UALG/TP/98-

Non-Perturbative Computation Of Gluon Mini-Jet Production In Nuclear Collisions At Very High Energies.

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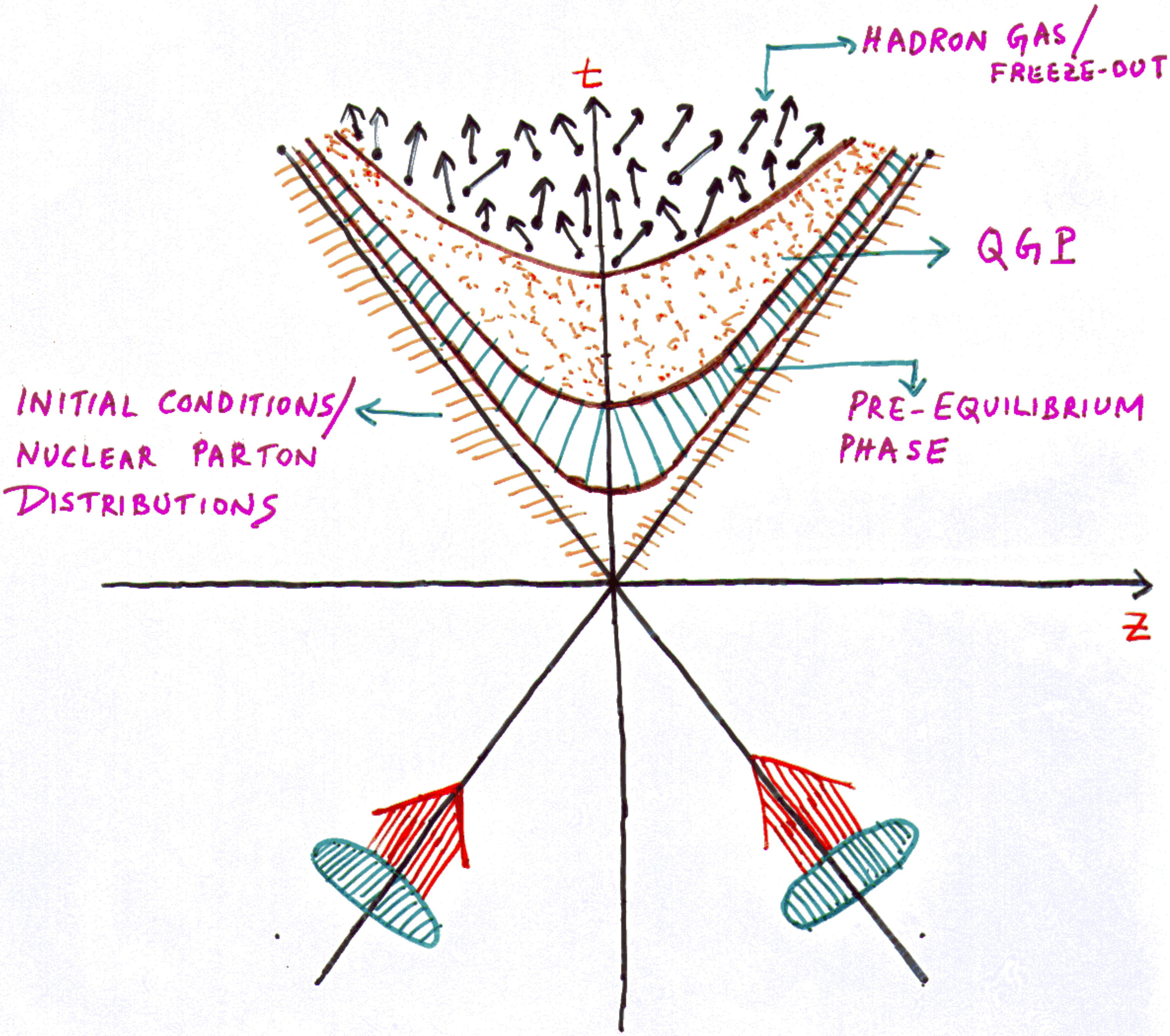
&
R. Venugopalan.

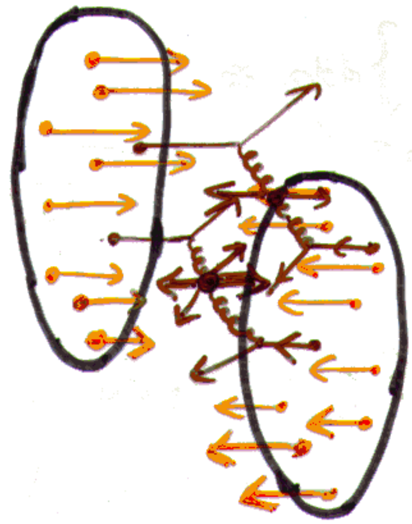
• DECONFINED quark gluon plasma

FORMED FLEETINGLY ($t \sim 10$ fm)

IN HIGH ENERGY NUCLEAR COLLISIONS ?



$$\boxed{\sqrt{s} \gg m_N \simeq 1\,\text{GeV}}$$



- NUCLEAR PARTON DISTRIBUTIONS PROVIDE THE INITIAL CONDITIONS FOR NUCLEI TO EVOLVE TO A QGP.

- MOST ENERGETIC PARTONS INTERACT WEAKLY AND PASS THROUGH $\longrightarrow$ LEADING PARTICLE EFFECT.

- SOFTER, "WEE" PARTONS INTERACT STRONGLY $\longrightarrow$ FORM PLASMA AT CENTRAL RAPIDITIES?

- AT RHIC & LHC, LARGE # OF MINI-JETS PRODUCED AT CENTRAL RAPIDITIES:

(Kajantie, Landshoff, Blaizot, Mueller, Lindfors, Eskola, Kajantie, Lindfors,

	$\bar{N}^f_{PbPb}$	$\bar{E}^f_T$ (GeV)
LHC :	4741	14.16×10^3
RHIC :	121	321

(Eskola)

$P_0 = 2$ GeV.
$|y| \le 0.5$.
$b = 0$.

- $$\bar{E}_T^f(b, \sqrt{s}, P_0, \Delta y) = T_{AA}(b)\, \sigma_{jet}(\sqrt{s}, P_0)\langle E_T^f\rangle_{\Delta y}$$
$$\bar{N}_{AA}^f(b, \sqrt{s}, P_0, \Delta y) = T_{AA}(b)\, \sigma_{jet}(\sqrt{s}, P_0)\langle n^f\rangle_{\Delta y}$$

$$\boxed{\sigma_{jet}(\sqrt{s}, P_0) = \frac{1}{2}\int_{P_0^2} dp_\perp^2\, dy_1\, dy_2\, x_1 F(x_1, Q^2)\, x_2 F(x_2, Q^2) * \frac{d\hat{\sigma}}{d\hat{t}}{}^{gg\to gg}}$$

$$\left(F(x, Q^2) = x g(x, Q^2) + \frac{4}{9}\sum_q x\left(q(x, Q^2) + \bar{q}(x, Q^2)\right)\right)$$

$$\hat{s} = x_1 x_2 s \; ; \; \hat{t} = -P_t^2\left(1 + e^{-(y_1 - y_2)}\right)$$

$$x_1 = \frac{P_\perp}{\sqrt{s}}\left(e^{y_1} + e^{y_2}\right) \; ; \; x_2 = \frac{P_\perp}{\sqrt{s}}\left(e^{-y_1} + e^{-y_2}\right)$$

$$T_{AA}(b) = \int d^2 s\, T_A(s)\, T_A(b - s)$$

$$\& \quad T_A(s) = \int dz\, n_A\left(\sqrt{s^2 + z^2}\right)$$

$$X \simeq \frac{2 p_\perp}{\sqrt{s}}$$

$$p_\perp = 2\ \mathrm{GeV} \Rightarrow \begin{cases} X \sim 10^{-3} \text{ at LHC} \\ X = 2 * 10^{-2} \text{ at RHIC} \end{cases}$$

AT SMALL x,

$$\sigma_{jet}(\sqrt{s}, p_0) \neq \frac{1}{2} \int_{p_0^2} dp_\perp^2\, dy_1\, dy_2\, x_1 F(x_1, Q^2) * x_2 F(x_2, Q^2) \frac{d\hat{\sigma}^{gg \to gg}}{d\hat{t}}$$

QCD COHERENCE EFFECTS BECOME IMPORTAN

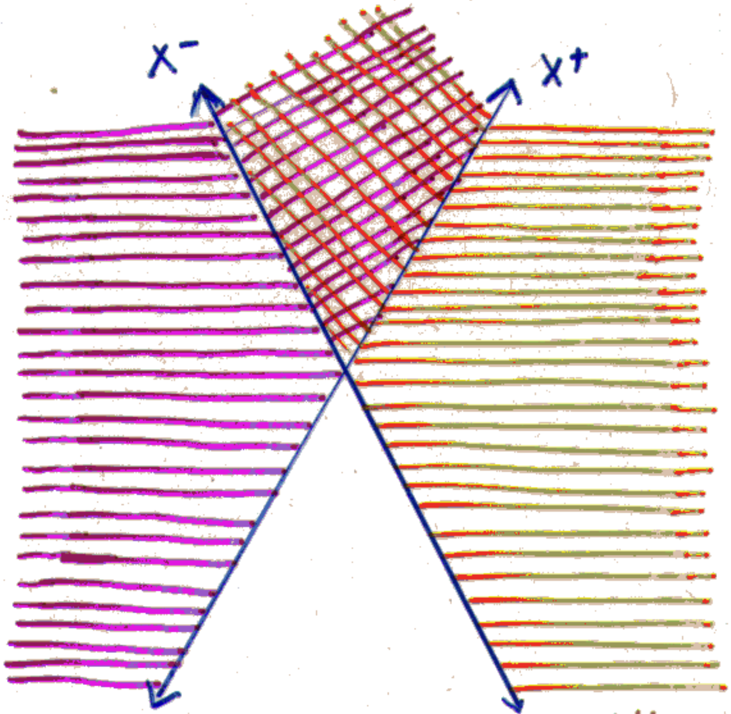
- Study Nuclear Collisions In Classical Effective Fields Approach.

a) Self-Consistent Space-Time Picture — Collision Of Non-Abelian Weizsäcker-Williams Fields.

b) Systematically Incorporates QCD Coherence Effects → Can Be Tested In e^--A Collider Expts.

c) Results For Dominant $qq \to qqg$ Process Agree With pQCD Predictions At High k_t.

COLLISION OF WEIZSÄCKER–WILLIAMS FIELDS.



(Kovner, McLerran, Weigert
Gyulassy–McLerran)

$t<0$: $\quad A_\perp = \theta(x^-)\,\theta(-x^+)\,\alpha_\perp^{(1)} + \theta(x^+)\,\theta(-x^-)\,\alpha_\perp^{(2)}$

$t>0$: NON-TRIVIAL SOLUTION OF CLASSICAL YANG-MILLS EQUATION.

$$D_\mu F^{\mu\nu} = J^\nu$$

$$J^\pm = \delta(x^\mp)\,\rho^\pm(x_\perp)$$

SOLUTIONS ARE BOOST INVARIANT.

- YANG-MILLS EQUATIONS IN FORWARD LIGHT CONE.

Schwinger gauge: $x^+A^- + x^-A^+ = 0 \; (A^\tau = 0)$

(see also Kovchegov-Rischke)

$$A^\pm = \pm x^\pm \alpha(\tau, x_\perp)$$
$$A^i = \alpha_\perp^i(\tau, x_\perp)$$

$$\frac{1}{\tau^3} \partial_\tau \tau^3 \partial_\tau \alpha + [D_i [D^i, \alpha]] = 0$$

$$\frac{1}{\tau} [D_i, \partial_\tau \alpha_\perp^i] + ig\tau [\alpha, \partial_\tau \alpha] = 0$$

$$\frac{1}{\tau} \partial_\tau (\tau \partial_\tau \alpha_\perp^i) - ig\tau^2 [\alpha, [D^i, \alpha]] - [D^j, F^{ji}] = 0$$

- INITIAL CONDITIONS FOR $\alpha(\tau, x_\perp)$ and $\alpha_\perp^i(\tau, x_\perp)$ IN TERMS OF SINGLE NUCLEUS SOLUTIONS.

$$\alpha_\perp^i \Big|_{\tau=0} = \alpha_1^i + \alpha_2^i$$

$$\alpha \Big|_{\tau=0} = \frac{ig}{2} [\alpha_1^i, \alpha_2^i]$$

- SOLUTIONS FOR THE TWO NUCLEI AT $t<0$:

$$\alpha_{1,2}^{i}(x_\perp) = \frac{1}{ig}\left(\mathcal{P}\, e^{-ig\int_{\pm\eta_{proj}}^{0} d\eta' \frac{1}{\nabla_\perp^2}\rho_\pm(\eta', x_\perp)}\right)$$

$$* \nabla^i \left(\mathcal{P} \exp\left(-ig\int_{\pm\eta_{proj}}^{0} d\eta' \frac{1}{\nabla_\perp^2}\rho_\pm(\eta', x_\perp)\right)\right)^{\dagger}.$$

$$\eta = \begin{cases} \eta_{proj} - \ln\left(\dfrac{x^-}{x^-_{proj}}\right), & p^+ \to \infty, \ \alpha_1 \\ -\eta_{proj} + \ln\left(\dfrac{x^+_{proj}}{x^+}\right), & p^- \to \infty, \ \alpha_2 . \end{cases}$$

- FOR REGULAR SOLUTIONS, MUST HAVE

$$\partial_\tau \alpha \Big|_{\tau=0} = 0$$

$$\partial_\tau \alpha_\perp^i \Big|_{\tau=0} = 0 .$$

- AVERAGING OVER CLASSICAL CHARGE DISTRIBUTIONS

$$\langle O \rangle_\rho = \int d\rho_+ \, d\rho_- \; O(\rho_+, \rho_-)$$

$$* \; \exp\left(- \int d^2x_\perp \frac{\mathrm{Tr}\left[\rho_+^2(x_\perp) + \rho_-^2(x_\perp)\right]}{2\mu^2} \right)$$

- $$\mu^2 = \frac{A^{1/3}}{\pi r_0^2} \int_{x_0}^{1} dx \left(\frac{1}{2N_c} q(x,Q^2) + \frac{N_c}{N_c^2-1} g(x,Q^2) \right)$$

(Gyulassy – McLerran)

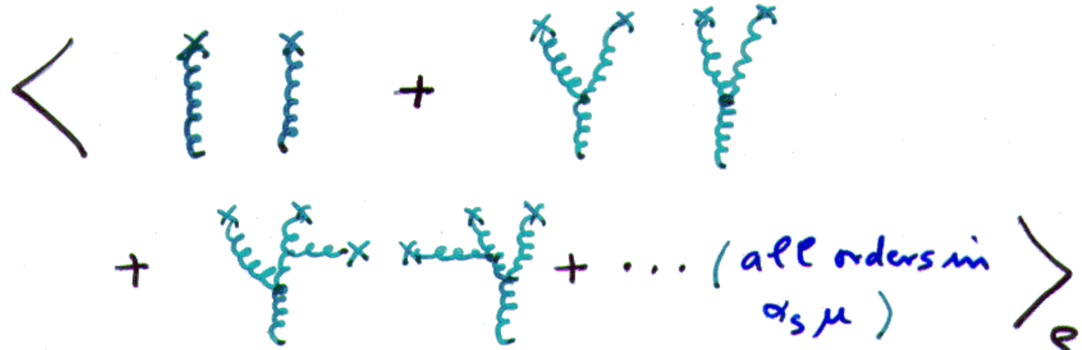
$r_0 = 1.12$ fm , $x_0 = Q/\sqrt{s}$; q, g ARE NUCLEON PARTON DISTRIBUTIONS

AT RHIC: $\mu \leq 0.5$ GeV

AT LHC: $\mu \leq 1$ GeV.

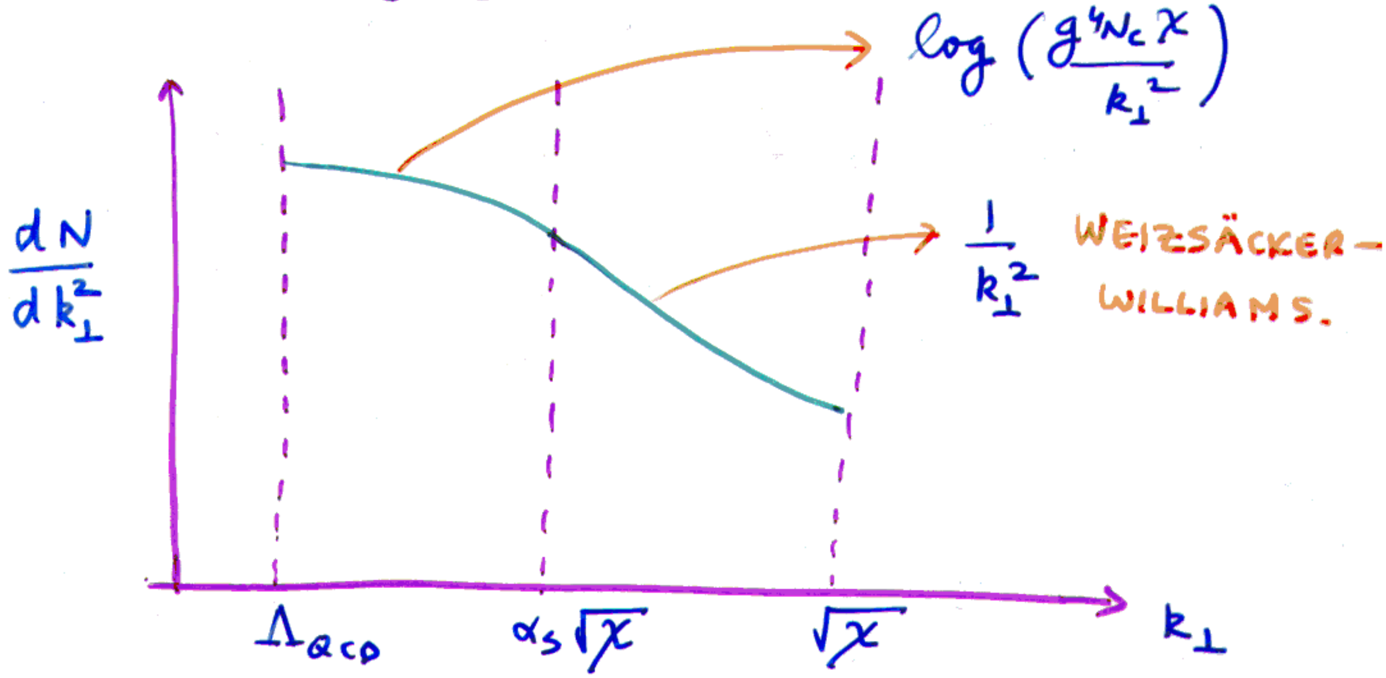
DEFINE $\boxed{\chi(y,Q^2) = \int_y dy\, \mu(y,Q^2).}$

• $\dfrac{dN}{d^2x_\perp} \equiv \Big\langle$



$+ \ldots$ (all orders in $\alpha_s \mu$) $\Big\rangle_\rho$

$$= \frac{4(N_c^2-1)}{N_c\, x_\perp^2}\left[1 - \left(x_\perp^2 \Lambda_{QCD}^2\right)^{\frac{g^4 N_c}{8\pi}\chi\, x_\perp^2}\right]$$



• RESULT FOR CLASSICAL GLUON RADIATION TO LOWEST ORDER IN α_s BUT ALL ORDERS IN $\alpha_s\sqrt{\chi}$.

(McLerran-Venugopalan
Jalilian-Marian, Kovner,
McLerran, Weigert)

- YANG–MILLS EQUATIONS SOLVED PERTURBATIVELY IN POWERS OF $\frac{\alpha_s \mu}{k_\perp}$

(KOVNER, McLERRAN, WEIGERT ; KOVCHÉGOV & RISCHKE ; MATINYAN, MÜLLER, RISCHKE ; GYULASSY, McLERRAN)

- TO SECOND ORDER IN ρ, SOLUTIONS OF SINGLE NUCLEUS EQUATIONS ARE

$$\alpha_m^i = -\partial^i \phi_m + \frac{ig}{2}\left(\delta^{ij} - \partial^i \frac{1}{\nabla_\perp^2}\partial^j\right)\left\{[\phi_m, \partial^j \phi_m] + O(\rho_m^3)\right\}$$

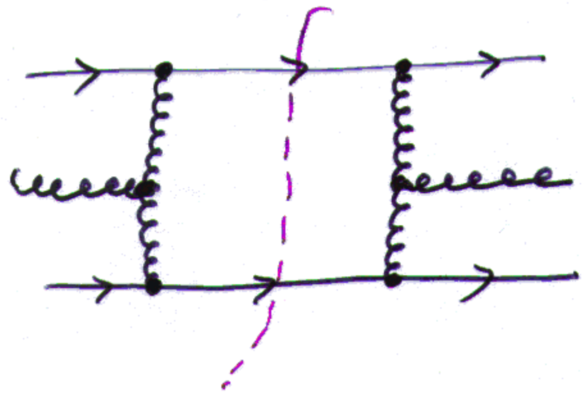
$m = 1, 2$ – labels of the nuclei

$$\phi_m = -g \frac{1}{\nabla_\perp^2} \rho_m$$

- TO SOLVE EQUATIONS IN FORWARD LIGHT CONE, EXPAND GAUGE FIELDS IN POWERS OF ρ_m.

$$\alpha = \sum_{n=0}^{\infty} \alpha_{(n)} \quad ; \quad \alpha_\perp^i = \sum_{n=0}^{\infty} \alpha_{\perp\,(n)}^i$$

A) COMPUTE GLUON PRODUCTION AS A FUNCTION OF τ.



PERTURBATIVE RESULT:

$$\frac{1}{\pi R^2}\frac{dN}{dy\,d^2k_\perp} = \frac{2g^6\chi^2(y,Q^2)}{(2\pi)^4}\,\frac{N_c(N_c^2-1)}{k_\perp^4}\,\ln\left(\frac{k_\perp^2}{(\alpha_s\sqrt{\chi})^2}\right)$$

AGREES EXACTLY WITH
PQCD BREMSSTRAHLUNG
EXPRESSION OF GUNION + BERTSCH.

KLW
Gyulassy-Mclerran
Kovchegov-Rischke
⋮

OUR GOAL: COMPUTE RESULT TO ALL ORDERS IN $\alpha_s \mu$.

B) STUDY ENERGY CORRELATIONS, THERMALIZATION,
D-Y PRODUCTION, ETC.

- FOR SOLUTION TO ALL ORDERS IN $\frac{\alpha_s \mu}{k_\perp}$,

CONSTRUCT LATTICE HAMILTONIAN AND SOLVE LATTICE EQUATIONS OF MOTION (ALEX KRASNITZ & R. VENUGOPALAN hep-ph/9706329)

$$\mathcal{H} = \int d\eta\, d\vec{r}_\perp\, \tau \left\{ \frac{1}{2} p^\eta p^\eta + \frac{1}{2\tau^2} p^r p^r + \frac{1}{2\tau^2} F_{r\eta} F_{r\eta} + \frac{1}{4} F_{xy} F_{xy} + J^\eta A_\eta + J^r A_r \right\}$$

$p^\eta = \frac{1}{\tau} \partial_\tau A_\eta$; $p^r = \tau \partial_\tau A_r$; $\tau = \sqrt{2x^+x^-}$

$\eta = \frac{1}{2} \ln\left(\frac{x^+}{x^-}\right)$

$\vec{r}_\perp = (x_\perp, y_\perp)$

- ASSUMING BOOST INVARIANCE,

$A_r(\tau, \eta, \vec{r}_\perp) \approx A_r(\tau, \vec{r}_\perp)$; $A_\eta(\tau, \eta, \vec{r}_\perp) \approx \Phi(\tau, \vec{r}_\perp)$

$\Rightarrow F^a_{\eta r} = -D_r \Phi^a$

$J^\eta, J^r = 0$ FOR $\tau > 0$. DEPENDENCE ON ρ's ENTIRELY THROUGH INITIAL CONDITIONS.

$\mathcal{H} = H_0 + H_I$.

$$H_0 = \int d\eta\, d\vec{r}_\perp\, \tau \left\{ \left(\frac{\partial A_i}{\partial \tau}\right)^2 + \frac{1}{4} F^a_{ij} F^a_{ij} \right\}$$

$$H_I = \int d\vec{r}_\perp \cdot d\eta\, \tau \cdot \frac{1}{2\tau^2} (D_\mu A_\eta)^2.$$

Lattice Hamiltonian: Kogut-Susskind In 2+1-D Coupled To Adjoint Scalar.

$$H_L = \frac{1}{2\tau} \sum_{\ell \equiv (j,\hat{n})} E_\ell^a E_\ell^a + \tau \sum_{\square} \left(1 - \tfrac{1}{2} \operatorname{Tr} U_\square \right)$$

$$+ \frac{1}{4\tau} \sum_{j,\hat{n}} \operatorname{Tr} \left(\phi_j - U_{j,\hat{n}}\, \phi_{j+\hat{n}}\, U_{j,\hat{n}}^\dagger \right)^2 + \frac{\tau}{4} \sum_j \operatorname{Tr} P_j^2$$

$U_{j,\hat{n}}$: SU(2) link variable from site j in direction $\hat{n}$.

E_ℓ : Generators of Right Covariant Derivatives On Group.

ϕ : Adjoint Scalar Field

P : Its conjugate momenta.

Eqn's of Motion Given By Solving $\dot{V} = \{ H_L, V \}$

V is a dynamical variable and $\{\cdot,\cdot\}$ are Poisson brackets.

$$\{ P_i^a, \Phi_j^b \} = \delta_{ij}\, \delta_{ab} \, ; \quad \{ E_\ell^a, U_m \} = -i\, \delta_{\ell m}\, U_\ell\, \sigma^a$$

$$\{ E_\ell^a, E_m^b \} = 2\, \delta_{\ell m}\, \epsilon_{abc}\, E_\ell^c$$

Gauss' Law:

$$C_j^a = \sum_{\hat{n}} \left[\frac{1}{2} E_{j,\hat{n}}^b \operatorname{Tr} \left(\sigma^a U_{j,\hat{n}}\, \sigma^b U_{j,\hat{n}}^\dagger \right) - E_{j-\hat{n},\hat{n}}^a \right]$$

$$- 2\, \epsilon_{abc}\, P_j^b A_j^c = 0 .$$

"SHADOWING" IN NUCLEI.

$$F_2^A < A F_2^N$$

E665-Preliminary

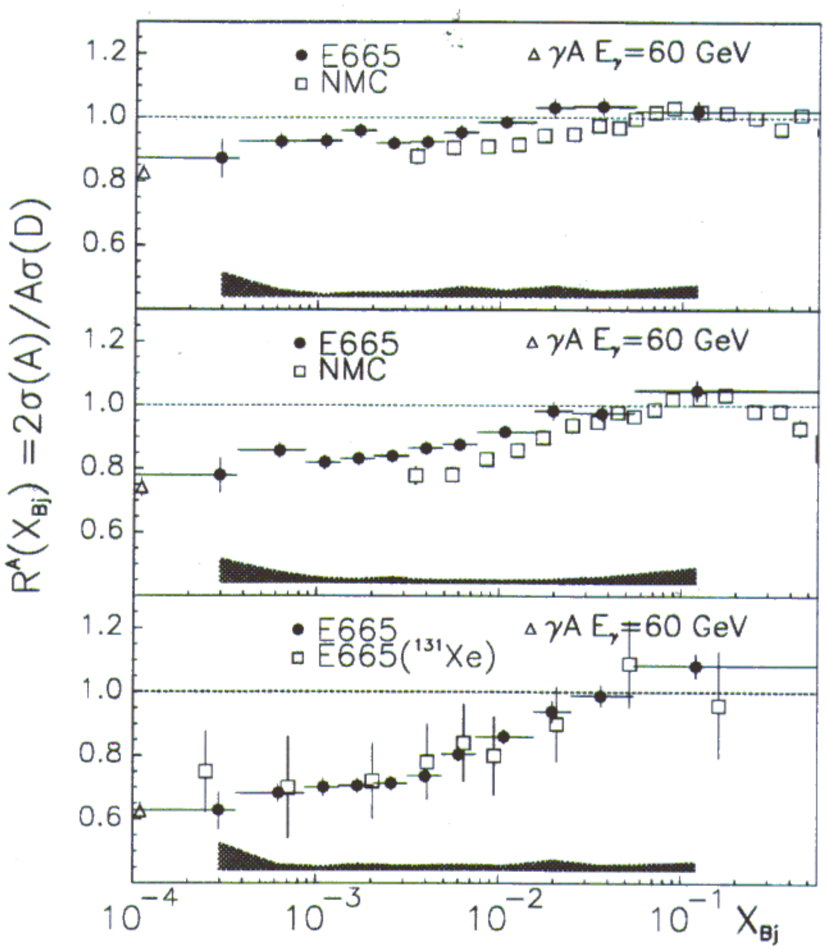


Figure 1: Per-nucleon cross-section ratios, $R^A(x_{Bj})$, for carbon, calcium and lead to deuterium. The shaded band represents the bin-to-bin systematic uncertainty. The overall normalization uncertainties of 2.06%, 2.08% and 2.23%, respectively, have not been included. The vertical error bars represent statistical errors only.

AGK:

$$\sigma_A^{(2)} = -A(A-1)\cdot 4\pi \int d^2b\, T_A^2(b) \int dM^2 \left.\frac{d\sigma^D_{\gamma^* p}}{dM^2 dt}\right|_{t=0} \times F_A(t_{min})$$

$$F_A(t_{min}) = \int dz\, e^{-iq_z z} \rho_A(b,z) / T_A(b)$$

- ## INITIAL CONDITIONS ON THE LATTICE.

MATCH LATTICE EQUATIONS OF MOTION FOR

$x^{\pm} = 0$; $x^{\pm} = 0,\ x^{\mp} > 0$.

TRANSVERSE LINK MATRICES

$$U_\perp = \theta(-x^+)\theta(-x^-)\,I + \theta(x^+)\,\theta(x^-)\,U(\tau)$$

$$+ \ \theta(-x^+)\,\theta(x^-)\,\underline{\underline{U_1}} + \theta(x^+)\,\theta(-x^-)\,\underline{\underline{U_2}}$$

$$U^q_{j,\hat{n}} = V_{q,j}\,V^\dagger_{q,j+\hat{n}}$$

MATCHING COEFFICIENTS OF $\delta(x^+)\delta(x^-)$

$\Downarrow$

$\tau = 0$: $$U = (U_1 + U_2)\,(U_1^\dagger + U_2^\dagger)^{-1}$$

Matching Coefficients of $\delta(x^+)\,\theta(x^-)$

$\tau = 0$:

$$\alpha_\gamma = \frac{i}{4N_c} \sum_n \mathrm{Tr}\Big(G_\gamma \Big\{ \big[(u_1 - u_2)(u^\dagger - I) - h.c. \big]_{j,n} - \big[(u^\dagger - I)(u_1 - u_2) - h.c. \big]_{j-n,n} \Big\} \Big)$$

- To summarize: Initial conditions for Hamilton's eqns.

$$u_\perp \big|_{\tau=0} = (u_1 + u_2)(u_1^\dagger + u_2^\dagger)^{-1}$$

$$E_\ell \big|_{\tau=0} = 0.$$

$$p_j \big|_{\tau=0} = 2\alpha$$

$$\Phi_j \big|_{\tau=0} = 0.$$

(Continuum initial conditions recovered for $a \to 0$.)

- <u>Conversion Between Lattice And Physical Units</u>

$$\mu_L = g^2 a \, \mu_c$$

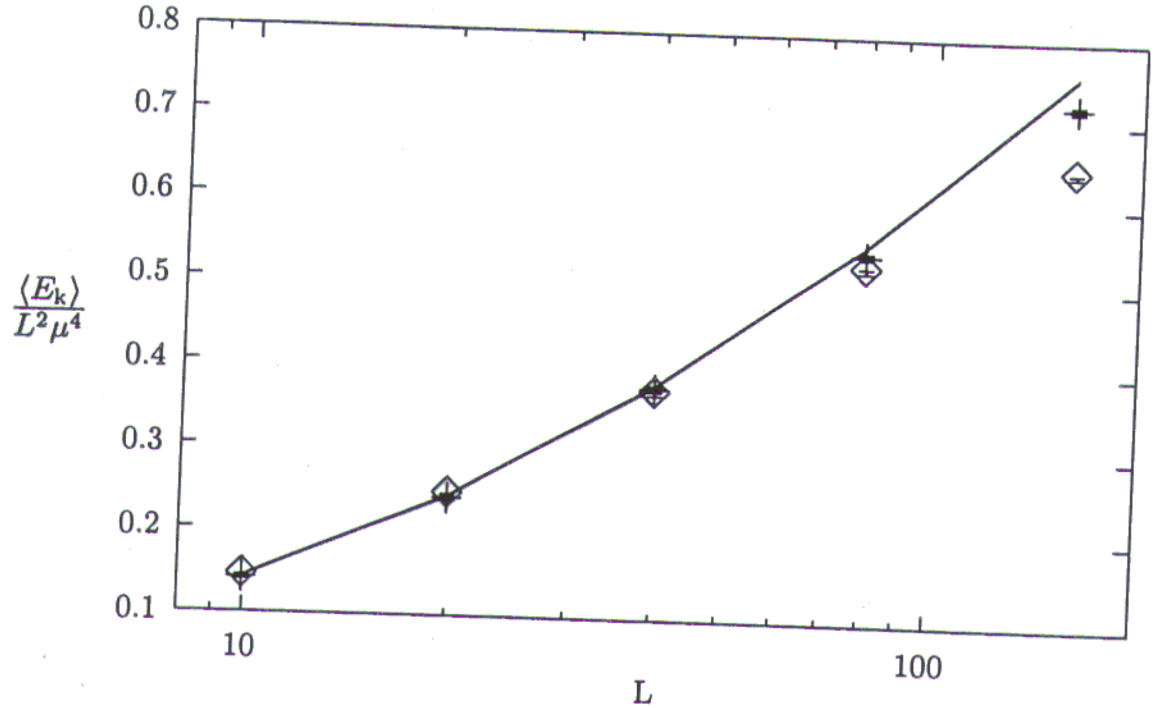
Can choose $g = 1$, $a = 0.07$ fm for $L = 160$. and $A = 200$.

$$\mu_L = 0.1 \Rightarrow \mu_c = \frac{\mu_L}{a} \sim 0.28 \text{ GeV}$$

$$\mu_c \equiv \mu_{physical} = 0.6 \text{ GeV} \Rightarrow \mu_L \cong 0.21$$

$$= 1 \text{ GeV} \Rightarrow \mu_L \simeq 0.36.$$

◇ $\mu_L = 0.05.$



SCALAR KINETIC ENERGY:

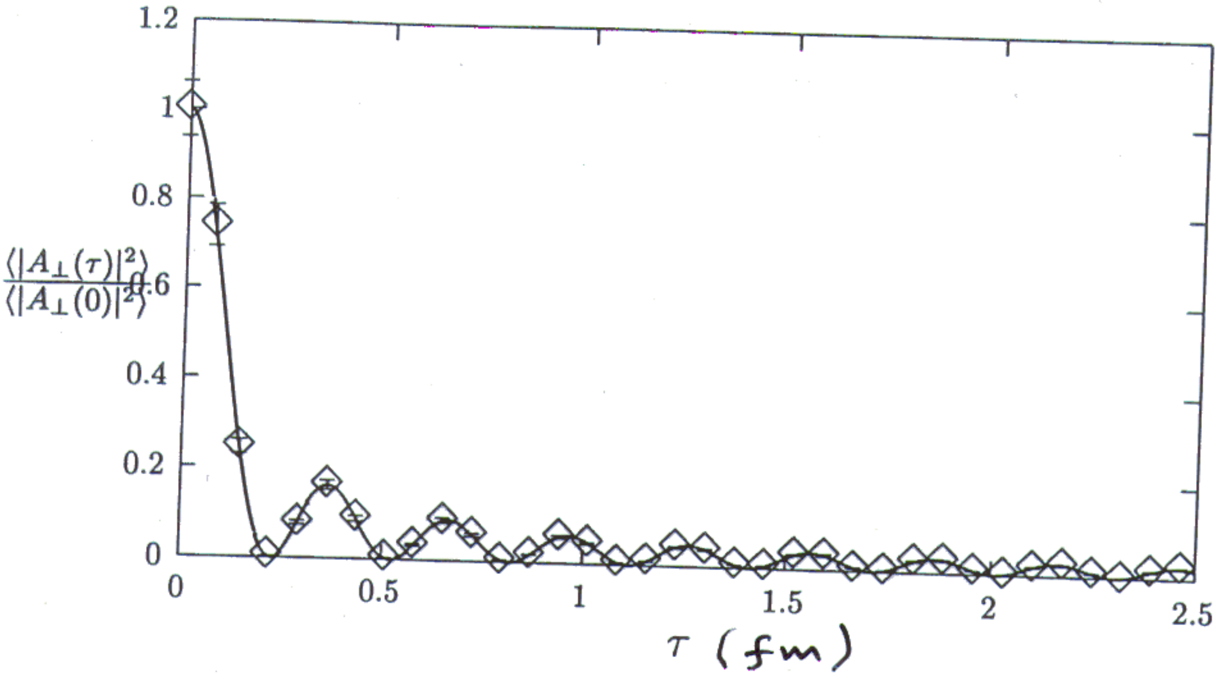
$$\frac{\langle E_{kin} \rangle}{L^2 \mu^4} \equiv p^a p^a = N_c (N_c^2 - 1) \frac{1}{N^4} \sum_{n,n'} \left[\left(\sum_{\vec{k}} \frac{\sin(\ell_n) \sin(\ell_{n'})}{\Delta^2(\ell)} \right)^2 + 16 \left(\sum_{\vec{k}} \frac{\sin^2(\frac{\ell_n}{2}) \sin^2(\frac{\ell_{n'}}{2})}{\Delta^2(\ell)} \right)^2 \right]$$

$$\Delta(\ell) = 2 \sum_{n=1,2} (1 - \cos(\ell_n))$$

$$\boxed{p^a p^a \longrightarrow A + B \log^2\left(\frac{L}{a}\right)}$$

$(k_x, k_y) = (\frac{\pi}{4}, 0)$

$L \times L : 160 \times 160.$



LPTH PREDICTION: $J_0^2(\omega\tau)$

$$\omega = \sqrt{2(2-\cos(k_x)-\cos(k_y))}$$

$$\equiv \sqrt{\Delta}$$

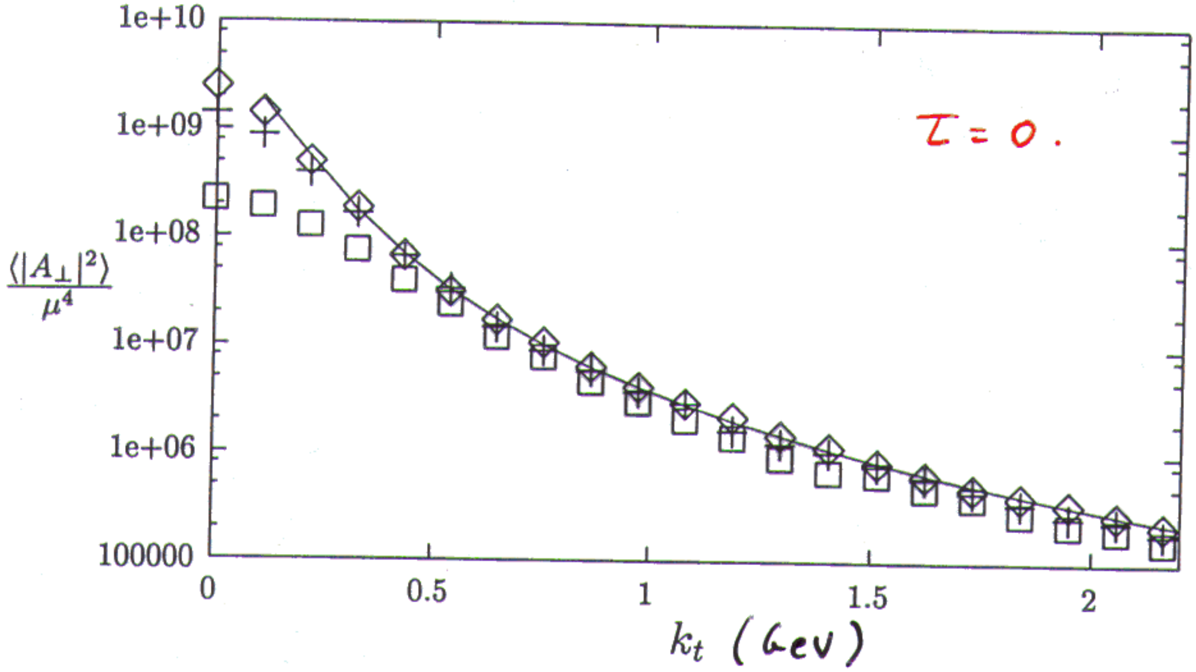
(Krasnitz & Venugopalan)

◇ $\mu_L = 0.025$.

+ $\mu_L = 0.05$.

□ $\mu_L = 0.1$

$L \times L = 160 \times 160$

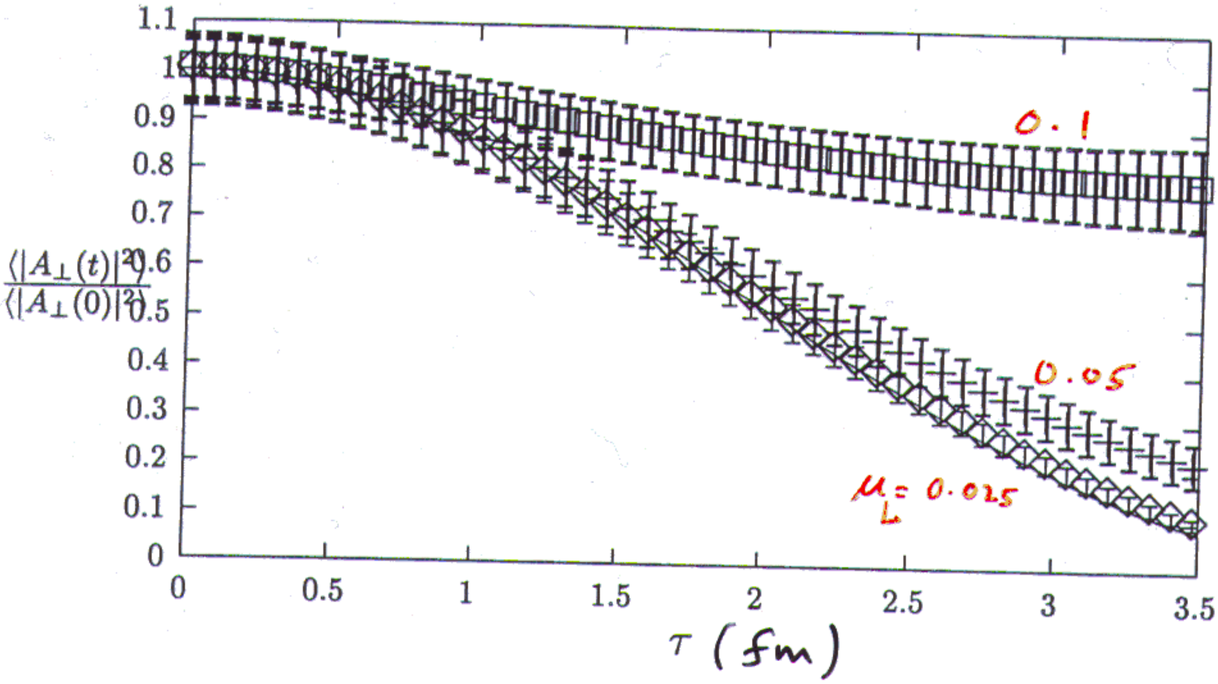


—— : LPTh PREDICTION.

$$|A'_\ell|^2 = \frac{g^6 N_c (N_c^2-1)\mu^4}{4\,\Delta(\ell)} \sum_{\ell'} \frac{\Delta(2\ell'-\ell)\Delta(\ell) - [\Delta(\ell'-\ell) - \Delta(\ell')]^2}{\Delta^2(\ell')\,\Delta^2(\ell'-\ell)}$$

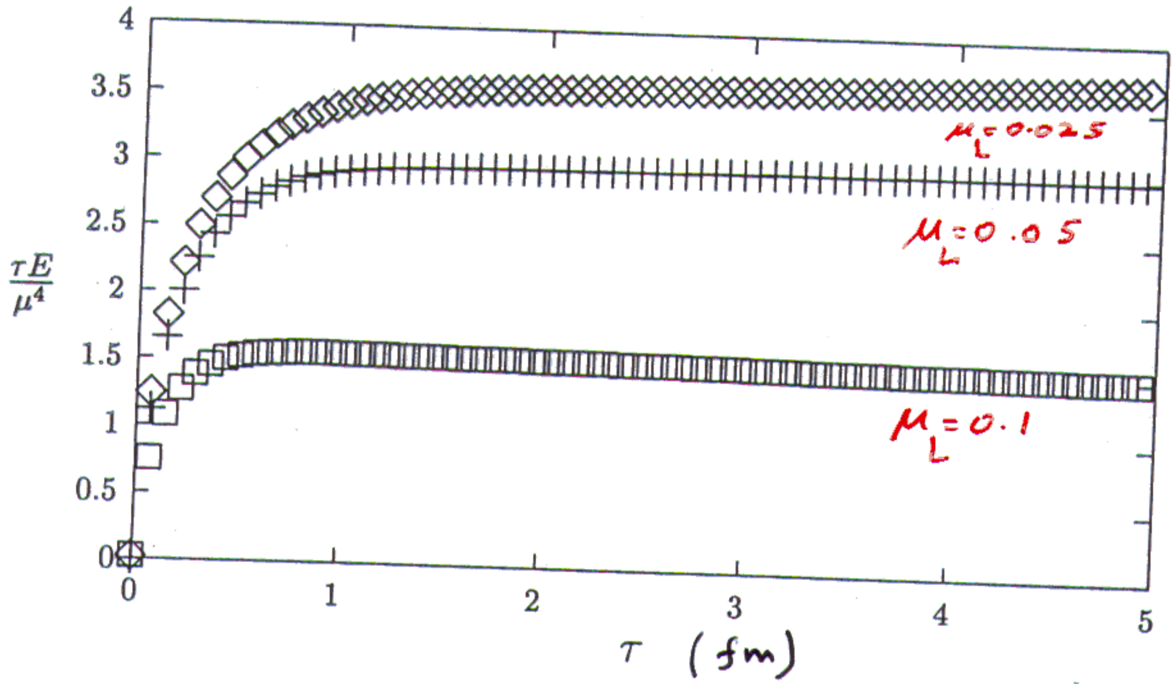
$\xrightarrow[(a \to 0)]{}$ CONTINUUM YANG-MILLS RESULT

160x160.



1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\frac{\langle |A_\perp(t)|^2\rangle}{\langle |A_\perp(0)|^2\rangle}$
0
0.5
1
1.5
2
2.5
3
3.5
τ (fm)
0.1
0.05
μ_L = 0.025

160 x 160.



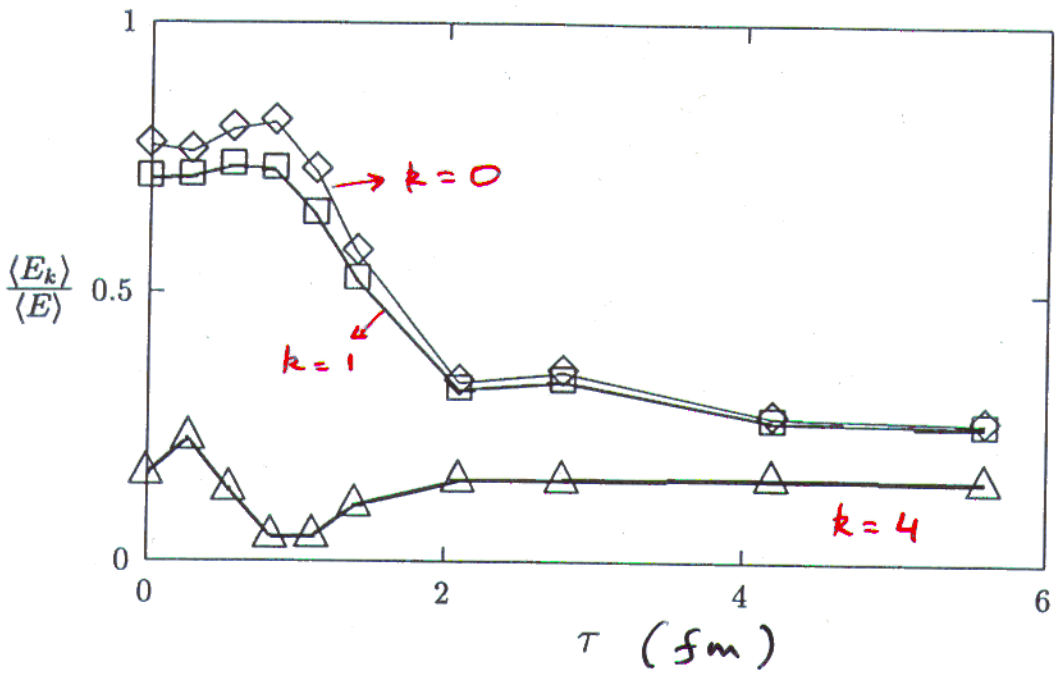
AT LATE TIMES : $E \propto 1/\tau$

TIME AT WHICH '$1/\tau$' BEHAVIOUR SETS IN

$\tau \propto 1/g^2\mu$.

80×80

$\mu = 0.41$ GeV



$$E = \mathcal{E}^2 + B^2 + \Phi^2 + P^2.$$

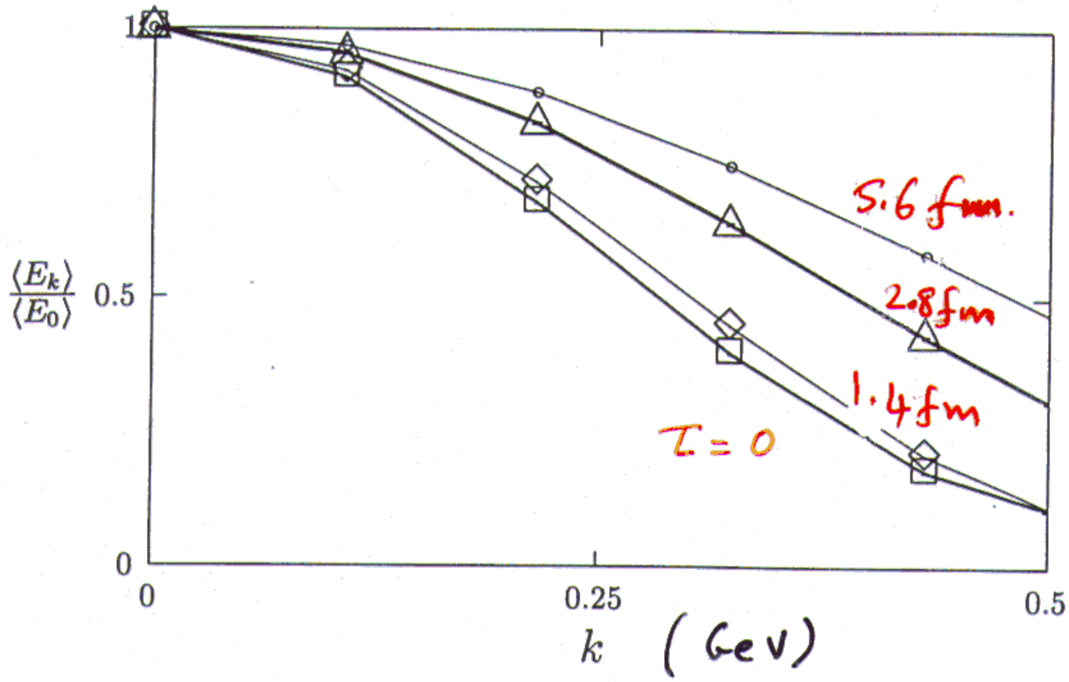
EXAMPLE:

$$\overline{B^2}(x_\perp) = \sqrt{\langle B^2(x_\perp)\, B^2(0) \rangle_\rho}$$

$$\overline{B^2}(k_\perp) = \int d^2x_\perp\, e^{ik_\perp \cdot x_\perp}\, \overline{B^2}(x_\perp).$$

80 × 80

$\mu = 0.41\,GeV$



- **OUTLOOK:**

→ COMPUTE POYNTING VECTOR — ENERGY FLUX — (in progress)

→ STRAIGHT FORWARD EXTENSION TO $N_c = 3$ & 3+1-D.

→ INCLUDE FERMIONS IN APPROACH — CAN COMPUTE RATIO OF QUARKS TO GLUE...

→ FINAL STAGE OF COMPUTATION

$\equiv$ INITIAL CONDITIONS FOR PARTONIC CASCADE.

→ SELF-CONSISTENT RG FOR NUCLEAR COLLISIONS.