Recent Developments in Weak Coupling Color Superconductivity

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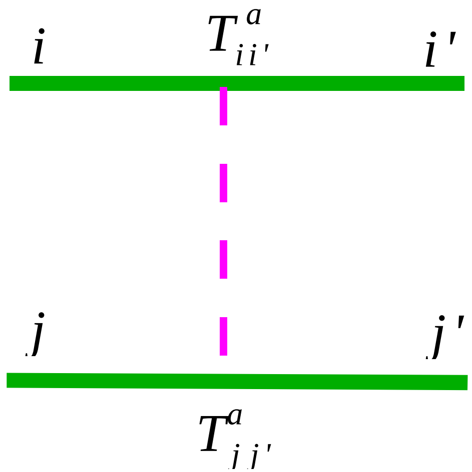
Based works

- *Q. Wang, D. H. Rischke*, Phys. Rev. D65, 054005 (2002).
- *A. Schmitt, Q. Wang, D. H. Rischke*, Phys. Rev. D66, 114010(2002); Phys. Rev. Lett. 91, 242301 (2003); nucl-th/0311006.
- *D.-F. Hou, Q. Wang, D. H. Rischke*, in preparation.
- *A. Mishra, Q. Wang, D. H. Rischke*, in preparation.
- *P. Reuter, Q. Wang, D. H. Rischke*, in preparation.

Concept of CSC



$$T^a_{ii'}T^a_{jj'} = \frac{1}{2}\delta_{ij'}\delta_{i'j} - \frac{1}{2N_c}\delta_{ii'}\delta_{jj'}$$

$$= \frac{1}{6}\left(\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'}\right) - \frac{1}{3}\left(\delta_{ii'}\delta_{jj'} - \delta_{ij'}\delta_{ji'}\right)$$

- **Attractive interaction in anti-symmetric channel**
- **CSC: Robust and Simpler**
  - **Any attractive interaction will lead to Cooper instablity**
  - **The attractive interaction in QCD is directly between two quarks**

Concept of CSC: Gap equation

Schematically gap equation is

$$\phi = g^2 \phi_0 \left[\boxed{\zeta \ln^2\left(\frac{\mu}{\phi_0}\right)} + \boxed{\beta \ln\left(\frac{\mu}{\phi_0}\right)} + \alpha \right]$$

O(1/g^2): long range static magnetic gluon —leading

O(1/g): short range non-static magnetic, static electric gluon, quark selfenergy —subleading

O(g^2): Gauge dependent term —subsubleading

Concept of CSC: Gap equation

The solution of gap eqn

$$\phi_0 = 2 b \mu \exp\left(-\boxed{\frac{\pi}{2}\frac{1}{\bar{g}}}\right)\left[1 + \boxed{O(g^2)}\right]$$

Son, 1999;
Schafer & Wilczek, 2000;
Pisarski & Rischke, 2000;
Hong etc., 2000;

$$b = 256\pi^4 [2/(N_f g^2)]^{5/2} (\pi^2+4)/8\, b'_0$$

Factor $(\pi^2+4)/8$ **from selfenergy; No contr. from vertex to prefactor;** $b_0{}'$ **depends on specific phase. [Brown et al., 2000; Wang & Rischke, 2002; Mishra et al., 2004] ==>subleaing**

$$\bar{g} = \frac{g}{3\sqrt{2}\pi}$$

==>leading

gauge dependent ==>subsubleading

Concepts of CSC: Transition temperature

For 2SC, transition-T is related to zero-T gap at Fremi surface in the same way as in BCS theory

$$T_c = \frac{e^{\gamma}}{\pi} \phi_0 \simeq 0.57 \phi_0$$

Brown, et al., 2000;
Pisarski & Rischke, 2000

BCS relation is violated in CFL and CSL phases, due the two gap structure. [Schafer, 2000; Schmitt, Wang & Rischke, 2000;]

Gap eqn. and subleading solution

Gap values at Fermi surface and T=0 and transition temperatures are given by

	2SC	CFL	CSL	Polar
Spin	0	0	1	1
ϕ_0/ϕ_0^{2SC}	1	$2^{-1/3}$	$2^{-2/3}e^{-5}$	$e^{-d(\theta)}$
T_c/ϕ_0	0.57	$2^{1/3}\times 0.57$	$2^{2/3}\times 0.57$	0.57

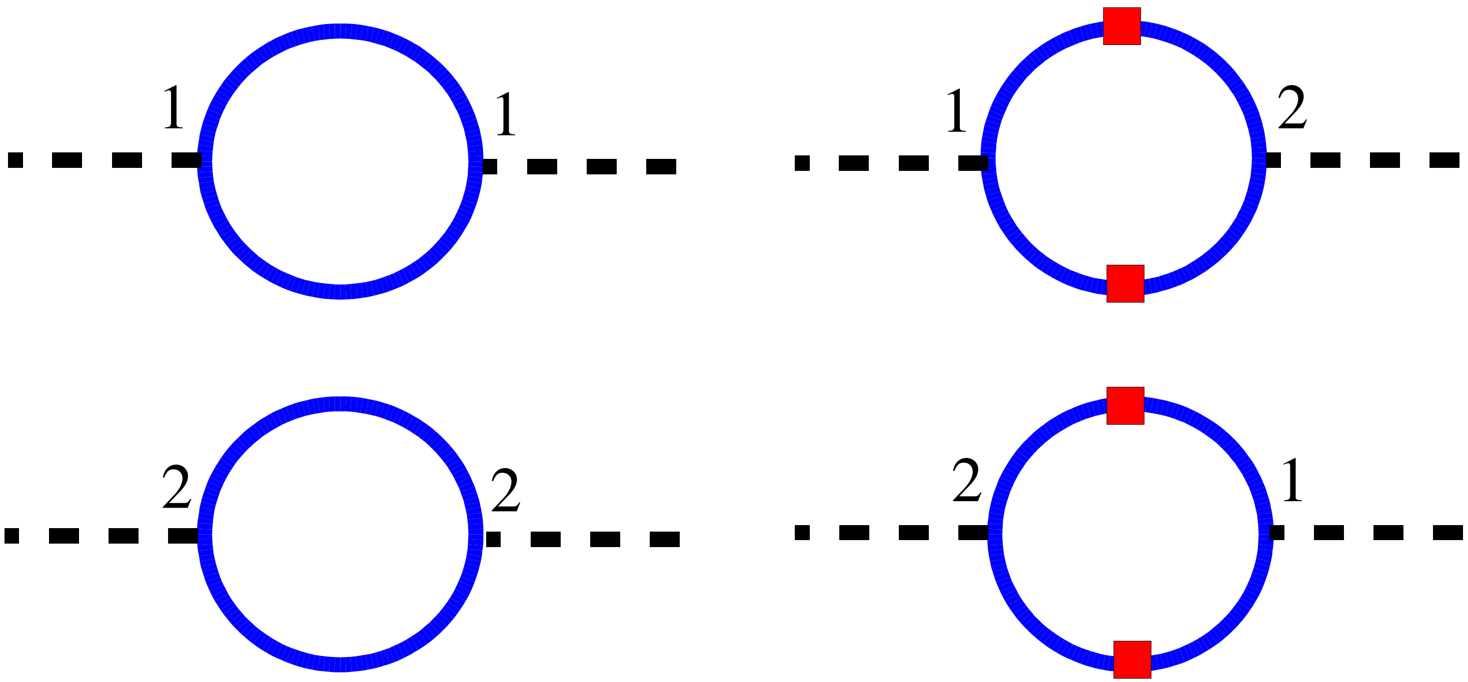
Two gap structure (non-zero gap) leads to violation of BCS relation; Sepcial two gap structure in 2SC with one zero gap--gapless mode. More gapless cases in 2SC and CFL by disparity of chemical potentials of different quarks. [Shovkovy, 2003, Alford, Kouvaris, Rajagopal, 2003]

BCS relation

angular structure of the CSL gap [Schafer 2000; Schmitt, Wang & Rischke 2002]

Meissner & Debye mass of gluon & photon in CSC

Meissner and Debye mass is the inverse penetration length of chromoelectric and chromomagnetic field into the body of a color superconductor. In NG basis, there are four diagrams



Meissner & Debye mass of gluon & photon in CSC

Question: whether all eight gluons and photons become massive in super phase?

Answer: this depends on the pattern of local symmetry breaking. If there are residual symmetries, the corresponding gauge bosons remain massless. A residual symmetry group is the one that leaves gap parameter invariant. Consider $SU(3)_c \times U(1)_{em}$

$$(g_c \times g_{em})\Delta(g_c^T \times g_{em}^T) = \Delta \qquad \text{where} \quad g_c \in SU(3)_c, \quad g_{em} \in SU(3)_{em}$$

	Representation	generators of residual group	η
2SC	$\bar{3}_c \times 1_f \times 1_J$	$T_1, T_2, T_3, Q + \eta T_8$	$-1/\sqrt{3}$
CFL	$\bar{3}_c \times \bar{3}_f \times 1_J$	$Q + \eta T_8$	$2/\sqrt{3}$
CSL	$\bar{3}_c \times 3_J$	--	--
polar	$\bar{3}_c \times 3_J$	$T_1, T_2, T_3, Q + \eta T_8$	$2/\sqrt{3}q$

Meissner & Debye mass of gluon & photon in CSC

- Zero-temperature rotated Debye masses

	$m^2_{D,\tilde{8}\tilde{8}}$	$m^2_{D,\tilde{\gamma}\tilde{\gamma}}$	$\cos(\theta_D)$
2SC	$3\,g^2$	$2\,e^2$	1
CFL	$(4\,e^2+3\,g^2)\zeta$	0	$3\,g^2/(4\,e^2+3\,g^2)$
Polar	$3\,g^2$	$18\,q^2e^2$	1
CSL	$3\,\beta\,g^2$	$18\,q^2e^2$	1

$Unit=N_f\mu^2/6\pi$
$\zeta\equiv(21-8\ln 2)/54$
$\alpha\equiv(3+4\ln 2)/27$
$\beta\equiv(6-4\ln 2)/9$

- Zero-temperature rotated Meissner masses

	$m^2_{M,\tilde{8}\tilde{8}}$	$m^2_{M,\tilde{\gamma}\tilde{\gamma}}$	$\cos(\theta_M)$
2SC	$g^2/3+e^2/9$	0	$3\,g^2/(e^2+3\,g^2)$
CFL	$(4\,e^2/3+g^2)\zeta$	0	$3\,g^2/(4\,e^2+3\,g^2)$
Polar	$g^2/3+4\,q^2e^2/9$	0	$g^2/(12\,q^2e^2+g^2)$
CSL	$\beta\,g^2$	$6\,q^2e^2$	1

Meissner & Debye mass of gluon & photon in CSC

- Rotated photon can penetrate into 2SC and CFL phases => <u>no</u> electromagnetic Meissner effect. [Alford, Berges & Rajagopal 2000; Manuel & Rajagopal 2002].
- Rotated photon in CSL phase has a non-vanishing mass =>electromagnetic Meissner effect.
- Although rotated photon in polar phase has a zero mass but a system with 2 or 3 flavors still exhibits electromagnetic Meissner effect because of different chemical potential or no single mixing angle for all flavors.
- Spin-1 color-superconductor (CSL or polar phase) is expected to be a type-I superconductor.

Gauge parameter independence of the gap

- Gauge parameter

$$D_{\mu\nu}(P)=\frac{P^L_{\mu\nu}}{P^2+\Pi_l}+\frac{P^T_{\mu\nu}}{P^2+\Pi_t}-\xi\frac{P_\mu P_\nu}{P^4}$$

- Gauge parameter independence

$$\phi(\xi')=\phi(\xi)$$ [Gerhold & Rebhan 2003]

- Gauge independence in mean field approx.

It is believed that the gap is only invariant at subleading order [Rajagopal & Shuster 2000, Pisarski & Rischke 2002]

$$\phi(\xi')=\phi(\xi)+\Delta\phi,\quad \Delta\phi\sim O(g^2\phi)$$

Guage parameter inependence of the gap

- **There is a problem with covariant gauge: the gap is not invariant at subleading order [Hong et al., 2000]**

$$\phi = e^{3\xi/2}\phi_0$$

where $e^{3\xi/2}$ **is from 1-loop correction**

- **Hint to the solution: impose on-shell condition and use Ward identity in NG-basis!**

$$K_{on} = (\epsilon_{\boldsymbol{k}}, \boldsymbol{k}) \rightarrow (\phi, \mu\hat{k})$$

$$S^{-1}(K_{on})\Psi(K_{on}) = 0$$

$$\overline{\Psi}(K_{on})S^{-1}(K_{on}) = 0$$

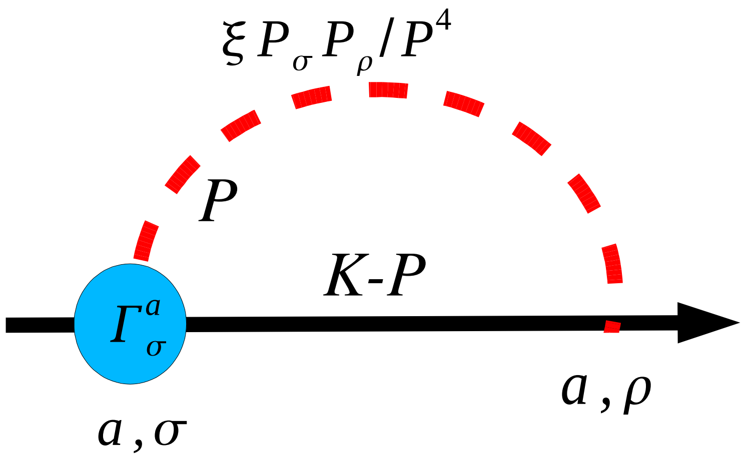
$$P_\sigma \Gamma^a_\sigma = g[\boldsymbol{S}^{-1}\boldsymbol{T}^a - \boldsymbol{T}^a\boldsymbol{S}^{-1}]$$

$$where \;\; \boldsymbol{T}^a = \begin{pmatrix} T^a & 0 \\ 0 & -T^{Ta} \end{pmatrix}$$

Gauge parameter independence of the gap

Generalized Ward identity:

$$P_\sigma \Gamma^a_\sigma = g\left[\boldsymbol{S}^{-1}(K)\,\boldsymbol{T}^a - \boldsymbol{T}^a\,\boldsymbol{S}^{-1}(K-P)\right]$$



On-shell condition:

$$\boldsymbol{S}^{-1}\Psi(K_{on})=0,\quad \overline{\Psi}(K_{on})\,\boldsymbol{S}^{-1}(K_{on})=0$$

$$I_\xi \sim -\xi g^2 \int d^4P\,\frac{1}{P^4}\left[\boxed{\boldsymbol{S}^{-1}(K)\,\boldsymbol{T}^a} - \boxed{\boldsymbol{T}^a\,\boldsymbol{S}^{-1}(K-P)}\right]\boldsymbol{S}(K-P)\,\boldsymbol{T}^a\,\gamma_\rho P_\rho$$

$$\boxed{\begin{aligned}&\overline{\Psi}(K_{on})\,I_\xi\,\Psi(K_{on})\\ \to\ &\overline{\Psi}(K_{on})\,\boldsymbol{S}^{-1}(K_{on})=0\end{aligned}}$$

$$\boxed{\int d^4P\,\frac{P_\rho}{P^4}=0}$$

$$\boxed{I_\xi=0}$$

Our perturbative calculation also show that the gauge part is of subsubleading order

Summary of recent developments

- A general formalism is proposed to derive and solve the gap equation up to subleading order.
- The gap parameters and transition temperatures are calculated for all three types of color superconductors in the number of flavors.
- Two gap structure of the order parameter is found to lead to violation of the BCS relation.
- By imposing quasi-particle mass-shell conditions and with the help of generalized Ward identity in super phase, we prove in the mean field approx. that the gap on the Fermi surface is gauge independent up to subleading order.