Gluon Production in Nucleon–Nucleus Collisions in a Quasi–Classical Approximation

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talk based on the paper hep-ph/9802440

Plan of the talk:

1. History of the problem. Hopes and expectations.

2. Current–nucleus ($j = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$) and pA gluon production in covariant gauge $\partial \cdot A = 0$ (multiple rescatterings).

3. Light cone gauge (everything is in the wave function):

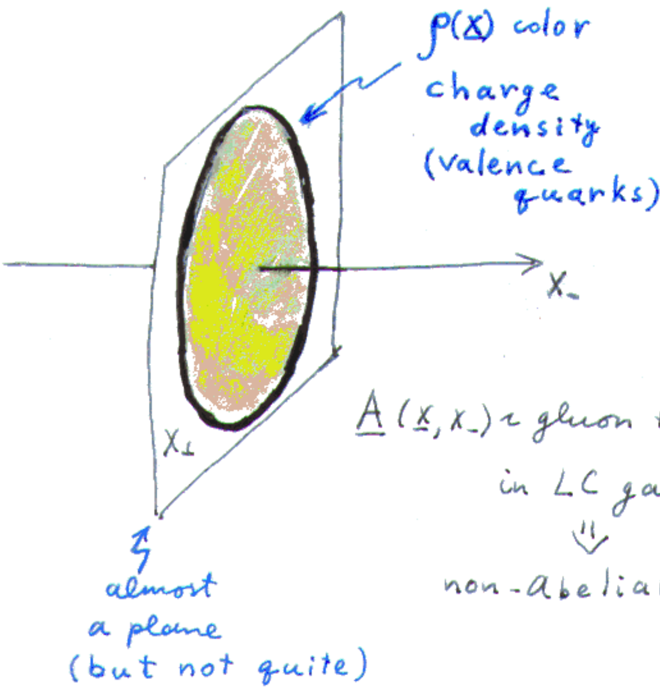
a. An example of the nucleus–currents scattering ($j = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$).

b. pA in light cone gauge.

1. History of the problem. Hopes and expectations.

Approach of L. McLerran and R. Venugopalan

(Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **50**, 2225 (1994))



- large nucleus, ultrarelativistic
- small x gluons, longitudinally coherent
- As the nucleus is large, can assume that $k_\perp \gg \Lambda_{QCD}$ and can use PQCD

$\underline{A}(\underline{x}, x_-) \sim$ gluon field in LC gauge

$\Downarrow$

non-abelian Weizsäcker-Williams field

⇒ Treat nucleus as a classical source for light cone gauge calculations to obtain non–Abelian Weizsäcker–Williams field of a nucleus

⇒ Use the higher momentum components as a source for the softer fields, iterate this procedure to obtain BFKL equation, and subleading corrections to it (see hep-ph/9706377 and hep-ph/9709432)

But: very hard to solve.

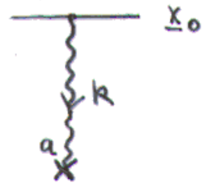
J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert

↳ alternative (Yu.K.) ~ quadratic integral eqn.

Non-Abelian Weizsäcker–Williams field for a large nucleus.

found simultaneously by McLerran et al in Phys.Rev. D **55** 5414 (1997) and Yu. K. in Phys. Rev. D **54**, 5463 (1996)

In our approach we start with the exact solution of the classical QCD equations of motion in covariant gauge, which corresponds to a single gluon exchange



$$A_+^{cov} = -\frac{g}{2\pi}(T^a)\,\delta(x_-)\ln(|\underline{x}-\underline{x}_0|\mu) \Rightarrow$$

for the nucleus just superimpose different valence quarks contribution

and make a gauge transformation to the $A_+ = 0$ light cone gauge:

$$\underline{A}(\underline{x}, x_-) = \frac{g}{2\pi}\sum_{a=1}^{8}\sum_{i=1}^{N}(T_i^a)\left(S(\underline{x}, x_{-i})T^a S^{-1}(\underline{x}, x_{-i})\frac{\underline{x}-\underline{x}_i}{|\underline{x}-\underline{x}_i|^2}\theta(x_- - x_{-i})\right.$$

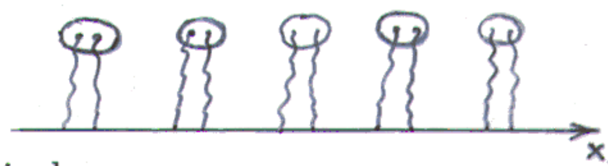
$$\left. -S(\underline{x}, x'_{-i})T^a S^{-1}(\underline{x}, x'_{-i})\frac{\underline{x}-\underline{x}'_i}{|\underline{x}-\underline{x}'_i|^2}\theta(x_- - x'_{-i})\right).$$

This is the non-Abelian Weizsäcker-Williams field generated by valence quarks. "Nucleons" were taken to be quark–antiquark pairs (onia).

$$S(\underline{x}, x_-) = \mathrm{P}\exp\left(-ig\int_{-\infty}^{x_-} dx'_- A'_+(\underline{x}, x'_-)\right)$$

$$= \prod_{i=1}^{N}\exp\left[\frac{ig^2}{2\pi}\sum_{a=1}^{8}T^a(T_i^a)\ln\left(\frac{|\underline{x}-\underline{x}_i|}{|\underline{x}-\underline{x}'_i|}\right)\theta(x_- - x_{-i})\right].$$

NOTE: can have different $S(\underline{x}, x_-)$ matrices, say $S = P\exp\left\{ig\int_{x_-}^{\infty} dx'_- A'_+(\underline{x}, x'_-)\right\}$



$\langle A(\underline{x}, x_-)\rangle = 0$, so the field is not quite classical,

but $\langle A(\underline{x}, x_-)A(\underline{y}, y_-)\rangle \neq 0$.

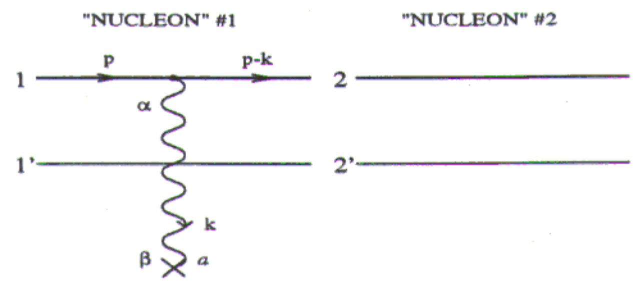
small in covariant gauge

The Feynman diagrams corresponding to this non-Abelian Weizsäcker–Williams field are:

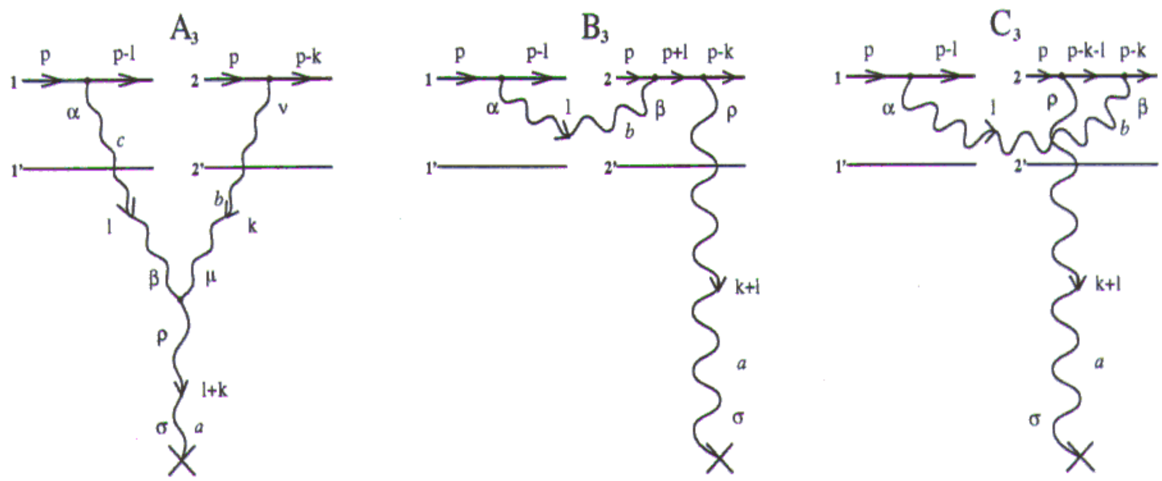
see Yu. K. , Phys.Rev. D **55** 5445 (1997)

Order g: Regularization of the LC propagator:

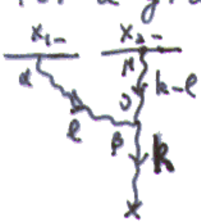
$$\frac{-i}{k^2+i\varepsilon}\left[g_{\alpha\beta}-\frac{\eta_\alpha k_\beta}{k_+}-\frac{\eta_\beta k_\alpha}{k_+}\right]\longrightarrow\frac{-i}{k^2+i\varepsilon}\left[g_{\alpha\beta}-\frac{\eta_\alpha k_\beta}{k_+-i\varepsilon}-\frac{\eta_\beta k_\alpha}{k_++i\varepsilon}\right]$$



Order g^3 (assume that $x_{1-} > x_{2-}$, opposite to the notations in the above reference):



for a graph like A, the relevant propagators are:



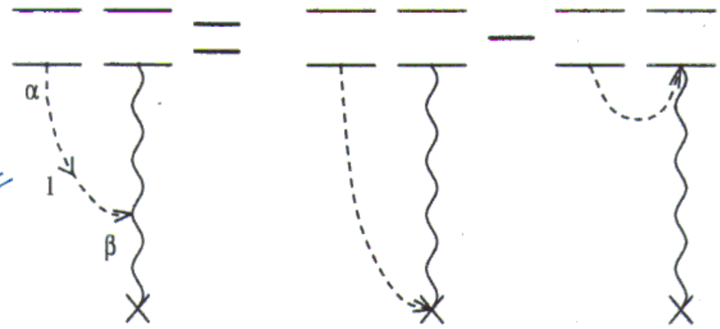
$$\int_{-\infty}^{\infty}\frac{dl_+}{2\pi}\,\frac{+i}{l^2}\left[g_{\alpha\beta}-\frac{\eta_\alpha l_\beta}{l_+-i\varepsilon}-\cancelto{0}{\frac{\eta_\beta l_\alpha}{l_++i\varepsilon}}\right]\frac{i}{(k-l)^2}\left[g_{\mu\nu}-\frac{(k-l)_\nu\eta_\mu}{k_+-l_+-i\varepsilon}-\cancelto{0}{\frac{(k-l)_\mu\eta_\nu}{k_+-l_++i\varepsilon}}\right].$$

$$\cdot\, e^{-il_+(x_{2-}-x_{1-})}\propto\int_{-\infty}^{\infty}\frac{dl_+}{2\pi}\,e^{il_+ 0 x_-}\,\frac{\eta_\alpha l_\beta}{l_+-i\varepsilon}\times\ldots$$

$\Rightarrow$ can apply Ward identity

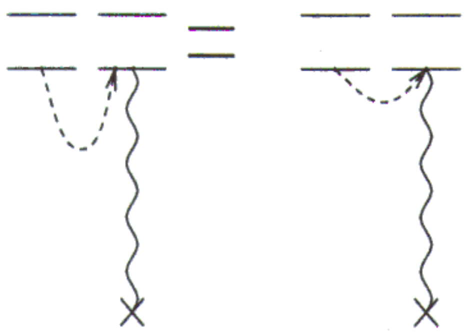
The principle behind is the Ward identity:

A$_3$:

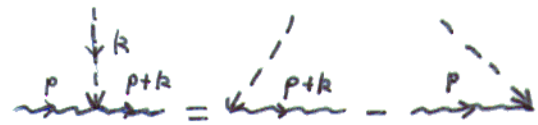


$$\frac{\eta_\alpha l_\beta}{l_+ - i\varepsilon}$$

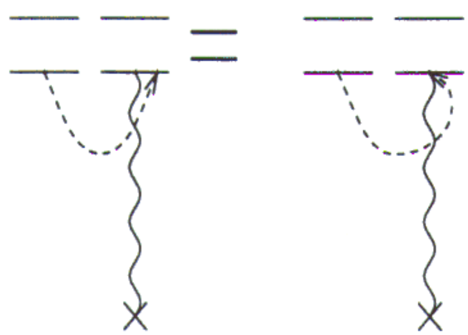
B$_3$:



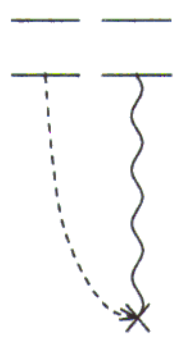
Ward identity in LC gauge:



C$_3$:



A$_3$+ B$_3$+ C$_3$:



At the higher orders (g^5):

Color average in the color space of the left nucleon to get classical field. Two gluons per nucleon limit.

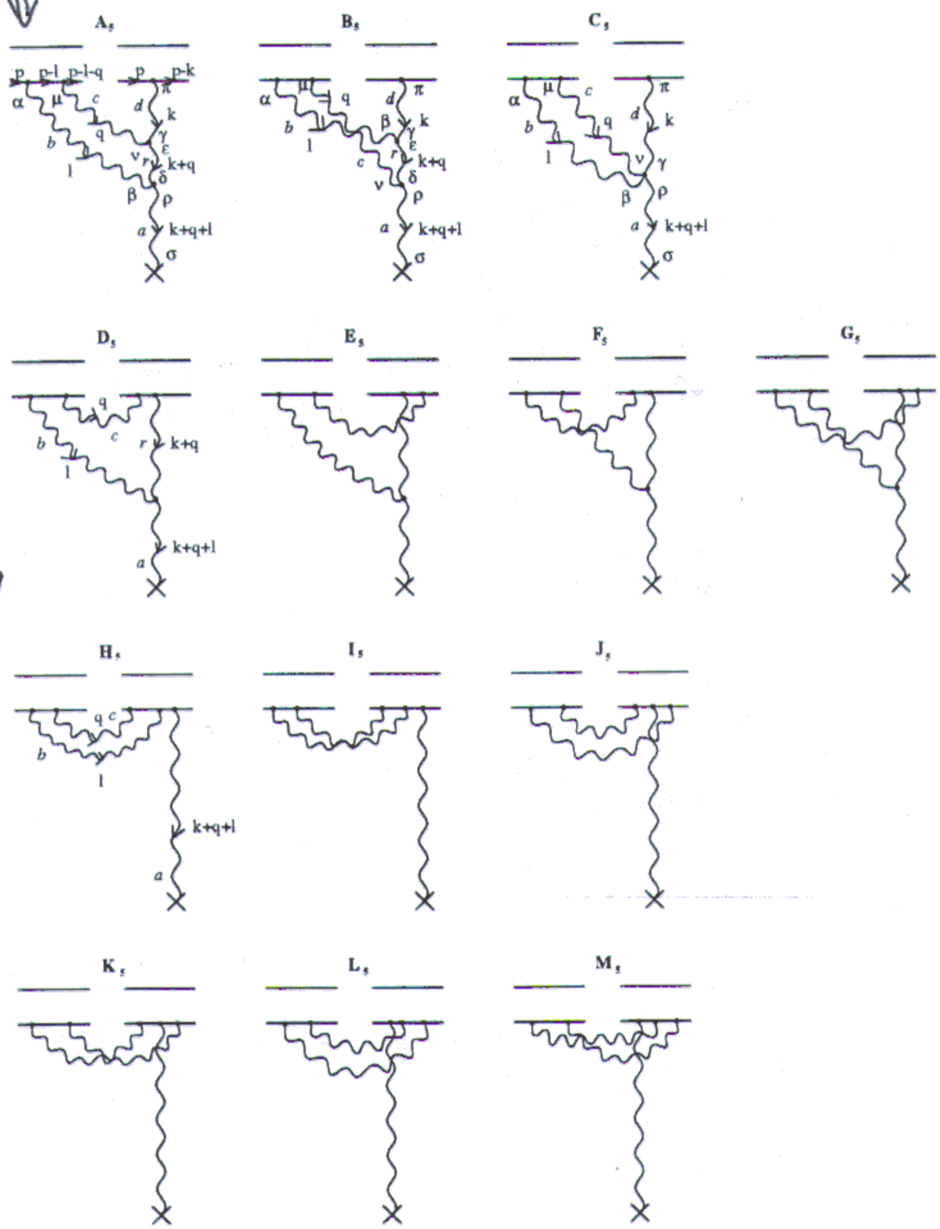
color averaged, otherwise would have to include  which is not classical

$\alpha^2 A^{1/3}$ is the effective coupling

$\alpha^2 A^{1/3} \sim 1$ strong field limit

$\alpha^2 A^{1/3} \ll 1$ weak field limit

$A^{1/3}$ is # interacting nucleons.



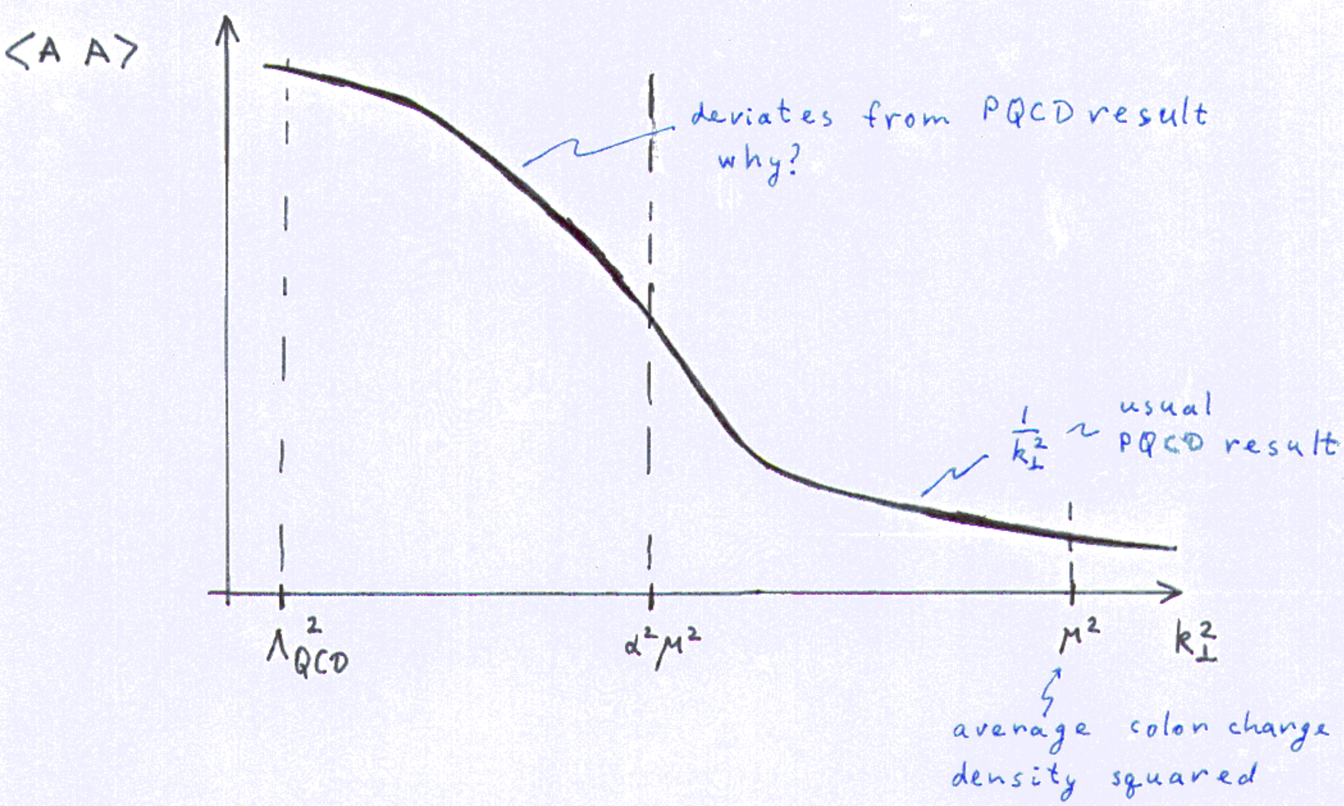
L. McLerran and collaborators found in Phys.Rev. D **55** 5414 (1997) the expectation in the nuclear wavefunction of the product of two gluon fields:

$$\langle A_i^a(\underline{x}, x_-)A_i^a(\underline{y}, y_-)\rangle = \frac{4(N_c^2-1)}{N_c x_\perp^2}\left\{1 - \exp\left[-\frac{1}{2}\alpha N_c \chi(y) x_\perp^2 \ln\left(\frac{1}{x_\perp^2 \Lambda_{QCD}^2}\right)\right]\right\},$$

where $x_\perp = |\underline{x} - \underline{y}|$ and $\chi(y)$ is some function of the longitudinal coordinates.

This object was associated with the nuclear gluon distribution function $xG_A(x, Q^2)$.

The plot of the function versus the gluon's transverse momentum is



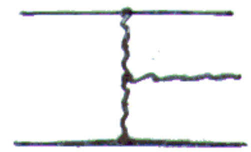
shadowing??? NO, and we'll see why

Nuclear Collisions in the Quasi–Classical Approximation

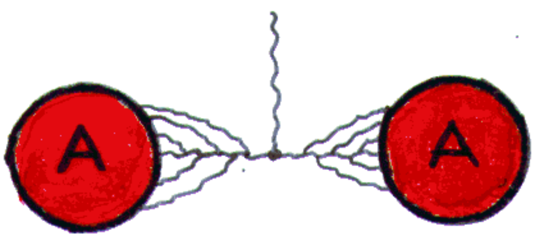
(for some efforts to attack the problem see: Dirk Rischke and Yu. K. in Phys.Rev. C **56** 1084 (1997) and A. Kovner et al in Phys. Rev. D **52**, 6231 (1995); **52**, 3809 (1995))

$$A_\pm = \frac{1}{\sqrt{2}}(A_0 \pm A_z)$$

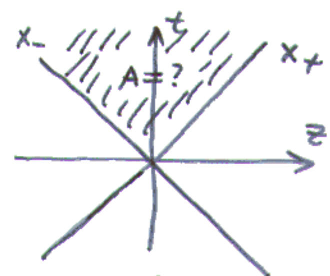
L. McLerran and collaborators tried to solve the classical QCD equations of motion in $x_- A_+ + x_+ A_- = 0$ gauge perturbatively to the next to lowest order. They interpreted the result diagrammatically as



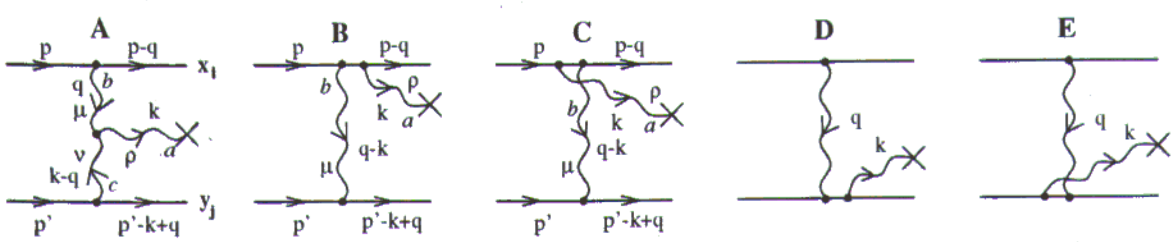
However: the idea is attractive:



but: hard to check as this gauge has no momentum space Feynman rules



Dirk Rischke and myself solved the same problem in $\partial \cdot A = 0$ gauge and gave an exact connection to the diagrams:

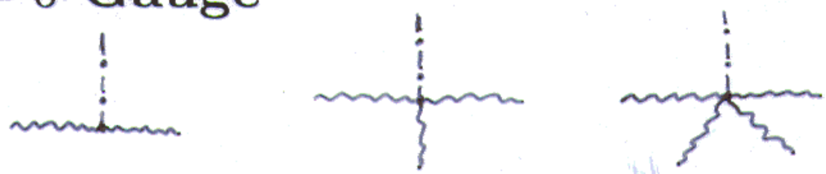


However, all these calculations still describe the weak field limit. It also would be quite interesting if the fields fusion assumptions were true.

S. Matynyan, B. Müller, D. Rischke

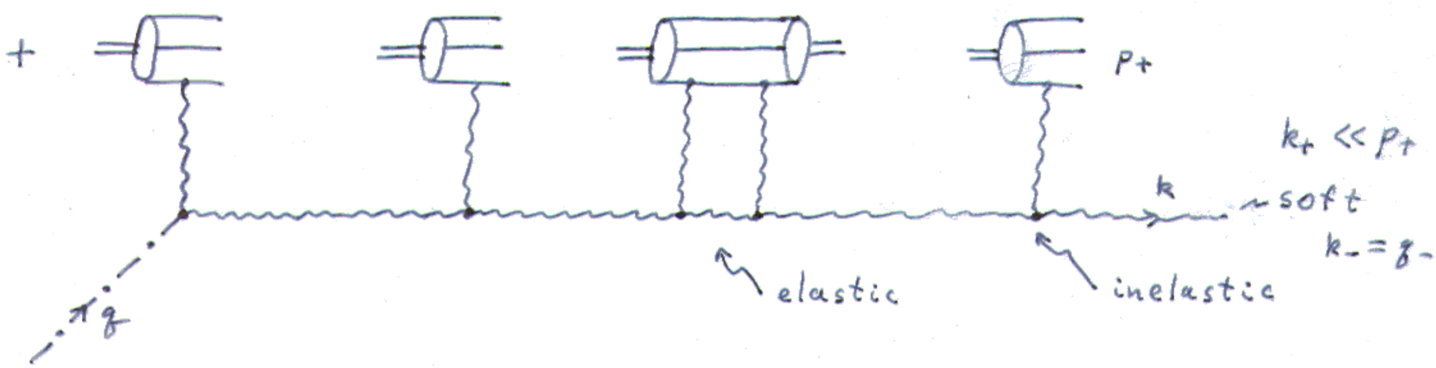
2. Gluon Production in Current–Nucleus and Nucleon–Nucleus Collisions in Covariant Gauge or in $A_- = 0$ Gauge

$$j = -\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$



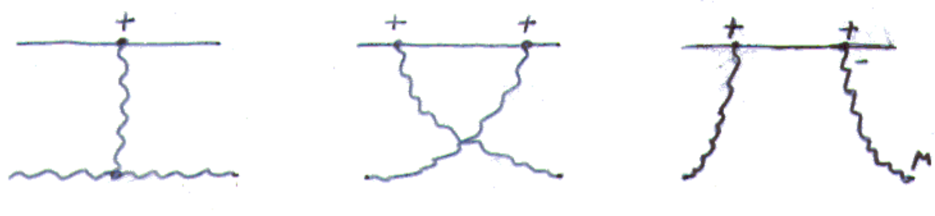
In our model we neglect QCD evolution of the gluon structure functions in the nucleons. The gluon structure function in the nucleons is taken to be $xG(x, Q^2) = \frac{\alpha C_F}{\pi} \ln(Q^2/\mu^2)$.

We limit the interactions of the gluon propagating through the nucleus with the nucleons by two gluons per nucleon limit. The strong field limit corresponds to $\alpha^2 A^{1/3} \propto 1$, with $A^{1/3}$ the number of nucleon's along the gluon's path.



This is Glauber expansion in terms of multiple scatterings. The gluon produced in the current–nucleon interaction rescatters in the nucleus both elastically and inelastically.

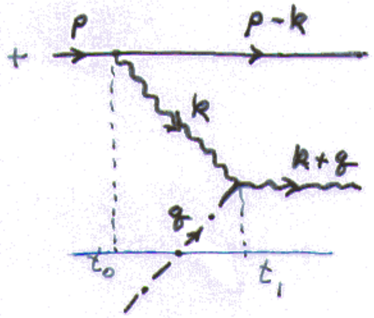
The gluon–nucleon interactions can be given by the following graphs in covariant gauge (but only the first of them survives in the $A_- = 0$ gauge):



$\Rightarrow$ need $\gamma_+ \Rightarrow$ get $D_{-\mu} = 0$ in $A_- = 0$ gauge

~ same in $\partial_\mu A^\mu = 0$ gauge and $A_- = 0$ gauge

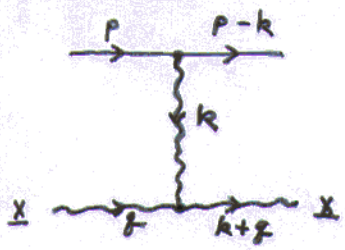
The gluon production in current–nucleon interaction is local:



OFPT: $\tau = t_1 - t_0 \approx \frac{1}{k_+} = \frac{2k_-}{\underline{k}^2} \approx 0$

as $(p-k)^2 = 0 \Rightarrow -2p_+ k_- = 0 \Rightarrow k_- = 0.$

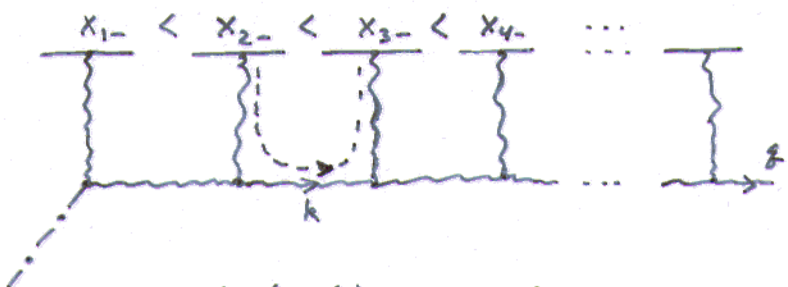
Th gluon–nucleon interaction is also local:

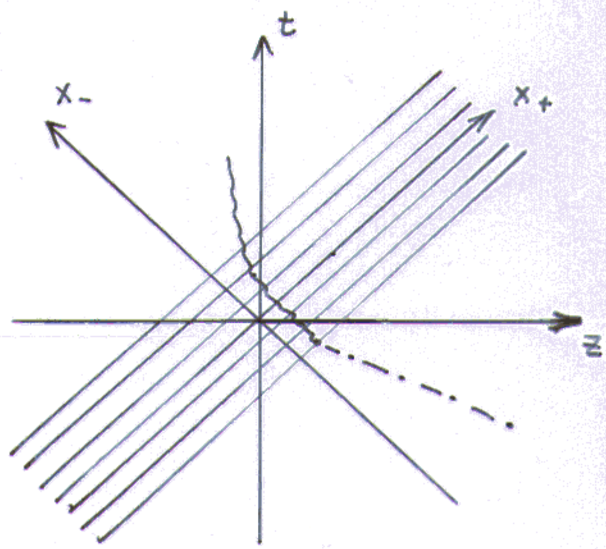


$\tau \approx \frac{2k_-}{\underline{k}^2} \approx 0$ $\Rightarrow$ the gluon hasn't gained any "minus" component of the momentum

the propagator $\frac{-i}{k^2} \rightarrow \frac{i}{\underline{k}^2}$ $\Rightarrow$ the gluon's transverse coordinate remains the same

Therefore we can distort the k_+–integration contour such that we'll pick up the poles of the covariant gluons' propagators, putting them on–shell:

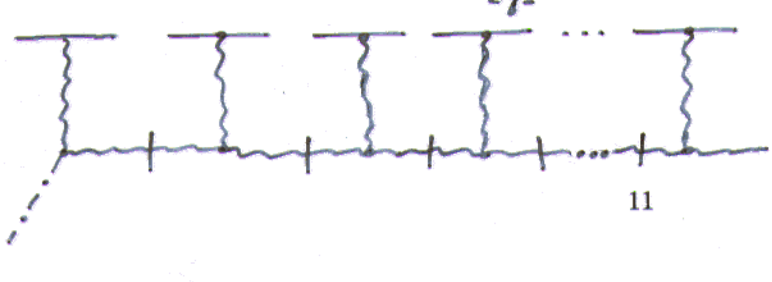




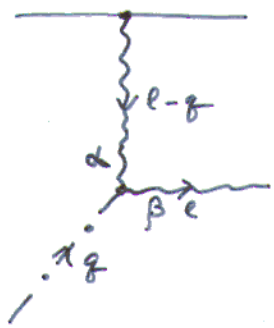
for each k ($k_- = q_- > 0$)

$$\int_{-\infty}^{\infty}\frac{dk_+}{(2\pi)}\, e^{-ik_+(x_{3-}-x_{2-})}\,\frac{1}{2k_+k_- - \underline{k}^2 + i\epsilon} =$$

$$= \frac{1}{2q_-}\int_{-\infty}^{\infty}\frac{dk_+}{2\pi}\, e^{-ik_+\Delta x_-}\,\frac{1}{k_+ - \frac{\underline{k}^2}{2q_-} + i\epsilon}$$ $\Leftarrow$ pick that pole



First stage is gluon production in current–nucleon interaction. In covariant gauge it's given by the graph:



The Feynman rules for the vertex:

$$v_{\alpha\beta} = g_{\alpha\beta}\, l\cdot(l-q) - l_\alpha\,(l-q)_\beta$$

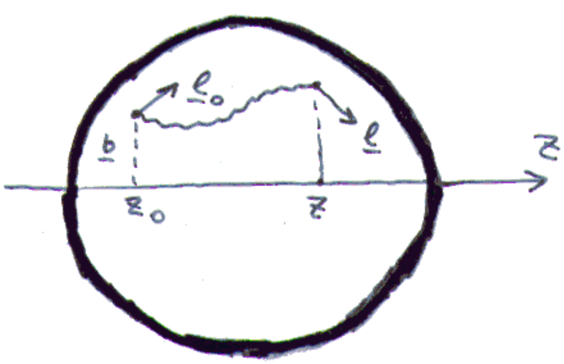
The number density of the produced gluons is given by the unintegrated gluon distribution of the nucleon:

$$\frac{dN_0(l)}{d^2l} = \frac{1}{\pi}\frac{dN_0(l)}{dl^2} = \frac{1}{\pi}\frac{\partial xG(x,\underline{l}^2)}{\partial \underline{l}^2}$$

where, in our approximation, we neglect the QCD evolution in the structure functions.

For a gluon produced with a transverse momentum $\underline{l}_0$, longitudinal coordinate z_0 and impact parameter $\underline{b}$ define the probability distribution $f(\underline{b}, z_0, \underline{l}_0, z, \underline{l})$ for the gluon to have transverse momentum $\underline{l}$ at a longitudinal position z. The probability conservation yields:

$$\int d^2l\, f(\underline{b}, z_0, \underline{l}_0, z, \underline{l}) = 1$$



Knowing the probability distribution of the gluons we may write down an expression for the number density of gluons produced in the nucleus:

CM ↓ can always boost

$$\frac{dN(l)}{d^2l} = \int d^2l_0 \frac{dN_0(l_0)}{d^2l_0} d^2b dz_0 \rho(\underline{b}, z_0) f(\underline{b}, z_0, \underline{l}_0, z, \underline{l})|_{z=\sqrt{R^2-b^2}},$$

for the gluon away from the fragmentation region. The normal nuclear density $\rho(\underline{b}, z_0)$ is normalized according to

$$\int d^2b\, dz_0\, \rho(\underline{b}, z_0) = A.$$

Our goal now is to find f. It obeys the following equation:

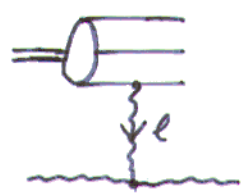
$$\frac{\partial}{\partial z} f(z, \underline{l}) = -\frac{1}{\lambda} f(z, \underline{l}) + \rho\sigma \int d^2l'\, V(\underline{l}')\, f(z, \underline{l} - \underline{l}')$$

with

$$f(z_0, \underline{l}) = \delta(\underline{l} - \underline{l}_0)$$

and

$$V(\underline{l}) = \frac{1}{\sigma}\frac{d\sigma}{d^2l}$$



the normalized gluon–nucleon scattering amplitude with l the momentum transfer.

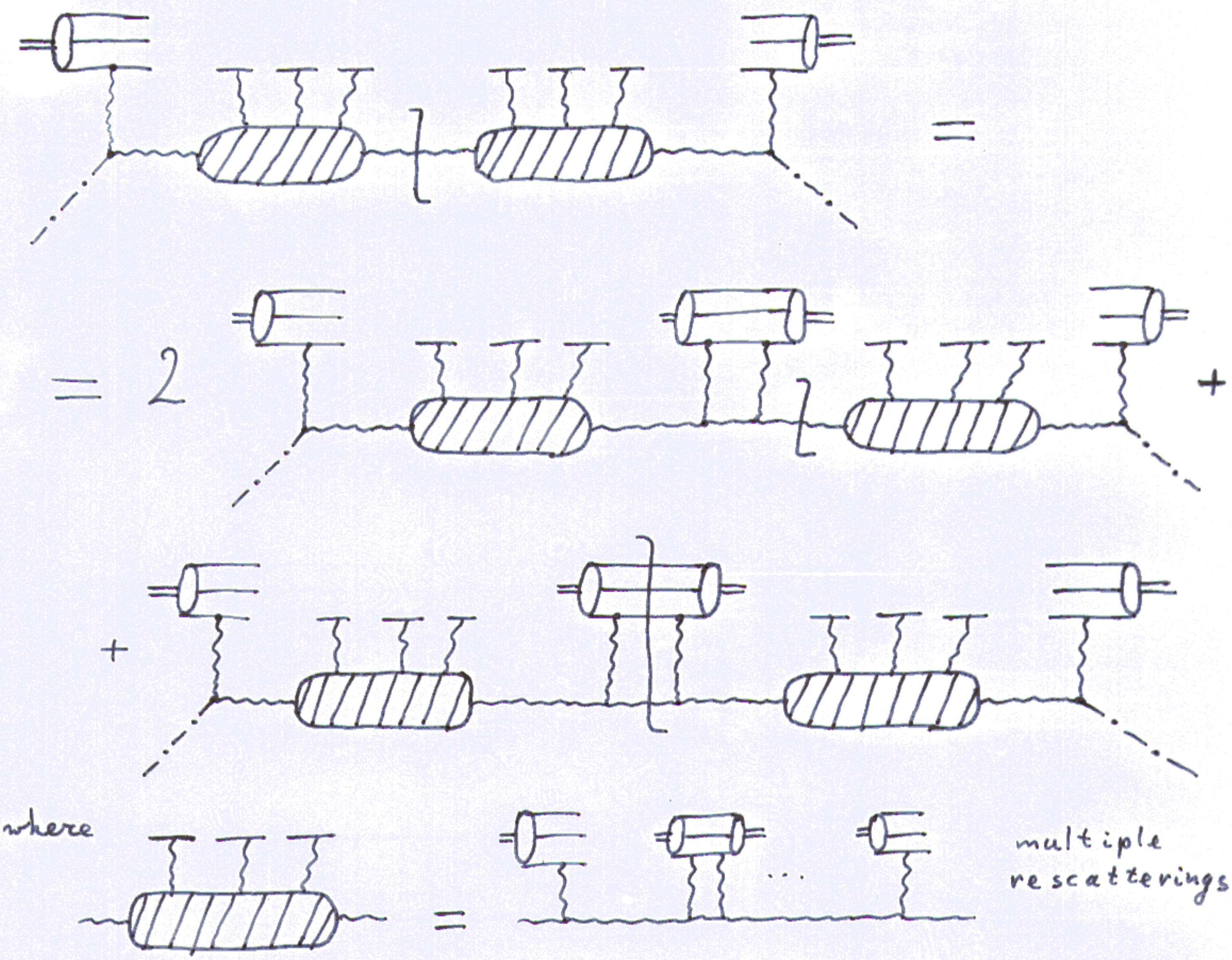
$$\lambda = \frac{1}{\rho\sigma}$$

is the mean free path of gluons in nuclear matter.

This equation can be derived from Glauber multiple scattering formalism (see R. Baier et al in Nucl. Phys. **B 484** 265 (1997)) or can be derived from the field–theoretical approach:

$$\frac{\partial}{\partial z} f(z, \underline{l}) = -\frac{1}{\lambda} f(z, \underline{l}) + \rho\sigma \int d^2 l' \, V(\underline{l}') \, f(z, \underline{l} - \underline{l}')$$

The first term on the right–hand side corresponds to elastic scattering (virtual term), the second one corresponds to inelastic scattering.



Solving the equation:

$$\frac{\partial}{\partial z} f(z,\underline{l}) = -\frac{1}{\lambda} f(z,\underline{l}) + \rho\sigma \int d^2 l'\, V(\underline{l}')\, f(z,\underline{l}-\underline{l}'),$$

going to transverse coordinate space

$$\tilde{f}(z,x_\perp^2) = \int d^2 l\, e^{-i(\underline{l}-\underline{l}_0)\cdot \underline{x}} f(z,\underline{l})$$

we obtain

$$\frac{\partial}{\partial z} \tilde{f}(z,x_\perp^2) = -\frac{1}{4\lambda} x_\perp^2 \tilde{v}(x_\perp^2) \tilde{f}(z,x_\perp^2),$$

with

$$\tilde{f}(z_0,x_\perp^2) = 1,$$

and where

$$\tilde{v}(x_\perp^2) = \frac{4}{x_\perp^2}(1 - \tilde{V}(x_\perp^2)),$$

with

$$\tilde{V}(x_\perp^2) = \int d^2 l\, e^{-i\underline{l}\cdot\underline{x}} V(\underline{l}).$$

$$\tilde{v}(x_\perp^2) = \frac{4}{x_\perp^2} \int d^2 l \left(1 - e^{-i\underline{l}\cdot\underline{x}}\right) V(\underline{l}) = \frac{4\pi}{x_\perp^2} \int_0^\infty dl^2 \left(1 - J_0(lx)\right) V(l^2) \approx$$

$$\approx \pi \int_0^{1/x_\perp^2} dl^2\, \underbrace{V(l^2)}_{\frac{1}{\pi\sigma}\frac{d\sigma}{dl^2}}\, l^2 \approx \frac{1}{\sigma} \int_0^{1/x_\perp^2} dl^2 \cdot l^2 \frac{d\sigma}{dl^2} \propto$$

$$\propto x\,G\left(x, \frac{1}{x_\perp^2}\right)$$

Apparently $\tilde{f}(z,x_\perp^2)$ is given by

$$\tilde{f}(z,x_\perp^2) = \exp\left(-\frac{z-z_0}{4\lambda} x_\perp^2 \tilde{v}(x_\perp^2)\right).$$

One can show (see R. Baier et al in Nucl. Phys. **B 484** 265 (1997)) that for small $x_\perp^2$

$$\frac{\tilde{v}(x_\perp^2)}{\lambda} = \frac{4\pi^2 \alpha N_c}{N_c^2 - 1} \rho\, x G(x, Q^2).$$

$$Q^2 = \frac{1}{x_\perp^2}$$

Defining

$$\tilde{N}(x_\perp^2) = \int d^2l e^{-i\underline{l}\cdot\underline{x}} \frac{dN(l)}{d^2l}$$

we get

$$\tilde{N}(x_\perp^2) = \int d^2b\, dz_0\, \rho(\underline{b}^2, z_0)\, \tilde{N}_0(x_\perp^2)\, \tilde{f}(z, x_\perp^2)|_{z=\sqrt{R^2-b^2}}.$$

Using $\tilde{f}(z, x_\perp^2)$ from above we get the answer:

$$\tilde{N}(x_\perp^2) = \int d^2b\, \frac{N_c^2 - 1}{\pi^2 \alpha N_c x_\perp^2} \left[1 - \exp\left(-\frac{\sqrt{R^2 - b^2}}{2\lambda} x_\perp^2 \tilde{v}(x_\perp^2) \right) \right],$$

or, equivalently,

$$\tilde{N}(x_\perp^2) = \int d^2b\, \frac{N_c^2 - 1}{\pi^2 \alpha N_c x_\perp^2}$$

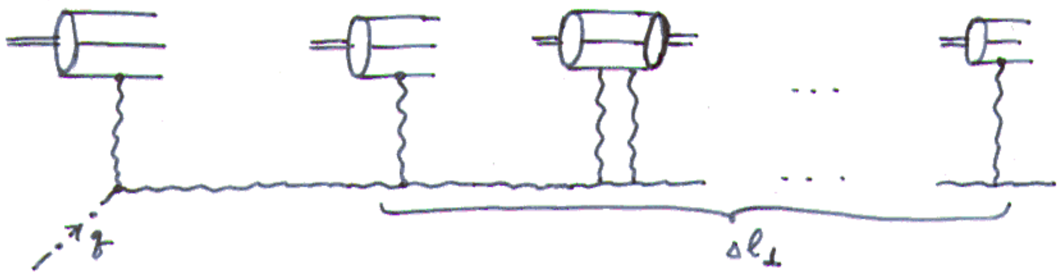
$$\times \left[1 - \exp\left(-\frac{2\pi^2 \sqrt{R^2 - b^2} x_\perp^2 \alpha N_c}{N_c^2 - 1} \rho x G(x, 1/x_\perp^2) \right) \right].$$

Amazingly enough, this result is EQUIVALENT to the result of McLerran and collaborators for

$$\langle A_i^a(\underline{x}, x_-) A_i^a(\underline{y}, y_-) \rangle !!!$$

$\Rightarrow$ NO SHADOWING, only FINAL STATE INTERACTIONS

Looks like the expression McLerran and collaborators have found is more likely to describe the gluon production, than the nuclear gluon distribution function.



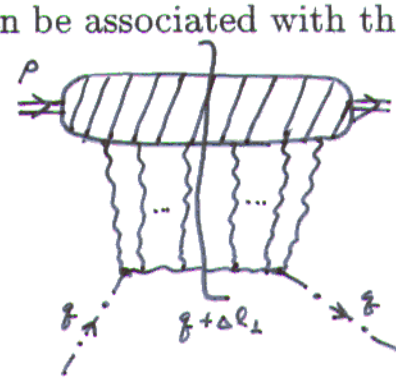
The current carries momentum q, and $Q^2 = -q \cdot q$. Let $\triangle l_\perp$ is a typical transverse momentum a high energy gluons obtains by multiple rescatterings with the nucleons.

$\Rightarrow$ What is usually understood as a gluon distribution of a nucleus $xG_A(x, Q^2)$ is measured at the scale Q^2. $xG_A(x, Q^2) = AxG(x, Q^2)$ holds for any Q^2 (in our approximation, without gluon loops).

$\Rightarrow$ When $Q^2 \leq \triangle l_\perp^2$, which can be achieved by going to a large nucleus or small Q^2, the produced gluon has transverse momentum very much different from Q^2, and , therefore, we can not associate the gluon production with the gluon distribution.

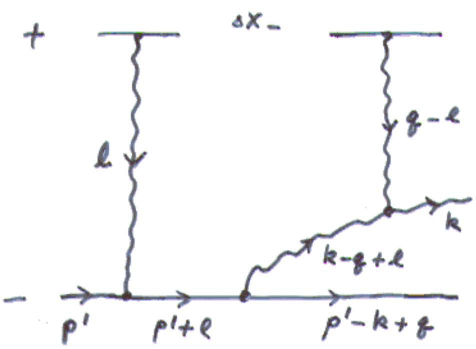
$\Rightarrow$ The distribution of the produced gluons does not reflect shadowing, but rather the probability conserving final state interactions, which modify the transverse momentum distribution of the produced gluon.

$\Rightarrow$ When $Q^2 \gg \triangle l_\perp^2$ the gluon production can be associated with the gluon distribution.



Gluon production in nucleon–nucleus collisions.

The nucleon–nucleus collision looks a little different from the current–nucleus collision. The incoming nucleon now may interact with the nucleus before and after it emits a gluon. One can show that for a particular graph all the interactions are either before or after the gluon's emission.



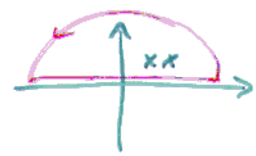
$k_+ = q_+\,, \quad \ell_- = q_- = 0$

The relevant propagators are:

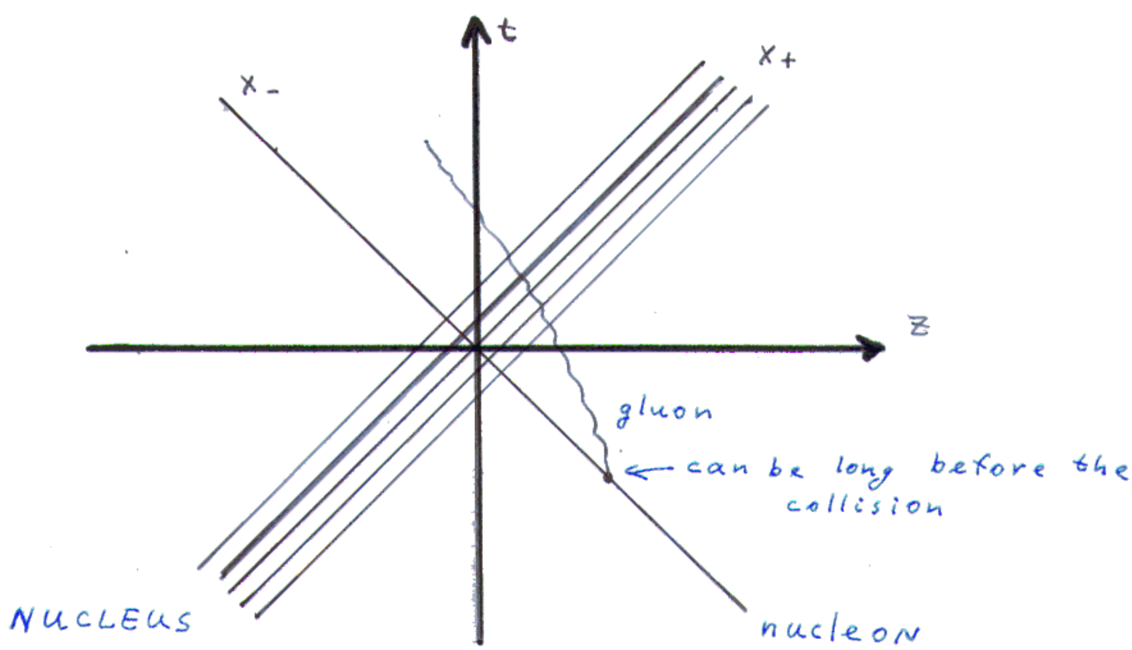
$$\propto \int_{-\infty}^{\infty} \frac{d\ell_+}{2\pi}\, e^{-i\ell_+\Delta x_-} \frac{1}{(p'+\ell)^2 + i\varepsilon} \, \frac{1}{(k-q+\ell)^2 + i\varepsilon} \propto$$

$$\propto \int_{-\infty}^{\infty} d\ell_+\, e^{-i\ell_+\Delta x_-} \frac{1}{\ell_+ + i\varepsilon} \, \frac{1}{\ell_+ - \frac{(\underline{k}-\underline{q}+\underline{\ell})^2}{2k_-} + i\varepsilon} \propto$$

$$\propto 1 - e^{-i\Delta x_- \frac{(\underline{k}-\underline{q}+\underline{\ell})^2}{2k_-}} \approx 0$$

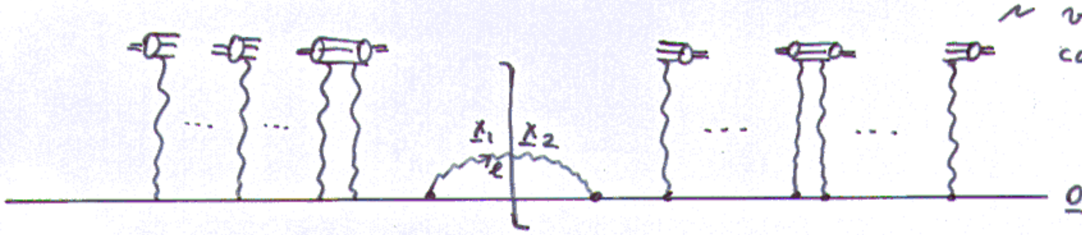


This is due to the fact that the gluon–nucleon system passes the nucleus instantaneously compared to the magnitude of the gluon emission time.

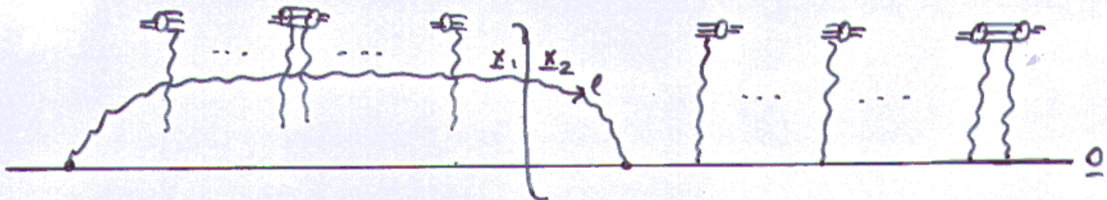


Using the light–cone perturbation theory we can calculate the cross–section. There are three cases:

real – virtual cancellation

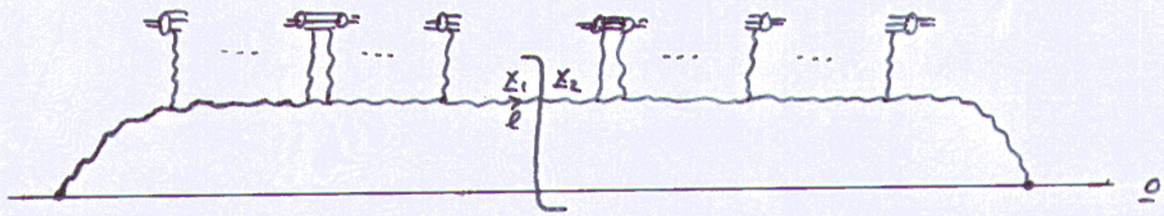


$$\frac{d\sigma^{(a)}}{d^2l\,dy} = \frac{1}{\pi}\int d^2b\, d^2x_1\, d^2x_2 \frac{1}{(2\pi)^2}\,\frac{\alpha C_F}{\pi}\,\frac{\underline{x}_1\cdot\underline{x}_2}{\underline{x}_1^2\underline{x}_2^2}\, e^{i\underline{l}\cdot(\underline{x}_1-\underline{x}_2)}$$



$$\frac{d\sigma^{(b+c)}}{d^2l\,dy} = -\frac{1}{\pi}\int d^2b\, d^2x_1\, d^2x_2\, \frac{1}{(2\pi)^2}\,\frac{\alpha C_F}{\pi}\frac{\underline{x}_1\cdot\underline{x}_2}{\underline{x}_1^2\underline{x}_2^2}\, e^{i\underline{l}\cdot(\underline{x}_1-\underline{x}_2)}$$

$$\times\left[\exp\left(-\frac{\underline{x}_1^2\tilde{v}\sqrt{R^2-b^2}}{2\lambda}\right)+\exp\left(-\frac{\underline{x}_2^2\tilde{v}\sqrt{R^2-b^2}}{2\lambda}\right)\right]$$



$$\frac{d\sigma^{(d)}}{d^2l\,dy} = \frac{1}{\pi}\int d^2b\, d^2x_1\, d^2x_2\, \frac{1}{(2\pi)^2}\,\frac{\alpha C_F}{\pi}\,\frac{\underline{x}_1\cdot\underline{x}_2}{\underline{x}_1^2\underline{x}_2^2}\, e^{i\underline{l}\cdot(\underline{x}_1-\underline{x}_2)}$$

$$\exp\left(-\frac{(\underline{x}_1-\underline{x}_2)^2\tilde{v}\sqrt{R^2-b^2}}{2\lambda}\right)$$

Performing the integration:

$$\frac{d\sigma^{(a)}}{d^2l dy} = \frac{1}{\pi}\frac{\alpha C_F}{\pi}\int d^2b\,\frac{1}{\underline{l}^2}$$

$$\frac{d\sigma^{(b+c)}}{d^2l dy} = -\frac{2}{\pi}\frac{\alpha C_F}{\pi}\int d^2b\,\frac{1}{\underline{l}^2}\left\{1-e^{-\frac{\underline{l}^2}{\langle l_\perp^2\rangle}}\right\}$$

$$\frac{d\sigma^{(d)}}{d^2l dy} = \frac{1}{\pi}\frac{\alpha C_F}{\pi}\int d^2b\,\frac{e^{-\frac{\underline{l}^2}{\langle l_\perp^2\rangle}}}{\langle l_\perp^2\rangle}\left\{2\ln(\langle l_\perp^2\rangle L^2) - \Gamma\left(0,-\frac{\underline{l}^2}{\langle l_\perp^2\rangle}\right)\right.$$

$$\left. - \ln(L^2\underline{l}^2) + \ln\left(-\frac{1}{4}\right)\right\}$$

with

$$\langle l_\perp^2(b)\rangle = \frac{2\tilde{v}(\langle l_\perp^2(b)\rangle)}{\lambda}\sqrt{R^2-b^2},$$

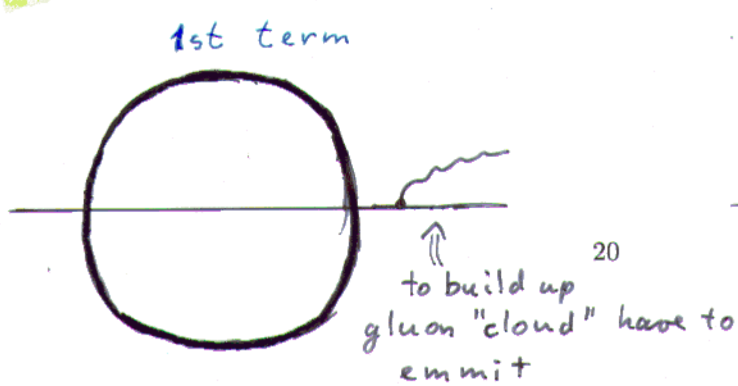
and L some infrared cutoff.

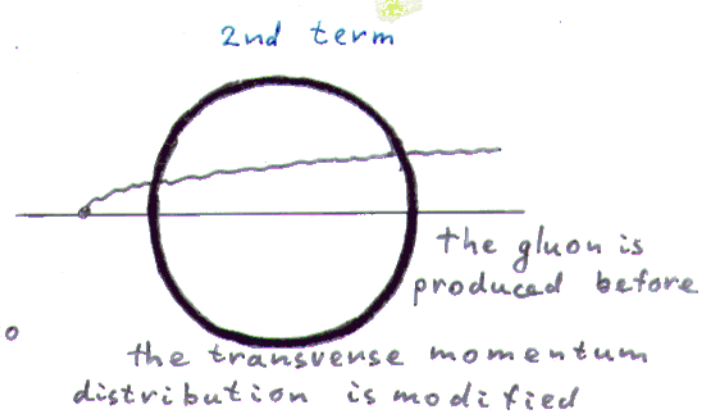
Generalizing to a nucleon, in the LLA we obtain:

$$\frac{d\sigma}{d^2l dy} = \frac{1}{\pi}\int d^2b\left\{\frac{\partial}{\partial l^2}xG(x,l^2) + xG(x,\langle l_\perp^2\rangle)\,\frac{e^{-\frac{l^2}{\langle l_\perp^2\rangle}}}{\langle l_\perp^2\rangle}\right\}.$$

Note that

$$\frac{d\sigma}{dy} = \int\frac{d\sigma}{d^2l dy}d^2l = 2\int d^2b\,xG(x,\langle l_\perp^2\rangle).$$

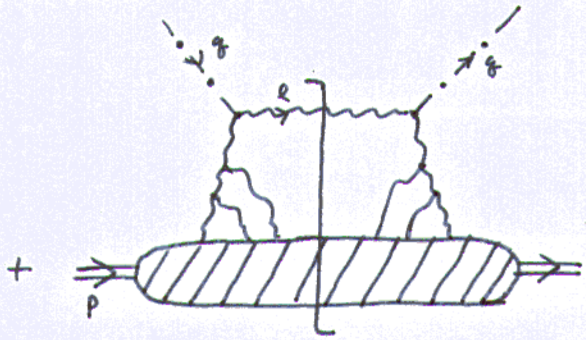




3. Light Cone Gauge $A_+ = 0$

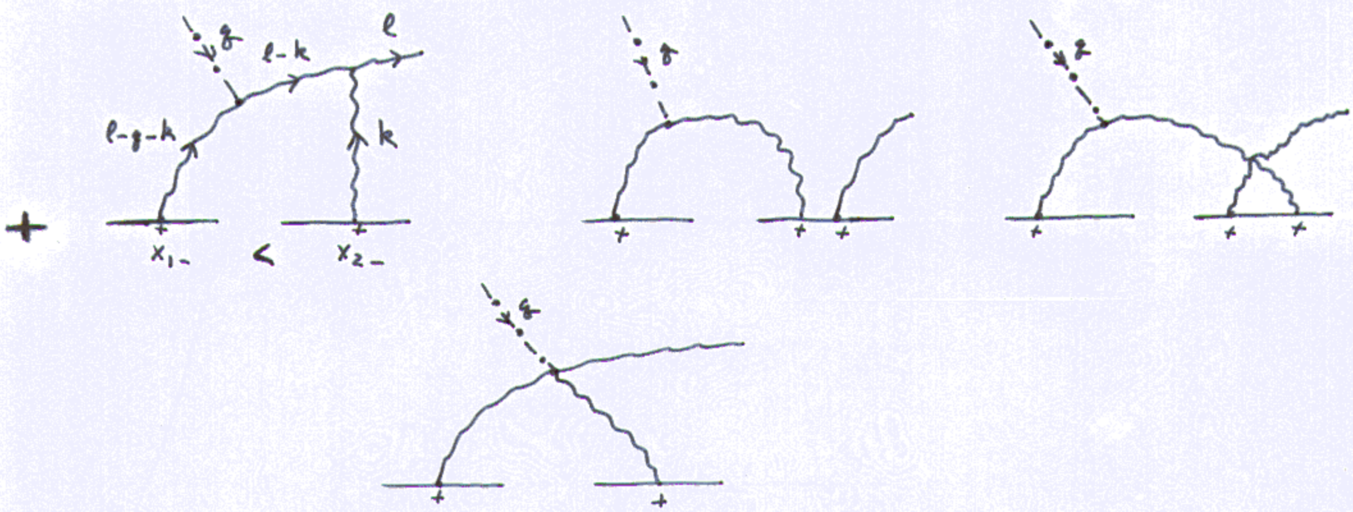
a. Nucleus–current scattering with $j = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$.

We consider the current–nucleus interaction:

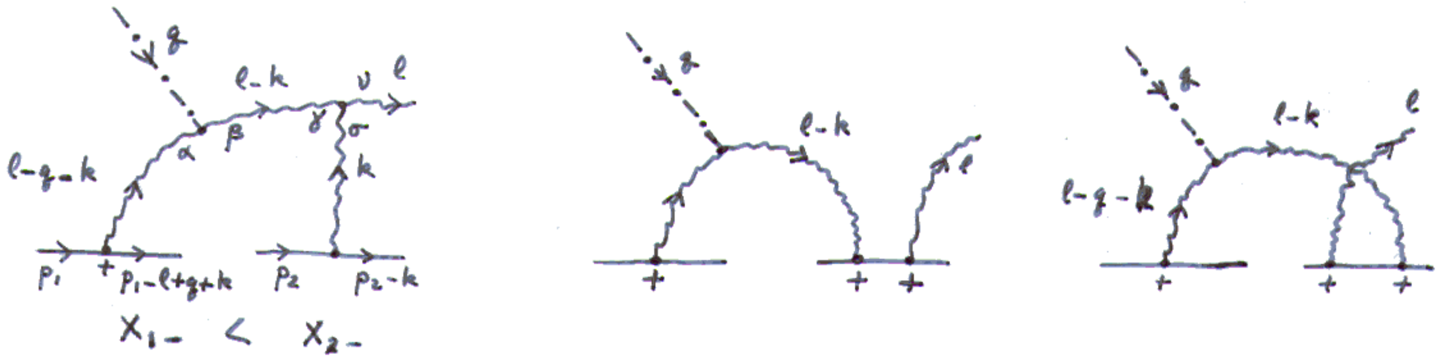


Let's assume that the current has $\underline{q} = 0$ and $Q^2 = -2q_+q_- \gg \underline{l}^2$. Therefore $l_+ \ll -q_+ \ll p_+$.

We want to show that there is no final state interactions in the light cone gauge. Consider the graphs:



For simplicity we'll take the polarization of the outgoing gluon to satisfy $\epsilon_- = 0$.



(a) (b) (c)

pick this pole

$$G_a = \frac{-i}{(\underline{k}+\underline{q}-\underline{l})^2}\boxed{\frac{(k+q-l)^{\perp}_{\alpha}}{(k+q-l)_+ + i\varepsilon}}\Big[(k-l+q)_\beta\,(k-l)_\alpha - g_{\alpha\beta}\,(k-l)\cdot(k+q-l)\Big]\cdot$$

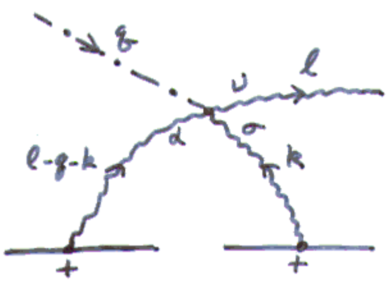
$$\cdot\frac{i}{2l_-\Big[(k-l)_+ + \frac{(\underline{k}-\underline{l})^2}{2l_-} - i\varepsilon\Big]}\Big[g_{\beta\gamma} - \cancel{\frac{\eta_\beta(k-l)_\gamma}{(k-l)_+ + i\varepsilon}} - \frac{\eta_\gamma(k-l)_\beta}{(k-l)_+ - i\varepsilon}\Big]\Gamma_{\gamma\sigma\nu}\,\frac{-i\,k^{\perp}_{\sigma}}{\underline{k}^2\,(k_+ - i\varepsilon)}\,\varepsilon_\nu(l)$$

$\to 0$ if we sum (a) + (b) + (c)

from $\varepsilon_- = 0$

$$\cdot\frac{dk_+}{(2\pi)}\,e^{-ik_+(x_{1-}-x_{2-})}$$

$k_+ = (l-q)_+ \approx -q_+ \Rightarrow \frac{1}{q_+}$ is like $\frac{1}{Q^2}$ ~ small $\Rightarrow \int dk_+\, G_a = 0$



(d)

$$G_d = \frac{-i\,(k-l+q)^{\perp}_{\alpha}}{(\underline{k}-\underline{l}+\underline{q})^2\,[(k+q-l)_+ + i\varepsilon]}\;\frac{-i\,k^{\perp}_{\sigma}}{\underline{k}^2\,(k_+ - i\varepsilon)}\;\varepsilon_\nu\,\Gamma^{q}_{\alpha\sigma\nu}$$

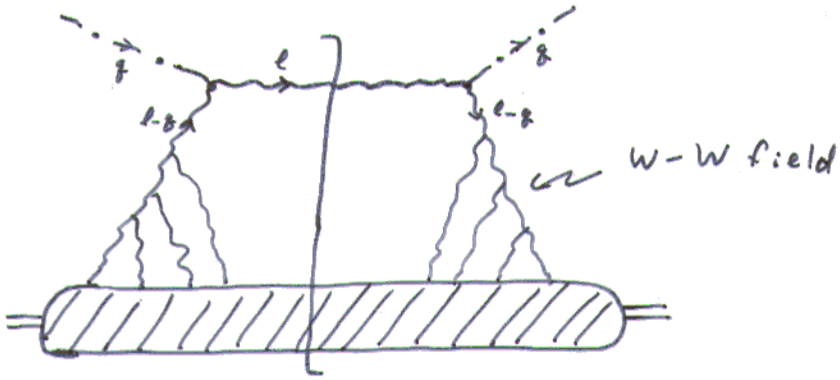
where $\Gamma^{q}_{\alpha\sigma\nu} = g_{\alpha\sigma}(l-q-2k)_\nu - g_{\alpha\nu}(2l-q-k)_\sigma + g_{\nu\sigma}(k+l)_\alpha$

$\Rightarrow$ also $\int dk_+\, G_d = 0.$

True in general, for higher order graphs: we pick $k_+ \approx -q_+$ pole and since Q^2 is big get suppression.

The current's lifetime is $\Delta x_- \approx \frac{2q_-}{Q^2}$ ~ small $\Rightarrow$ we don't have enough range in x_- integration to generate a small light cone denominator.

Now that we have proved that in the large Q^2 limit in the light cone gauge there is no final state interaction and no multiple current–gluons vertices, the current–nucleus interaction looks like:

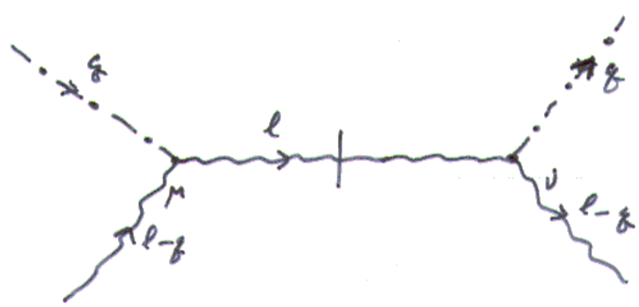


One can show that the number density of the produced gluons is :

$$\pi \frac{dN}{d^2 l} = -2\mathrm{Tr}\left\langle A_\mu^\perp(\underline{l}, l_+ - q_+) A_\nu^\perp(-\underline{l}, -l_+ + q_+)\right\rangle$$

$$\times \left[g_{\mu\nu}^\perp \frac{(Q^2 - \underline{l}^2)^2}{4} - l_\mu^\perp l_\nu^\perp Q^2 \right] \frac{1}{\underline{l}^2},$$

where the term in the square brackets comes from the following graph:



Taking the leading Q^2 term yields:

$$\tilde{N}(x_\perp^2) = -\frac{2}{\pi} \int d^2 b \mathrm{Tr}\left\langle A_\mu^\perp(\underline{b}) A_\mu^\perp(\underline{b} + \underline{x})\right\rangle .$$

Rewriting the non-Abelian Weizsäcker–Williams field result from Phys. Rev. D **54**, 5463 (1996) as

$$A_\mu^\perp(\underline{x}, x_-) = \int d^2b\, db_-\, S(\underline{x}, b_-) T^a S^{-1}(\underline{x}, b_-)$$

$$\times \frac{\underline{x} - \underline{b}}{|\underline{x} - \underline{b}|^2} \hat{\rho}^a(\underline{b}, b_-) \theta(b_- - x_-),$$

with

$$S(\underline{x}, x_-) = \mathrm{P} e^{\left\{ i s T^a \int \ln(|\underline{x} - \underline{b}|\mu) \hat{\rho}^a(\underline{b}, b_-) \theta(b_- - x_-) d^2b\, db_- \right\}}$$

and

$$\left\langle \hat{\rho}^a(\underline{b}, b_-) \hat{\rho}^{a'}(\underline{b}', b'_-) \right\rangle = \frac{\rho(\underline{b}, b_-)}{N_c^2 - 1} Q^2 \frac{\partial}{\partial Q^2} x G(x, Q^2)$$

$$\times \delta(\underline{b} - \underline{b}') \delta(b_- - b'_-) \delta^{aa'},$$

where the expectation is taken in the nuclear wave function, we can calculate the gluon field correlation function and the number density of the produced gluons:

$$\tilde{N}(x_\perp^2) = \frac{N_c^2 - 1}{\pi^2 \alpha N_c x_\perp^2} \int d^2b$$

$$\times \left[1 - \exp\left(- \frac{2\pi^2 \alpha N_c x_\perp^2 \sqrt{R^2 - b^2}}{N_c^2 - 1} \rho x G(x, 1/x_\perp^2) \right) \right].$$

This exactly corresponds to the covariant gauge result!!!

The details of calculations:
plugging in the non-Abelian Weizsäcker–Williams field

$$\tilde{N}(x_\perp^2) = -\frac{2}{\pi}\int d^2b \int d^2b' db'_- d^2b'' db''_-$$

$$\times \left\langle \frac{\underline{b}-\underline{b}'}{|\underline{b}-\underline{b}'|^2} \cdot \frac{\underline{b}+\underline{x}-\underline{b}''}{|\underline{b}+\underline{x}-\underline{b}''|^2} \hat{\rho}^a(\underline{b}', b'_-)\hat{\rho}^b(\underline{b}'', b''_-)\right.$$

$$\left. \times \, \mathrm{Tr}[S(\underline{b}, b'_-)T^a S^{-1}(\underline{b}, b'_-) S(\underline{b}+\underline{x}, b''_-) T^a S^{-1}(\underline{b}+\underline{x}, b''_-)]\right\rangle .$$

Using the density correlation function:

$$\tilde{N}(x_\perp^2) = 4\frac{\rho_{rel}}{N_c^2-1}\ln(|\underline{x}|\mu) Q^2 \frac{\partial}{\partial Q^2} xG(x, Q^2)$$

$$\times \int d^2b db'_- \left\langle \mathrm{Tr}[S(\underline{b}, b'_-)T^a S^{-1}(\underline{b}, b'_-) S(\underline{b}+\underline{x}, b'_-) T^a S^{-1}(\underline{b}+\underline{x}, b'_-)]\right\rangle .$$

By the definition of $S(\underline{b}, b_-)$

$$S(\underline{b}, b_-) = \prod_i \left[1 + igT^a \hat{\rho}^a(\underline{y}, y_-)\ln(|\underline{b}-\underline{y}|\mu) d^2y \triangle y_{i-}\right.$$

$$\left. -(1/2)g^2 T^a T^b \hat{\rho}^a(\underline{y}, y_-)\hat{\rho}^b(\underline{y}', y'_-)\ln(|\underline{b}-\underline{y}|\mu)\ln(|\underline{b}-\underline{y}'|\mu) d^2y \triangle y_{i-} d^2y' \triangle y'_{i-}\right] .$$

Going term by term we get:

$$\left\langle \mathrm{Tr}[S(\underline{b}, b'_-)T^a S^{-1}(\underline{b}, b'_-) S(\underline{b}+\underline{x}, b'_-) T^a S^{-1}(\underline{b}+\underline{x}, b'_-)]\right\rangle =$$

$$\left(1 - g^2 \frac{\pi \rho_{rel} N_c x_\perp^2}{4(N_c^2-1)} xG(x, 1/x_\perp^2)\triangle y_-\right)$$

$$\times \left\langle \mathrm{Tr}[S(\underline{b}, b'_- - \triangle y_-)T^a S^{-1}(\underline{b}, b'_- - \triangle y_-) S(\underline{b}+\underline{x}, b'_- - \triangle y_-) T^a S^{-1}(\underline{b}+\underline{x}, b'_- - \triangle y_-)]\right\rangle ,$$

This yields for the trace:

$$\left\langle \mathrm{Tr}[S(\underline{b}, b'_-)T^a S^{-1}(\underline{b}, b'_-) S(\underline{b}+\underline{x}, b'_-) T^a S^{-1}(\underline{b}+\underline{x}, b'_-)]\right\rangle =$$

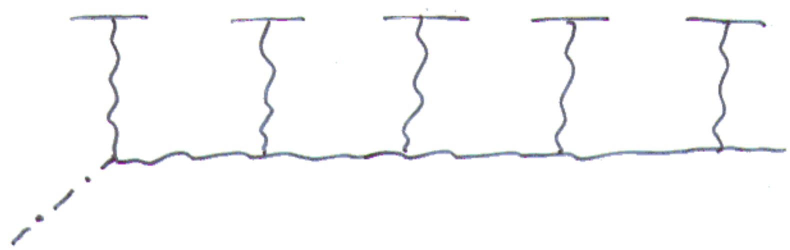
$$\exp\left(-g^2 \frac{\pi \rho_{rel} N_c x_\perp^2}{4(N_c^2-1)} xG(x, 1/x_\perp^2)(b'_- + b'_{0-})\right) .$$

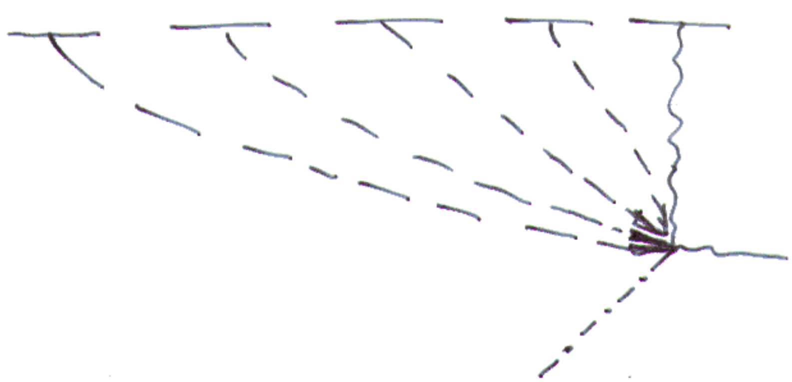
Integrating over b'_- we easily obtain the answer.

We observe that the final state interactions, which are present in covariant gauge, are absent in the light cone gauge.

There they become encoded in the wave function of the nucleus.

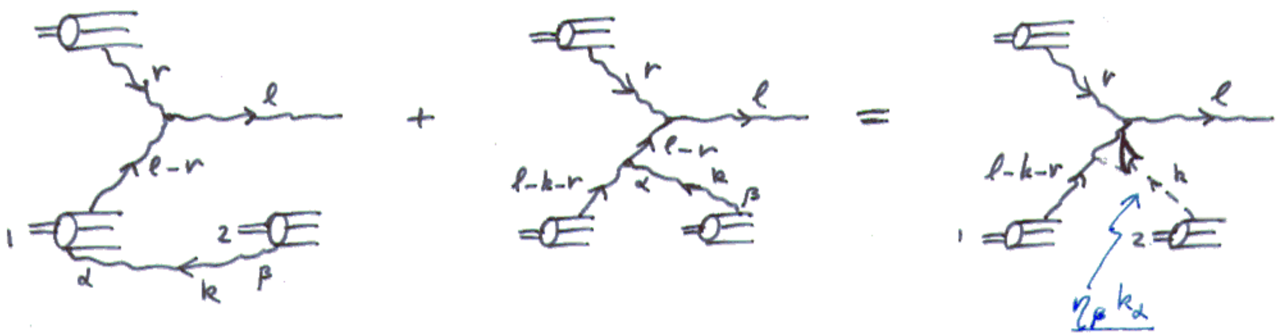
Glauber expansion in light cone gauge becomes a property of the wave function.



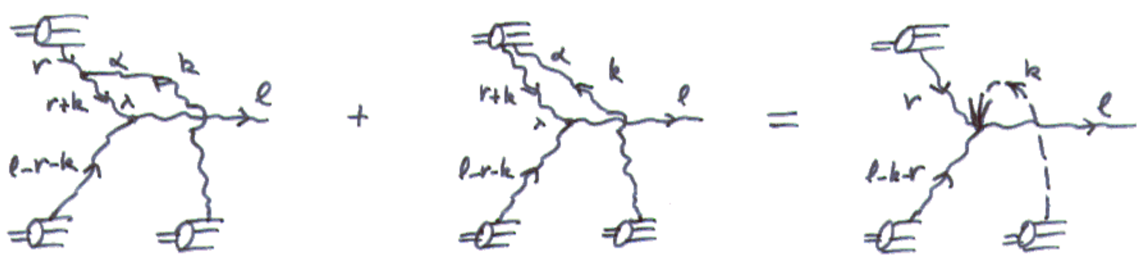


b. Nucleon–nucleus scattering in light cone gauge.

choose $A_+ = 0$ gauge with $\varepsilon_- = 0$ (for simplicity)
start with two nucleons:



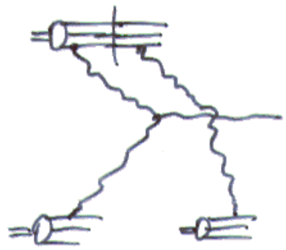
just like with non-Abelian Weizsäcker–Williams field, only $\frac{\eta_\beta k_\alpha}{k_+ - i\varepsilon}$ part of the $D_{\alpha\beta}$ propagator contributes; have $e^{ik_+(x_{2-} - x_{1-})}$ factor which makes us pick up the pole.



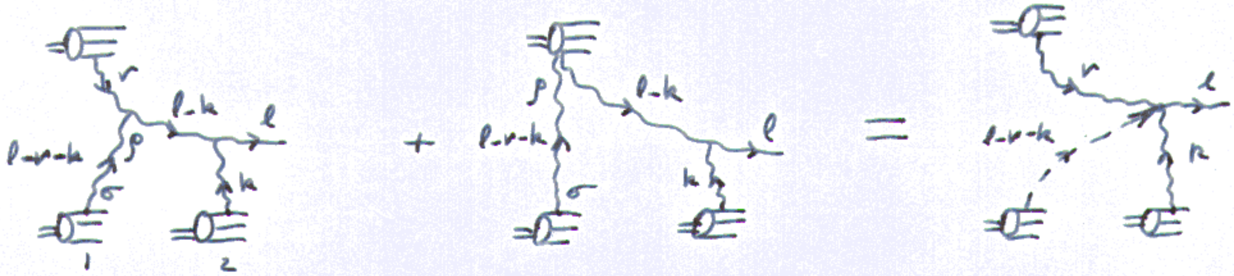
There's also a $\frac{(r+k)_\lambda}{(r+k)_+ - i\varepsilon}$ term, but it cancels.

Therefore, again we pick $\frac{\eta_\beta k_\alpha}{k_+ - i\varepsilon}$ pole & use Ward identity

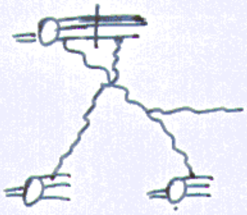
We also have a term like



but it goes away due to real-virtual cancellation



Here we have to distort the k_+ integration contour in the lower half plane to pick $\dfrac{\eta_{\sigma}(l-r-k)_\rho}{l_+-r_+-k_+-i\varepsilon}$ pole. It's not natural for $e^{ik_+(x_{2-}-x_{1-})}$, but we have enough factors of k_+ in the denominator.

We also get  but it's canceled as before.

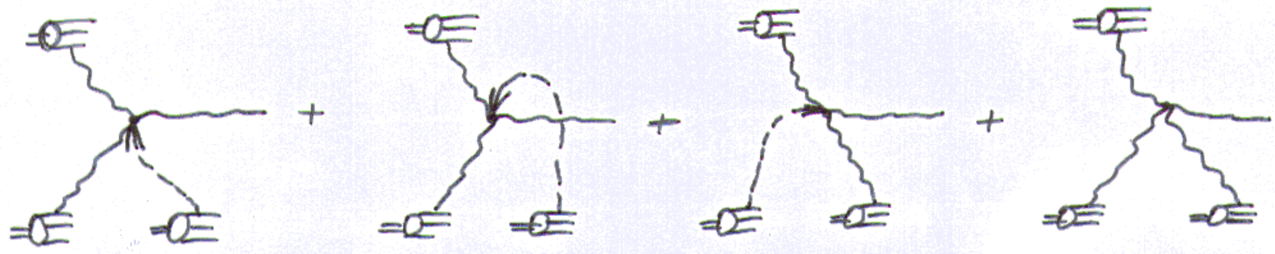
Finally, there's  graph.

In general

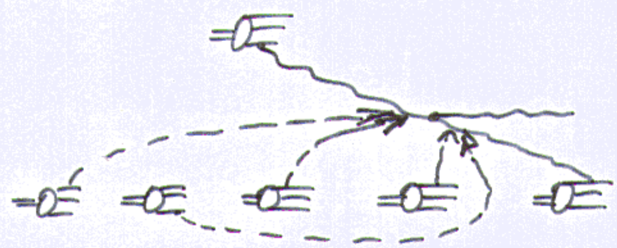
$\eta_\alpha k_\beta$'s cancel after summing over all connection

either longitudinally polarized, or becomes a final state interaction and goes away.

Therefore, all the non–zero contributions of the graphs at this order are:



The same is true for all orders. One can picture the final answer as:



We denote the non-Abelian Weizsäcker–Williams field of the nucleus by

$$A_\mu^\perp(\underline{x}, x_-).$$

The "classical" field of the nucleon is

$$A'_\mu(\underline{x}, x_+, \underline{b}) = -\eta_\mu S(\underline{x}) T^a S^{-1}(\underline{x}) \delta(x_+) \ln(|\underline{x} - \underline{b}|\mu) \hat{\rho}_N^a,$$

with

$$\langle \hat{\rho}_N^a \hat{\rho}_N^{a'} \rangle = \frac{\delta^{aa'}}{N_c^2 - 1} Q^2 \frac{\partial}{\partial Q^2} x G(x, Q^2).$$

Define free quantized gluon field $A_\mu^{free}(x)$, normalized according to

$$(0|A_\mu^{free}(x)|l\lambda a) = \frac{\epsilon_\mu^{(\lambda)}(l) e^{il\cdot x}}{\sqrt{(2\pi)^3 2\omega_k}} T^a.$$

To write a compact formula for the cross–section define a total gluon field $A_\mu^{tot}(x)$:

$$A_\mu^{tot}(x) = A_\mu^\perp(x) + A'_\mu(x, \underline{b}) + A_\mu^{free}(x).$$

Then the gluon production cross–section is

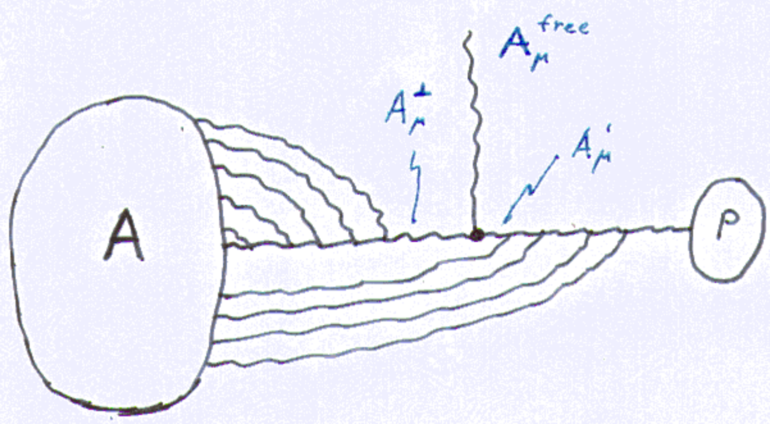
$$\frac{d\sigma}{d^2l\,dy} = \omega_l \int d^2b \left\langle \sum_{\lambda=1}^{2} \sum_{a=1}^{N_c^2-1} (0|S|l\lambda a)(l\lambda a|S|0) \right\rangle,$$

with

$$S = -\frac{1}{2} \int d^4x \,\mathrm{Tr} F_{\mu\nu}(x) F_{\mu\nu}(x)$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu^{tot} - \partial_\nu A_\mu^{tot} - ig[A_\mu^{tot}, A_\nu^{tot}.]$$



Conclusions:

⋆ We found an answer for the quasi–classical gluon production in current–nucleus and pA scattering in covariant gauge.

⋆ In light cone gauge multiple rescatterings in the final state turn into the property of the wave function. Glauber expansion is encoded in light cone wave function.

⋆ Found a partonic interpretation of the process in light cone gauge.