

Parton Equilibration, Energy Loss and Brute Force Quenching

1. Intro:
 - a) Glue thermal or chem equilib is not assured at RHIC
 - b) Need $p_{\perp} \gtrsim 3 \text{ GeV}$ data to probe QCD energy loss mech
2. Why Few $N_{\text{scatt}} = 1, 2, 3$ collision energy loss is important
 - a) color coherence hides collinear glue
 - b) Nuclear corona hides $N_{\text{scatt}} \gg 1$
 - c) $N=0$ Self Quench is huge
 - d) Hadronization reduces sensitivity
3. Brute Force pQCD $N_{\text{scatt}} = 1, 2, 3$
P. Levai, I. Vitev, M.G.
(work in progress)

Part 1:

Status of Parton (Non) Equilib ratio

- 1) ZPC B. Zhang
- 2) DMPC D. Molnar
- 3) Hydro Dumitru
- 4) HISING quenching X. Wang, M.

Parton Cascade Event Generators and the Story of ET

Miklos GYULASSY, Yang PANG and Bin ZHANG

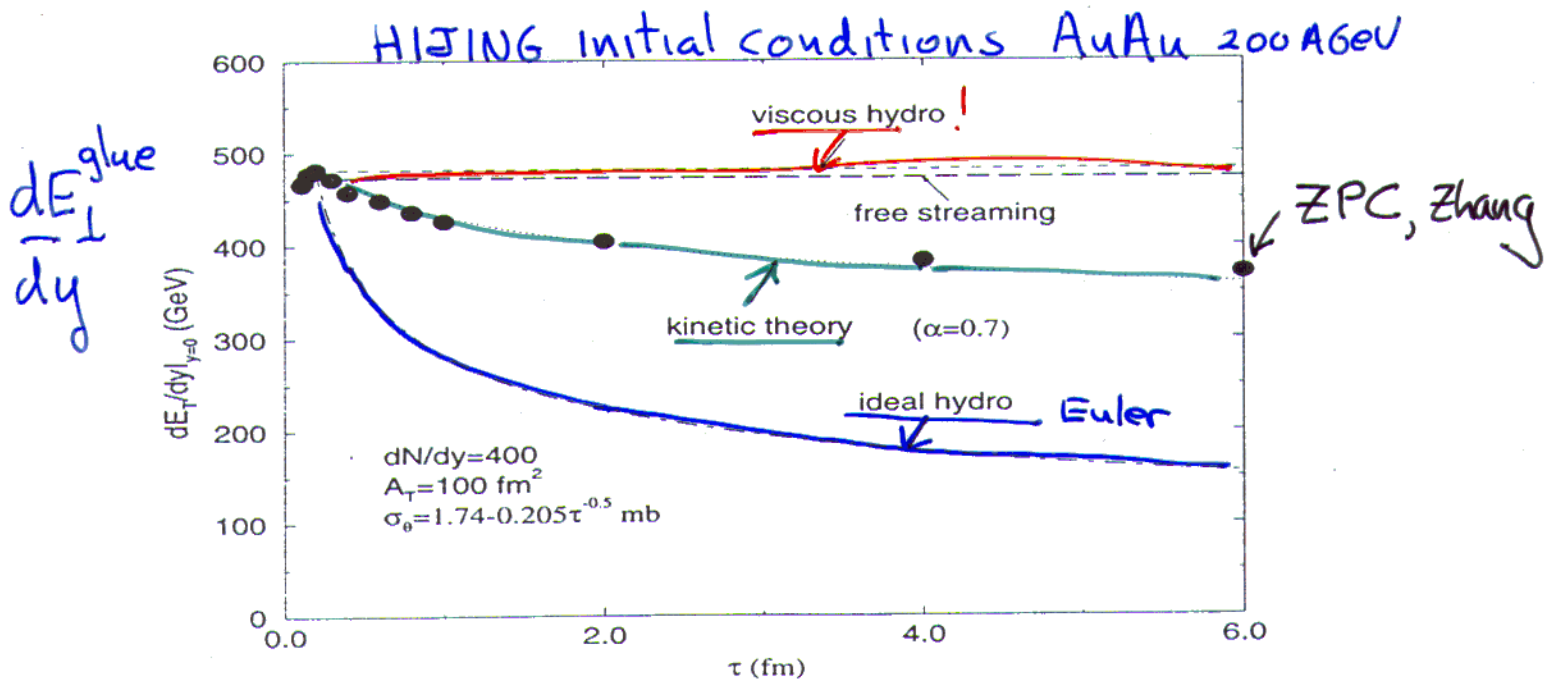


Fig. 2. Comparison of analytic kinetic theory results to numerical ZPC code²⁰ results obtained by averaging 20 events. A periodic transverse grid of dimensions 10 fm was used. Initially (at $\tau = 0.1 \text{ fm}$), $T_0 = 500 \text{ MeV}$, in an interval $-5 < \eta < 5$, with $\frac{dN}{d\eta} = 400$. The screening mass was $\mu = 3 \text{ fm}^{-1}$ and the initial mean free path was $\approx 0.3 \text{ fm}$. Note agreement of ZPC and analytic results in this case where Navier Stokes fails.

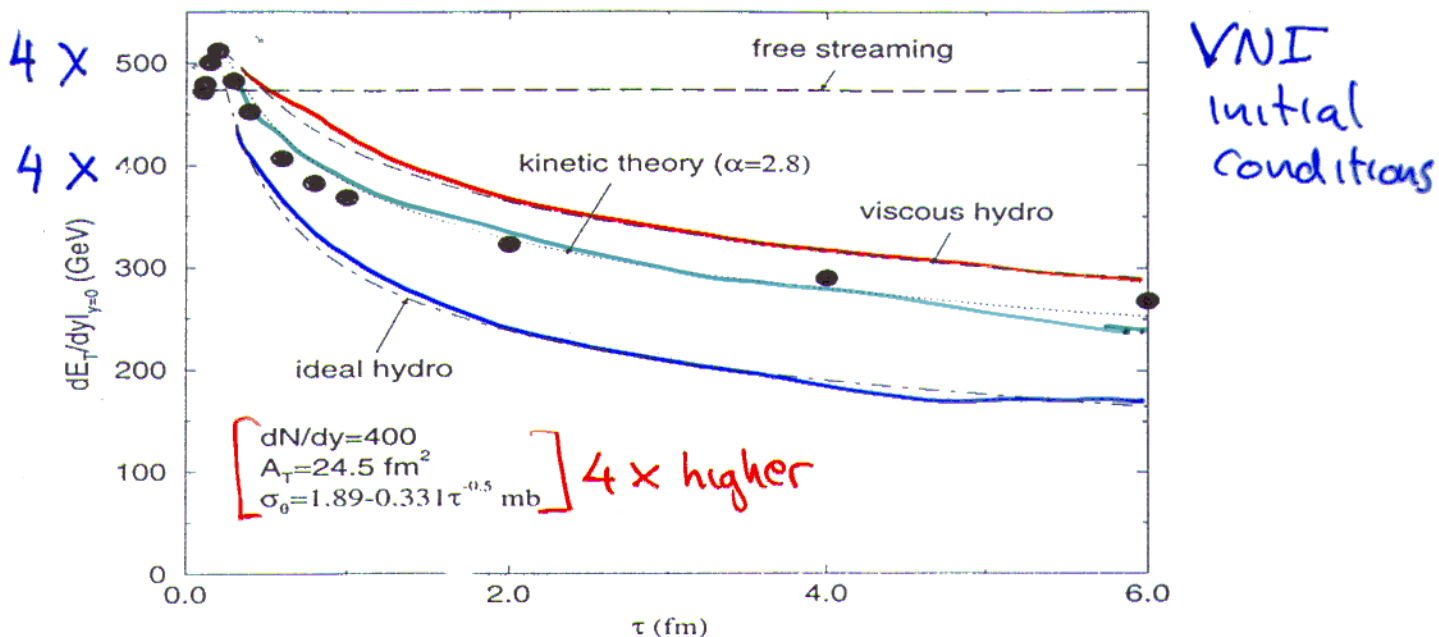
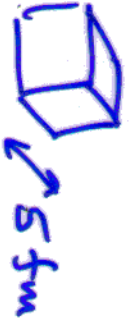


Fig. 3. Comparison of ZPC results with analytic kinetic theory and scaling Navier-Stokes for initial conditions with the parameter $\alpha = 2.8$. This demonstrates the ability of the ZPC cascade model to approach the Navier-Stokes dissipative hydrodynamic domain under extreme initial conditions corresponding to four times the default HIJING parton density.



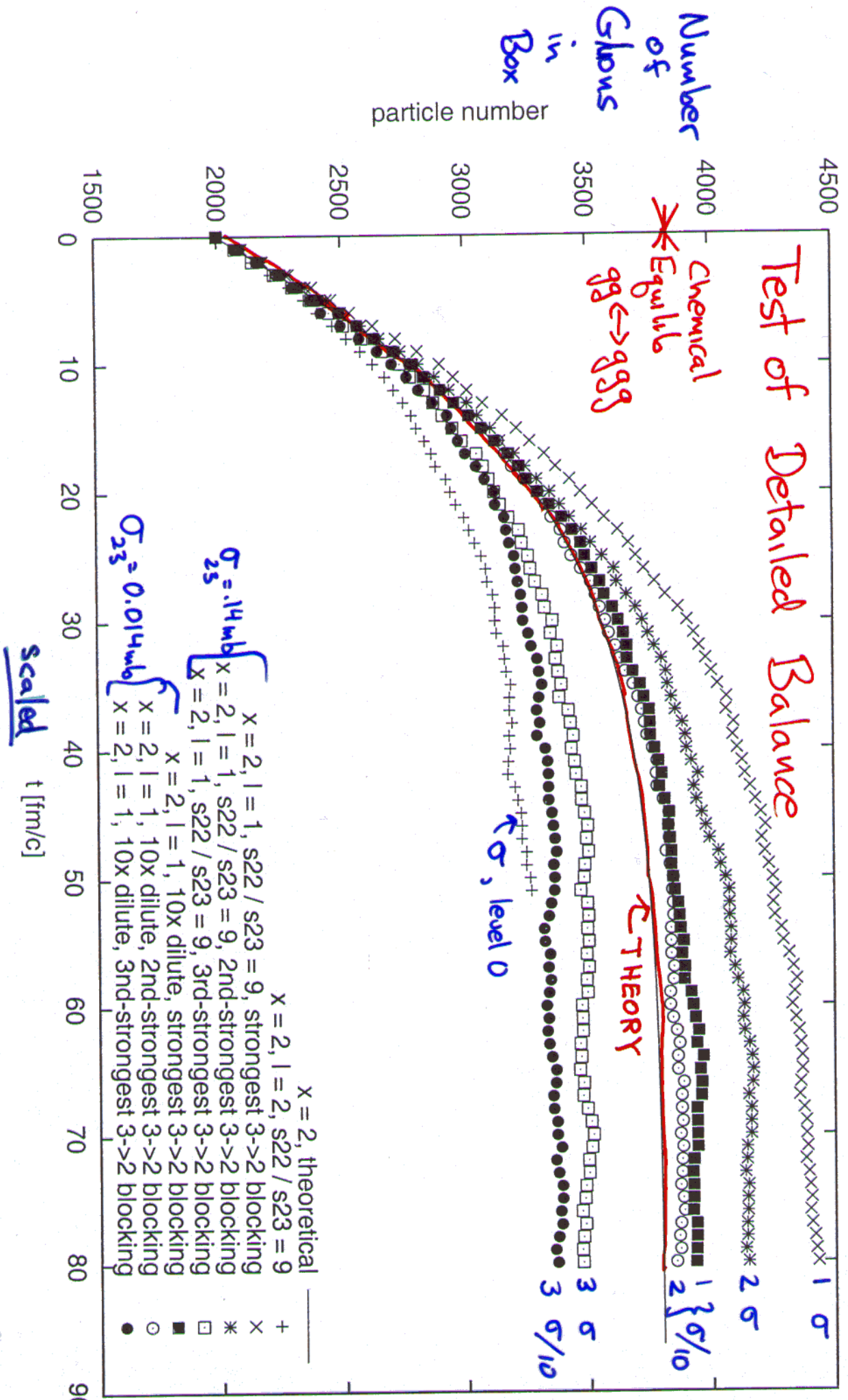
$$\left(\begin{array}{l} \sigma_{22} = 1.8 \text{ mb} \\ \sigma_{32} = 0.14 \text{ mb} \end{array} \right)$$

Chemical equilibration - best try of
($T_0 = 200 \text{ MeV}$, $L = 5 \text{ fm}$, $s_{23} = 0.014 \text{ fm}^2$)

Deves Malvar

Parton Cascade

block level

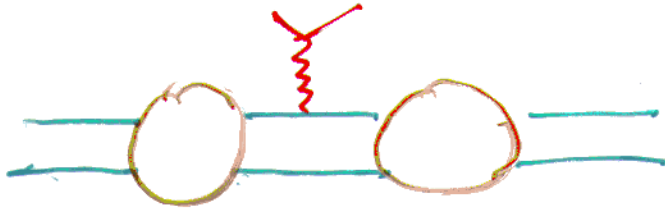


Blocking descriptions

2 ↔ 2

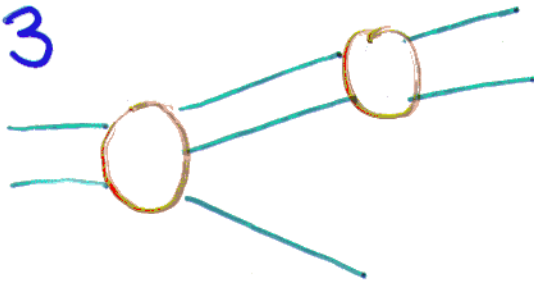


no block level 0

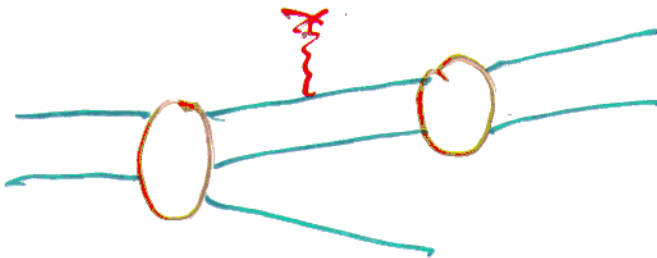


require at least on extra scatt level 1

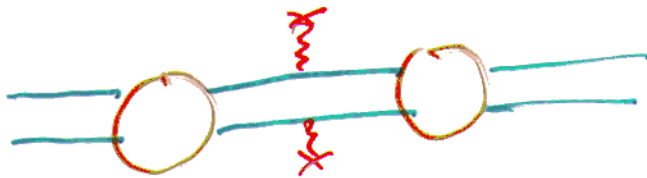
2 → 3



no block allows 2 of 3 to rescatt again

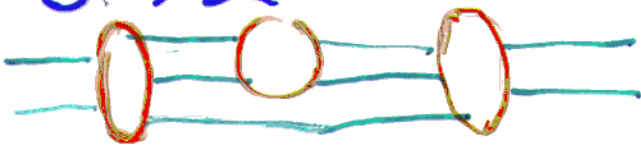


level 1 block



level 2 block

3 → 2



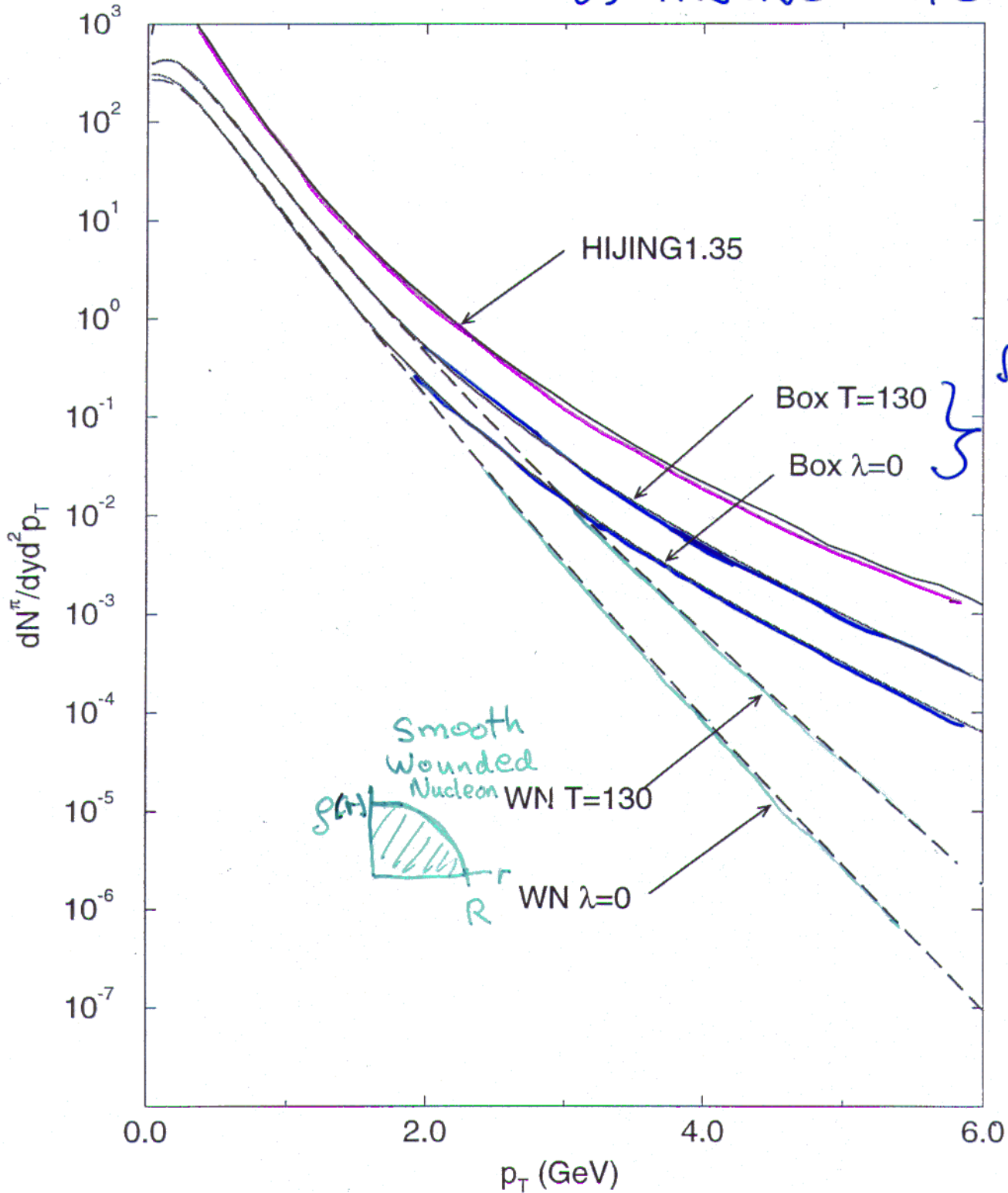
no block levels 1 ...

Au+Au RHIC 2+1 Hydro

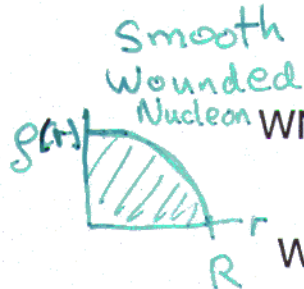
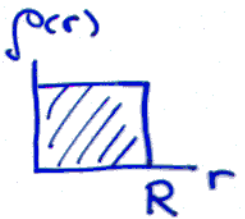
Dumitru, Rischke

vs HIJING

M.G., P. Levai



sharp
initial
radial



How to separate soft (phenomenology)
and hard (pQCD calculable) probes?

$\frac{dN}{dp_{\perp}}$ is hard probe at $p_{\perp} > ??$

Proposed Answer

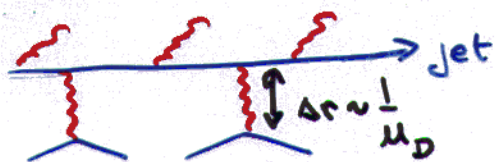
Compare pQCD to Hydro
(quantum non-eq) vs (local equilibrium)

At SPS p_{\perp} spectra have no break

At RHIC there is good news

pQCD \gg Hydro

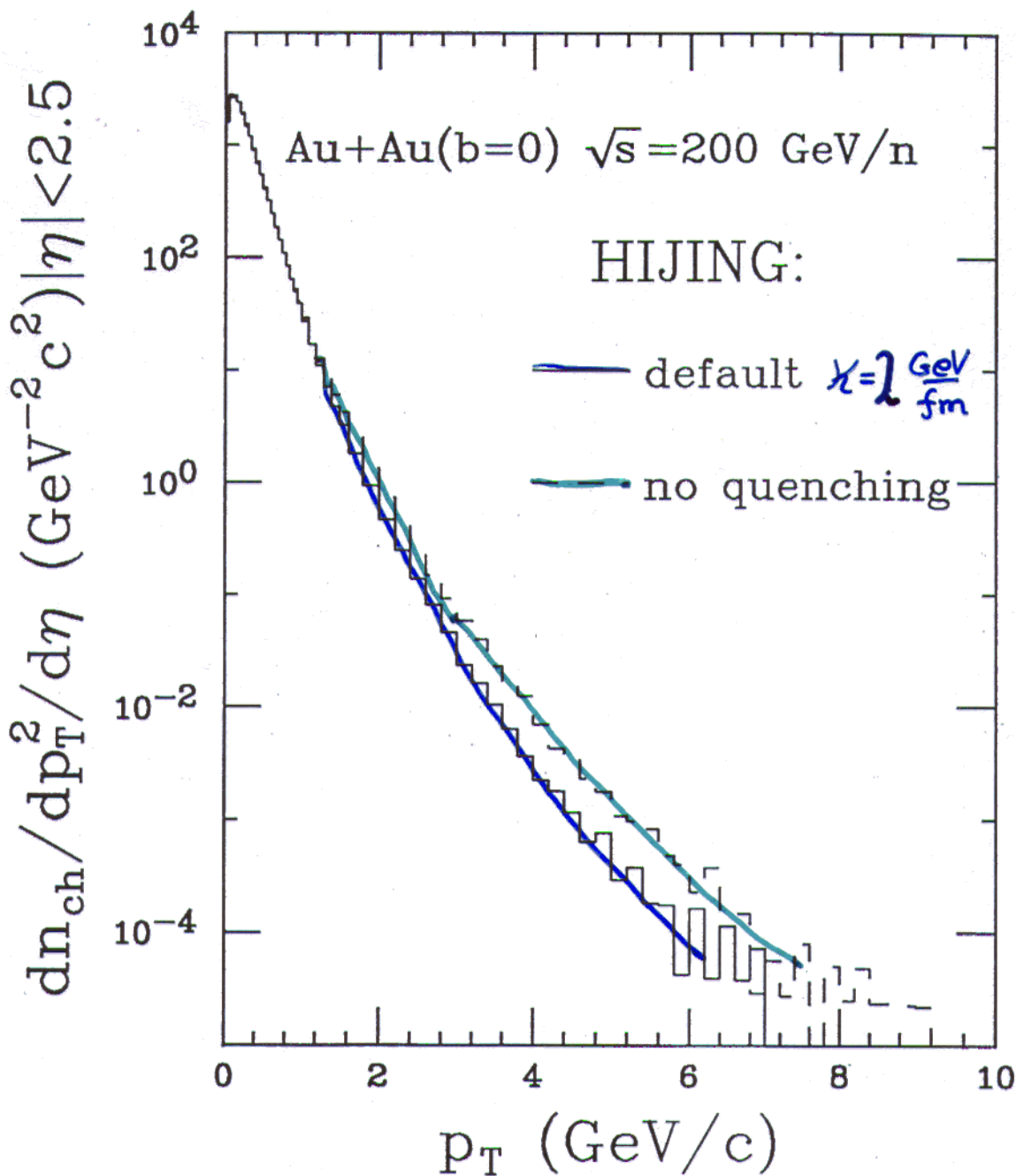
$p_{\perp} \gtrsim 3 \text{ GeV}$



$$\frac{dE}{dx} \approx \alpha_s \mu_D^2 \log^2 \frac{s}{4\mu_D^2} \quad \text{energy loss}$$

$$\sim 1-2 \text{ GeV/fm}$$

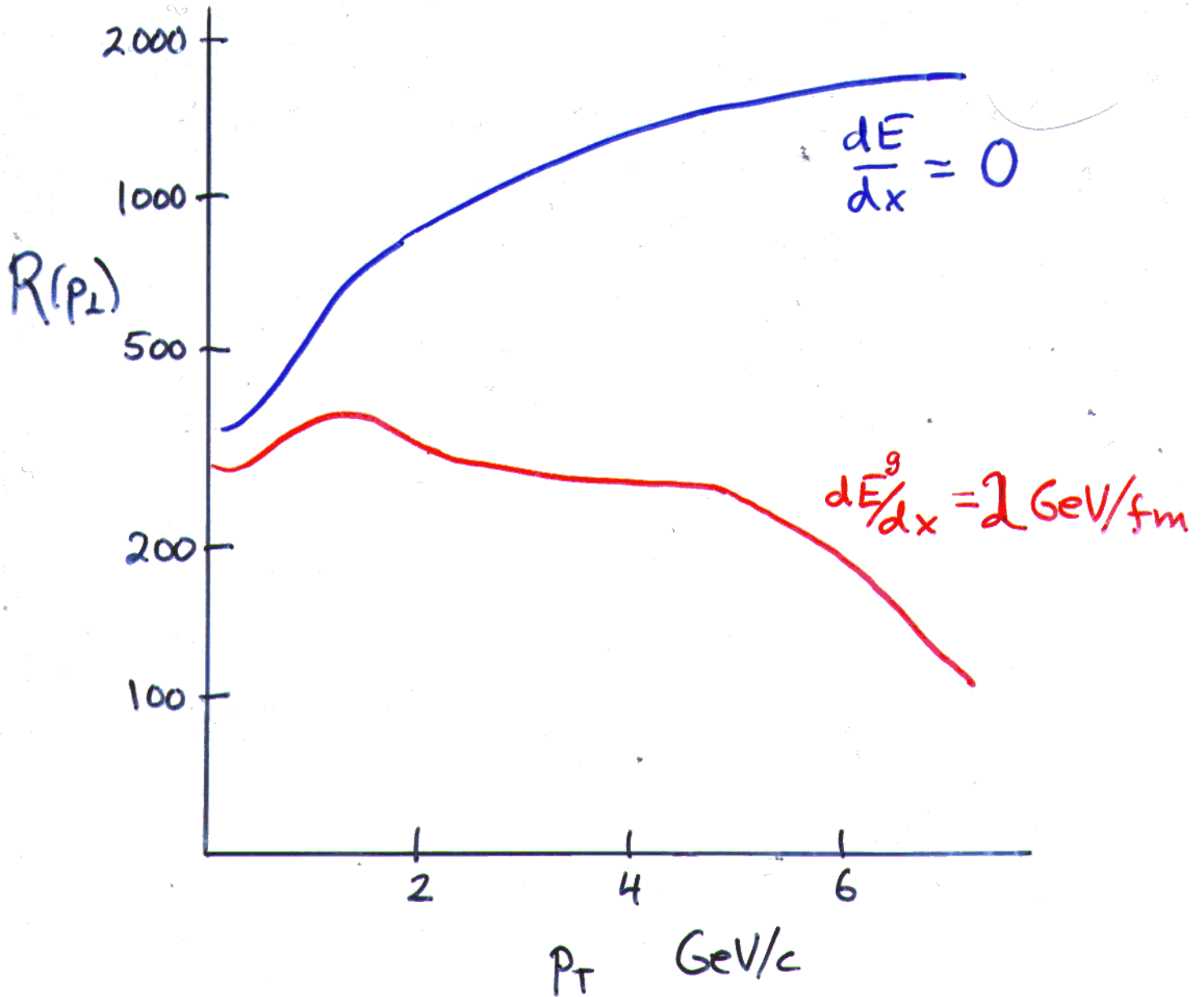
Effects of Energy Loss



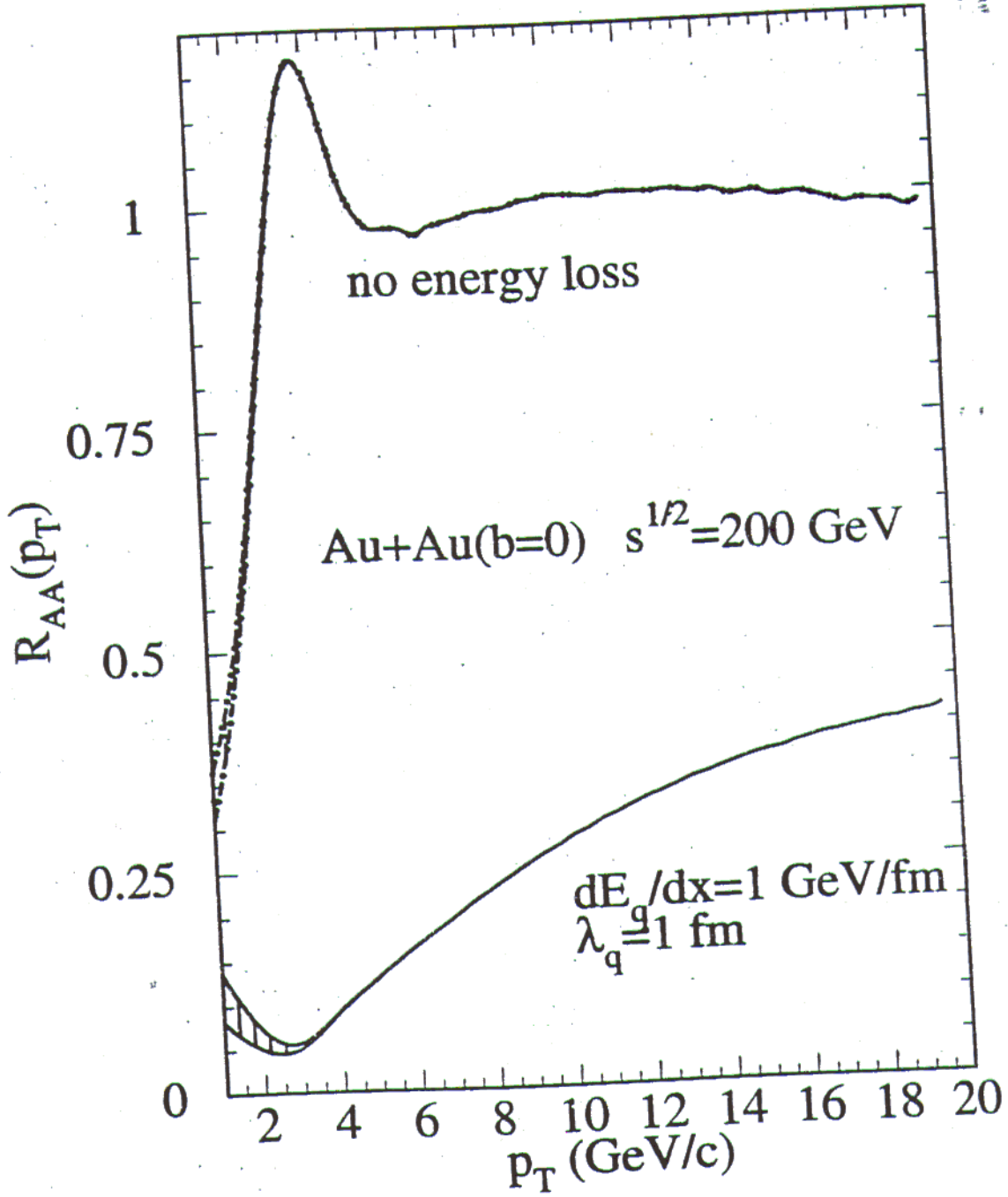
Jet quenching provides info on non abelian energy loss

$$R = \frac{dN_{AA}/dp_{\perp}}{dN_{pp}/dp_{\perp}}$$

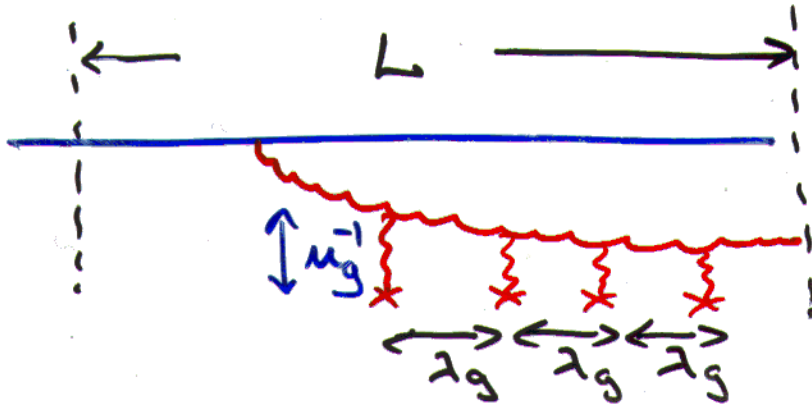
Au+Au (RHIC)



Cronin + Quench



Random Walk in \vec{p}_\perp shortens formation time



$$\langle k_\perp^2 \rangle_L \approx \frac{L}{\lambda_g} \mu_g^2$$

$$\frac{dE}{dx} \approx \bar{\rho} L \left(\frac{\mu^2}{\lambda} \right)_g$$

Non linear $\Delta E(L)$!

Mueller, BDPS (98)

$$\bar{\rho} = \frac{2}{3} \alpha \frac{N_c}{8} \log \frac{L}{\lambda} \sim \frac{1}{3} \left(\frac{2}{3} \right)$$

In QGP $\mu = \mu_{\text{Debye}} \approx gT \approx 2T \sim 0.6 \text{ GeV}$

$$\lambda_g^{-1} = \sigma_g \rho_T \approx \frac{4\pi\alpha^2}{\mu^2} (2T^3)$$

$$\left(\frac{\mu^2}{\lambda} \right)_{\text{QGP}} \approx 4\pi\alpha^2 \rho_T \approx 3T^3 \sim 2 \frac{\text{GeV}}{\text{fm}^2}$$

$T = 300 \text{ MeV}$

In cold nuclei use $p+A \rightarrow \pi/\eta + X$

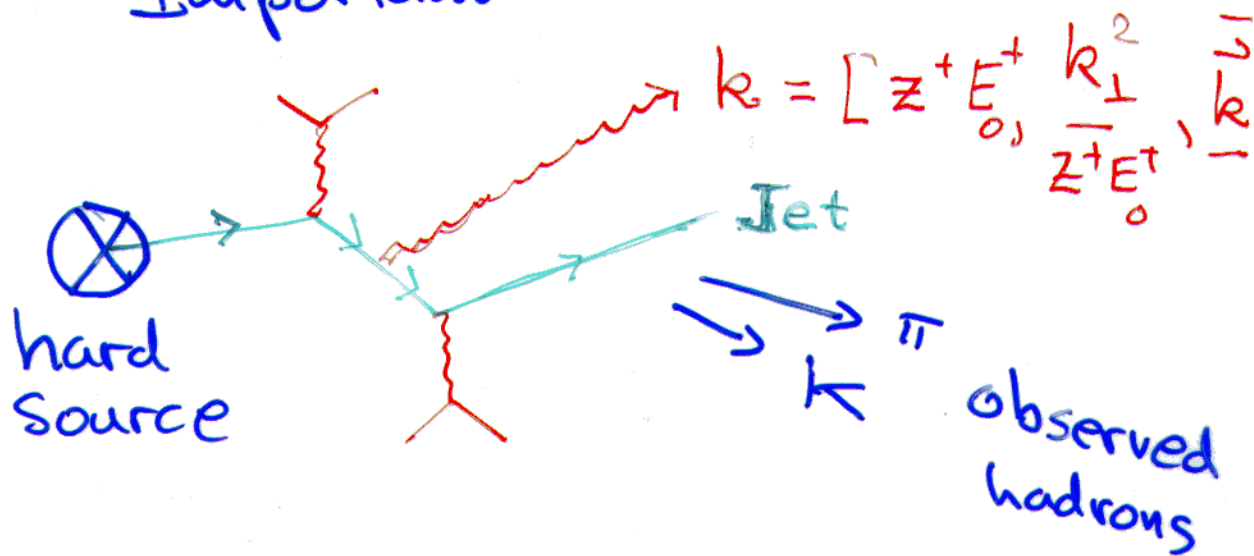
$$\langle p_\perp^2 \rangle_A = p_0^2 + \nu_A \lambda_N \left(\frac{\mu_g^2}{\lambda_g} \right)_{T=0} \quad \nu_A \lambda_N \approx 1.8 A^{1/3} \text{ fm}$$

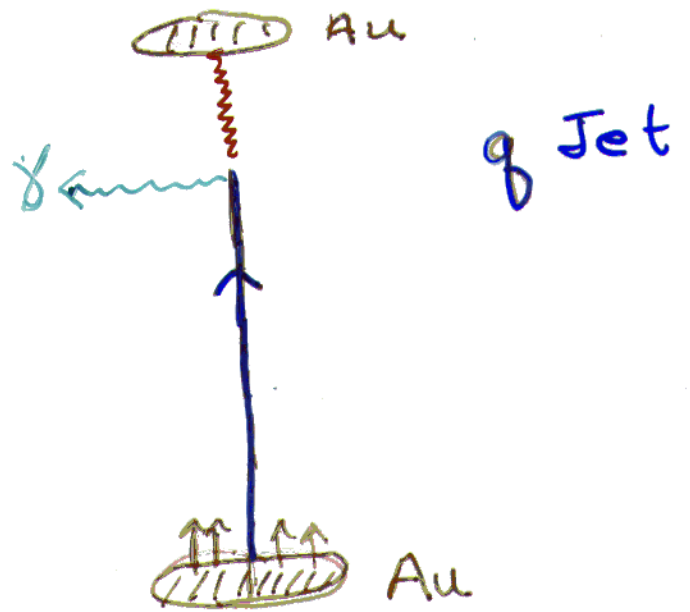
Badier et al $\langle p_\perp^2 \rangle_{pT} - \langle p_\perp^2 \rangle_p \approx 0.36 \text{ GeV}^2$

$$\left(\frac{\mu_g^2}{\lambda_g} \right)_{T=0} \approx 0.05 \frac{\text{GeV}^2}{\text{fm}} \approx \Lambda_{\text{QCD}}^3 \approx \frac{1}{10} \left(\frac{\mu_g^2}{\lambda_g} \right)_{\text{QGP}}$$

Part II:

Why Few Collision Processes are Important



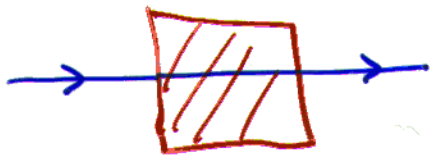


At RHIC $AA \rightarrow \pi + X$

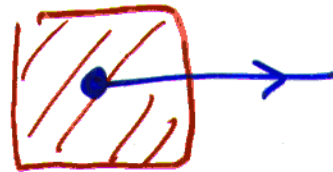
In order to test non-linear $\frac{dE(\omega, \mathbf{k}_\perp)}{dx}$
at observable hadronic distrib level
we need

1) $N_{\text{scatt}} < \infty$, $N_c < \infty$, angular info $\frac{dN_g}{dy d^2k_\perp}$

2) Must consider $(\text{Hard} + \text{Soft})^2$
destructive interference



"Jackson Problem"

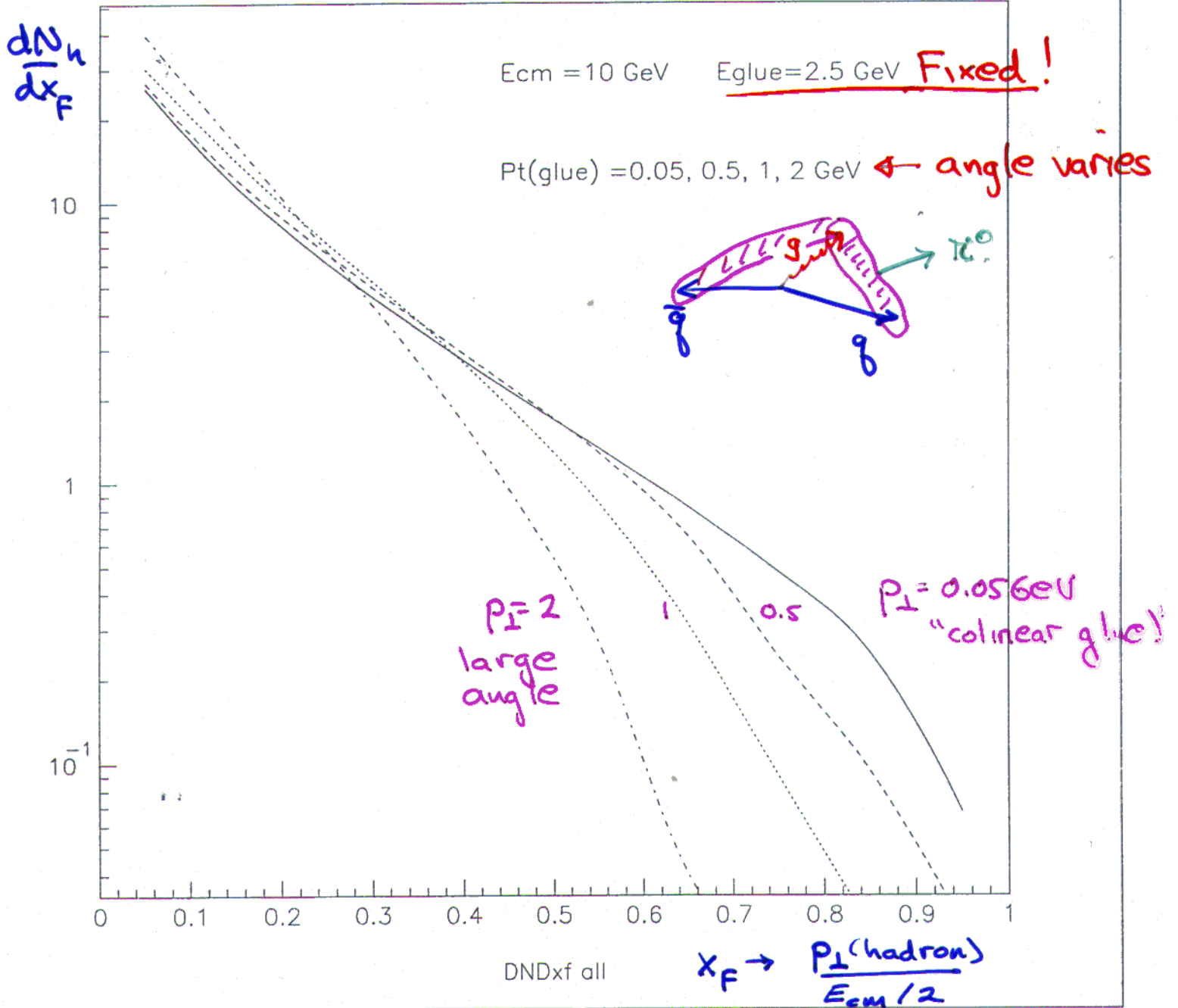


Our nuclear problem
Jets are born naked
and take time to dress

3) Need Hadronization Model

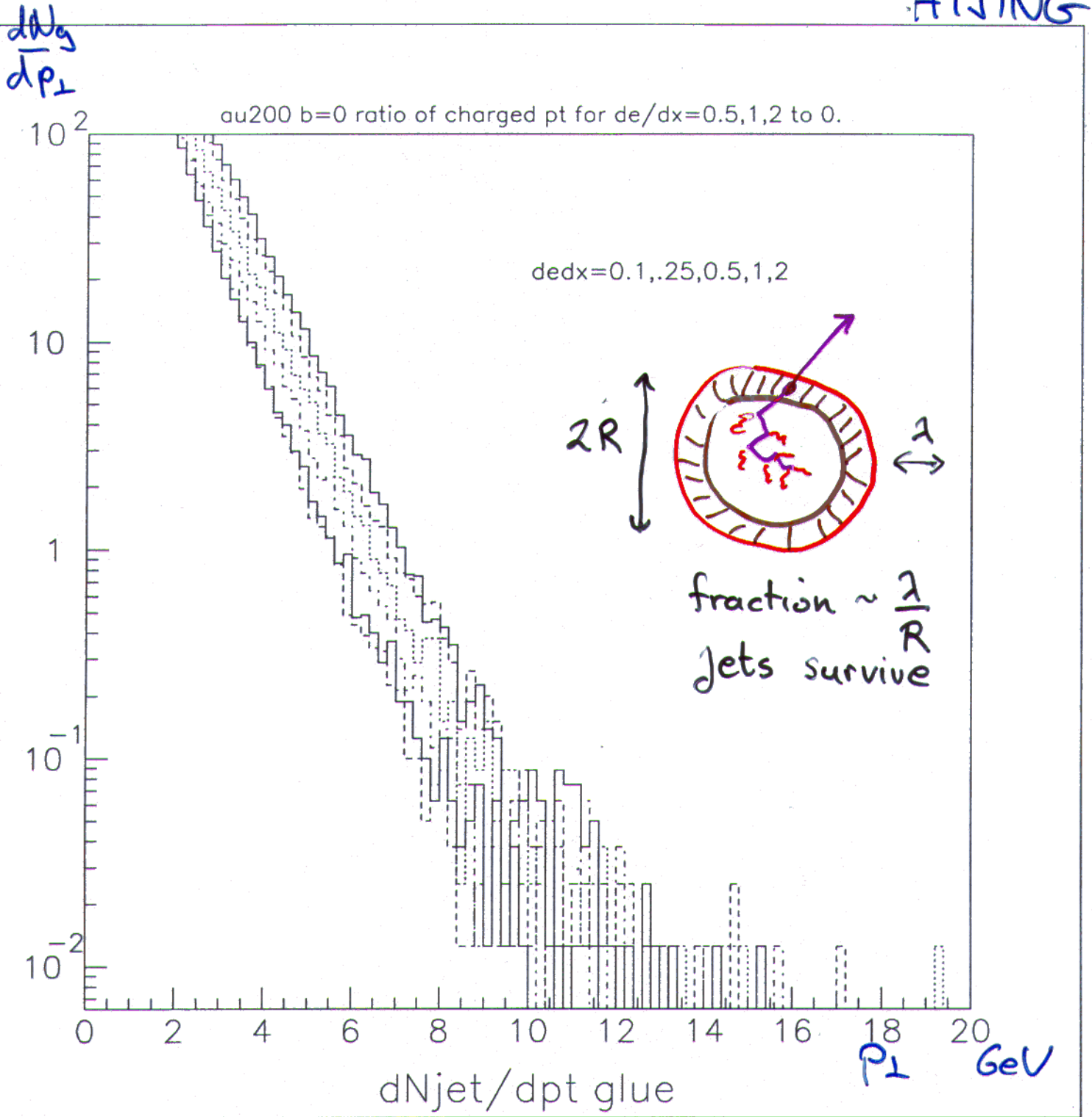
Dependence of hadron quenching on gluon angular distribution

Lund Fragmentation of $q\bar{b}-g-q$ Strings – charged hadrons



Saturation of Jet Quench

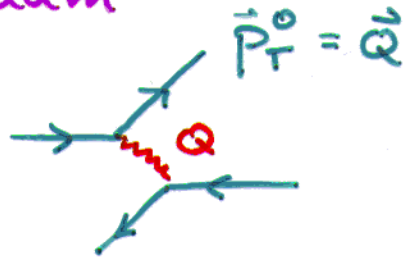
HIJING



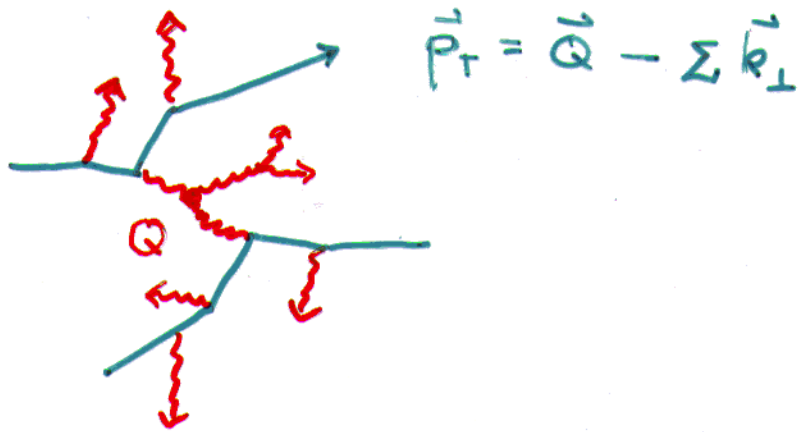
Result sensitive to
finite $N_{scat} = 1, 2, 3$

"Self Quenching" of Hard Processes in Vacuum

Bare Born



Radiative Hair

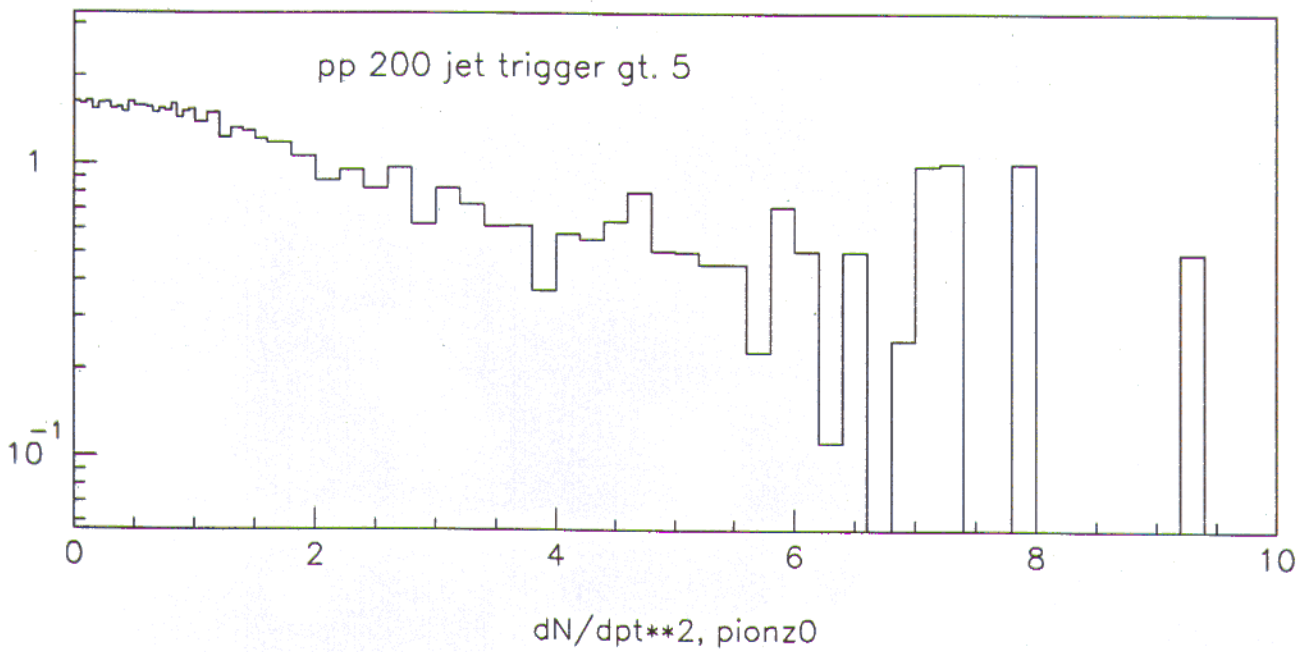
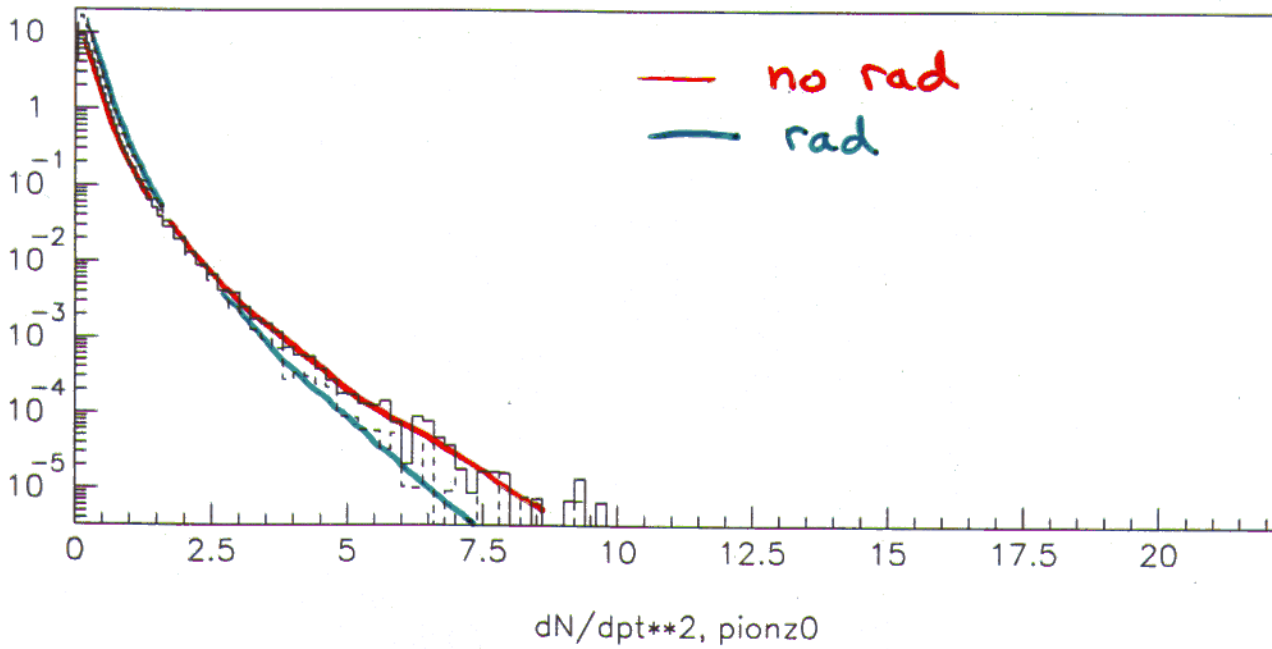


Fragmentation of $q \rightarrow \pi$
softens at high Q

$\sqrt{s} = 200 \text{ GeV}$

$pp \rightarrow \pi^0$

Trigger $p_{\perp} \sim 5 \text{ GeV}$



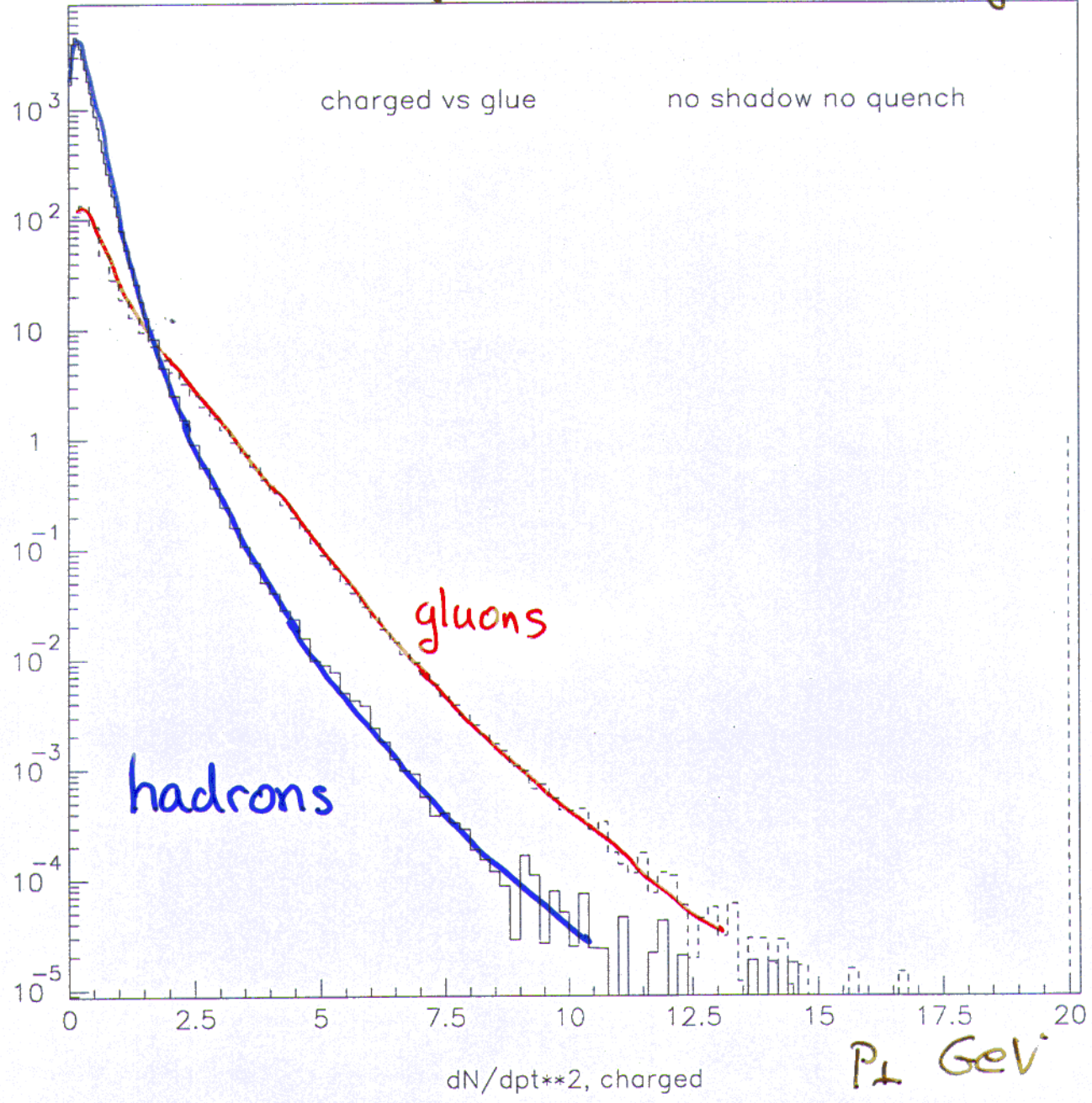
Jet Quench via HIJING

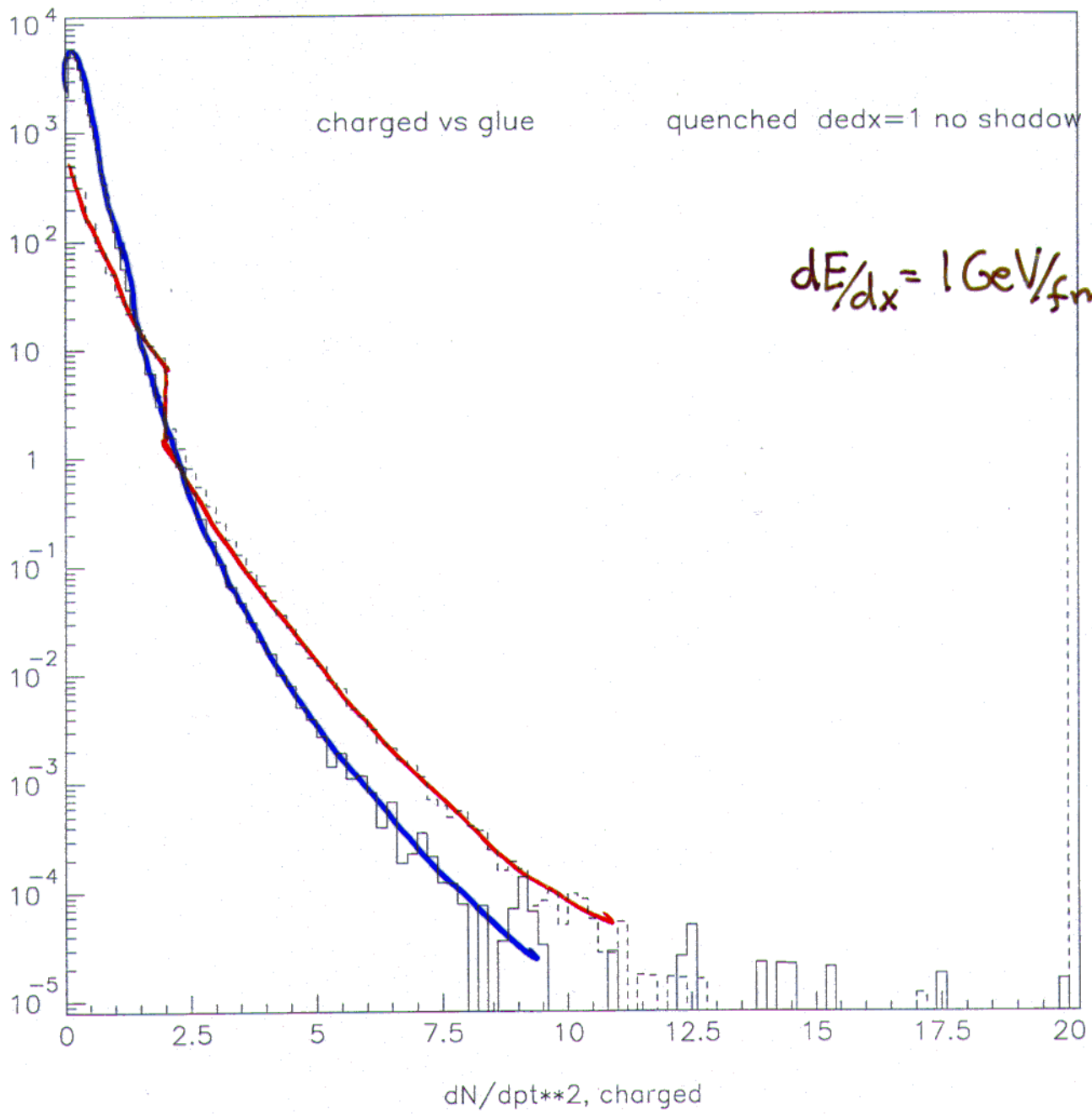
Au+Au $\sqrt{s} = 200$ AGeV

X.N.Wang, M.G

$$\frac{dN}{dy dp_{\perp}^2}$$

Single Particle Inclusive ($y=0$)





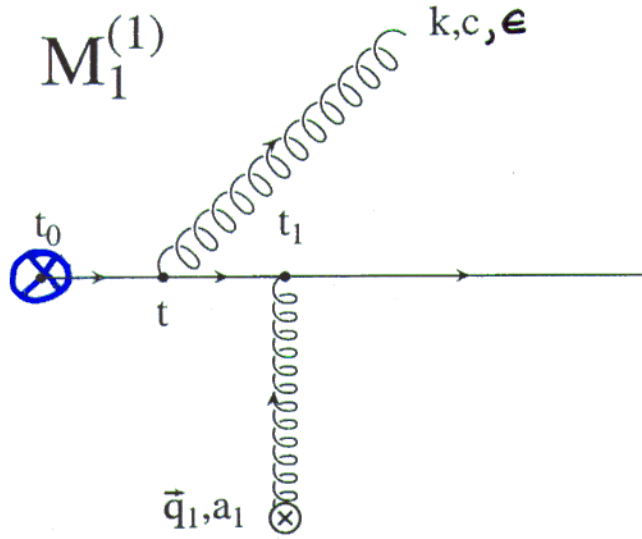
Part 3:

Brutus Forcus pQCD

Peter Levai Ivan Vitev MG
progress report

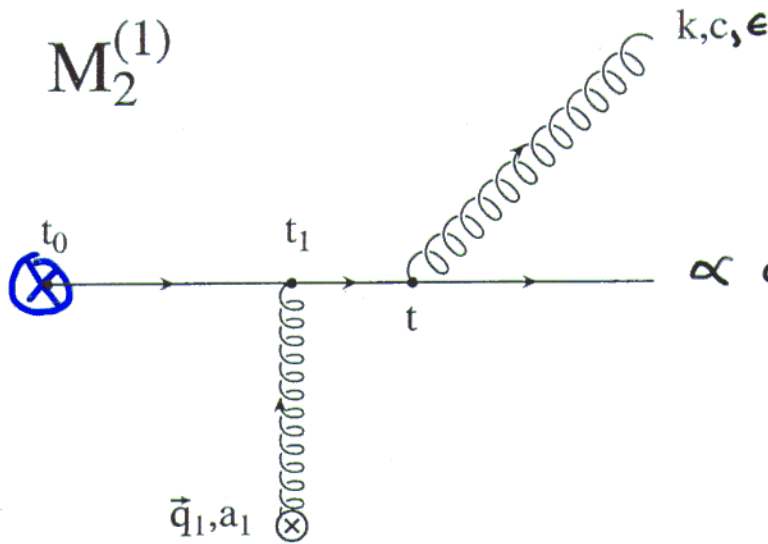
Hard Jet Source at t_0 + $N=1$ Rescattering at t_1

use O.F.P.T.
à la BDMPS

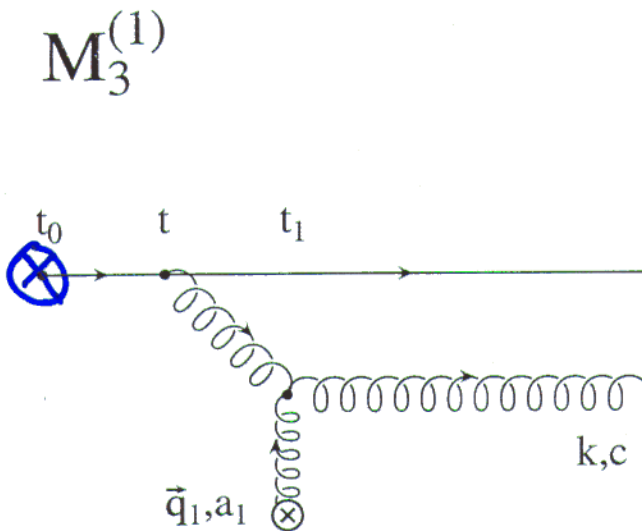


$$\propto g \vec{\epsilon}_\perp \cdot \vec{H} (1 - e^{i\phi_1}) T_a T_c$$

$$\begin{cases} \vec{H} \equiv \vec{k}_\perp / k_\perp^2 \\ \phi_1 \equiv (t_1 - t_0) \frac{k_\perp^2}{2\omega} \end{cases}$$



$$\propto g \vec{\epsilon}_\perp \cdot \vec{H} e^{i\phi_1} T_c T_a$$



$$\propto g \vec{\epsilon}_\perp \cdot \vec{C}_1 (e^{i\phi_2} - e^{i\phi_1}) [T_c, T_a]$$

$$\begin{cases} \vec{C}_1 \equiv \frac{\vec{k}_\perp - \vec{q}_{\perp 1}}{(k_\perp - q_{\perp 1})^2} \\ \phi_2 = \phi_1 - \Delta t \frac{(k_\perp - q_\perp)^2}{2\omega} \end{cases}$$

Gluon Radiation with Two Scattering Centers
BDPMS, NPB484 (97) 291

$$\mathcal{M}_1 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k_\perp^2} (e^{it_1 \frac{k_\perp^2}{2\omega}} - e^{it_0 \frac{k_\perp^2}{2\omega}}) a_2 a_1 c,$$

$$\mathcal{M}_2 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k_\perp^2} (e^{it_2 \frac{k_\perp^2}{2\omega}} - e^{it_1 \frac{k_\perp^2}{2\omega}}) a_2 c a_1,$$

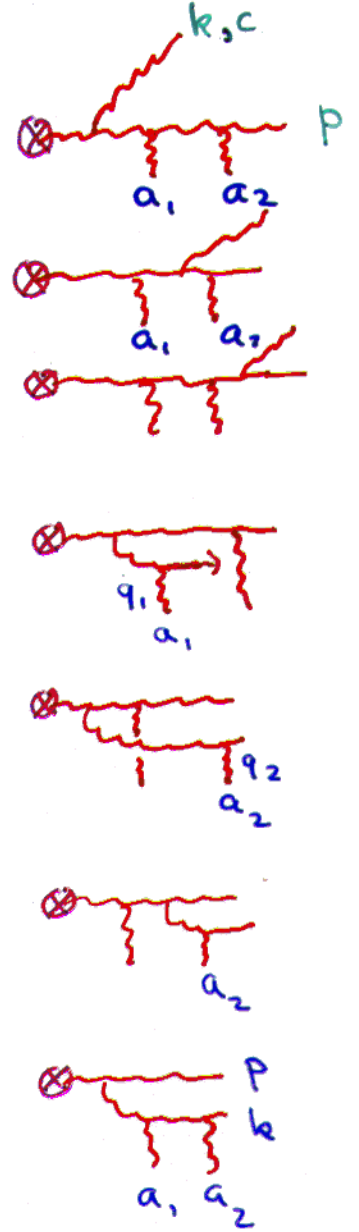
$$\mathcal{M}_3 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k_\perp^2} (-e^{it_2 \frac{k_\perp^2}{2\omega}}) c a_2 a_1,$$

$$\mathcal{M}_4 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot (\vec{k} - \vec{q}_1)_\perp}{(k - q_1)_\perp^2} e^{it_1 \frac{k_\perp^2 - (k - q_1)_\perp^2}{2\omega}} \times \\ \times (e^{it_1 \frac{(k - q_1)_\perp^2}{2\omega}} - e^{it_0 \frac{(k - q_1)_\perp^2}{2\omega}}) a_2 [c, a_1],$$

$$\mathcal{M}_5 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot (\vec{k} - \vec{q}_2)_\perp}{(k - q_2)_\perp^2} e^{it_2 \frac{k_\perp^2 - (k - q_2)_\perp^2}{2\omega}} \times \\ \times (e^{it_1 \frac{(k - q_2)_\perp^2}{2\omega}} - e^{it_0 \frac{(k - q_2)_\perp^2}{2\omega}}) a_1 [c, a_2],$$

$$\mathcal{M}_6 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot (\vec{k} - \vec{q}_2)_\perp}{(k - q_2)_\perp^2} e^{it_2 \frac{k_\perp^2 - (k - q_2)_\perp^2}{2\omega}} \times \\ \times (e^{it_2 \frac{(k - q_2)_\perp^2}{2\omega}} - e^{it_1 \frac{(k - q_2)_\perp^2}{2\omega}}) [c, a_2] a_1,$$

$$\mathcal{M}_7 = 2ig_s \frac{\vec{\epsilon}_\perp \cdot (\vec{k} - \vec{q}_1 \vec{q}_2)_\perp}{(k - q_1 - q_2)_\perp^2} [[c, a_2], a_1] \\ e^{it_2 \frac{k_\perp^2 - (k - q_2)_\perp^2}{2\omega}} e^{it_1 \frac{(k - q_2)_\perp^2 - (k - q_1 - q_2)_\perp^2}{2\omega}} \\ \times (e^{it_1 \frac{(k - q_1 - q_2)_\perp^2}{2\omega}} - e^{it_0 \frac{(k - q_1 - q_2)_\perp^2}{2\omega}}).$$



$$M = M_1 + M_2 + M_3$$

$$= e^{i\phi_0} \vec{E}_\perp \cdot \left\{ \underbrace{\vec{H} T^a T^c}_{\text{initial hard}} + \underbrace{(\vec{B} e^{i\phi_1} + \vec{C} e^{i\phi_2})}_{\text{Bertsch-Gunion}} [T^c, T^a] \right\}$$

gluon rescattering

$$t_0 \frac{k_\perp^2}{2\omega}$$

$$\vec{H} = \frac{\vec{k}_\perp}{k_\perp^2}$$

$$\vec{C} = \frac{\vec{k}_\perp - \vec{q}_\perp}{(k_\perp^0 - q_\perp^0)^2}$$

$$\phi_1 = (t_1 - t_0) k_\perp^2$$

$$\phi_2 = \phi_1 - \frac{(t_1 - t_0)}{2\omega} (k_\perp^0 - q_\perp^0)^2$$

$$\vec{B} = \vec{H} - \vec{C} = \frac{\vec{k}_\perp}{k_\perp^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{(k_\perp^0 - q_\perp^0)^2}$$

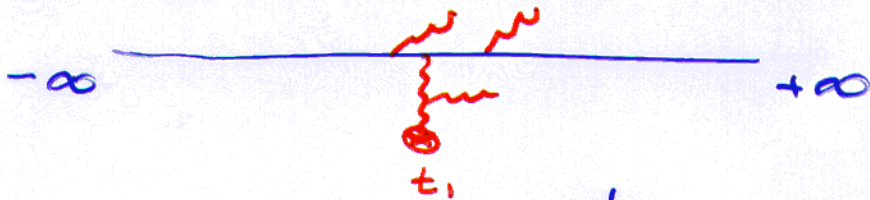
Limits:

1) $t_0 \rightarrow -\infty$

$e^{i\phi_0}$ and $e^{i\phi_0 + \phi_2} \rightarrow 0$

But $e^{i\phi_0 + \phi_1} \rightarrow e^{it_1 k_\perp^2 / 2\omega}$ finite

$$M \rightarrow M_{BG} \propto \vec{E}_\perp \cdot \vec{B} [T^c, T^a]$$



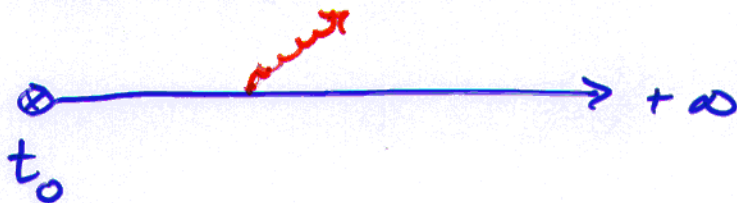
isolated
Bertsch
Gunion

2) $t_1 \rightarrow +\infty$
 t_0 fixed

$e^{i\phi_1}$ and $e^{i\phi_2} \rightarrow 0$

But $e^{i\phi_0} = e^{it_0 k_\perp^2 / 2\omega}$ finite

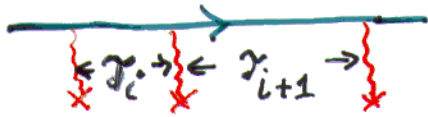
$$M \rightarrow M_H \propto \vec{E}_\perp \cdot \vec{H} T^a T^c$$



pure
Hard
process

Multiple Collision Ensemble Average: H.G., X. Wang

① over collision times $\tau_i = t_i - t_{i-1}$



$$\langle dN \rangle_\lambda = \int \prod_i \pi \frac{d\tau_i}{\lambda_i} e^{-\tau_i/\lambda_i} dN$$

$$\langle \cos \tau_1 \omega_1 \rangle = \frac{1}{1 + \lambda_1^2 \omega_1^2}$$

$$\langle \cos (\tau_1 \omega_1 + \tau_2 \omega_2) \rangle = \frac{1 - \lambda_1 \lambda_2 \omega_1 \omega_2}{(1 + \lambda_1^2 \omega_1^2)(1 + \lambda_2^2 \omega_2^2)}$$

(Converts oscillating terms to rational fncs)

② Over $\vec{q}_\perp i$ via Yukawa cross sec

$$u_i \equiv \frac{\mu^2}{q_{\perp i}^2 + \mu^2} \quad \mu = \text{screening scale}$$

$$\langle dN \rangle_\mu = \int \prod_i \frac{1}{1 - u_0} \frac{du_i}{1 - u_0} dN$$

$$u_0 \equiv \frac{\mu^2}{Q^2 + \mu^2} \quad \text{where} \quad Q^2 = 6 E_{\text{jet}} T$$

requires numerical evaluation

N=0 Distribution

$$dN^0 \propto \frac{dz^+}{z^+} \frac{dk_{\perp}^2}{k_{\perp}^2} = d \log z^+ d \log k_{\perp}^2$$

$$\frac{\mu}{E_0^+} < z^+ = (\omega + k_z)/E_0^+ < 1, \quad 0 < k_{\perp} < \omega \left(\frac{E_0^+}{2} \right)$$

$$k_z > 0 \Rightarrow z^+ > \frac{k_{\perp}}{2E_0^+}$$

N=1 Distribution (ave over $t_1 - t_0$)

$$\langle dN^1 \rangle_1 = dN^0 \left(1 + \tilde{f}_1 \left(\xi = \frac{\mu^2 \lambda}{2\omega}, k_{\perp}, \vec{q}_{\perp} \right) \right)$$

$$\tilde{f}_1 = \frac{C_A}{C_R} \xi^2 \left\{ \begin{aligned} & \frac{k_{\perp}^4}{\mu^4 + \xi^2 k_{\perp}^4} \\ & - \frac{\vec{k}_{\perp} \cdot (\vec{k}_{\perp} - \vec{q}_{\perp}) (2k_{\perp}^2 - \vec{k}_{\perp} \cdot \vec{q}_{\perp})}{(\mu^4 + \xi^2 k_{\perp}^4) (\mu^4 + \xi^2 (q_{\perp}^2 + 4(\vec{k}\vec{q}) (\vec{k}\vec{q} - q^2))} \\ & + \frac{2(\vec{k}\vec{q}) (\vec{k}_{\perp} - \vec{q}_{\perp})^2}{\mu^4 + \xi^2 (\vec{k}_{\perp} - \vec{q}_{\perp})^4} \end{aligned} \right\}$$

Finite singularity free due to time ave

$$\vec{q}_{\perp} \rightarrow 0$$

fixed k_{\perp} large $\omega \Rightarrow$ small ξ

$$\rightarrow 0$$

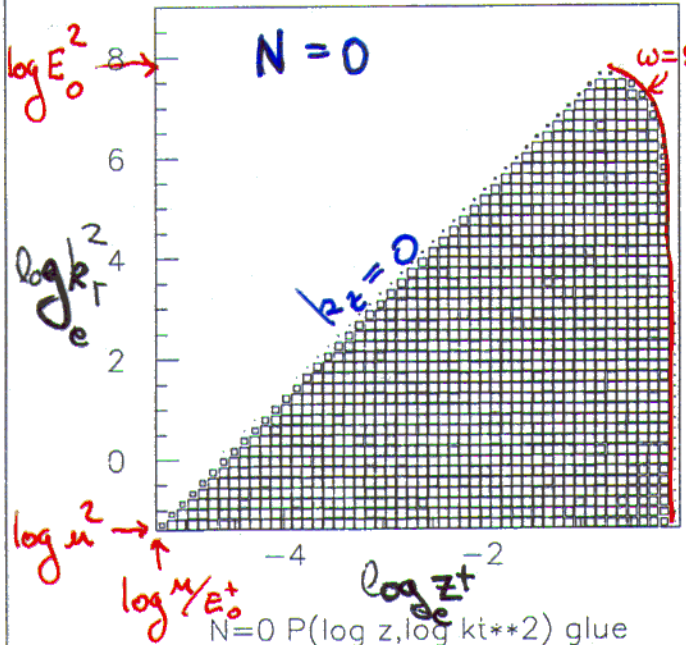
fixed ω small k_{\perp}

$$\langle dN^1 \rangle_{1,\mu} = dN^0 \left(1 + f_1(\xi, k_{\perp}) \right)$$

$N=0$ vs 1 Phase space

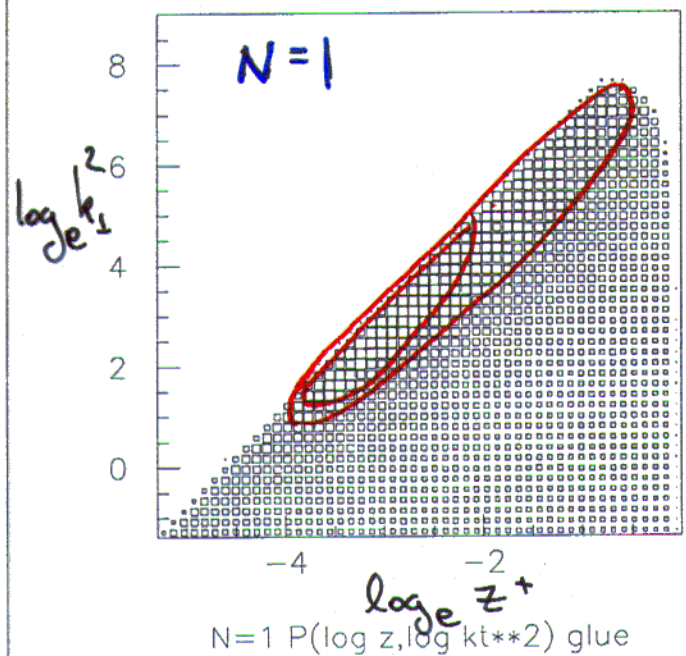
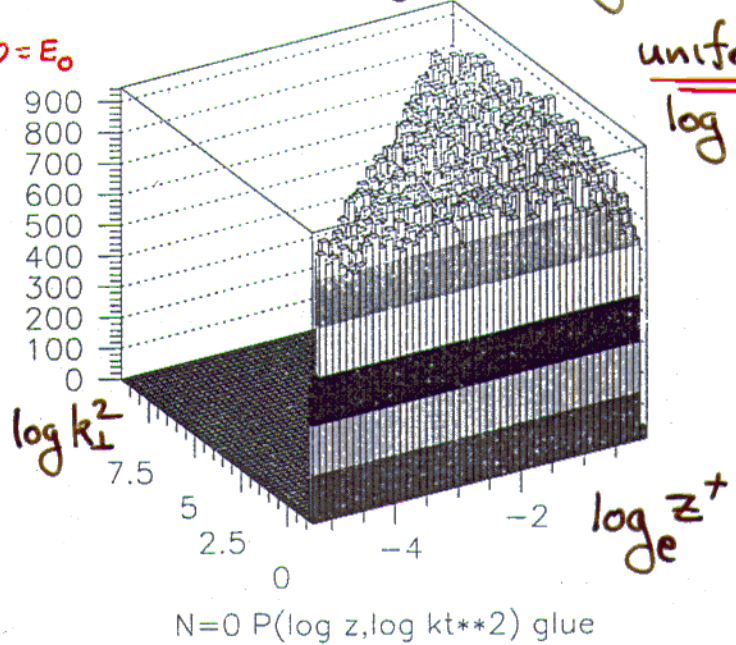
$E_{jet} = 50 \text{ GeV}$

$$z^+ = \frac{\omega + k_z}{E_0^+}$$



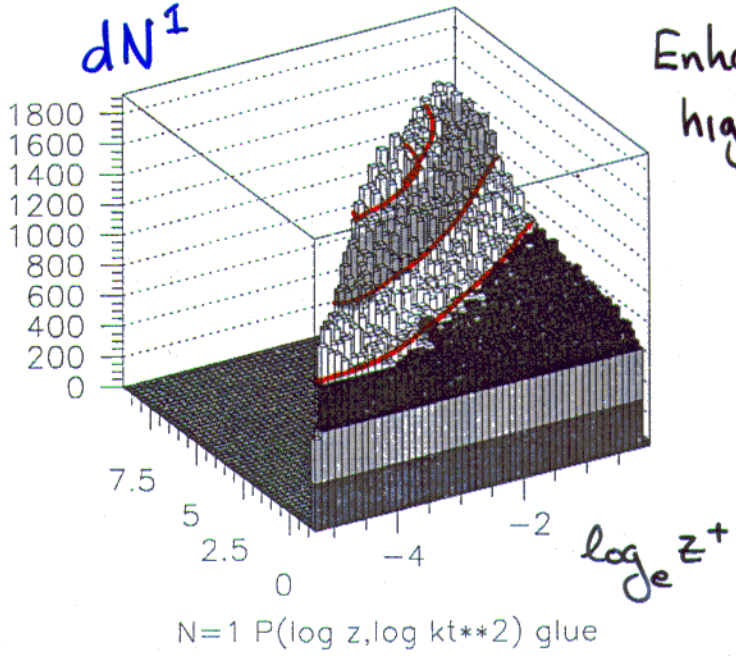
$$dN^0 \propto d \log z^+ d \log k_{\perp}^2$$

uniform
log cake

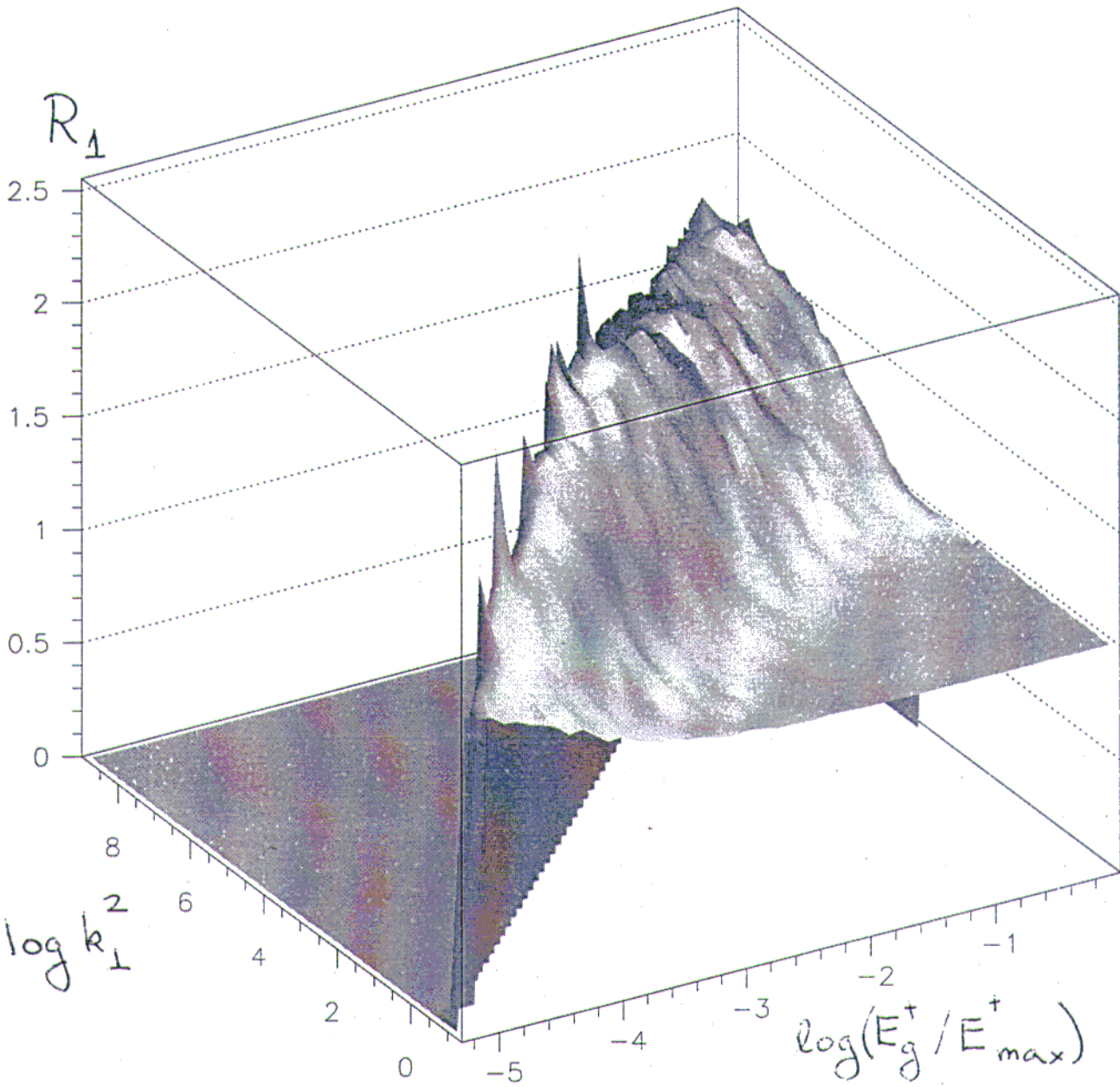


$$dN^1$$

Enhanced
high k_{\perp}



$$dN_g^z = dN_g^0 R_L(k_\perp, z)$$



N=1 P(log z, log kt**2) glue

Radiation Phase Space

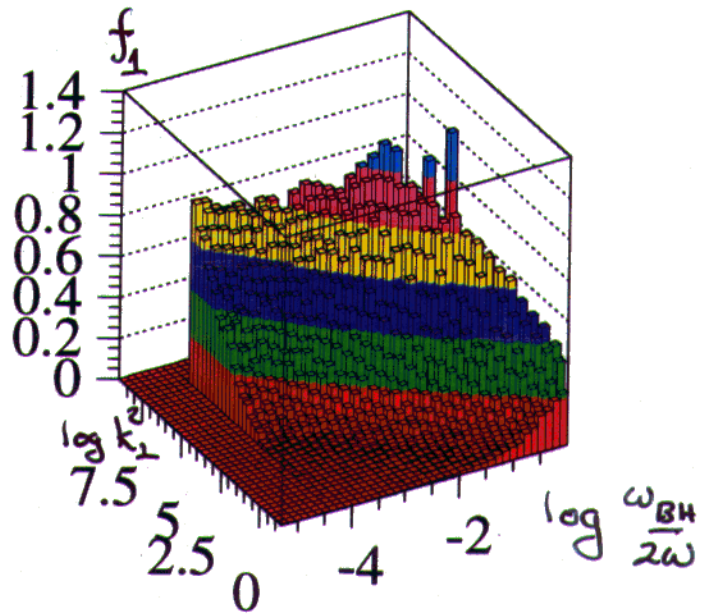
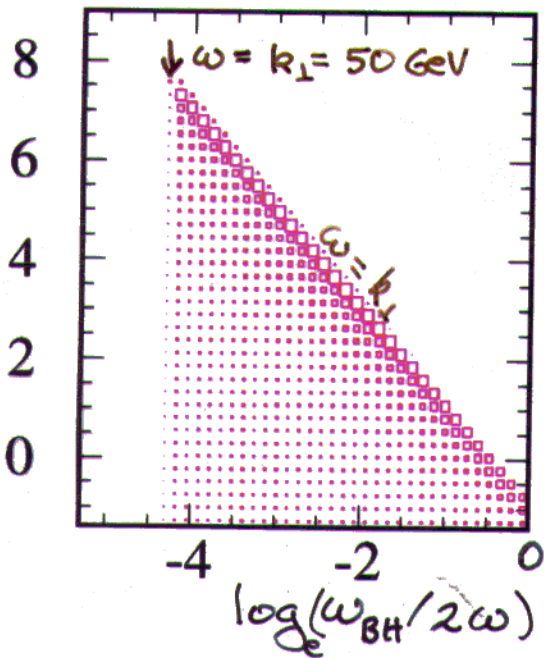
$$dN_1 = dN_0 (1 + f_1(z, k_1))$$

$$E_g = 50 \text{ GeV}$$

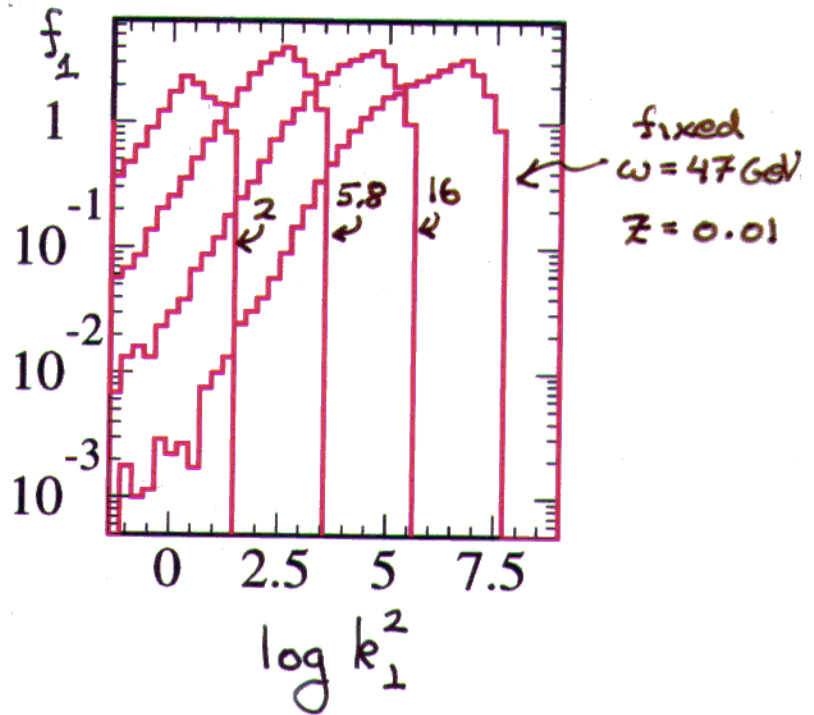
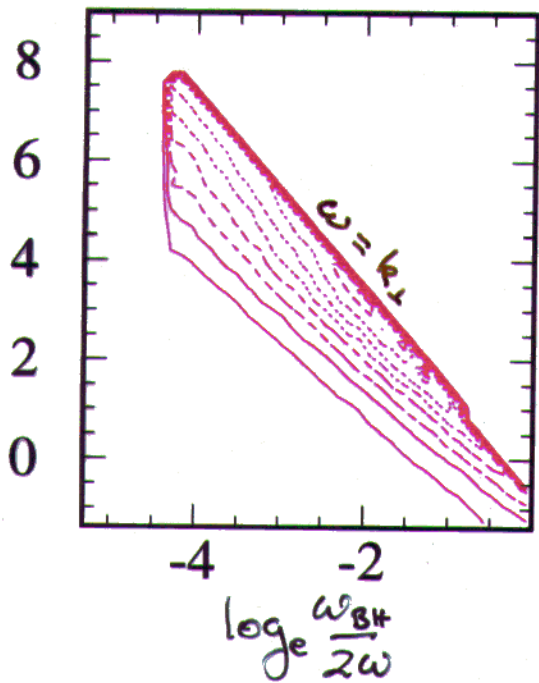
$$\omega_{\text{BH}} = 1.3 \text{ GeV}$$

$$\mu = 0.5 \text{ GeV}$$

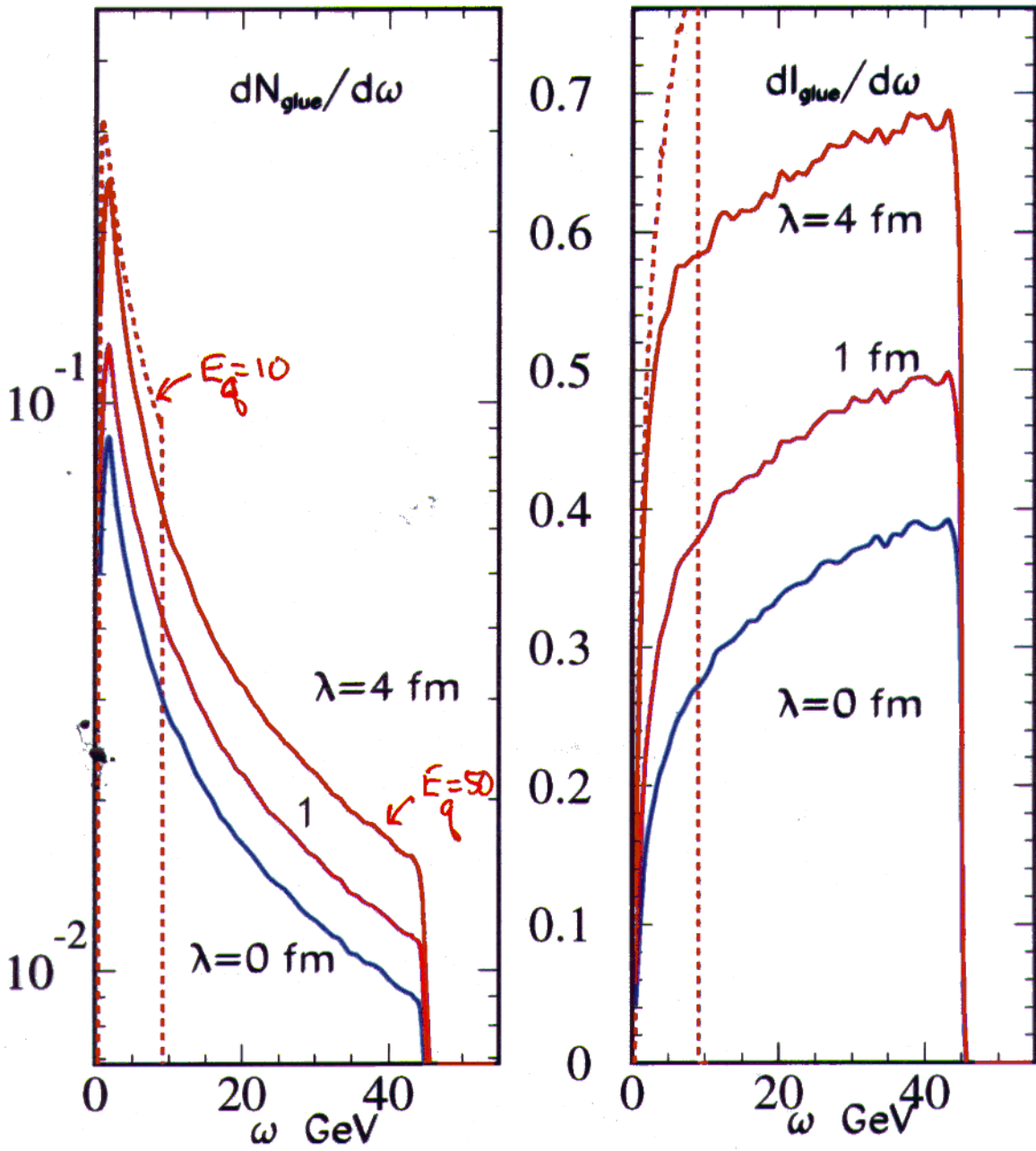
$\log_e k_1^2$



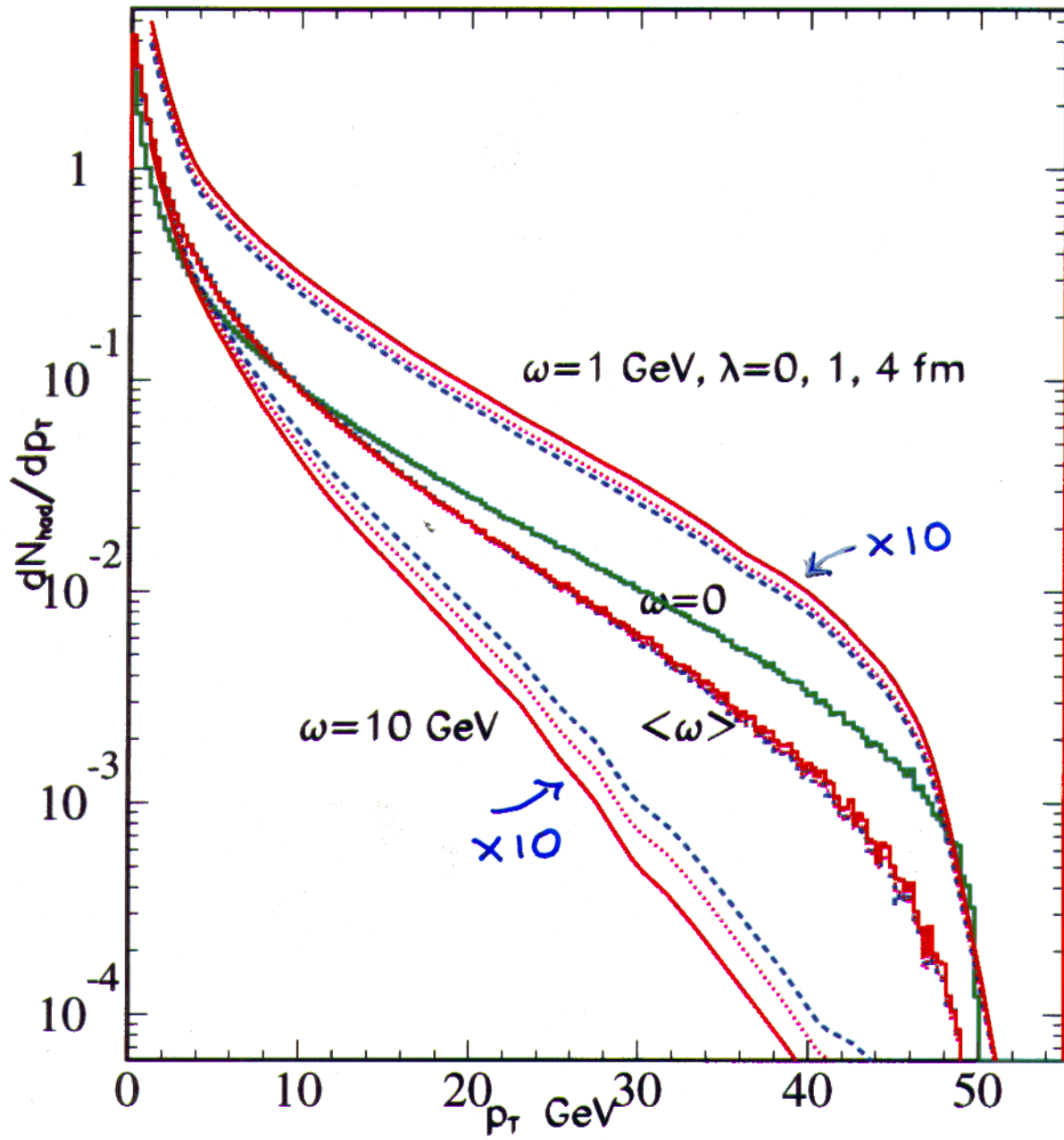
$\log_e k_1^2$



Hard + $N_{sc}=1$ Gluon Spectrum, $E_q=10,50$ GeV



$E_q=50$ GeV – hadrons $N=0$ vs 1



Summary (Part 3)

1) $\omega \frac{dN_g}{d\omega d\vec{k}_\perp}$ computed for $N=0, 1, 2, 3$

2) Interference Hard + B.G. + Cascade limits enhancement to large k_\perp

$$\sqrt{\frac{\omega}{\lambda}} < k_\perp < \omega$$

approximations used
($k_\perp < \omega$) are poor and need improvement!

Bertsch-Gunion induced radiation terms are greatly suppressed

Cascade terms dominate

3) Lund Hadronization of $N=1$ $\frac{dN_1}{dy d\vec{k}_\perp}$ is totally insensitive to modifications at high k_\perp

4) But $\frac{dI}{d\omega}$ is \approx uniformly enhanced \Rightarrow need to go to multiple glue showers

(via modified Altarelli-Parisi evolution)

5) Small N_{scat} Fluctuations (very skewed distrib) important, Ave $\langle \frac{dE}{dx} \rangle$ is not enough