Simple heat.xmcd

Assume sandwich with inner layer at a given temperature, some insulation and an outer layer exposed to air cooling flow. Assume that air flow is always completely mixed such that the outer skin of the insulator is exposed to air with the final temperature. Now as a function of air flow find the power, final air temperature and the skin temperature at the air insulator interface. This assumption of complete mixing will give the maximum skin temperature and the minimum air temperature.

Treat a cylindrical geometry as a plane for this analysis

$r = \frac{40mm + 66mm}{2}$ middle radius of the insulating cylinder

$L = 1070mm$ length of the cylinder

$A = 2 \cdot \pi \cdot r \cdot L$ Area at the middle radius

$t = (66 - 40)mm$ Thickness of the insulator

$Cp_{air} = \left(1.005 \times 10^3 \cdot \frac{J}{kg \cdot K}\right)$ air heat capacity

$\rho_{air} = 1.1644 \frac{kg}{m^3}$ density of air

$hc = 20 \frac{W}{m^2 K}$ convective heat tranfer coefficient, not forced

$Cg = .058 \frac{W}{m \cdot K}$ heat conductivity of the insulator

$Tsin = 250K$ Inner skin layer temperature

$Tairin = 22K$ Input cooling air temperature

$P = (Tairout - Tairin) \cdot F \cdot \rho_{air} \cdot Cp_{air}$ eq. 1 power carried away by cooling air

where:
Tairout is the exhaust air temperature
F is the flow of cooling air

$$P = (Tsin - Tsout)\cdot\frac{A\cdot Cg}{t}$$ eq. 2 power flowing through the insulator

$$P = (Tsout - Tairout)\cdot A\cdot hc$$ eq. 3 Convective heat transfer from the outer skin to the air.

where:
Tsout is the outer skin temperature.

Again assuming that the cooling air is completely mixed so that the ambient temperature that the skin sees is the exhaust air temperature.

Solve these 3 equations to find Tsout, Tairout and P as a function of flow F

From eq. 1

$$Tairout = \frac{P + Cp_{air}\cdot F\cdot Tairin\cdot \rho_{air}}{Cp_{air}\cdot F\cdot \rho_{air}}$$

substitute into eq. 3

$$P = \left(Tsout - \frac{P + Cp_{air}\cdot F\cdot Tairin\cdot \rho_{air}}{Cp_{air}\cdot F\cdot \rho_{air}}\right)\cdot A\cdot hc$$

or

$$Tsout = \frac{P + \frac{A\cdot hc\cdot\left(P + Cp_{air}\cdot F\cdot Tairin\cdot \rho_{air}\right)}{Cp_{air}\cdot F\cdot \rho_{air}}}{A\cdot hc}$$

and substitute this into eq. 2 gives

$$P = \left[Tsin - \frac{P + \frac{A\cdot hc\cdot\left(P + Cp_{air}\cdot F\cdot Tairin\cdot \rho_{air}\right)}{Cp_{air}\cdot F\cdot \rho_{air}}}{A\cdot hc}\right]\cdot\frac{A\cdot Cg}{t}$$

or or

$$P = \frac{A \cdot Cg \cdot (Tsin - Tairin)}{t \cdot \left[\frac{Cg \cdot \left(\frac{A \cdot hc}{Cp_{air} \cdot F \cdot \rho_{air}} + 1 \right)}{hc \cdot t} + 1 \right]} \qquad \text{eq. 4}$$

Substitute eq. 4 into 1

$$\frac{A \cdot Cg \cdot (Tsin - Tairin)}{t \cdot \left[\frac{Cg \cdot \left(\frac{A \cdot hc}{Cp_{air} \cdot F \cdot \rho_{air}} + 1 \right)}{hc \cdot t} + 1 \right]} = (Tairout - Tairin) \cdot F \cdot \rho_{air} \cdot Cp_{air}$$

gives:

$$Tairout = \frac{A \cdot Cg \cdot Tsin \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot Tairin \cdot \rho_{air} + Cp_{air} \cdot F \cdot Tairin \cdot hc \cdot t \cdot \rho_{air}}{A \cdot Cg \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot \rho_{air} + Cp_{air} \cdot F \cdot hc \cdot t \cdot \rho_{air}} \qquad \text{eq. 5}$$

and Subtiute eq. 4 into eq. 2

$$\frac{A \cdot Cg \cdot (Tsin - Tairin)}{t \cdot \left[\frac{Cg \cdot \left(\frac{A \cdot hc}{Cp_{air} \cdot F \cdot \rho_{air}} + 1 \right)}{hc \cdot t} + 1 \right]} = (Tsin - Tsout) \cdot \frac{A \cdot Cg}{t}$$

gives:

$$Tsout = \frac{A \cdot Cg \cdot Tsin \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot Tsin \cdot \rho_{air} + Cp_{air} \cdot F \cdot Tairin \cdot hc \cdot t \cdot \rho_{air}}{A \cdot Cg \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot \rho_{air} + Cp_{air} \cdot F \cdot hc \cdot t \cdot \rho_{air}} \qquad \text{eq. 6}$$

Now writing eqs. 4,5,6 as a function of F

$$P(F) = \frac{A \cdot Cg \cdot (Tsin - Tairin)}{t \cdot \left[\frac{Cg \cdot \left(\frac{A \cdot hc}{Cp_{air} \cdot F \cdot \rho_{air}} + 1 \right)}{hc \cdot t} + 1 \right]}$$

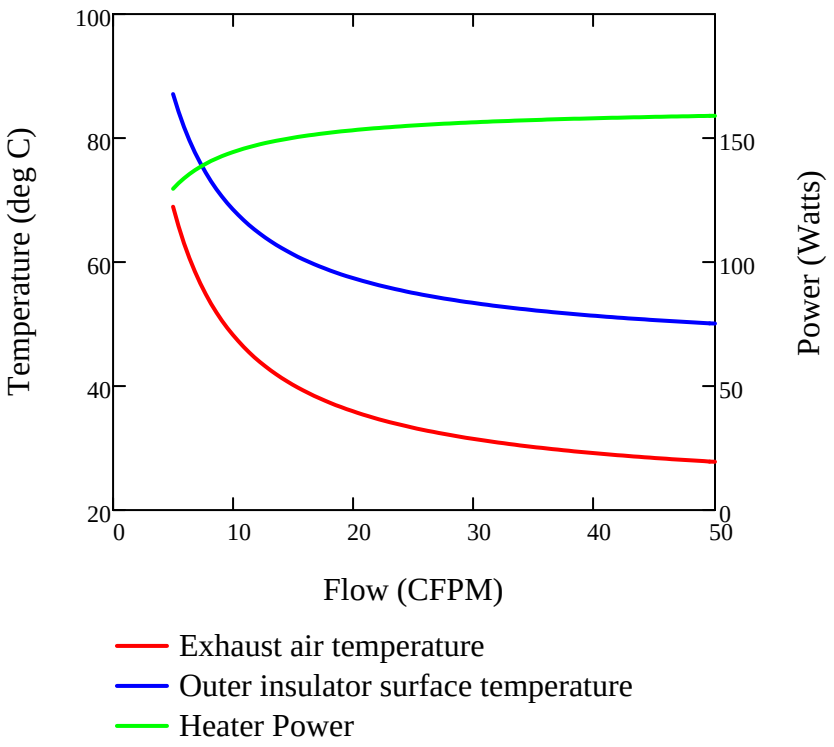
$$Tairout(F) = \frac{A \cdot Cg \cdot Tsin \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot Tairin \cdot \rho_{air} + Cp_{air} \cdot F \cdot Tairin \cdot hc \cdot t \cdot \rho_{air}}{A \cdot Cg \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot \rho_{air} + Cp_{air} \cdot F \cdot hc \cdot t \cdot \rho_{air}}$$

$$Tsout(F) = \frac{A \cdot Cg \cdot Tsin \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot Tsin \cdot \rho_{air} + Cp_{air} \cdot F \cdot Tairin \cdot hc \cdot t \cdot \rho_{air}}{A \cdot Cg \cdot hc + Cg \cdot Cp_{air} \cdot F \cdot \rho_{air} + Cp_{air} \cdot F \cdot hc \cdot t \cdot \rho_{air}}$$

$$N = 100$$

$$n = 0 .. N$$

$$F_n = \left(5 + \frac{45 \cdot n}{N} \right) \cdot \frac{ft^3}{min}$$


100
80
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Temperature (deg C)
Power (Watts)
Flow (CFPM)
Exhaust air temperature
Outer insulator surface temperature
Heater Power

$$P = (Tairout - Tairin) \cdot F \cdot \rho_{air} \cdot Cp_{air}$$