

Hydrodynamic Models of Heavy-Ion Collisions

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Parallel Talks Based on Hydro

Jan. 13

- H. Niemi, Photon production from non-equilibrium QGP in heavy-ion collisions
- M. Csanad, Indication for quark deconfinement and evidence for a Hubble flow in Au+Au collisions at RHIC

Jan. 15

- Y. Nara, CGC, hydrodynamics and the parton energy loss
- E. Shuryak, Why does the QGP behaves like a perfect fluid?
- U. Heinz, Rapidity dependence of momentum anisotropies in nuclear collisions
- D. Teaney, Viscosity and thermalization

Outline

1. Why hydrodynamics?
2. How hydrodynamics works at RHIC
3. Hybrid models based on hydrodynamics
 - Information of the inside (jet quenching , EM probe)
 - Improvement of initial stage
4. Improvement of ideal hydro (viscosity)
5. Summary

1. Why Hydrodynamics?

Once we accept local thermalization ansatz, life becomes very easy.

Static

- EoS from Lattice QCD
- Finite T, μ field theory
- Critical phenomena
- Chiral property of hadron

Energy-momentum: $\partial_\mu T^{\mu\nu} = 0,$

Conserved number: $\partial_\mu n_i^\mu = 0$

Dynamic Phenomena in HIC

- Expansion, Flow
- Space-time evolution of thermodynamic variables

Caveat: Thermalization in HIC is a tough problem like building the Golden Gate Bridge!

1. Why Hydrodynamics (contd.)

Space-time evolution of energy density in
 $\sqrt{s_{NN}}=200$ GeV Au+Au collision at $b=7.2$ fm

Animation is here in the presentation.
If you need, please ask me (hirano@bnl.gov).

A full 3D hydrodynamic simulation with a CGC initial condition
Talk by Y.Nara

Hydrodynamics provides
us a very intuitive
and simple description of
relativistic heavy ion collisions.

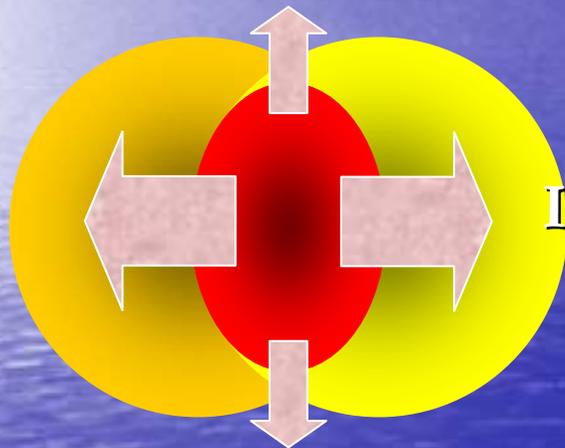
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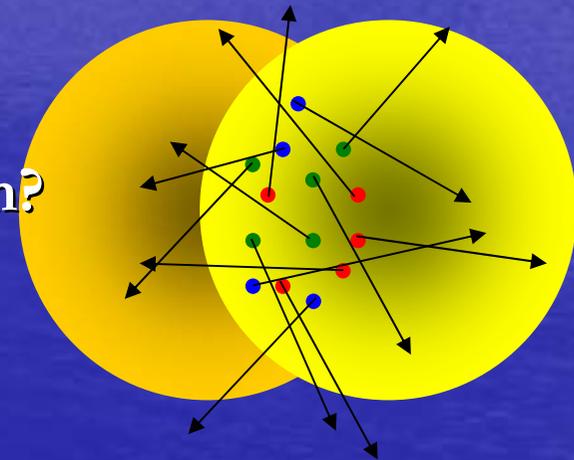
2. How Hydrodynamics Works at RHIC

Elliptic flow (J.-Y. Ollitrault ('92))

How does the system respond to initial spatial anisotropy?



Dense or dilute?
If dense, thermalization?
If thermalized, EoS?



$$\frac{dN}{p_T dp_T dy d\phi} = \frac{1}{2\pi} \frac{dN}{p_T dp_T dy} (1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \dots)$$

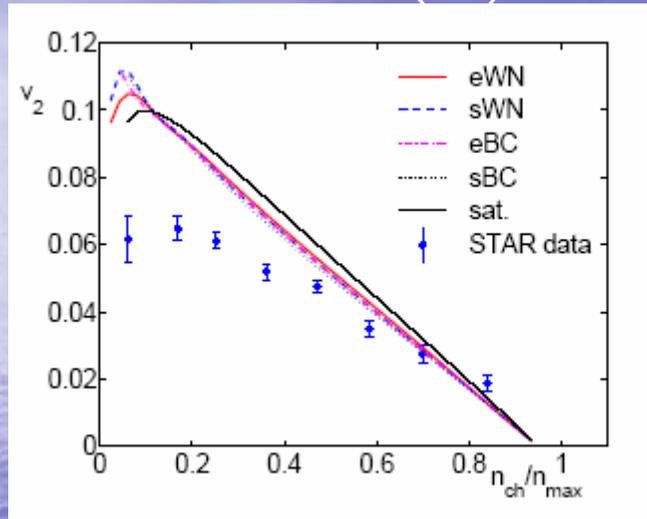
A. Poskanzer & S. Voloshin ('98)

$$v_2(p_T, y) = \frac{\int d\phi \cos(2\phi) \frac{dN}{p_T dp_T dy d\phi}}{\int d\phi \frac{dN}{p_T dp_T dy d\phi}} = \langle \cos(2\phi) \rangle$$

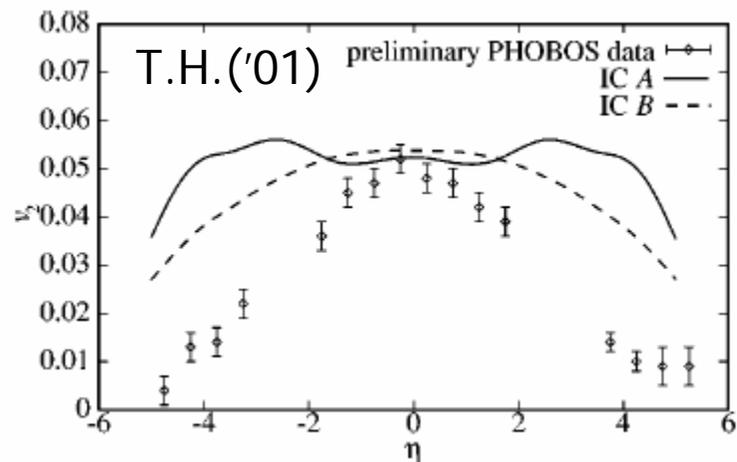
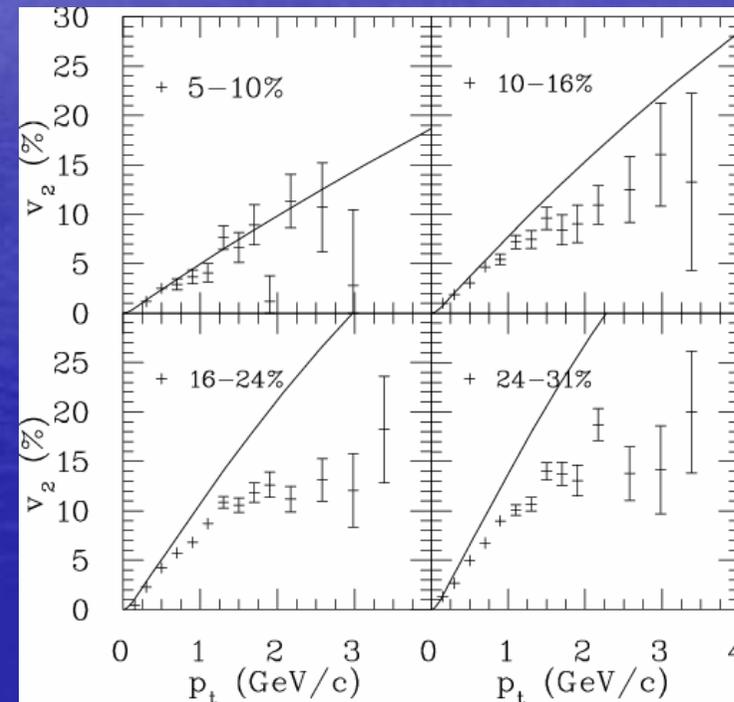
v4: P. Kolb ('03), talk by A. Poskanzer

Elliptic Flow of Charged Particles

P.Kolb et al.('01)



P.Huovinen('03)



Tetsufumi Hirano (RBRC)

Roughly speaking,
ideal hydro gives a good description

$$b \lesssim 5 \text{ fm},$$

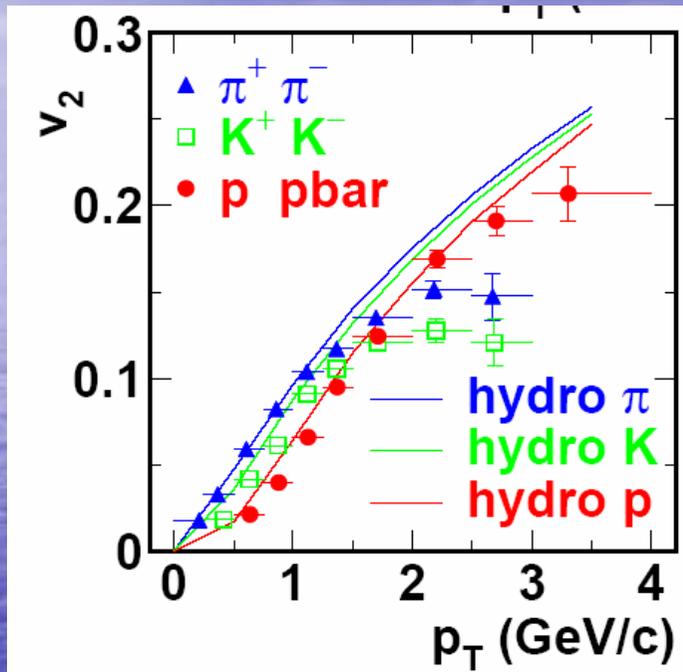
$$p_T \lesssim 1-1.5 \text{ GeV}/c,$$

$$|\eta| \lesssim 1-2$$

For improvement of models, talk by U.Heinz

More on Elliptic Flow

PHENIX, PRL91('03)182301.



Hydro: P.Huovinen et al.('01)

See recent excellent reviews,

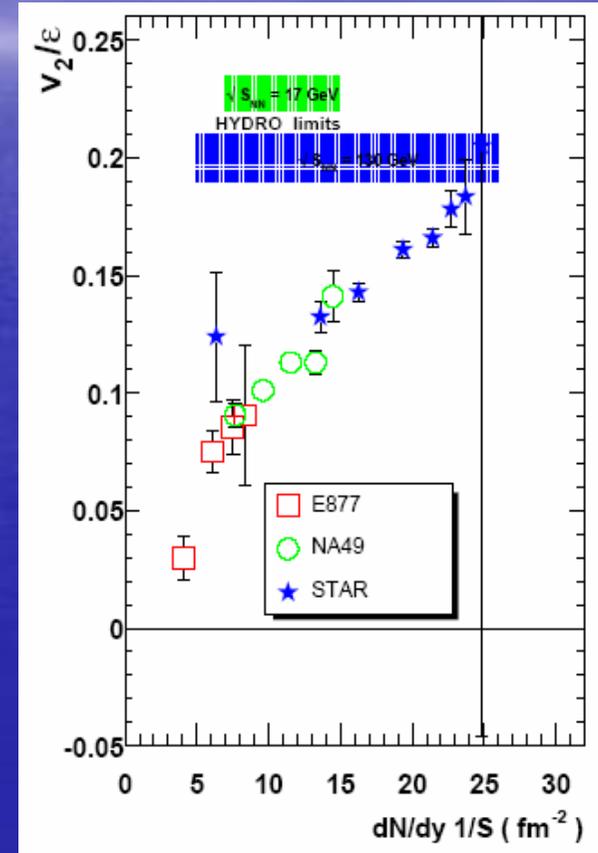
P.Huovinen (QM2002), nucl-th/0305064;

P.Kolb and U.Heinz, nucl-th/0305084;

E.Shuryak, hep-ph/0312227, today's talk.

Tetsufumi Hirano (RBRC)

STAR, PRC66('02)034904



Hydro: P.Kolb et al.('99)

(Note: Hydro+RQMD gives a better description.

D.Teaney et al.('01)

Ideal hydro seems to give a
good description at RHIC

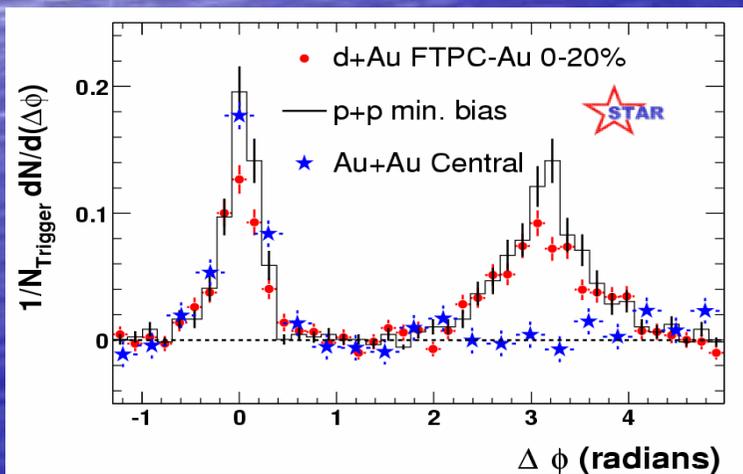
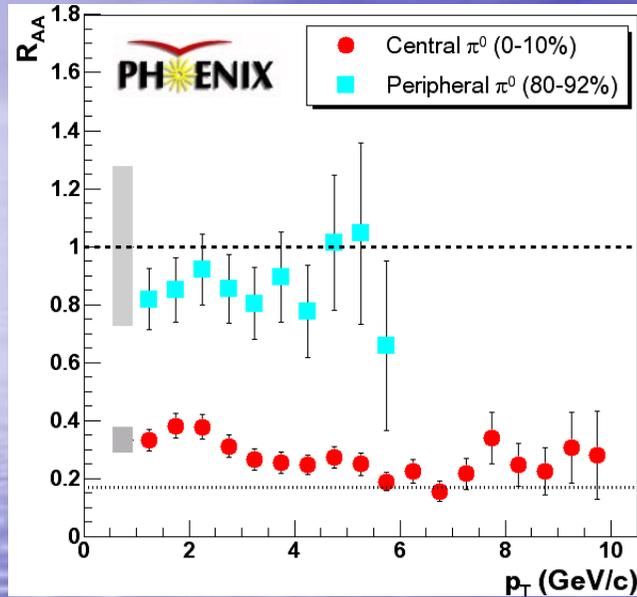
What's next?

1. *Making the most use of hydro models to study the RHIC physics*
2. *Checking how robust the current results are when hydro models are improved*

Outline

1. Why hydrodynamics?
2. How hydrodynamics works at RHIC
3. Hybrid models based on hydrodynamics
 - Information inside fluids
(jet quenching , EM probe)
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3.1 Information inside fluids



Jet quenching is a manifestation of interaction between **matter** and partons
(Talks by G. Moore and I. Vitev)

For quantitative analysis, the information about **the space-time evolution of matter** is indispensable!

3.1.1 Hydro as a Tool to Analyze Jet Quenching

Jet quenching analysis taking account of (2+1)D hydro results

(M.Gyulassy et al.('02))

Hydro+Jet model (T.H. & Y.Nara ('02))

GLV 1st order formula (M.Gyulassy et al.('00))

$$\Delta E = C \int_{\tau_0} d\tau (\tau - \tau_0) \times \rho(\tau, \mathbf{x}(\tau)) \ln \left(\frac{2p_0^\mu u_\mu}{\mu^2 L} \right)$$

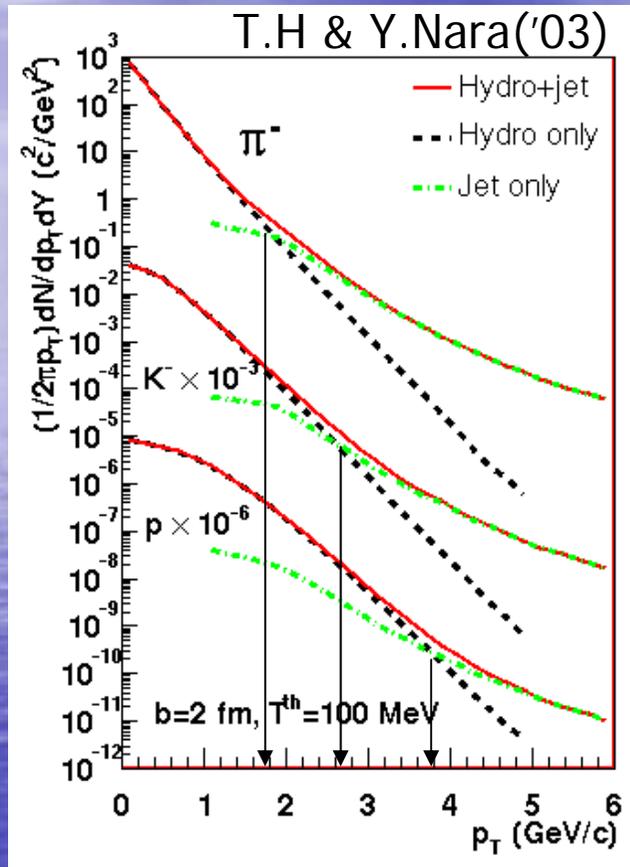
Parton density $\rho(\mathbf{x})$ taken from full 3D hydro simulations

Animation is here in the presentation. If you need, please ask me hirano@bnl.gov

Movie and data of $\rho(\mathbf{x})$ are available at <http://quark.phy.bnl.gov/~hirano/>

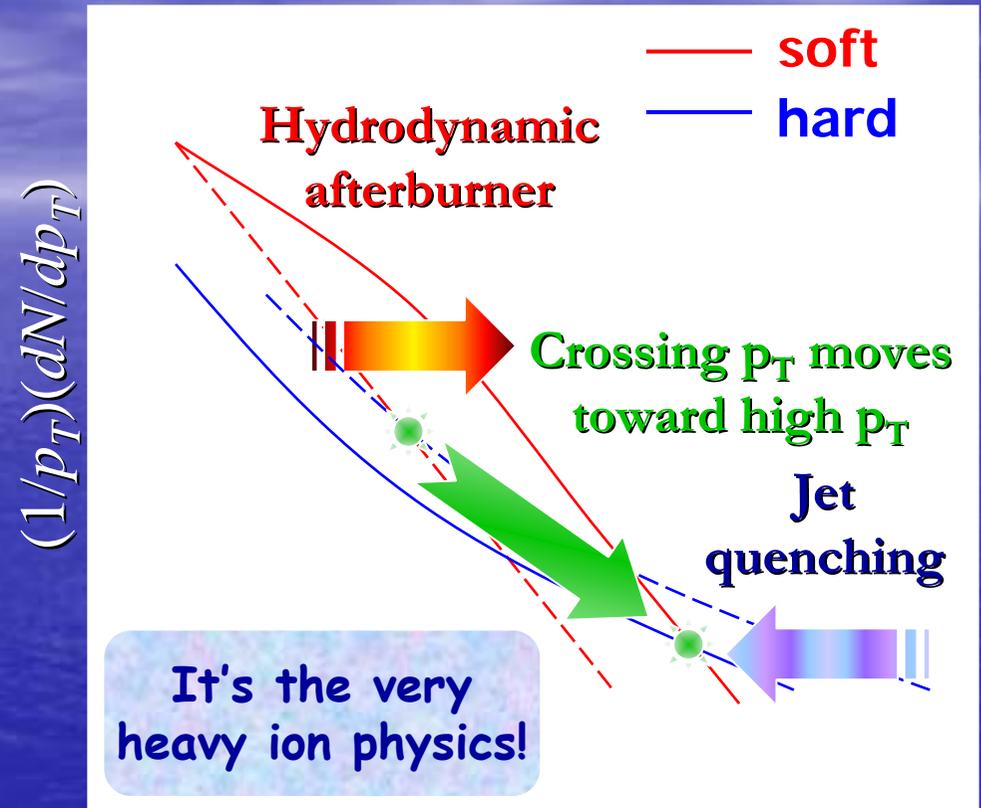
Interplay between Soft and Hard

Au+Au at b=2 fm



$$N_{\text{SOFT}} \sim N_{\text{HARD}}$$

$$p_{T,\text{cross}} \sim \begin{cases} 1.8 \text{ GeV}/c & \text{for } \pi \\ 2.7 \text{ GeV}/c & \text{for } K \\ 3.7 \text{ GeV}/c & \text{for } p \end{cases}$$

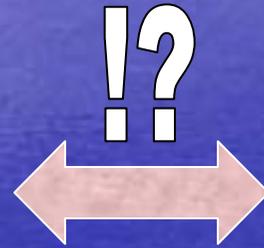
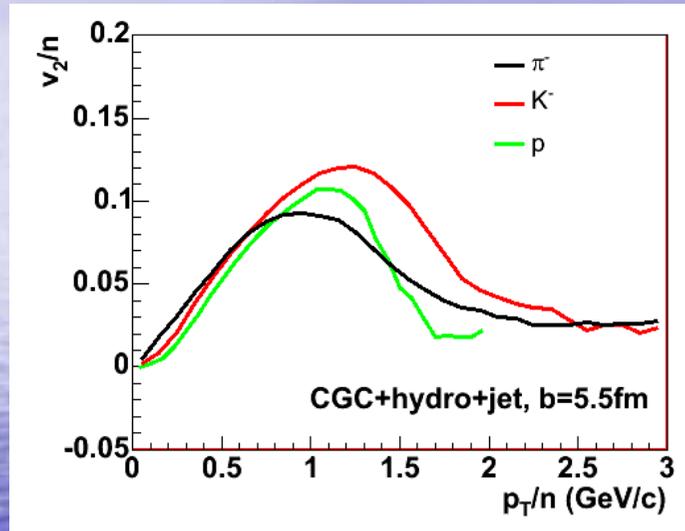


Interesting region
 → Intermediate p_T
 ($2 < p_T < 3.5$ GeV/c)
 Pion ← hard, Proton ← soft

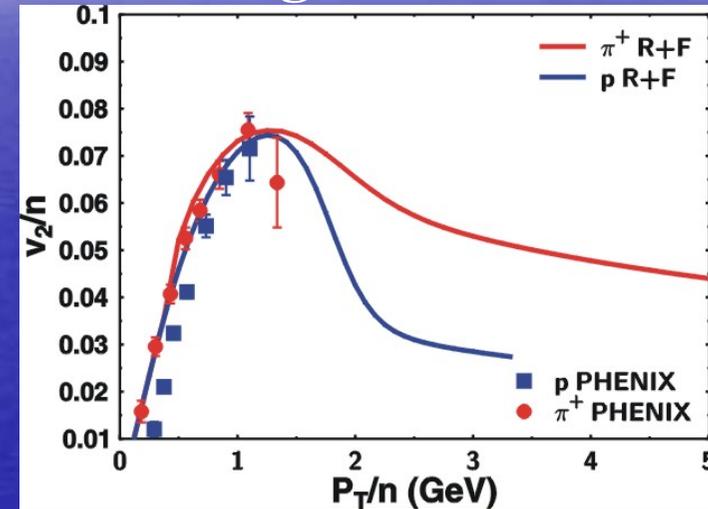
p_T

Consequence from hadron species

dependent $p_{T,cross}$
Hydro+Jet

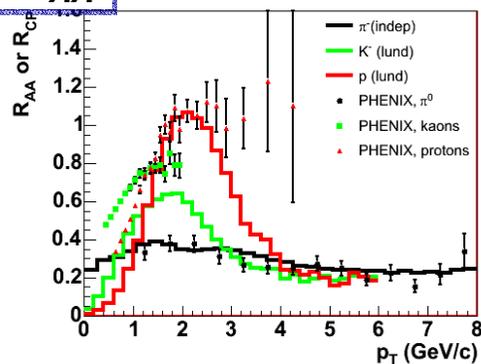


Recombination
+Fragmentation

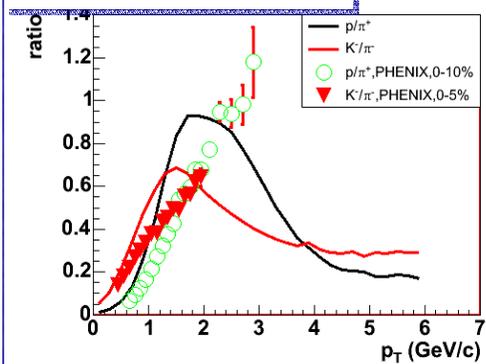


Talk by R.Fries

R_{AA}



Particle ratio



Poster by R. Fries (RBRC)

“Scaling v_2 ”

Interplay between soft and hard?

Recombination mechanism?

3.1.2 Hydro as a Tool to Analyze Electromagnetic Radiation

Thermal photon is a penetrating probe of QGP (E.Shuryak('78))

- Production rate (Number per unit space-time volume)

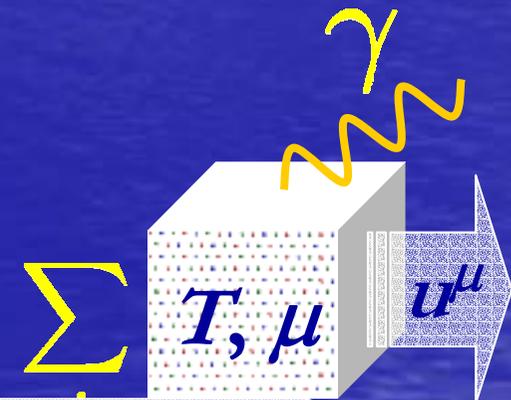
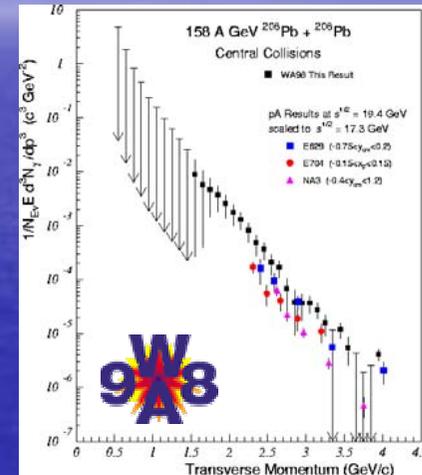
$$E \frac{dR_\gamma}{d^3p}(T, \mu) = \frac{-1}{(2\pi)^3} n(T, \mu) \text{Im}\Pi^R_\mu{}^\mu$$

H.A.Weldon ('83), L.McLerran & T.Toimela ('84)
C.Gale & J.Kapusta ('91)
Talk by G.Moore

- Invariant spectrum of photons

$$E \frac{dN_\gamma}{d^3p} = \sum_i \Delta V_i \tilde{E} \frac{dR_\gamma}{d^3\tilde{p}}(T(x_i), \mu(x_i), \tilde{E} = p_\mu u^\mu(x_i))$$

D.K.Srivastava & B.Sinha('94), J.Sollfrank et al.('97),
J.Alam et al.('01) and a lot of work



Importance of temperature profile

Chemical Non-Equilibrium

QGP phase	“Gluon Plasma (GP)” → QGP
Hadron phase	Chemical freezeout $T^{\text{th}} < T < T^{\text{ch}}$

QGP phase

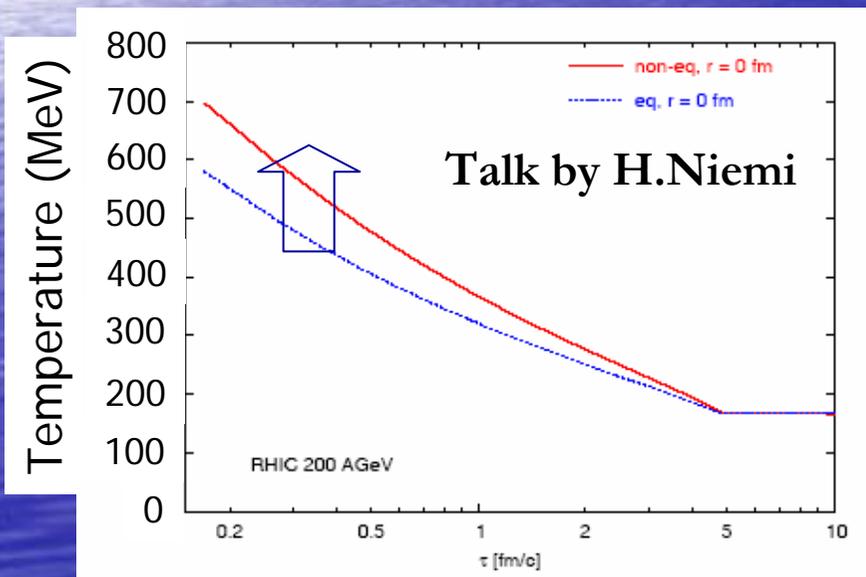
Hydro + rate eq.

Smaller d.o.f. → Larger initial T

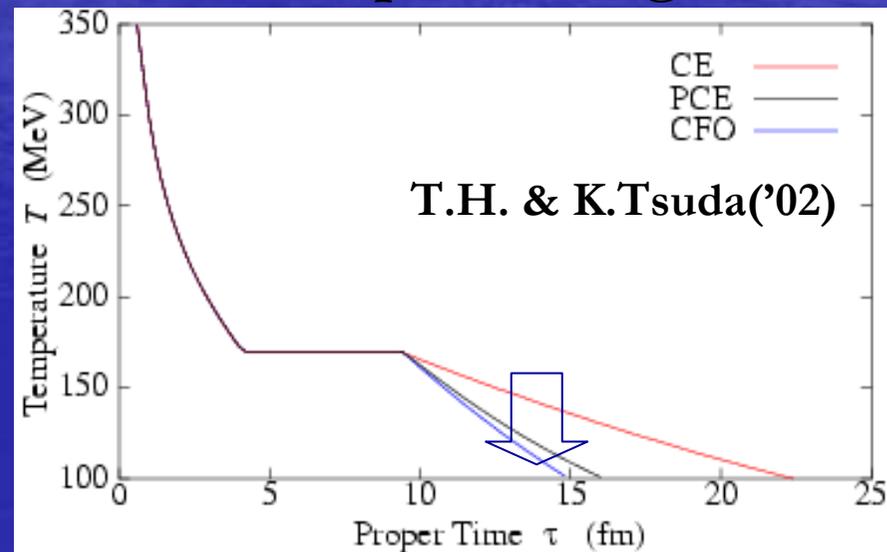
Hadron phase

Hydro ($T^{\text{th}} \neq T^{\text{ch}}$)

Overpopulation of resonance
→ Rapid cooling

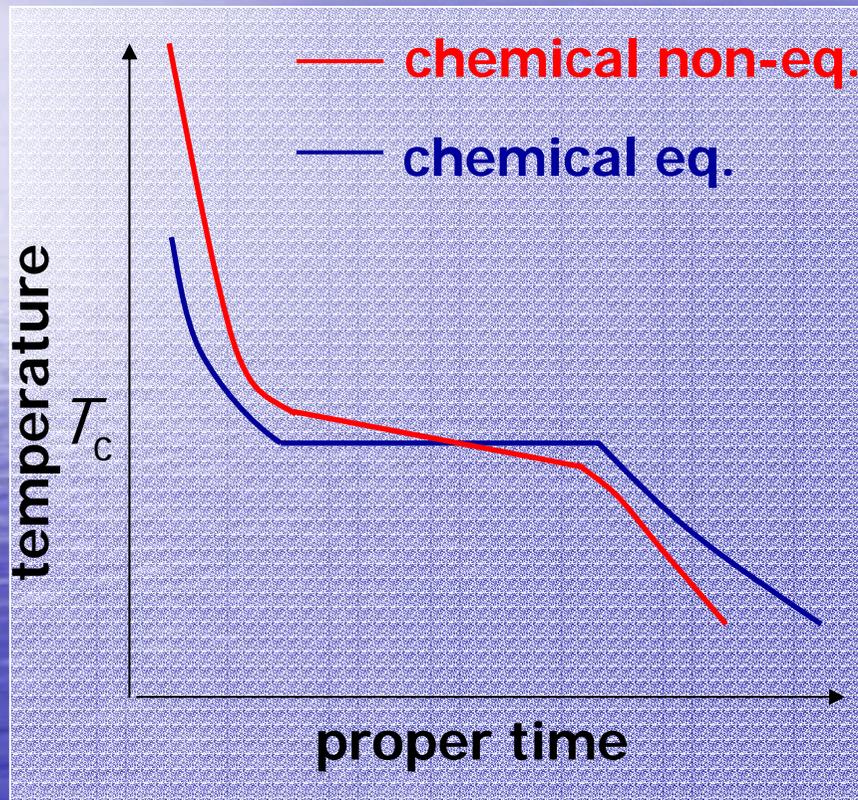


T.S.Biro et al.('93), D.K.Srivastava et al.('97),
A.K.Chaudhuri('00), D.M.Elliott & D.Rischke('00) Tetsufumi Hirano (RBRC)



N.Arbex et al.('01), T.H. & K.Tsuda('02),
D.Teaney('02), P.Kolb & R.Rapp('03)

Novel Temperature Evolution



Caveat:

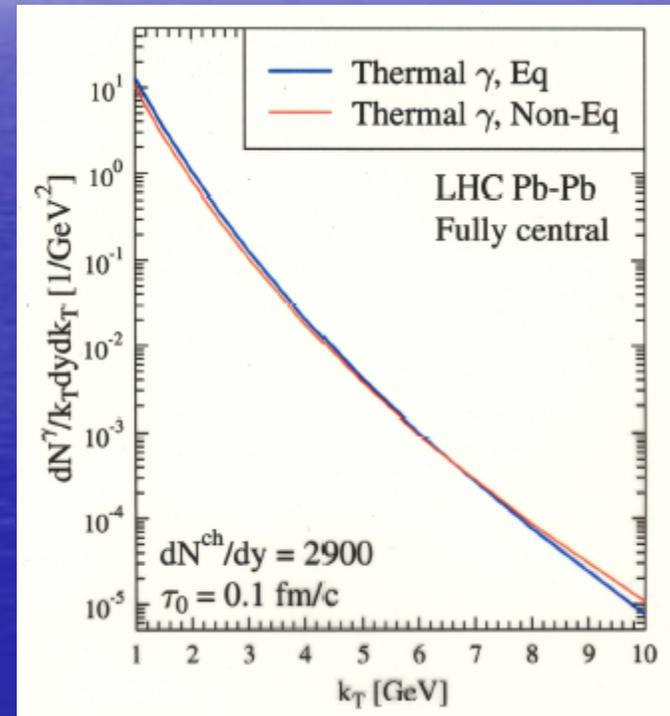
one has to take account of **fugacity** λ in calculating EM spectra.

- QGP phase: $\lambda < 1$
- Hadron phase: $\lambda > 1$

$$n(T, \lambda) \sim \lambda n_0(T)$$

Compensation between T and λ ?

Talk by H.Niemi



3.2 Improvement of Initial Condition -Toward an unified model in HIC-

$$e(\tau_0, x), n_B(\tau_0, x), u^\mu(\tau_0, x)$$

Group	Hydro	Initial condition
M.Gyulassy et al.	SHASTA (2+1D, Bjorken)	HIJING, event-by-event
C.Nonaka et al.	Lagrangian hydro (full 3D)	URASiMA, event average
B.Schlei et al.	HYLANDER (2+1D)	VNI, event average
C.E.Aguiar et al.	SPheRIO (full 3D)	NeXus, event-by-event
L.P.Csernai et al.	Particle-in-cell (full 3D)	String ropes, flux tubes, classical YM
K.Eskola et al.	SHASTA (2+1D, Bjorken)	pQCD + final state saturation
T.H. & Y.Nara	τ-η coordinate (full 3D)	CGC, $\phi(k_T^2, x)$ a la Kharzeev & Levin
...

C.E.Aguiar, R.Andrade, F.Grassi, Y.Hama,
T.Kodama, T.Osada, O.Socolowski Jr....

Poster by F.Grassi

3.2.1 SPheRIO*

Main features:

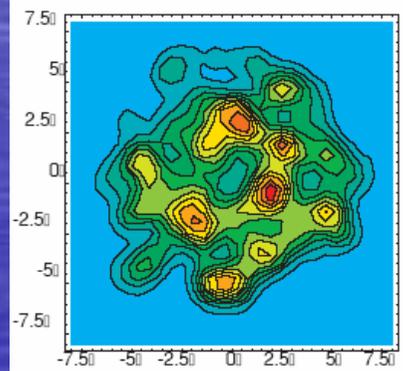
- “Particle” method (a kind of Lagrangian hydro)
- Numerical cost cheaper than conventional finite grids method (Even in 3+1 D, any geometry)
- **Event-by-event physics (NeXus + SPheRIO=NeXSPheRIO)**
(NeXus: parton based Gribov-Regge theory)

SPheRIO

Conventional approach

Spectra
from

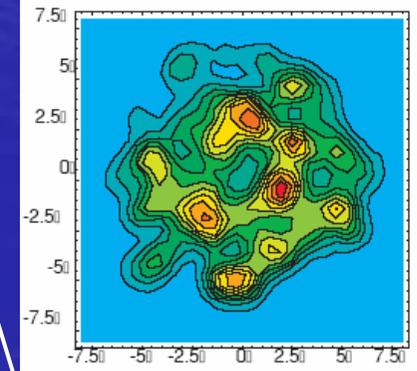
Energy density
of single event



?

Spectra
from

Energy density
of single event



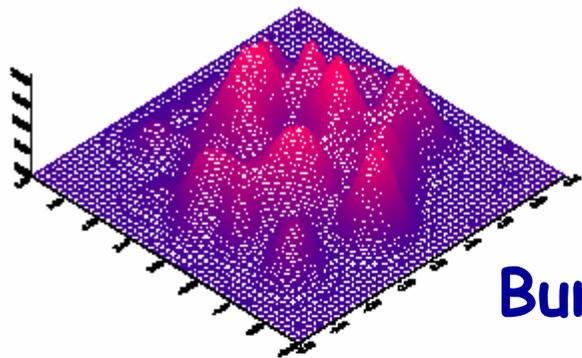
Similar approach based on HIJING:

M.Gyulassy et al.(’97)

Tetsufumi Hirano (RBRC)

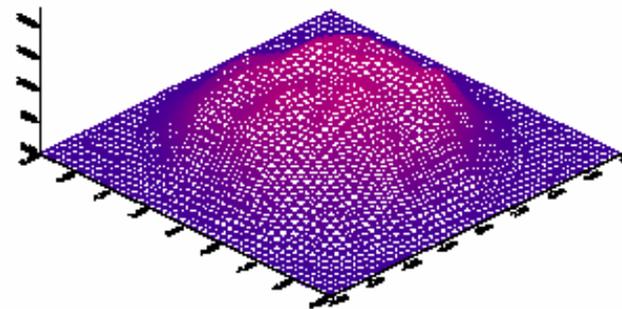
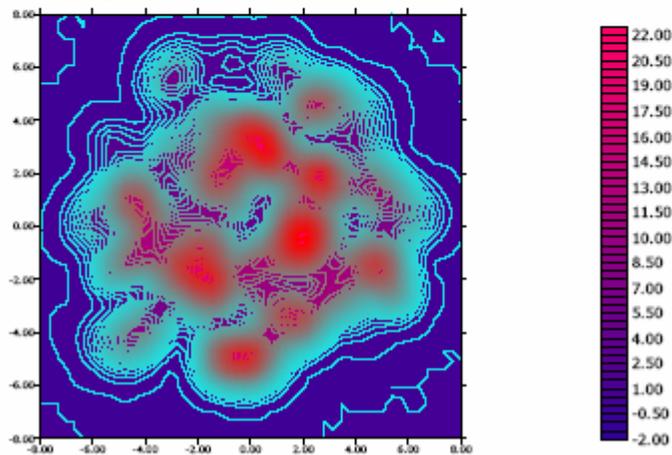
Initial Conditions in NeXSPheRIO

Energy density in the transverse plane ($z=0$)

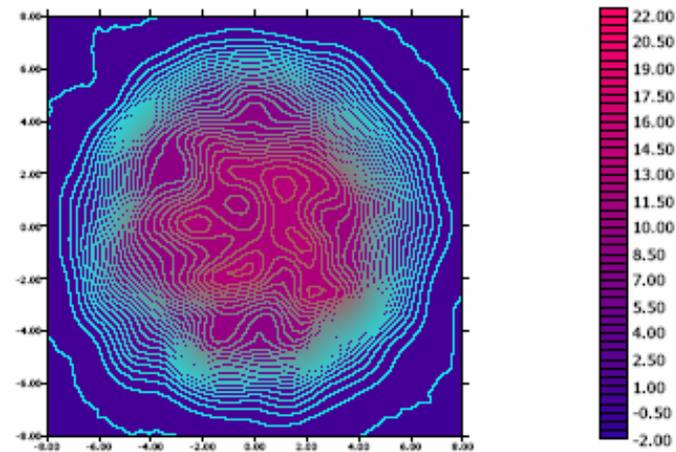


Bumpy!

Single event ($b=0\text{fm}$)

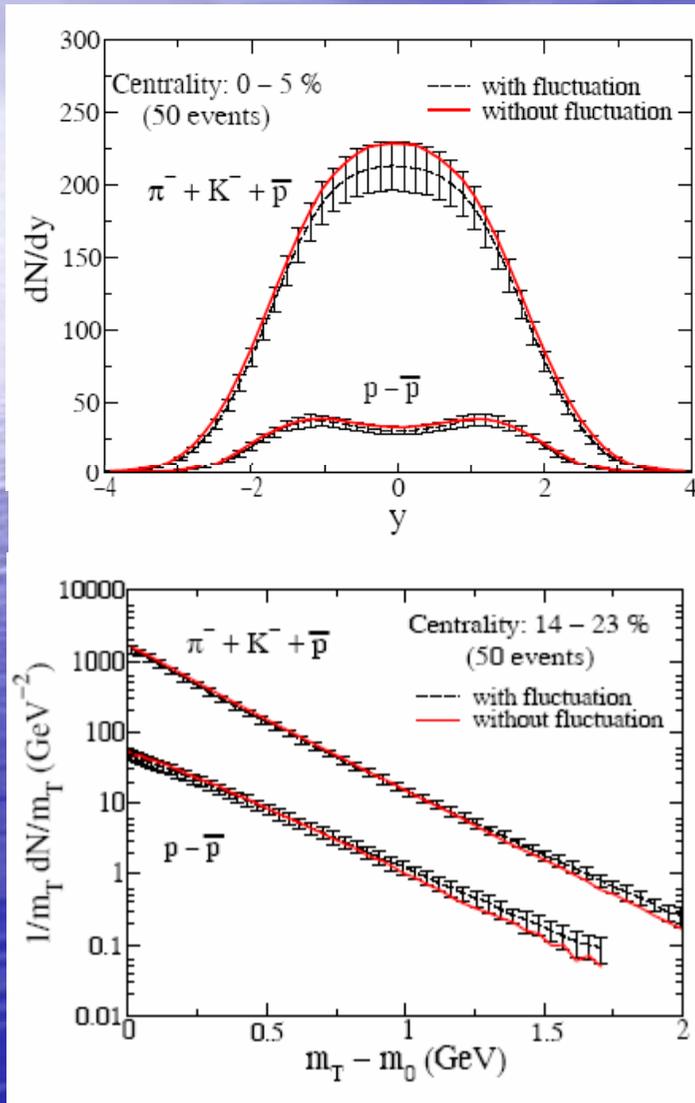


Average over 30 events ($b=0\text{fm}$)



Results from NeXSPheRIO

Pb+Pb 17.3A GeV



Effect of *initial energy density* fluctuation (simple EoS case):

$$\langle \dots \rangle = \frac{1}{N} \sum_i \dots \quad : (\text{event average})$$

$$\langle S \rangle_{\text{final}} \sim \langle S \rangle_{\text{initial}} \left[1 - \alpha \frac{\langle \Delta E^2 \rangle}{\langle E \rangle^2} \right]$$

Negative!

Multiplicity is reduced by ~10%!

p_T slope is not affected largely.

→ $v_2(p_T)$ and its fluctuation?

Now the hydro simulation becomes close to experimental situations like event-generators!

3.2.2 CGC+Hydro+Jet Model

T.H. & Y.Nara

Talk by Y.Nara

Color Glass Condensate

Talk by

J.Jalilian-Marian

Dense Medium

Elliptic flow

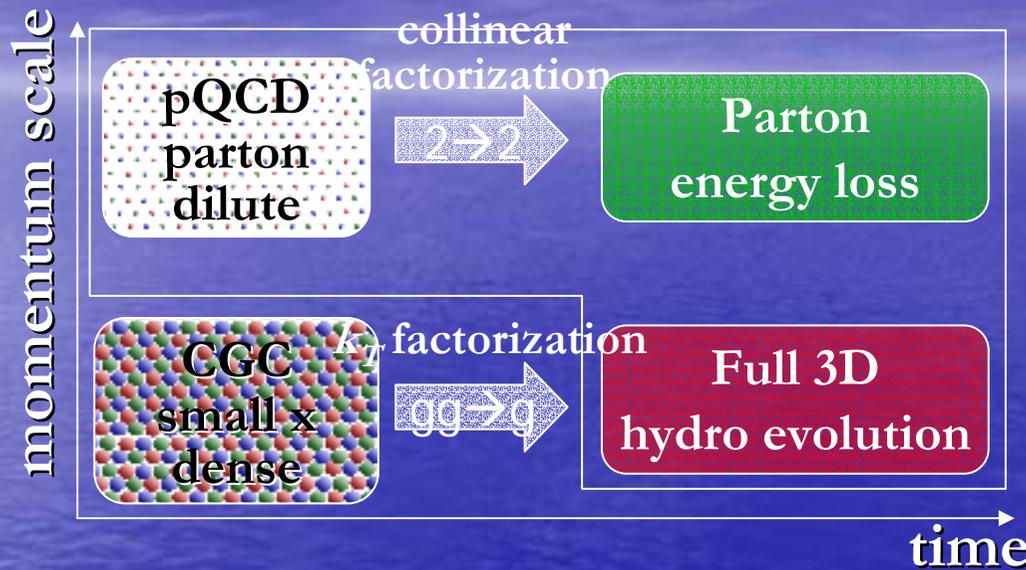
Jet quenching

Talk by

I.Vitev

These three physics closely related with each other!

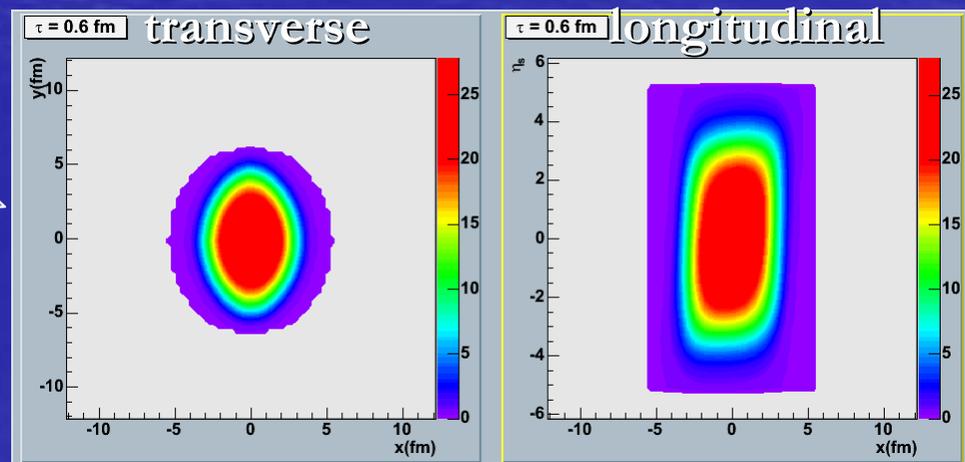
CGC+Hydro+Jet Model (contd.)



Initial condition of energy density from CGC

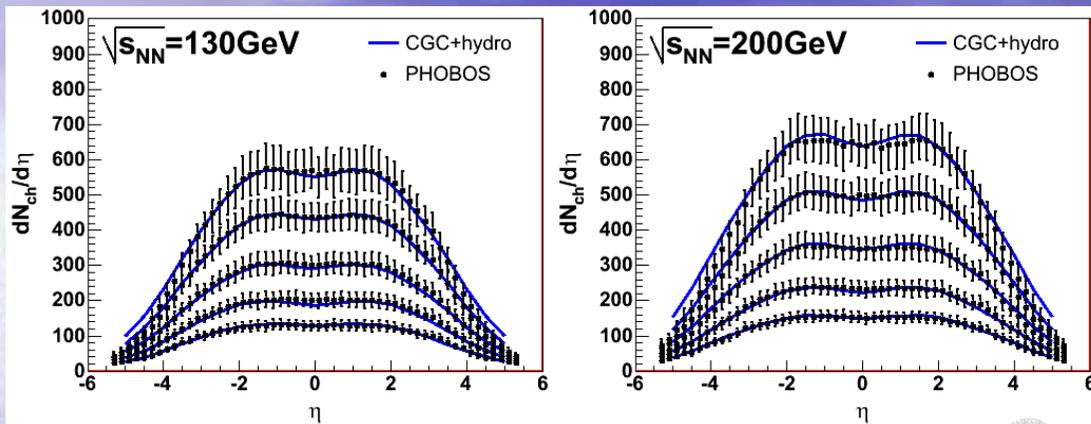
Au+Au 200A GeV

$b=7.2\text{fm}$, $\tau_0=0.6\text{fm}$

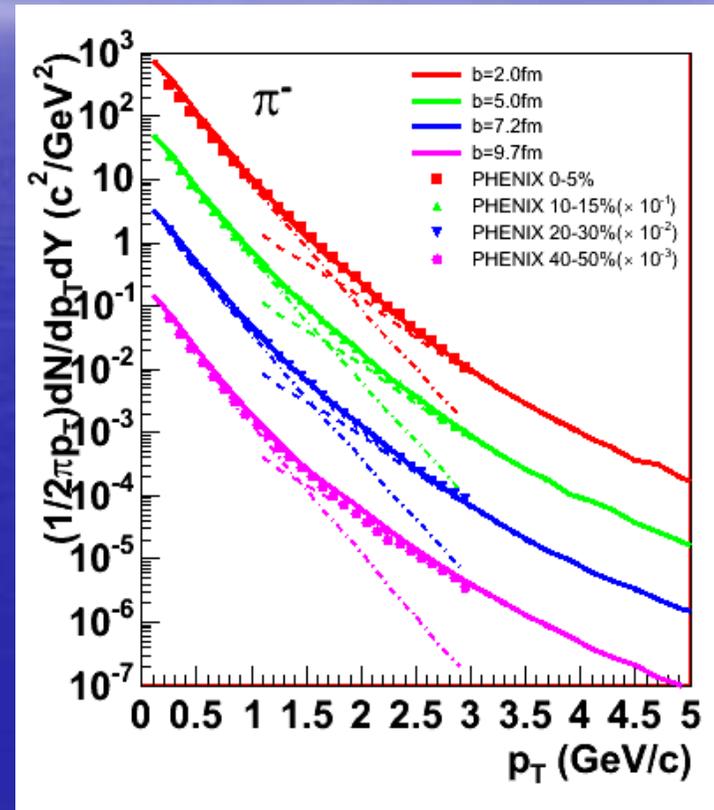


Tetsufumi Hirano (RBRC)

Results from CGC+hydro+jet



Au+Au $\sqrt{s_{NN}} = 200$ GeV



CGC initial condition
works very well!
(Energy, rapidity, centrality
dependences)

For details, talk by Y.Nara

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4. Viscosity

Talks by E.Shuryak and D.Teaney

Change not only the equations of motion
but the local thermal distribution function

A.Dumitriu('02), D.Teaney('03)

- Blast wave model + dist. fn. with viscous correction

1st order correction to dist. fn.:

$$\delta f \propto \frac{\Gamma_s}{T^2} f_0 (1 + f_0) p^\mu p^\nu X_{\mu\nu}$$

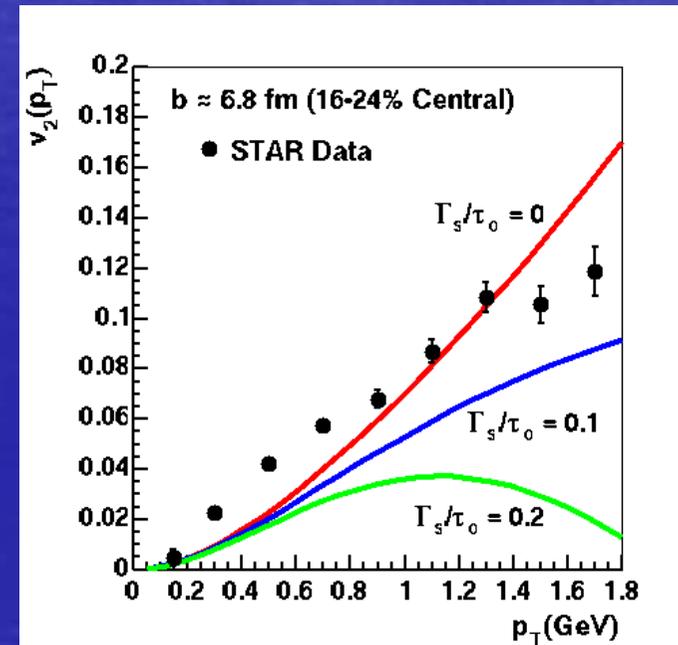
Γ_s : Sound attenuation length

$X_{\mu\nu}$: Tensor part of thermodynamic force

Reynolds number in Bjorken flow

$$R^{-1} \approx \Gamma_s / \tau$$

Nearly ideal hydro !?



D.Teaney('03)

Break Down of Naive Navier-Stokes Eq. and a Relaxation Method

- Non-relativistic case (Based on discussion by Cattaneo (1948))

Balance eq.: $\dot{T} = -\vec{\nabla} \cdot \vec{q}$

Constitutive eq.: $\tau \dot{\vec{q}} + \vec{q} = -\kappa \vec{\nabla} T$ $\tau \rightarrow 0$: Fourier's law

$$\tau \ddot{T} + \dot{T} = \kappa \Delta T, \quad c = \sqrt{\kappa/\tau}$$

τ : "relaxation time"

Parabolic equation (heat equation)

ACAUSAL!!



Finite τ

Hyperbolic equation (telegraph equation)

Talk by D. Teaney
See also, A. Muronga ('02)

5. Summary

Hydrodynamics is one of the valuable tools at RHIC energies

- Open our mind !
Hydrodynamics can be used even for "high p_T physics in HIC".
 - Jet tomography
 - EM probe
 - (J/Ψ suppression)
 - ...
- Keep in mind !
How robust is the current agreement of hydro?:
 - Chemical non-eq.?
 - Initial fluctuation?
 - Viscosity?
 - Thermalization?
 - EoS?
 - (Freeze-out?)