POSSIBILITY OF SPONTANEOUS P, CP VIOLATION IN HOT QCD

Based on work with
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Outline

1. Brief reminder about the $U_A(1)$ problem

   mini-
   (reminder about $U_A(1)$:

   $\mathcal{L}_{QCD}$ is invariant under $q \to e^{i\gamma_5 \alpha} q$;
   where are the parity doublets in hadron spectrum?
   if broken, where is the Goldstone boson?)

2. θ-vacua 

3. Large N effective Lagrangian

4. Finite temperatures:
   possibility of spontaneous P, CP violation ?!

   $\Downarrow$

5. Signatures at RHIC

I Brief reminder about the $U_A(1)$ problem

1. Consider pseudoscalar flavour-singlet field (the "would-be" ninth Goldstone η_0')

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle$$

why it is not, if $U_A(1)$ is spontaneously broken? $q \to e^{i\gamma_5\alpha} q$

divergence of the corresponding current

$$\partial^\mu J^0_{5\mu} = 2i\sum_f m_f \bar{q}_f \gamma_5 q + \underline{2N_f \frac{g^2}{16\pi^2} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})}$$

interaction with gluons => anomalous part; does not vanish in the chiral limit $m_f \to 0$

2. introduce (gauge-dependent) topological current

$$K_\mu = 2N_f \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\lambda\rho} \mathrm{Tr}(G^{\nu\lambda} A^\rho)$$

in the chiral limit,

$$\partial^\mu J^0_{5\mu} = \partial^\mu K_\mu ,$$

and we can define a new axial current

$$J_{5\mu} \equiv J^0_{5\mu} - K_\mu ,$$

which is now explicitly conserved in the chiral limit:

$$\partial^\mu J_{5\mu} = 2i \sum_f m_f \bar{q}_f \gamma_5 q_f \xrightarrow[m_f \to 0]{} 0$$

$\Rightarrow$ naively, we expect the corresponding charge conservation

$$Q_5 = \int d^3x\, J_{50} \qquad \frac{dQ_5}{dt} = 0 \quad ?$$

3. Let us check this:

(we expect $\int_{-\infty}^{\infty} dt \, \frac{dQ_5}{dt} = 0$ for a conserved charge)

We get

$$\int_{-\infty}^{\infty} dt \, \frac{dQ_5}{dt} = 2N_f \, \nu[G],$$

with

$$\nu[G] = \cancel{2N_f} \, \frac{g^2}{32\pi^2} \int d^4x \, \mathrm{Tr}\left(G_{\mu\nu} \tilde{G}^{\mu\nu}\right)$$

"topological charge"

In QED, $\nu = 0$

But in QCD $\nu \neq 0$;

for the one-instanton configuration, for example,

$$\nu[G_{inst}] = 1$$

$\Rightarrow$ Q_5 is not conserved;

from $t = -\infty$ to $t = +\infty$ it changes by

$$\Delta Q_5 = 2N_f \, \nu[G]$$

$\Rightarrow$ Non-perturbative topological solutions explicitly break the $U_A(1)$ symmetry

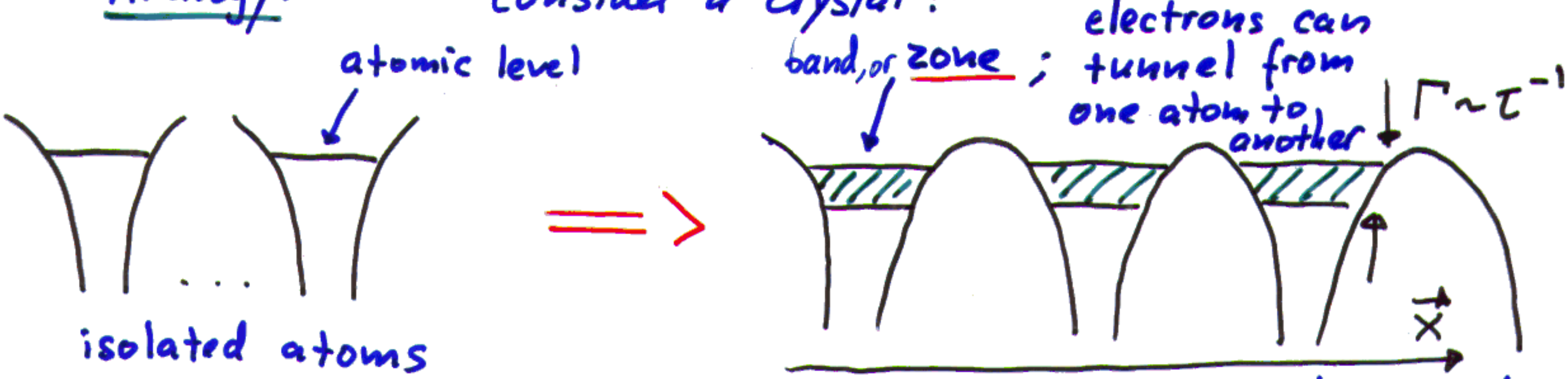
$\Downarrow$

There should be no Goldstone

2. Θ - Worlds

Non-conservation of Q_5 $\Longleftrightarrow$ Existence of vacua with different "winding numbers" ν

Analogy: consider a crystal:

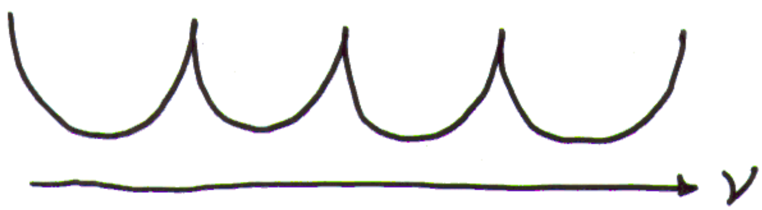


The wave function of the ground state:

$$|\vec{k}\rangle = \sum_{\vec{x}} e^{i\vec{k}\vec{x}} |\vec{x}\rangle$$

"quasi-momentum"

QCD:



$$|\theta\rangle = \sum_{\nu} e^{i\theta\nu} |\nu\rangle$$

To compute an observable, use

$$\langle O \rangle_{\Omega} = \frac{\sum_{\nu} e^{i\theta\nu} \int_{\nu} [d\varphi] \exp\left(i\int_{\Omega} d^4x\, \mathcal{L}\right) O(\varphi)}{\sum_{\nu} e^{i\theta\nu} \int_{\nu} [d\varphi] \exp\left(i\int_{\Omega} d^4x\, \mathcal{L}\right)}$$

$\Rightarrow$ this is equivalent to adding to the Lagrangian the term

$$\mathcal{L}_{\theta} \simeq \theta \cdot \int d^4x\, \mathrm{Tr}\left(G_{\mu\nu} \tilde{G}^{\mu\nu}\right)$$

Example:

an effective Lagrangian including $U_A(1)$ term
(non-linear σ-model)

G. Veneziano,
P. di Vecchia;
E. Witten

$$\mathcal{L} = \frac{F_\pi^2}{2}\Big\{ \underbrace{Tr\, \partial_\mu U \partial_\mu U^{-1}}_{U(3)\times U(3)\ \text{invariant}} + \underbrace{(Tr\, MU + Tr\, MU^+)}_{\text{under } SU(3)\times SU(3) \text{ transforms as quark mass term}} -$$

$$- \underbrace{\frac{a}{N}(-i \ln \det U - \theta)^2}_{\text{preserves } SU(3)\times SU(3),\ \text{reflects } U_A(1) \text{ anomaly}} \Big\}$$

$$U = \exp\left(i \frac{\Phi}{F_\pi}\right)$$

$$a \sim \int d^4x \left\langle T\{ G_{\mu\nu}\tilde{G}^{\mu\nu}(x), G_{\mu\nu}\tilde{G}^{\mu\nu}(0)\} \right\rangle_{YM}$$

The angle θ is severely constrained

by D_n, $\eta \to \pi\pi$ $\qquad |\theta| < 10^{-9}$

We will assume $\theta = 0$

Effective potential (vacuum energy)

$$V_{eff}(U) = F_\pi^2\left(-\frac{1}{2}\,\mathrm{Tr}\, MU - \frac{1}{2}\,\mathrm{Tr}\, MU^+ + \frac{a}{2N}\left(-i\ln\det U\right)^2\right)$$

assume $m_u = m_d$

$$M = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \equiv \mathrm{diag}(\mu^2; \mu^2; \mu_s^2)$$

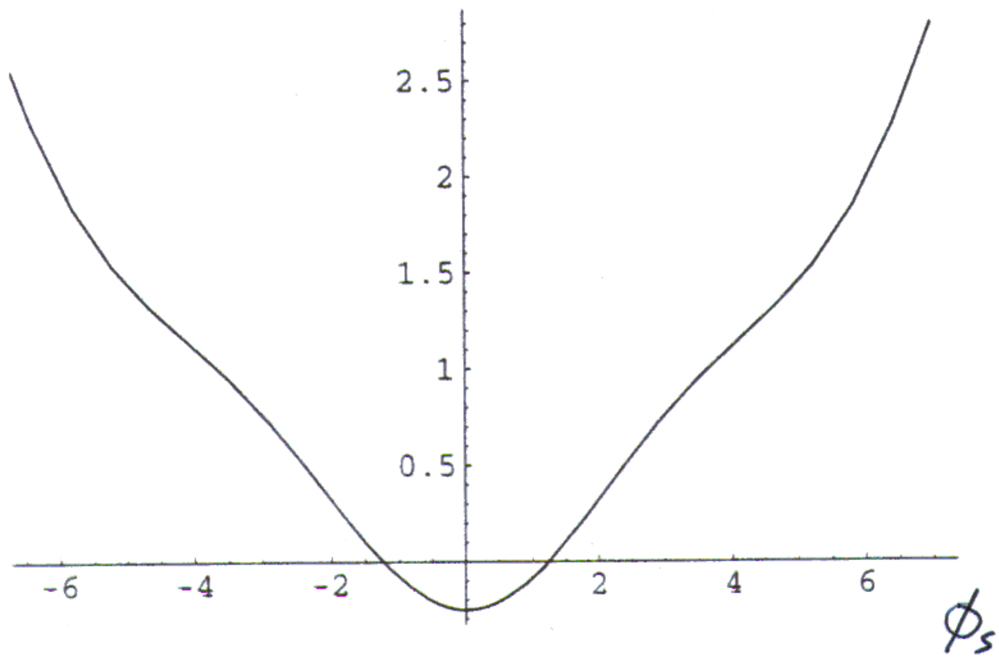
$$U = \begin{pmatrix} e^{i\phi_1} & & 0 \\ & e^{i\phi_2} & \\ 0 & & e^{i\phi_3} \end{pmatrix}$$

In terms of ϕ's, the effective potential

$$V_{eff}(\phi_i) = F_\pi^2\left[-\sum_i \mu_i^2 \cos\phi + \frac{a}{2N}\left(\sum_i \phi_i - \theta\right)^2\right]$$

How does it look like?

$\rightarrow$ Fig



2.5
2
1.5
1
0.5
-6
-4
-2
2
4
6
ϕ_s

B. Allés
M. D'Elia,
A. Di Giacomo
P.W. Stephenson
hep-lat/9808004

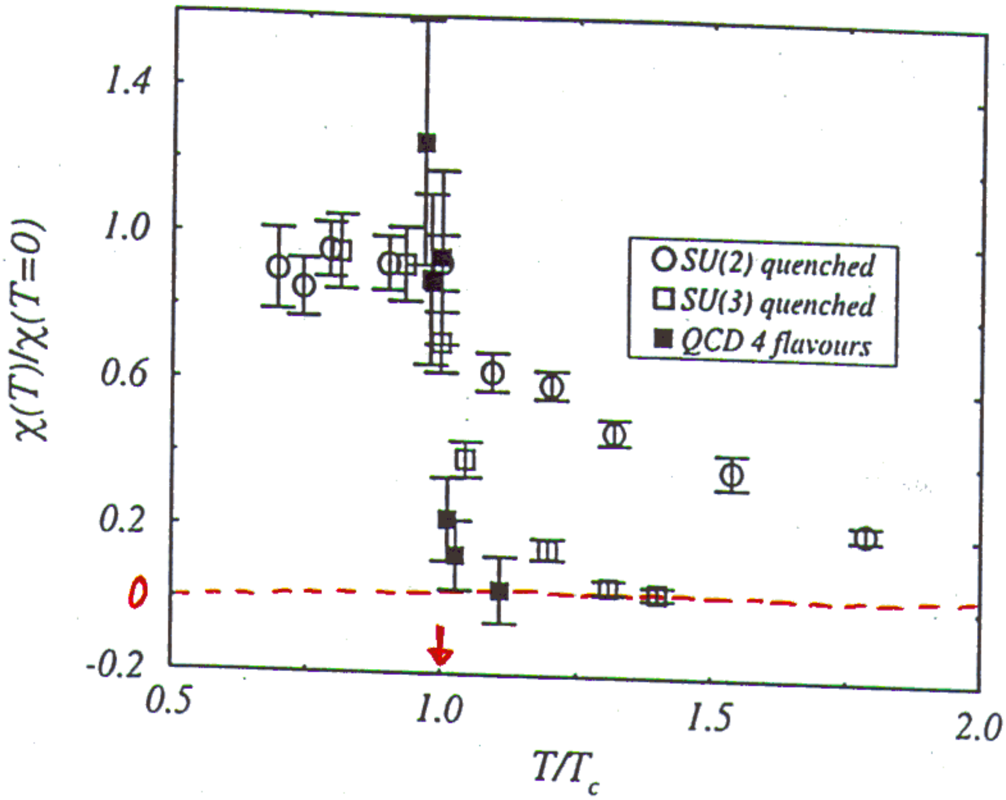


Figure 3. Behaviour of the topological susceptibility as a function of the normalized temperature T/T_c.

large N_c:

- below T_c, interactions are suppressed by $1/N_c$,
  \# of degrees of freedom $\sim N_c^0$
  $T_c \sim N_c^0$
  $\Rightarrow$ "cold" gas of glueballs and mesons
- above T_c, \# of degrees of freedom $\sim N_c^2$

$\Rightarrow$ huge change of the free energy at T_c

$\Downarrow$

any phase transition occurs at T_c

At $\theta = 0$, (we do not consider $\theta \neq 0$, Dashen phenomena, etc)

only trivial solution

$$\langle \phi_u \rangle = \langle \phi_d \rangle = \langle \phi_s \rangle = 0$$

But: @ high density, instantons are screened away

+ large N arguments:

$\Downarrow$ $T_d \simeq T_{U(1)}$

D. Gross
R. Pisarski
L. Yaffe

R. Pisarski
F. Wilczek

E. Shuryak
M. Velkovsky
. . . . .

+ Lattice?
$\rightarrow$ fig

$a \rightarrow 0.4a$ @ $T_c - \epsilon$?

When density grows,

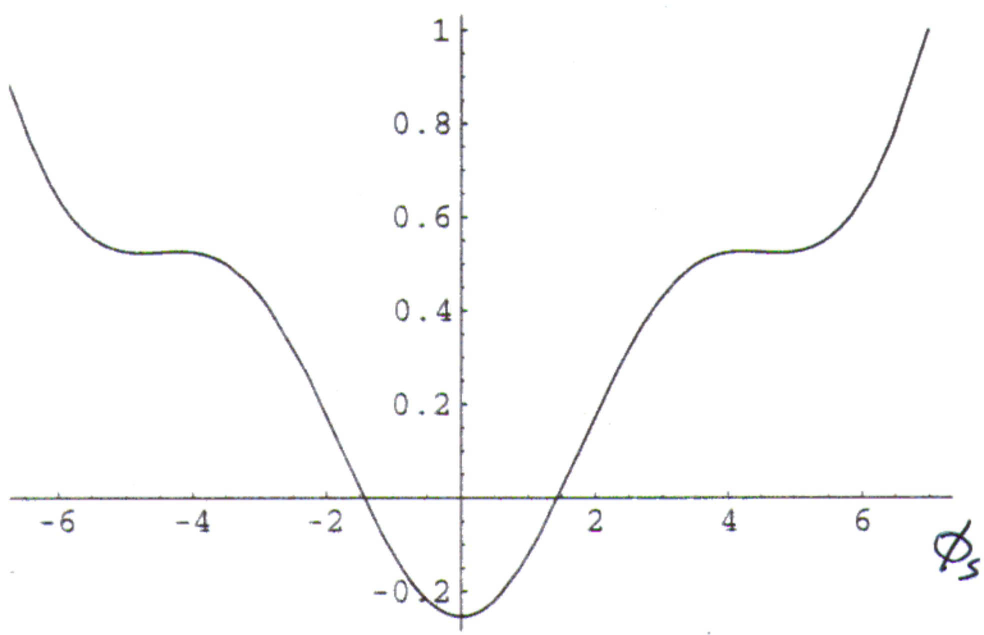
$$a \sim \int d^4x \langle T\{ G_{\mu\nu}\tilde{G}^{\mu\nu}(x), G_{\mu\nu}\tilde{G}^{\mu\nu}(0) \} \rangle$$

should decrease

Does the behavior of the effective potential change?

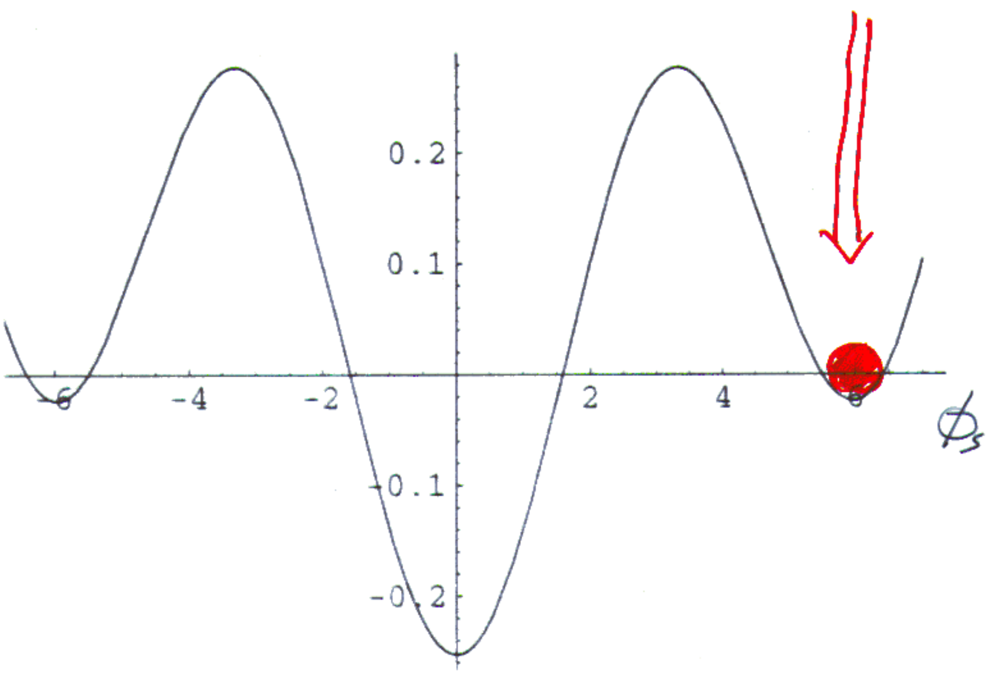
YES

$\rightarrow$ figure



1
0.8
0.6
0.4
0.2
-0.2
-6
-4
-2
2
4
6
ϕ_s

Metastable,
CP & P odd, vacuum!



The additional minima are <u>local</u>;
they have the energy density $\epsilon > \epsilon_{\text{true vacuum}}$,
so they do not contribute to the partition function in the $V \to \infty$.

$\Rightarrow$ $\boxed{\text{does not contradict to Vafa-Witten theorem}}$

<u>But</u>: they describe metastable, "false" vacua which can be excited (at RHIC, for example.)

These metastable vacua contain $\eta - \eta'$ condensate $J^{PC} = 0^{-+}$

$\Downarrow$

$\boxed{\text{Massive violation of P, CP, and \sout{(possibly)} isospin}}$

Signatures at RHIC

1) "False" vacua will decay with the emission of η, η' $\Rightarrow$ enhanced η, η' yields

J. Kapusta, D.K. L. McLerran; Z. Huang X.-N. Wang

How to detect?

$\left.\begin{array}{l}\eta' \to \gamma\gamma \\ \eta \to \gamma\gamma\end{array}\right)$ difficult at small p_T

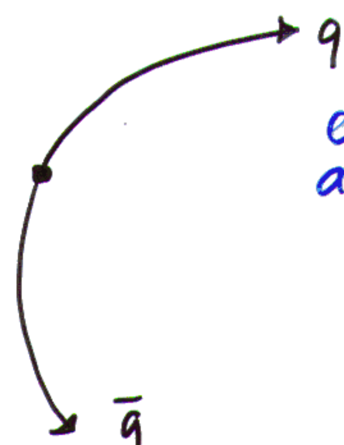
$\eta' \to \pi^+\pi^-\eta \Rightarrow$ HBT! S. Vance, T. Csörgo, D.K.

2) Parity-violating decays, e.g. $\eta \to \pi\pi$

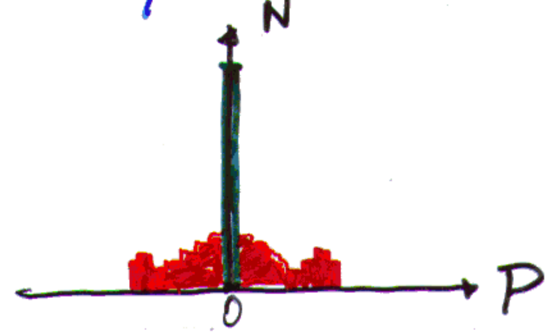
3) Global P, CP-odd observables,

e.g. $$\mathcal{P} = \sum_{\pi^+\pi^-} \frac{[\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}] \cdot \vec{z}}{|\vec{p}_{\pi^+}|\,|\vec{p}_{\pi^-}|}$$

$G\tilde{G} \sim \vec{E} \cdot \vec{H}$



event-by-event analysis:



+ cosmological implications? $\to$ magnetic fields

+ baryons