Student Lecture at Quark Matter 2004
Oakland – January 11, 2004

Perturbative QCD in Nuclear Environment

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See the mini-review by J.W. Qiu and G. Sterman (2003), and references therein

Quantum Chromodynamics (QCD)

❑ Fields:

$\psi_i^f(x)$ Quark fields, Dirac fermions (like e^-)
Color triplet: $i = 1,2,3=N_C$
Flavor: $f = u,d,s,c,b,t$

$A_{\mu,a}(x)$ Gluon fields, spin-1 vector field (like γ)
Color octet: $a = 1,2,\ldots,8 = N_C^2-1$

❑ Lagrangian density:

$$L_{QCD}(\psi,A)=\sum_f \overline{\psi}_i^f\left[\left(i\partial_\mu - gA_{\mu,a}(t_a)_{ij}\right)\gamma^\mu - m_f\right]\psi_i^f$$

$$-\frac{1}{4}\left[\partial_\mu A_{\nu,a}-\partial_\nu A_{\mu,a}-gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2$$

\+ *gauge fixing* + *ghost terms*

Color matrix: $[t_a,t_b]=iC_{abc}t_c$

❑ Gauge invariance:

$$\psi_i \to \psi'_j = U_{ji}(x)\psi_i$$

$$A_\mu \to A_\mu' = U(x)A_\mu U^{-1}(x) + \frac{i}{g}\left[\partial_\mu U(x)\right]U^{-1}(x)$$

where $A_\mu = A_{\mu,a}t_a$

Perturbative QCD

- Physical quantities can't depend on the renormalization scale - *μ*:

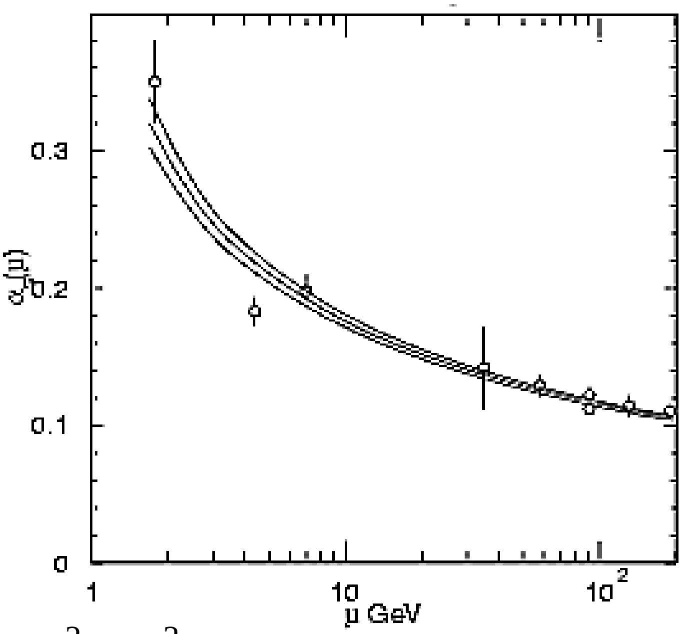
$$\mu^2 \frac{d}{d\mu^2}\sigma_{\text{phy}}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0$$

$$\Longrightarrow \qquad \sigma_{\text{phy}}(Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- Asymptotic freedom

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

$$\propto \frac{1}{\ell n(\mu^2 / \Lambda_{\text{QCD}}^2)}$$



Can we choose μ^2 as large as we want?

No!

$$\sigma^{(n)}(Q^2, \mu^2) \propto \ell n(Q^2 / \mu^2) + \ldots$$

$$\Longrightarrow \qquad \boxed{\mu^2 \sim Q^2}$$

Larger Q^2, larger effective μ^2, smaller $\alpha_s(\mu)$

⇨ Perturbative QCD works better for physical quantities with a large momentum exchange

PQCD Factorization

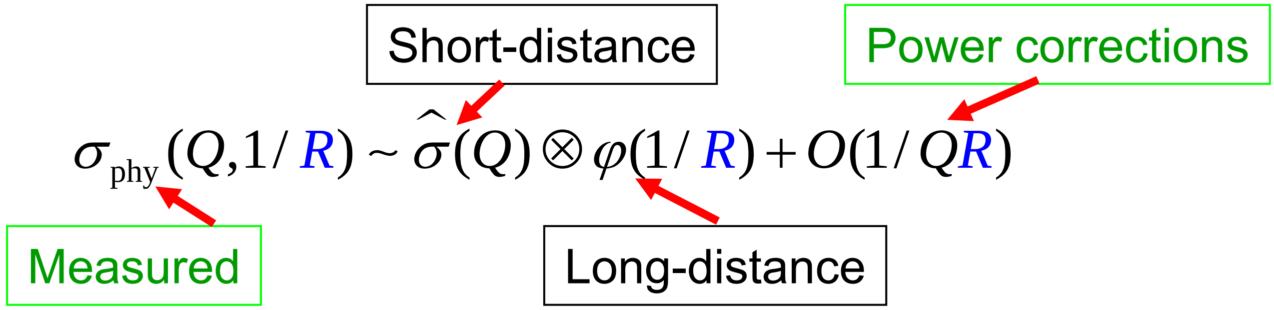
- Can pQCD work for calculating x-sections involving hadrons?

Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

Energy exchange in hard collisions: $Q \gg \Lambda_{QCD}$

⇒ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

- PQCD can be useful iff quantum interference between perturbative and nonperturbative scales can be neglected



Short-distance

Power corrections

$$\sigma_{\text{phy}}(Q,1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

Measured

Long-distance

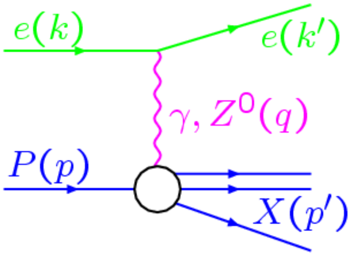
⇒ Factorization - Predictive power of pQCD

- short-distance and long-distance are separately gauge invariant
- short-distance part is Infra-Safe, and calculable
- long-distance part can be defined to be universal

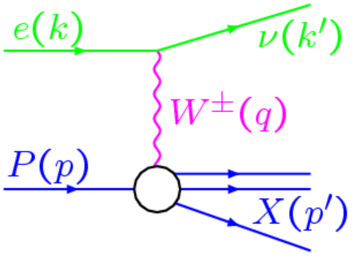
Lepton-Hadron DIS

❑ Kinematics:

Neutral Current (NC)



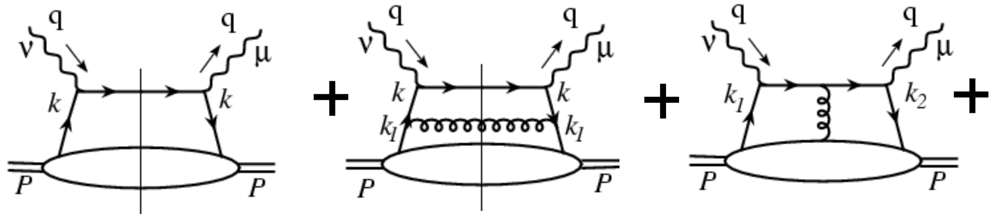
Charged Current (CC)



negative four-momentum transfer squared	$Q^2 = -q^2 = -(k-k')^2$
fraction of proton momentum	$x = \frac{Q^2}{2p\cdot q}$
inelasticity	$y = \frac{p\cdot q}{p\cdot k}$
squared cms energy	$s = (k+p)^2 = \frac{Q^2}{xy}$

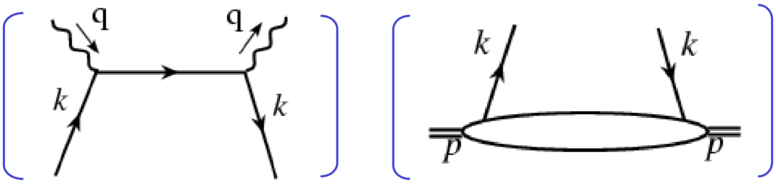
❑ Feynman diagram representation:

$W^{\mu\nu} \propto$



❑ Perturbative pinch singularities:

$\int d^4k$

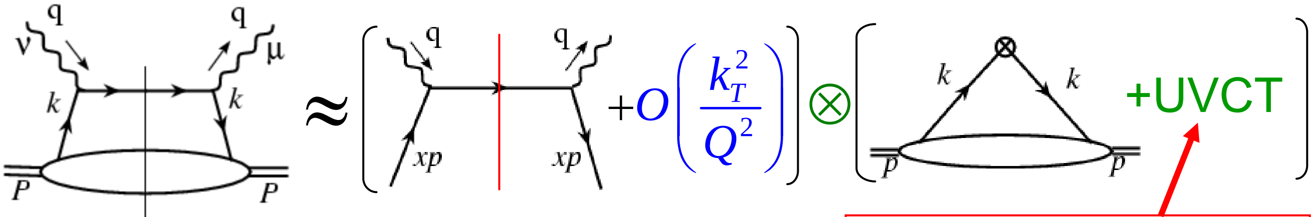


$$\Longrightarrow \int d^4k \frac{i}{k^2+i\varepsilon}\frac{i}{k^2-i\varepsilon} \Rightarrow \infty$$

$\Longrightarrow$ “long-lived parton state if $k^2 \ll Q^2$

Factorization in DIS

- ❑ Collinear approximation, if $\langle k_T^2 \rangle \ll xp$



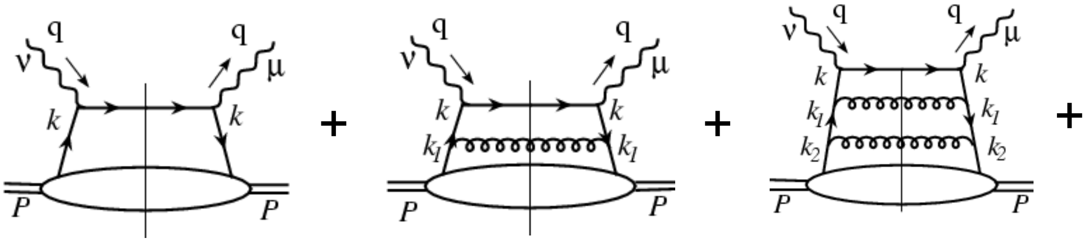
Scheme dependence

- ❑ DIS limit: $\nu, Q^2 \to \infty,$ while x_B fixed

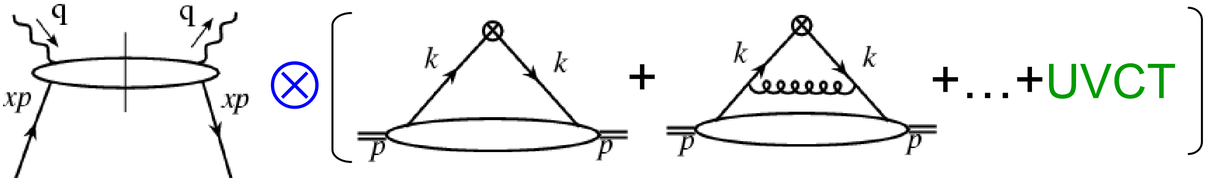
⟹ Feynman's parton model and Bjorken scaling

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \varphi(x_B) + O(\alpha_s) + O\left(\frac{\Lambda_{\text{QCD}}^2}{\text{Q}^2}\right)$$

- ❑ QCD corrections: pinch singularities in $\int d^4 k_i$



- ❑ resum leading logarithms into parton distributions

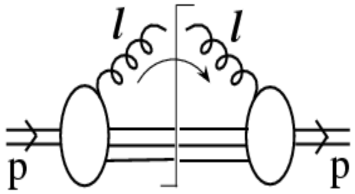


$$\Longrightarrow \quad F_2(x_B, Q^2) = \sum_q C_q\left(\frac{x_B}{x}, \frac{Q^2}{\mu^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu^2\right) + O\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

Parton Distributions

- Gluon distribution in collinear factorization:

$$g(x,\mu^2)=\int d^4l\ \delta\left(x-\frac{l\cdot n}{p\cdot n}\right)\left[\quad\right.$$



$$\left. + \text{UV CT}\quad\right]$$

❖ Integrate over all transverse momentum!
❖ μ²-dependence from the UV counter-term (UVCT)

- μ²-dependence determined by DGLAP equations

$$\mu^2\frac{\partial}{\partial\mu^2}\varphi_i(x,\mu^2)=\sum_j P_{i/j}\left(\frac{x}{x'},\alpha_s\right)\otimes\varphi_j(x',\mu^2)$$

Boundary condition extracted from physical x-sections

$$F_2(x_B,Q^2)=\sum_q C_q\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_f\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)$$

- extracted parton distributions depend on the perturbatively calculated C_q and power corrections

❖ Leading order (tree-level) C_q
❖ Next-to-Leading order C_q

$$\left.\begin{matrix}\text{Leading order (tree-level) } C_q\\ \text{Next-to-Leading order } C_q\end{matrix}\right\}\Longleftrightarrow\left\{\begin{matrix}\text{LO PDF's}\\ \text{NLO PDF's}\end{matrix}\right.$$

LO PDF's
NLO PDF's

❖ Calculation of C_q at NLO and beyond depends on the UVCT $\Longrightarrow$ the scheme dependence of C_q

$\Longrightarrow$ the scheme dependence of PDFs

Factorization in hadronic collisions

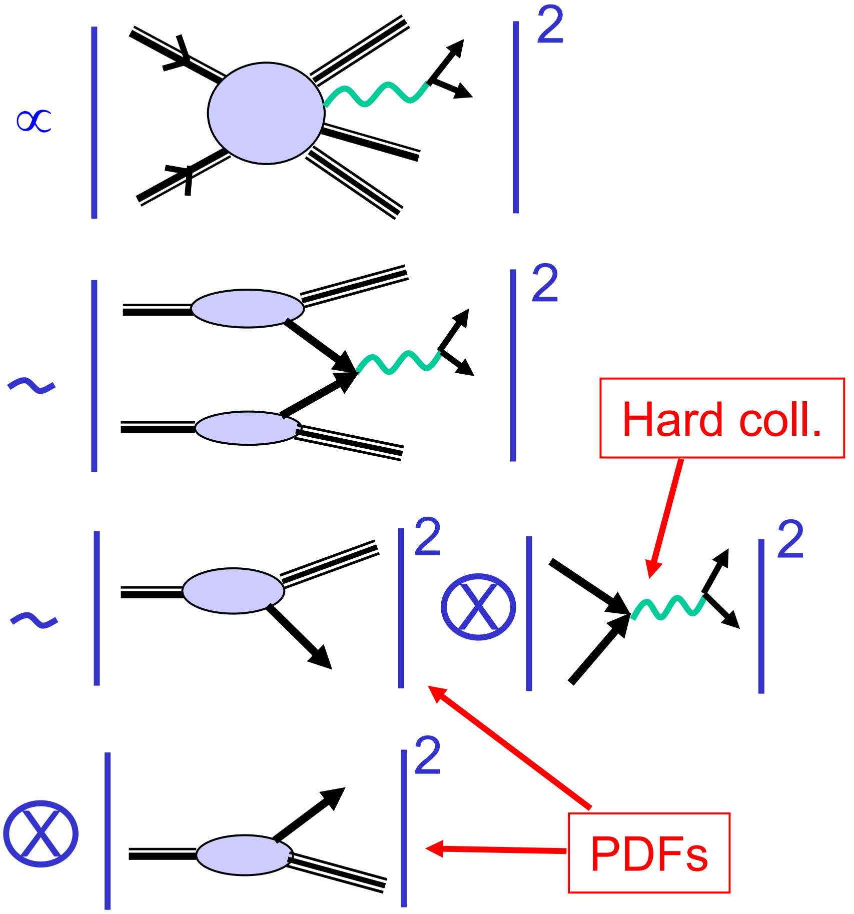
❑ Basic assumptions:

$$d\alpha^{\text{Drell-Yan}}_{AB\to\ell^+\ell^- X}(Q^2) \propto$$

❖ no interaction between A & B before hard coll.

❖ single parton

❖ no quantum interference between hard collision & distributions



$$\Rightarrow \quad \frac{d\sigma^{\text{DY}}_{AB}}{dQ^2} = \sum_{a,b}\int dx_1\varphi_{a/A}(x_1)\,dx_2\varphi_{b/B}(x_2)\,\frac{d\hat{\sigma}^{\text{DY}}_{ab}}{dQ^2}$$

How well can we justify above assumptions?

Heuristic Arguments for the Factorizations

- There are always soft interaction between two hadrons
  - Gauge field A_μ is not Lorentz contracted

    ⇨ Long range soft gluon interaction between hadrons
  - a "pure gauge field" is gauge-equivalent to a zero field

    ⇨ Perturbation theory to "mask" factorization, except at the level of gauge invariant quantities
  - Field strength contracted more than a scalar field

    Factorization should fail at $\gamma^{-2} \sim Q^{-4}$

    It does!

$$\sigma^{\mathrm{DY}}(Q^2) = \sigma^{\mathrm{LP}}(Q^2) + \frac{1}{Q^2}\sigma^{\mathrm{NLP}}(Q^2) + \frac{1}{Q^4}\sigma^{\mathrm{NNLP}}(Q^2) + \ldots$$

factorized

Not factorized

- Single parton interaction:

$$\frac{x\varphi(x,Q^2)\cdot(1/Q^2)}{\pi R^2} \sim \frac{x\varphi(x,25\mathrm{GeV}^2)}{\pi\cdot 25\cdot 25} \ll 1$$

- If x is not too small, hadron is very transparent!

- Extra parton interaction is suppressed by *$1/Q^2$*

3. MAGIC OF NUCLEAR TARGETS

- Facts:
  - Nuclear binding energy is about 8 MeV per nucleon $\ll$ typical energy exchange in hard collisions
  - But, large and non-trivial nuclear dependence have been observed in almost all processes involving nuclear targets
- EMC effect and nuclear shadowing (since 1983)
  - Ratio of structure functions

$$R_{F_2} = \frac{\frac{1}{A}\, F_2^A(x,Q^2)}{\frac{1}{2}\, F_2^D(x,Q^2)} \neq 1$$

- Cronin effect (since 1975)

  Anomalous nuclear dependence in hadronic single particle transverse momentum distribution

  - process: $h + A \to \text{particle}(p) + X$
  - definition: $E\frac{d\sigma^{hA}}{d^3p} \equiv E\frac{d\sigma^{hN}}{d^3p}\, A^{\alpha(p)}$
  - $\alpha(p) < 1$ for low p_T, and > 1 for large p_T

Note: $\alpha_{AA}(p) < 1$ for all observed p_T at RHIC

- Nuclear dependence in acorplanarity:
  - process: $p(\text{or } \gamma) + A \to \text{jet}(\ell) + \text{jet}(\ell') + X$
  - definition: $\vec{k}_T \equiv \vec{\ell}_T + \vec{\ell}'_T$
  - momentum imbalance:

    $\langle k_T^2 \rangle$ shows large nuclear dependence

- Transverse momentum broadening:
  - process:

    $$\begin{aligned} A + B \longrightarrow\ & \gamma^*[\to \ell^+\ell^-(q)] + X \\ & \text{J/}\psi(q) + X \\ & \ldots \end{aligned}$$

  - averaged q_T:

    $$\langle q_T^2 \rangle = \frac{\int dq_T^2\, q_T^2\, \frac{d\sigma}{dq_T^2}}{\int dq_T^2\, \frac{d\sigma}{dq_T^2}}$$

  - broadening:

    $$\Delta\langle q_T^2 \rangle \equiv \langle q_T^2 \rangle^{AB} - \langle q_T^2 \rangle^{NN} \neq 0$$

    shows strong nuclear dependence

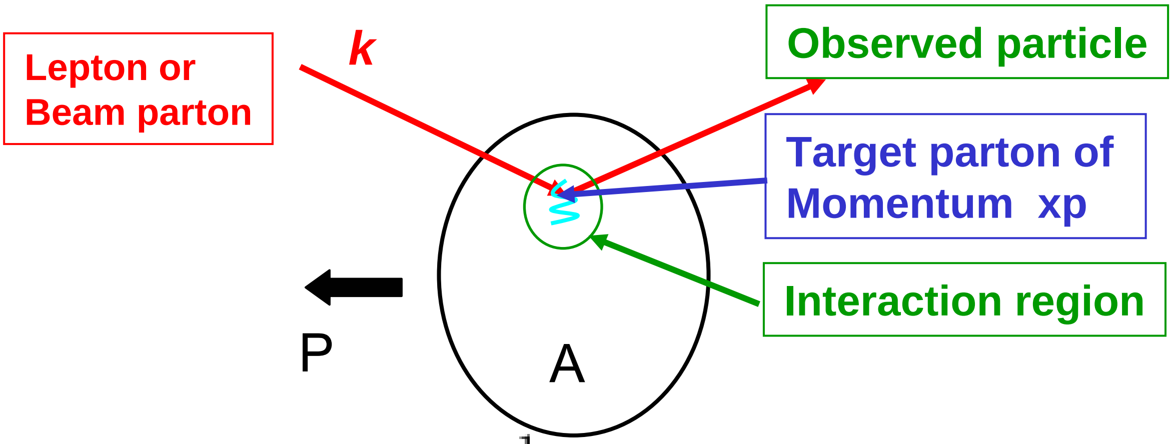
- J/ψ suppression:
  - process: $A + B \to \text{J}/\psi + X$
  - ratio of x-section:

    $$R_{\text{J}/\psi} \equiv \frac{\sigma^{AB}}{\sigma^{NN}} < 1$$

- . . .

SOURCES OF ANOMALOUS NUCLEAR DEPENDENCE

- Distance scales where hard collision took place:



Lepton or Beam parton

Observed particle

Target parton of Momentum xp

Interaction region

  - transverse size: $\sim \frac{1}{Q} \ll 1$ fm $\Leftrightarrow$ localized
  - longitudinal size: $\sim \Delta z(x) \sim \frac{1}{xp}$
  - longitudinal size of a nucleon: $\sim \Delta z_n \sim 2r\left(\frac{m}{p}\right)$

- Binding energy should have very little effect on observed anomalous nuclear dependence

- Fact that nucleons in a nucleus are very close to each other should be a key in any potential explanation of the nuclear dependence

- critical parton momentum fraction: x_c

$$\Delta z_n = \Delta z(x) \Leftrightarrow x_c = \frac{1}{2\,m\,r} \approx 0.1$$

  - Small x physics: $x \leq x_c \Leftrightarrow \Delta z(x) \geq \Delta z_n$

    more than one nucleon “involved” in collision
  - Large x physics: $x \geq x_c \Leftrightarrow \Delta z(x) \leq \Delta z_n$

    single scattering is localized within one nucleon

"CONCLUSIONS" WITHOUT DETAILED CALCULATIONS:

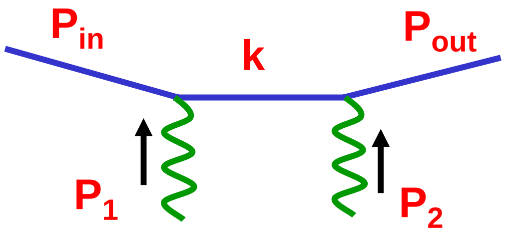
- Small x case:
  - multiple nucleons are involved in the region of hard interaction
  - Coherence between partons from different nucleons leads to strong nuclear dependence
  - Examples: shadowing, gluon saturation, etc.
- Large x case:
  - Single hard scattering is localized in all direction
  - Any anomalous nuclear dependence is a consequence of elastic multiple scattering covering different nucleons

    $\Rightarrow$ nuclear dependence should be proportional to nuclear medium size

    $\Rightarrow$ change spectrum, but, not total cross section
  - Examples: GLV approach to Cronin Effect
- Coherent multiple scattering is suppressed by power of $1/Q^2$ for each additional scattering
  - Examples: Resummed all power corrections to DIS structure functions – shadowing

Multiple Scattering in QCD

❑ Classical multiple scattering – cross section level:



⇨

Kinematics fix only $P_1 + P_2$

either P_1 or P_2 can be ~ zero

$$d\sigma^{\text{Double}} \sim \sigma^{\text{single}}(p_{in}, p_1)\cdot\sigma^{\text{single}}(p_2, p_{out})$$
$$\times\, dp_1 dp_2 \delta(p_1 + p_2 + p_{in} - p_{out})$$

Finite

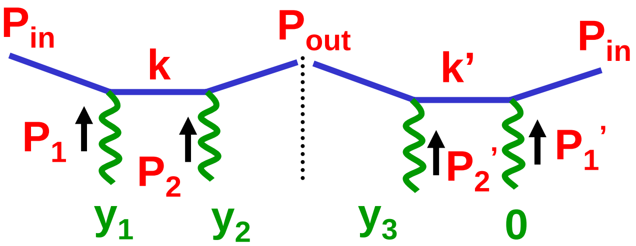
❑ Parton level multiple scattering (incoherent/indep.)

In pQCD, above $d\sigma^{\text{double}} \to \infty$ as p_1 or $p_2 \to 0$

- ❖ parton distribution at x=0 is ill-defined
- ❖ pinch poles of k in above definition

❑ Quantum mechanical multiple scattering

- Amplitude level



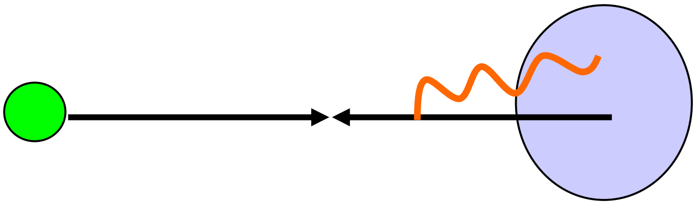
$$\langle A|\phi^{+}(0)\phi^{+}(y_3)\phi(y_2)\phi(y_1)|A\rangle$$

- ❖ 3-independent parton momenta
- ❖ no pinched poles
- ❖ depends on 4-parton correlation functions

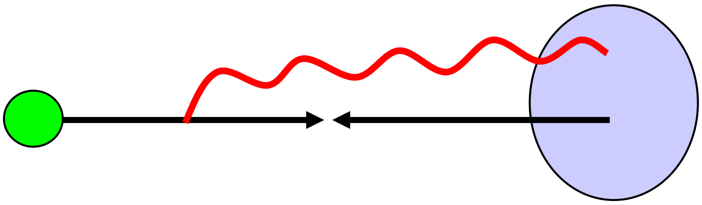
Need to include interference diagrams

Classification of nuclear dependence

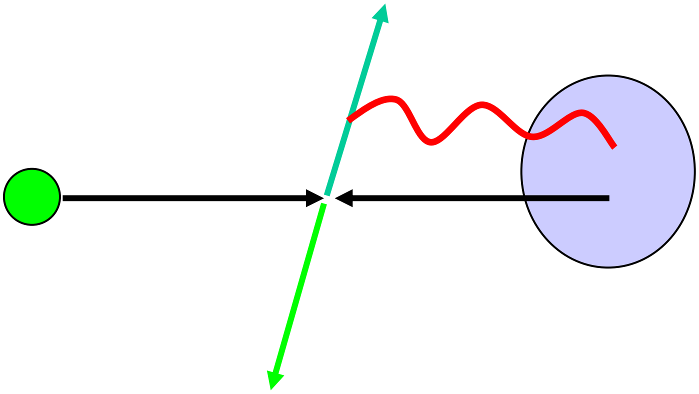
- ❑ Universal nuclear dependence from nuclear wave functions (in PDFs):



- ❑ Process-dependent nuclear dependence (power corrections):
  - ❖ Initial-state:

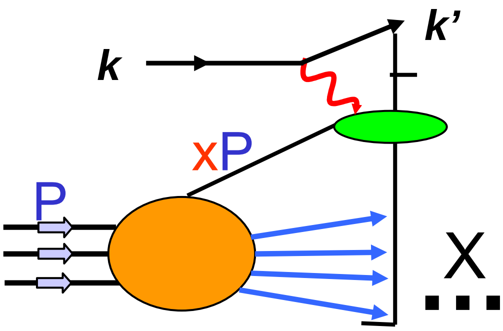


  - ❖ Final-state:



- ❑ Separation of medium-induced nuclear effect (process-dependent) from that in nuclear PDFs (process-independent)

All twist contributions to shadowing



Variables:

$$q = k - k', \ \nu = E - E',$$

$$y = (E - E')/E, \ Q^2 = -q^2,$$

$$x = Q^2/(2p \cdot q)$$

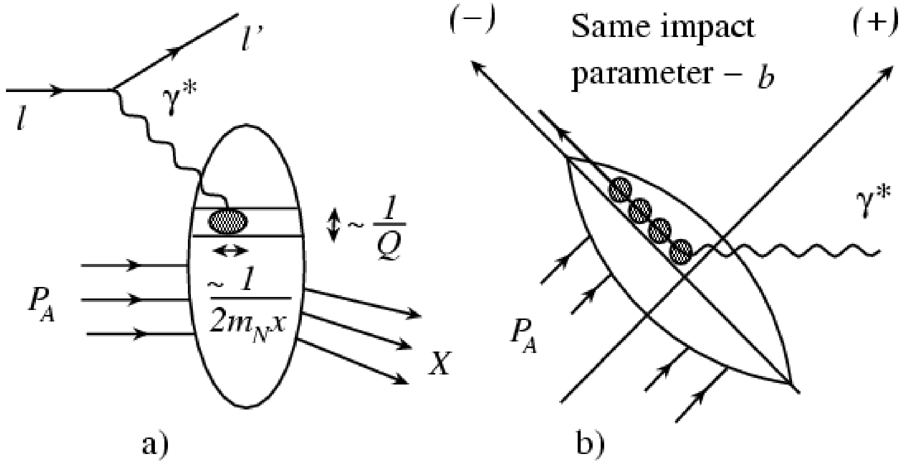
$$\frac{d\sigma_{lh}}{dx\,dy} = \frac{4\pi\alpha_{\mathrm{em}}}{Q^2}\frac{1}{xy}\left[\frac{y^2}{2}2xF_1(x,Q^2) + \left(1 - y - \frac{m_N xy}{2E}\right)F_2(x,Q^2)\right]$$

$F_1(x,Q^2), \ F_2(x,Q^2)$ **- the DIS structure functions**

- **Lightcone gauge:** $A \cdot n = A^+ = 0$
- **Frame:** $\bar{n} = [1,0,0_\perp], \ n = [0,1,0_\perp]$

$$q = -xp^+\bar{n} + \frac{Q^2}{2xp^+}n, \ \ p = \bar{n}p^+, \ \ xp + q = \frac{Q^2}{2xp^+}n$$

When $x < x_c$, virtual photon probes more than one nucleon at the given impact parameter

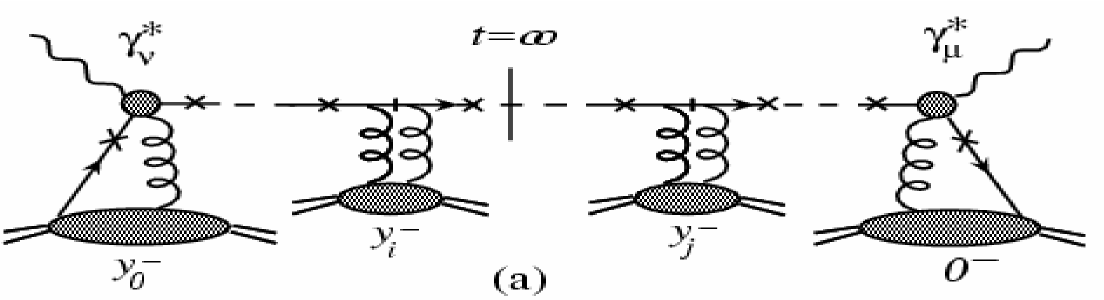


a) b)

J.W. Qiu and I. Vitev, hep-ph/0309094

Calculating power corrections

- ❑ When $x_B < 0.1/A^{1/3}$, the DIS probe covers all nucleons at the same impact parameter



- ❑ Fully coherent multiple scattering

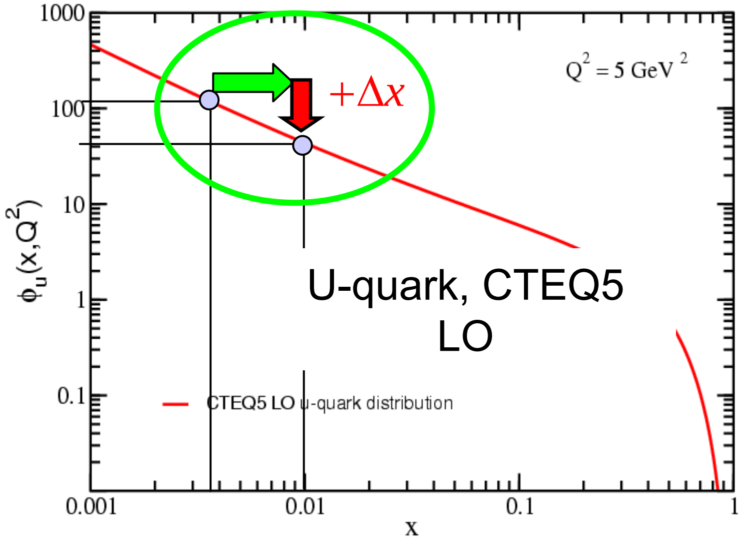
⇨ Take all possible insertions and cuts

$$F_T^A(x,Q^2)=\sum_{n=0}^{N}\frac{A}{n!}\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n\frac{d^nF_T^{(LT)}(x,Q^2)}{d^nx}\approx A F_T^{(LT)}\left(x+\frac{x\xi^2(A^{1/3}-1)}{Q^2},Q^2\right)$$

$$F_L^A(x,Q^2)=A\,F_L^{(LT)}(x,Q^2)+\sum_{n=0}^{N}\frac{A}{n!}\left(\frac{4\xi^2}{Q^2}\right)\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n\frac{d^nF_T^{(LT)}(x,Q^2)}{d^nx}$$

$$\approx A\,F_L^{(LT)}(x,Q^2)+\frac{4\xi^2}{Q^2}F_T^A(x,Q^2)$$

- ❖ slope of PDF's determines the shadowing
- ❖ Valence and sea have different suppression



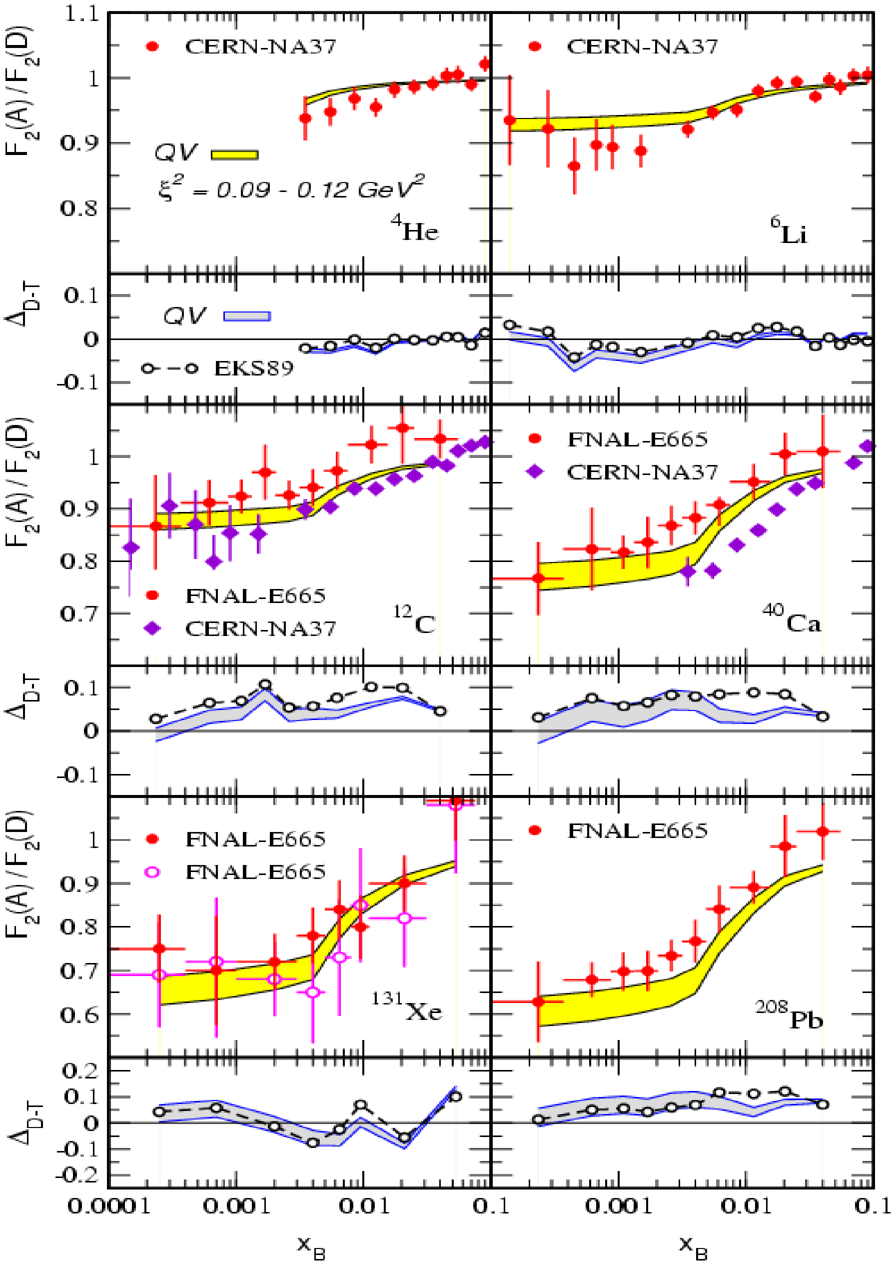
U-quark, CTEQ5 LO

Comparison with existing data

- ❑ Characteristic scale of power corrections:

$$\xi^2 = \left(\frac{3\pi\alpha_s(Q^2)}{8r_{0\perp}^2}\right)\boxed{\left\langle p\left|\hat{F}^2(\lambda_i)\right|p\right\rangle} \rightarrow \frac{1}{2}\lim_{x\to 0} xG\left(x,Q^2\right)$$

For $\xi^2 \approx 0.09 - 0.12 \ GeV^2$



The Gross-Llewellyn Smith and Adler Sum Rules

- Apply the same calculation to neutrino-nucleus DIS
  -- predictions without extra free parameter
- Gross-Llewellyn Smith sum rule:

D.J.Gross and C.H Llewellyn Smith , Nucl.Phys. B 14 (1969)

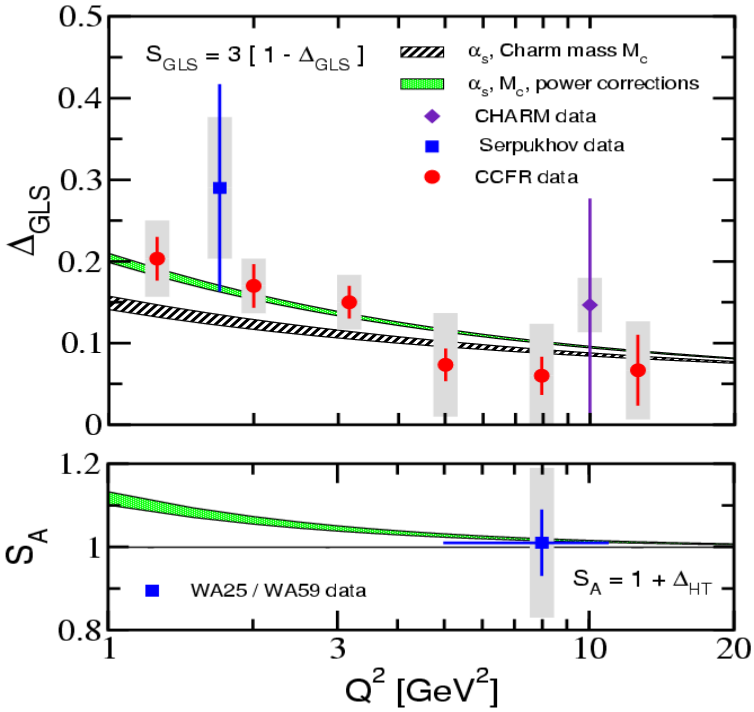
$$S_{GLS} = \int_0^1 dx\ \frac{1}{2x}\left(xF_3^{\nu N}(x,Q^2) + xF_3^{\bar{\nu} N}(x,Q^2)\right) = 3(1-\Delta_{GLS})$$

$$S_{GLS} = \#U + \#D = 3$$

- To one loop in $\alpha_s(Q^2)$

$$\Delta_{GLS} = \alpha_s(Q^2)/\pi$$

- Nuclear-enhanced power corrections are important



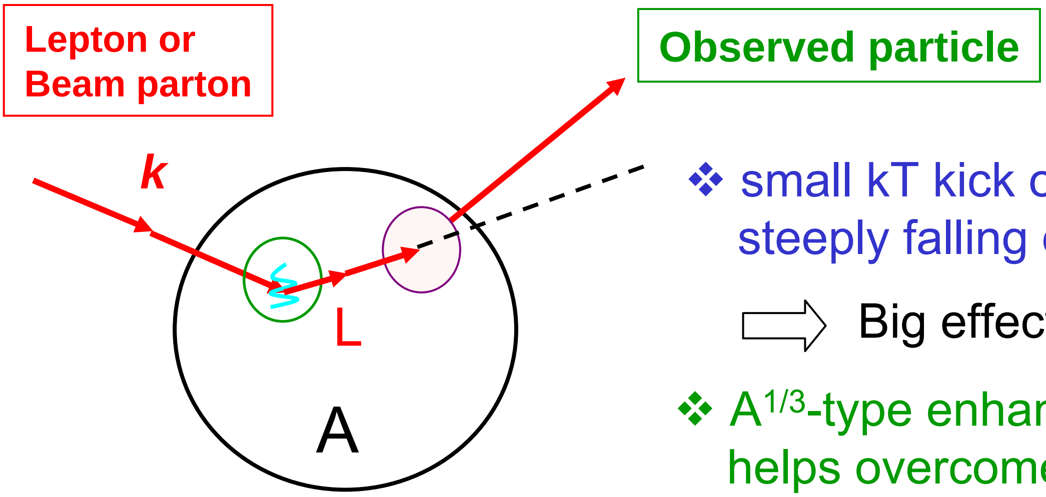
- Adler sum rule:

S.Adler , Phys.Rev. 143 (1964)

$$S_A = \int_0^1 dx\ \frac{1}{2x}\left(F_2^{\nu n}(x,Q^2) - F_2^{\bar{\nu} n}(x,Q^2)\right) = 1 + \Delta_{HT}$$

Predictions are compatible with the trend in the current data

Transverse momentum broadening



- ❖ small kT kick on a steeply falling distribution

⇨ Big effect

- ❖ $A^{1/3}$-type enhancement helps overcome the $1/Q^2$ power suppression

❑ Data are concentrated in small p_T region, but, $d\sigma/dQ^2dp_T^2$ for Drell-Yan is Not perturbatively stable (resummation is necessary)

❑ The moments are perturbatively stable (infrared safe)

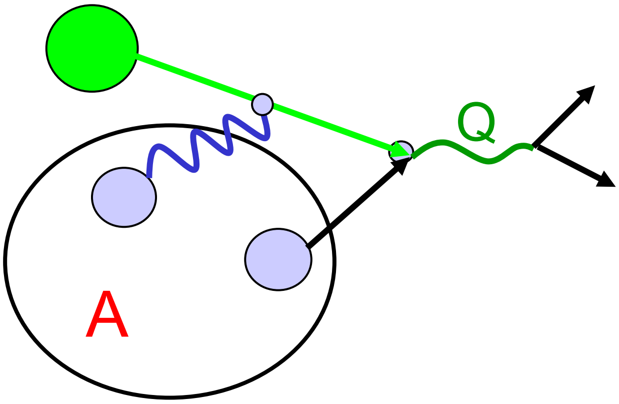
$$\int dp_T^2 \left(p_T^2\right)^N \left[\frac{d\sigma}{dQ^2dp_T^2}\right] \quad \text{with} \quad N \geq 0$$

$$\left\langle p_T^2\right\rangle \equiv \int dp_T^2\, p_T^2 \left(\frac{d\sigma}{dQ^2dp_T^2}\right) \Big/ \int dp_T^2 \left(\frac{d\sigma}{dQ^2dp_T^2}\right)$$

❑ Transverse momentum broadening

$$\Delta\left\langle p_T^2\right\rangle \equiv \left\langle p_T^2\right\rangle_{pA(\text{or } AA)} - \left\langle p_T^2\right\rangle_{pN}$$

Drell-Yan transverse momentum broadening



Plus interference diagrams

☐ Broadening:

$$\Delta\left\langle p_T^2\right\rangle = \left(\frac{4\pi^2\alpha_s}{3}\right)\frac{\sum_q e_q^2\int dx\,\varphi_{q/h}(x)T_{\bar{q}/A}(\tau/x)/x}{\sum_q e_q^2\int dx\,\varphi_{q/h}(x)\overline{\varphi}_{\bar{q}/N}(\tau/x)/x}$$

X. Guo (2001)

☐ Four-parton correlation functions:

$$T_{q/A}(x)=\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\int\frac{dy_1^-dy_2^-}{2\pi}\theta(y^- - y_1^-)\theta(-y_2^-)$$

$$\times\ \left\langle A\right|F_\alpha^+(y_2^-)\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y^-)F^{+\alpha}(y_1^-)\left|A\right\rangle$$

$$\approx \lambda^2 A^{1/3}\varphi_{q/A}(x)$$

☐ Predictions:

$$\Delta\left\langle p_T^2\right\rangle \approx \frac{4\pi^2\alpha_s}{3}\lambda^2 A^{1/3}$$

- $A^{1/3}$-type dependence
- Small energy dependence to the broadening
- $\lambda^2 \sim \xi^2$

☐ Fermilab and CERN data

Show small energy dependence and give $\lambda^2 \sim 0.01$ GeV^2

Summary and outlook

- ❑ Predictive power of QCD perturbation theory relies on the factorization theorem

- ❑ The Theory has been very successful in interpreting data from high energy collisions

- ❑ PQCD can also be used to calculate anomalous nuclear dependence in terms of parton-level multiple scattering, if there is a sufficiently large energy exchange in the collision

- ❑ In nuclear collisions, we need to deal with both coherent inelastic as well as incoherent elastic multiple scattering

  - ❖ elastic scattering re-distribute the particle spectrum without change the total cross section
  - ❖ inelastic scattering changes the spectrum as well as the total cross sections

- ❑ nuclear dependence is a unique observable to parton-parton correllations, the properties of the medium.