A new model for heavy ion collisions *) at RHIC **)

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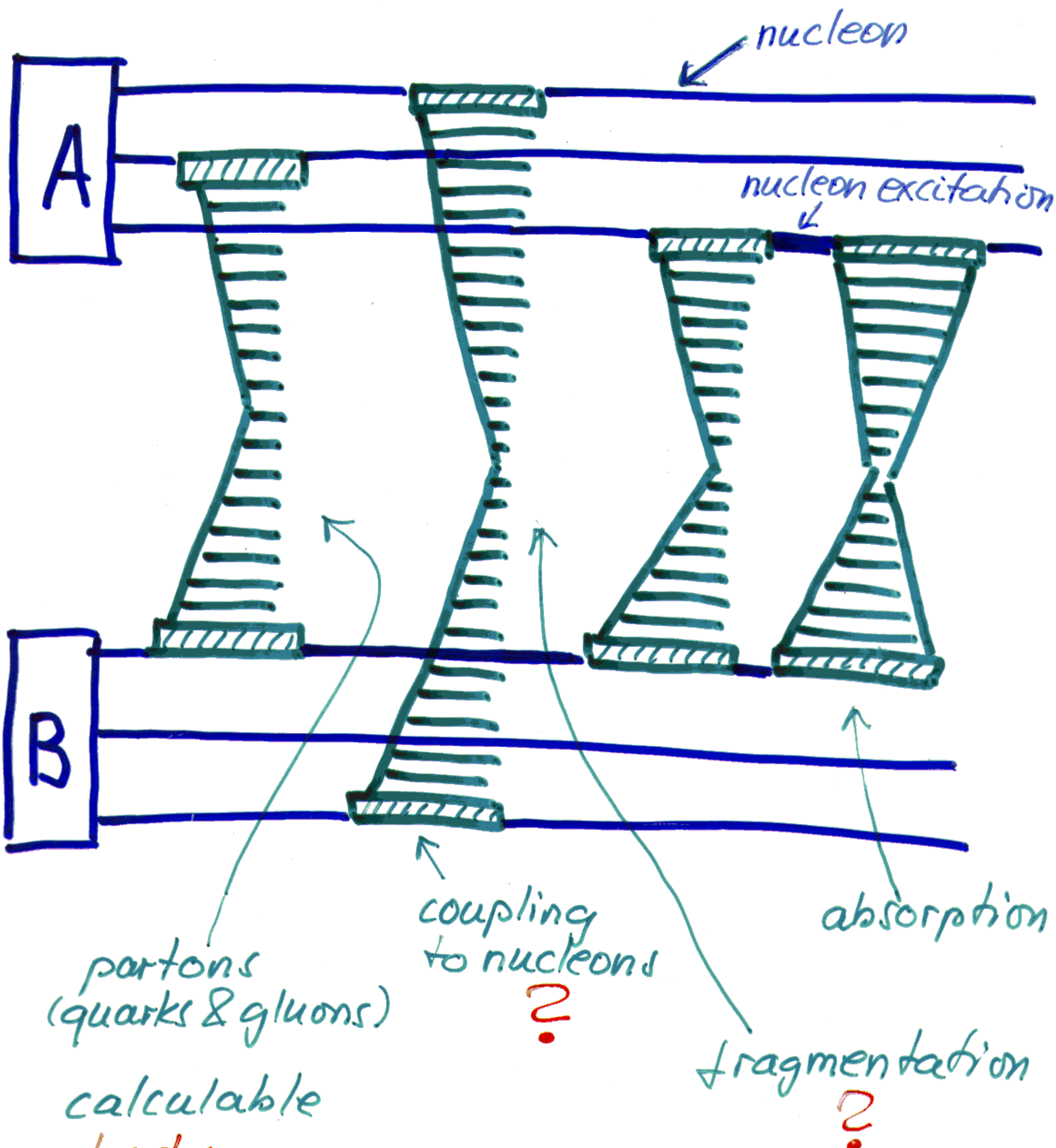
* including pp, pA

** SPS - RHIC

Primary AB Interaction



calculable

but:

cutoffs ?

variables ?

higher order ?

validity of LLA ?

many ???

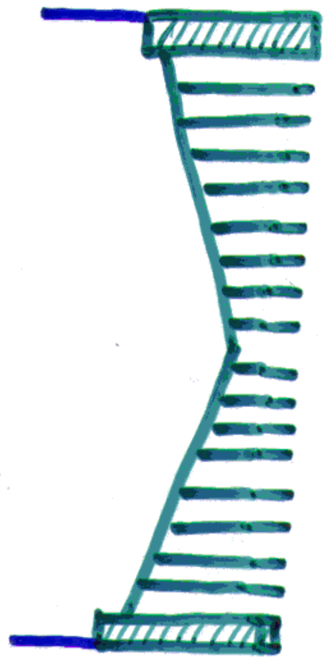
fundamental hypothesis:

$\boxed{\text{Behaviour of high energy interactions is universal}}$

otherwise: hopeless

$\boxed{\text{All the details of nuclear interactions can be determined by: studying simpler systems}}$

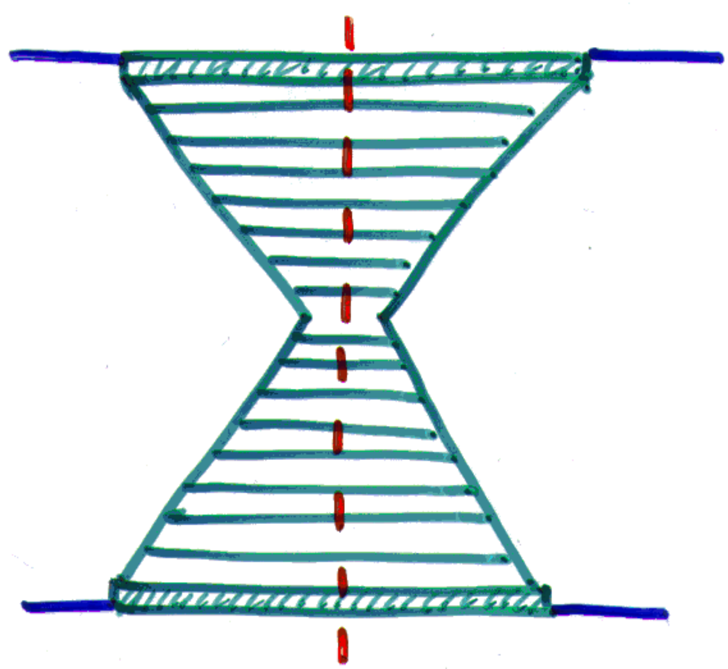
Wanted: understanding
one elementary interaction:



and corresponding cross section

$$= \int |\text{amplitude}|^2 \, d\Phi$$

= cut diagram



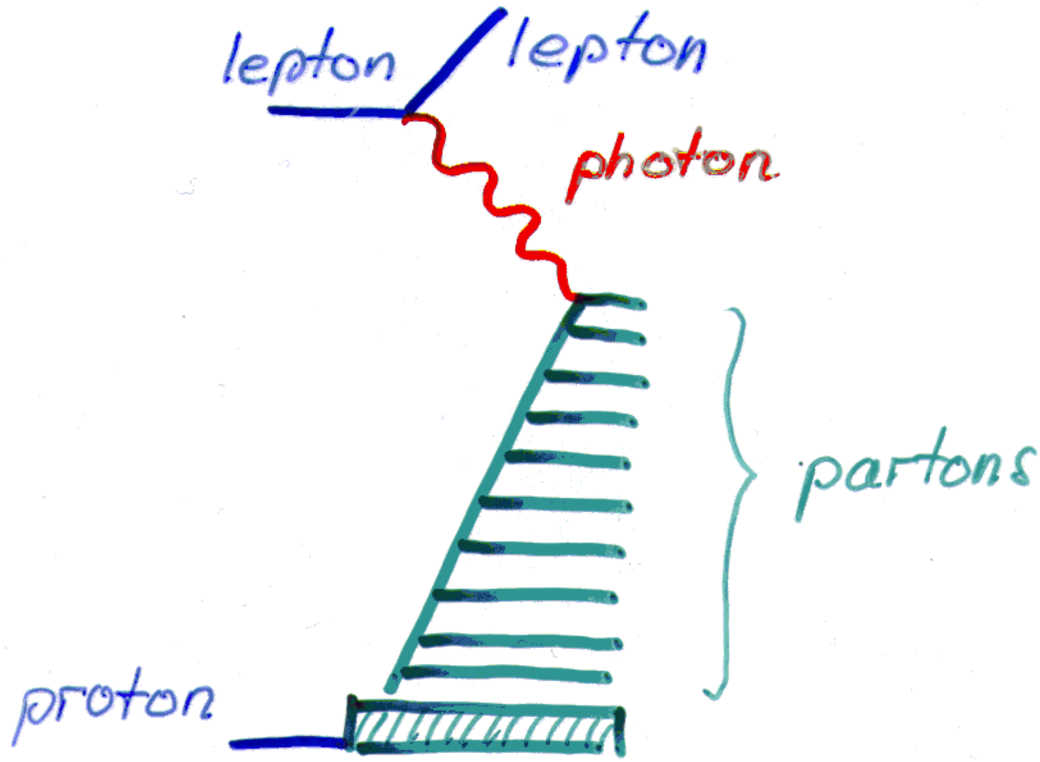
$\widehat{=}$ semihard Pomeron

The Semihard Pomeron

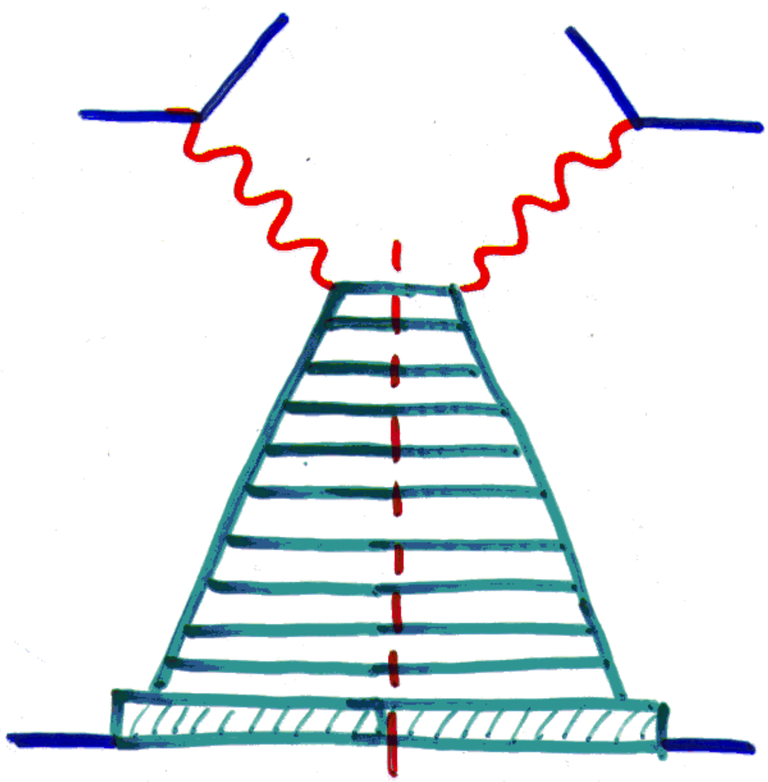
→ studying DIS

deep inelastic lepton-proton scattering (DIS)

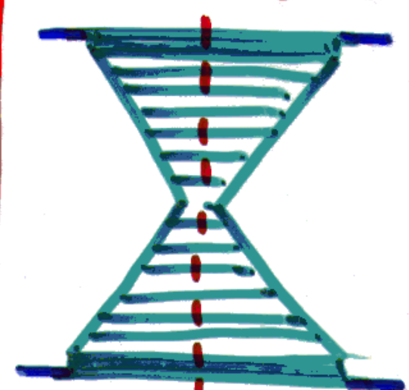
DIS :



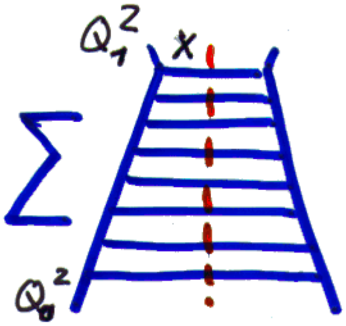
cross section $= \int |\text{amplitude}|^2 =$ cut diagram



compare:
elementary pp:

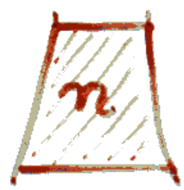


The evolution function E_{QCD}

$$E_{QCD}(Q_0^2, Q_1^2, x) = \sum$$ 

E_{QCD} represents the QCD evolution of a parton cascade from scale Q_0^2 to scale Q_1^2

Calculation of E:

$$E_{QCD} = \lim_{n \to \infty} E_{QCD}^{(n)} =$$ 

$\hat{=}$ ladder with at most n rungs

iterative equation:

calculated initially, for discrete values of Q_0^2, Q_1^2, x

later used via interpolation

Coupling to the Nucleon

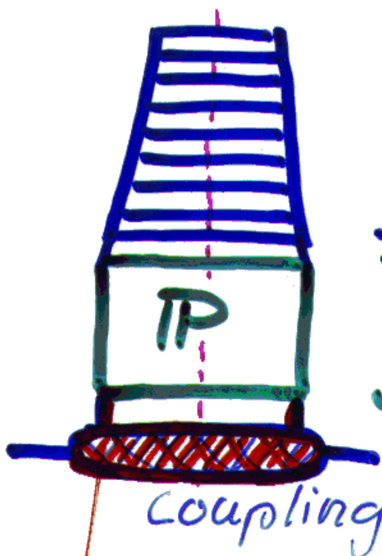
← scale $Q_0^2 \approx 1\,\mathrm{GeV}^2$

} small mom. fraction x

→ large mass$^2 \sim 1/x$

→ Pomeron ($I\!P$)

→ the complete diagram :



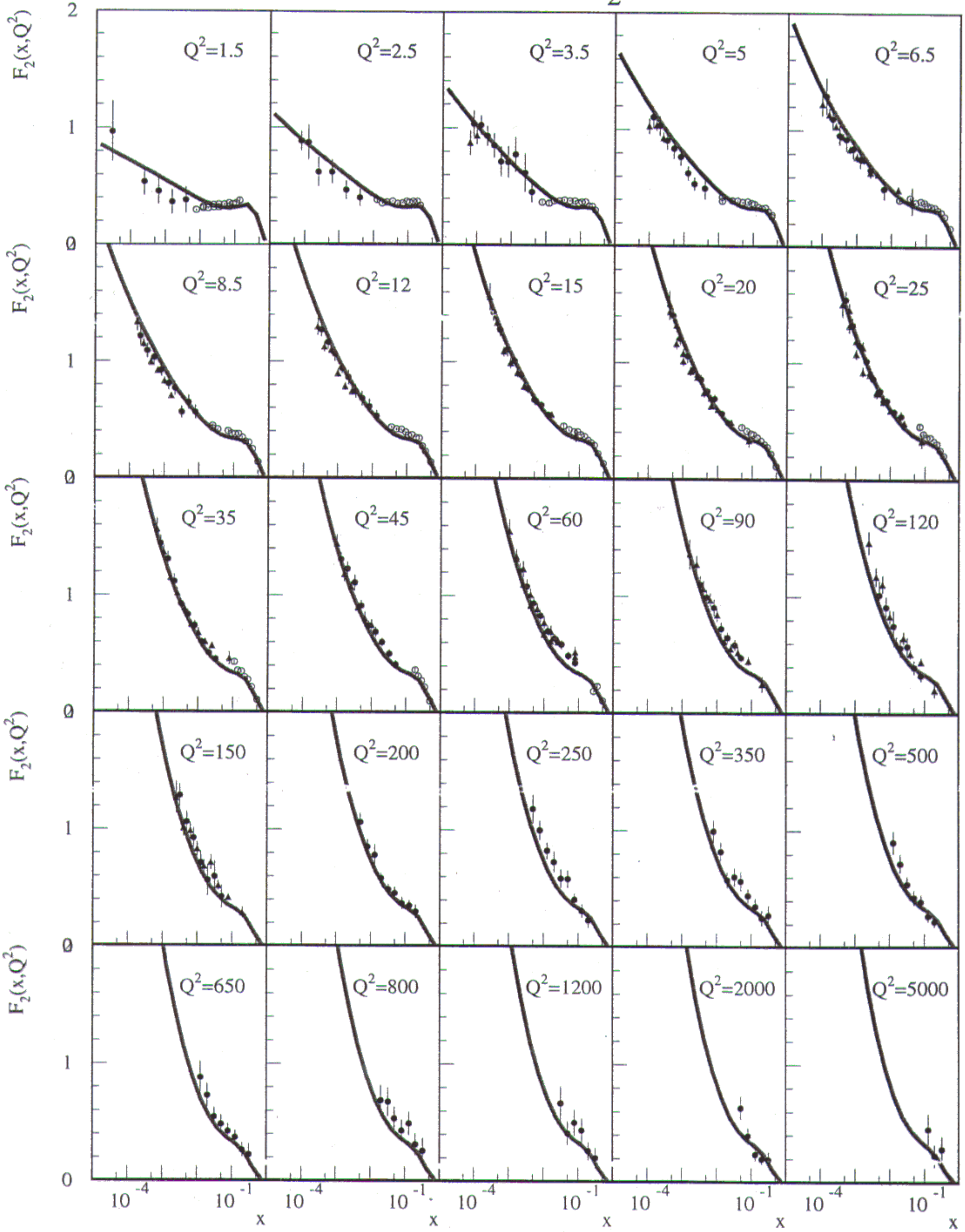
} $\boxed{\text{QCD evolution } E_{QCD}}$

} $\boxed{\text{Regge theory : } E_{soft}(x) \sim x^{-\alpha_{I\!P}}}$

quark-antiquark :

$$\boxed{C_{I\!P}(x) \sim x^{-\alpha_{NP}}}$$

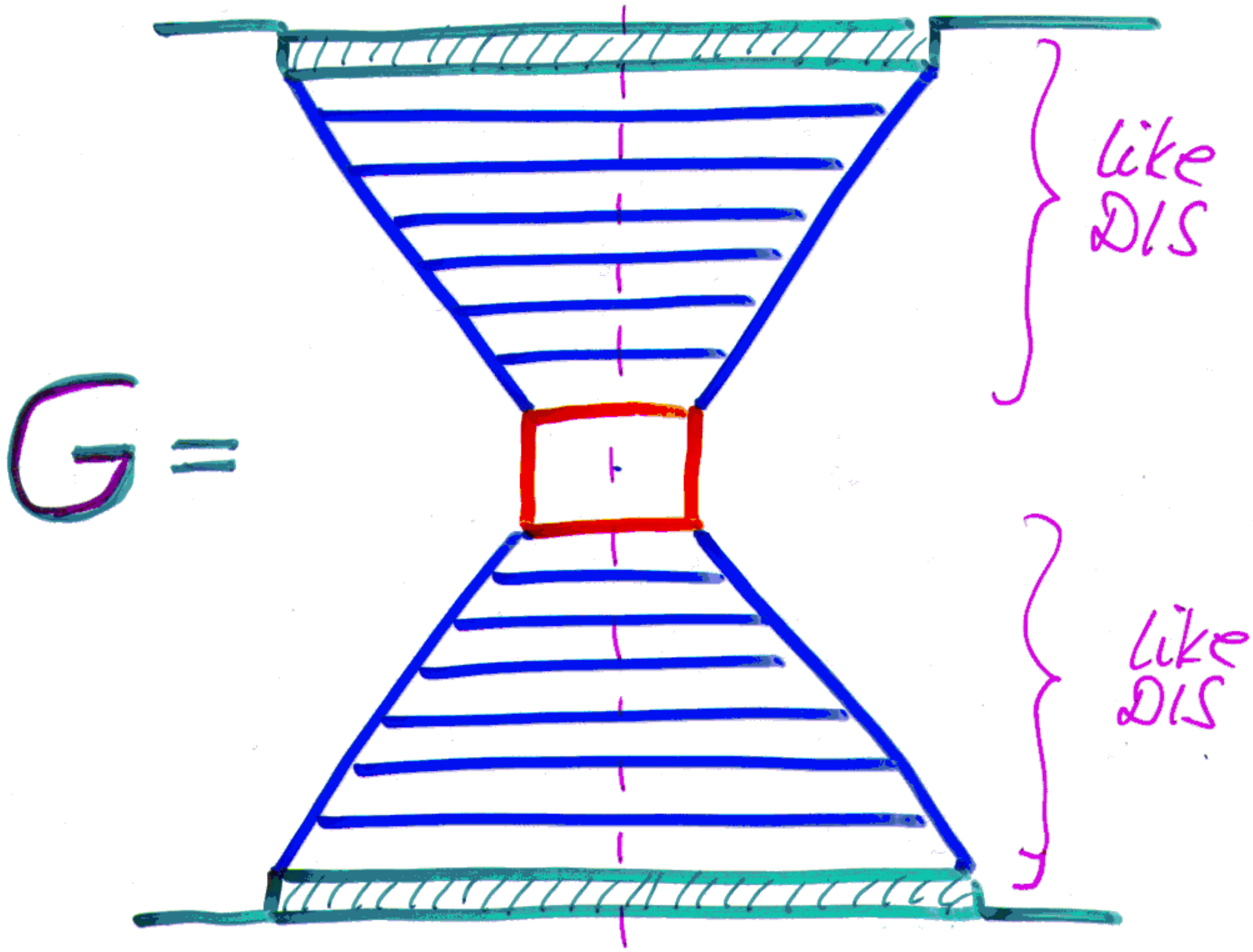
Structure Function $F_2(x,Q^2)$



$\boxed{C_{IP} \sim x^{-0.6}}$

The semihard Pomeron (elementary pp interaction)

based on DIS results



upper and lower "half ladder" identical to DIS diagram

(same parameters, cutoffs, ...)

Low energy behavior

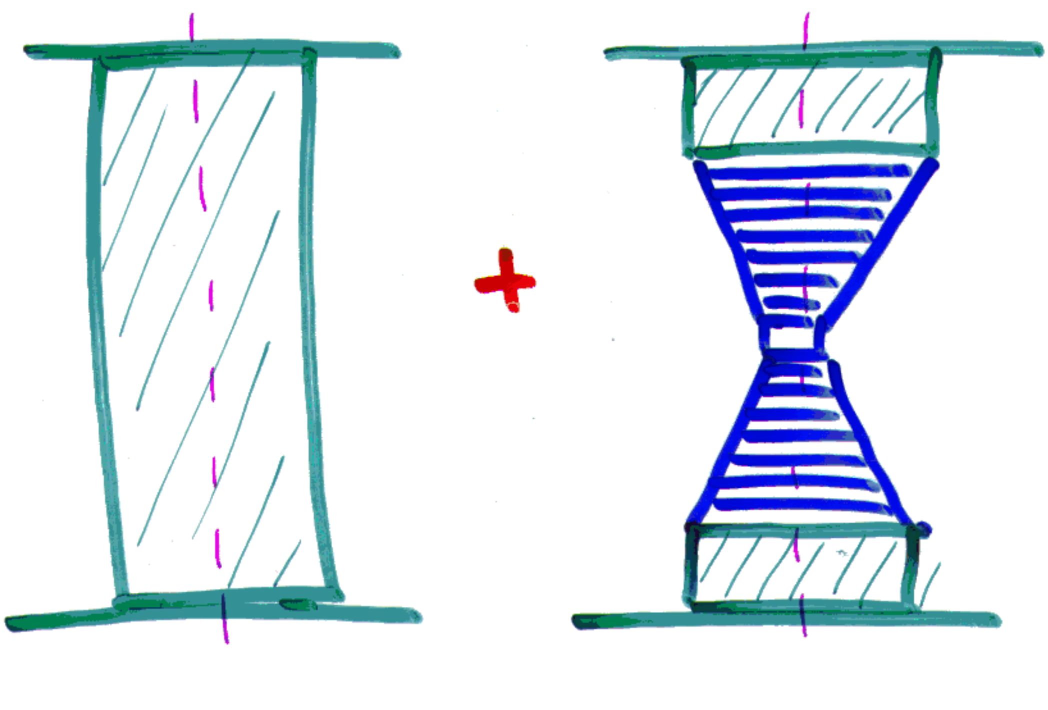
We require a smoth transition towards low energies (SPS)

The dominant contribution should be

$\mathbb{P}_{soft}$ at low energies

$\mathbb{P}_{semi}$ at high energies

$$\rightarrow \quad \boxed{\mathbb{P} = \mathbb{P}_{soft} + \mathbb{P}_{semi}}$$



Multiple scattering in pp and AB collisions

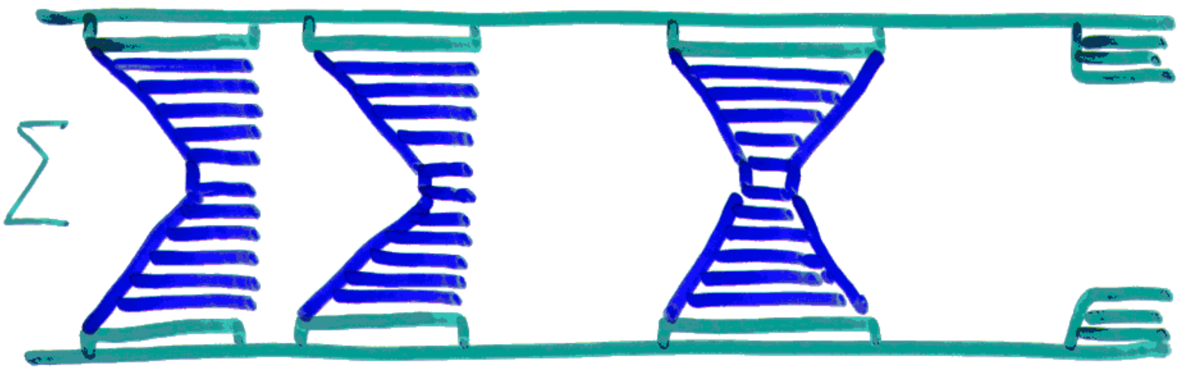
based on
Pomeron exchange

↳ based on DIS results

pp scattering

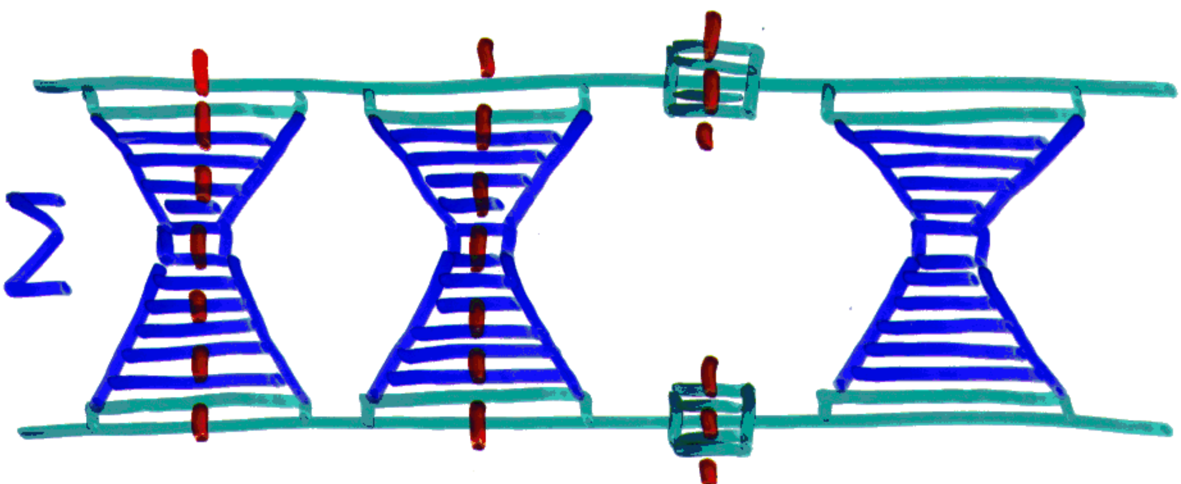
amplitude :

$$A = \sum$$



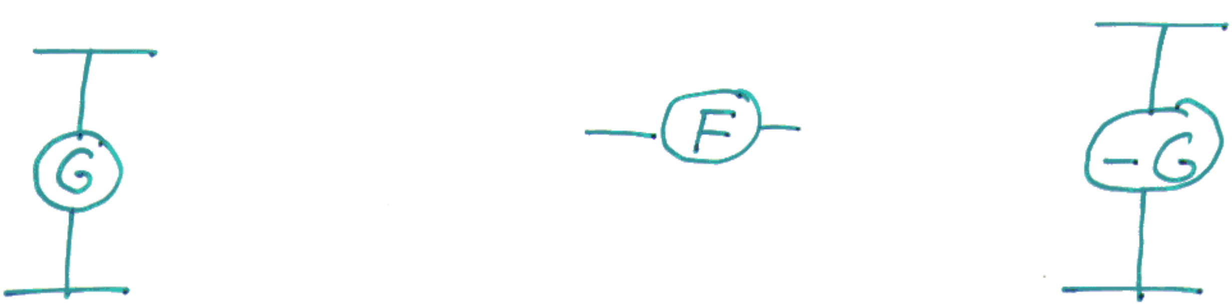
integrated squared amplitude :

$$\int |A|^2 dP = \sum$$



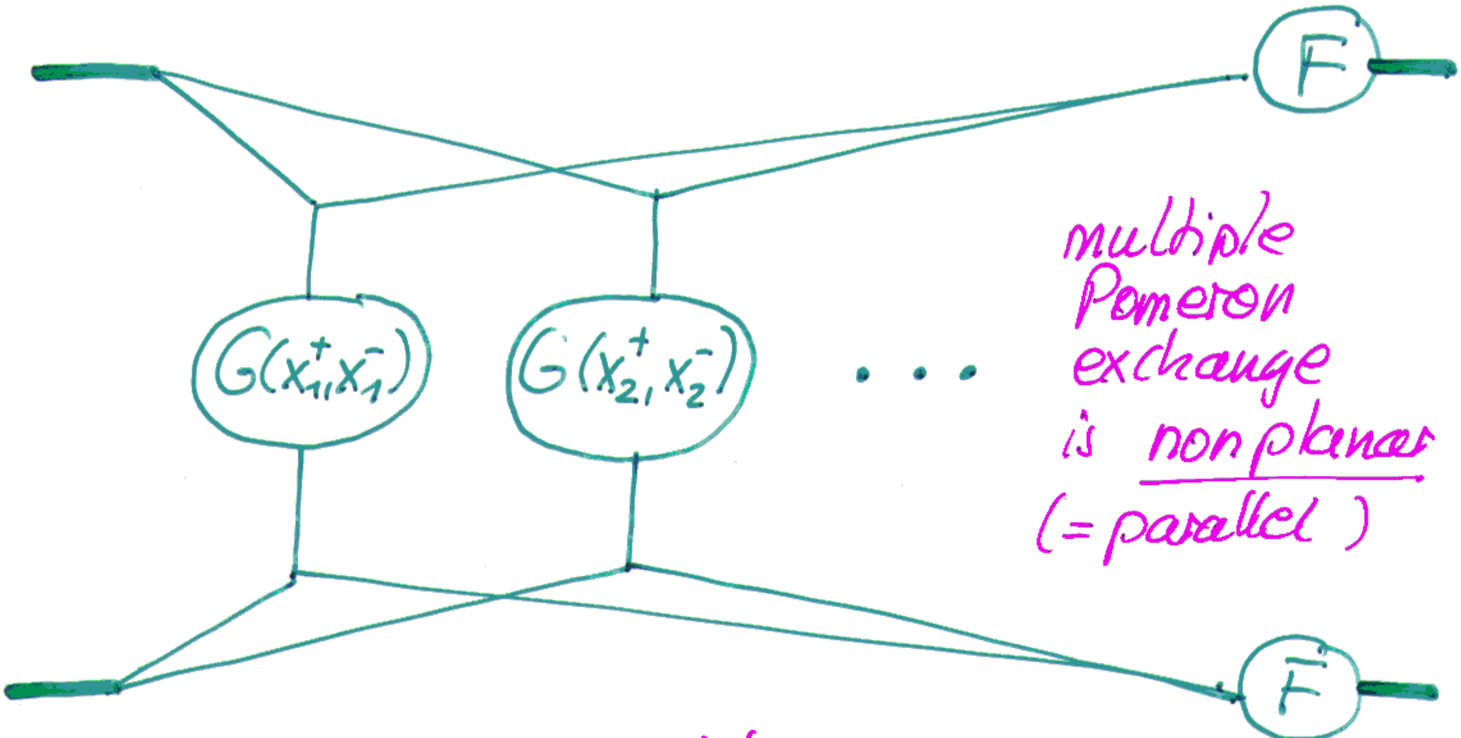
cut Pomeron	cut remnant	uncut Pomeron
$\rightarrow G$	$\rightarrow F$	$\rightarrow -G$

symbols:



important: energy conservation

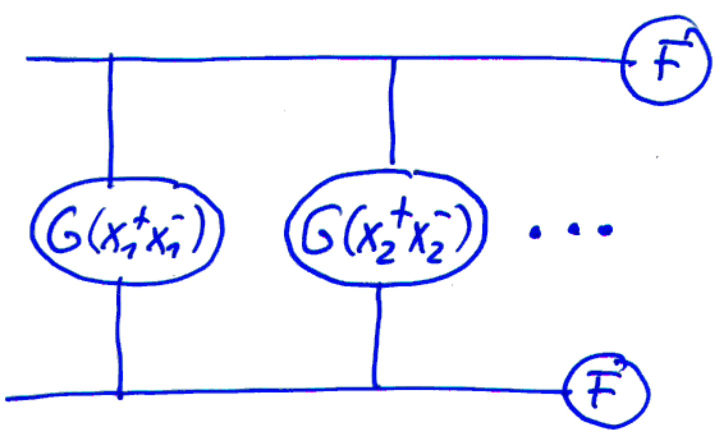
(the total energy must be shared between all Pomerons)



multiple Pomeron exchange is <u>nonplanar</u> (= parallel)

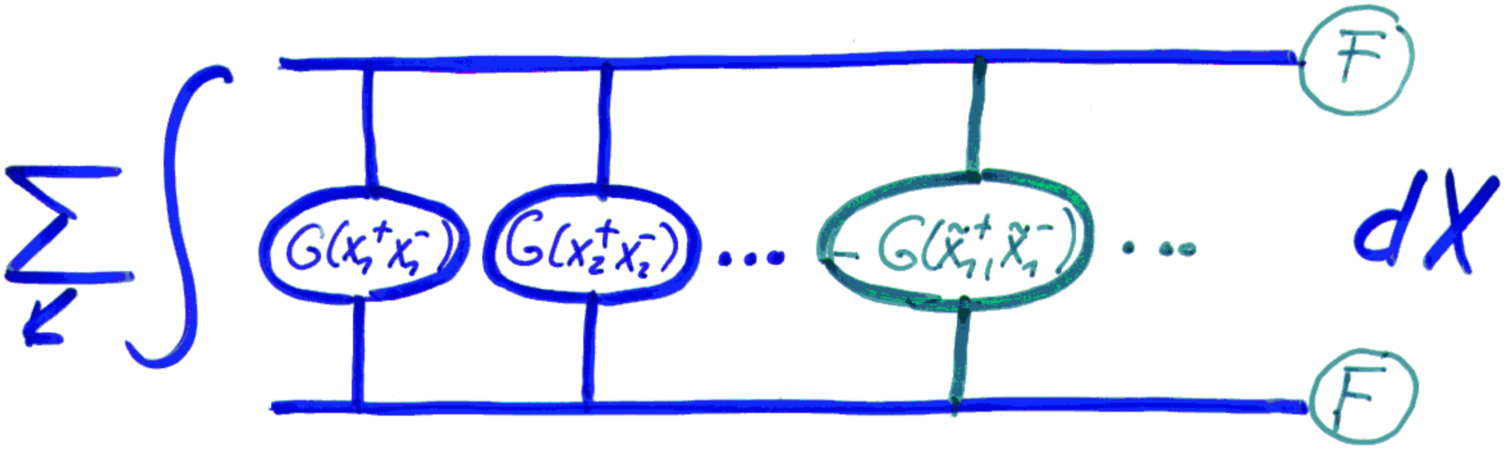
x_i^+, x_i^- : light cone momentum fractions of i^{th} Pomeron

symbolic :



the nonplanar structure is not shown to simplify graphs (but taken into account !)

pp cross section:



$K =$ configuration
(given number of cut and uncut Pomerons)

$X = x_1^+ x_1^- \, x_2^+ x_2^- \cdots \tilde{x}_1^+ \tilde{x}_1^- \cdots$

(light cone momentum fractions)

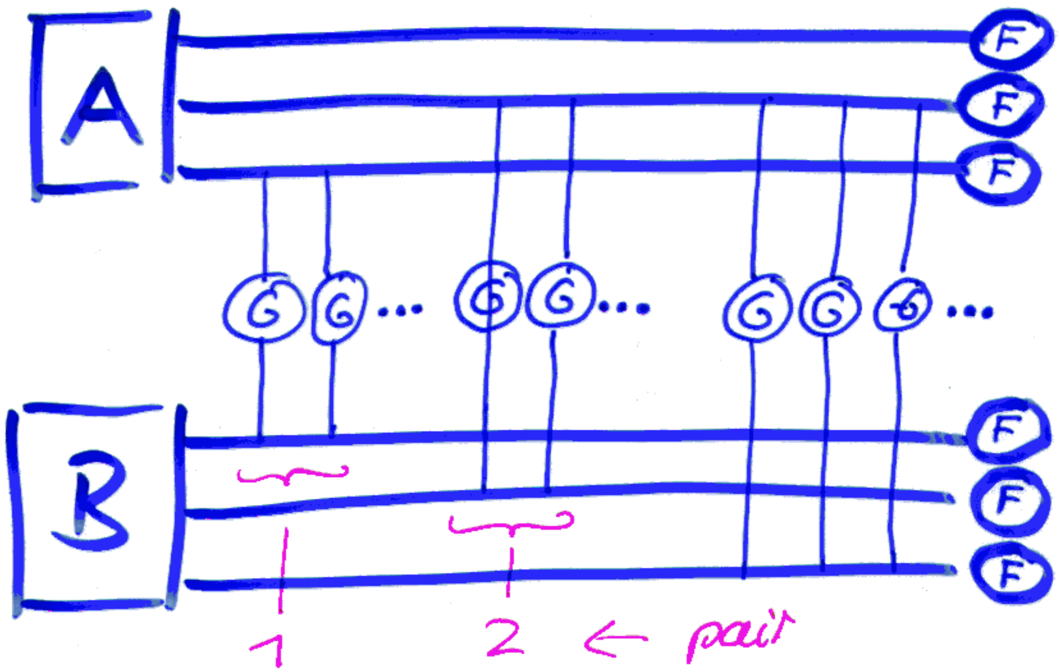
not explicitely shown
(but taken into account):

p_t, flavours, b, s

Nucleus-Nucleus

$$\sigma_{inel} = \int dT_{AB} \sum_{M} \int dX$$

$\times$



$$G = G(x_i^+, x_i^-, p_{ti}, t_i, b_k, s)$$

$$X = x_{11}^+ x_{11}^- x_{12}^+ x_{12}^- \cdots$$

Pomeron 1

pair 1

M = interaction type
(which pairs involved,
number of IP's per pair)

$$\sigma_{inel} = \sum \int \Omega(M, X)\, dX$$

The complete formula :

avec le paramètre d'impact b entre les deux noyaux. La section efficace pour l'échange de $l_1 \dots l_N$ Pomerons non coupés ($l = \sum_k l_k$) et $m_1 \dots m_N$ Pomerons coupés ($m = \sum_k m_k$) dans N interactions avec au total $n = m + l$ Pomerons s'écrit

$$
\begin{aligned}
&\tilde{\sigma}^{i_1 j_1 \dots i_N j_N}_{l_1 \dots l_N, m_1 \dots m_N}(s,b) = C^{n-N} \sum_{I_{\mathrm{el}} I_{\mathrm{eb}}} \int dT_{AB} \\
&\quad \int \Big\{ \prod_{k=1}^{N} \prod_{\nu_k=1}^{n_k} \sum_{q^+_{\nu_k} q^-_{\nu_k}} dx^+_{\nu_k}\, dx^-_{\nu_k}\, d^2k^+_{\nu_k}, d^2k^-_{\nu_k} \Big\} \\
&\quad \int \Big\{ \prod_{i=1}^{A} \sum_{e_{i,\mathrm{a}} q_{i,\mathrm{a}}} dx^+_{i,\mathrm{a}}\, dx^-_{i,\mathrm{a}}\, d^2k_{i,\mathrm{a}} \Big\} \int \Big\{ \prod_{j=1}^{B} \sum_{e_{j,\mathrm{b}} q_{j,\mathrm{b}}} dx^+_{j,\mathrm{b}}\, dx^-_{j,\mathrm{b}}\, d^2k_{j,\mathrm{b}} \Big\} \\
&\quad \int \Big\{ \prod_{i=1}^{A} \prod_{j=1}^{B} d\hat{x}^+_{ij}\, d\hat{x}^-_{ij}\, d^2\hat{k}_{ij} \Big\} \\
&\quad \prod_{k \in I_{\mathrm{inel}}} \frac{1}{l_k! m_k!} \\
&\quad \prod_{\mu_k=1}^{m_k} \tilde{G}(q^+_{\mu_k}, q^-_{\mu_k}, x^+_{\mu_k}, x^-_{\mu_k}, k^+_{\mu_k}, k^-_{\mu_k}, s, b_k) \prod_{\lambda_k=m_k+1}^{m_k+l_k} -\tilde{G}(q^+_{\lambda_k}, q^-_{\lambda_k}, x^+_{\lambda_k}, x^-_{\lambda_k}, k^+_{\lambda_k}, k^-_{\lambda_k}, s, b_k) \\
&\quad \prod_{k \in I_{\mathrm{diff}}} \Big(1 - \frac{1}{C}\Big)\Big(1 - \frac{1}{2^{l_k-1}}\Big) \frac{1}{l_k!} \prod_{\lambda_k=1}^{l_k} -\tilde{G}(q^+_{\lambda_k}, q^-_{\lambda_k}, x^+_{\lambda_k}, x^-_{\lambda_k}, k^+_{\lambda_k}, k^-_{\lambda_k}, s, b_k) \\
&\quad \prod_{k \in I_{\mathrm{el}}} \frac{1}{C}\Big(1 - \frac{1}{2^{l_k-1}}\Big) \frac{1}{l_k!} \prod_{\lambda_k=1}^{l_k} -\tilde{G}(q^+_{\lambda_k}, q^-_{\lambda_k}, x^+_{\lambda_k}, x^-_{\lambda_k}, k^+_{\lambda_k}, k^-_{\lambda_k}, s, b_k) \\
&\quad \prod_{k \in I_{\mathrm{eb}}} \frac{1}{2^{l_k-1}} \frac{1}{l_k!} \prod_{\lambda_k=1}^{l_k} -\tilde{G}(q^+_{\lambda_k}, q^-_{\lambda_k}, x^+_{\lambda_k}, x^-_{\lambda_k}, k^+_{\lambda_k}, k^-_{\lambda_k}, s, b_k) \\
&\quad \prod_{i=1}^{A} F_{\mathrm{a}}(e_{i,\mathrm{a}}, x^+_{i,\mathrm{a}}, x^-_{i,\mathrm{a}}, k_{i,\mathrm{a}})\, \Theta\Big(x^+_{i,\mathrm{a}} - \sum_{\substack{k'=1 \\ \mathrm{proj}(k')=i}}^{N} \sum_{\lambda_{k'}=m_{k'}+1}^{m_{k'}+l_{k'}} x^+_{\lambda_{k'}}\Big) \\
&\quad \delta\Big(x^+_{i,\mathrm{a}} + \sum_{\substack{j=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{B} \hat{x}^+_{ij} + \sum_{\substack{k'=1 \\ \mathrm{proj}(k')=i}}^{N} \sum_{\mu_{k'}=1}^{m_{k'}} x^+_{\mu_{k'}} - 1\Big)\, \delta\Big(x^-_{i,\mathrm{a}} - \sum_{\substack{j=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{B} \hat{x}^-_{ij}\Big) \\
&\quad \delta^2\Big(k_{i,\mathrm{a}} + \sum_{\substack{j=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{B} \hat{k}_{ij} + \sum_{\substack{k'=1 \\ \mathrm{proj}(k')=i}}^{N} \sum_{\mu_{k'}=1}^{m_{k'}} k^+_{\mu_{k'}}\Big)\, \delta_{q_{i,\mathrm{a}} (q_{i,\mathrm{a},0} - \sum^{N}_{\substack{k'=1 \\ \mathrm{proj}(k')=i}} \sum^{m_{k'}}_{\mu_{k'}=1} q^+_{\mu_{k'}})} \\
&\quad \prod_{j=1}^{B} F_{\mathrm{b}}(e_{j,\mathrm{b}}, x^+_{j,\mathrm{b}}, x^-_{j,\mathrm{b}}, k_{j,\mathrm{b}})\, \Theta\Big(x^-_{j,\mathrm{b}} - \sum_{\substack{k'=1 \\ \mathrm{targ}(k')=j}}^{N} \sum_{\lambda_{k'}=m_{k'}+1}^{m_{k'}+l_{k'}} x^-_{\lambda_{k'}}\Big) \\
&\quad \delta\Big(x^-_{j,\mathrm{b}} + \sum_{\substack{i=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{A} \hat{x}^-_{ij} + \sum_{\substack{k'=1 \\ \mathrm{targ}(k')=j}}^{N} \sum_{\mu_{k'}=1}^{m_{k'}} x^-_{\mu_{k'}} - 1\Big)\, \delta\Big(x^+_{j,\mathrm{b}} - \sum_{\substack{i=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{A} \hat{x}^+_{ij}\Big) \\
&\quad \delta^2\Big(k_{j,\mathrm{b}} + \sum_{\substack{i=1 \\ \mathrm{coll}(i,j) \in I_{\mathrm{in,di}}}}^{A} \hat{k}_{ij} + \sum_{\substack{k'=1 \\ \mathrm{targ}(k')=j}}^{N} \sum_{\mu_{k'}=1}^{m_{k'}} k^-_{\mu_{k'}}\Big)\, \delta_{q_{j,\mathrm{b}} (q_{j,\mathrm{b},0} - \sum^{N}_{\substack{k'=1 \\ \mathrm{targ}(k')=j}} \sum^{m_{k'}}_{\mu_{k'}=1} q^-_{\mu_{k'}})},
\end{aligned}
\tag{3.123}
$$

everything should be based on the expression for Ω

possible but difficult

so far:

problem solved with additional assumptions

1st step for complete solution (work in progress) approximations :

- uncut Pomerons summed up ignoring energy conservation

- separation of M and X dependence

We write

$$\sigma_{inel} = \sum_M \Omega_1(M) \int \Omega_2(M,X)\, dX$$

where Ω_1 represents Ω in case of ignoring energy conservation

M is generated according to Ω_1
X is then generated according to $\Omega_2(M,X)$

same numerical techniques used for exact and approximate solution

approximate treatment = test case for the exact treatment

A new numerical technique

to treat multiple scattering

in pp and AB

Using Monte Carlo technique, we need to generate

configurations K acc. to $\Omega(K)$

with $K = (M, X)$ — M = interaction type
— X = momentum fractions of all Pomerons

$\Omega(K)$ is a given distribution (as discussed)

Problem: the configuration space is enormous:

anything between 1 and $A \cdot B$ nucleon-nucleon interactions is possible
any number of Pomerons may be involved for each of these ...

Solution: Monte Carlo via Markov chains

(PhD thesis of M. Hladik)

La configuration sur réseau

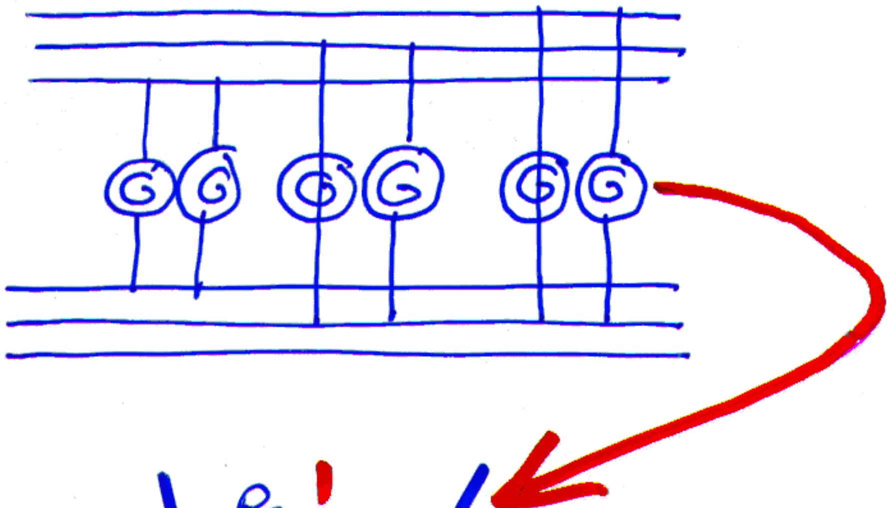
inspirée par les modèles des spins :

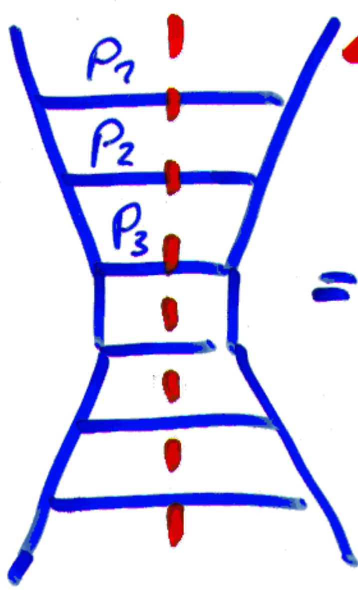
$k \quad m_k$	1	2	$\dots$	$\dots$	ρ	$\dots$	$\dots$	$\dots$	M
1	0	$\mathbb{P}$	$\mathbb{R}$	0	0	0	$\mathbb{P}$	0	0
2	$\mathbb{P}$	0	0	$\mathbb{P}$	0	$\mathbb{P}$	0	0	0
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
r	$\mathbb{R}$	$\mathbb{P}$	0	0	$\boxed{\mathbb{P}}$	$\mathbb{R}$	$\mathbb{P}$	0	0
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
$\vdots$					$\vdots$				
AB	0	0	$\mathbb{P}$	$\mathbb{R}$	0	$\mathbb{P}$	0	0	0

Parton configurations

again
based on $\Omega(K)$

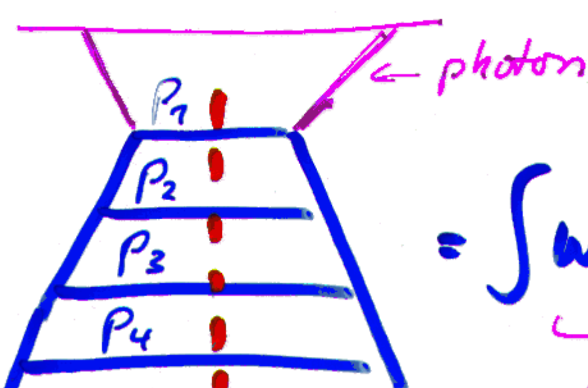
One has to consider the Pomeron substructure:





$$= \int \underbrace{W(p_1, p_2, \cdots)}_{\substack{\text{probability} \\ \text{distribution} \\ \text{for partons } p_1, p_2 \cdots}} dp_1 \, dp_2 \cdots$$

same procedure in DIS:



$$= \int \underbrace{W(p_1, p_2 \cdots)}_{\substack{\text{prob. distr.} \\ \text{for partons}}} dp_1 \, dp_2 \cdots$$

Hadron Production

parton configuration

$\Downarrow$

phenomenological procedure

$\Downarrow$

hadron configuration

* not a fundamental theory
* intelligent parametrisation based on symmetries

Hadronisation via "Kinky Strings"

phenomenological model ($\neq$ QCD)

- data: fragmentation roughly one-dimensional
- theory: any procedure should be Lorentz-invariant

$\rightsquigarrow$ Relativistic string

+ some reasonable assumptions (symmetries)

$\rightsquigarrow$ dynamics (up to few parameters)

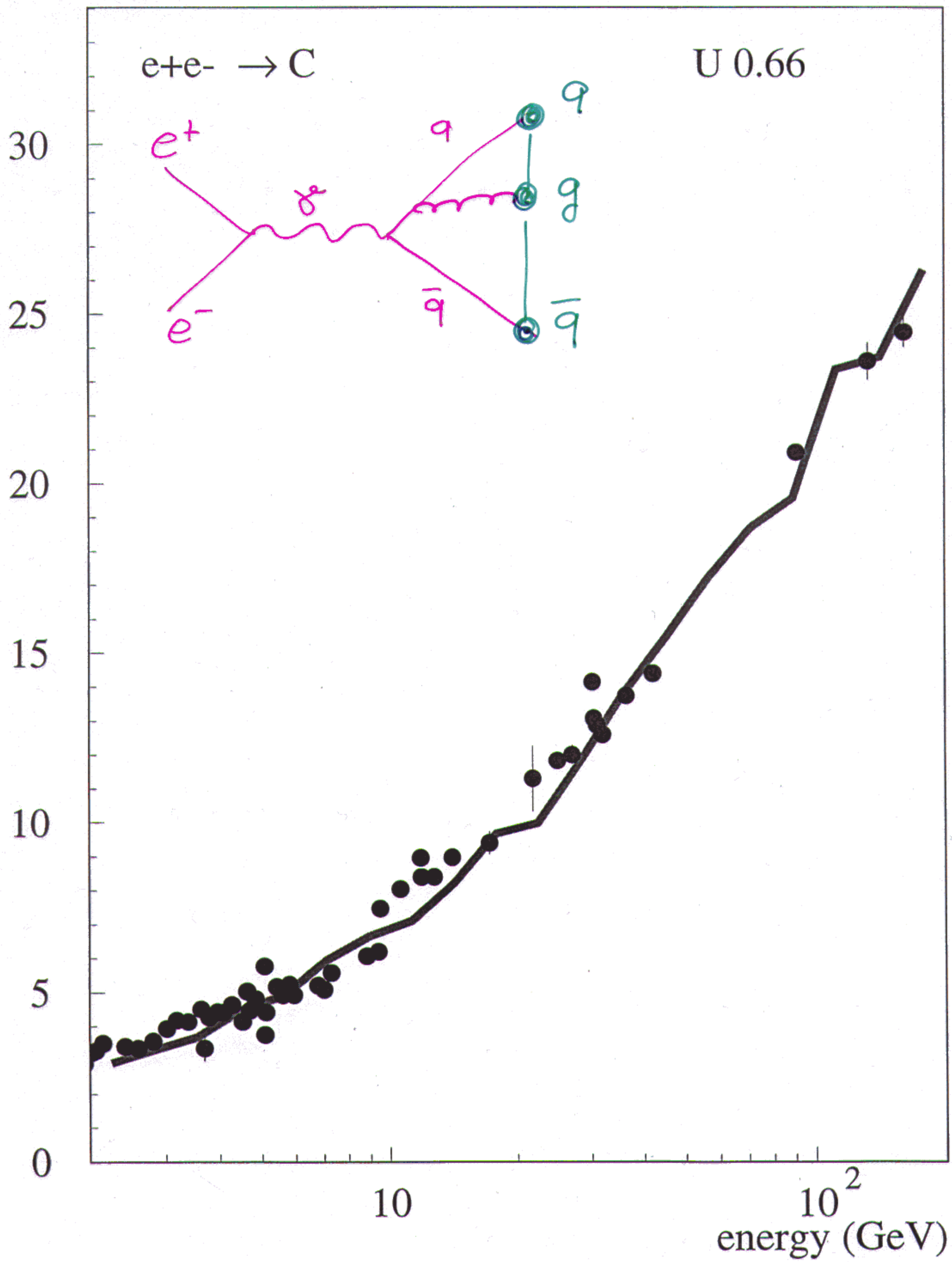
two steps:

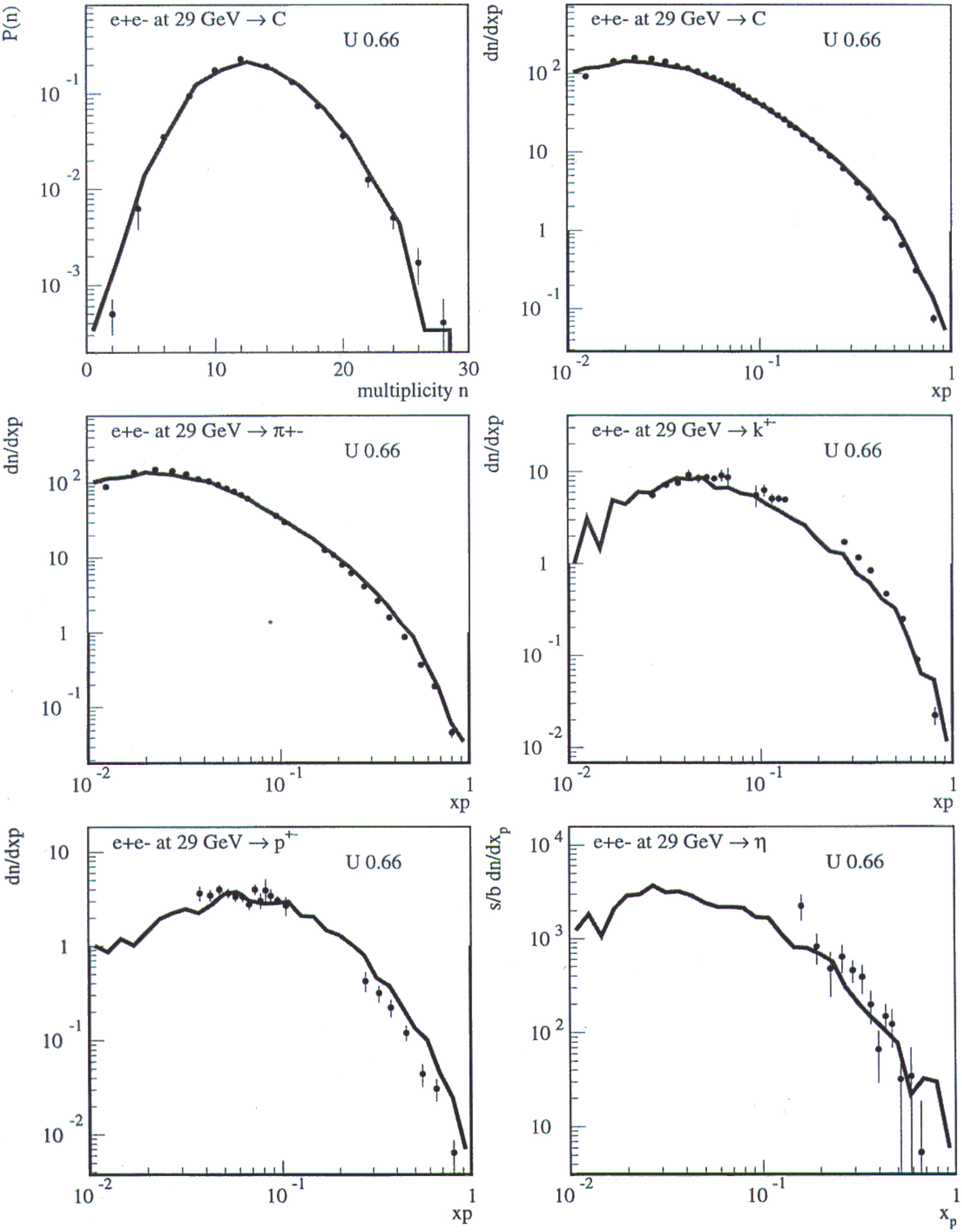
1) string formation
2) string fragmentation (breaking)

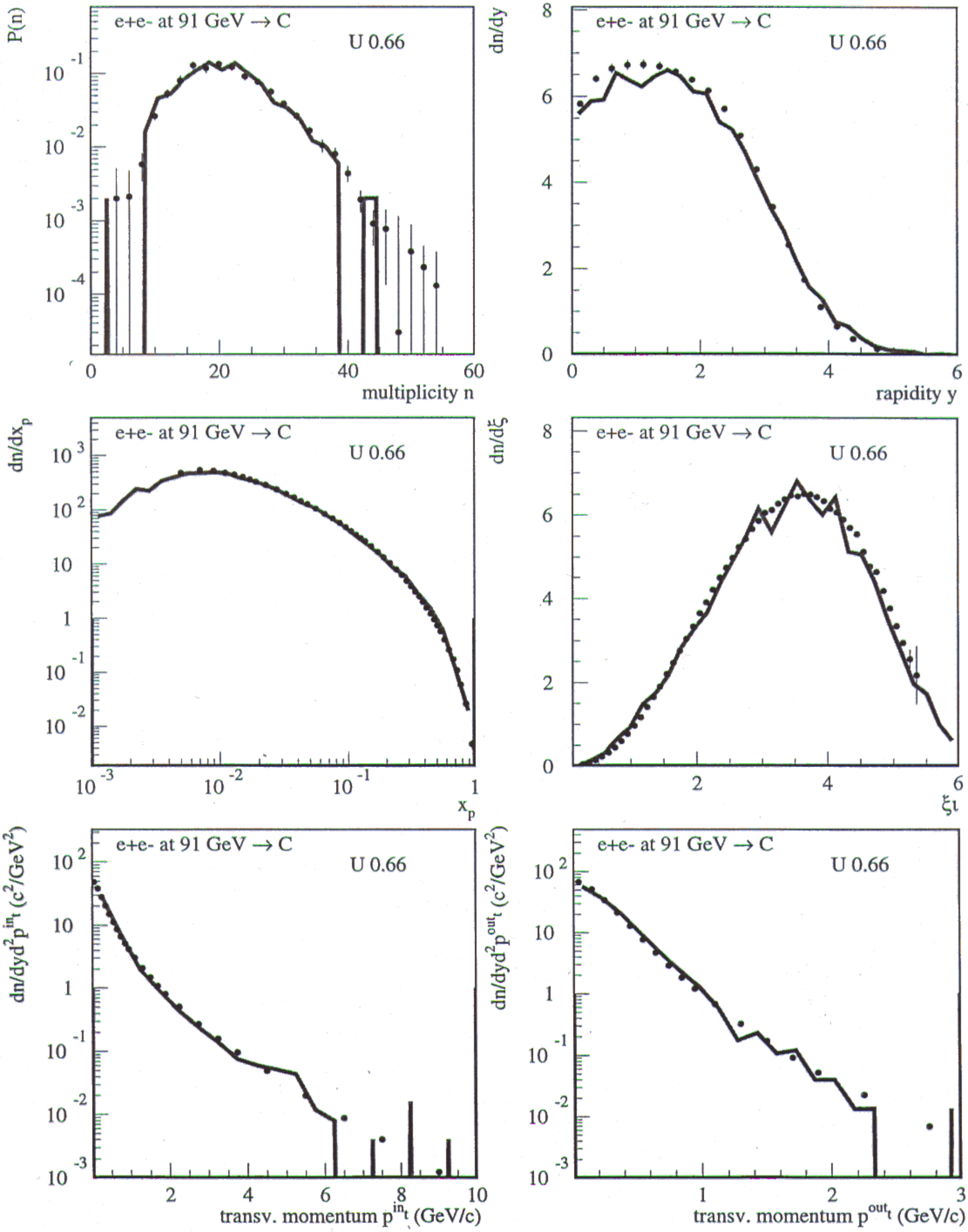
$\boxed{\rightarrow \text{H.J. Drescher}}$

inirj 5

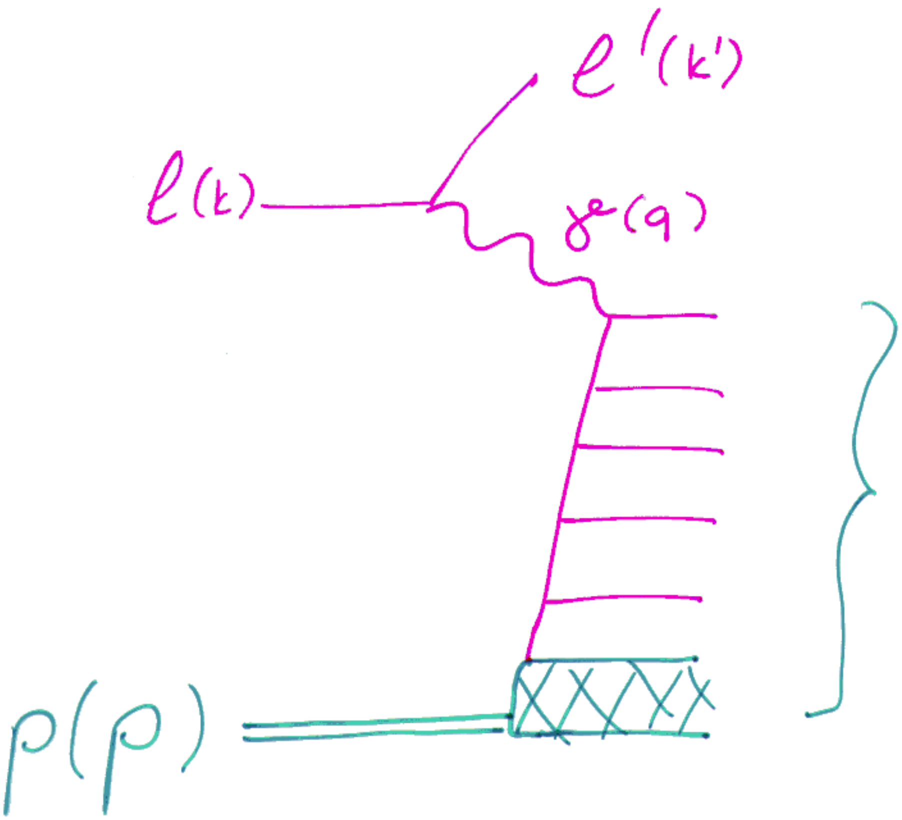
fragmentation parameters are fixed using electron-positron (e^+e^-) annihilation




P(n)
e+e- at 29 GeV → C
U 0.66
multiplicity n
dn/dxp
e+e- at 29 GeV → C
U 0.66
xp
dn/dxp
e+e- at 29 GeV → π+-
U 0.66
xp
dn/dxp
e+e- at 29 GeV → k+-
U 0.66
xp
dn/dxp
e+e- at 29 GeV → p+-
U 0.66
xp
s/b dn/dx_p
e+e- at 29 GeV → η
U 0.66
x_p


P(n)
e+e- at 91 GeV → C
U 0.66
multiplicity n
dn/dy
e+e- at 91 GeV → C
U 0.66
rapidity y
dn/dx_p
e+e- at 91 GeV → C
U 0.66
x_p
dn/dξ
e+e- at 91 GeV → C
U 0.66
ξι
dn/dyd²p^in_t (c²/GeV²)
e+e- at 91 GeV → C
U 0.66
transv. momentum p^in_t (GeV/c)
dn/dyd²p^out_t (c²/GeV²)
e+e- at 91 GeV → C
U 0.66
transv. momentum p^out_t (GeV/c)

Results DIS

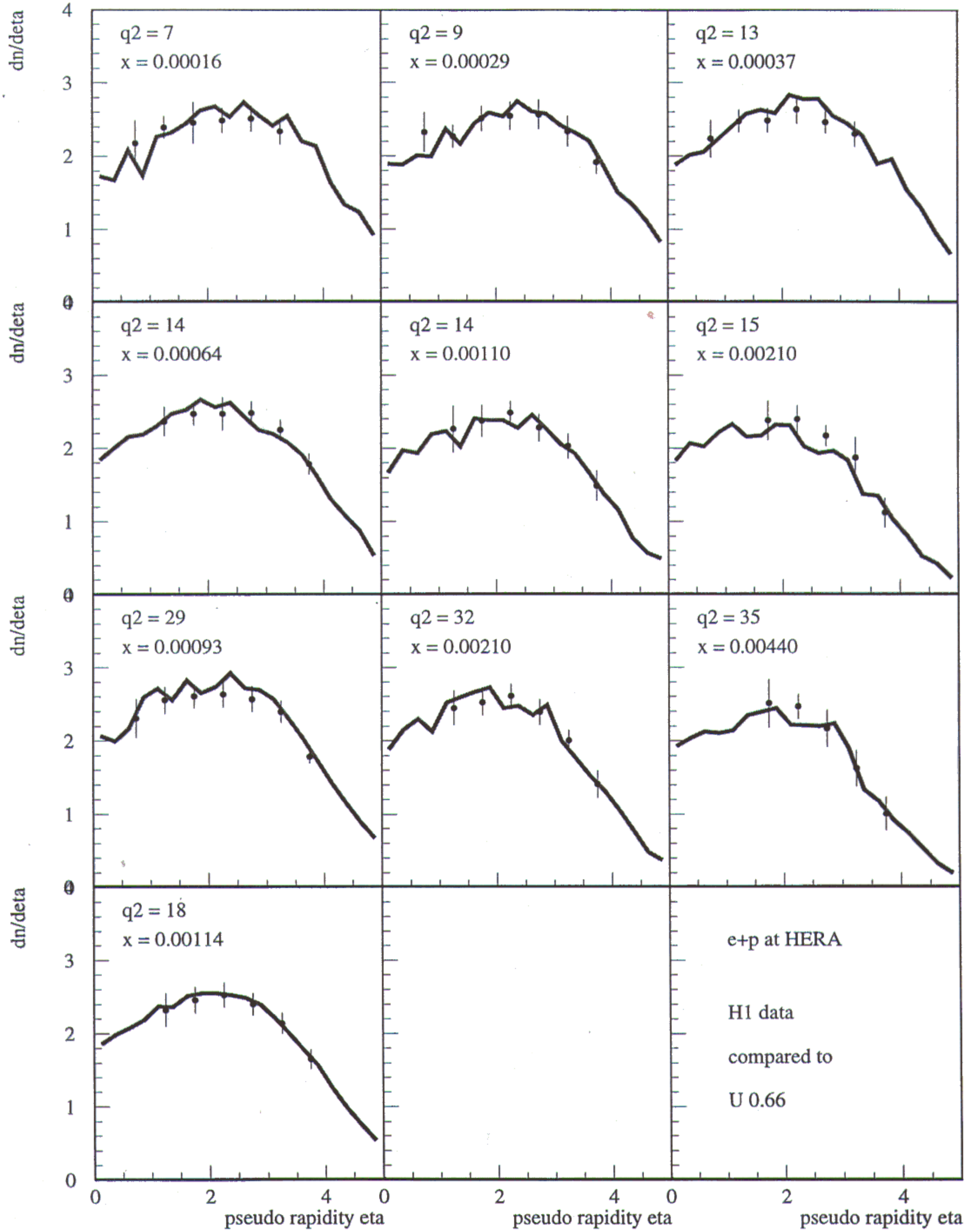


$q = k - k'$
$Q^2 = -q^2$
$x = 2pq/Q^2$
energy W

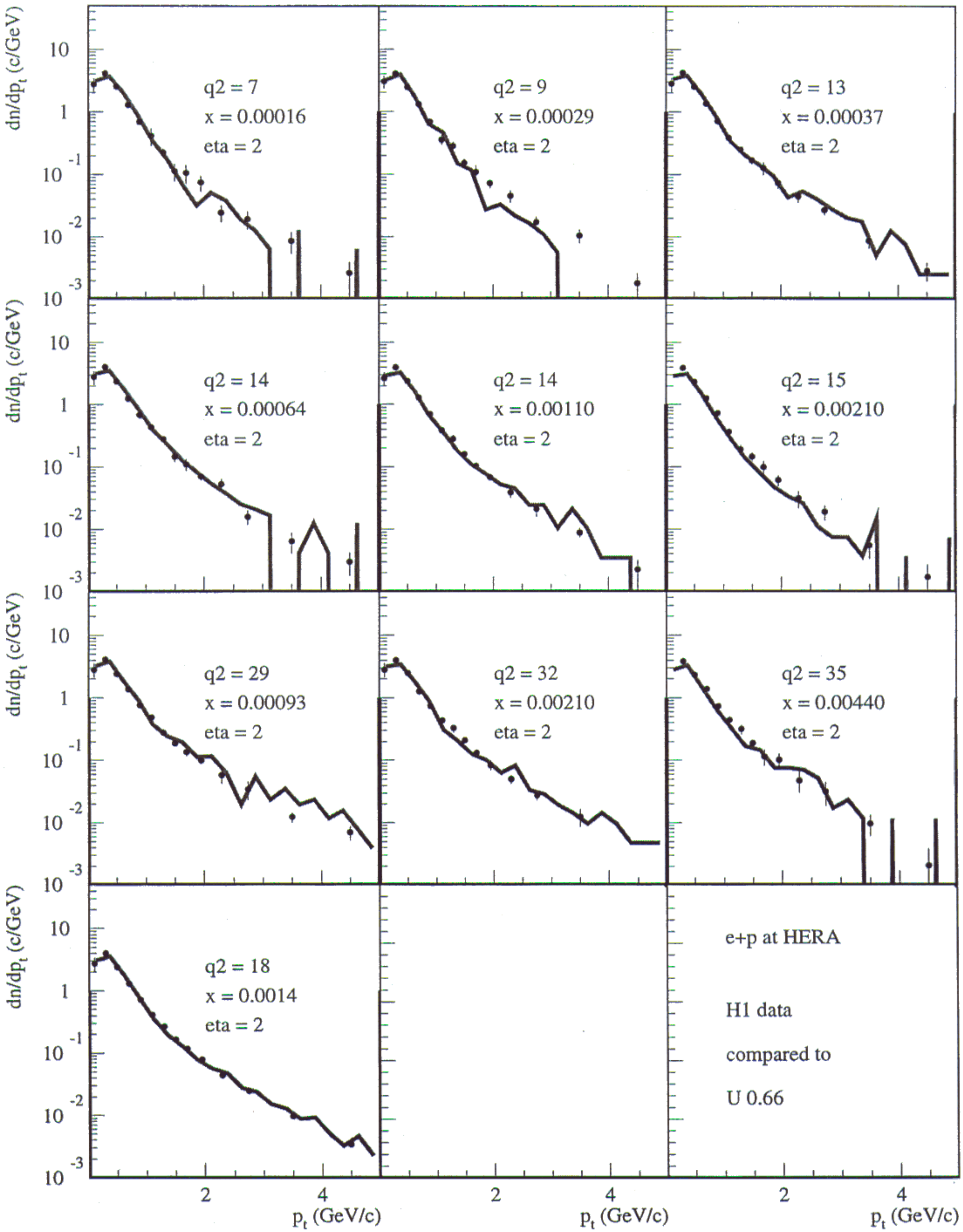
$$W^2 = (q+p)^2$$

$$\xrightarrow{\text{large } \frac{1}{x}} \quad W^2 \approx \frac{Q^2}{x}$$

$\rightarrow$ small x $\leftrightarrow$ large energy



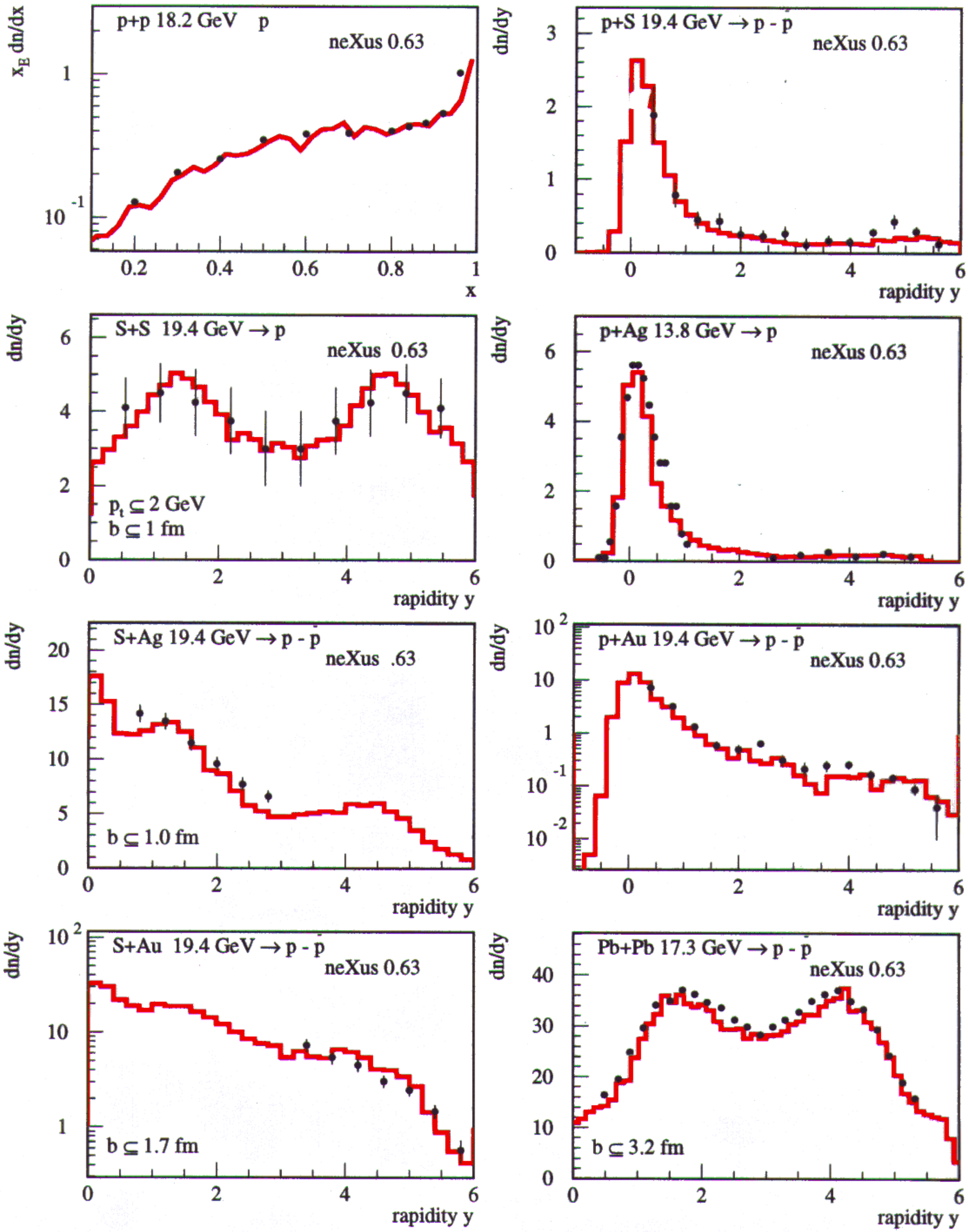
q2 = 7
x = 0.00016
q2 = 9
x = 0.00029
q2 = 13
x = 0.00037
q2 = 14
x = 0.00064
q2 = 14
x = 0.00110
q2 = 15
x = 0.00210
q2 = 29
x = 0.00093
q2 = 32
x = 0.00210
q2 = 35
x = 0.00440
q2 = 18
x = 0.00114
e+p at HERA
H1 data
compared to
U 0.66
dn/deta
pseudo rapidity eta



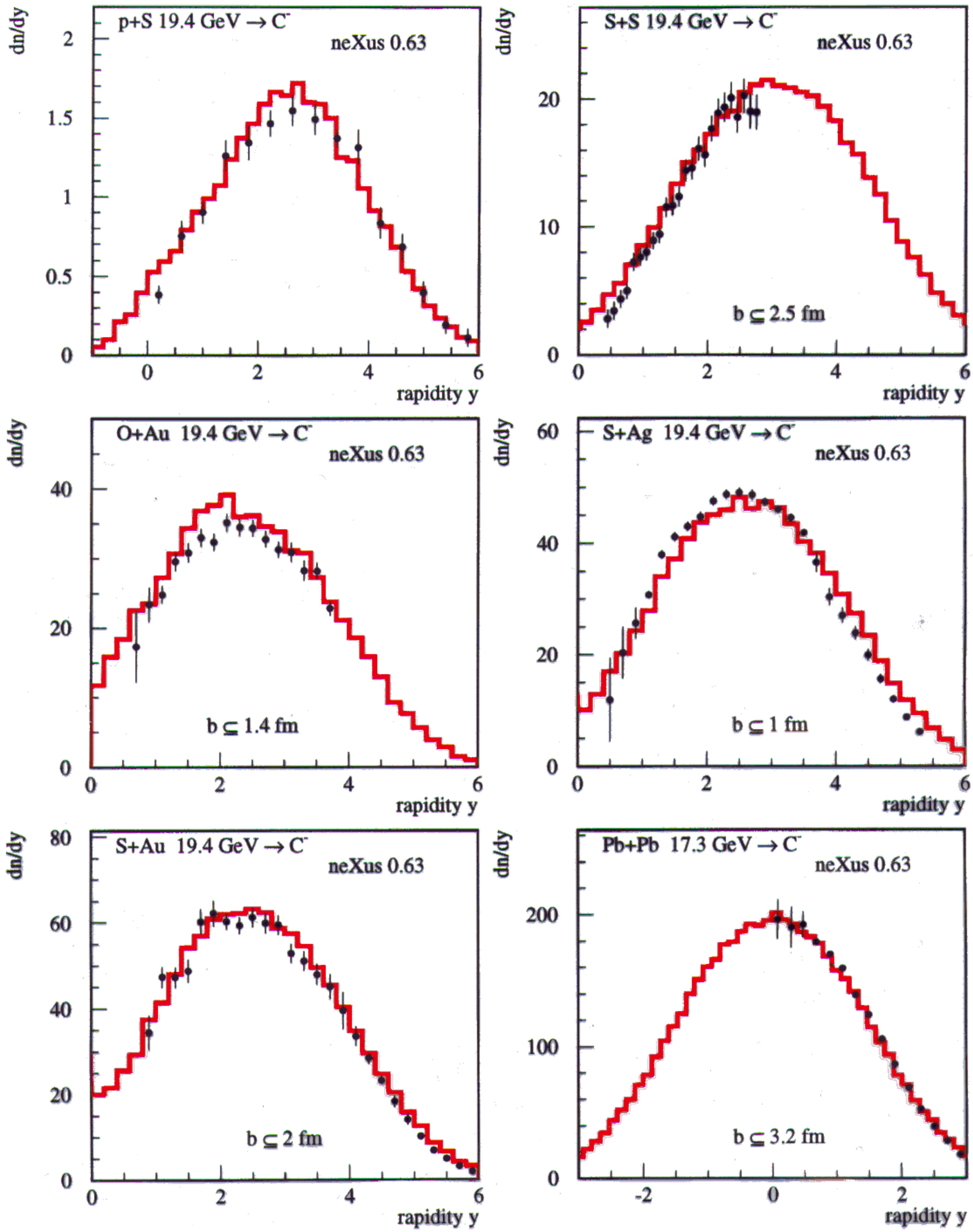
dn/dp$_t$ (c/GeV)
q2 = 7
x = 0.00016
eta = 2
q2 = 9
x = 0.00029
eta = 2
q2 = 13
x = 0.00037
eta = 2
q2 = 14
x = 0.00064
eta = 2
q2 = 14
x = 0.00110
eta = 2
q2 = 15
x = 0.00210
eta = 2
q2 = 29
x = 0.00093
eta = 2
q2 = 32
x = 0.00210
eta = 2
q2 = 35
x = 0.00440
eta = 2
q2 = 18
x = 0.0014
eta = 2
e+p at HERA
H1 data
compared to
U 0.66
p$_t$ (GeV/c)

Results

pp and AA



p+p 18.2 GeV p
neXus 0.63
x_E dn/dx
x
p+S 19.4 GeV → p - p̄
neXus 0.63
dn/dy
rapidity y
S+S 19.4 GeV → p
neXus 0.63
p_t ⊆ 2 GeV
b ⊆ 1 fm
p+Ag 13.8 GeV → p
neXus 0.63
S+Ag 19.4 GeV → p - p̄
neXus .63
b ⊆ 1.0 fm
p+Au 19.4 GeV → p - p̄
neXus 0.63
S+Au 19.4 GeV → p - p̄
neXus 0.63
b ⊆ 1.7 fm
Pb+Pb 17.3 GeV → p - p̄
neXus 0.63
b ⊆ 3.2 fm



p+S 19.4 GeV → C⁻
neXus 0.63
S+S 19.4 GeV → C⁻
neXus 0.63
b ⊆ 2.5 fm
O+Au 19.4 GeV → C⁻
neXus 0.63
b ⊆ 1.4 fm
S+Ag 19.4 GeV → C⁻
neXus 0.63
b ⊆ 1 fm
S+Au 19.4 GeV → C⁻
neXus 0.63
b ⊆ 2 fm
Pb+Pb 17.3 GeV → C⁻
neXus 0.63
b ⊆ 3.2 fm
dn/dy
rapidity y