Searching for the Phase Transition at the AGS Energies

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- Introduction
- Relativistic Landau Theory
- Thermodynamic Properties
- Transport Theory
- Collision Processes
- Elliptic-Flow Data
- Utility of v_2
- Conclusions

Introduction

One of important goals of H-I collisions:
Detection of q-g Plasma

- Folklore on the approach to phase transition:
- As hadrons increase in density they push out from their region more and more of the standard vacuum.
- With the fraction of perturbative vacuum increasing, the average hadron masses decrease.
- At the phase transition the number of degrees of freedom dramatically increases; the masses vanish.

QCD ⇒ Lattice Calcs ⇒ Thermodynamic Properties at $\mu = 0$

Not enough for reactions. . .

Low energy density: individual hadrons interacting in a relatively straightforward fashion; hadronic transport theory had successes at moderate beam energies

Ground-State Nuclear Matter

Properties of q-g plasma out-of-equilibrium uncertain. . .

Hadronization outside of comprehension.

Presumably hadronic distances/time-scales involved. . .

HADRONIC

Idea: Model that is consistent with established limits, such as $\mu = 0$ and low-density hadron matter, that can be applied in more general situations.

Masses: $m_0 \quad \rightarrow \quad m = m_0\, S$

As particle density increases $S \rightarrow 0$.

d.o.f. in $\mu = 0$ q-g plasma: 24 q's + 16 g's = 40

We take N, $\overline{\text{N}}$, Δ, $\overline{\Delta}$, π, ρ

When these pctles become light, the # d.o.f.:

8 + 32 + 3 + 9 = 52

? Formulation of the dynamics ?

At low densities collisions and mean field matter.
The mean field might be used to lower the masses.

Common approach: Lagrangian + mean-field approx.

Baym, Chin NP A262,527 (76)

Relativistic Landau Theory

$$T^{00} = e \equiv e\{\underset{\sim}{f}\}$$

$$\epsilon^i_{\mathbf{p}} = \frac{\delta e}{\delta f^i(\mathbf{p}, \mathbf{r}, t)}$$

SINGLE-PTCLE ENERGY

e – volume energy density, f^i – phase-space density, $(\mathbf{p}, \epsilon_{\mathbf{p}})$ – 4-vector

Simple parametrization of the energy density in the local rest-frame:

$$e = \sum_i \int d\mathbf{p}\, \epsilon^i_{\mathbf{p}}\, f^i(\mathbf{p}) + e_s(\rho_s) + e_v(\rho_v)$$

where

SCALAR – $$\rho_s = \sum_i \int d\mathbf{p}\, \frac{m^i\, m^i_0}{\sqrt{m^{i2} + p^2}}\, f^i(\mathbf{p})$$

VECTOR ≡ BARYON DENSITY – $$\rho_v = \sum_i B^i \int d\mathbf{p}\, f^i(\mathbf{p})$$

Thermodynamic Properties

General picture at $\mu = 0$:

As T increases so does the hadron density.

As the density increases, the hadron masses decrease, leading to an additional increase in the density.

SYSTEM UNSTABLE

Eventually a phase transition occurs.

Consistency condition (as in the Walecka m.) at fixed T:

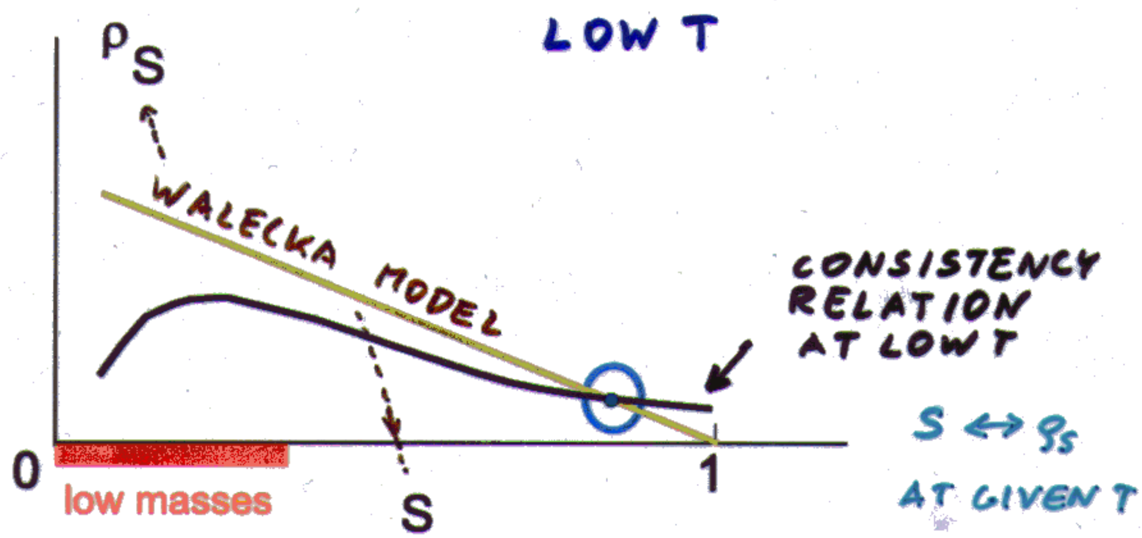
S-REDUCTION FACTOR FOR MASSES
$m = m_0 \times S$, $S(\rho_s)$ - SPECIFIED

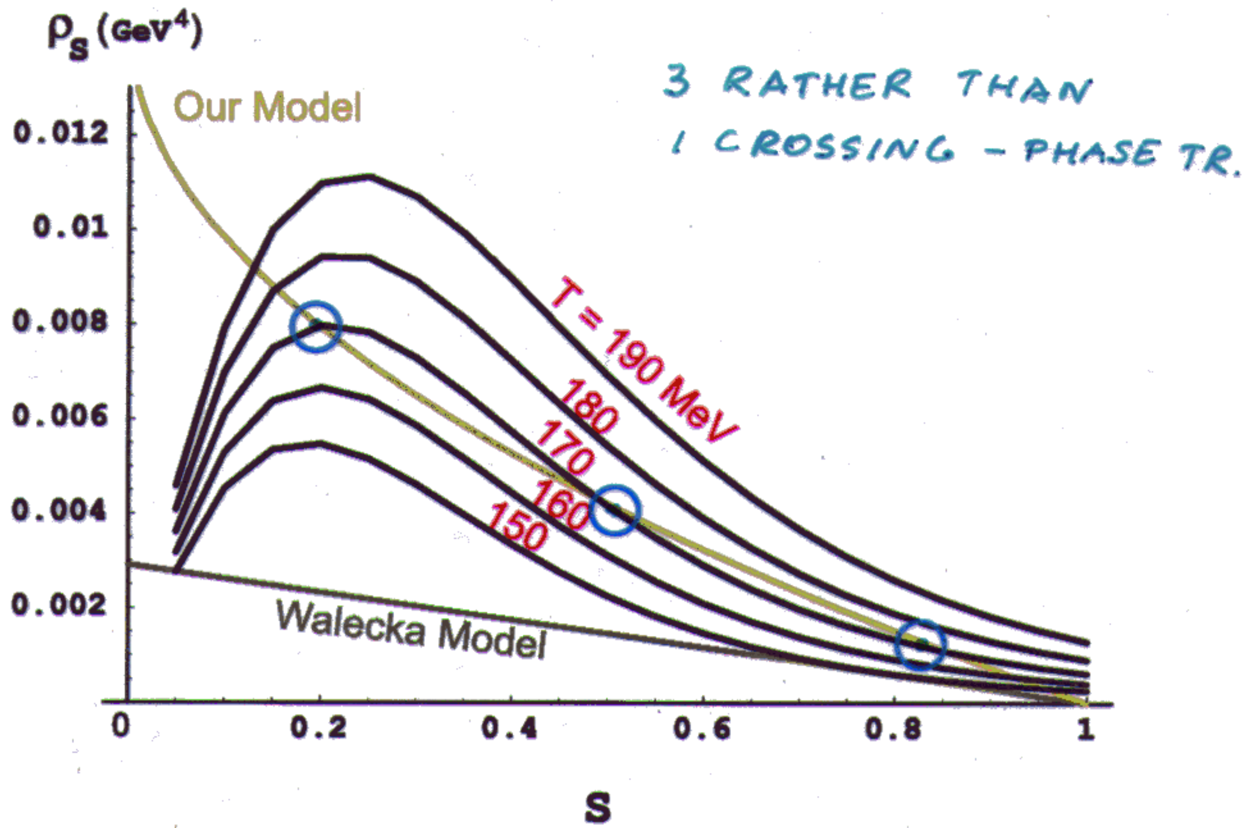
$$\rho_s \equiv \rho_s(S) = \sum_i \int d\mathbf{p} \frac{m_0^{i2}\, S}{\sqrt{m_0^{i2}\, S^2 + p^2}} \times \frac{1}{\exp\left(\sqrt{m_0^{i2}\, S^2 + p^2}/T\right) \pm 1}$$

↑ THERMAL OCCUPATION $= f(p)$

Best investigated in the ρ_s–S plot.

$S(\rho_s) \simeq 1 - a\,\rho_s$ AT LOW ρ_s


LOW T
ρ_S
WALECKA MODEL
CONSISTENCY RELATION AT LOW T
S ↔ ρ_S
AT GIVEN T
0
1
low masses
S


ρ_S (GeV^4)
3 RATHER THAN 1 CROSSING - PHASE TR.
Our Model
0.012
0.01
0.008
0.006
0.004
0.002
T = 190 MeV
180
170
160
150
Walecka Model
0
0.2
0.4
0.6
0.8
1
S

Interactions are v. strong in the Walecka model:

$$S = 1 - 2.6\,(\mathrm{fm}^3/\mathrm{GeV})\,\rho_s$$

The phase transition occurs at a low temperature $T < 100$ MeV.

We use a weaker dependence,

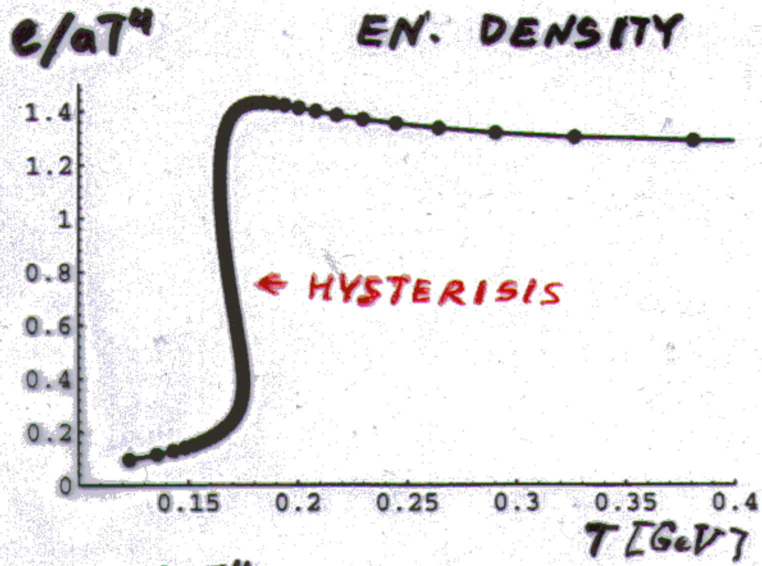
$$S = \left(1 - 0.54\,(\mathrm{fm}^3/\mathrm{GeV})\,\rho_s\right)^2 ,$$

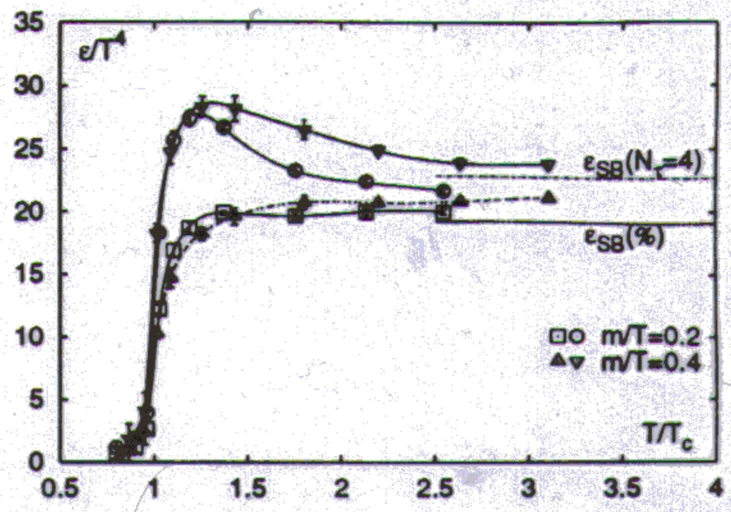
getting the phase transition at $T \approx 170$ MeV.

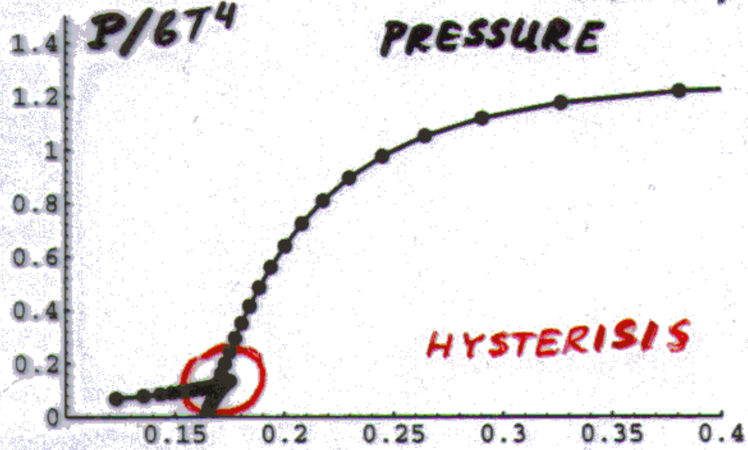
On taking care of $\mu = 0$, we can turn to $T = 0$.

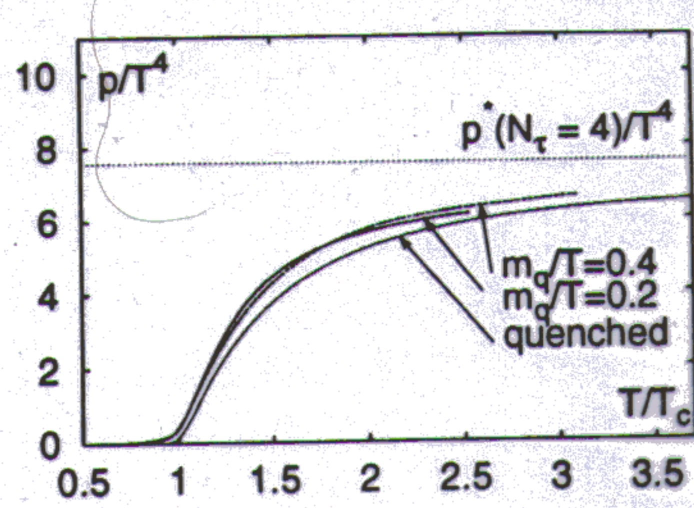
OUR MODEL

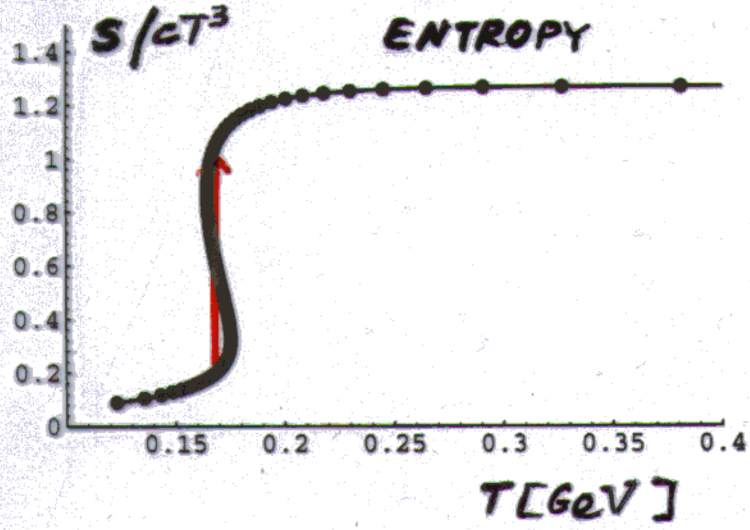
KARSCH ET AL. ↓ LATTICE



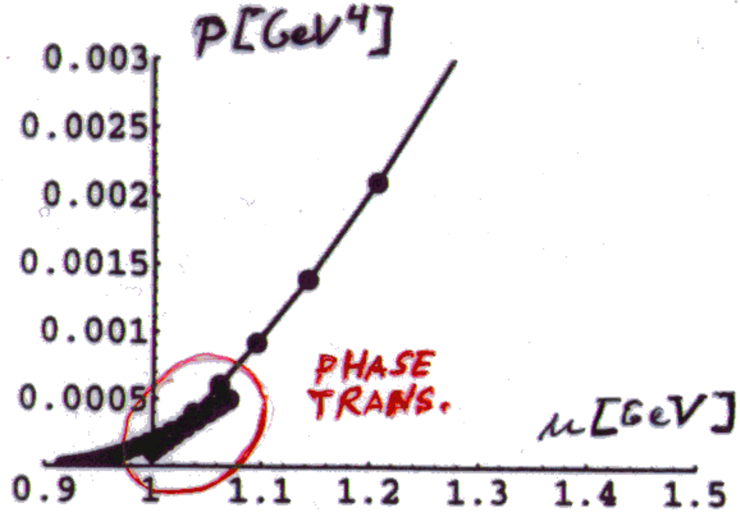




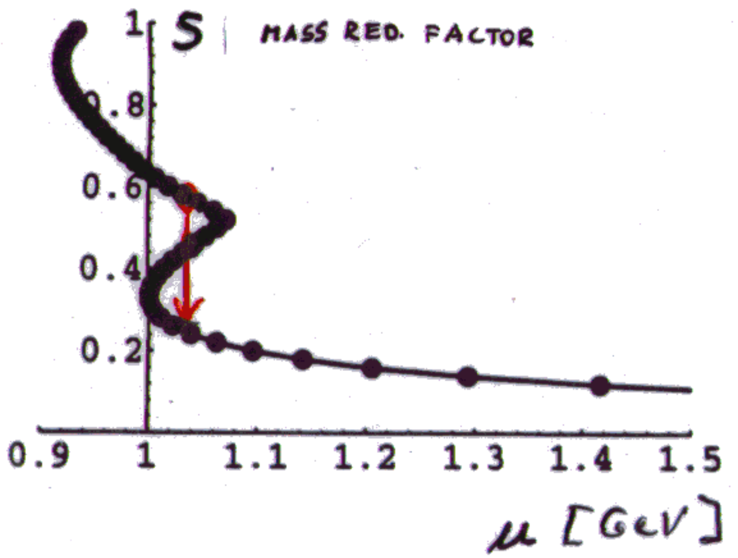




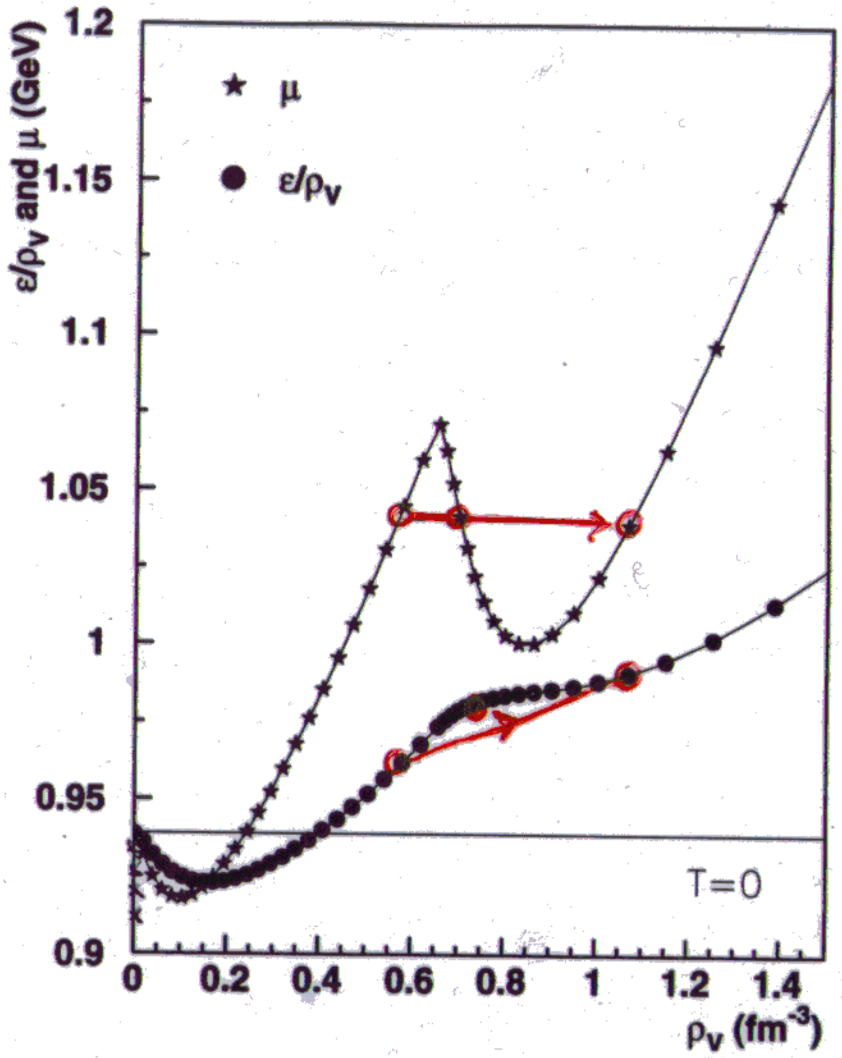
T=0



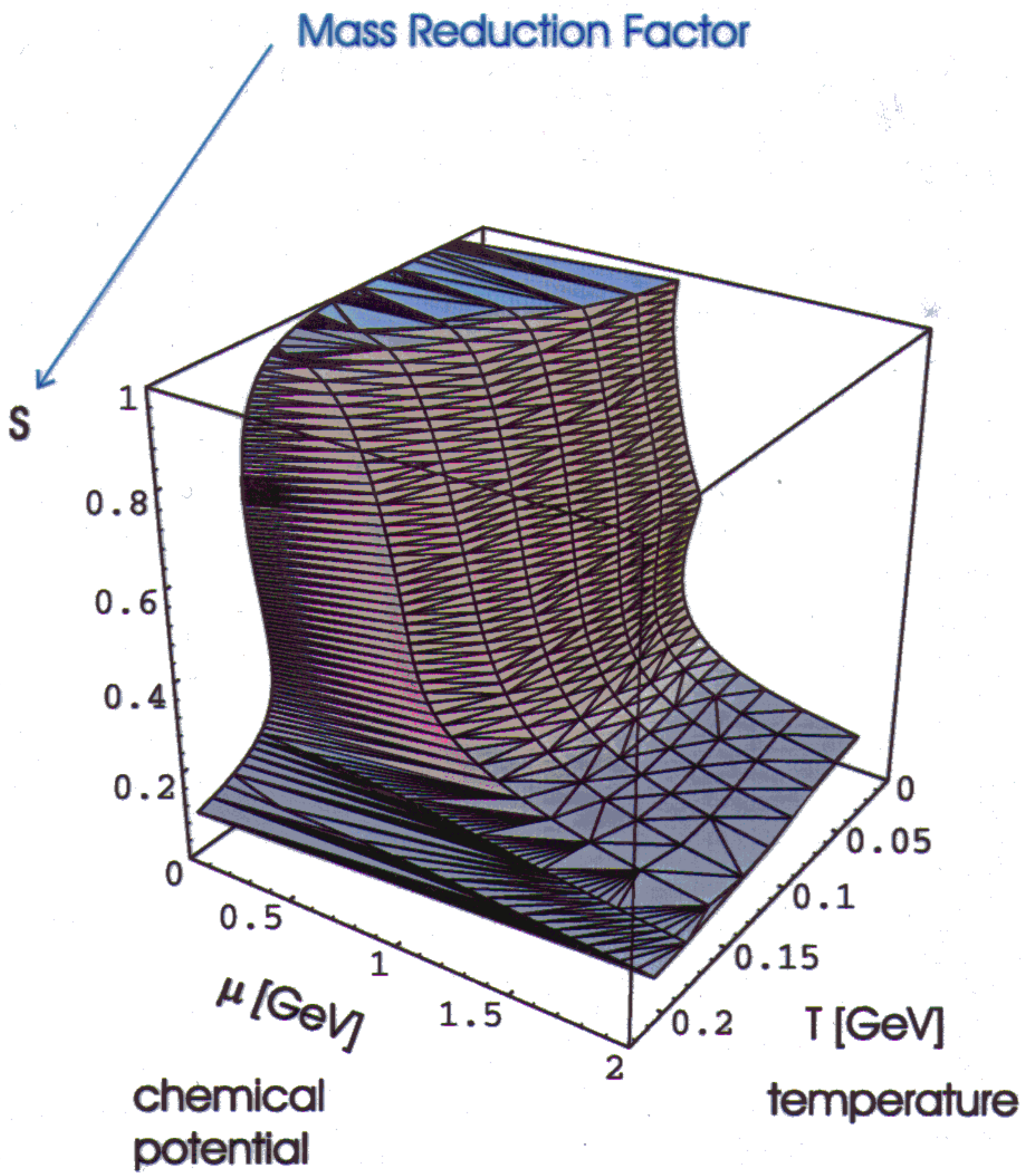
P [GeV^4]
0.003
0.0025
0.002
0.0015
0.001
0.0005
PHASE TRANS.
μ [GeV]
0.9
1
1.1
1.2
1.3
1.4
1.5



S
MASS RED. FACTOR
1
0.8
0.6
0.4
0.2
0.9
1
1.1
1.2
1.3
1.4
1.5
μ [GeV]



Thermodynamical Functions at T=0
ε/ρ_V and μ (GeV)
μ
ε/ρ_V
T=0
ρ_V (fm^-3)
1.2
1.15
1.1
1.05
1
0.95
0.9
0
0.2
0.4
0.6
0.8
1
1.2
1.4


Mass Reduction Factor
S
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
μ [GeV]
chemical potential
0
0.05
0.1
0.15
0.2
T [GeV]
temperature

Transport Theory

Boltzmann Eq.: same general form relativistically as nonrelativistically:

COLL RATE

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = I$$

VELOCITY FORCE

In terms of kinematic vbles:

KIN ENERGY

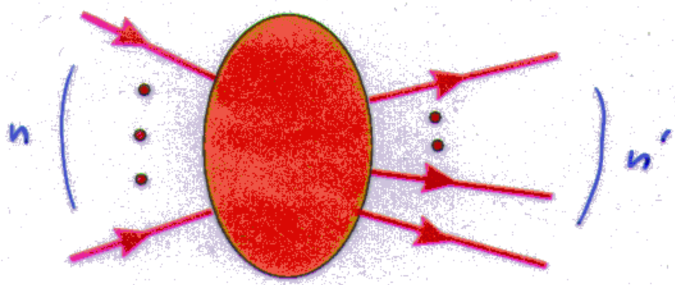
$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}^*}{\epsilon^*_{\mathbf{p}}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial}{\partial \mathbf{r}} \left(\epsilon^*_{\mathbf{p}} + V^0 \right) \frac{\partial f}{\partial \mathbf{p}^*} = I$$

POTENTIAL

I – collision rate. All functions refer to one location $(\mathbf{r}, t)$ in space-time.

Walecka m. before: Ko, Li, Wang, PRL59, 1084 (87)

Collision Processes



$$I = \sum_{n,n'\geq 2} \int \frac{d\mathbf{p}_2}{\gamma_2} \cdots \frac{d\mathbf{p}_n}{\gamma_n} \int \frac{d\mathbf{p}'_1}{\gamma'_1} \cdots \frac{d\mathbf{p}'_{n'}}{\gamma'_{n'}} |\mathcal{M}|^2$$

$$\times \delta\left(\sum_{i'=1}^{n'} p'_{i'} - \sum_{i=1}^{n} p_i\right) (f'_1 \ldots f'_{n'} - f_1 \ldots f_n)$$

GAIN LOSS

STATISTICS SUPPRESSED

$$= \sum_{n,n'\geq 2} \int \frac{d\mathbf{p}^*_2}{\gamma_2} \cdots \frac{d\mathbf{p}^*_n}{\gamma_n} \int \frac{d\mathbf{p}^{*\prime}_1}{\gamma'_1} \cdots \frac{d\mathbf{p}^{*\prime}_{n'}}{\gamma'_{n'}} |\mathcal{M}|^2$$

$$\times \delta\left(\sum_{i'=1}^{n'} p^{*\prime}_{i'} - \sum_{i=1}^{n} p^*_i\right) (f'_1 \ldots f'_{n'} - f_1 \ldots f_n)$$

Practical simplifications due to scaling of all masses by the same factor S.

Early high-energy processes: production only, $2 \to N$. Longitudinal phase-space model.

$$I \propto \prod_{j=1} \frac{d\mathbf{p}'_j}{\gamma'_j} \mathrm{e}^{-B\,E'_{\perp j}}\, W_{\parallel j} \times \delta\left(p_1 + p_2 - \sum_{j=1}^{N} p'_j\right)$$

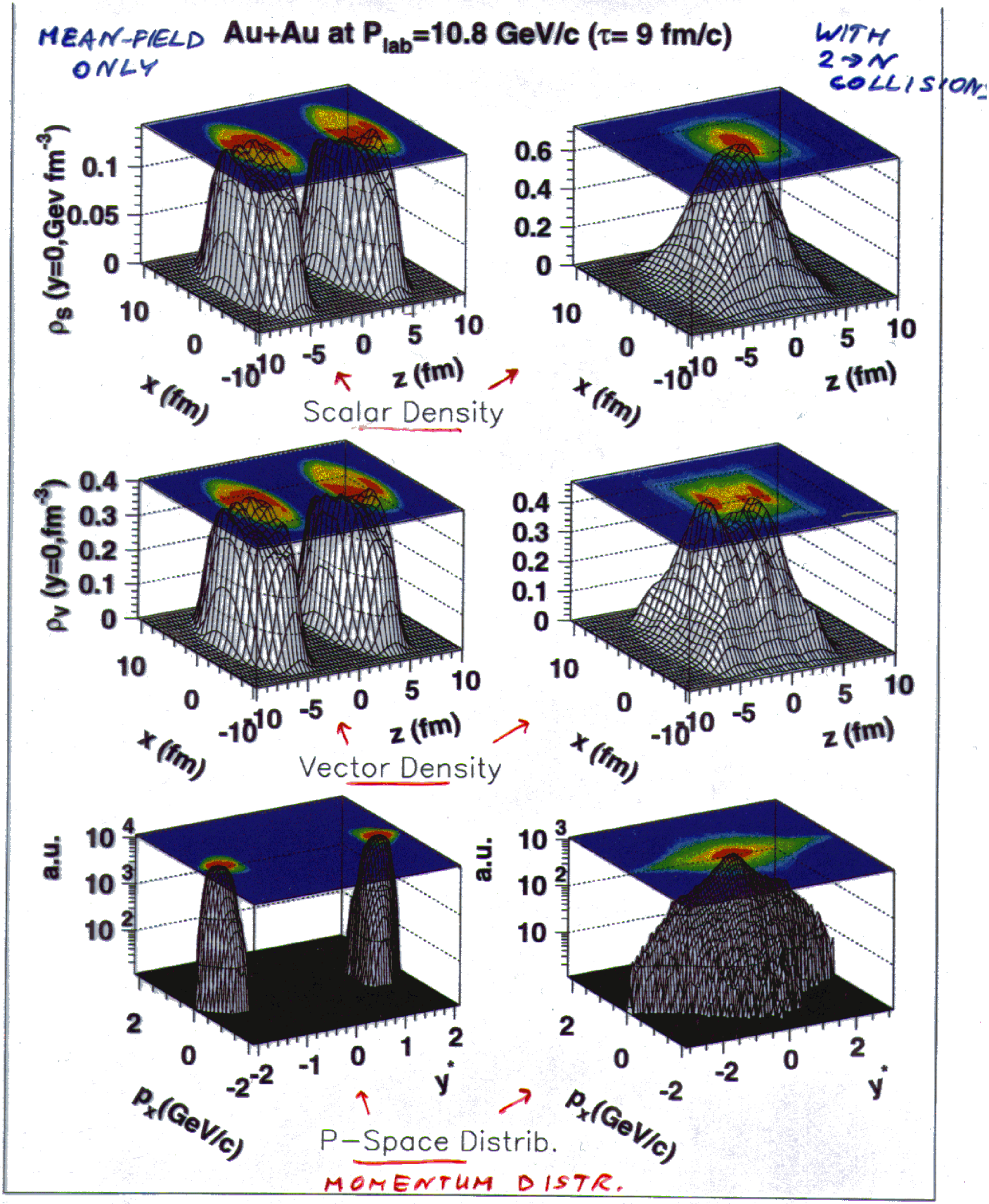
$W_\parallel = \mathrm{e}^{-|y-y_i|}$ for leading ptcles, and $W_\parallel = 1$ for central

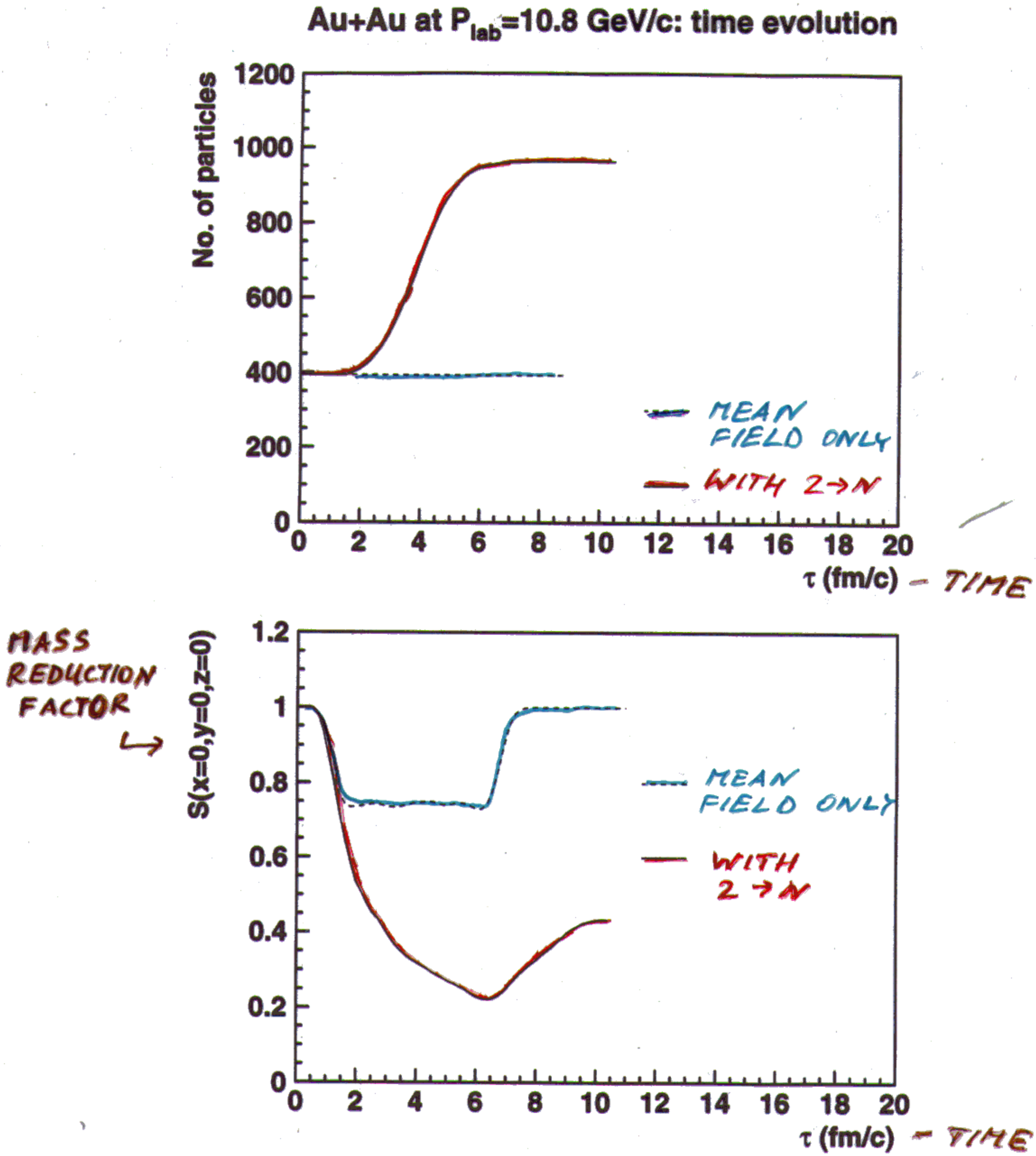
Similar to ARC: Pang, Schlagel, Kahana

Later lower-energy processes treated preserving detailed balance, $2 \leftrightarrow 2$, $2 \leftrightarrow 1$:

elastic, $\pi + N \leftrightarrow \Delta$, $\pi + B \leftrightarrow \rho + B$
$\pi + \pi \leftrightarrow \rho$, $\pi + \pi \leftrightarrow \rho + \rho$, $N + N \leftrightarrow N + \Delta$
$N + N \leftrightarrow \Delta + \Delta$, $N + \Delta \leftrightarrow \Delta + \Delta$, $B + \overline{B} \leftrightarrow \pi + \pi$
$B + \overline{B} \leftrightarrow \rho + \rho$, $B + \overline{B} \leftrightarrow \rho + \pi$

Parametrization of the processes completed, but a full implementation into the dynamic scheme not yet.


MEAN-FIELD ONLY
Au+Au at P_{lab}=10.8 GeV/c (τ= 9 fm/c)
WITH 2→N COLLISIONS
ρ_S (y=0,Gev fm^{-3})
x (fm)
z (fm)
Scalar Density
ρ_V (y=0,fm^{-3})
x (fm)
z (fm)
Vector Density
a.u.
p_x(GeV/c)
y*
P–Space Distrib.
MOMENTUM DISTR.


Au+Au at P_{lab}=10.8 GeV/c: time evolution
No. of particles
MEAN FIELD ONLY
WITH 2→N
τ (fm/c) - TIME
MASS REDUCTION FACTOR
S(x=0,y=0,z=0)
MEAN FIELD ONLY
WITH 2→N
τ (fm/c) - TIME

PRL 81 (98) 2438

Elliptic Flow

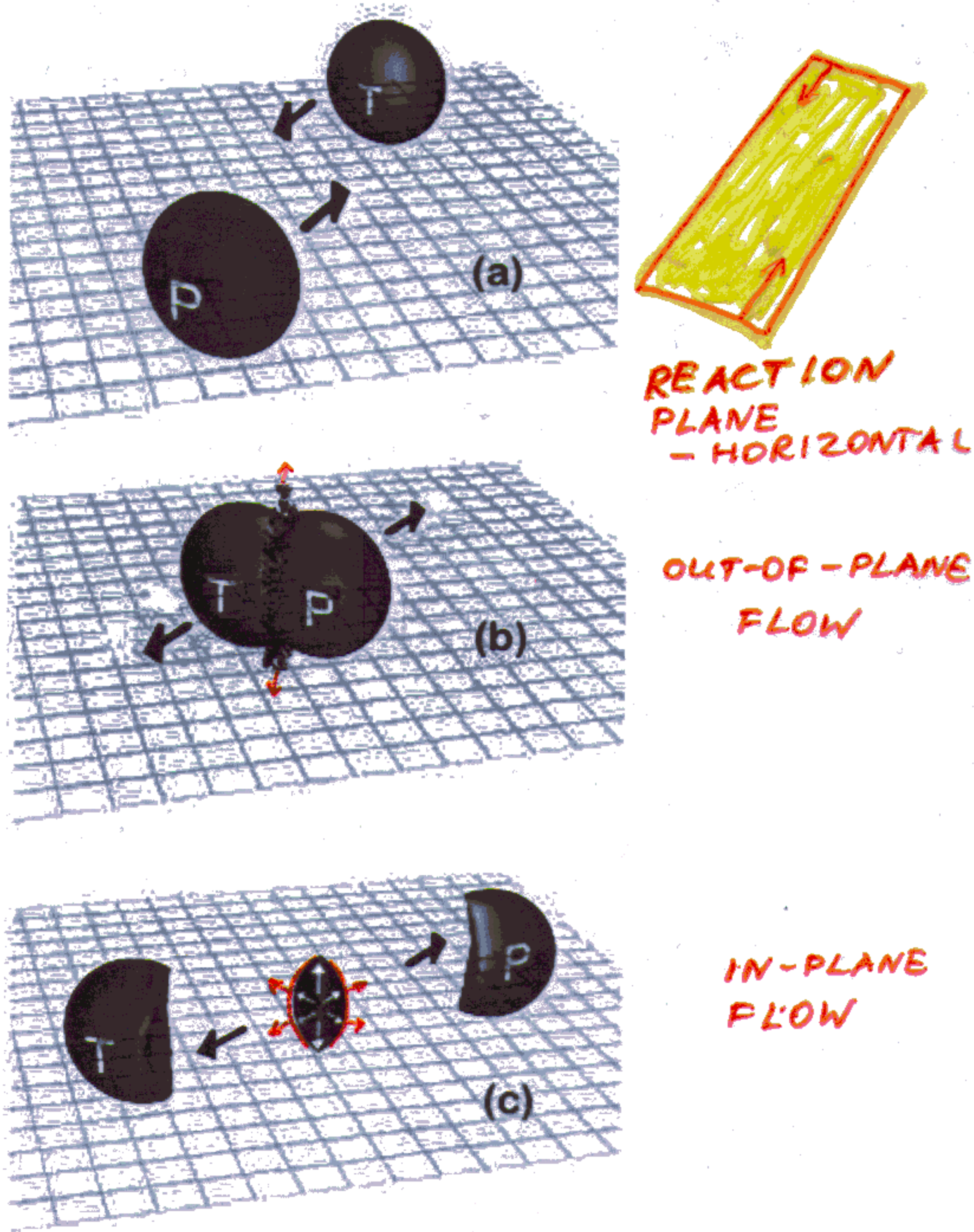
ELLIPTIC ANISOTROPY OF TRANSVERSE EMISSION AT MIDRAPIDITY

General Idea:

2-nd order elliptic flow is more sensitive to the pressure reached in the reactions, than the 1-st order sidewards flow

At AGS energies $\sim 1 - 11$ GeV/nucl: the elliptic flow results from a strong competition between squeeze-out and in-plane flow ;

- Early in a reaction, the spectator nucleons block the path of participants emitted towards the reaction plane $\Longrightarrow$ squeeze out $\perp$ to the plane
- Late in a reaction, geometry favors in-plane emission

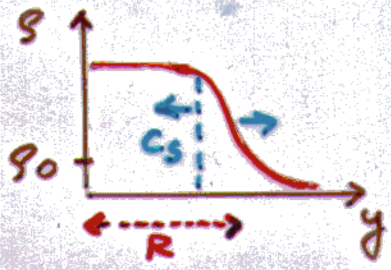

T
P
(a)
REACTION PLANE - HORIZONTAL
T
P
(b)
OUT-OF-PLANE FLOW
T
P
(c)
IN-PLANE FLOW

Sign/magnitude of the elliptic flow depends on:

– pressure build-up early on COMPARED TO ENERGY DENSITY e
  (p)

– passage time for spectators

characteristic time for the development of expansion $\perp$ to the plane: R/c_s



$c_s = \sqrt{\partial p / \partial e}$

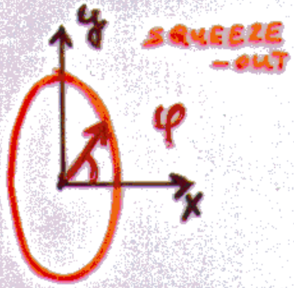
↑ SPEED OF SOUND

SPECTATOR passage time: $2R/(\gamma_0\, v_0)$

v_0 – c.m. spectator velocity

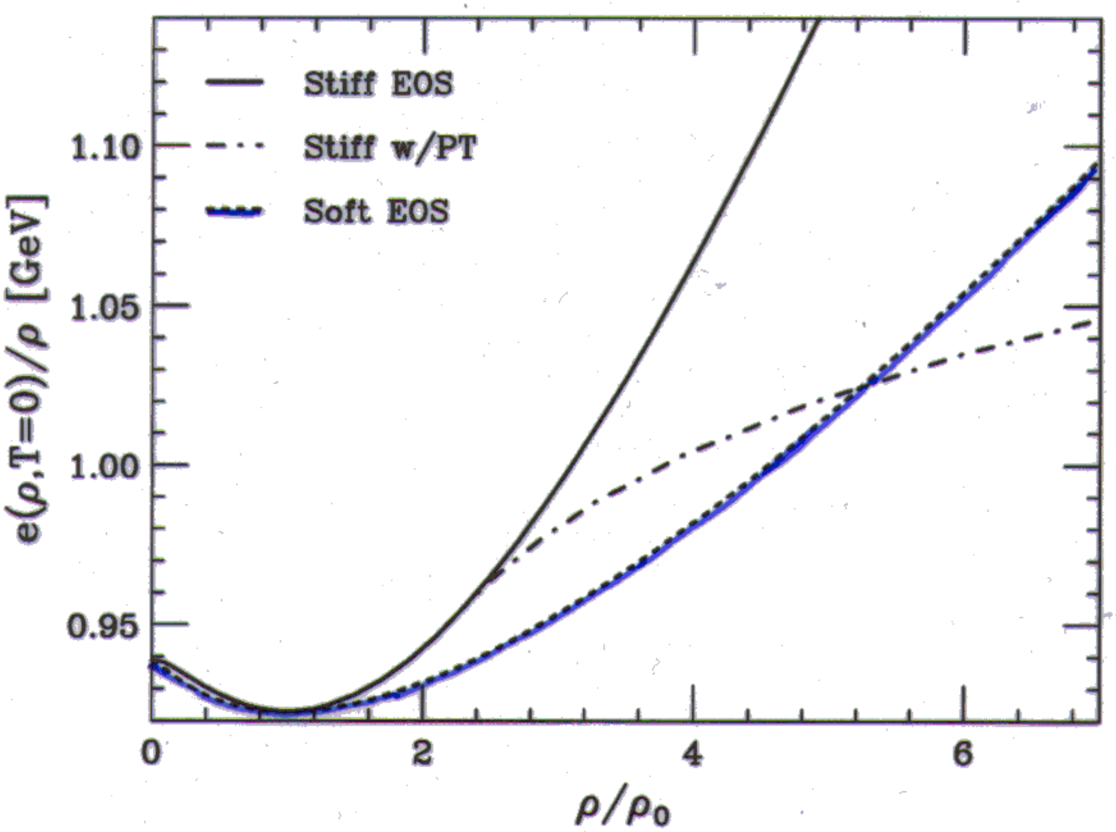
$\Rightarrow$ EARLY squeeze-out contribution to the elliptic flow

TIME RATIO $\sim \dfrac{c_s}{\gamma_0\, v_0}$

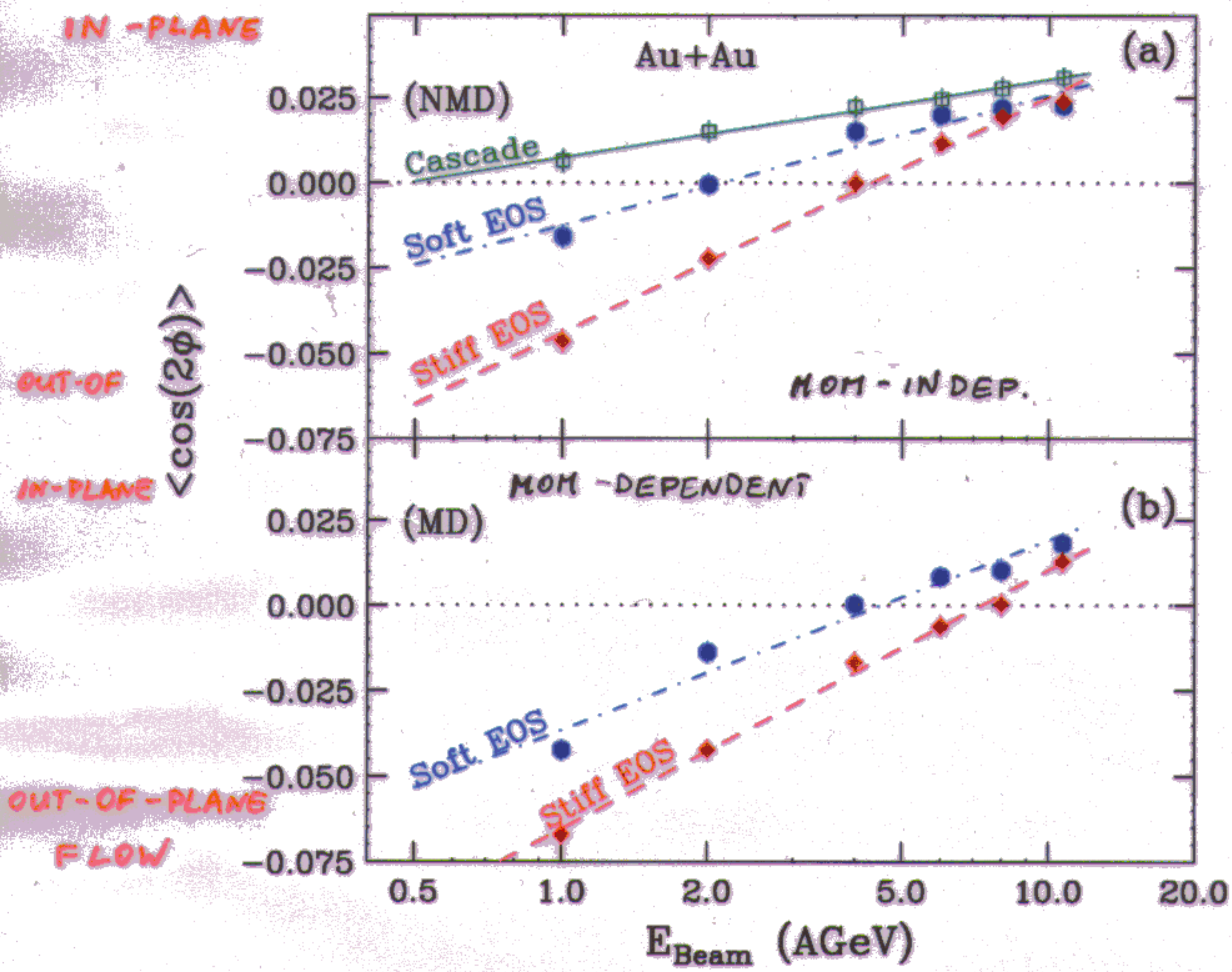


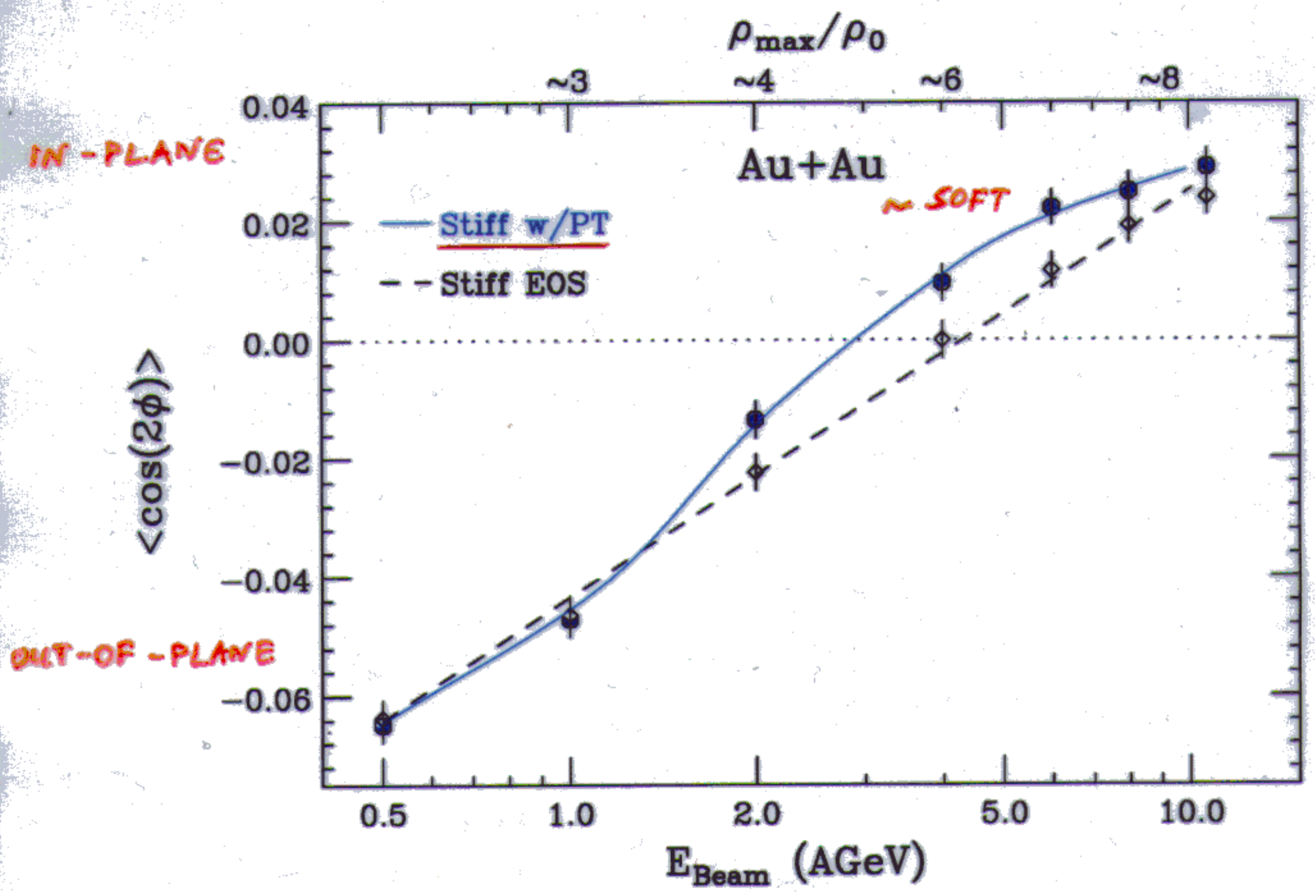
Flow measure: $\langle \cos 2\phi \rangle \equiv v_2$

< 0 – squeeze-out, > 0 – in-plane flow


Stiff EOS
Stiff w/PT
Soft EOS
1.10
1.05
1.00
0.95
e(ρ,T=0)/ρ [GeV]
0
2
4
6
ρ/ρ0

6 ~ 5 fm

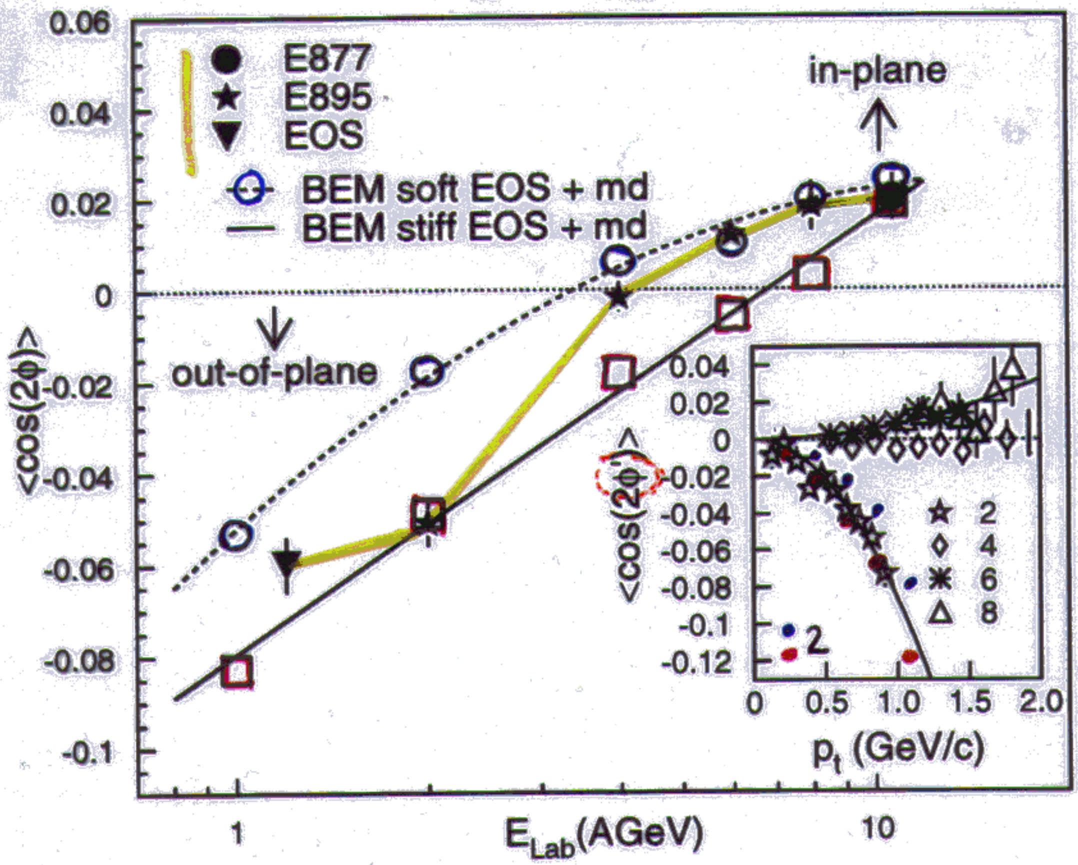

IN-PLANE
OUT-OF
IN-PLANE
OUT-OF-PLANE
FLOW
Au+Au
(a)
(NMD)
Cascade
Soft EOS
Stiff EOS
MOM - INDEP.
MOM - DEPENDENT
(b)
(MD)
Soft EOS
Stiff EOS
<cos(2φ)>
0.025
0.000
−0.025
−0.050
−0.075
0.5
1.0
2.0
5.0
10.0
20.0
E_Beam (AGeV)



ρ_{max}/ρ_0
~3
~4
~6
~8
IN - PLANE
OUT-OF - PLANE
Au+Au
~ SOFT
Stiff w/PT
Stiff EOS
0.04
0.02
0.00
−0.02
−0.04
−0.06
<cos(2φ)>
0.5
1.0
2.0
5.0
10.0
E_{Beam} (AGeV)

COMPARISON TO DATA

Au + Au

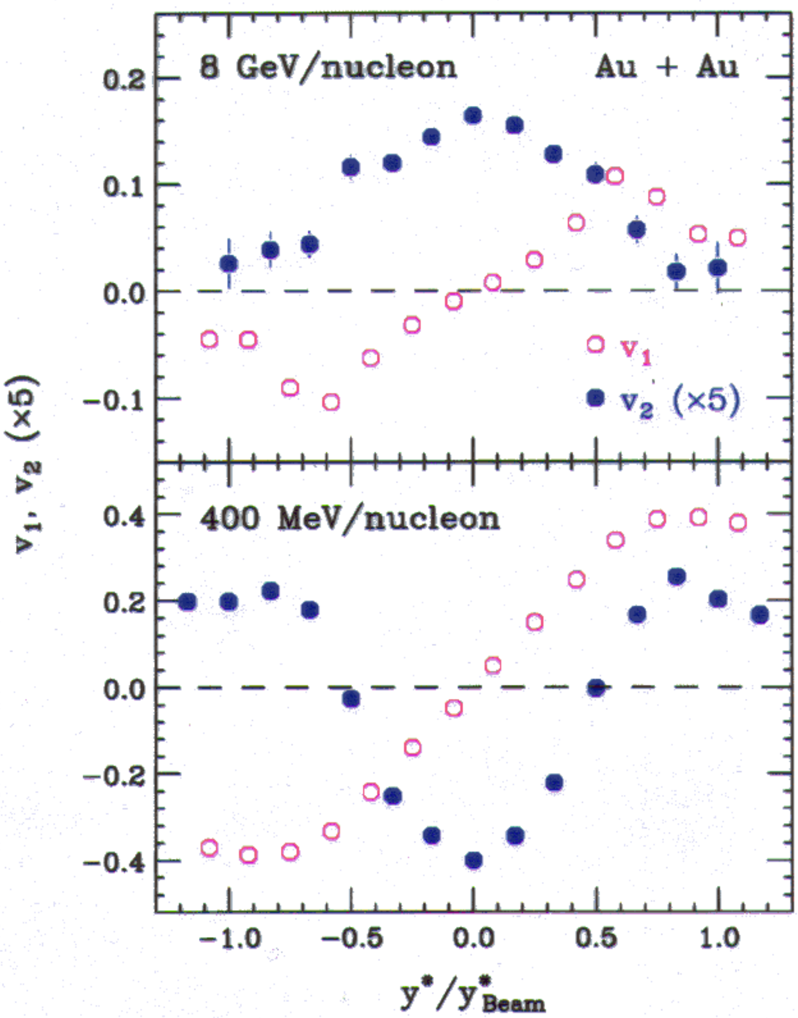


b ~ 6 fm

Is $v_2 = \langle \cos 2\phi \rangle$ an optimal observable for studying the changes in the elliptic flow with energy?

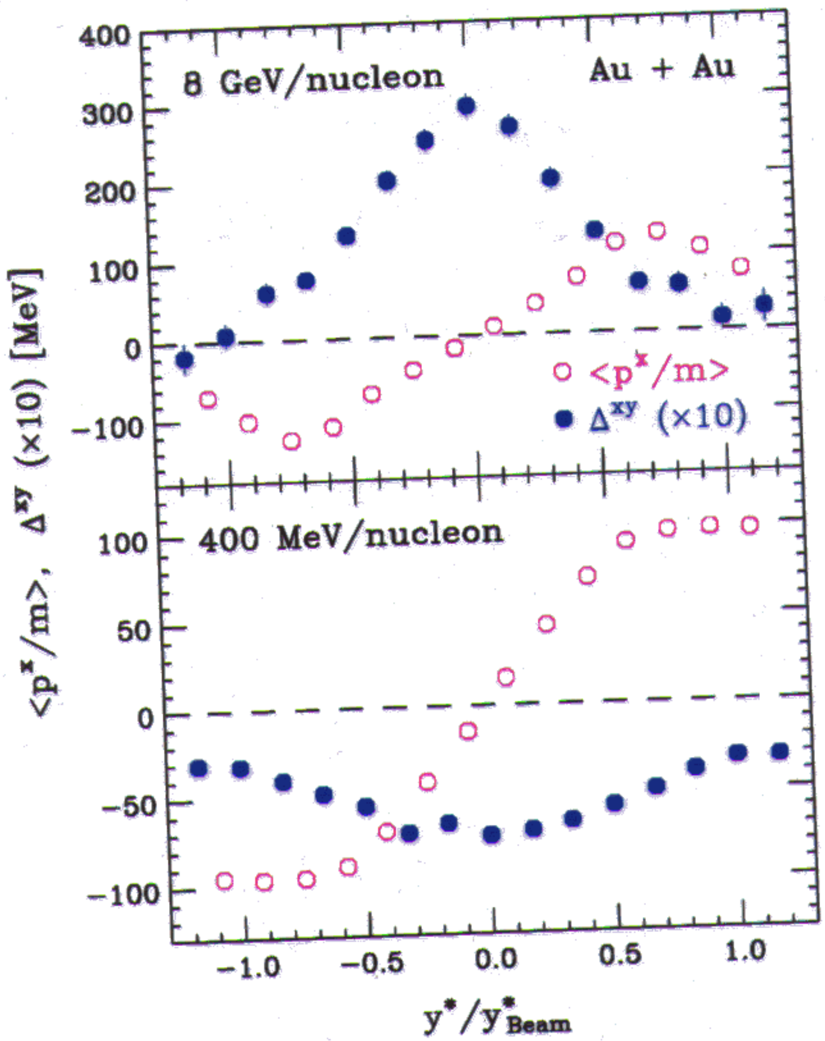
– Rapidity dependence of v_2

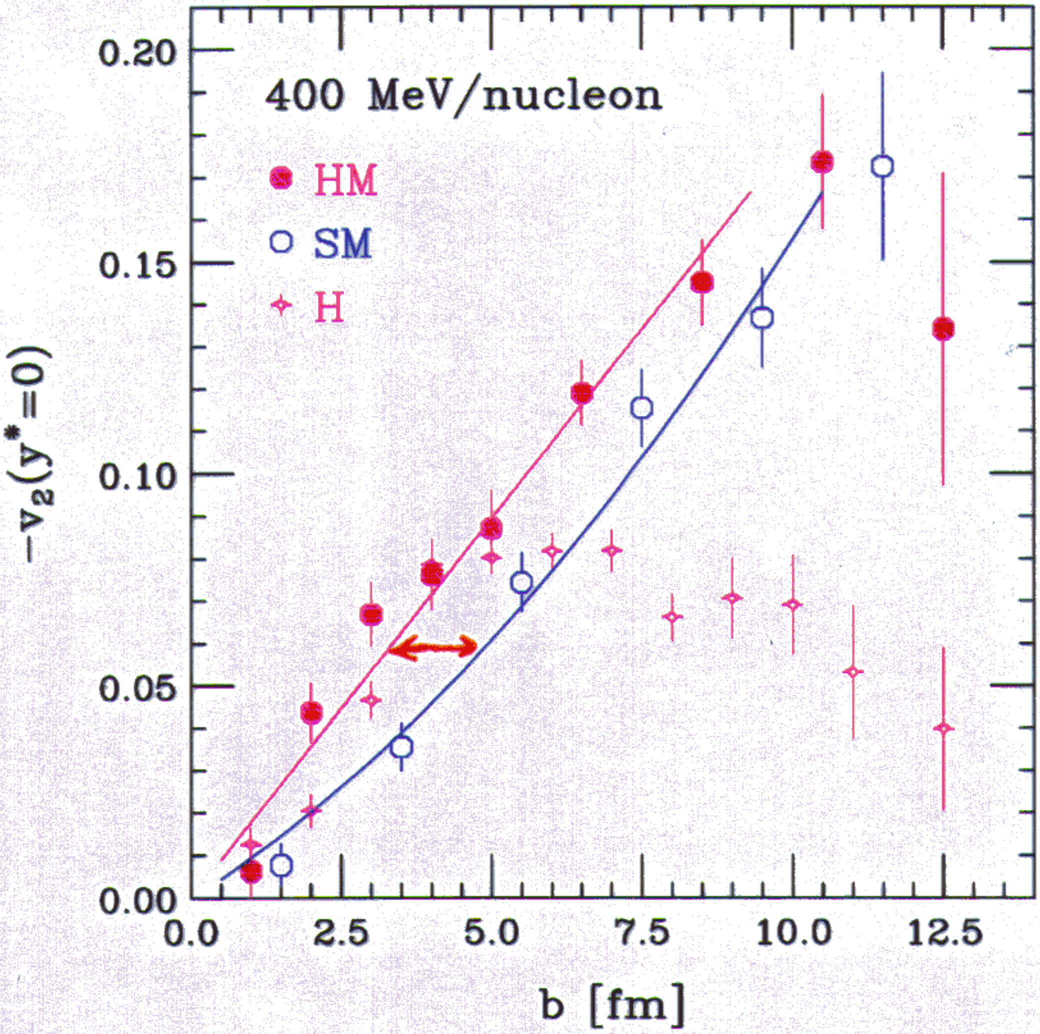
– Impact-parameter dependence



Away from $y^* = 0$, v_2 mixes vector and elliptic flows.

$$\Delta^{xy} = \left\langle \frac{(p^x - m\langle p^x/m\rangle)^2 - p^{y2}}{2m} \right\rangle$$



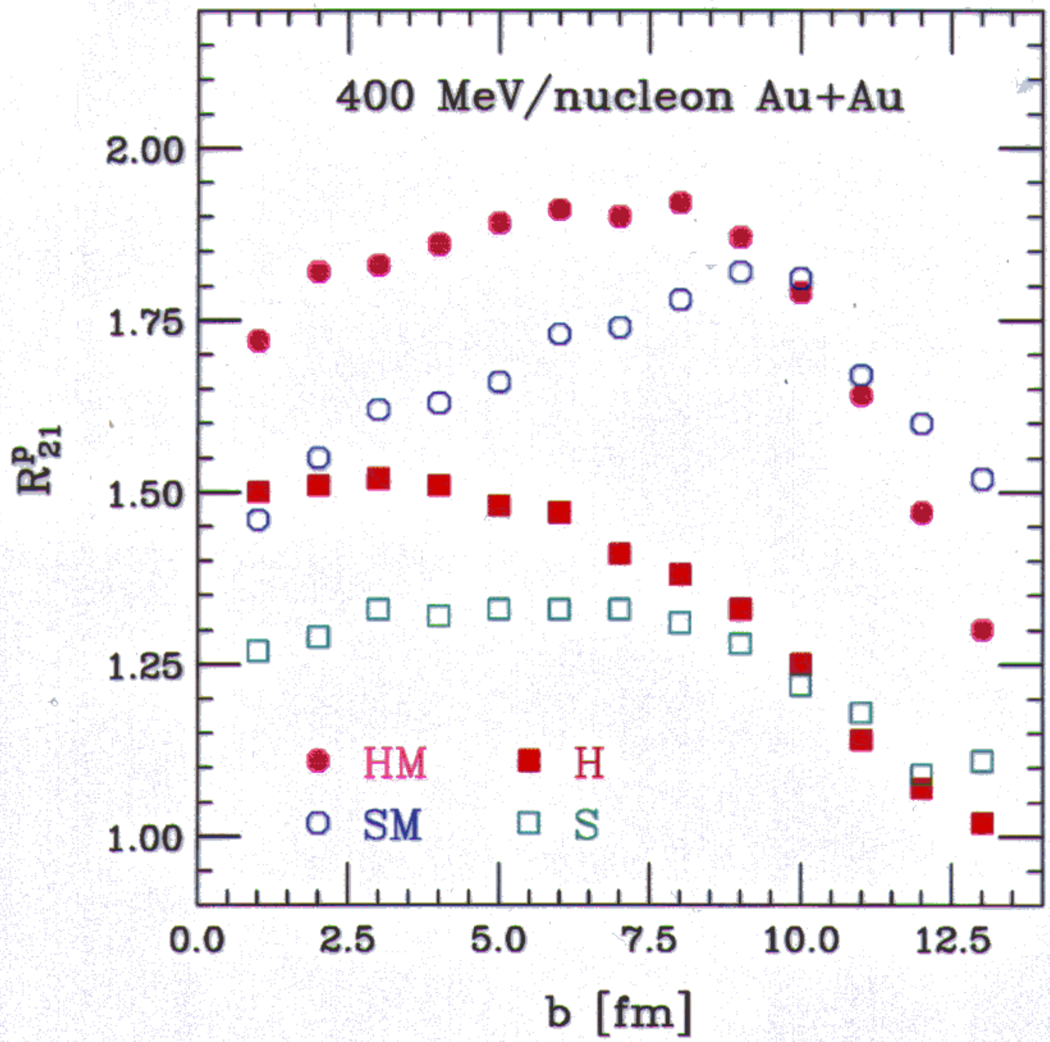


Better than 2 fm absolute resolution is needed in the impact parameter or the data need to be analysed consistently at the different energies.

ANISOTROPY OF THE KINETIC ENERGY TENSOR

$$R^{ij} = \left\langle \frac{p^i p^j}{m+E} \right\rangle$$

OUT-OF-PLANE AXIS TO SHORTER IN-PLANE AXIS



Conclusions

- We have specified a tractable transport model w/thermodynamic properties close to those known for the strongly-interacting matter.
- In the phase-transition region, along the $\mu = 0$ axis, the masses become low and the number of the degrees of freedom rapidly increases.
- The flow measurements can decide about the presence or absence of the phase transition.
- Present data point to a variation in the stiffness of EOS around $E_{Beam} \sim 2$ AGeV (baryon density of $\rho \sim 3\,\rho_0$).
- A comprehensive assessment of the EOS requires a full access to the data. Single sweeps with theory will not do.