What have we learnt from HBT interferometry at the SPS?

U. Heinz, CERN/Regensburg

- Introduction
- Emission fctn, spectra & correlations
- HBT radii and homogeneity lengths
- Pb + Pb data analysis & interpretation
- Average freeze-out phase-space density
- Expectations for RHIC

Collaborators:

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Heavy ion collisions:

2 colliding cold nuclei ⇒ primary NN collisions ⇒ rescattering among secondaries ⇒ dense stage

⇒ expansion thermalization flow ⇒ freeze-out ⇒ measurement

- Large theoretical uncertainties in computing dynamical evolution

  ⇒ Need constraints, empirical & theoretical

- At SPS energies, initial conditions are not reliably calculable

  ⇒ most severe constraints come from observation of the final state

- Richest pool of available data: Hadrons (yields, spectra, 2-particle correlations)

  ⇒ reconstruct hadronic freeze-out stage

HBT: access to space-time structure: geometry <u>and</u> dynamics @ $T_{f.o.}^{therm}$

Fundamental Relations

Shuryak
Pratt
Csörgö
Chapman + u.a.
⋮

no FSI!

(1) $$E_p \frac{dN}{d^3p} = \int d^4x \; S(x,p) \qquad @ \; p^0 = E_p$$

(2) $$C(\vec{q},\vec{K}) \cong 1 \pm \frac{\left| \int d^4x \; e^{iq\cdot x} \, S(x,K) \right|^2}{\left| \int d^4x \; S(x,K) \right|^2}$$

$\vec{q} = \vec{p}_1 - \vec{p}_2$
$\vec{K} = (\vec{p}_1 + \vec{p}_2)/2$

$@ \; q^0 = E_1 - E_2$
$K^0 = \frac{1}{2}(E_1 + E_2) \cong E_K$

momentum space info. $\Longleftrightarrow$ phase space (includes coordinate space info.)

measurable — theoretical interest

The mass-shell constraint:

$q^0 \neq E_q$ off-shell, but fixed by "mass-shell constraint" $K \cdot q = 0$:

$$\boxed{q^0 = \vec{\beta} \cdot \vec{q} \quad \text{with} \quad \vec{\beta} = \frac{\vec{K}}{K^0} \cong \frac{\vec{K}}{E_K}}$$

$\rightarrow$ Fourier transform $\int d^4x \, e^{iq\cdot x} S(x,K)$

not invertible!

$$\boxed{C(\vec{q},\vec{K}) \approx 1 \pm \frac{\left|\int d^4x \, e^{i\vec{q}\cdot(\vec{x}-\vec{\beta}t)} S(x,K)\right|^2}{\left|\int d^4x \, S(x,K)\right|^2}}$$

- For time-independent sources $C(\vec{q},\vec{K})$ measures F.T. of spatial source distribution
- For time-dependent source $C(\vec{q},\vec{K})$ mixes spatial and temporal information of source.

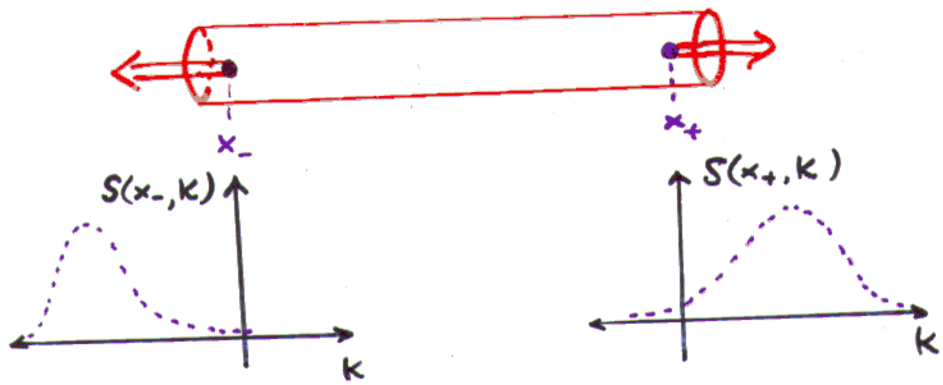
$\vec{K}$-dependence of

- If $S(x, K) \neq f(x)\, g(K)$, i.e. does not factorize

$$\longrightarrow \quad C(\vec{q}, \vec{K}) \text{ depends on } \vec{K}\,!$$

- Typical source with x – K – correlations:

Expanding sources



- $\boxed{\text{Collective dynamics} \quad \Downarrow \ \Uparrow ? \quad \vec{K}\text{-dependence of correlation fct.}}$

? Can one reconstruct the full space-time structure of a dynamical source from HBT correlation functions?

The Gaussian approximation:

Write

$$\boxed{S(x,K) = N(K)\, S(\bar{x}(K),K)\, e^{-\frac{1}{2}(x-\bar{x})_\mu B^{\mu\nu}(K)(x-\bar{x})_\nu} + \delta S(x,K)}$$

!

$N(K)$, $\bar{x}^\mu(K)$, $B^{\mu\nu}(K)$ fixed by

$$\int d^4x\, \delta S = \int d^4x\, x^\mu\, \delta S = \int d^4x\, x^\mu x^\nu\, \delta S \overset{!}{=} 0$$

$\rightsquigarrow$

$$N(\vec{K}) = \frac{\det^{1/2} B_{\mu\nu}(\vec{K})}{S(\bar{x}(\vec{K}),K)}\, E_K \frac{dN}{d^3K}$$

$\bar{x}^\mu(\vec{K}) = \langle x^\mu \rangle$ "saddle point" = point of max. emissivity @ $\vec{K}$

$\left(B^{-1}(\vec{K})\right)_{\mu\nu} = \langle (x-\bar{x})^\mu (x-\bar{x})^\nu \rangle =: \langle \tilde{x}^\mu \tilde{x}^\nu \rangle$

"Covariance matrix" = Gaussian width of space-time distribution @ $\vec{K}$!

Then

$$\boxed{C(\vec{q},\vec{K}) = 1 \pm e^{-q^\mu q^\nu \langle \tilde{x}_\mu \tilde{x}_\nu \rangle(\vec{K})} + \delta C(\vec{q},\vec{K})}$$

$\rightsquigarrow$ $\bar{x}(\vec{K})$, $N(\vec{K})$ not measurable !

Cartesian parametrization: eliminate $q^0 = \vec{\beta}\cdot\vec{q}$

$$\boxed{C(\vec{q},\vec{K}) = 1 + \lambda(\vec{K}) \exp\left[-R_s^2(\vec{K})\, q_s^2 - R_{out}^2(\vec{K})\, q_{out}^2 - R_l^2(\vec{K})\, q_l^2 - 2\, R_{ol}^2(\vec{K})\, q_{out}\, q_l\right]}$$

$$R_s^2 = \langle \tilde{y}^2 \rangle$$

$$R_{out}^2 = \langle (\tilde{x} - \beta_\perp \tilde{t})^2 \rangle = R_s^2 + \beta_\perp^2 \langle \tilde{t}^2 \rangle - 2\beta_\perp \langle \tilde{x}\tilde{t} \rangle + \langle \tilde{x}^2 - \tilde{y}^2 \rangle$$

$$R_l^2 = \langle (\tilde{z} - \beta_l \tilde{t})^2 \rangle$$

$$R_{ol}^2 = \langle (\tilde{x} - \beta_\perp \tilde{t})(\tilde{z} - \beta_l \tilde{t}) \rangle$$

frame dependent!

Yano – Koonin – Podgoretskiĭ parametrization:

$$\boxed{\begin{aligned} C(\vec{q},\vec{K}) &= 1 + \lambda(\vec{K}) \exp\left[-R_\perp^2(\vec{K})\, q_\perp^2 - R_\parallel^2(\vec{K})(q_l^2 - q^{0\,2}) - \left(R_\parallel^2(\vec{K}) + R_0^2(\vec{K})\right)(q\cdot u(\vec{K}))^2\right] \\ u(\vec{K}) &= \gamma(\vec{K})\,(1, 0, 0, v(\vec{K})); \qquad q_\perp^2 = q_{out}^2 + q_s^2 \end{aligned}}$$

$v(\vec{K}) \approx$ long. velocity of source of particles with $\vec{K}$

$R_\perp, R_\parallel, R_0$ invariant under long. boosts! $R_\perp \equiv R_s$

In YK frame ($v(\vec{K}) = 0$):

$$R_\parallel^2(\vec{K}) = \langle \tilde{z}^2 \rangle - 2\frac{\beta_l}{\beta_\perp}\langle \tilde{x}\tilde{z} \rangle + \frac{\beta_l^2}{\beta_\perp^2}\langle \tilde{x}^2 - \tilde{y}^2 \rangle$$

$$R_0^2(\vec{K}) = \langle \tilde{t}^2 \rangle - \frac{2}{\beta_\perp}\langle \tilde{x}\tilde{t} \rangle + \frac{1}{\beta_\perp^2}\langle \tilde{x}^2 - \tilde{y}^2 \rangle$$

Cross-check relations:

(1) $$R_s^2(K) = R_\perp^2(K) = \langle \tilde{y}^2 \rangle$$

(2) YKP $\rightarrow$ Cartesian:

$$R_{diff}^2 = R_{out}^2 - R_s^2 = \beta_\perp^2 \gamma^2 (R_0^2 + v^2 R_\parallel^2)$$

$$R_l^2 = (1-\beta_l^2) R_\parallel^2 + \gamma^2 (\beta_l - v)^2 (R_0^2 + R_\parallel^2)$$

$$R_{ol}^2 = \beta_\perp \left(-\beta_l R_\parallel^2 + \gamma^2 (\beta_l - v)(R_0^2 + R_\parallel^2)\right)$$

(3) Cartesian $\longrightarrow$ YKP

$$A = \frac{R_{diff}^2}{\beta_\perp^2} \qquad B = R_l^2 - 2\frac{\beta_l}{\beta_\perp} R_{ol}^2 + \frac{\beta_l^2}{\beta_\perp^2} R_{diff}^2$$

$$C = -\frac{1}{\beta_\perp} R_{ol}^2 + \frac{\beta_l}{\beta_\perp^2} R_{diff}^2$$

$$R_0^2 = A - vC \qquad R_\parallel^2 = B - vC$$

$$v = \frac{A+B}{2C}\left(1 - \sqrt{1 - \left(\frac{2C}{A+B}\right)^2}\right)$$

Caution: May become negative!

A model for a finite expanding source:

(Csörgő & Lørstad 1994, Chapman, Nix, Heinz, 1995)

$$S(x,K) = N\, m_\perp \operatorname{ch}(y-Y)\, e^{-K\cdot u(x)/T}$$
$$* \exp\left[-\frac{r^2}{2R^2} - \frac{(\eta-\eta_{cm})^2}{2(\delta\eta)^2} - \frac{(\tau-\tau_0)^2}{2(\delta\tau)^2}\right]$$

- Flow profile:

$$u^\mu(x) = \left(\operatorname{ch}\eta_\ell \operatorname{ch}\eta_t,\ \operatorname{sh}\eta_t\, \vec{e}_r,\ \operatorname{sh}\eta_\ell \operatorname{sh}\eta_t\right)$$

with

$$\eta_\ell = \eta = \frac{1}{2}\ln\frac{t+z}{t-z} \qquad \text{boost-inv. long. expansion}$$

$$\eta_t(r) = \eta_f \frac{r}{R} \qquad \text{linear transv. rapidity profile}$$

Parameters:

R transverse size of source $(\langle\tilde{x}^2\rangle = \langle\tilde{y}^2\rangle = R^2)$

τ_0 average freeze-out time

$\delta\eta$ long. size parameter $L = 2\tau_0 \operatorname{sh}\delta\eta$

$\delta\tau$ duration of particle emission

η_f transverse flow rapidity @ $r=R$.

T average freeze-out temperature

Analytical approximation: (qualitative only !!)

Compute $\langle \tilde{x}_\mu \tilde{x}_\nu \rangle$ by saddle point integration:

$$\boxed{\begin{array}{lll} R_\perp^2 = R_*^2 & & \dfrac{1}{R_*^2} = \dfrac{1}{R^2} + \dfrac{1}{R_{flow}^2} \\ R_o^2 = \Delta t_*^2 & & \Delta t_*^2 = (\delta\tau)^2 + 2\left(\sqrt{\tau_0^2 + L_*^2} - \tau_0\right)^2 \\ R_\parallel^2 = L_*^2 & & \dfrac{1}{L_*^2} = \dfrac{1}{(\tau_0 \delta\eta)^2} + \dfrac{1}{L_{flow}^2} \end{array}}$$

(for $Y=0$)

Dynamical lengths of homogeneity:

$$\boxed{\begin{array}{l} R_{flow} = \dfrac{R}{\eta_f}\sqrt{\dfrac{T}{m_\perp}} = \dfrac{1}{\left(\dfrac{\partial \eta_t(r)}{\partial r}\right)}\sqrt{\dfrac{T}{m_\perp}} \\ L_{flow} = \tau_0 \sqrt{\dfrac{T}{m_\perp}} = \dfrac{1}{(\partial \cdot u_\ell)}\sqrt{\dfrac{T}{m_\perp}} \end{array}}$$

Chapman, Scotto, UH (1995)

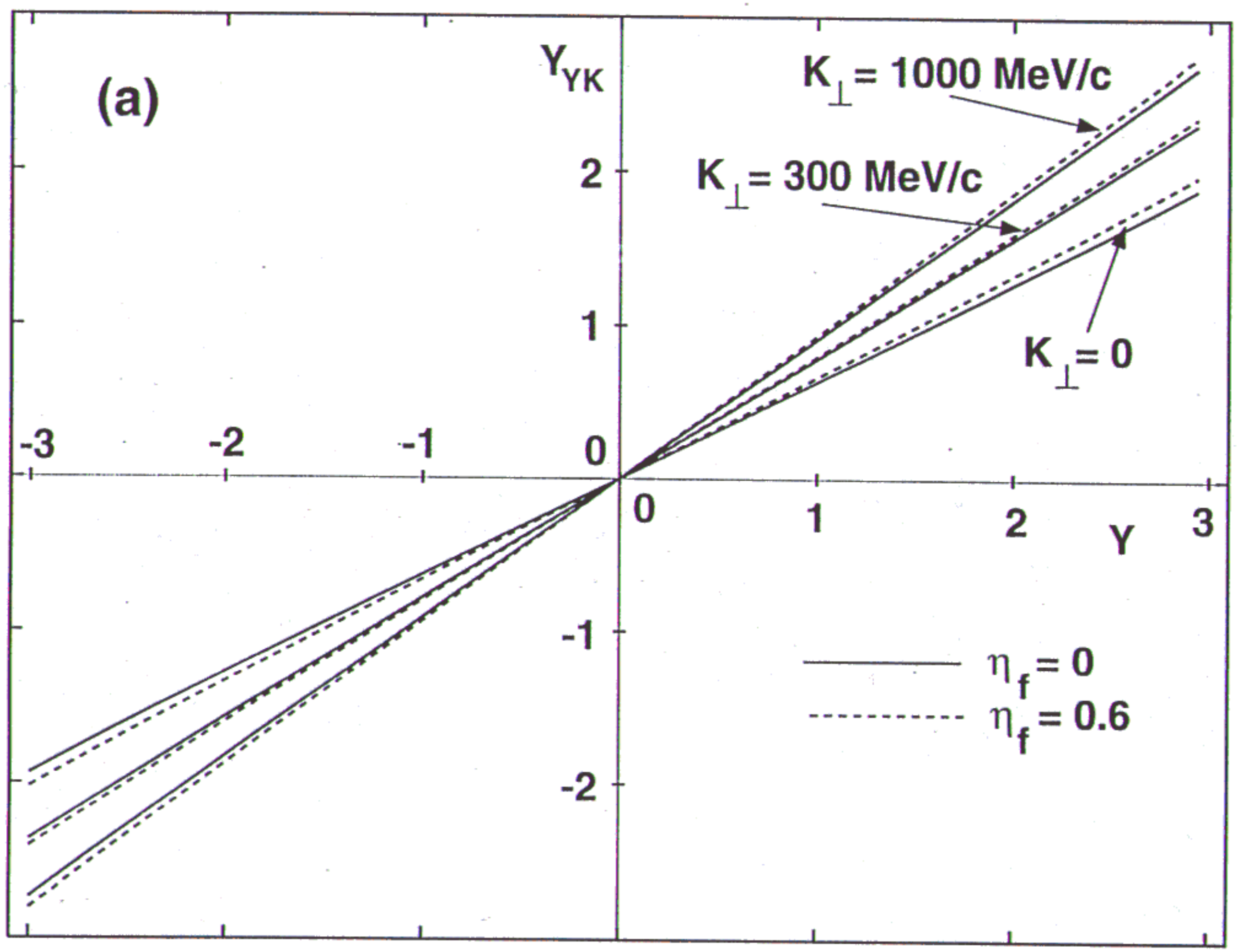
Makhlin + Sinyukov (1987)

(velocity gradient)$^{-1}$

thermal smearing

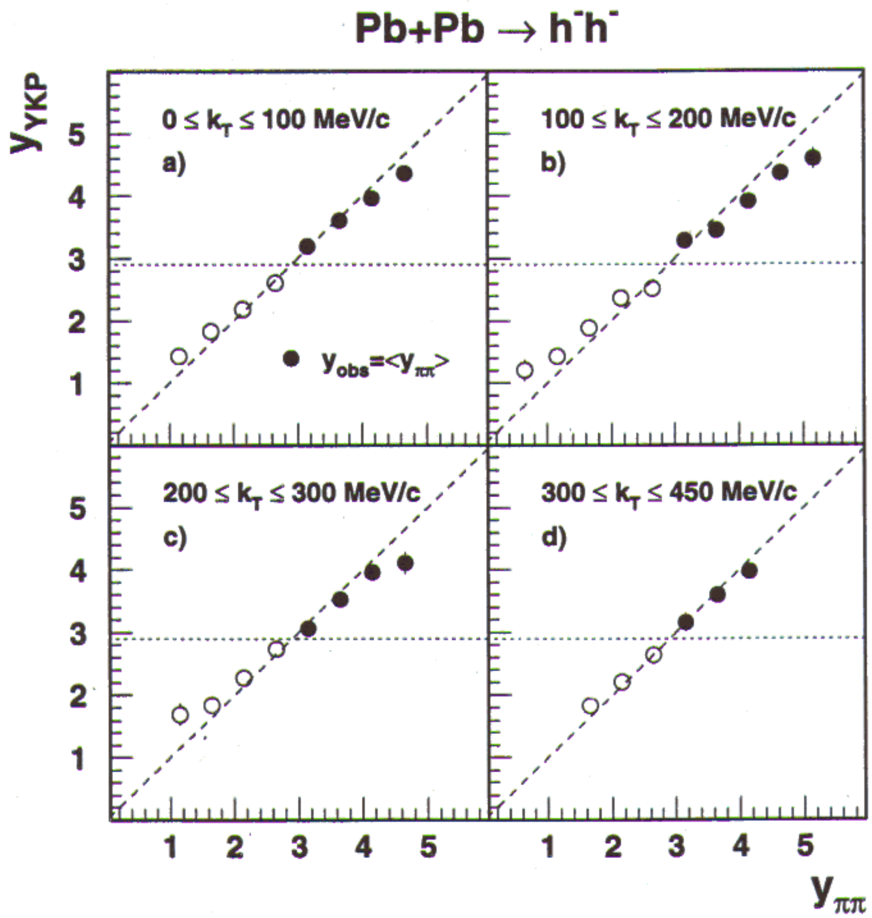
$\rightsquigarrow$ All HBT radii $m_\perp$ - dependent !

Yano - Koonin (source) velocity



Heinz, Wu, Tomášik, Wiedemann, PLB 382 ('96) 181

NA 49

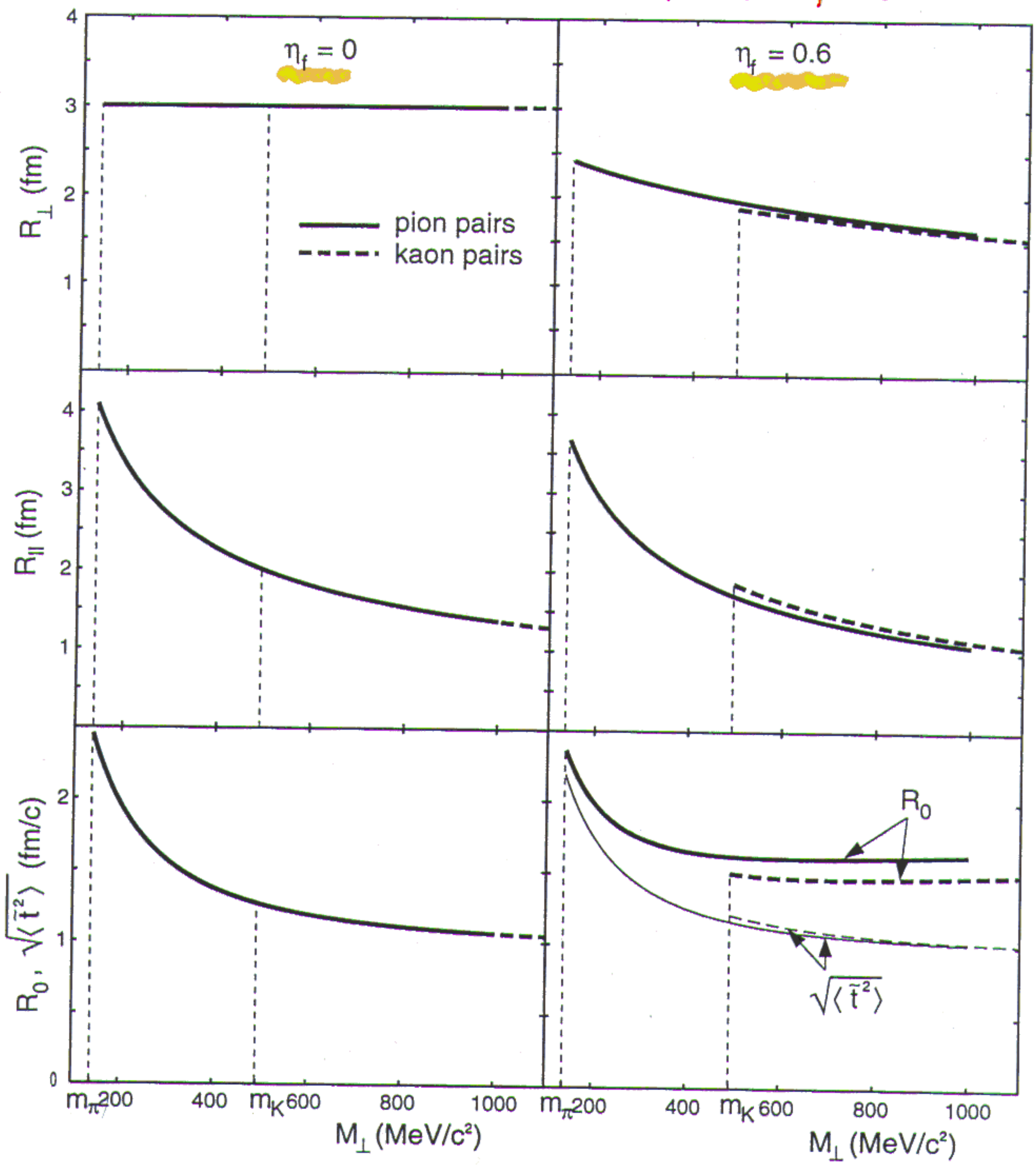


H. Appelshäuser, PhD thesis

Pions vs. kaons — YKP radii vs. $M_\perp$

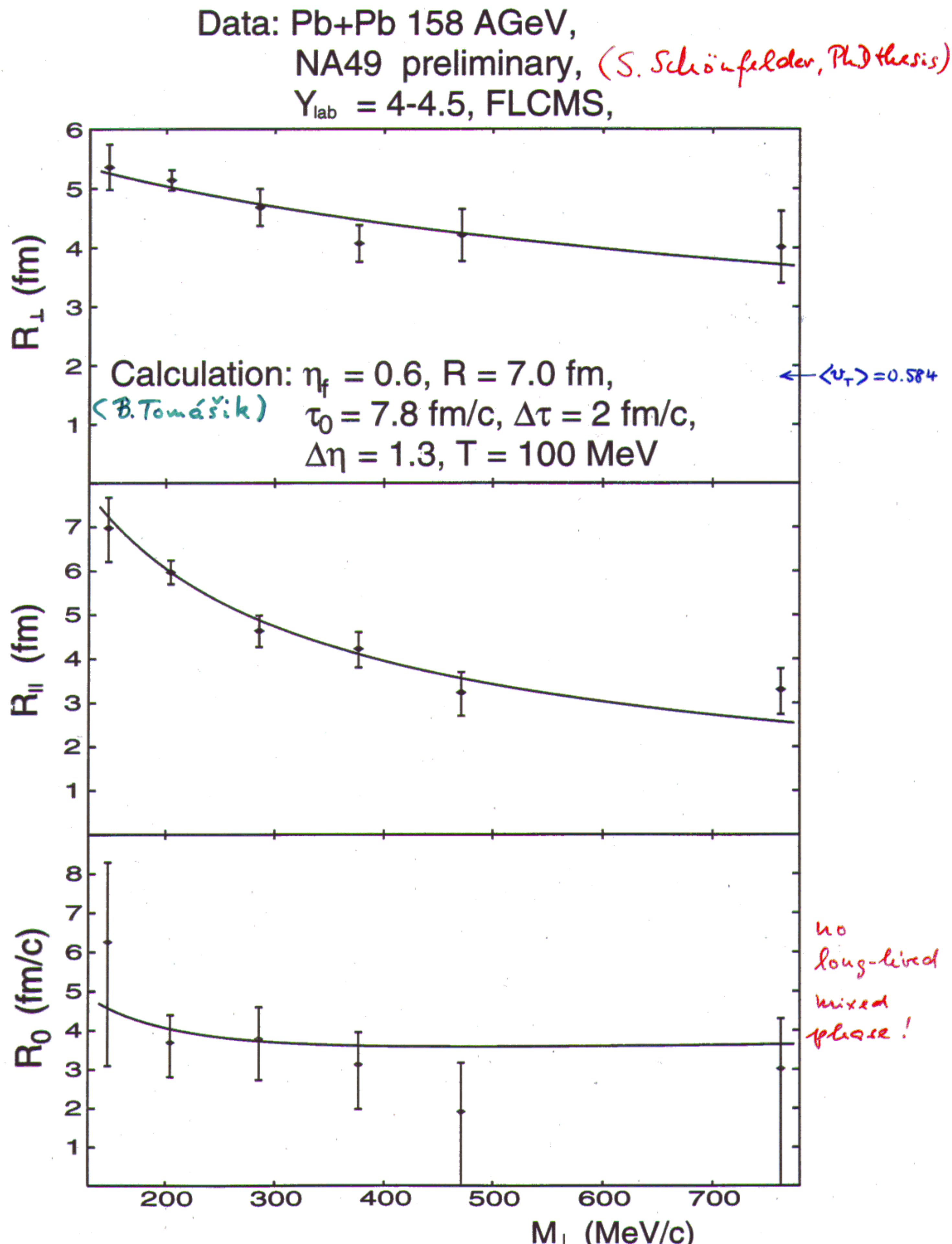
no transv. flow

transv. flow

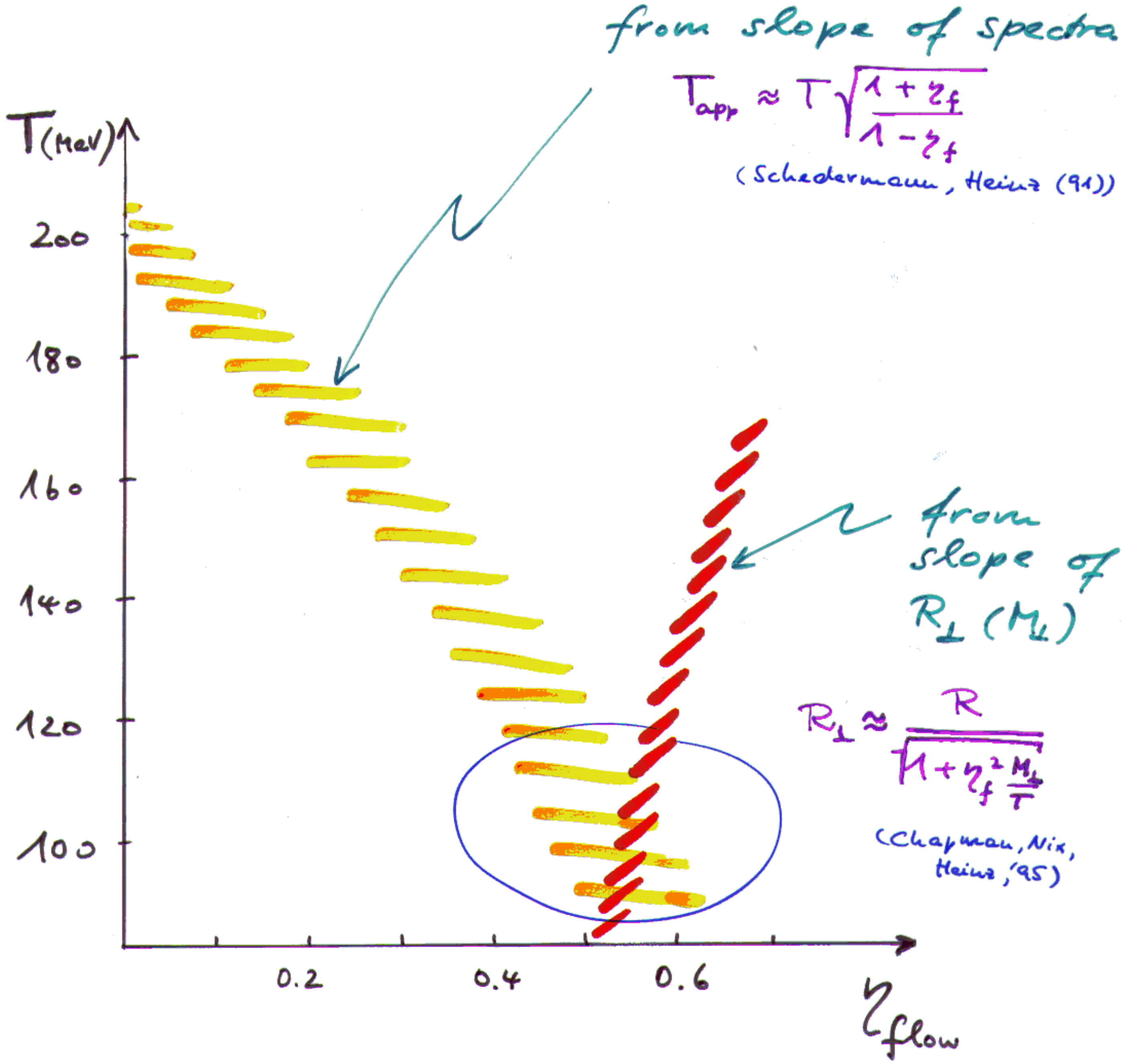


$M_\perp$ - scaling

Fig.10



Data: Pb+Pb 158 AGeV,
NA49 preliminary, (S. Schönfelder, PhD thesis)
Y_{lab} = 4-4.5, FLCMS,
$R_\perp$ (fm)
Calculation: η_f = 0.6, R = 7.0 fm,
τ_0 = 7.8 fm/c, $\Delta\tau$ = 2 fm/c,
$\Delta\eta$ = 1.3, T = 100 MeV
(B. Tomášik)
$\langle v_T \rangle$ = 0.584
$R_\parallel$ (fm)
R_0 (fm/c)
no long-lived mixed phase!
$M_\perp$ (MeV/c)



Spectra ⊕ HBT

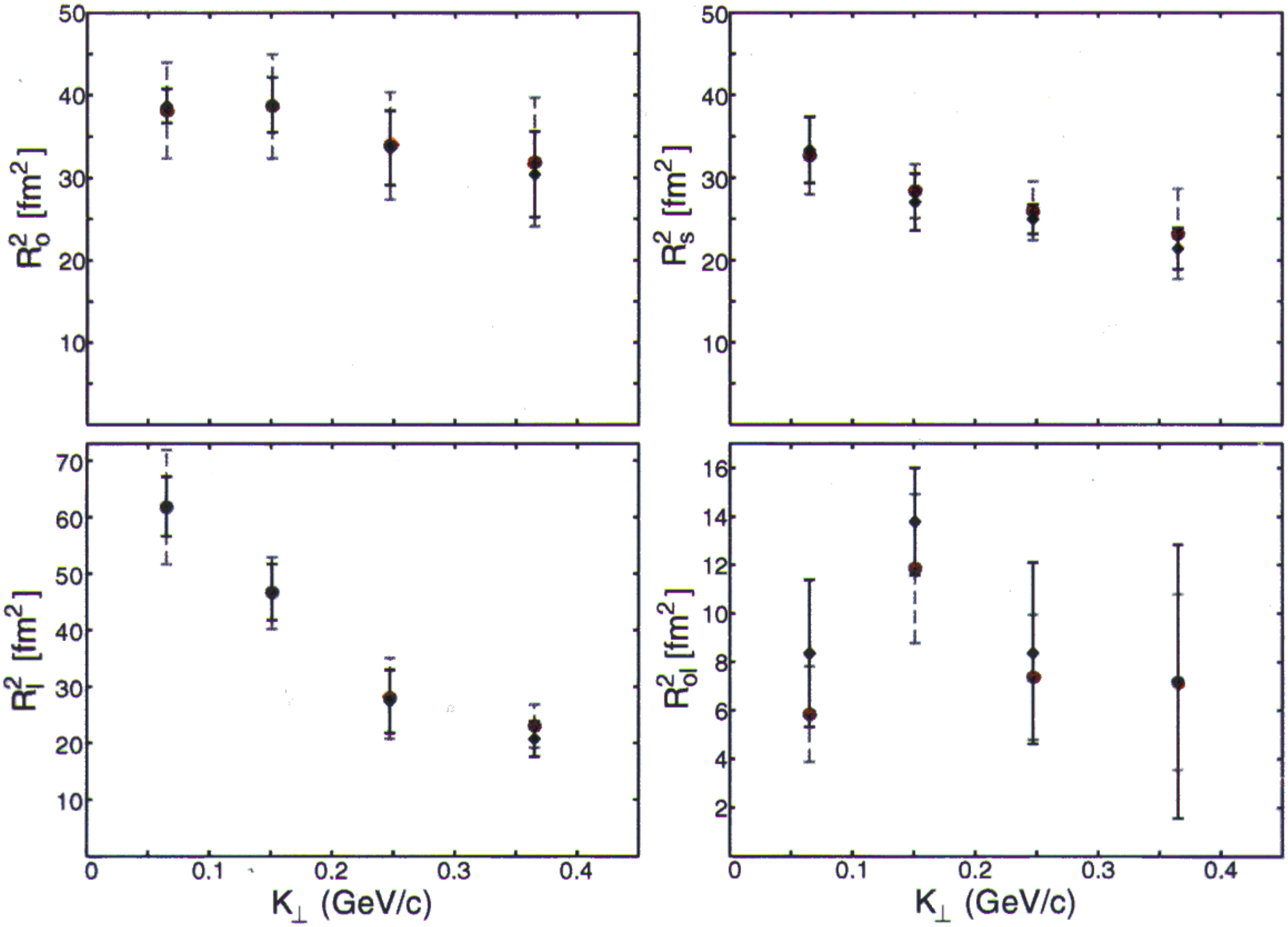
⇓

Separation of T and η_f

(thermal vs. collective)

NA49 Pb+Pb

Cartesian parameters in LCMS

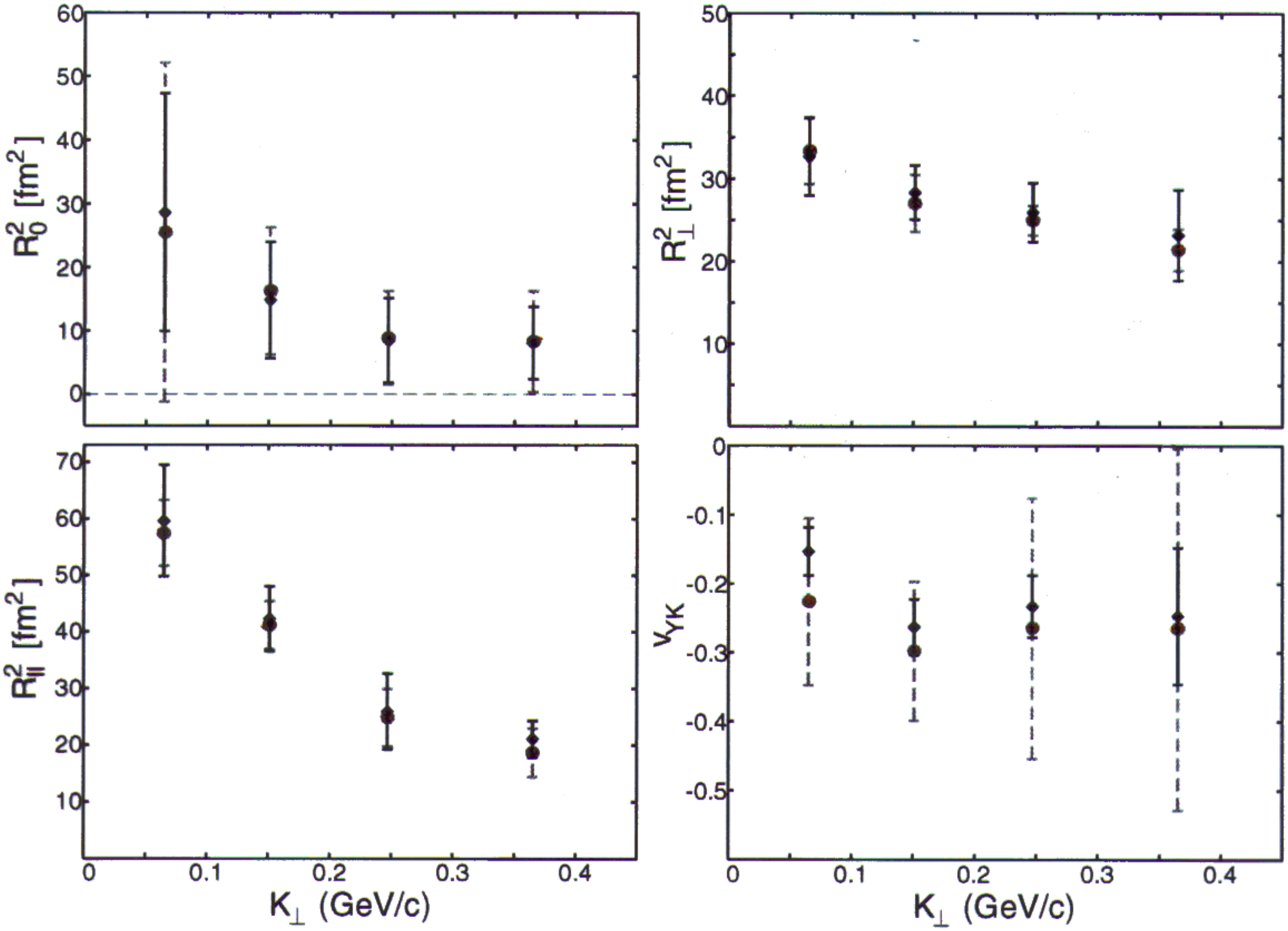


$1 < Y_{cM} < 1.5$

Data compiled from theses of H. Appelshäuser and S. Schönfelder (NA49) by B. Tomášik

NA 49 Pb + Pb

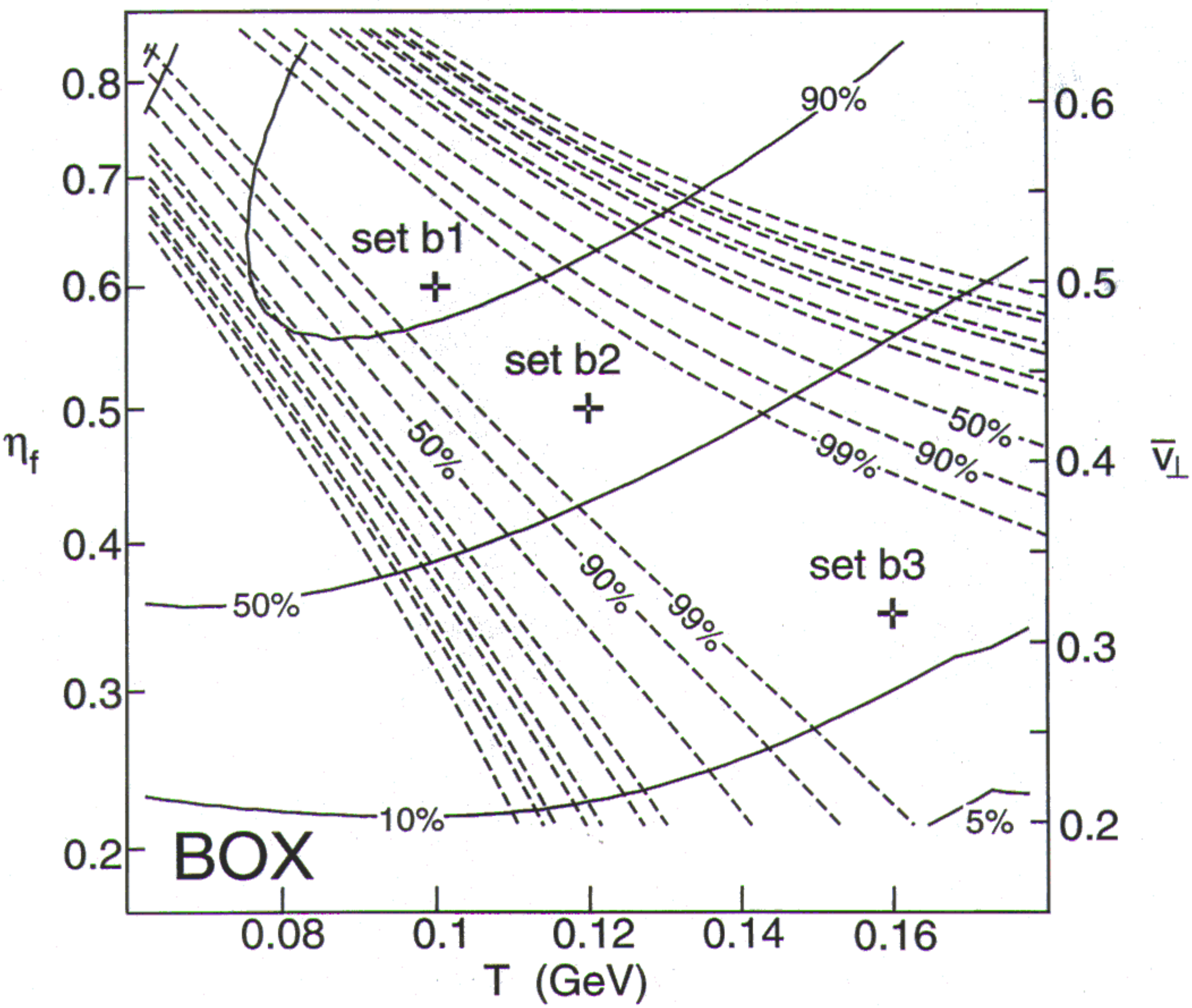
YKP parameters in LCMS



$1 < Y_{cm} < 1.5$

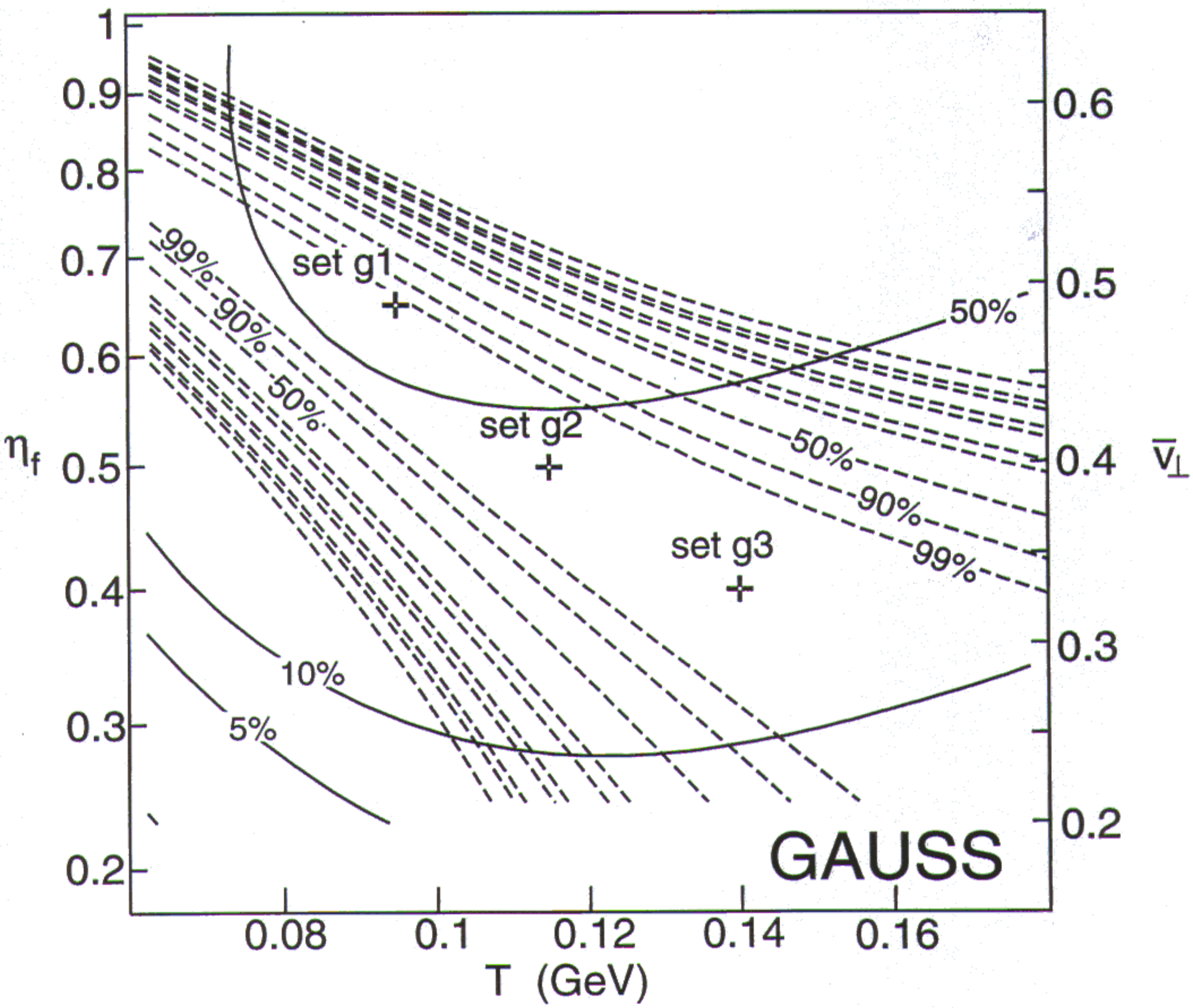
Data compiled from theses of
H. Appelshäuser and S. Schönfelder (NA49)
by B. Tomášik

box-shaped transverse density profile



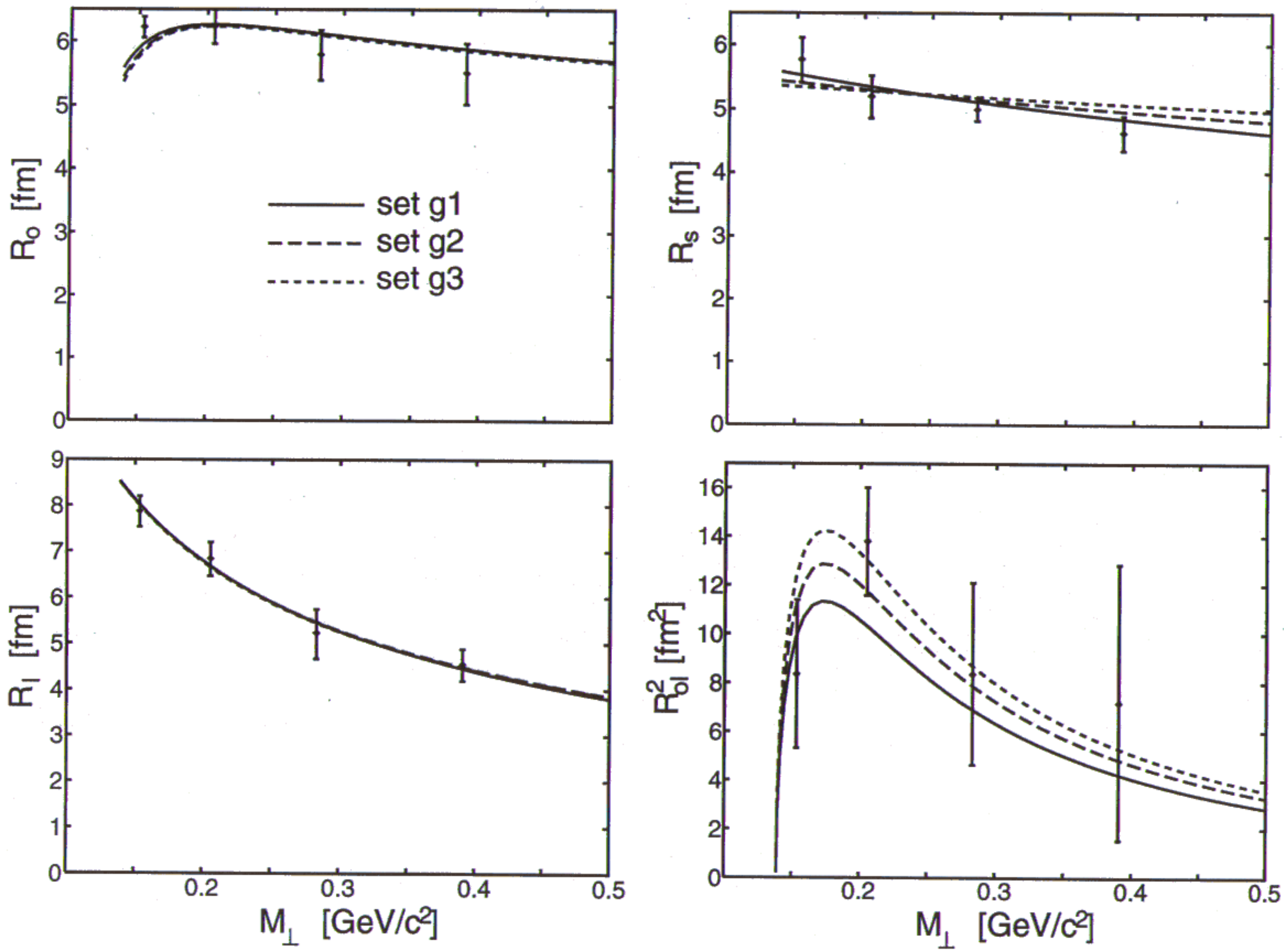
B. Tomášik, PhD thesis

Gaussian transverse density profile



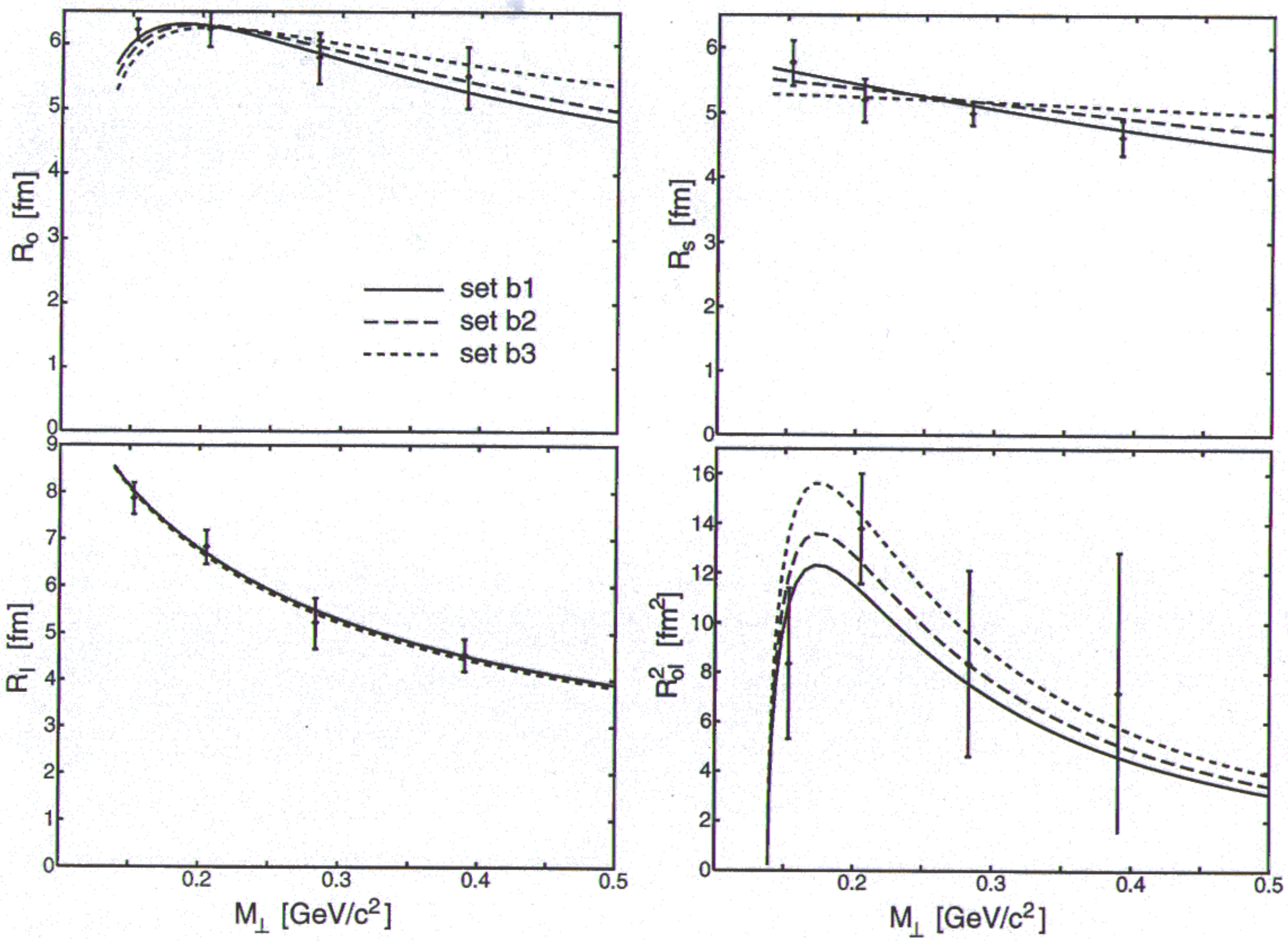
B. Tomášik, PhD thesis

Gaussian transverse density profile



B. Tomášik, PhD thesis

box-shaped transverse density profile



B. Tomášik, PhD thesis

Fit results for NA49 Pb+Pb

in $1 < Y_{cm} < 1.5$

set	box-shaped			Gaussian		
	b1	b2	b3	g1	g2	g3
T (MeV)	100	120	160	95	115	140
η_f	0.6	0.5	0.35	0.65	0.5	0.4
R_B/R_G (fm)	12.12 ± 0.23	11.45 ± 0.21	10.74 ± 0.20	6.72 ± 0.14	6.03 ± 0.13	5.7 ± 0.12
τ_0 (fm/c)	6.30 ± 1.05	5.51 ± 1.21	4.41 ± 3.52	8.35 ± 0.73	6.83 ± 0.89	5.85 ± 0.98
$\Delta\tau$ (fm/c)	3.64 ± 0.61	3.18 ± 0.7	2.55 ± 2.03	2.18 ± 0.75	2.32 ± 0.73	2.09 ± 0.76
$\Delta\eta$ (fixed)	1.3	1.3	1.3	1.3	1.3	1.3
$\bar{v}_\perp$	0.497	0.429	0.314	0.488	0.395	0.33
N	125 ± 21	226 ± 50	735 ± 587	84 ± 8	125 ± 17	262 ± 45

550-650 (Exp.)

(h^-: 716 ± 11)

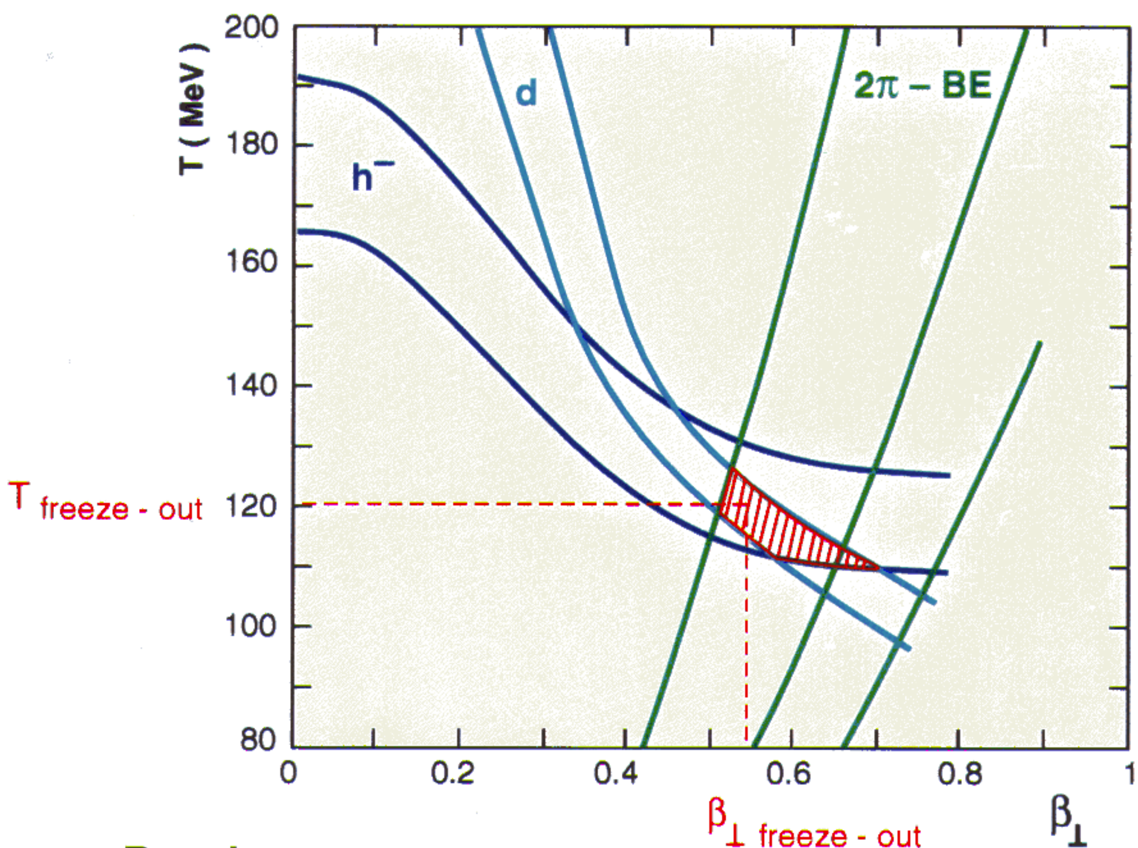
B. Tomášik, PhD thesis

NA49 central Pb+Pb at 158 GeV/Nucleon

Hadronic Expansion Dynamics

- Bose Einstein correlation of **negative pions** (2π - BE)
- and transverse mass spectra of **negative hadrons** (h^-) and **deuterons** (d)

→ **determine the conditions at hadronic decoupling**



Results

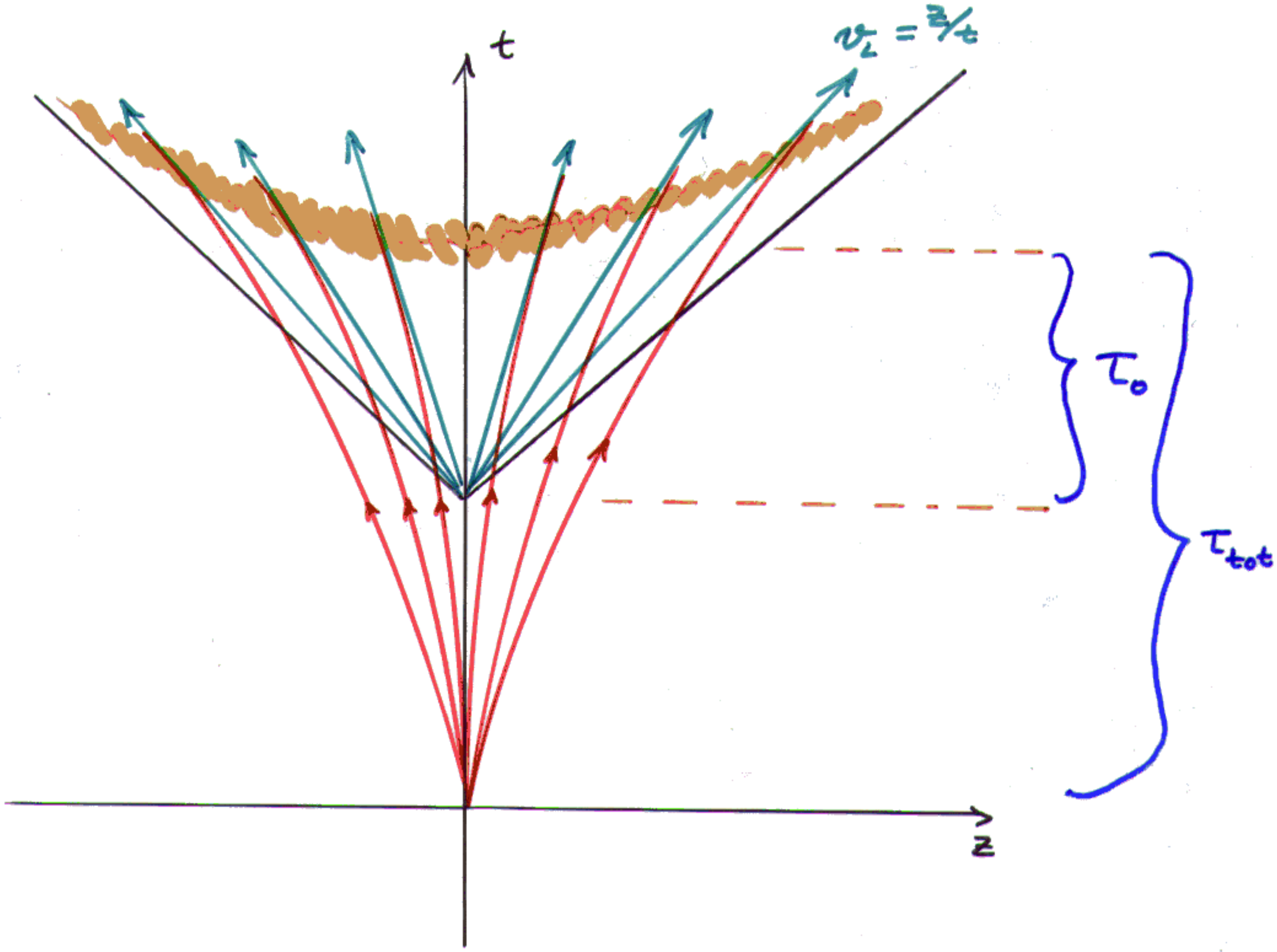
From initial hadronization stage at T = 190 MeV to final hadronic decoupling (freeze - out)

- **Source expanding radially and longitudinally**
- **Duration of expansion $<\tau>$ = 8 fm / c**
- **Local thermal equilibrium**

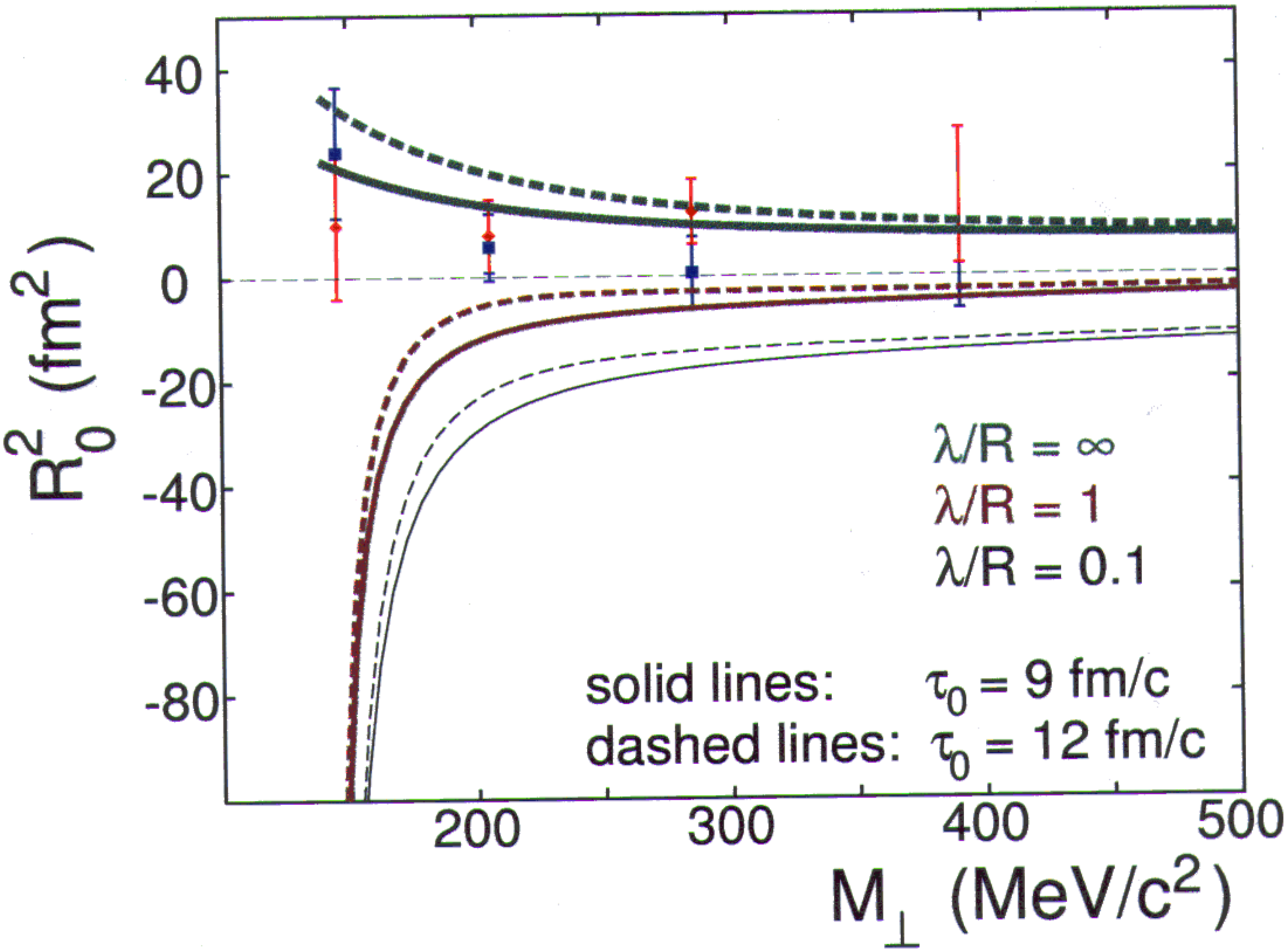
  $T_{freeze - out} = 120 \pm 10$ MeV

  $\beta_{\perp\ freeze - out} = 0.55 \pm 0.12$

  $\beta_{L\ freeze - out} = 0.90$


t
$v_L = z/t$
τ_0
τ_{tot}
z

B. Tomášik + U.H. '97



data: NA49 prelim.
(H. Appelshäuser, PhD. Thesis)
$2.9 < y < 3.4$: h^+h^+, h^-h^-

Average phase-space density at freeze-out:

$$f(\vec{x},\vec{p},t) = \frac{(2\pi)^3}{E_p} \int_{-\infty}^{t} dt'\, S(\vec{x}-\vec{\beta}(t-t'),t';\vec{p})$$

$$\langle f\rangle(\vec{p}) = \frac{\int d^3x\, f^2(\vec{x},\vec{p},t>t_f)}{\int d^3x\, f(\vec{x},\vec{p},t>t_f)}$$

time independent for $t>t_f$

Bertsch (1994): $\langle f\rangle$ can be calculated from $C(\vec{q},\vec{K})$:

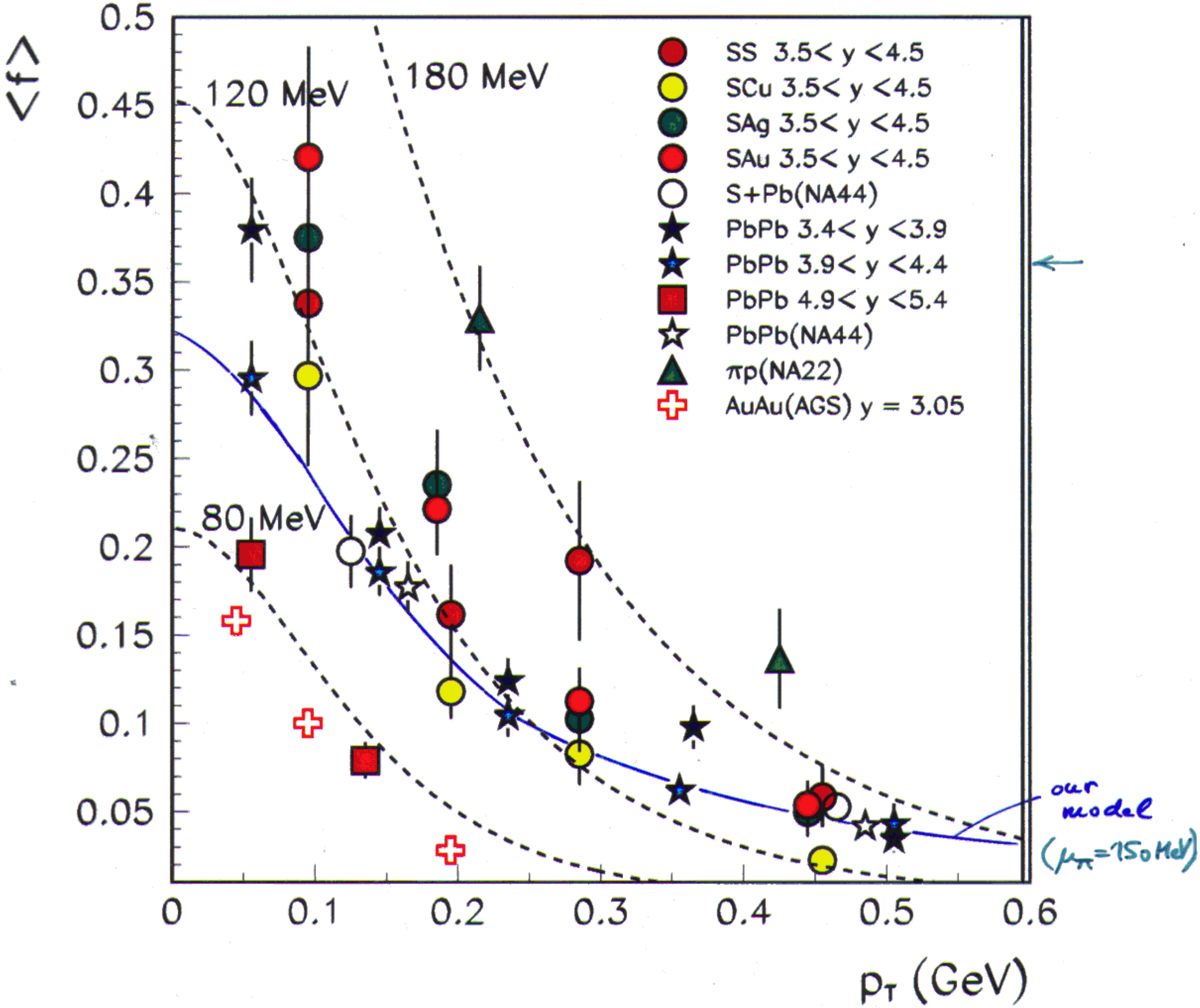
$$\langle f\rangle(\vec{K}) \cong P_1(\vec{K}) \underbrace{\int d^4q\, \delta(q\cdot K)\left(C(\vec{q},\vec{K})-1\right)}_{1/V_{\text{homogen.}}}$$

$$\leadsto \boxed{\langle f\rangle(K_\perp,Y) = \frac{dN}{dY\, M_\perp dM_\perp\, d\Phi} \cdot \frac{1}{V_{\text{eff}}(Y,K_\perp)} \cdot \sqrt{\lambda(K_\perp,Y)}}$$

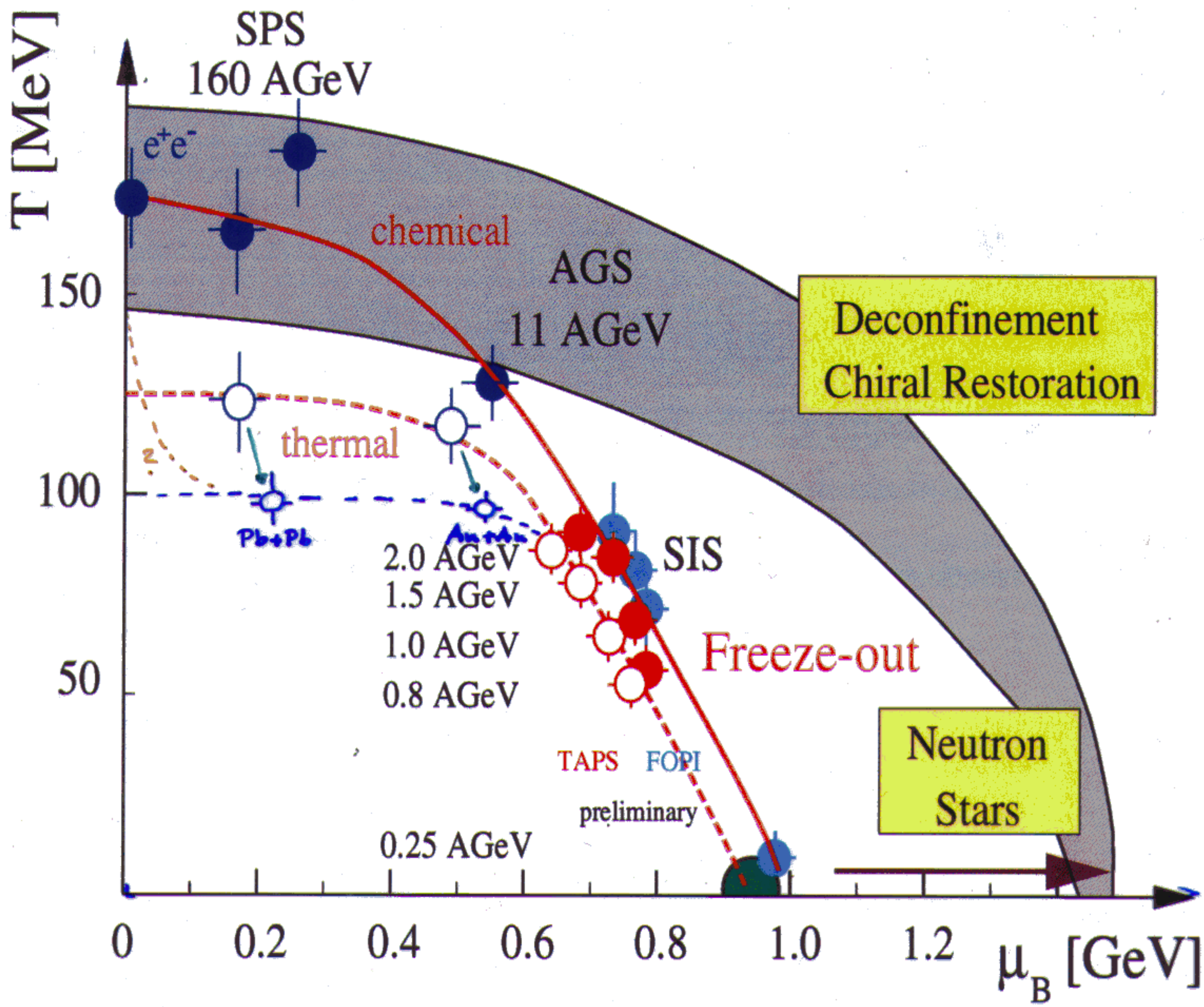
$$\boxed{V_{\text{eff}}(Y,K_\perp) = \frac{M_\perp \operatorname{ch} Y}{\pi^{3/2}}\left(R_s\sqrt{R_o^2R_l^2-(R_{ol}^2)^2}\right)(K_\perp,Y)}$$

(Bertsch / Miśkowiec)

Average freeze-out phase-space density



D. Ferenc, B. Tomášik, U.H., hep-ph/9901230



At RHIC expect thermal f.o. at higher T

(less baryonic "glue" in hadronic phase)

Expectations for RHIC:

$$R_\perp^2 \cong \frac{R^2}{1 + \eta_f^2 \frac{M_\perp}{T}} \qquad \eta(r) = \eta_f \frac{r}{R}$$

$$R_\parallel^2 = \frac{\tau_0^2 \; T/M_\perp}{1 + \frac{T}{M_\perp (\delta\eta)^2}}$$

Schlei et al, Pb + Pb or Au + Au :

$$\tau_0 \big|_{RHIC} \lessapprox 2\, \tau_0 \big|_{SPS}$$

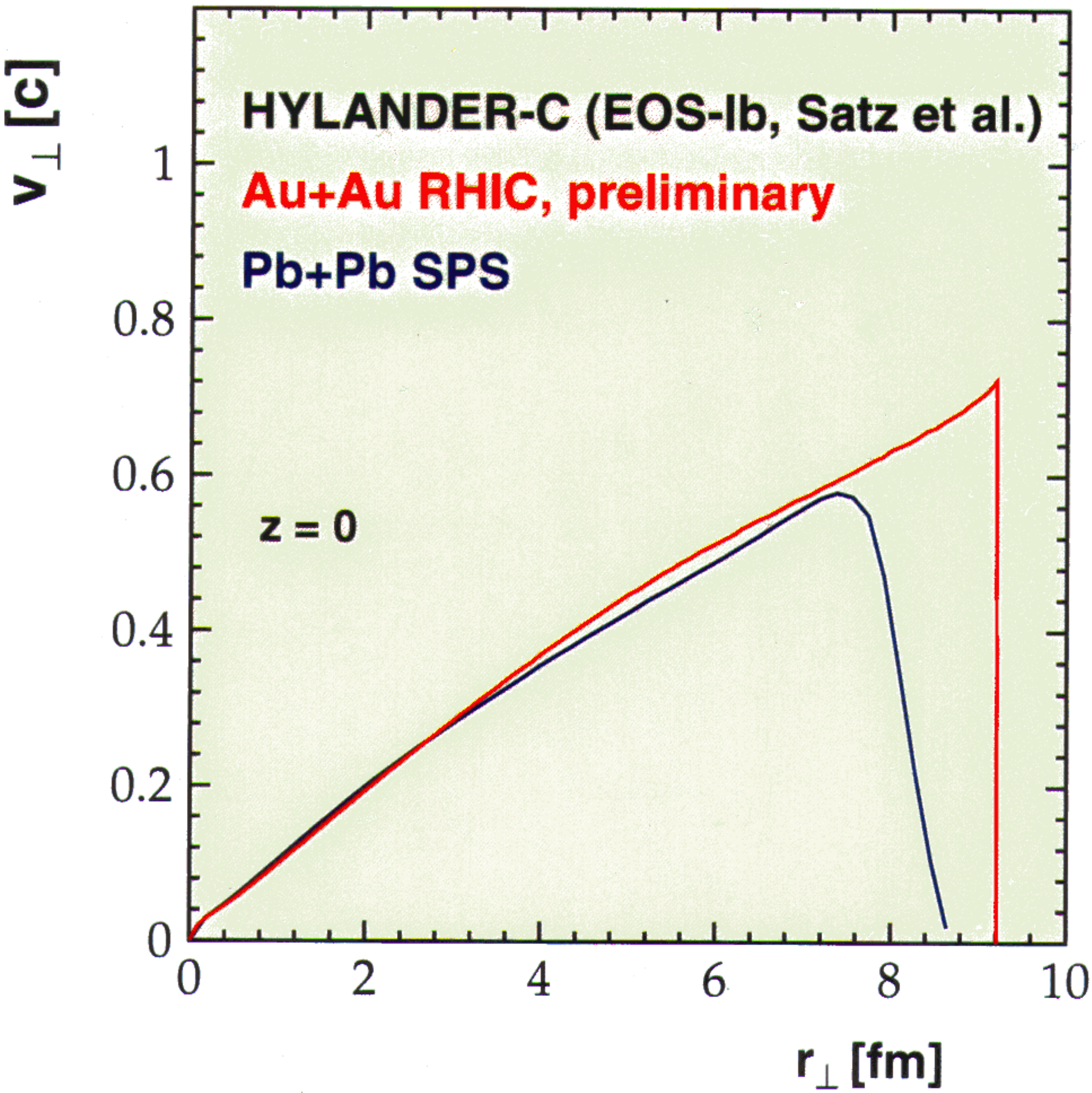
$$\left.\begin{aligned} R \big|_{RHIC} &\approx 1.3\, R \big|_{SPS} \\ \eta_f \big|_{RHIC} &\approx 1.3\, \eta_f \big|_{SPS} \end{aligned}\right\} \frac{\eta_f}{R} \approx \text{const.}\,!$$

$$\rightarrow \begin{cases} R_\perp^{RHIC} \lessapprox 1.2\, R_\perp^{SPS} \\ R_\parallel^{RHIC} \lessapprox 2\, R_\parallel^{SPS} \end{cases}$$

with slightly steeper $M_\perp$-dependence

$R_0^2 \sim (\delta\tau)^2$? Expect it to remain short ("sudden bulk freeze-out")

B.R. Schlei, in preparation.



SUMMARY:

- HBT $\longrightarrow$ Geometry AND Dynamics
- Single particle spectra $\oplus$ HBT $\longrightarrow$

  $\underbrace{\text{"complete"}}_{}$ reconstruction of final state

  up to unavoidable, but rather weak model dependence

  $\longrightarrow$ severe constraints for dynamical models

  $\longrightarrow$ starting point for extrapolations backward in time

- Pb + Pb at CERN SPS:
  - late thermal freeze-out at $T \approx 100$ MeV

    $\bar{v}_\perp \approx 0.5c$

    $T_{therm} \ll T_{chem}$ (100 MeV vs. 180 MeV)
  - freeze-out happens rather suddenly and in bulk
  - at T_{therm} need large μ_π $(N_\pi^{exp} \approx 4\, N_\pi^{therm})$

- $\langle f \rangle(\vec{K}) \approx$ "universal" (varies < 2 for $5 \lesssim \frac{dN^-}{dy} \lesssim 100$)