$\langle p_\perp \rangle$ and T^* systematics of hadrons produced in p+p and Au+Au / Pb+Pb

- Collective particle production in high-energy A+A collisions – "explosion" of a hypothetical quark-gluon fluid?

- High-$m_\perp$ hadron production in p+p at $\sqrt{s}=200\,\mathrm{GeV}$ by mini-jet fragmentation

- High-$m_\perp$ hadron production in Au+Au at $\sqrt{s}=200\,A\,\mathrm{GeV}$ – do they reflect parton equilibration?
  (necessary for formation of very hot QCD vacuum)

A.D., D.H. Rischke: nucl-th/9806003

A.D., C. Spieles: hep-ph/9810378

Hydrodynamics

$$\boxed{\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu j_i^\mu &= 0 \end{aligned}}$$

for ideal fluid

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$j_i^\mu = \rho_i u^\mu$$

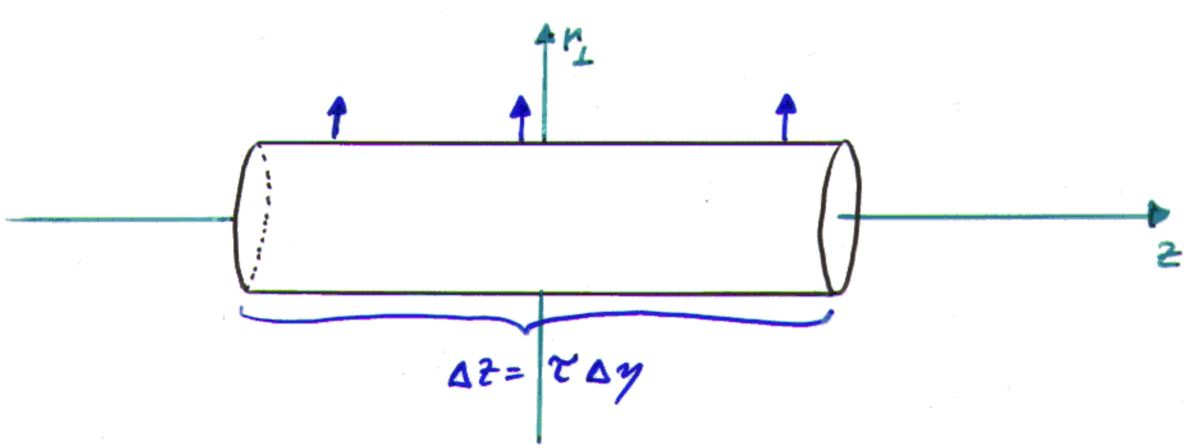
Longit. boost-invariant & cylindrically sym. $\perp$-expansion

$$\boxed{u^\mu(x^\mu) = \gamma_r \left(\cosh\eta,\ v_r \sin\phi,\ v_r \cos\phi,\ \sinh\eta\right)}$$

$$\tanh\eta \equiv v_\parallel = z/t$$

$\curvearrowright$ $\left.\dfrac{\partial p}{\partial \eta}\right|_\tau = 0$, no pressure between rapidity 'slices"

$\left.\dfrac{\partial R_i}{\partial \eta}\right|_\tau = 0$, charges conserved in each rapidity slice, $R_i = \int d^2 r_\perp\, \rho_i(\vec{r}_\perp)$



Equation Of State

Hadronic phase:

assume $T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$

and $T^{\mu\nu} = \sum_i \int \frac{d^3p}{p^0} p^\mu p^\nu f_i(x,p)$

$$\boxed{\begin{aligned} p &= q_\mu q_\nu T^{\mu\nu} \\ &= \sum_i \frac{1}{3} \int \frac{d^3p}{p^0} \vec{p}^{\,2} f_i \end{aligned}}$$

with $q^\mu = \frac{1}{\sqrt{3}}\left[(0,1,0,0) + (0,0,1,0) + (0,0,0,1)\right]$

$q^2 = -1$

and $s = \frac{\partial p(T,\mu_i)}{\partial T}$, $\quad s_i = \frac{\partial p(T,\mu_i)}{\partial \mu_i}$

all hadrons (strange + nonstrange) up to $m \approx 2$ GeV

QGP:

MIT bag-model EoS for ideal QGP ($\alpha_s = 0$)

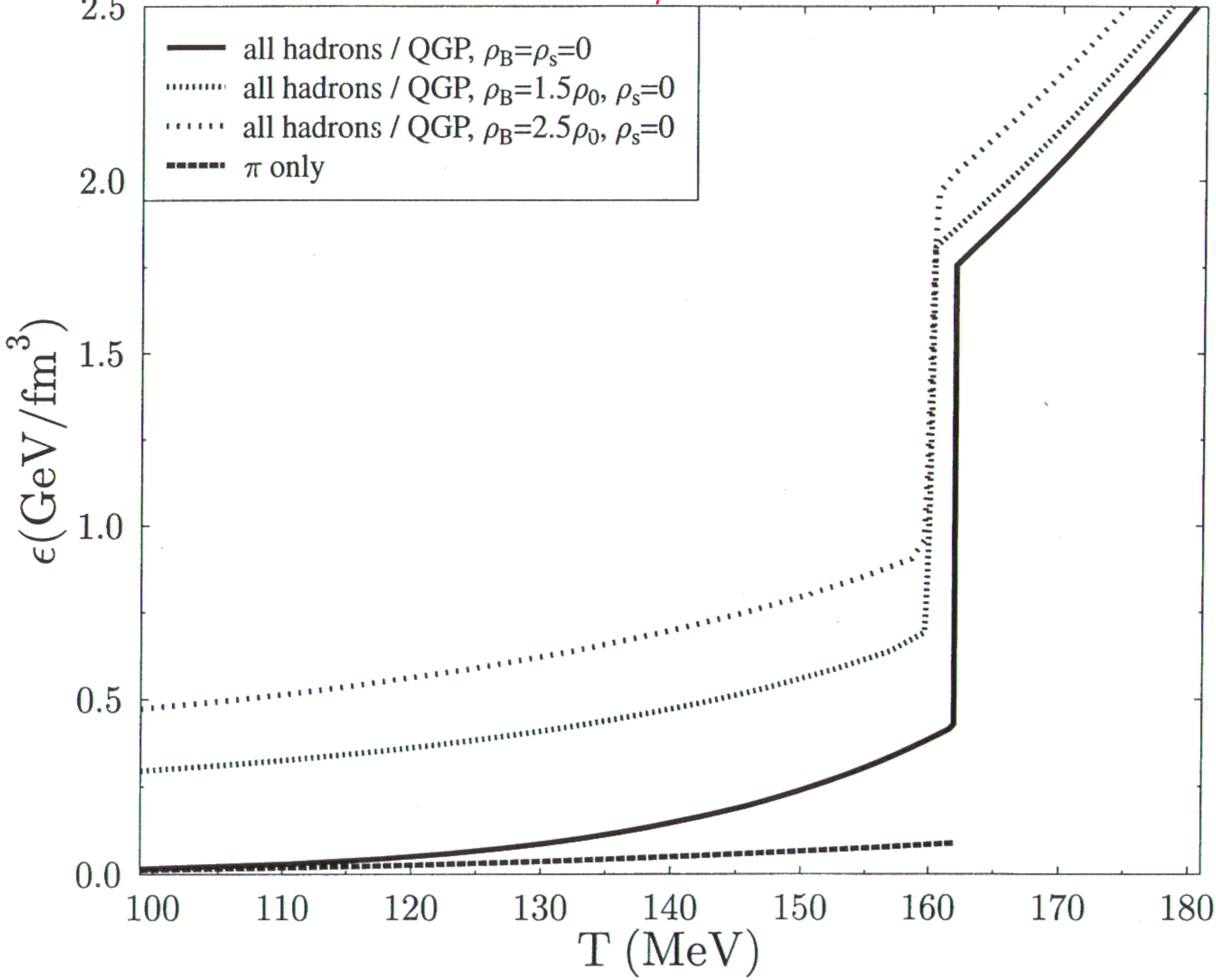
of u, d, s, g ($m_s = 150$ MeV, $m_u = m_d = 0$), $B = 380$ MeV/fm^3

Mixed phase:

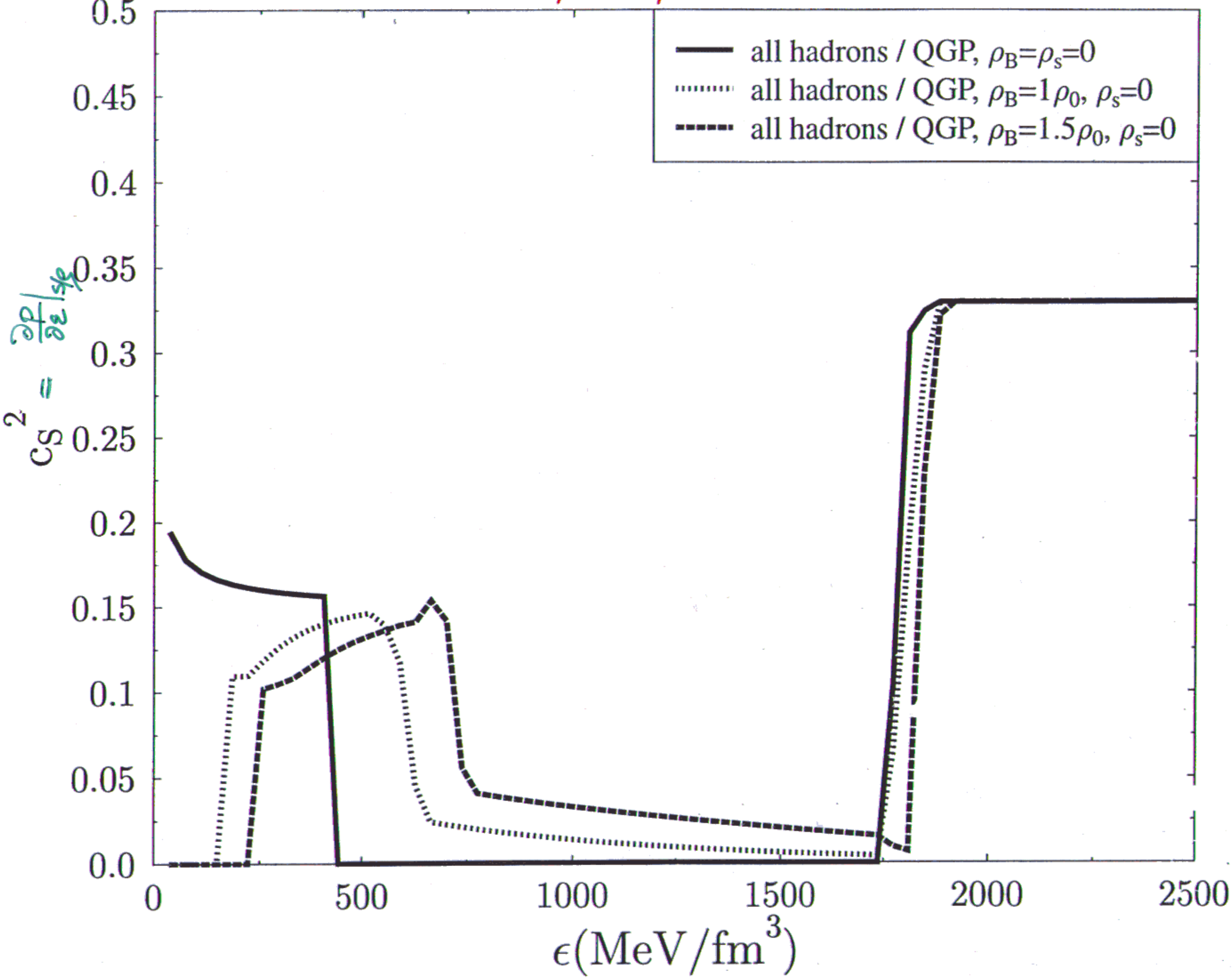
for all T, μ_q, μ_s where

$$p^{QGP}(T,\mu_i) = p^H(T,\mu_i)$$

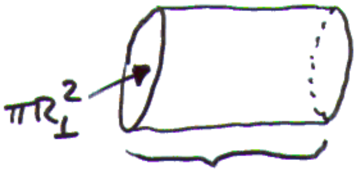
Equation of State

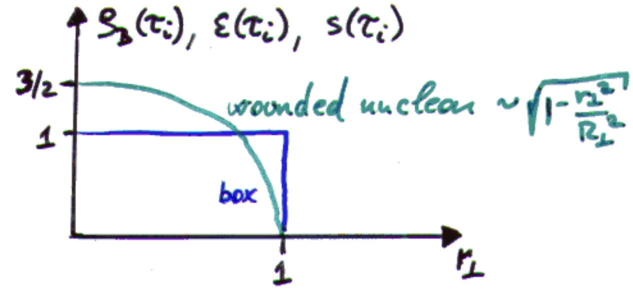


Velocity Of Sound



Initial Conditions for Hydro Expansion





$\Delta z = \tau\,\Delta y\ ,\quad \tau^2 = t^2 - z^2$

$V_{\Delta y=1} = \tau\,\pi R_\perp^2$

$$\rho_B(\tau_i) = \frac{dN_B}{dy}\,\frac{1}{\tau_i\,\pi R_\perp^2} \qquad\qquad \varepsilon(\tau_i) = \frac{dE_\perp}{dy}\,\frac{1}{\tau_i\,\pi R_\perp^2}$$

Pb+Pb at SPS, $\sqrt{s} = 18\,A\,\mathrm{GeV}$

$\frac{dN_B}{dy} = 80\,,\quad \frac{s}{\rho_B} = 40\text{-}50$ (cross-check : $\frac{dE_\perp}{dy}(\tau_f) = 400\,\mathrm{GeV}$)

$\tau_i = 1\,\mathrm{fm}$ $\qquad\qquad dE = T\cancel{dS} - p\,dV + \cancel{\mu\,dN_B}$

$\rightarrow \varepsilon(\tau_i) = 5.3\ \mathrm{GeV/fm^3},\quad T = 216\ \mathrm{MeV}$

$\mu_q = 167\ \mathrm{MeV},\quad \mu_s = 0$

p+p at RHIC, $\sqrt{s} = 200\ \mathrm{GeV}$:

$\frac{dE_\perp(\tau_i)}{dy} = 2.8\ \mathrm{GeV}$ (from PYTHIA) $\rightarrow \varepsilon(\tau_i) = 1.1\ \mathrm{GeV/fm^3}$

$\frac{dN_B}{dy} = 0$

Au+Au at RHIC, $\sqrt{s} = 200\,A\,\mathrm{GeV}$:

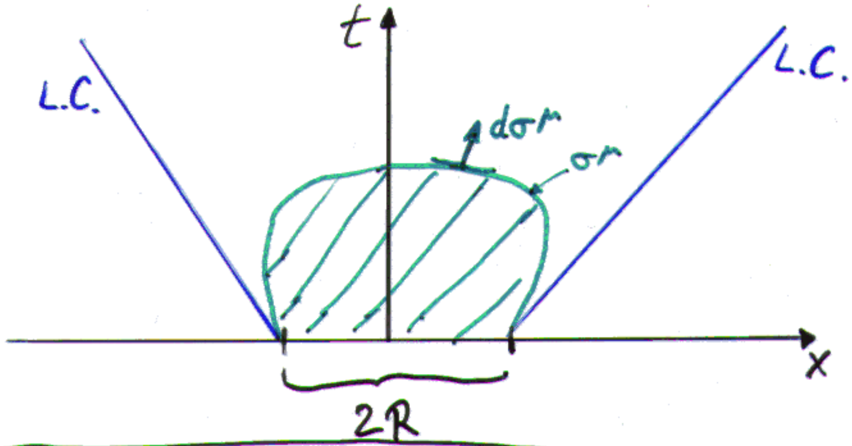
$\frac{dN_B}{dy} = 25\,,\quad \frac{s}{\rho_B} = 200$ $\left(\frac{dE_\perp}{dy}(\tau_i) = 1150\ \mathrm{GeV},\ \frac{dE_\perp}{dy}(\tau_f) = 630\ \mathrm{GeV}\right)$

$\tau_i = 0.6\,\mathrm{fm}$

$\rightarrow \varepsilon(\tau_i) = 17\ \mathrm{GeV/fm^3},\quad T = 300\ \mathrm{MeV}$

$\mu_q = 47\ \mathrm{MeV},\quad \mu_s = 0$

Particle Momentum Distributions



$$\boxed{E\frac{dN}{d^3p} = \int_\sigma d\sigma_\mu\, p^\mu\, f(\sigma_\mu, p_\mu)}$$

Cooper, Frye PRD 10 ('74) p.186

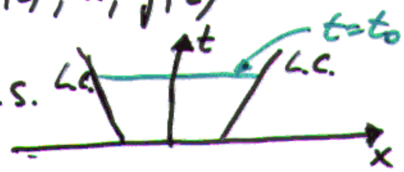
Hypersurface (3-parametic fct.) $\sigma^\mu(\xi, \eta, \phi)$

normal $d\sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\frac{\partial\sigma^\alpha}{\partial\xi}\frac{\partial\sigma^\beta}{\partial\eta}\frac{\partial\sigma^\gamma}{\partial\phi}\, d\xi\, d\eta\, d\phi$

(Missner, Thorne, Wheeler: Gravitation)

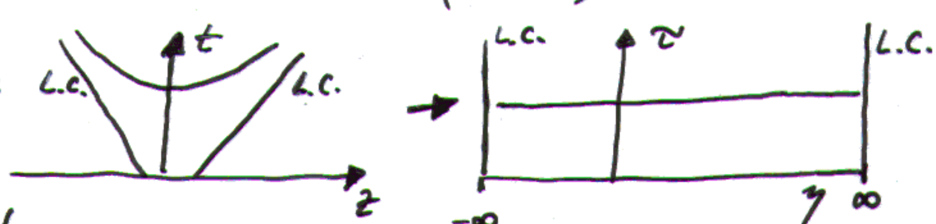
- Flat space: $\sigma^\mu = (t(x,y,z), x, y, z)$

For isochronous h.s.



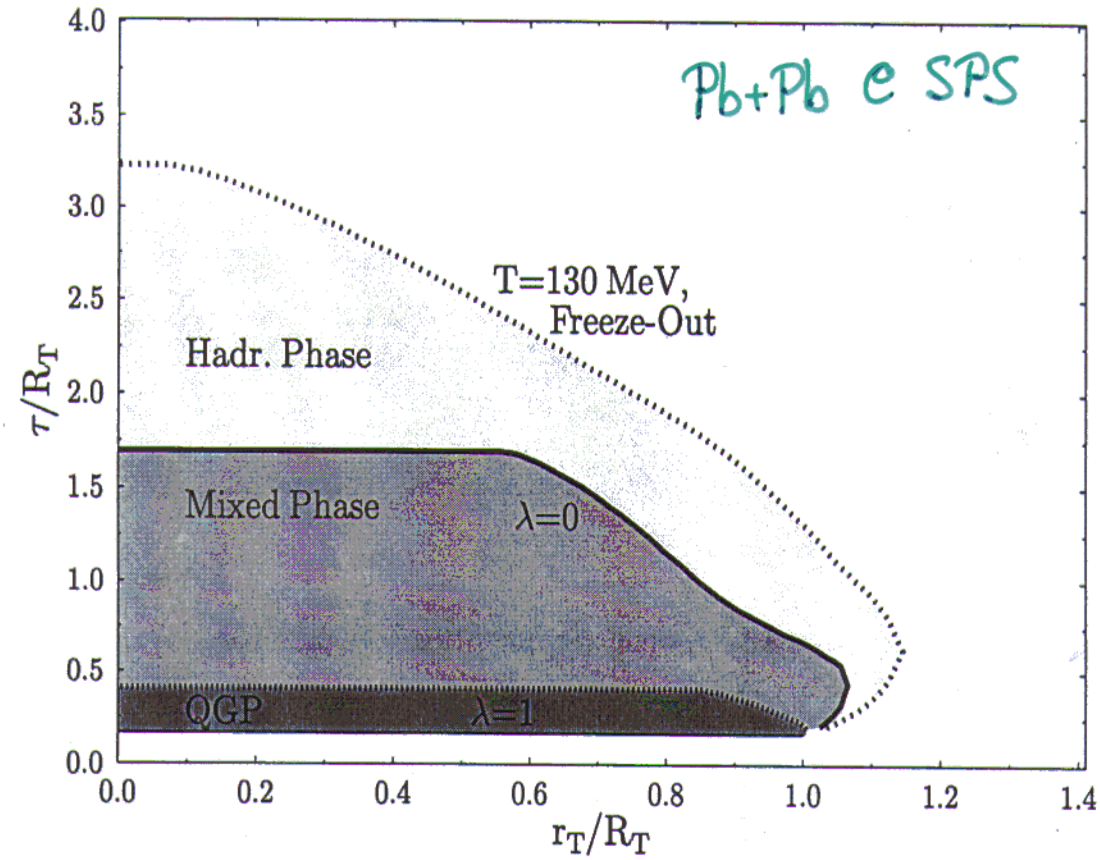
$\curvearrowright \sigma^\mu = (t_0, x, y, z), \quad d\sigma^\mu = (d^3x, \vec{0})$

- Bjorken expansion:

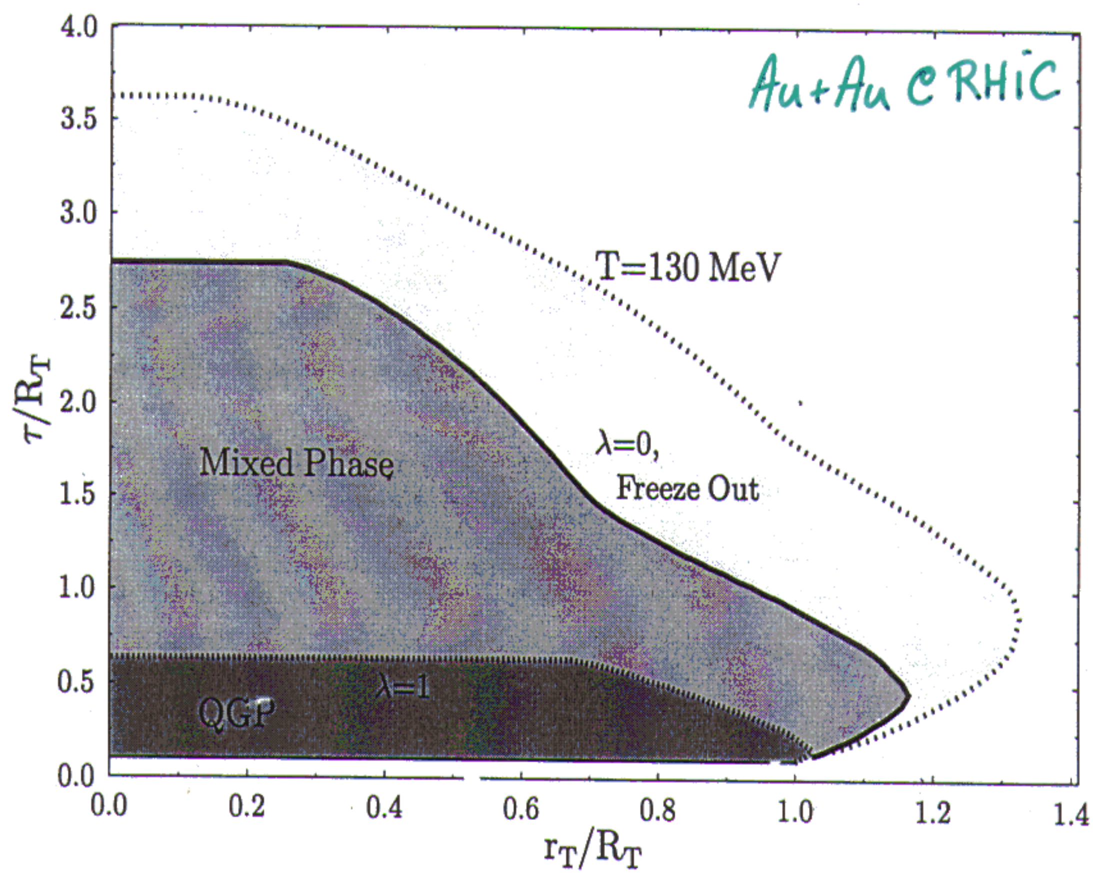


$$\left.\begin{array}{l}\tau = \sqrt{t^2 - z^2} \\ \text{th}\,\eta = z/t\end{array}\right\} \leftrightarrow \begin{array}{l} t = \tau\,\text{ch}\,\eta \\ z = \tau\,\text{sh}\,\eta\end{array} \qquad dt\,dz = \tau\, d\tau\, d\eta$$

$\sigma^\mu = (\tau\,\text{ch}\,\eta,\ r\cos\phi,\ r\sin\phi,\ \tau\,\text{sh}\,\eta) \to d\sigma^\mu = \left(\frac{dr}{d\xi}\text{ch}\,\eta, \frac{d\tau}{d\xi}\cos\phi, \frac{d\tau}{d\xi}\sin\phi,\ dr/d\xi\ \text{sh}\,\eta\right)$

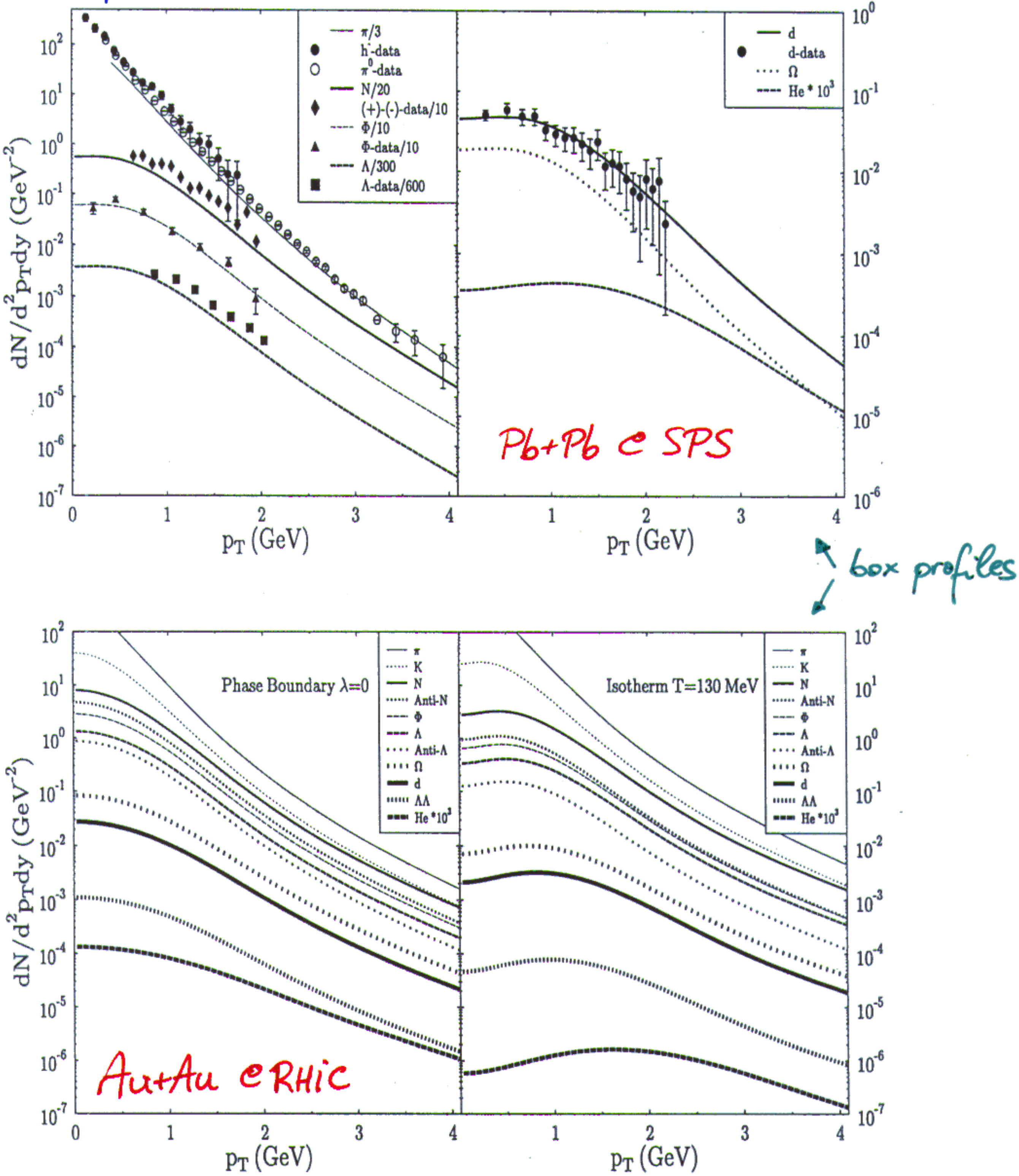


Pb+Pb @ SPS
T=130 MeV,
Freeze-Out
Hadr. Phase
Mixed Phase
λ=0
QGP
λ=1
τ/R_T
r_T/R_T

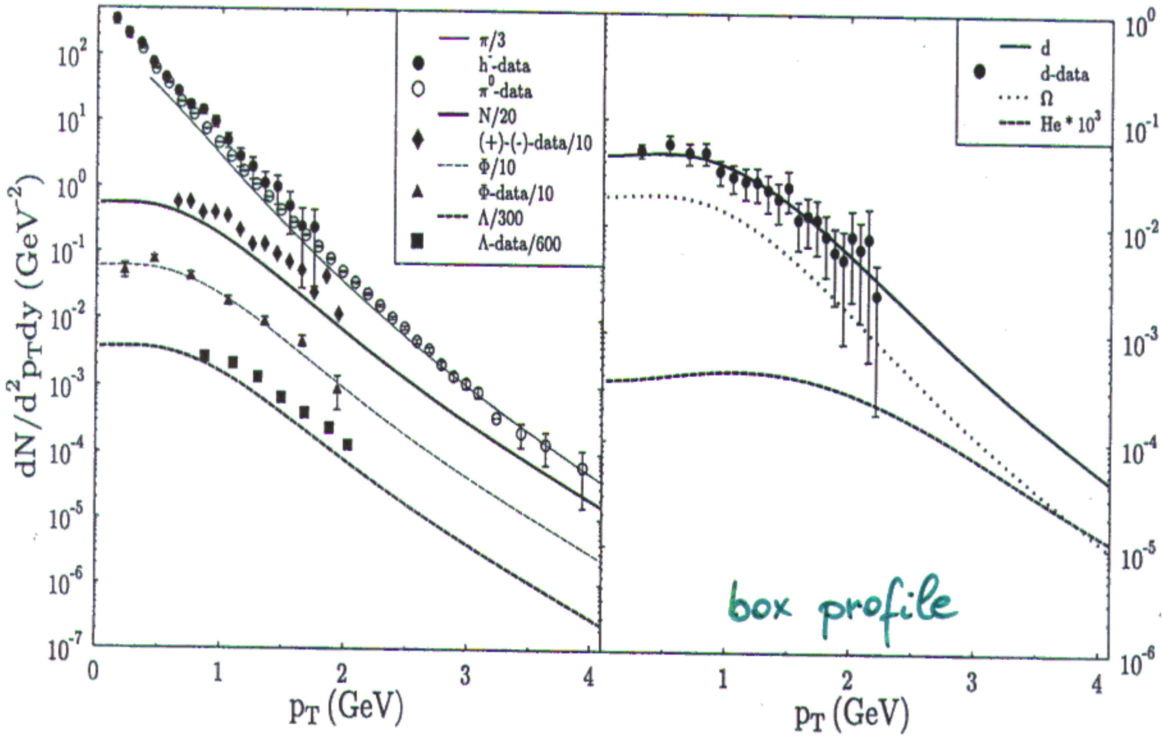


Au+Au @ RHIC
T=130 MeV
λ=0,
Freeze Out
Mixed Phase
QGP
λ=1
τ/R_T
r_T/R_T

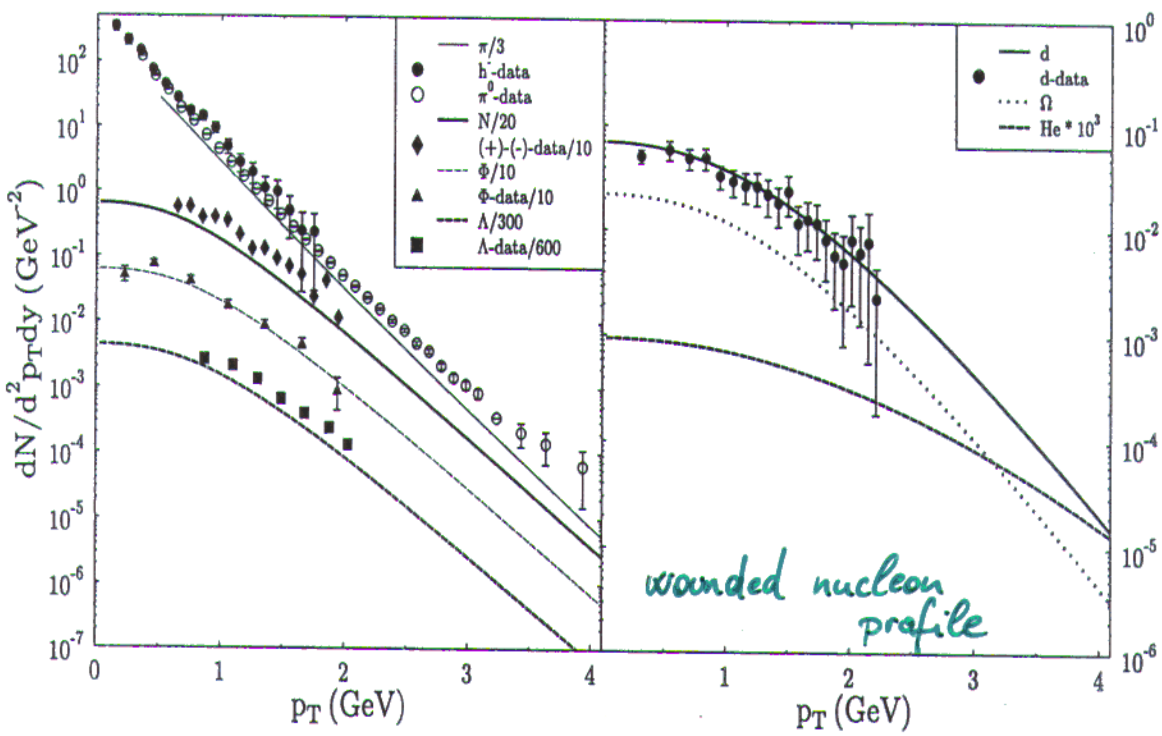
Comparison to prelim. NA49 data nucl-th/9806003



Comparison to preliminary NA49 & WA98 data

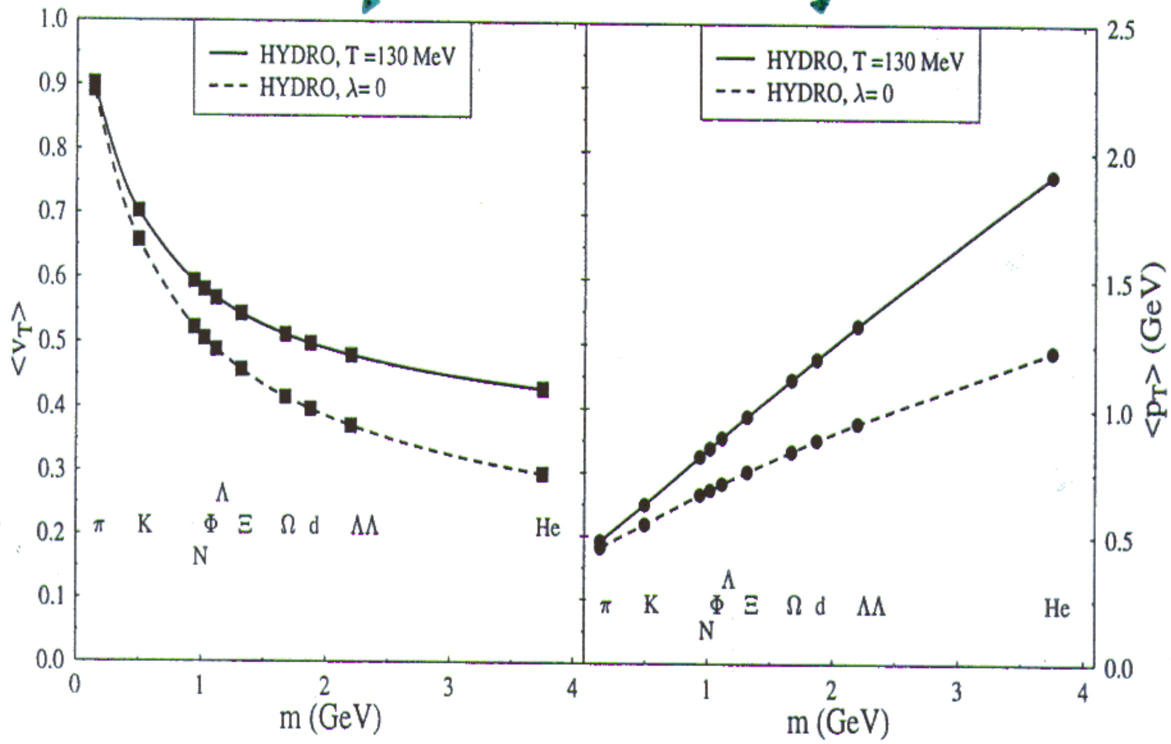


nucl-th/9806003

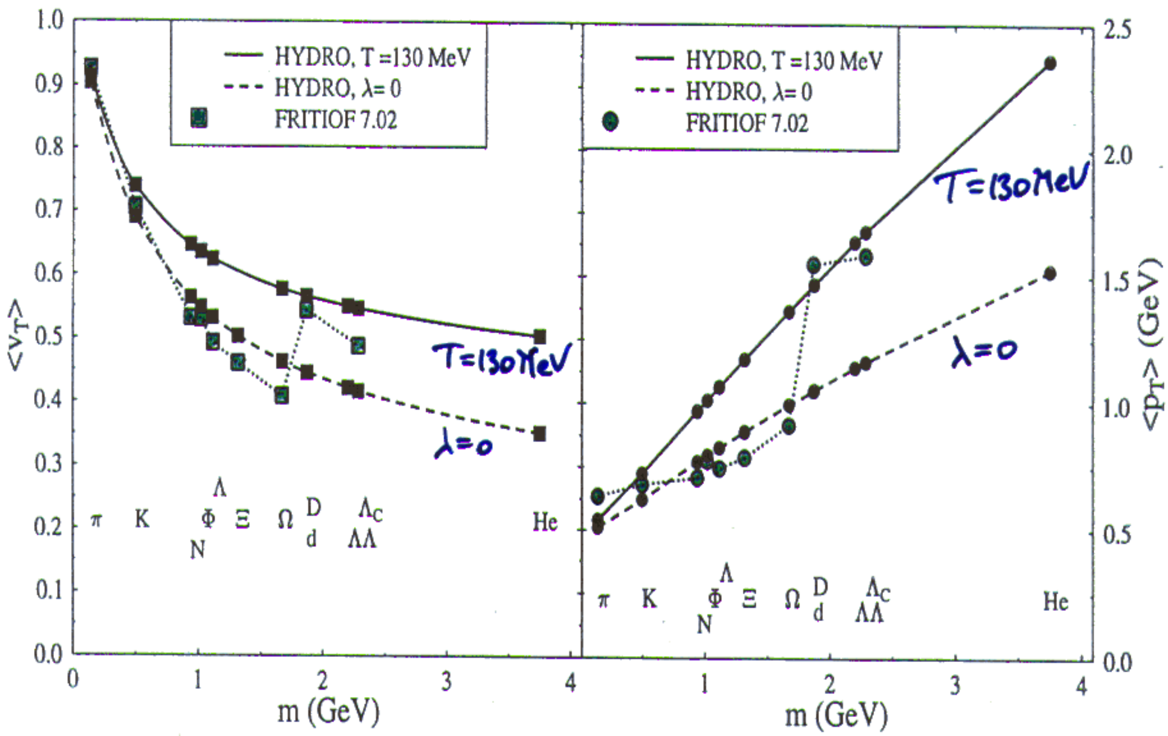


Mean $\perp$ velocities and momenta

SPS

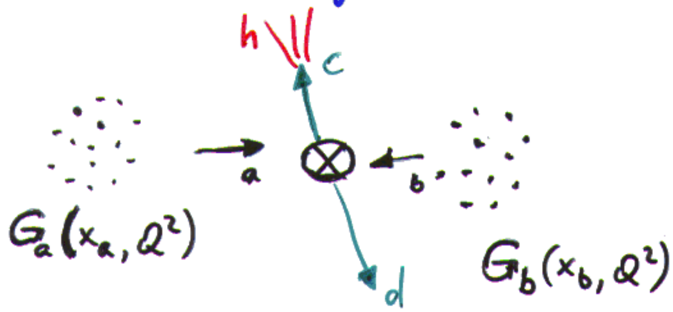


RHIC



nucl-th/9806003

Minijet Production



c.f. e.g. Owens, Rev. Mod. Phys. 59 ('87) p. 465

$$d\sigma(pp \to cd) = \frac{1}{2\hat{s}} \sum_{ab} G_a(x_a, Q^2)\, dx_a\, G_b(x_b, Q^2)\, dx_b \frac{d^3p_c}{(2\pi)^3 2E_c} \frac{d^3p_d}{(2\pi)^3 2E_d}$$

$$\times \sum |M_{ab\to cd}|^2 (2\pi)^4 \delta(p_a + p_b - p_c - p_d)$$

$$E\frac{d\sigma}{d^3p_c}(pp \to c + X) = \sum_{abd} \int dx_a\, dx_b\, G_a(x_a, \mu^2)\, G_b(x_b, \mu^2)$$

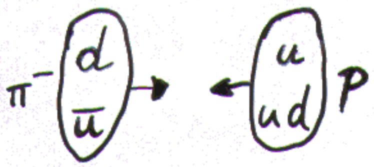
$$\times \frac{\hat{s}}{\pi} \frac{d\sigma^{ab\to cd}}{d\hat{t}}(Q^2)\, \delta(\hat{s} + \hat{t} + \hat{u} - \sum_i m_i^2)$$

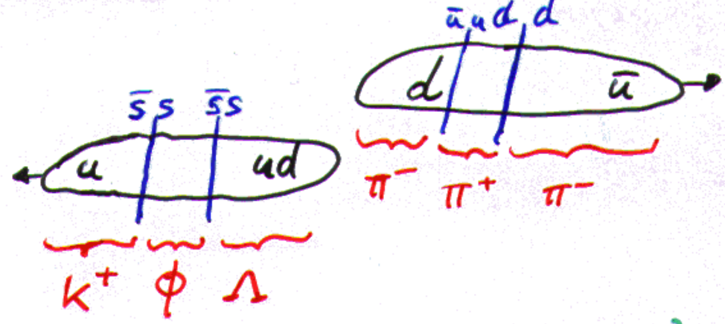
Frg.-Fct.: $D_{h/c}(z)$

$$E\frac{d\sigma}{d^3p_h}(pp \to h + X) = \sum_{abcd} \int dx_a\, dx_b\, dz\, G_a(x_a, \mu^2)\, G_b(x_b, \mu^2)$$

$$\times D_{h/c}(z, M^2) \frac{\hat{s}}{\pi z} \frac{d\sigma^{ab\to cd}}{d\hat{t}}(Q^2)\, \delta(\hat{s} + \hat{t} + \hat{u} - \sum_i m_i^2)$$

Small $m_\perp$

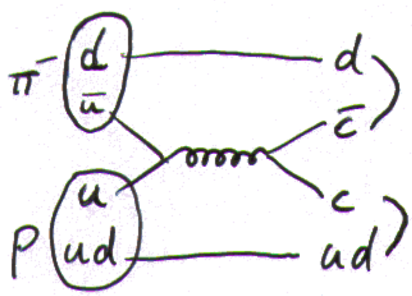


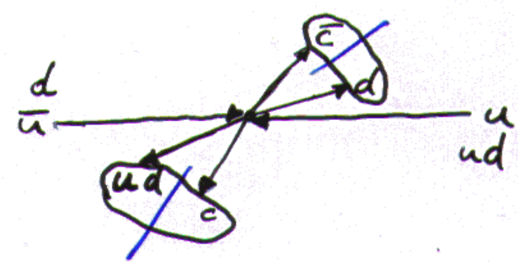


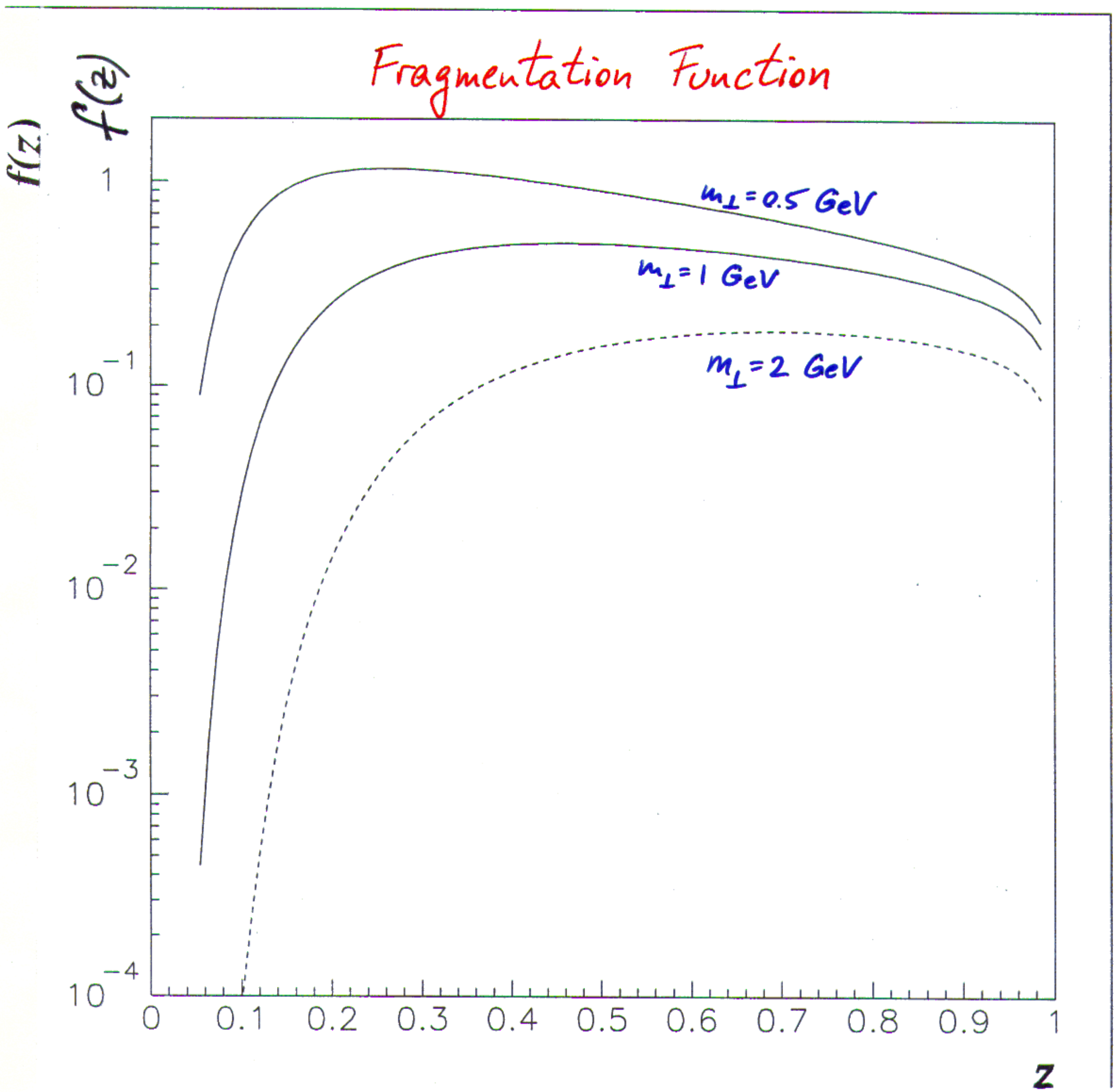
- Tunneling Probability $\sim e^{-\pi m_\perp^2/\kappa}$
- Fragmentation Function $f(z) \sim \frac{1}{z}(1-z)^n e^{-b m_\perp^2/z}$

  (z = Fraction of $P^+ = E + p_{\parallel}$)

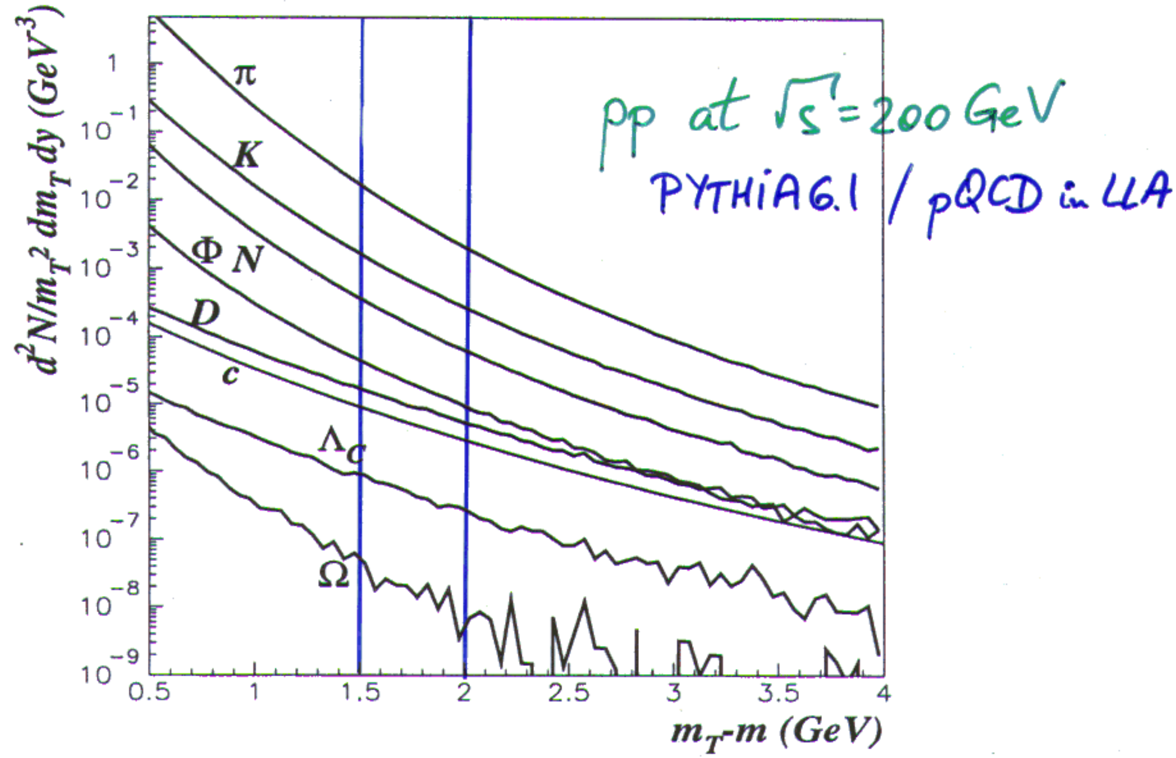
High $m_\perp$

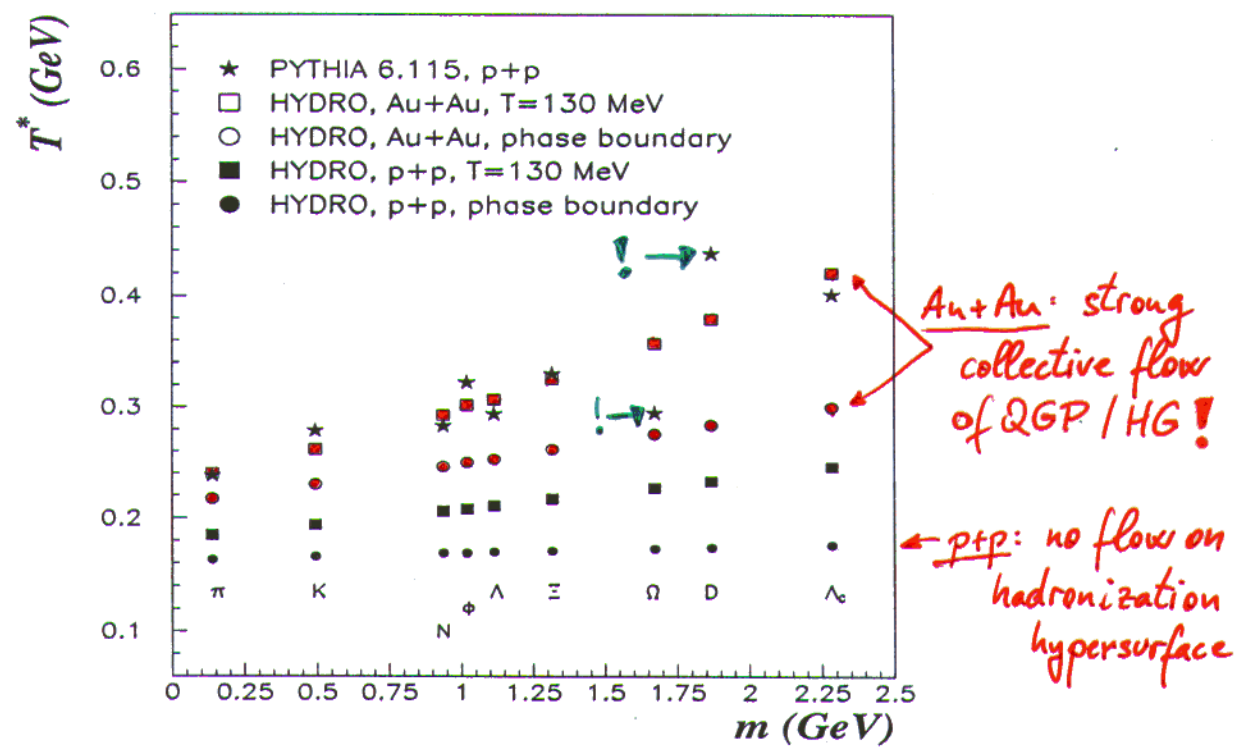


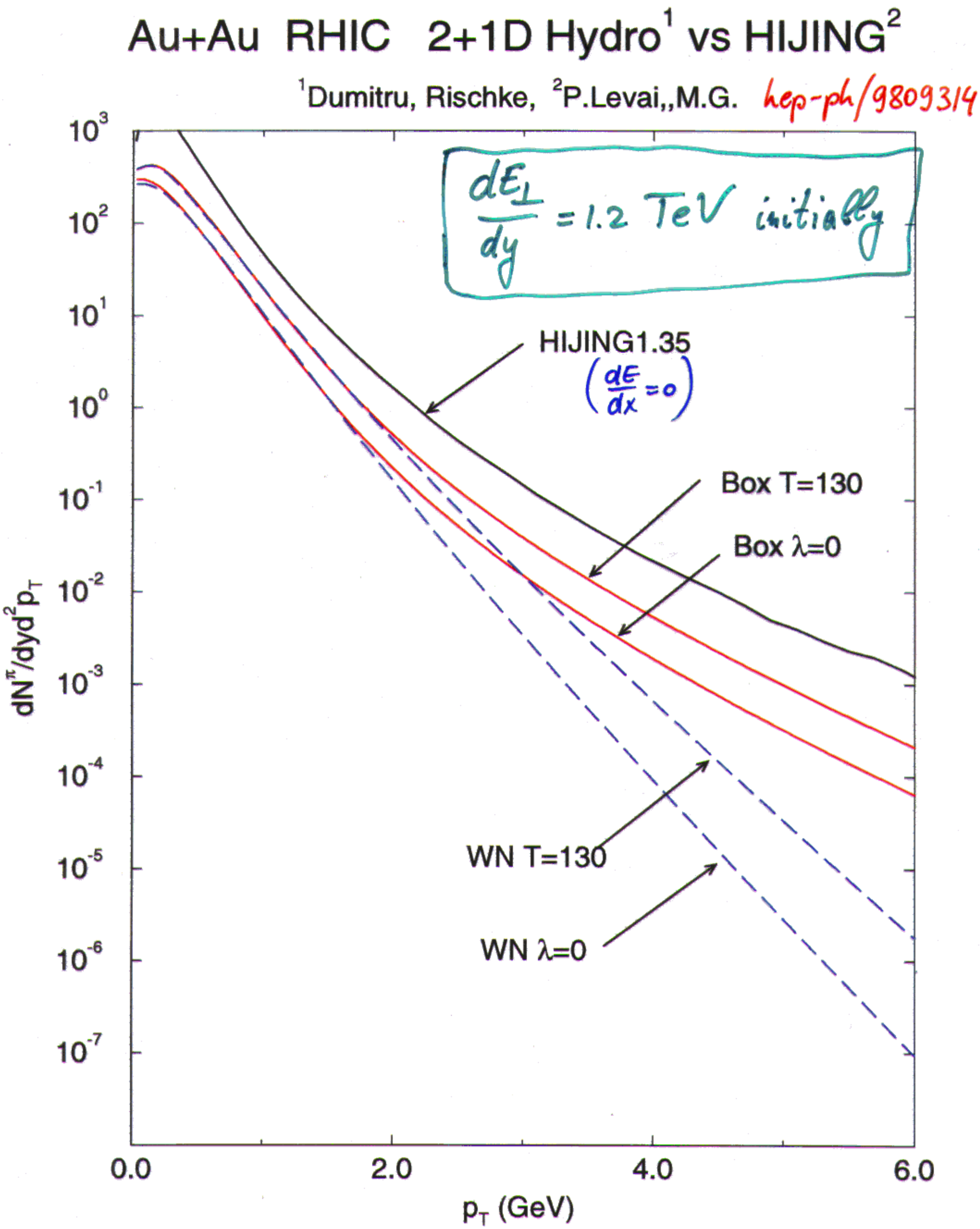




Fragmentation Function
f(z)
f(z)
$m_\perp = 0.5$ GeV
$m_\perp = 1$ GeV
$m_\perp = 2$ GeV
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
z


$d^2N/m_T^2\,dm_T\,dy\ (GeV^{-3})$
$m_T\text{-}m$ (GeV)
π
K
Φ N
D
c
Λ_c
Ω
pp at $\sqrt{s}$ = 200 GeV
PYTHIA6.1 / pQCD in LLA


T^* (GeV)
m (GeV)
PYTHIA 6.115, p+p
HYDRO, Au+Au, T=130 MeV
HYDRO, Au+Au, phase boundary
HYDRO, p+p, T=130 MeV
HYDRO, p+p, phase boundary
π
K
N
Φ
Λ
Ξ
Ω
D
Λ_c
Au+Au: strong collective flow of QGP / HG!
p+p: no flow on hadronization hypersurface


Au+Au RHIC 2+1D Hydro[1] vs HIJING[2]
[1]Dumitru, Rischke, [2]P.Levai,,M.G. hep-ph/9809314
$\frac{dE_\perp}{dy} = 1.2$ TeV initially
HIJING1.35
$\left(\frac{dE}{dx}=0\right)$
Box T=130
Box λ=0
WN T=130
WN λ=0
dN^π/dyd^2p_T
p_T (GeV)
0.0
2.0
4.0
6.0
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}

SUMMARY

Pb+Pb at SPS, $\sqrt{s} = 18\,A$ GeV:

- measured $dN/d^2p_\perp dy$ can be described within hydro; even π-spectra above $p_\perp = 1$ GeV
- large space-time volume of hadronic phase required ($T_{fo} \simeq 130$ MeV), i.e. a second "explosion" of the hadron gas
- $\langle p_\perp \rangle_\phi$, $\langle v_\perp \rangle_\phi$ similar to that of N, Λ
- $\langle v_\perp^{flow} \rangle \simeq 0.3$ on the phase boundary ($\lambda = 0$)
  $\simeq 0.4$ on the $T = 130$ MeV isotherm

p+p at RHIC, $\sqrt{s} = 200$ GeV:

- due to minijets, T^* increases with m (not reproduced by hydro)
- "discontinuity" at charm threshold, i.e. $T_D^* \gg T_\Omega^*$

Au+Au at RHIC, $\sqrt{s} = 200\,A$ GeV:

Hypothetical Quark-Gluon Fluid

- $\langle p_\perp \rangle$, T^* depend only on hadron mass (not on any quantum number)
- (nearly) linear increase of $\langle p_\perp \rangle$, T^* with m, no jump at charm threshold, T_D^* smaller than in pp
- although ε_i (RHIC) $\gg$ ε_i (SPS) 1st order phase transition leads to similar $\langle v_\perp^{flow} \rangle$, i.e. stall of $\perp$-flow above SPS
- if minijets thermalize, the number of pions above $p_\perp \sim 2$-3 GeV decreases by at least one order of magnitude (Wang, Gyulassy – "jet quenching")