Fit a circle in 3D

Following method of Craig M. Shakarji of NIST

http://nvlpubs.nist.gov/nistpubs/jres/103/6/j36sha.pdf

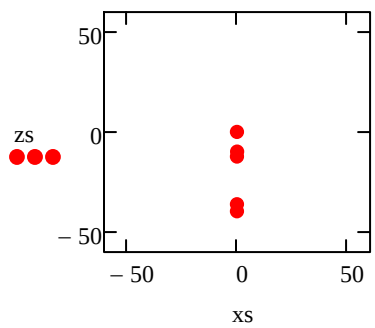
Steps:
1. Transform data points so that the centroid is at the origin
2. Fit a plane to the points
3. Rotate the date to that plane
4. Do a 2D circle fit to the data
5. Using the plane direction vector and the 2D circle values as the initial estimate of the fit parameters do a full 3D circle simultaneous fit with the 7 parameters (plane vector, circle center vector and radius). This fit is done in the centroid frame.
6. Transform back from the centroid frame to the lab frame.

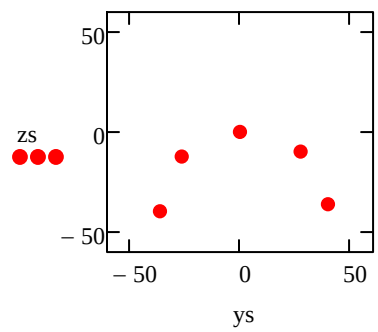
data input, turn off when using simulated data

$XG = \mathrm{READPRN}(\text{"CMMG2.txt"})$ units in mm

$X = XG$

$xs = X^{\langle 0 \rangle}$ $ys = X^{\langle 1 \rangle}$ $zs = X^{\langle 2 \rangle}$





each row is a point in x,y,z

$$X = \begin{pmatrix} -0.01852 & -0.03624 & 0.00365 \\ -0.00957 & -36.14912 & -39.67986 \\ -0.00299 & -26.29254 & -12.35234 \\ 0.00148 & 27.32683 & -9.85594 \\ -0.00203 & 39.67964 & -36.14912 \end{pmatrix}$$

data input, turn off when using simulated data

Simulation data for testing, turn off when using real data

Do the math

Results

units: mm

$$c = \begin{pmatrix} -0.00383 \\ 1.765123 \\ -37.910735 \end{pmatrix}$$

circle center

$$A1 = \begin{pmatrix} 1 \\ -0.0001 \\ 0.000132 \end{pmatrix}$$

circle normal

$r = 37.955014$

circle radius

$drv = dr(xc, yc, zc, Aa, Ab, Ac, r, Xs)$

radial distance of points from r

$$drv = \begin{pmatrix} 0.00214 \\ 0.000481 \\ -0.00157 \\ -0.001459 \\ 0.000405 \end{pmatrix}$$

$Stdev(drv) = 0.001547$

standard deviation of the points from r

$Stdev(drv) \cdot \sqrt{12} = 0.005358$

window spread

$$\frac{Stdev(drv) \cdot \sqrt{12}}{2} = 0.002679$$

or +- value

$sprd(drv) = 0.00371$

this sample spread

$dnv = dn(xc, yc, zc, Aa, Ab, Ac, Xs)$

distance of points from the circle plane

$$dnv = \begin{pmatrix} -0.009517 \\ -0.00217 \\ 0.007019 \\ 0.00644 \\ -0.00177 \end{pmatrix}$$

$Stdev(dnv) = 0.006877$ standard deviation distance from the plane

$Stdev(dnv)\cdot\sqrt{12} = 0.023822$ window spread of points from the plane

$sprd(dnv) = 0.016536$ this sample spread in distance from the plane

wobble of the rotation axis implied by the distance of the points from the circle plane

$$dwn = \frac{dnv}{r} = \begin{pmatrix} -0.000251 \\ -0.000057 \\ 0.000185 \\ 0.00017 \\ -0.000047 \end{pmatrix}\cdot rad$$

$Stdev(dwn) = 0.000181\cdot rad$ standard deviation of wobble

$Stdev(dwn)\cdot\sqrt{12} = 0.000628\cdot rad$ window spread

$Stdev(dwn)\cdot\sqrt{12} = 129.461961\cdot arcsec$ there is no spec given for axis wobble on the MTR rotary head, but the rotation angle spec is +- 5 arc sec which is 13 times less than this wobble.

$L = 300$ length of the sector

$$dwn\cdot L = \begin{pmatrix} -0.075227 \\ -0.017154 \\ 0.055479 \\ 0.050904 \\ -0.013993 \end{pmatrix}$$

projected error at the end of the sector due to axis wobble

$Stdev(dwn\cdot L)\cdot\sqrt{12} = 0.188295$ window spread in the projected error at the end of the sector due to axis wobble

$$p(c,X) = \begin{pmatrix} -0.01469 & -1.801363 & 37.914385 \\ -0.00574 & -37.914243 & -1.769125 \\ 0.00084 & -28.057663 & 25.558395 \\ 0.00531 & 25.561707 & 28.054795 \\ 0.0018 & 37.914517 & 1.761615 \end{pmatrix}$$

Measured position vectors relative to the fit center of the circle

$$\text{angmeasured} = \text{ang2}(c,X) = \begin{pmatrix} 0 \\ 89.951397 \\ 44.948715 \\ 45.057912 \\ 90.059945 \end{pmatrix}$$

angle between the first point vector and all the point vectors
in degrees

$$\text{angset} = \begin{pmatrix} 0 \\ 90 \\ 45 \\ 45 \\ 90 \end{pmatrix}$$

values set for rotary head

$$\delta\theta = \text{angmeasured} - \text{angset} = \begin{pmatrix} 0 \\ -0.048603 \\ -0.051285 \\ 0.057912 \\ 0.059945 \end{pmatrix}$$

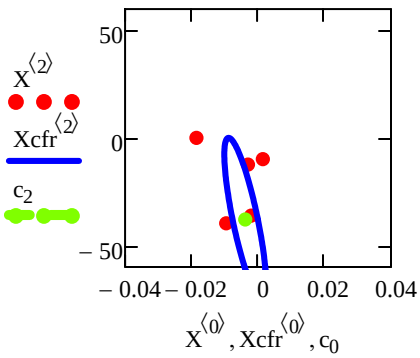
discrepancy between measured and set values in degrees

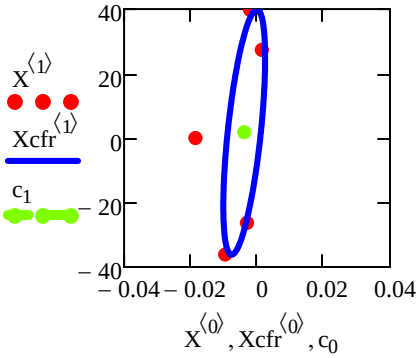
$$\theta\text{spread} = \text{sprd}(\delta\theta)\cdot\frac{\pi}{180} = 400.428427\cdot\text{arcsec}$$

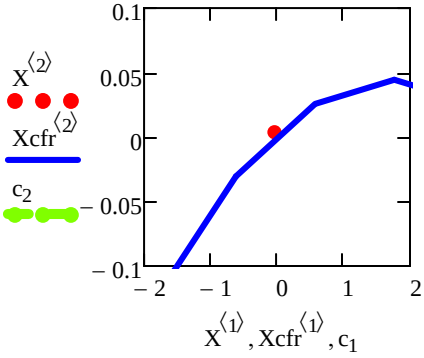
This is 40 times the +- 5 arcsec spec given for the MTR rotary head

units is mm

plotting x limits $\quad \text{xmin} = -.04 \quad \text{xmax} = .04$


X⟨2⟩
Xcfr⟨2⟩
c2
50
0
− 50
− 0.04
− 0.02
0
0.02
0.04
X⟨0⟩ , Xcfr⟨0⟩ , c0


X⟨1⟩
Xcfr⟨1⟩
c1
40
20
0
− 20
− 40
− 0.04
− 0.02
0
0.02
0.04
X⟨0⟩ , Xcfr⟨0⟩ , c0


X⟨2⟩
Xcfr⟨2⟩
c2
0.1
0.05
0
− 0.05
− 0.1
− 2
− 1
0
1
2
X⟨1⟩ , Xcfr⟨1⟩ , c1