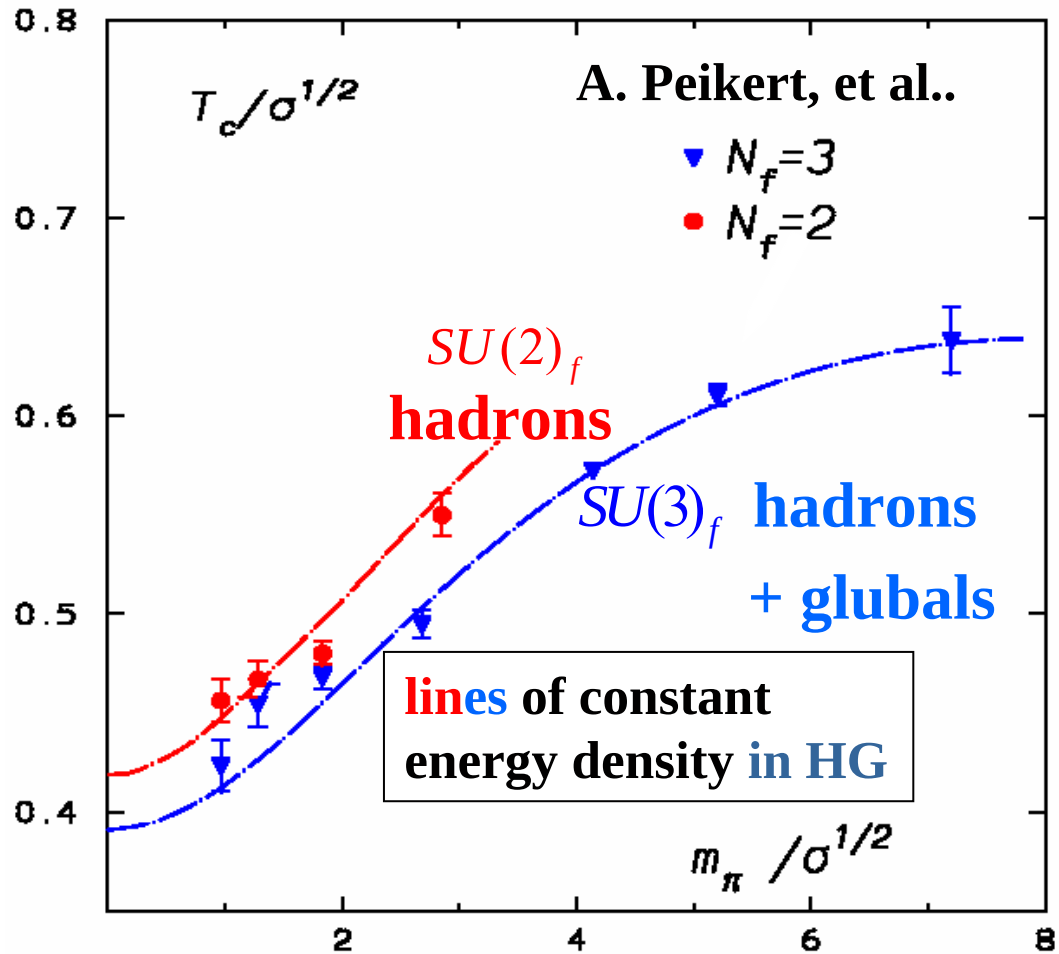
on N_f **and** m_q**, however**

$$\varepsilon(T_c) \approx (0.6 \pm 0.3) GeV / fm^3$$

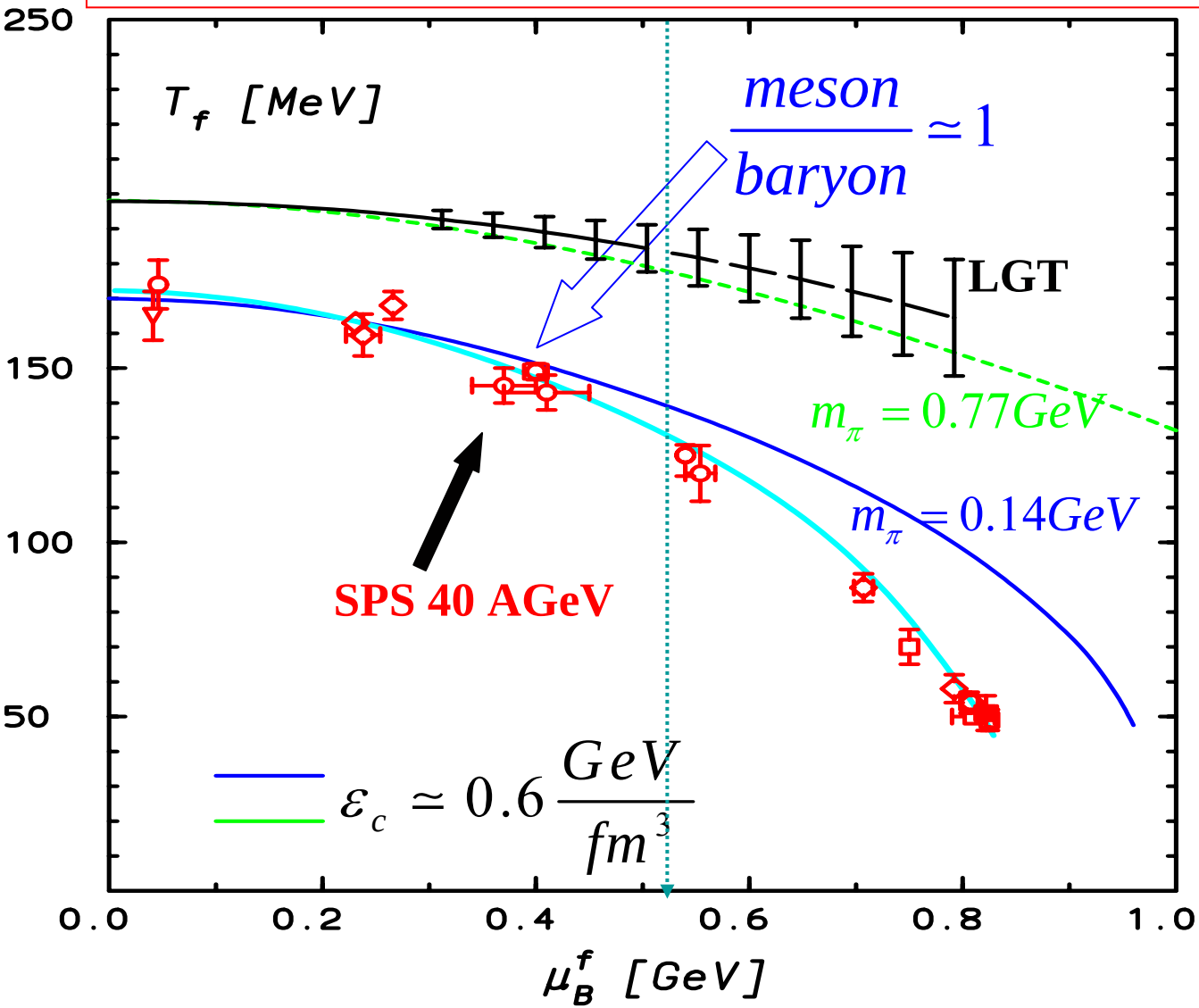
for $N_f = 2,3$ **and for all** m_q

Hadron resonance gas partition function

provides good descritpion of QCD thermodynamics



Phase boundary of fixed energy density versus chemical freezeout



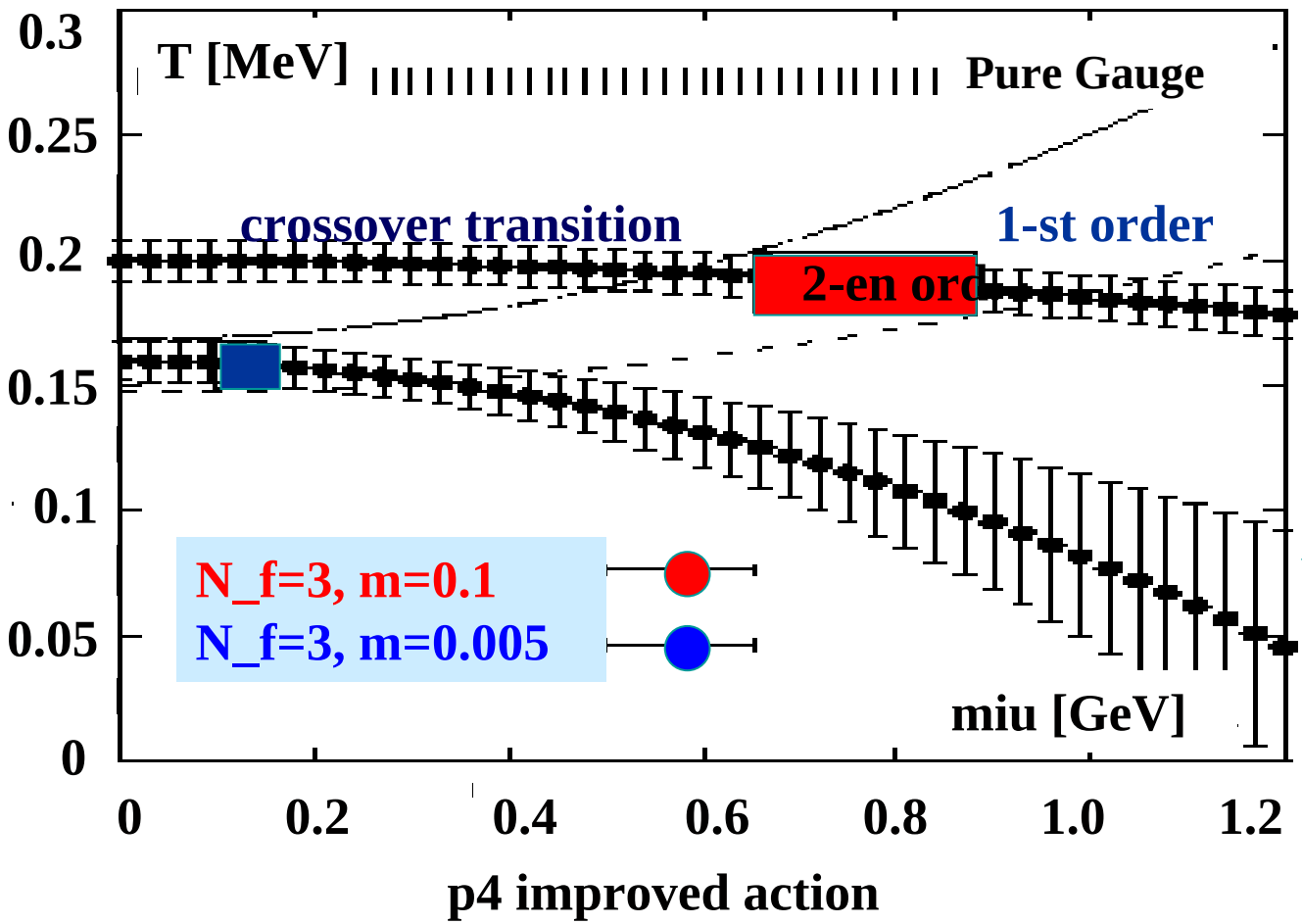
- Spliting of chemical freezeout and phase boundary surface most likely appears when the **densities of mesons and baryons are comparable**
- For E<40 AGeV strong collective effects in hadronic medium are to be expected

see also NJL results on critical conditions (T. Kunihiro et al.)

Chiral critical point in 3-flavour QCD

F. Karsch et al.

Strong dependence of the position of the second order endpoint on the quark mass!



$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \alpha(m_q)\left(\frac{\mu}{T_c(0)}\right)^2$$

$$\alpha = \begin{pmatrix} 0.025 & m_q a = 0.1 \\ 0.114 & m_q a = 0.005 \end{pmatrix}$$

Strong dependence of the slope on the quark mass

Conclusions

Hadron Resonance Gas Partition Function

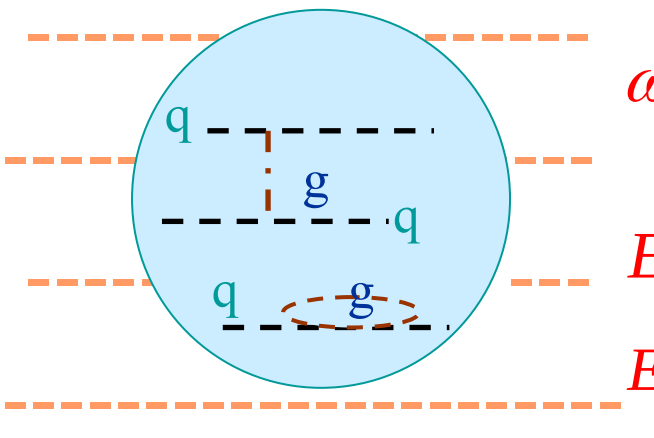
provides the consistent description of

particle yields obtained in heavy ion collisions from SIS->AGS->SPS->RHIC

the equation of state in confined phase obtained in LGT at $0 \leq \mu_B \leq 0.5\ GeV$

Thus: the system created in the late stage of heavy ion collisions is of thermal origin

Bag Model and Hadron Masses



$$E = VB + \frac{Z_0}{R} + \omega\left(m_q, R\right) + E_E + E_H$$

$$\omega = -\frac{1}{R}\sqrt{x^2(m_q R) + (m_q R)^2} \quad : kinetic \;\; energy$$

$$E_E = \frac{g^2}{2}\int d^3x \overrightarrow{E^a}\,\overrightarrow{E^a} \quad : electrostatic \;\; \text{int.} \; energy$$

$$E_H = -\frac{g^2}{2}\int d^3x \overrightarrow{B^a}\,\overrightarrow{B^a} \quad : magnetostatic \;\; \text{int.} \; energy$$

$$\vec{r}\cdot\overrightarrow{E^a} = 0$$

$$\vec{r}\times\overrightarrow{B^a} = 0$$

:surface boundary conditions

Hadron Masses from: $\frac{\partial E}{\partial R} = 0 \Rightarrow P_R = 0$

for $m_q = 0$ and $g^2 = 0 \Rightarrow$: $\boxed{M_{had.} = \frac{16\pi}{3} R_0^3 B}$