Particle Correlations

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Outline

• Why particle correlations?
  – Few-body (jet-like) correlations
  – Many-body (flow) correlations
  – Analysis techniques

• Particle correlations in heavy-ion collisions
  – Near-side ridge correlation
  – Away-side double-peak correlation
  – Triangular flow background

• Particle correlations in small systems
  – Revisit two-particle acceptance correction

• Flow correlations
  – Some new idea: initial state anisotropy, quantum mechanics
Artist’s view of heavy-ion collisions
BRAHMS

STAR Detector
Why particle correlations?

• Single particles can only measure production rates and kinematic distributions
• High-energy collisions are complex—need particle correlations to measure the complex structure of the collision system
• Particle correlations measure jet-like correlations, flow, etc.
• Majority of measurements in heavy-ion collisions are done by particle correlations
Two categories of correlations

- **Few-body, e.g.**
  - Jets
  - Resonance decays

- **Many-body, event-wise**
  - Collective flow
Analysis techniques

\[ \Delta \phi = \phi_{\text{assoc}} - \phi_{\text{trig}}, \Delta \eta = \eta_{\text{assoc}} - \eta_{\text{trig}} \]

\[ S(\Delta \eta, \Delta \phi) = \frac{1}{N_{\text{trig}}} \frac{d^2N_{\text{same}}}{d\Delta \eta d\Delta \phi} \]

\[ B(\Delta \eta, \Delta \phi) = \frac{1}{N_{\text{trig}}} \frac{d^2N_{\text{mix}}}{d\Delta \eta d\Delta \phi} \]

- Tracking efficiency is corrected for associated particles.
- Trigger particles are often uncorrected, because correlations are normalized per trigger. Better to have trigger particle correction as well.
- Two-particle acceptance often corrected by mixed-events: \( B(\Delta \eta, \Delta \phi) / B(0, \Delta \phi) \).

\[ \frac{dN}{d\eta} = \text{const.} \quad (-\eta_{\text{max}} < \eta < \eta_{\text{max}}) \]

\[ \frac{dN}{d\Delta \eta} = \int d\eta_1 \int d\eta_2 \left( \text{const} \times \text{const} \right) \delta(\eta_2 - \eta_1 - \Delta \eta) \]

\[ \propto 1 - \frac{|\Delta \eta|}{2\eta_{\text{max}}} \]
Particle correlations in heavy-ion collisions
Jet correlations

- Calculable by pQCD
- Fragmentation of partons produces back-to-back jets of hadrons.
- Jets are clustered in angle and rich in high-$p_T$ particles.

- Jets produced in AA traverse and interact with the medium, lose energy and thus carry information of the medium.
Particle correlations: focus on **away side**

**Au+Au**

hadrons

leading particle suppressed

away-side particles enhanced at low $p_T$

away-side particles suppressed at high $p_T$

$p_T^{trig}>4$ GeV/c, $2<p_T^{assoc}<4$ GeV/c

$d$+Au FTPC-Au 0-20%

**p+p**

$p_T^{trig}=4-6$ GeV/c, $p_T^{assoc}=0.15-4$ GeV/c

**STAR**

STAR PRL91 (2003) 072304

STAR PRL95 (2005) 152301
The **near-side** is also interesting

STAR, PRL 95 (2005); PRC82 (2010)

\[ \Delta \phi \text{ (radians)} \]

- d+Au FTPC-Au 0-20% (preliminary)
- p+p
- Au+Au 0-5%

\[ p_T^{\text{trig}} = 4-6 \text{ GeV/c} \]
\[ p_T^{\text{assoc}} = 0.15-4 \text{ GeV/c} \]

\[ p_T > 0.7 \text{ GeV/c} \]

\[ \Delta \phi \text{ (radians)} \]

1/N trigger \[ dN/d(\Delta \phi) \]

\[ \Delta \phi \text{ (radians)} \]

- trigger jet
- AuAu
- dAu

\[ p_T^{\text{trig}} = 2.5-4 \text{ GeV/c} \]
\[ p_T^{\text{assoc}} = 1-2 \text{ GeV/c} \]

\[ p_T^{\text{trig}} > 4 \text{ GeV/c} \]
\[ p_T^{\text{assoc}} > 2 \text{ GeV/c} \]

\[ \Delta \phi \text{ (radians)} \]

\[ \Delta \eta \text{ (radians)} \]

1/N trigger \[ dN/d(\Delta \phi) \]

**Away-side**
Triangular flow in heavy-ions

- Double-peak away-side correlations
- Long-range near-side ridge
- Triangular flow, $v_3$
- Other odd harmonics
$v_n$ are measured by two-particle correlations

- $V_n$ from two-particle correlation
- Subtract $v_n$ from two-particle correlation
- Almost a tautology
- Comparison to hydro gives us confidence that $v_n$ are mostly from flow
- Quantitatively how much is flow and how much is nonflow—still an open question.
- Hydro has some tension to simultaneously describe $v_2$ and $v_3$
- Important to reduce/eliminate nonflow contributions to flow; do as best a job as we can.

EP-dep. correlation with $v_n$ subtraction

Strategy:

- Measure $v_n$ by two-particle correlation with one particle at as low $p_T$ as feasible, to maximally reduce nonflow contaminations.
- Subtract $v_n$ measurements from two-particle correlations at high and intermediate $p_T$.

Open questions:

- Effect of jets on event plane reconstruction?
- Are any remaining correlations still coming from hydro flow, i.e. jets are completely gone?
Particle correlations in small systems
Ridge in small systems

usual p-p collision

High multiplicity p-p collision

Minimum Bias
no cut on multiplicity

High multiplicity data set
and N>110

CMS, JHEP 1009 (2010) 091

New “ridge-like” structure extending to large $\Delta \eta$ at $\Delta \phi \sim 0$
• Why wasn’t it discovered long ago by HEP?

• Two types of discoveries:
  – Theoretically predicted, and experimentally verified
  – Surprises

• HEP moved on to more exclusive processes

• There may be still important physics that were missed in last half century
p-p collision (high Mult.)

Physical origin unclear

CMS, JHEP 1009 (2010) 091

\[ N > 110, 1.0 \text{GeV}/c < p_T < 3.0 \text{GeV}/c \]

pp 7 TeV

CMS, PLB718 (2013) 795

p-Pb collision (high Mult.)

CMS Preliminary

\[ \sqrt{s_{NN}} = 5.02 \text{ TeV}, N_{\text{offline}}^\text{trg} \geq 110 \]

1 < \( p_T < 3 \) GeV/c

Much bigger than pp
There is an intrinsic correlation in azimuthal angle coming from the two-particle production process, such as the one shown in Fig. 11 [92]. There is only a single loop momentum $k_T$ in this two-particle production process, causing correlations. Because the single gluon distribution peaks at the saturation scale $Q_s$, large probability is found for production of two particles with their momenta $p_T$ and $q_T$ parallel to each other such that $|p_T - k_T| \sim Q_s$ and $|q_T - k_T| \sim Q_s$. These processes therefore cause small angle correlations at $\Delta \phi = 0$. Because the correlations originate from the very early times of the collision, $\tau_{\text{init.}}$, they can persistent to large rapidity differences, $\Delta y = 2 \ln(\tau_{\text{f.o.}}/\tau_{\text{init.}})$ where $\tau_{\text{f.o.}}$ is the particle freeze-out proper time.
Another explanation: Hydro flow

- In heavy-ions, subtract $v_2 \rightarrow$ non-zero finite correlation: near-side large $\Delta\eta$ ridge, away-side double peak $\rightarrow v_3$

- In pp, pA (and possibly dA) systems, subtract uniform pedestal $\rightarrow$ non-zero finite correlation: large $\Delta\eta$ ridge $\rightarrow v_2$ (and $v_3$)
Acceptance correction revisited


- Two-particle acceptance correction by mixed-events is, in principle, wrong.
  \[
  \frac{1}{N_{\text{trig}}} \frac{d^2 N^{\text{same}}}{d\Delta \eta d\Delta \varphi} / \frac{1}{N_{\text{trig}}} \frac{d^2 N^{\text{mix}}}{d\Delta \eta d\Delta \varphi}
  \]
- Should just be corrected by single particle efficiencies:
  \[
  \frac{1}{N_{\text{trig}}} \frac{d^2 N}{d\Delta \eta d\Delta \varphi} / \mathcal{E}_{\text{trig}} \mathcal{E}_{\text{assoc}}
  \]
- How much error it makes?
Dihadron per trigger pair density

Low multiplicity

High multiplicity

Near-side jet

Shape reflects single particle dN/d\eta

L. Xu (CMS) QM 2014
Ridge yield vs $\eta_{\text{assoc}}$

Near-side ridge after jet subtraction

- Near-side ridge yield: different $\eta$ dependences for p-going and Pb-going triggers

CMS Preliminary \[ p\text{Pb } \sqrt{s_{NN}}=5.02 \text{ TeV} \]

Not normalized by single particle $dN/d\eta$

220 \leq N_{\text{trk}} < 260

0.3 < p_T < 3 \text{ GeV/c}

L. Xu (CMS) QM 2014
η-dependence of $\nu_2(\eta)/\nu_2(0)$

- $\nu_2$ shape is $\eta$ dependent in p+Pb!
- $\nu_2$ asymmetric about mid-rapidity
Flow correlations
Anisotropy Parameter $v_2$

$\mathcal{E} = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$

$v_2 = \langle \cos 2\varphi \rangle$, $\varphi = \tan^{-1} \left( \frac{p_y}{p_x} \right)$

Initial/final conditions, EoS, degrees of freedom
Collectivity, Deconfinement at RHIC

- Low $p_T (\leq 2$ GeV/c): hydrodynamic mass ordering
- High $p_T (> 2$ GeV/c): number of constituent quarks scaling

→ Quark degrees of freedom, deconfinement, Partonic Collectivity,
Comparison with Hydrodynamics

Small value of viscosity to entropy density ratio $\eta/s$

Model uncertainty dominated by initial eccentricity $\varepsilon$

Model: Song et al. arXiv:1011.2783
Low $\eta/s$ for QCD Matter at RHIC

- $\eta/s \geq 1/4\pi$
- $\eta/s$ (QCD matter) < $\eta/s$ (QED matter)
Viscosity quantum limit

\[
\eta = \frac{1}{3} n p l_{mfp}
\]

\[
l_{mfp} = \frac{1}{n \sigma}
\]

\[
pl_{mfp} \geq \hbar
\]

\[
s \sim 4nk_B
\]

\[
\frac{\eta}{s} > \frac{\hbar}{4\pi k_B}
\]

\[
\frac{\eta}{s} > \frac{1}{4\pi}
\]

Kovtun, Son, Starinets, PRL 94 (2005) 111601
Schafer, arXiv:0912.4236
Does it have to be all pressure-driven hydro flow?
Uncertainty principle

\[ \Delta x \cdot \Delta p > \frac{\hbar}{2} \]

\[ p_x > p_y \]

\[ \varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} \quad v_2 = \langle \cos 2\varphi \rangle = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} \]

Infinite square well

\[-\frac{\hbar^2}{2m} \nabla^2 \psi = E\psi \quad \Rightarrow \quad \psi \propto \begin{cases} \cos \frac{n_{\text{odd}} \pi}{a} x \\ \sin \frac{n_{\text{even}} \pi}{a} x \end{cases}\]

Take even mode for example:

\[\langle p_x^2 \rangle = \hbar^2 k^2 \quad \langle x^2 \rangle = a^2 \frac{2}{k^2} \quad k = \frac{n_{\text{odd}} \pi}{a}\]

\[\sqrt{\langle p_x^2 \rangle \cdot \langle x^2 \rangle} = \hbar \sqrt{a^2 \frac{k^2}{4} - 2} = \hbar \sqrt{\frac{\pi^2}{4} n_{\text{odd}}^2 - 2} > \hbar / 2\]

\[v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{b^2 - a^2}{b^2 + a^2} = \varepsilon \quad \text{for all } n.\]
Harmonic oscillator

\[ \left( -\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m\omega^2 x^2 \right) \psi = E\psi \ , \quad E = \left( n \frac{1}{2} \right) \hbar \omega \]

\[ \langle p_x^2 \rangle = \left< \frac{1}{2} m\omega^2 x^2 \right> = \frac{E}{2} = \frac{1}{2} \left( n \frac{1}{2} \right) \hbar \omega \]

\[ \sqrt{\langle p_x^2 \rangle \langle x^2 \rangle} = \left( n \frac{1}{2} \right) \hbar \]

\[ v_2 = \frac{\left< p_x^2 \right> - \left< p_y^2 \right>}{\left< p_x^2 \right> + \left< p_y^2 \right>} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y} \]

\[ \varepsilon = \frac{\left< y^2 \right> - \left< x^2 \right>}{\left< y^2 \right> + \left< x^2 \right>} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y} \]

\[ v_2 = \varepsilon \quad \text{for each and all } n \]
Bose-Einstein Condensate

Single ground state in anisotropic trap $\rightarrow$ large momentum anisotropy

D. S. Jin and C. A. Regal
Thermal probability

\[ \text{x, y at same Fermi energy, so different number of filled energy levels.} \]

At high temperature, classical limit, sum is approximated by integral:

\[
\frac{dN}{dP} = N \frac{\int d\mathbf{r} e^{-H_1(\mathbf{p}, \mathbf{r})/T}}{\int d\mathbf{r} d\mathbf{p} e^{-H_1(\mathbf{p}, \mathbf{r})/T}} = N \frac{\int e^{-K(\mathbf{p})/T}}{\int d\mathbf{p} e^{-K(\mathbf{p})/T}}
\]

then it’s independent of potential.
It’s isotropic at all temperature because \( K = (p_x^2 + p_y^2)/2m \) is isotropic.
Is QGP hot?

Size $r \sim 1 \text{ fm}$
Intrinsic momentum/energy scale $\sim 1/r \sim 200 \text{ MeV}$

QGP temperature $T \sim 300 \text{ MeV}$
Typical momentum/energy $\sim T \sim 300 \text{ MeV}$

QGP is not hot at all.
Quantum effect must be present.
Thermal probability weight

\[
\rho(r) \equiv \frac{dN}{dr} = \frac{1}{Z} \sum_j |\psi_j(r)|^2 e^{-E_j/T}
\]

\[
f(p) \equiv \frac{dN}{dp} = \frac{1}{Z} \sum_j |\psi_j(p)|^2 e^{-E_j/T}
\]

\[
Z \equiv \sum_j e^{-E_j/T}
\]

\[
\langle p_i^2 \rangle = \frac{M \omega_i}{2} \coth \frac{\omega_i}{2T}
\]

\[
\langle r_i^2 \rangle = \frac{1}{2M \omega_i} \coth \frac{\omega_i}{2T}
\]
Initial $v_2$ from QM

\[ \bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1 + \varepsilon} \]

\[ \bar{v}_2 \approx \frac{\hbar^2}{12 k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1 + \varepsilon} \]

\[ v_{2n}(p_T) = h_n \left( \frac{p_T^2}{2MT} (S_y - S_x) \right), \quad S_i = \frac{T}{\omega_i} \tanh \frac{\omega_i}{2T} \]

Typical Au+Au collisions

$b = 8$ fm: $\langle r_x^2 \rangle^{1/2} = 1.5$ fm and $\langle r_y^2 \rangle^{1/2} = 2.2$ fm.

Transverse profile from SHO

\[ \rho(r) \propto \exp \left( - \sum_i \frac{r_i^2}{2\langle r_i^2 \rangle} \right), \quad f(p) \propto \exp \left( - \sum_i \frac{p_i^2}{2\langle p_i^2 \rangle} \right) \]

This may not correspond exactly to heavy-ion collision energy density profile, but close.
Cold atoms
Strong elliptic anisotropy


Lithium atoms $M \sim 6000$ MeV
Temperature $T \sim 1 \, \mu K \sim 10^{-16}$ MeV
Trap size $x \sim 20 \, \mu m$, $y \sim 100 \, \mu m$

Typical momentum $(TM)^{1/2} \sim 10^{-6}$ MeV
Intrinsic momentum quantum $\sim 1/r \sim 10^{-8}$ MeV, negligible.

Typical energy $\sim T \sim 10^{-16}$ MeV
Intrinsic energy quantum $1/(mr^2) \sim 10^{-20}$ MeV, negligible.

Cold Lithium atoms are actually “hotter” than the hot QGP.

$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B TM \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1 + \varepsilon} \sim 10^{-5}$$

The observed large $v_2$ is indeed due to strong interactions.
Is quantum $v_2$ real?

- It should be... but need experiment to verify
- Would be neat to verify QM and uncertainty principle

Cold atom experiment

- Need trap size \textit{x100 smaller}
- Or need \textit{nano-Kelvin} temperature

Proposing a cold atom quantum simulator for high-energy nuclear physics
Control the interaction

• Hydrodynamics is only an assumption
• Is initial QM $v_2$ important after hydro evolution?
• When does hydro sets in and takes over?
• Will the initial QM $v_2$ be washed out by hydro?
• Current hydro implementation is classical
• Need to incorporate QM into hydro: quantum hydrodynamics
Shooting fast atoms through trap

- jet-quenching partonic energy loss mechanisms are far from clear. A very active and extensive field

- Can we gain insights from cold atoms?

- Shoot fast atoms through cold atom system

**Probing strongly interacting atomic gases with energetic atoms**

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We investigate properties of an energetic atom propagating through strongly interacting atomic gases. The operator product expansion is used to systematically compute a quasiparticle energy and its scattering rate both in a spin-1/2 Fermi gas and in a spinless Bose gas. Reasonable agreement with recent quantum Monte Carlo

- External hard probes under full control
Summary

• Particle correlations are a powerful tool to study pp, pA, AA collisions

• Unambiguous signal of strongly interacting QGP from high-$p_T$ jet-quenching data.

• Low $p_T$ anisotropic flow data indicate hydrodynamic behavior of sQGP. Extracting transport properties (such as $\eta/s$) from measured data still need extra effort. Initial anisotropy may not be neglected.

• There should be indispensable information at intermediate $p_T$ from jet-medium interactions (not discussed in this lecture). Need creative mind and novel approaches.