Rescattering Effects on HBT Interferometry

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What does HBT measure?

- For stars it measures the radius. Rescattering in the photosphere is irrelevant since the thickness is so small compared to the radius. Opaque.
- For elementary particle collisions it measures the space-time distribution of the production points. Rescattering effects are usually negligible. Transparent.

What does HBT measure?

- For heavy ion collisions, rescattering after production may be significant. The source is neither opaque nor transparent. The usual interpretation is that HBT measures the distribution of last scatterings. What if the last scattering is very soft? What counts as the "last scattering"?

Assumptions:

- Nonrelativistic
- Localized wave-packet produced at t_P and momentum eigenstate observed at $t_2 \gg t_P$
- Different wavepackets are produced with uncorrelated phases
- No interactions between produced particles, only with static localized potentials

$$\psi_{\text{produced}}(\mathbf{x}) = \int d^3p/(2\pi)^3\ \varphi(\mathbf{p},x_P)\ \exp\{i\chi(x_P)\}\ \exp\{i[\mathbf{p}\bullet(\mathbf{x}-\mathbf{x}_P) - E_p(t-t_P)]\}$$

Wave packet with phase χ centered at $\mathbf{x}_P$ at time t_P.
Momentum space wave function φ may be chosen real.

$$\psi_{\text{observed}}(\mathbf{x}) = \exp\{i[\mathbf{k}\bullet\mathbf{x} - E_k t_2)]\}$$

Momentum eigenstate observed by detector at time $t_2 >> t_P$.

$$A = \int d^3x_1 d^3x_2\ \psi^*_{\text{obs}}(\mathbf{x}_2)\ G(\mathbf{x}_2,t_2;\ \mathbf{x}_1,t_1)\ \psi_{\text{prod}}(\mathbf{x}_1)$$

Single particle amplitude with G = propagator
that includes the effects of rescattering.

For bosons that do not interact with each other, and for wavepackets produced with random phases, the correlation function is

$$\boxed{C(\mathbf{k}_1,\mathbf{k}_2) = 1 + \left|\int d^4x_P \exp(iq\bullet x_P)\ \rho_{eff}(\mathbf{k}_1,\mathbf{k}_2;x_P)\right|^2}$$

$$\rho_{eff} = \left[K(\mathbf{k}_1;x_P;t_2,t_1)/\sqrt{P(\mathbf{k}_1)}\right]\left[K^*(\mathbf{k}_2;x_P;t_2,t_1)\ /\sqrt{P(\mathbf{k}_2)}\right]\rho(x_P)$$

$$K(\mathbf{k};x_P;t_2,t_1) = \int d^3p/(2\pi)^3 \exp[ix_P\bullet(p\text{-}k_1)] \exp[-iE_p t_1]$$
$$*\ G(\mathbf{k},t_2;\mathbf{p},t_1)\ \varphi(\mathbf{p},x_P)$$

The effective density of wavepacket sources.

$$P(\mathbf{k}) = \int d^4x_P\ \rho(x_P)\ \left|K(\mathbf{k};x_P;t_2,t_1)\right|^2$$

The single particle momentum distribution.

When there is no rescattering

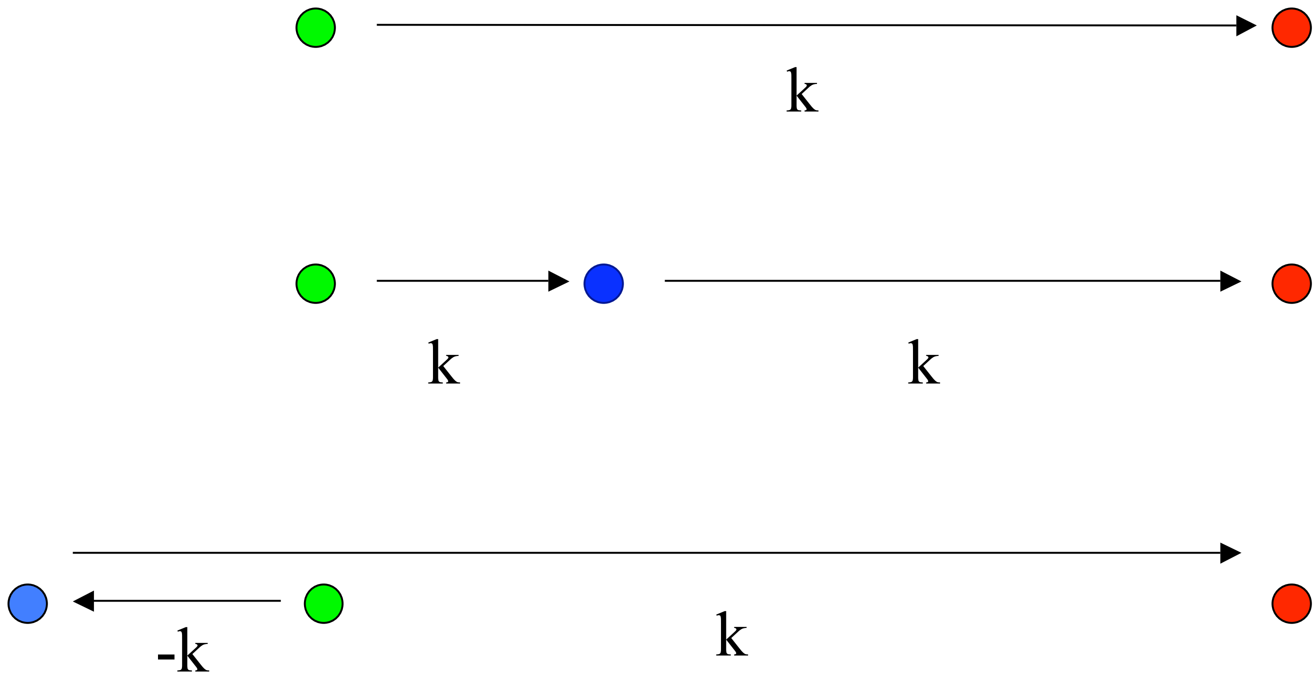
$K = K_{free}$ and $\rho_{eff} = \rho$.

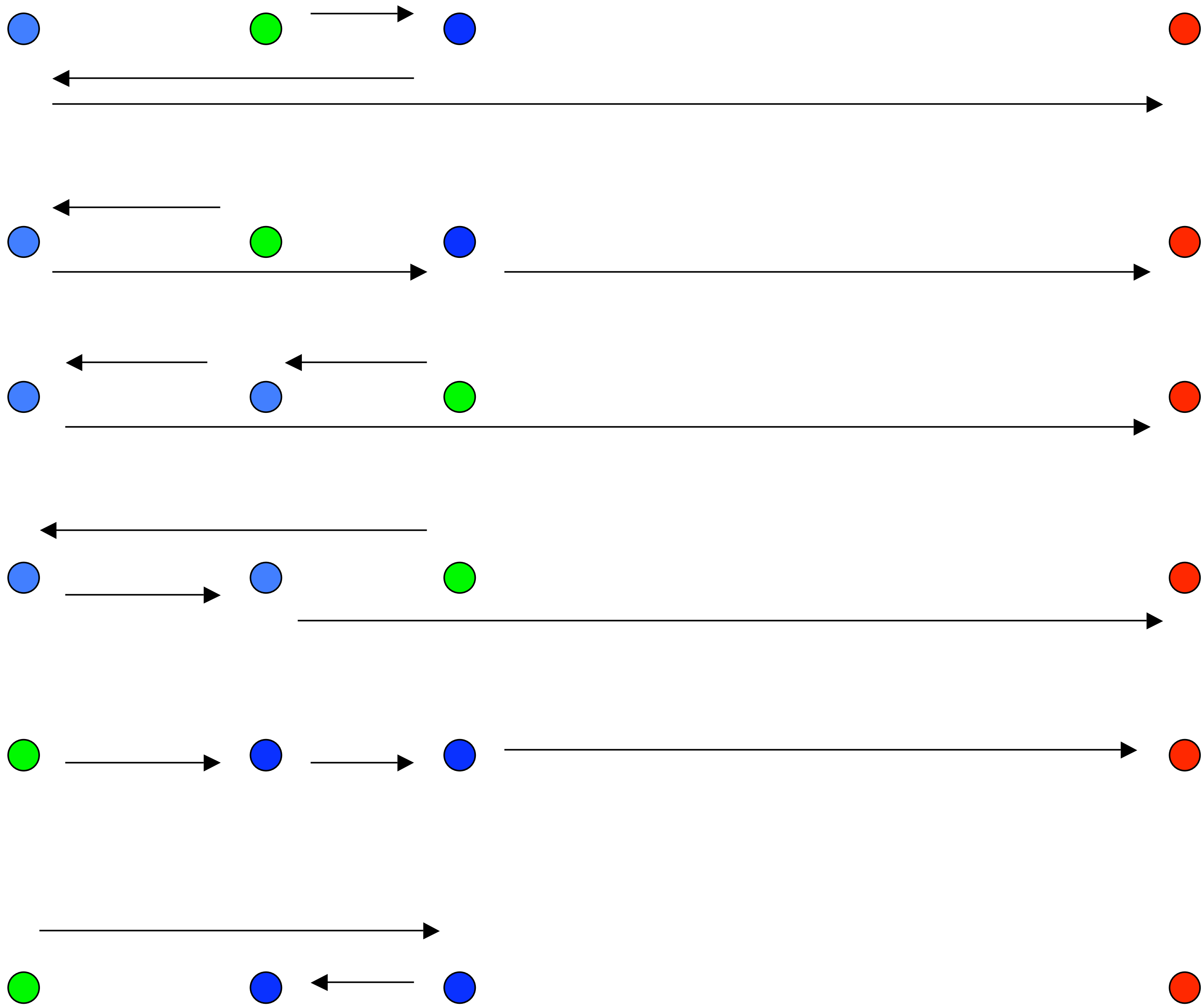
Standard results are reproduced.

As a first illustration consider the one dimensional problem.

$\psi_{produced}(x;t_1)$

$\psi_{observed}(x;t_2) = \exp[i(kx-E_k t_2)]$





When the transmission coefficient is much greater than the reflection coefficient backscattering can be neglected.

$$K/K_{free} = 1 - imv_{FT}(0)/2\pi k - m^2v^2_{FT}(0)/2k^2$$

Since this is independent of x_P the correlation function is unaffected by rescattering and $\rho_{eff} = \rho$. This is true even when backscattering is included.

In one dimension the HBT correlation function measures the size of the production source, where the wavepackets are produced, and is not affected by elastic rescattering on static localized potentials.

Three dimensional example with Gaussian distribution of wavepacket source

$$\rho(x) = 1/(4\pi^2 R_P{}^3 \sigma_t)\ \exp[-\mathbf{x}^2/2R_P{}^2]\ \exp[-t^2/2\sigma_t{}^2]$$

and scattering potentials

$$\rho_{scat}(\mathbf{x}_c) = N_c\,/(\sqrt{2\pi}\,R_c)^3\ \exp[-\mathbf{x}_c{}^2/2R_c{}^2]$$

with the Fourier transform of the potential parametrized as

$$v_{FT}(\mathbf{q}) = v_{FT}(0)\ \exp[-\mathbf{q}^2 r_0{}^2]$$

With $\varphi(\mathbf{k},x_P) = \varphi(\mathbf{k})$ and defining the effective source size via the curvature in momentum space we find

$$R_{out}^2 = R_P^2 \left\{ 1 + \frac{2\pi}{k^4} \frac{\alpha}{(1+2\alpha)(1+\alpha)^2} \rho_{scat}^2(0) \frac{d\sigma(0)}{d\Omega} \right\} + \frac{k^2}{m^2} \sigma_t^2$$

$$R_{side}^2 = R_P^2 \left\{ 1 - \frac{4\pi}{k^2} \frac{R_P^2}{(1+2\alpha)^2} \rho_{scat}^2(0) \frac{d\sigma(0)}{d\Omega} \right\}$$

$$\alpha = R_P^2 / R_c^2$$

valid at high momentum and to first order in the cross section.

Conclusions:

- We calculated elastic rescattering effects on the HBT correlation function.
- Of crucial importance is the space-time position of the wavepacket when its phase is taken to be random.
- In a model calculation R_{out} increases and R_{side} decreases but its generality is not yet known.
- Obvious generalizations to moving potentials, resonances, relativistic motion, fermions, partial coherence, flow, and so on.