Space Charge Distortion in the STAR TPC, An Approximate Formula

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Abstract:
A simple expression for the space charge distortion is derived which is based on the assumption that interactions produce tracks with dN/dη equal to a constant and that all the tracks are straight lines radiating out from the interaction point. This is for primary ionization in the TPC volume and does not include back ground tracks or positive ion leaks from the wire chambers. A closed form expression for the distortion is obtained by using a one dimensional solution to Poisson's equation. Results are shown for the highest projected AuAu luminosities at RHIC II. This note is based on work done in 1996 to predict space charge effects that we might encounter at RHIC startup.

▶ units

The positive ion generation rate per unit volume:

$$S(r) = L \cdot \sigma \cdot \frac{dN}{d\eta} \cdot \frac{dq}{ds} \cdot \frac{1}{2 \cdot \pi \cdot r^2}$$

Derived in Appendix A1, this is eq. A1-8

Note, since this is a Mathcad document it has been convenient to express variable names for derivative values as follows, for example:

$\frac{dN}{d\eta}$ appears as $dNd\eta$

$dNd\eta = 170$ Measured number from James Dunlop for min-bias AuAu with the corresponding trigger cross section for the ZDC detectors

$\sigma = 10 barn$

$L = 90 \cdot 10^{26} \cdot \frac{1}{cm^2 \cdot sec}$ RHIC II predicted max luminosity with Au+Au

http://hepwww.physics.yale.edu/star/tpc-study/Luminosity.pdf

$minI = 96 \cdot \frac{qe}{cm}$ charge density along track for minimum Ionizing

$dqds = 146 \cdot \frac{qe}{cm}$ average charge density along track length from HIJET total charge (I. Sakrejda).

So use the HIJET number for ionization density along the track which is

50% higher than MinI

$$S(50\cdot cm) = 1\times 10^{5}\,\frac{qe}{cm^{3}\cdot sec}$$

positive ion creation rate at the inner radius, note the creation rate is uniform in z independent of where the interaction occurred. This is a consequence of the uniform dN/dη assumption.

To give a quantitative feeling for this process, the positive ion creation density at the inner radius from a single average min bias AuAu event is:

$$\frac{S(50\cdot cm)}{L\cdot\sigma\cdot qe} = 1.6\,\frac{1}{cm^{3}}$$

The positive ions created through out the TPC volume drift to the central membrane where they are collected. The steady state positive ion density is obtained by integrating the positive ion source rate over z and by knowing the positive ion drift velocity.

$$Zmax = 210\cdot cm$$

drift length of TPC

$$v = 250\cdot\frac{cm}{sec}$$

positive ion drift velocity in the z direction, this comes from Fig. 2.5, p. 63 in Blume and Rolandi. It is the positive ion drift velocity in P10 at 130 V/cm

The charge density at points in the TPC caused by positive ions flowing from their points of creation to the central membrane is given as:

$$\rho(r,z) = \int_{z}^{Zmax}\frac{S(r,z)}{v}\,dz = L\cdot\sigma\cdot dNd\eta\cdot dqds\cdot\frac{1}{2\cdot\pi\cdot r^{2}}\cdot\frac{1}{v}\cdot\int_{z}^{Zmax}1\,dz$$

$$\rho(r,z) = L\cdot\sigma\cdot dNd\eta\cdot dqds\cdot\frac{1}{2\cdot\pi\cdot r^{2}}\cdot\frac{1}{v}\cdot(Zmax - z)$$

steady state positive ion density (space charge)

$$\rho(50cm,0cm) = 1.9\times 10^{-14}\,\frac{coul}{cm^{3}}$$

$$\rho(50cm,0cm) = 1.2\times 10^{5}\,\frac{qe}{cm^{3}}$$

With the much higher luminosity expected for RHIC II this positive ion density is 200 times what was predicted originally in Gulshan Rai's STAR note #3

In the following an approximation of the distortion is obtained from this space charge density by using a one dimensional solution to Poison's equation. The approximation over predicts the distortion and is ~50% larger than a full 3D Poison solution.

The radial E field (Er) can be approximated using Gausses law if we treat the TPC tube as ∞ length with a uniform charge density along the tube. Then:

$$\int_{surface} \vec{E}\, d\vec{s} = \frac{1}{\varepsilon_0} \cdot \int_{volume} \rho_t(r,z)\, dx^3$$

The integration surface is the inner field cage metal where the field is 0 and the outer cylinder is set at radius r where we wish to evaluate the radial field. The charge included in the volume of integration is the space charge derived above plus an induced surface charge, σ_{in} ,on the inner field cage.

The integrals above reduce to:

$$Er(r,z) \cdot 2 \cdot \pi \cdot r \cdot \Delta z = \frac{\Delta z}{\varepsilon_0} \cdot \int_{r_{in}}^{r} \rho(rp,z) \cdot 2 \cdot \pi \cdot rp\, drp + \frac{\Delta z \cdot 2 \cdot \pi \cdot r_{in}}{\varepsilon_0} \cdot \sigma_{in}$$

$$Er(r,z) \cdot 2 \cdot \pi \cdot r \cdot \Delta z = \frac{\Delta z}{\varepsilon_0} \cdot L \cdot \sigma \cdot dN d\eta \cdot dqds \cdot \frac{1}{v} \cdot (Zmax - z) \cdot \int_{r_{in}}^{r} \frac{1}{2 \cdot \pi \cdot rp^2} \cdot 2 \cdot \pi \cdot rp\, drp + \frac{\Delta z \cdot 2 \cdot \pi \cdot r_{in}}{\varepsilon_0} \cdot \sigma_{in}$$

$$Er(r,z) = \frac{1}{2 \cdot \pi \cdot r \cdot \varepsilon_0} \cdot L \cdot \sigma \cdot dN d\eta \cdot dqds \cdot \frac{1}{v} \cdot (Zmax - z) \cdot \int_{r_{in}}^{r} \frac{1}{rp}\, drp + \frac{r_{in}}{\varepsilon_0 \cdot r} \cdot \sigma_{in}$$

$$Er(r,z) = \frac{1}{2 \cdot \pi \cdot r \cdot \varepsilon_0} \cdot L \cdot \sigma \cdot dN d\eta \cdot dqds \cdot \frac{1}{v} \cdot (Zmax - z) \cdot \ln\left(\frac{r}{r_{in}}\right) + \frac{r_{in}}{\varepsilon_0 \cdot r} \cdot \sigma_{in}$$

Keep in mind the confusing variable names, namely σ is the average min bias AuAu cross section and σ_{in} is the surface charge density on the inner field cage induced by the space charge.

The surface charge, σ_{in} , is obtained knowing that the potential between the inner and outer field cage is 0 at any given z.

$$\int_{r_{in}}^{r_{out}} Er(r,z)\,dr = \int_{r_{in}}^{r_{out}} \frac{1}{2\cdot\pi\cdot r\cdot\varepsilon_0}\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot\frac{1}{v}\cdot(Zmax - z)\cdot\ln\left(\frac{r}{r_{in}}\right) dr + \frac{\sigma_{in}\cdot r_{in}}{\varepsilon_0}\cdot\int_{r_{in}}^{r_{out}} \frac{1}{r}\,dr = 0$$

$$\int_{r_{in}}^{r_{out}} Er(r,z)\,dr = \frac{1}{4}\cdot\ln\left(\frac{r_{out}}{r_{in}}\right)^2\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot\frac{Zmax - z}{\pi\cdot(\varepsilon_0\cdot v)} + \sigma_{in}\cdot\frac{r_{in}}{\varepsilon_0}\cdot\ln\left(\frac{r_{out}}{r_{in}}\right) = 0$$

$$\sigma_{in} = \frac{-1}{4}\cdot(Zmax - z)\cdot dqds\cdot dN d\eta\cdot L\cdot\sigma\cdot\frac{\ln\left(\frac{r_{out}}{r_{in}}\right)}{r_{in}\cdot\pi\cdot v}$$

$$Er(r,z) = \frac{1}{2\cdot\pi\cdot r\cdot\varepsilon_0}\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot\frac{1}{v}\cdot(Zmax - z)\cdot\ln\left(\frac{r}{r_{in}}\right) + \frac{r_{in}}{\varepsilon_0\cdot r}\cdot\sigma_{in}$$

$$Er(r,z) = \frac{1}{4}\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot(Zmax - z)\cdot\frac{\ln\left(\frac{r^2}{r_{in}\cdot r_{out}}\right)}{\pi\cdot r\cdot\varepsilon_0\cdot v}$$

finally the radial component of the electric field caused by space charge

The distortion in the $r\phi$ direction comes from the EXB term integrated along the drift path and is:

$$r\phi(r,z) = \frac{\omega\tau}{1 + \omega\tau^2}\cdot\int_{z}^{Zmax} \frac{Er(r,z)}{Ez}\,dz = \frac{\omega\tau}{1 + \omega\tau^2}\cdot\frac{1}{4}\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot\frac{\ln\left(\frac{r^2}{r_{in}\cdot r_{out}}\right)}{\pi\cdot r\cdot\varepsilon_0\cdot v\cdot Ez}\cdot\int_{z}^{Zmax} (Zmax - zp)\,dz$$

$$r\phi(r,z) = \frac{\omega\tau}{1+\omega\tau^2}\cdot L\cdot\sigma\cdot dN d\eta\cdot dqds\cdot\frac{\ln\left(\frac{r^2}{r_{in}\cdot r_{out}}\right)}{8\cdot\pi\cdot r\cdot\varepsilon_0\cdot v\cdot Ez}\cdot(Zmax - z)^2$$

distortion in the $r\phi$ direction

assume $\omega\tau$

$\omega\tau = 2.28$ — for 0.5 Tesla in P10, this may not be the latest value

$r_{in} = 48\cdot cm$ — inner field cage radius

$r_{out} = 200\cdot cm$ — outer field cage radius

$Ez = 130\cdot\frac{volt}{cm}$ — drift field

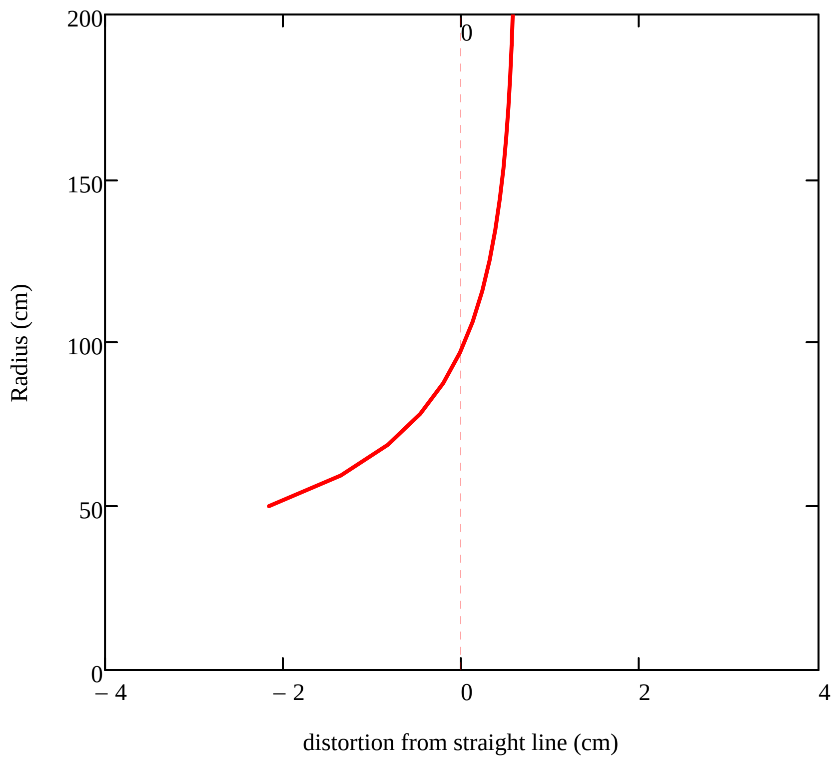
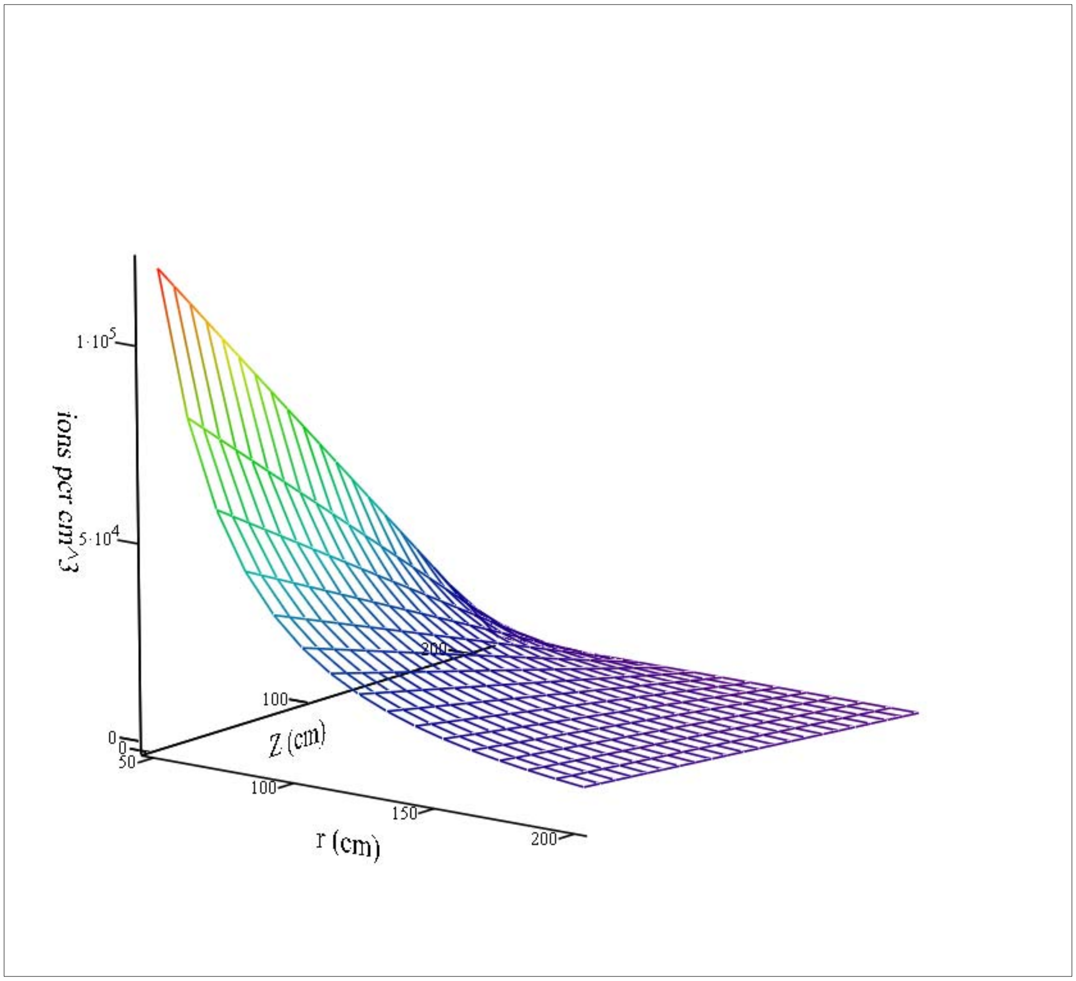


Fig. 1 $r\phi$ distortion for a straight line track. A straight line vertical track at the center of TPC (dotted line) will be distorted by space charge and appears as the solid line.



ρ1

Fig. 2 Steady state positive ion density as a function of r and z in the TPC volume for RHIC II predicted AuAu luminosity. This is the positive ion density that produced the distortion shown in Fig. 1

Appendix
A1

Derivation of

$$S(r) = \frac{dq}{dVdt} = L \cdot \sigma \cdot \frac{dN}{d\eta} \cdot \frac{dN_i}{ds} \cdot \frac{q_e}{2 \cdot \pi \cdot r^2}$$ Eq. A1-1 the rate of positive ion generation per volume

using a cylindrical coordinate geometry (Fig. A1-1).

$\frac{dN}{d\eta}$	number of tracks per unit pseudo rapidity, assumed constant
$\frac{dN_i}{ds}$	number of ion-electron pairs created per unit length along the track
L	Luminosity
σ	cross section
q_e	electron charge

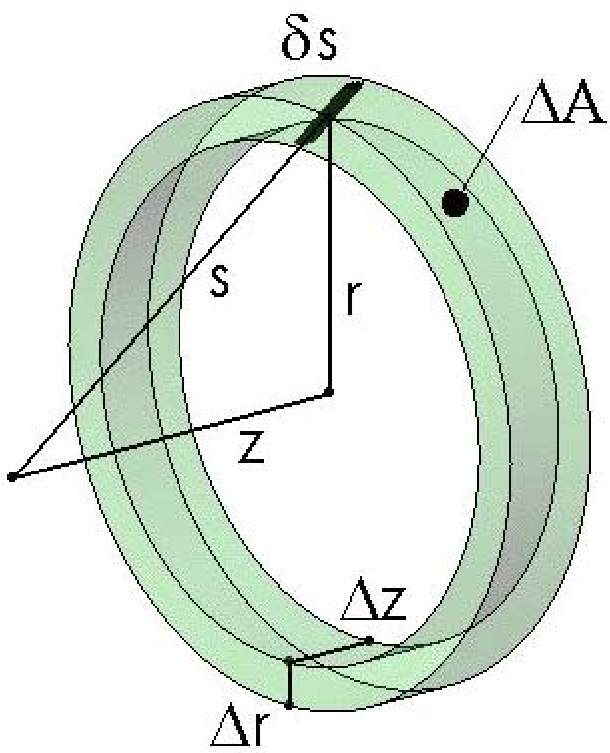


Fig. A1-1 Elemental volume with track along s leaving ionization segment δs

from the geometry illustrated in Fig. A1-1

$$\Delta q = \frac{dN}{dA} \cdot \Delta A \cdot \frac{dN_i}{ds} \cdot \delta s \cdot q_e$$ Eq. A1-2

where

$$\frac{dN}{dA} = \frac{dN}{dz} \cdot \frac{1}{2 \cdot \pi \cdot r} = \frac{dN}{d\eta} \cdot \frac{d\eta}{dz} \cdot \frac{1}{2 \cdot \pi \cdot r}$$ Eq. A1-3

is for a collision, the average number of tracks per area impinging on the inside surface of the elemental volume in Fig. A1-1

and $$\delta s = \Delta r \cdot \frac{s}{r} = \Delta r \cdot \frac{\sqrt{r^2 + z^2}}{r}$$ Eq. A1-4

is the length of the track segment passing through the elemental volume

$$\Delta V = \Delta A \cdot \Delta r$$ Eq. A1-5

is the elemental volume

combining Eq. A1-2 through Eq. A1-5 gives

$$\frac{dq}{dV} = \frac{\Delta q}{\Delta V} = \frac{\frac{dN}{d\eta} \cdot \frac{d\eta}{dz} \cdot \frac{1}{2 \cdot \pi \cdot r} \cdot \Delta A \cdot \frac{dN_i}{ds} \cdot \Delta r \cdot \frac{\sqrt{r^2 + z^2}}{r} \cdot q_e}{\Delta A \cdot \Delta r} = \frac{dN}{d\eta} \cdot \frac{d\eta}{dz} \cdot \frac{dN_i}{ds} \cdot \frac{\sqrt{r^2 + z^2}}{2 \cdot \pi \cdot r^2} \cdot q_e$$ Eq. A1-6

the charge volume density left by a single collision

To find $\frac{d\eta}{dz}$ use the definition of pseudo rapidity

$$\eta = -\ln\left(\tan\left(\frac{\theta}{2}\right)\right)$$ and $$\theta = \operatorname{atan}\left(\frac{r}{z}\right)$$

to get

$$\eta(r, z) = \ln\left(\frac{\sqrt{z^2 + r^2} + z}{r}\right)$$

and taking the derivative

$$\frac{d\eta}{dz} = \frac{1}{\sqrt{r^2 + z^2}}$$ Eq. A1-7

combine Eq. A1-7 and A1-6 to eliminate $\frac{d\eta}{dz}$

and multiply by the event rate $L \cdot \sigma$

gives the following expression for the generation rate of positive ions per volume

$$\frac{dq}{dVdt} = L \cdot \sigma \cdot \frac{dN}{d\eta} \cdot \frac{dN_i}{ds} \cdot \frac{q_e}{2 \cdot \pi \cdot r^2}$$ Eq. A1-8

which is uniform in z, i.e. it does not matter where in z the collision occurred.

A similar and perhaps more straight forward derivation has been done starting with spherical coordinates which gives the same expression. This will be added later.

References:

1. C:\Documents and Settings\Howard Wieman\My Documents\space charge distortion STAR\ TwoD5.MCD