THE

PHASES OF QCD MATTER

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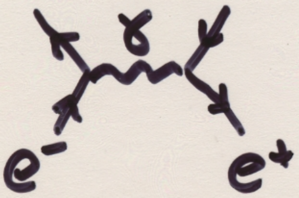
WHAT IS QCD?

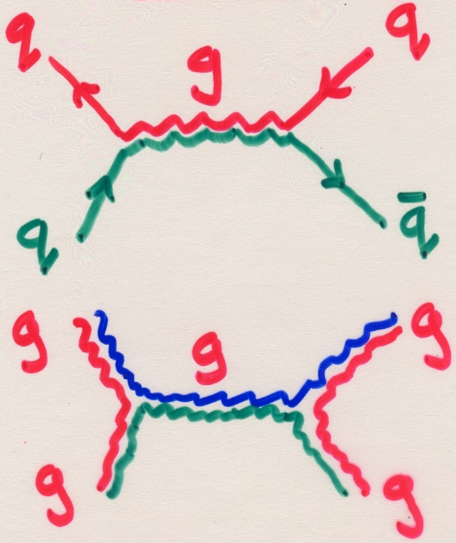
Its Lagrangian suggests it is a theory of quarks and gluons, not too different from QED which is a theory of electrons and photons:

QED



e⁻: charge -1
γ: neutral

QCD



q: charge r, g or b
gluons: also colored.

Quarks come in six flavors:

Flavor	Mass (MeV)	
u	5	light. treat as massless to first approx.
d	10	
s	100	← middleweight
c	1500	too heavy to play a role in this talk
b	5000	
t	175000	

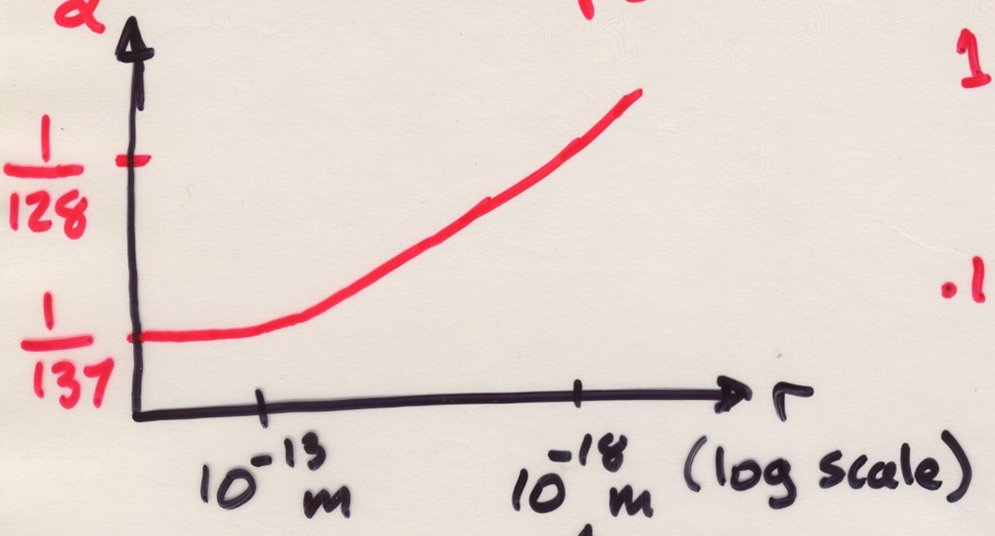
ASYMPTOTIC FREEDOM

Gross, Wilczek, Politzer (1973)

In quantum field theory, the vacuum is a medium which can screen charge.

QED

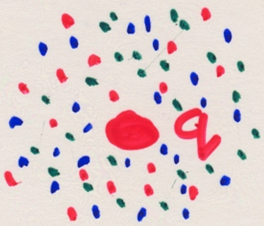
α: Force between electrons ~ $\frac{\alpha(r)}{r^2}$

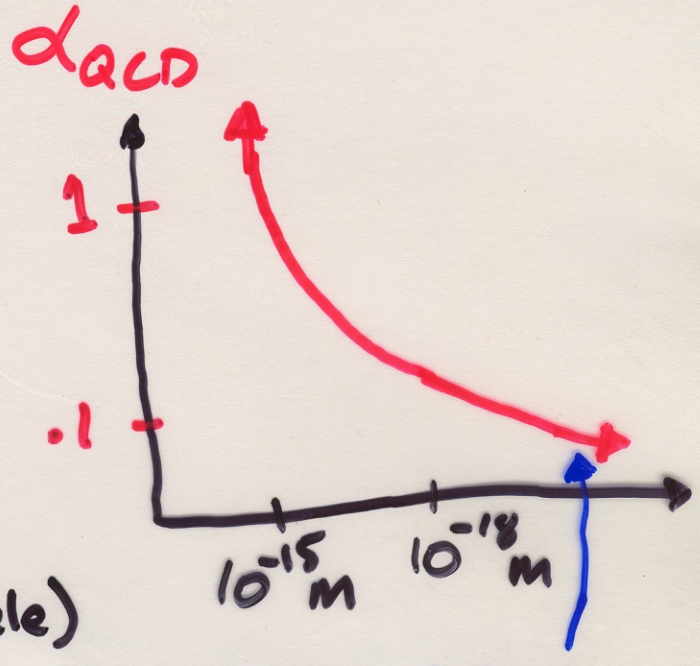


experiments at CERN

Coupling "constants" not constant. Depend on scale at which you probe.

QCD



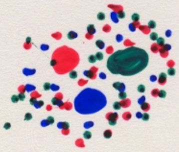


asymptotic freedom, or anti-screening. (That's why Friedman, Kendall, Taylor were able to see weakly interacting quarks.)

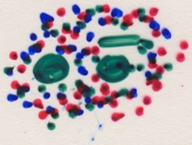
WHAT DOES QCD DESCRIBE?

It is an experimental fact that in the world around us, quarks and gluons occur only in colorless packages:

Protons, neutrons,...



Pions, kaons,....



These hadrons are the quasiparticles of the QCD vacuum.

They, in turn, make up everything from nuclei to neutron stars, and thus most of the mass of you and me.

Why no colored quasiparticles?

- would disturb vacuum out to ∞, and $\therefore$ have ∞ mass.
- NB: growth of $\alpha(r)$ with $r \Rightarrow$ force between colored objects does not fall off with distance.
- their absence confirmed by direct calculation. (Lattice gauge theory.)

NB: hadrons are heavy.

$M_{\text{proton}} = 938$ MeV

$M_{u+u+d} \cong 20$ MeV

WHAT IS QCD?

A theory of quarks and gluons....

WHAT DOES QCD DESCRIBE?

Colorless, heavy, hadrons...

Hadrons are the (rather complicated) quasi-particles of the QCD vacuum.

The vacuum, whose excitations are the hadrons, is therefore quite a nontrivial [confinement; chiral symmetry breaking; strong coupling; ...] phase of the theory.

BUT: QCD is asymptotically free ....

DO OTHER (SIMPLER?) PHASES EXIST?

Do other phases exist whose quasiparticles look more like the quarks and gluons of the QCD Lagrangian? And look more like phases familiar from QED?

Asymptotic freedom: quarks and gluons weakly interacting

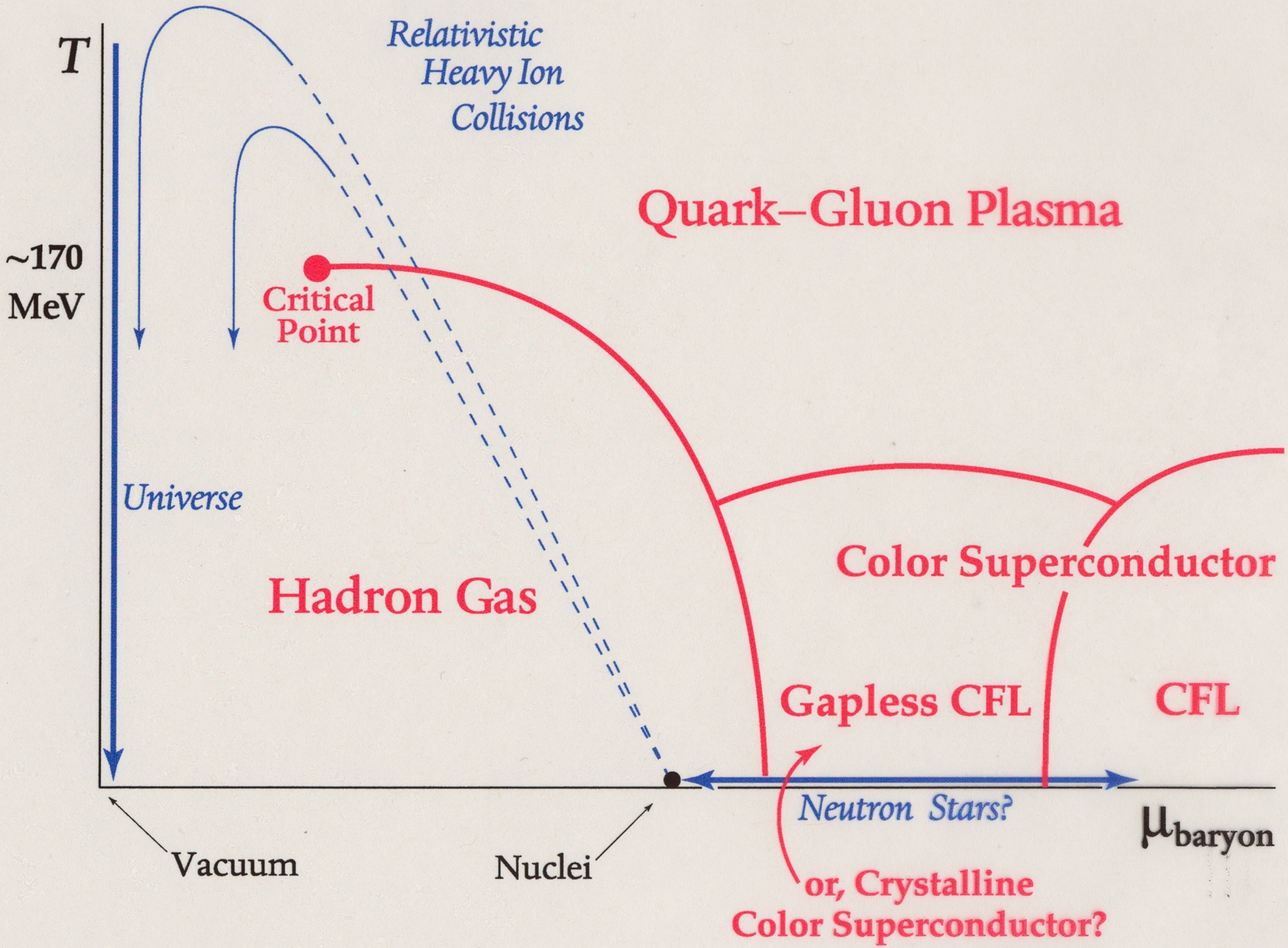
i) when close together

ii) when interact at large momentum.

Suggests look at high density or high temperature.

NB: condensed matter physics teaches us that phases may be far from simple even for α as small as $\frac{1}{137}$.

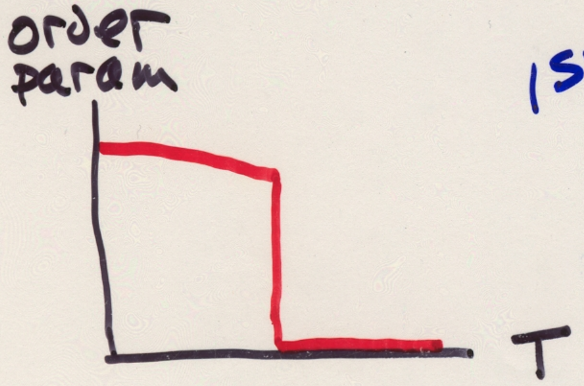
EXPLORING *the* PHASES *of* QCD



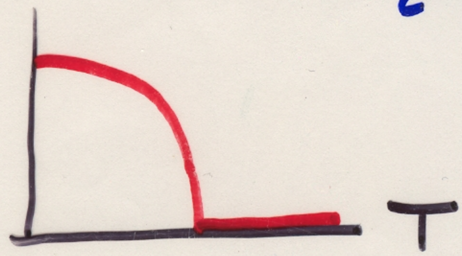
PHASE TRANSITIONS

i) Look for an order parameter.
- zero on one side of transition, non-zero on the other
- change in symmetry?

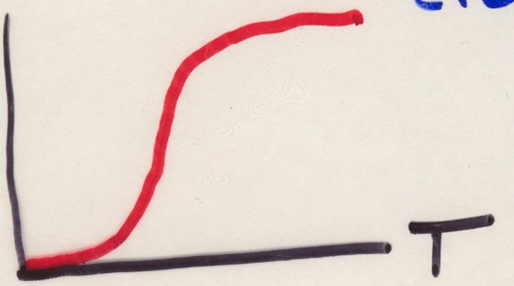
ii)



1st order: thermodynamic quantitities discontinuous
- latent heat; bubbles
- eg: boiling water



2nd order: continuous, but not smooth.
- long wavelength fluctuation
- no length scale at T_c
- eg: Curie transition



Crossover: smooth. No order parameter. No change in symmetry.
eg: ionization of a gas

SECOND ORDER PHASE TRANSITIONS

Physics is scale invariant at $T=T_c$.

⇒ fluctuations on all length scales.

⇒ coarsen your microscope, and the world looks the same.

(fancy way to say this: you are at an infrared fixed point of the renormalization group.)

⇒ long wavelength physics independent of microscopic physics — UNIVERSAL.

⇒ many microscopic theories → same long wavelength physics.

CHIRAL SYMMETRY

There is another qualitative difference between $T \ll T_c$ and $T \gg T_c$, associated with a qualitative feature of the QCD vacuum.

$$\mathcal{L}_{QCD} = \sum_i \bar{q}^i_L \, i\not{D} \, q^i_L + \sum_i \bar{q}^i_R \, i\not{D} \, q^i_R$$

$$+ \mathcal{L}_{\text{gluons only}}$$

i is a flavor index. $i = u, d$ (2 massless flavors, for now.)

$\mathcal{L}_{QCD}$ is symmetric under:

$$SU(2)_L \times SU(2)_R$$

but: predictions of this symmetry fail.

eg predicts 4 pions and only 3 exist.

RESOLUTION: $\mathcal{L}$ invariant, but $|0\rangle$ not:

$$\langle 0 | \bar{q}^i_L \, q^j_R | 0 \rangle \neq 0$$

$$= \sigma \mathbb{1}^{ij} + i \vec{\pi} \cdot \vec{\tau}^{ij}$$

- only symmetric under $SU(2)_{L+R}$
- can point in one of four directions

ie: $\bar{u}u, \bar{d}d, \bar{u}d, \bar{d}u$ or: $\sigma, \pi^1, \pi^2, \pi^3$

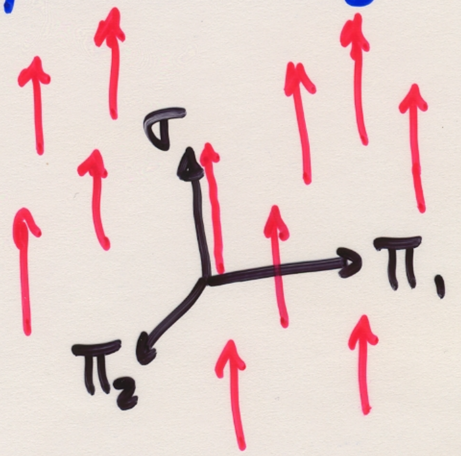
CHIRAL SYMMETRY BREAKING....

(for now, $m_u = m_d = 0$; $m_s = \infty$)

The QCD vacuum (the $\bar{q}q$ pairs therein) is ordered in flavor space.

$\langle \bar{q}_L q_R \rangle \neq 0$ condensate "picks a direction" among 4 previously equivalent options.

- called σ-direction.
- points in same direction everywhere.



$\langle \sigma \rangle \neq 0$ $\langle \vec{\pi} \rangle = 0$

Could have pointed any direction. ∴ waves in which direction of ↑ undulates associated with massless pions.
(Goldstone's theorem)

$m_\pi = 140$ MeV. Lightest hadron.
($m_\pi \neq 0 \leftrightarrow m_q \neq 0$)

NB: Heaviness of other hadrons (eg p, n) can be seen as due to their interaction with (disturbance of) condensate.

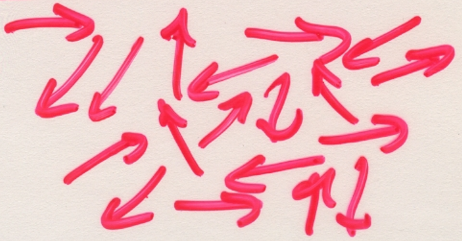
... CHIRAL SYMMETRY RESTORATION

T ≠ 0

"waves on the condensate", but $\langle \bar{q}q \rangle$ still nonzero. Still a preferred direction. Symmetry still broken.

a.k.a. a gas of pions

T ABOVE SOME T_c...



Entropy wins over order. "Condensate scrambled." Disordered. $\langle \bar{q}q \rangle \rightarrow 0$

All directions equivalent.

...CHIRAL SYMMETRY RESTORED

What is T_c? Lattice calculations indicate $T_c \sim 140 - 190$ MeV

$\sim 2 \times 10^{12}$ Kelvin

THE QCD PHASE TRANSITION

$T \ll T_c$	$T \gg T_c$
hadrons confinement	plasma of quarks and gluons, which is weakly interacting for $T \to \infty$.

(associated with change in symmetry if $m_{\text{all quarks}} \to \infty$)

chiral symmetry spontaneously broken	chiral symmetry restored

(associated with change in symmetry if $m_{\text{2 or more quarks}} \to 0$)

$T_c \sim$ ~~140-190~~ MeV

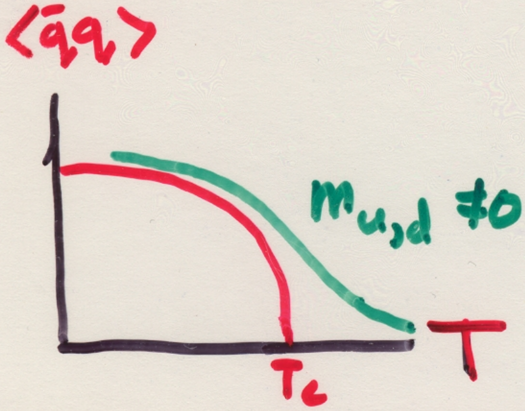
165-180

APPLICATION OF UNIVERSALITY

QCD near T_c ↔ 4-component magnet near its T_c.

↑

Has 2nd order transition.



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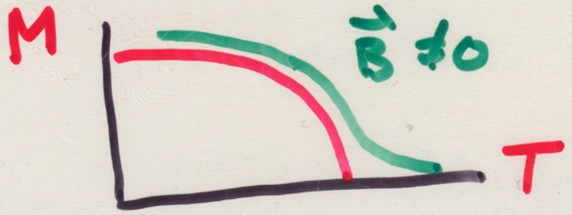


Calculations tested in expt, at least for 3-component magnets.

$\langle \bar{q}q \rangle \sim (T_c - T)^{\beta}$ at $m_{u,d} = 0$

$\beta = .383 \pm .005$

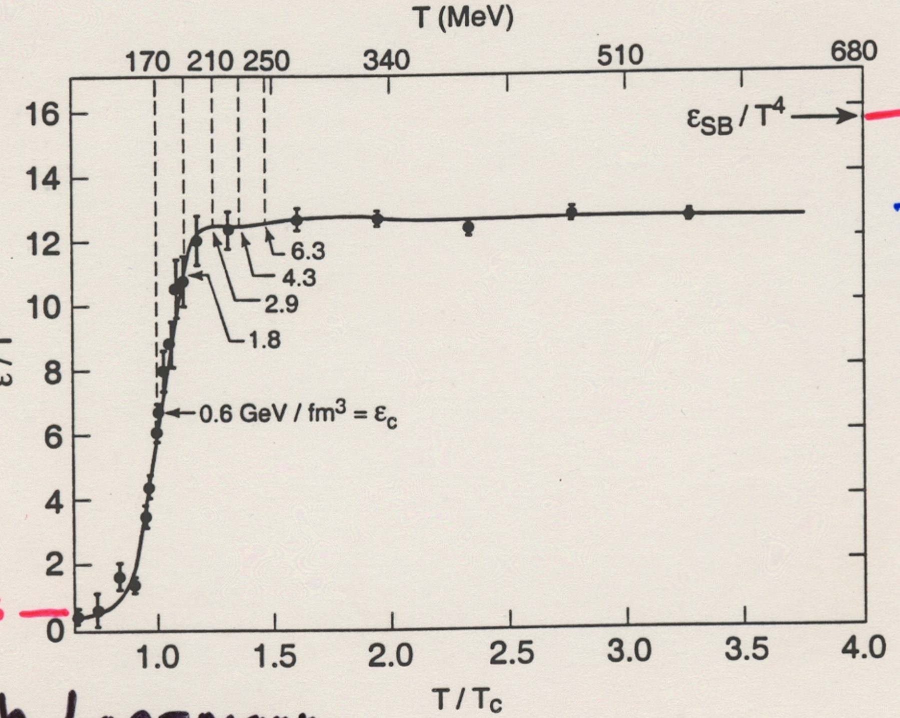
$\langle \bar{q}q \rangle \sim (m_{u,d})^{1/\delta}$ at $T = T_c$

$1/\delta = .208 \pm .001$

These predictions from magnets for QCD being tested by simulation of quarks and gluons on worlds biggest computers.

T(MeV), assuming $T_c = 170$ MeV.
(estimate is $140 < T_c < 190$)

$\frac{\varepsilon}{T^4}$



ideal QGP

pion gas

DECONFINEMENT

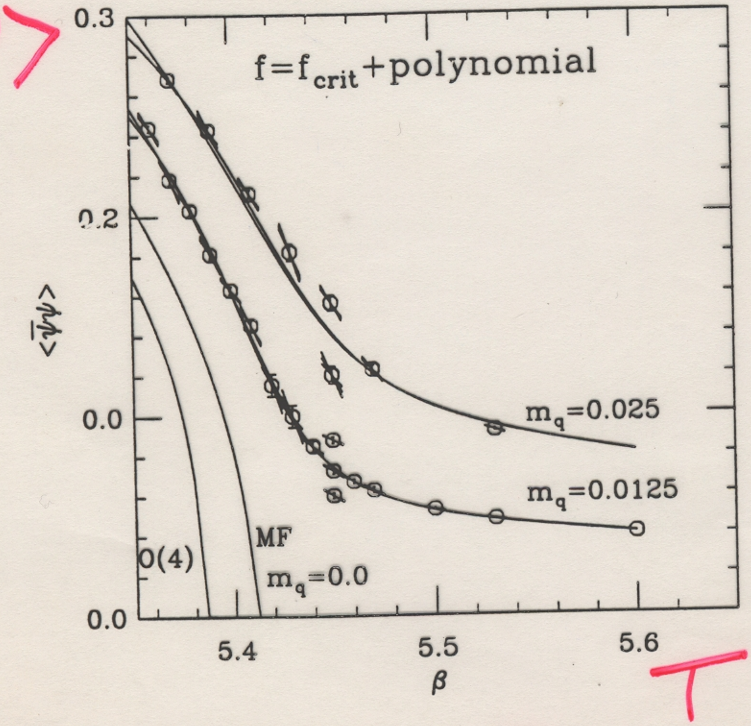
(IONIZING THE HADRONS)

Karsch Laermann
Peikert (Heine)

T/T_c

&

$\langle \bar{\psi}\psi \rangle$



Blum
DeTar
MILC
collab.

CHIRAL SYMMETRY RESTORATION

(MELTING THE VACUUM)

ON THE LATTICE

T

(funny units)

$N_f = 2$
$m_q \neq 0$
∴ smooth crossover

WHAT ABOUT THE STRANGE QUARK?

	$M_{u,d}=0$	$M_{u,d}\neq 0$
$M_s=\infty$:	2nd order	Crossover
$M_s=0$:	1st order	1st order

↳ lattice calculations and universality arguments agree.

M_s AS IN NATURE?

Best lattice calculations suggest crossover, not 1st order. (∃ caveats)

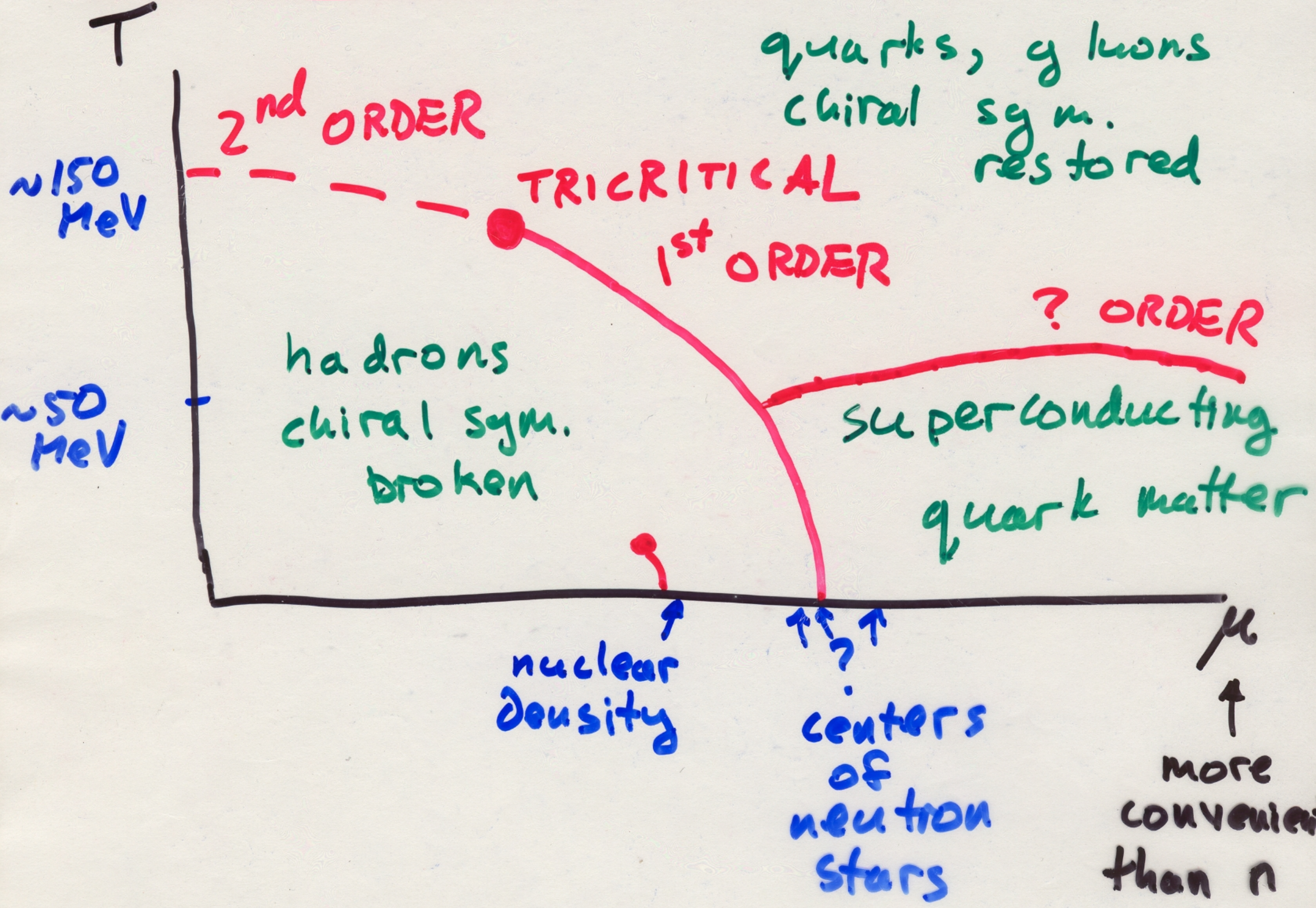
ASIDE: Strong 1st order phase trans. ruled out by cosmological data, as it upsets big bang nucleosynthesis.

CAN EXPERIMENTS DETERMINE WHETHER CROSSOVER OR 1st ORDER?

Yes, in a sense that I will describe. But, to talk about expts need to introduce nonzero baryon density....

THE QCD PHASE DIAGRAM

i) $m_u = m_d = 0$ $m_s = \infty$



Alford KR Wilczek

Rapp Schaefer Shuryak Velkovsky

Berges KR

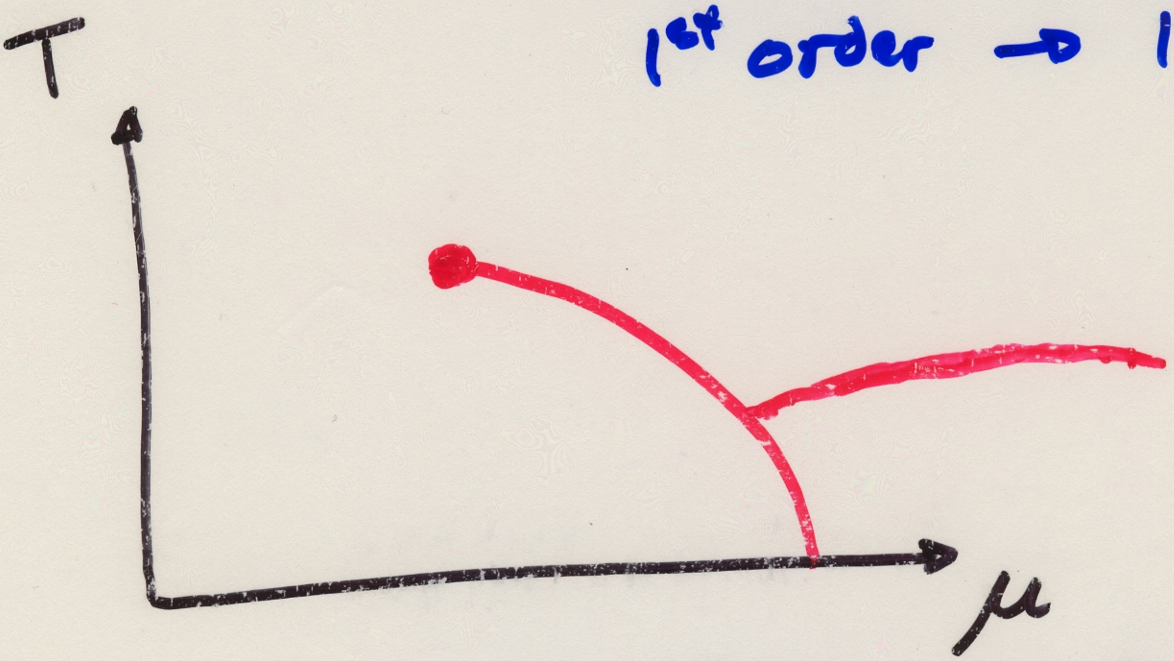
Halasz Jackson Shrock Stephanov
Verbaarschot

$m_{u,d} \neq 0$

2nd order → crossover

Tricritical → 2nd order

1st order → 1st order



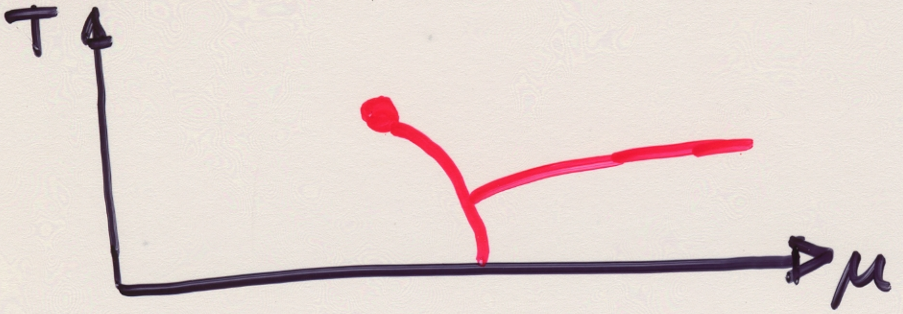
At ● : $m_\pi \neq 0$ (because of $m_{u,d} \neq 0$)

$m_\sigma = 0$

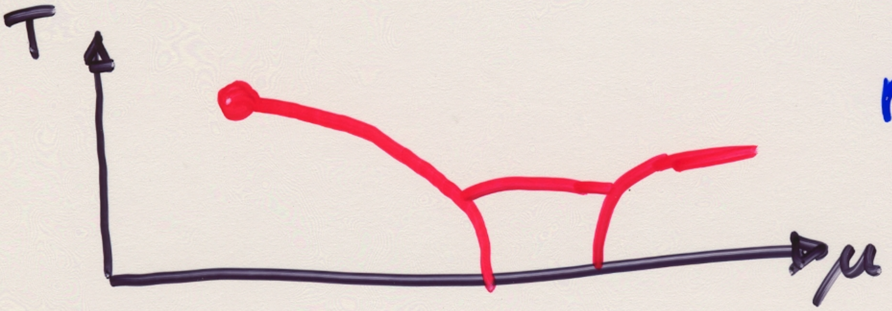
3D Ising model universality class

eg liquid-gas critical point

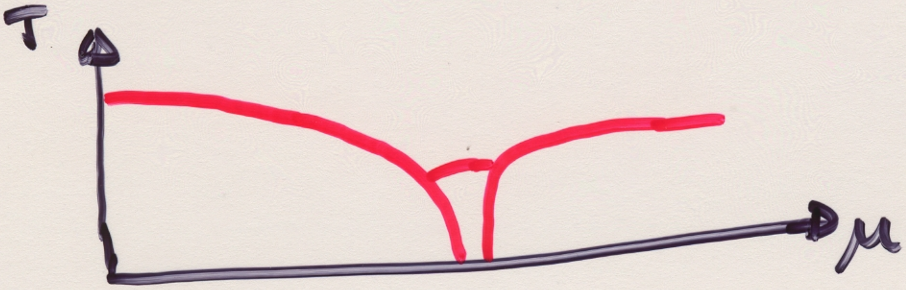
WHAT ABOUT THE STRANGE QUARK?



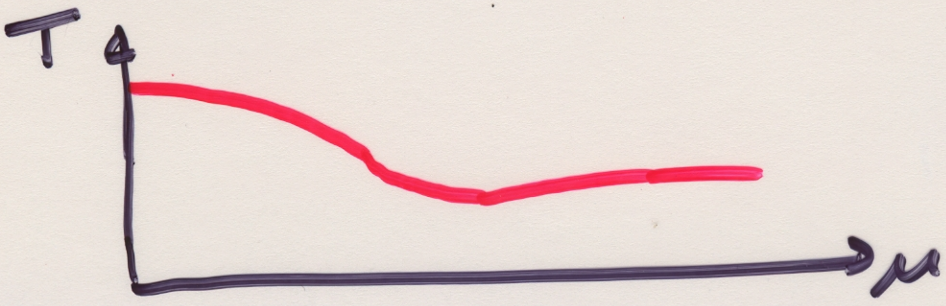
$m_s = \infty$



$m_s \sim$ physical ?



$m_s <$ physical



$m_s = 0$

Effects of reducing m_s:

i) ● sucked to the left

ii) funny business at large μ. (WAIT)

If experiments were to detect signatures of ●, learn that cosmological phase transition not first order, ie crossover.

$T \neq 0$; $\mu \neq 0$; μ / T NOT LARGE

- regime explored by heavy ion collisions
- very recently, we are starting to learn about this regime from lattice calculations that rely on smallness of μ / T to keep fermion sign problem under control.
- these methods may be used to locate the ....

CRITICAL POINT, a 2nd order point in the phase diagram where a line of 1st order transitions ends. (Location is sensitive to quark masses. Moves leftward as masses ↓.)

THREE NEW LATTICE METHODS

① Reweighting. Fodor + Katz

Want physics at $\textcircled{a} \equiv (\mu, T_a)$.

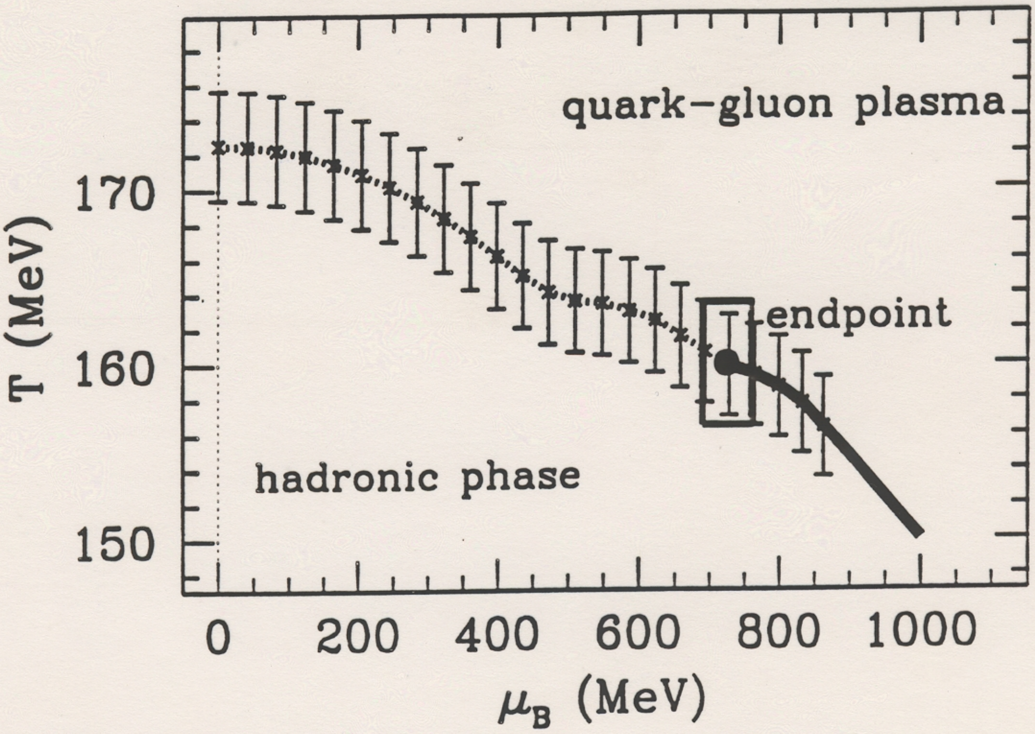
Simulate at $\textcircled{b} \equiv (0, T_b)$, and "reweight": lump difference betwee physics at $\textcircled{b}$ and $\textcircled{a}$ into observables.

$$\text{Difficulty} \sim \exp\left[\frac{|F_{\textcircled{b}} - F_{\textcircled{a}}|V}{T}\right]$$

F + K choose T_b to minimize ⤴.

BUT: cannot use method at large volumes.

Fodor+Katz



- $T_{\text{crossover}}(\mu)$ quite flat.
- claim to locate end point!

CAVEATS: $V = 4^3, 6^3, 8^3$ is small.

(makes me wonder how they located end point so accurately.)

- recall: <u>can't</u> go to $V \to \infty$
- no continuum extrapolation yet
- light quarks not light enough ($\Rightarrow$ end point too far right)

LOCATING THE CRITICAL POINT

- Best guess at present is that critical point has μ_B somewhere around 600 MeV.
- error estimates uncertain and large. (Not at all like calculating Tc. Yet.)
- progress is all of a sudden occurring very rapidly....
- race between lattice QCD & experiment to locate critical point
- Convincing determination that Nature's phase diagram features critical point ⇒ crossover at lower μ_B.

WHAT WE WANT TO LEARN ABOUT FROM HEAVY ION COLLISIONS (AND FROM LATER SPEAKERS)

1. Physics of initial nuclei: lots of gluons at small x in initial wave function. How are they liberated in collision? → ask Raju

2. Is there experimental evidence that all those gluons equilibrate early enough, while ~~energies are~~ energy density still high?

   Yes. V_2 at low p_T

   → ask Thomas and Jamie

   If answer were to prove to be "no", then heavy ion collisions cannot be used to explore QCD phase diagram.

3. Can expts. measure energy density and/or pressure at early times?

Data on v_2 & "jet quenching" take us towards this goal. Lets see what we learn at the conference.

4. Can expts. measure TEMPERATURE at same early time as above?

NOT YET. In future, we hope γ or dileptons or J/ψ will give a handle on T.

Why care? MEASURING ε/T^4 or P/T^4 TELLS YOU WHETHER MATERIAL IS DECONFINED

5. What other properties can be measured?

V_2 & "jet quenching" both point to unexpectedly short mean free paths. Quark gluon plasma turns out to be a Quark gluon liquid, with m.f.p. comparable to spacing between particles not much longer as in a plasma. (Will become a plasma at higher T, where coupling gets weaker.)

Lets watch for theoretical postdictions related to this at the conference ....

6. Can experiments locate the critical point?

AGS SPS RHIC

$\mu_B^{\text{freezeout}}$: 550 MeV $\longleftrightarrow$ 30 MeV

Vary $\sqrt{s}$, and hence μ_B, and look for enhancement of event-by-event fluctuations

of: i) # & mean p_T of low p_T pions / hadrons.

ii) observables that are proxies for baryon number, like # of protons − antiprotons.

NA49 has data at lower energy → higher μ than before. Lets see whether they report any event-by-event fluctuation data at these energies....

Chemical freeze–out in the T–μ_B plane

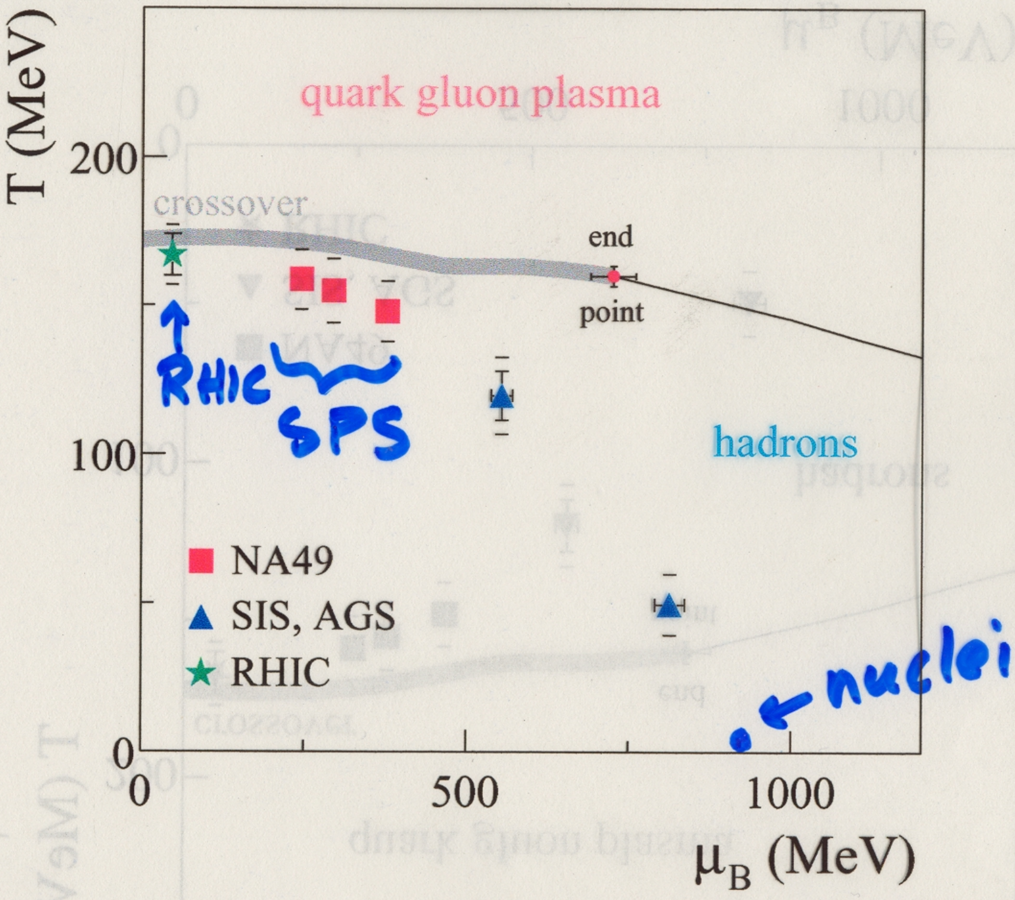
40 and 80 *A*GeV yields also fitted

	40 *A*GeV	80 *A*GeV	158 *A*GeV
T (MeV)	148 ± 2	155 ± 4	159 ± 2
μ_B (MeV)	377 ± 7	294 ± 15	244.5 ± 4.7
γ_S	0.75 ± 0.02	0.72 ± 0.03	0.82 ± 0.02
χ^2/NDF	14.8/4	10.4/4	23.5 / 11

$\sqrt{s}$: 9 12 17

fits by F. Becattini

- Freeze–out parameters on a (relatively) smooth curve
- Curve approaches phase boundary in the SPS energy range
- Even at RHIC, the parameters do not enter QGP–phase



Cross–over line from Z. Fodor, S.D. Katz hep–lat/0204029

CERES uses a variable $\Sigma_{p_T} \sim \frac{1}{2}(F-1)$

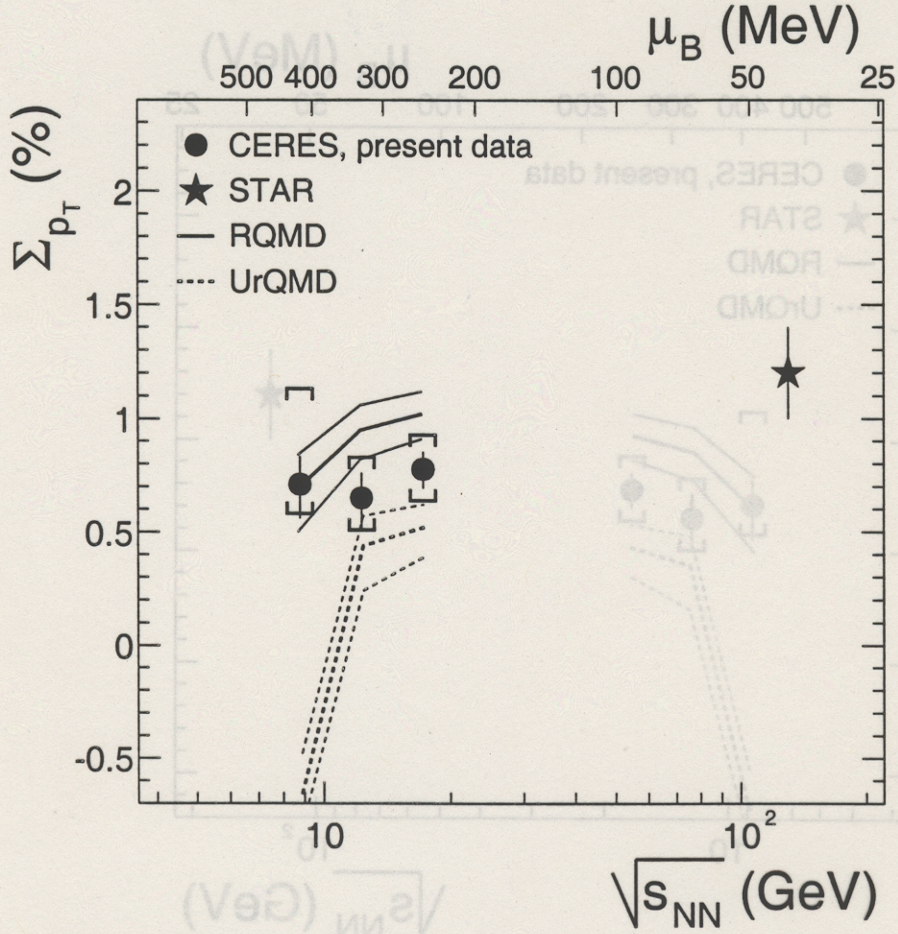


Fig. 10. The fluctuation measure Σ_{p_T} as function of $\sqrt{s_{NN}}$ and of μ_B at chemical freeze-out [30]. The full circles show CERES results (after SRC removal) in central events at 40, 80, and 158 A GeV/c. The brackets indicate the systematic errors. Also shown is the STAR result [31] at $\sqrt{s_{NN}} = 130$ GeV which is not corrected for SRC. Results and statistical errors from RQMD and URQMD calculations (with rescattering) are indicated as solid and dashed lines, respectively.

the critical point of the QCD phase diagram. At SPS energies and for the finite rapidity acceptance window of the CERES experiment, the fluctuations should reach values of about 2%, i.e. more than three times larger than observed in the present data[3]. Most important, no indication for a non-monotonic behaviour as function of the beam energy has been observed. This suggests that the critical point may not be located in the μ_B regime below 450 MeV.

The results from RQMD and URQMD show rough agreement with the data, except for the URQMD calculation at 40 A GeV/c where Σ_{p_T} is negative (see Fig. 10). We note that a positive value of $\Sigma_{p_T} = 0.38^{+0.17}_{-0.48}\%$ is obtained from

---

[3] The predicted fluctuations in the measure $\sqrt{F} = 1.1$ in [13] corresponds to about 2% in Σ_{p_T} in the CERES acceptance [33].

CERES nucl-ex/0305002

LARGE μ; SMALL T

Whereas at high T entropy wins → quark-gluon plasma with symmetries of the QCD Lagrangian manifest....

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

How can we use astrophysical observations of compact stars to determine the QCD phase diagram?

THE DIFFICULTY WITH DENSITY

Why are we still asking basic questions about QCD at high μ, low T, like "what is symmetry of ground state?"

NO LATTICE CALCULATIONS

$\mu \neq 0$ → complex Euclidean action
→ sign problem that makes difficulty of standard Monte Carlo $\sim e^V$.

Equally nasty sign problems can be solved in simpler systems. Chandrasekharan Wiese

Sign problem may also be evaded:

- at small V, small μ/T Fodor Katz; Hands Karsch et al
- calculate at Im μ; continue observables. Works at $\mu/T < \pi/3$. V can be large. de Forcrand Philipsen; d'Elia Lombardo

→ may be used to locate critical point.

- modify the theory. (color superconductivit studied on lattice for NJL & QCD w̄ $N_c=2$ Hands et al Kogut et al

NO EVASION POSSIBLE FOR QCD at $\mu \gg T$

- use smallness of g at $\mu \to \infty$
- use models at accessible μ.

WHY COLOR SUPERCONDUCTIVITY?

Large $\mu \rightarrow$ quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom $\rightarrow$ <u>weak</u> interactions between quarks at Fermi surface.

<u>BUT</u> <u>any</u> attractive interaction, no matter how weak, $\rightarrow$ COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color 3. (no need to resort to phonons; $\therefore$ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,

$\Rightarrow$
- electric & color currents superconduct
- mass for photon & (some) gluons(?)
- Meissner effects. (Magnetic & color magnetic fields excluded.)

Barrois; Bailin & Love

GAP AND T_c

Much work (that I will not review) suggests that @ $\mu_q \sim 500$ MeV ⌈~10 × nuclear density⌋

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ THIS is high T_c s.c.!

Two classes of methods ~ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R., Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

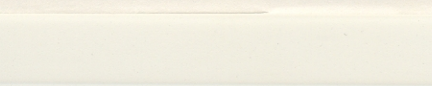
ii) weak-coupling QCD calculations, valid for $\mu \rightarrow \infty$; $g \rightarrow 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^8$ MeV K.R., Shuster)

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{}$$

Schaefer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; K.R., Shuster; Rischke, Wang; ....

Son

⌈$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:



$$\rightarrow 1 = g^2 \underbrace{\ln\frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln\frac{\Delta}{\mu}}_{\text{collinear divergence}}$$