field wire array Gene.xmcd

In this exercise potentials and fields from an array of wires in a gating grid are calculated. Here we apply potential setting of the wires to capture the positive ions although the potentials can be applied arbitrarily to achieve what ever condition is desired. The ion paths are then tracked through the grid to study whether complete capture is archived and also to find the time to capture under given conditions.

The length units used are mm and potentials are in volts. The permittivity of vacuum, $\varepsilon 0$, is dropped since it cancels out. The result is that surface charge units become fields V/mm. Fields naturally come out as Volts/mm, but we apply the factor of 10 in our results so that fields are in the usual Volts/cm.

The formulas for potentials and fields are obtained from:
W. Blum and L. Rolandi, ***Particle Detection with Drift Chambers***, Springer-Verlag, 2nd printing 1994, p. 99-106.

units mm

$w := 2$ wire spacing in the plane

$s := 4$ twice the wire spacing in the plane, the pitch between like potential wires in the alternating potential pattern used in the closed state of the gating grid.

$\Delta z := 3$ separation of the planes

$r := .1$ radius of the wires

$zc := 100$ distance to the cathode plane of wires

To find the potential function one solves for the charge on each of the wire elements.

The charge on the wires is solved where

$V = A \cdot \sigma$ where V is a vector representing the potentials on the different elements of the wire planes and σ is a vector giving the charges on the elements. A is an N by N matrix based on geometry.

and σ is found from

$\sigma = A^{-1} \cdot V$

The A matrix below represents a ground plane followed by 4 equally spaced planes of alternating potential wires and finally a cathode plane of wires at a larger distance defining the field in the open drift volume. A solid plane could be used for the cathode, but in the interest of using the same formulas a wire plane was chosen.

$$A := \begin{pmatrix} \Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & \Delta z - \frac{s}{2\pi}\cdot\ln(2) & \Delta z & \Delta z & \Delta z & \Delta z \\ \Delta z - \frac{s}{2\pi}\cdot\ln(2) & \Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & \Delta z & \Delta z & \Delta z & \Delta z \\ \Delta z & \Delta z & 2\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & 2\Delta z - \frac{s}{2\pi}\cdot\ln(2) & 2\cdot\Delta z & 2\cdot\Delta z \\ \Delta z & \Delta z & 2\Delta z - \frac{s}{2\pi}\cdot\ln(2) & 2\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & 2\cdot\Delta z & 2\cdot\Delta z \\ \Delta z & \Delta z & 2\cdot\Delta z & 2\cdot\Delta z & 3\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & 3\Delta z - \frac{s}{2\pi}\cdot\ln(2) \\ \Delta z & \Delta z & 2\cdot\Delta z & 2\cdot\Delta z & 3\Delta z - \frac{s}{2\pi}\cdot\ln(2) & 3\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) \\ \Delta z & \Delta z & 2\cdot\Delta z & 2\cdot\Delta z & 3\cdot\Delta z & 3\cdot\Delta z \\ \Delta z & \Delta z & 2\cdot\Delta z & 2\cdot\Delta z & 3\cdot\Delta z & 3\cdot\Delta z \\ \Delta z & \Delta z & 2\cdot\Delta z & 2\cdot\Delta z & 3\cdot\Delta z & 3\cdot\Delta z \end{pmatrix}$$

$$A = \begin{pmatrix} 4.178 & 2.559 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2.559 & 4.178 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 7.178 & 5.559 & 6 & 6 & 6 & 6 & 6 \\ 3 & 3 & 5.559 & 7.178 & 6 & 6 & 6 & 6 & 6 \\ 3 & 3 & 6 & 6 & 10.178 & 8.559 & 9 & 9 & 9 \\ 3 & 3 & 6 & 6 & 8.559 & 10.178 & 9 & 9 & 9 \\ 3 & 3 & 6 & 6 & 9 & 9 & 13.178 & 11.559 & 12 \\ 3 & 3 & 6 & 6 & 9 & 9 & 11.559 & 13.178 & 12 \\ 3 & 3 & 6 & 6 & 9 & 9 & 12 & 12 & 101.178 \end{pmatrix}$$

$$AI := A^{-1}$$

$$
AI = \begin{pmatrix}
0.437 & -0.18 & -0.055 & -0.055 & -5.46 \times 10^{-3} & -5.46 \times 10^{-3} & -5.952 \times 10^{-4} & -5.952 \times 10^{-4} & -4.92 \times 10 \\
-0.18 & 0.437 & -0.055 & -0.055 & -5.46 \times 10^{-3} & -5.46 \times 10^{-3} & -5.952 \times 10^{-4} & -5.952 \times 10^{-4} & -4.92 \times 10 \\
-0.055 & -0.055 & 0.432 & -0.186 & -0.055 & -0.055 & -6.036 \times 10^{-3} & -6.036 \times 10^{-3} & -4.989 \times 1 \\
-0.055 & -0.055 & -0.186 & 0.432 & -0.055 & -0.055 & -6.036 \times 10^{-3} & -6.036 \times 10^{-3} & -4.989 \times 1 \\
-5.46 \times 10^{-3} & -5.46 \times 10^{-3} & -0.055 & -0.055 & 0.431 & -0.186 & -0.061 & -0.061 & -5.009 \times 1 \\
-5.46 \times 10^{-3} & -5.46 \times 10^{-3} & -0.055 & -0.055 & -0.186 & 0.431 & -0.061 & -0.061 & -5.009 \times 1 \\
-5.952 \times 10^{-4} & -5.952 \times 10^{-4} & -6.036 \times 10^{-3} & -6.036 \times 10^{-3} & -0.061 & -0.061 & 0.379 & -0.239 & -5.03 \times 10 \\
-5.952 \times 10^{-4} & -5.952 \times 10^{-4} & -6.036 \times 10^{-3} & -6.036 \times 10^{-3} & -0.061 & -0.061 & -0.239 & 0.379 & -5.03 \times 10 \\
-4.92 \times 10^{-6} & -4.92 \times 10^{-6} & -4.989 \times 10^{-5} & -4.989 \times 10^{-5} & -5.009 \times 10^{-4} & -5.009 \times 10^{-4} & -5.03 \times 10^{-3} & -5.03 \times 10^{-3} & 0.011
\end{pmatrix}
$$

voltage vector

$$V = \begin{pmatrix} V_{1p} \\ V_{1m} \\ V_{2p} \\ V_{2m} \\ V_{3p} \\ V_{3m} \\ V_{4p} \\ V_{4m} \\ V_c \end{pmatrix}$$

voltage on more positive wires of plane 1

voltage on more negative wires of plane 1

voltage on more positive wires of plane 2

voltage on more negative wires of plane 2

voltage on more positive wires of plane 3

voltage on more negative wires of plane 3

voltage on more positive wires of plane 4

voltage on more negative wires of plane 4

voltage on the cathode plane

Normal voltages for open drift field

$$V_o := -400 \cdot \begin{pmatrix} .3 \\ .3 \\ .6 \\ .6 \\ .9 \\ .9 \\ 1.2 \\ 1.2 \\ 10 \end{pmatrix}$$

Added voltages to close the gate and flush out the ions. Values for checker board pattern.

$$V_{c\Delta} := \begin{pmatrix} 200 \\ -200 \\ -200 \\ 200 \\ 200 \\ -200 \\ -200 \\ 200 \\ 0 \end{pmatrix}$$

The voltages for closing the grid are added on to the normal open voltages that establish the drift field.

$$V_c := V_o + V_{c\Delta} = \begin{pmatrix} 80 \\ -320 \\ -440 \\ -40 \\ -160 \\ -560 \\ -680 \\ -280 \\ -4 \times 10^3 \end{pmatrix}$$

voltages for closed state

$$\sigma = \begin{pmatrix} \sigma_{1p} \\ \sigma_{1m} \\ \sigma_{2p} \\ \sigma_{2m} \\ \sigma_{3p} \\ \sigma_{3m} \\ \sigma_{3p} \\ \sigma_{3m} \\ \sigma_c \end{pmatrix}$$

wire charges

$\sigma_c := AI \cdot V_c$ solving to get wire charges.

$$\sigma_c = \begin{pmatrix} 123.483 \\ -123.483 \\ -123.485 \\ 123.481 \\ 123.459 \\ -123.507 \\ -123.72 \\ 123.246 \\ -39.473 \end{pmatrix}$$

the solution values

$$x_0 := \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

x locations of the wire plane patterns. The more positive wire grid is placed 1 mm negative and the more negative wire grid is placed 1 mm positive. The cathode plane is arbitrary for this exercise and is set at x = 0.

$$z_0 := \Delta z \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \\ 3 \\ 4 \\ 4 \\ 0 \end{pmatrix}$$

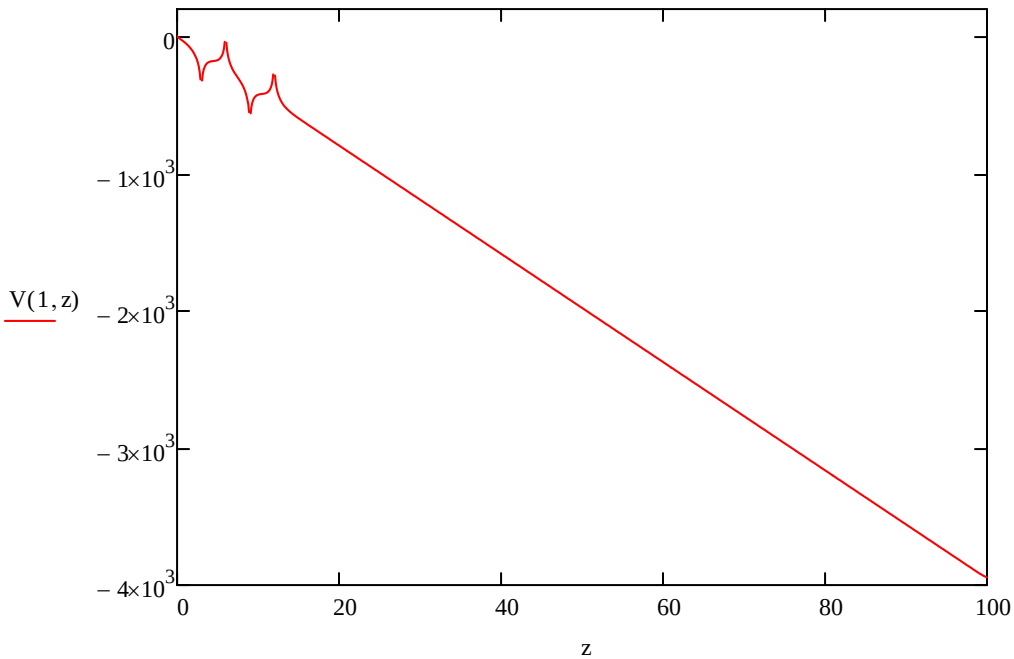
z positions of the planes

$$z_{0_8} := 100$$

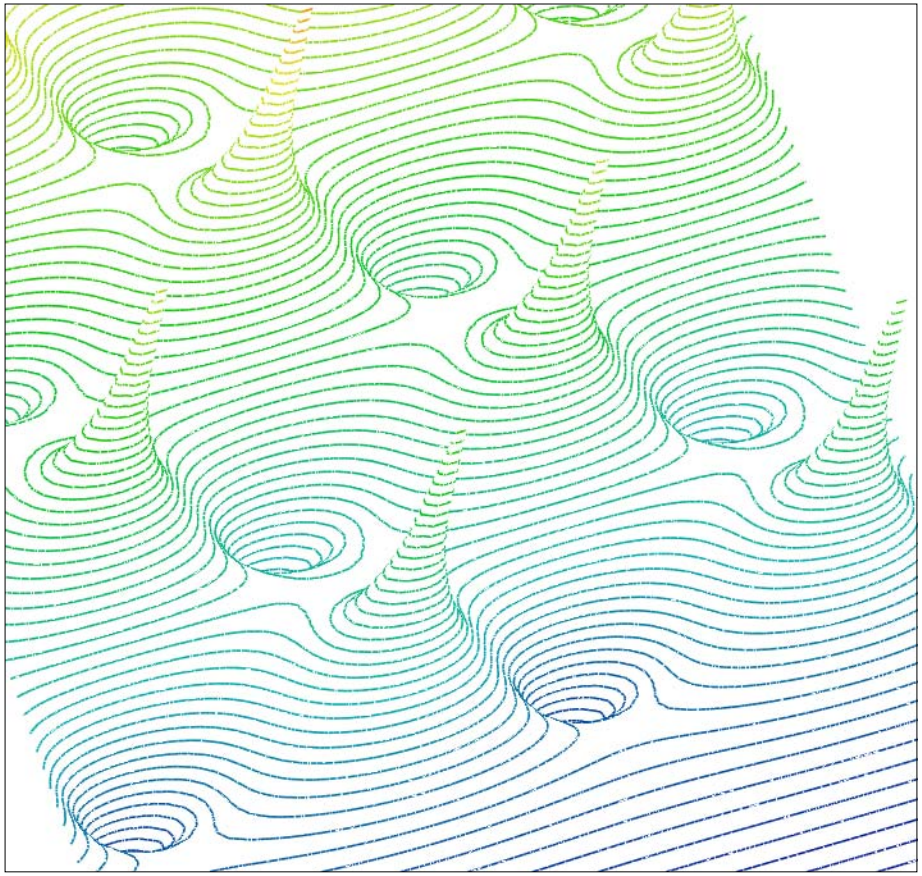
$$z_0 = \begin{pmatrix} 3 \\ 3 \\ 6 \\ 6 \\ 9 \\ 9 \\ 12 \\ 12 \\ 100 \end{pmatrix}$$

$$V(x,z) := \frac{-1}{4 \cdot \pi} \cdot \sum_{n=0}^{8} \left[s \cdot \sigma_{c_n} \cdot \ln\left[\frac{\sin\left[\frac{\pi}{s} \cdot \left(x - x_{0_n}\right)\right]^2 + \sinh\left[\frac{\pi}{s} \cdot \left(z - z_{0_n}\right)\right]^2}{\sin\left[\frac{\pi}{s} \cdot \left(x - x_{0_n}\right)\right]^2 + \sinh\left[\frac{\pi}{s} \cdot \left(z + z_{0_n}\right)\right]^2} \right] \right]$$

The potential function

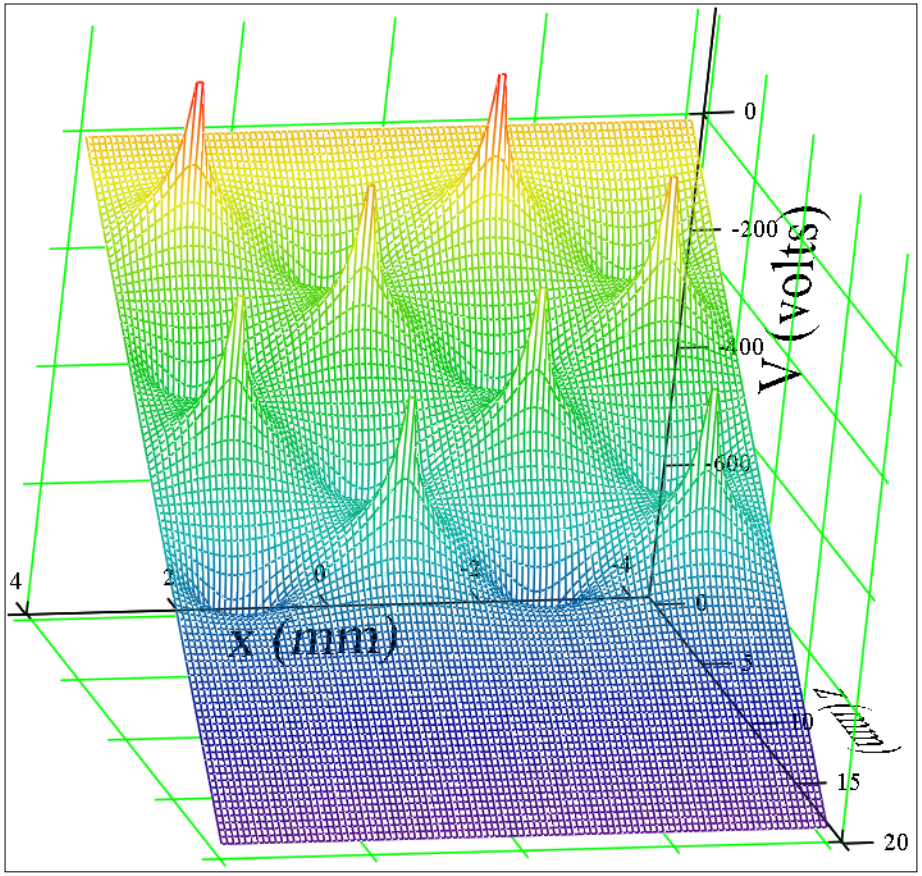


Plot of the potential at x = 1 mm as a function of z. Potential units volts and z units mm.



V

Contour plot of potential in the vicinity of the wires, closed condition.



V Surface plot of potential, closed, flush condition.

Electric field in x direction, factor 10 added to give units of V/cm

$$E_x(x,z) := \frac{10}{2}\sum_{n=0}^{8}\left[\sigma_{c_n}\cdot\left[\sin\left[\frac{2\cdot\pi}{s}\left(x - x_{0_n}\right)\right]\cdot\left[\frac{1}{\cosh\left[\frac{2\cdot\pi}{s}\left(z - z_{0_n}\right)\right] - \cos\left[\frac{2\cdot\pi}{s}\cdot\left(x - x_{0_n}\right)\right]} - \frac{1}{\left[\cosh\left[\frac{2\cdot\pi}{s}\left(z + z_{0_n}\right)\right] - \cos\left[\frac{2\cdot\pi}{s}\cdot\left(x - x_{0_n}\right)\right]\right]}\right]\right]\right]$$

Electric field in z direction.

$$E_z(x,z) := \frac{10}{2}\cdot\sum_{n=0}^{8}\left[\sigma_{c_n}\cdot\left[\frac{\sinh\left[\frac{2\cdot\pi}{s}\left(z - z_{0_n}\right)\right]}{\left[\cosh\left[\frac{2\cdot\pi}{s}\left(z - z_{0_n}\right)\right] - \cos\left[\frac{2\cdot\pi}{s}\cdot\left(x - x_{0_n}\right)\right]\right]} - \frac{\sinh\left[\frac{2\cdot\pi}{s}\left(z + z_{0_n}\right)\right]}{\left[\cosh\left[\frac{2\cdot\pi}{s}\left(z + z_{0_n}\right)\right] - \cos\left[\frac{2\cdot\pi}{s}\cdot\left(x - x_{0_n}\right)\right]\right]}\right]\right]$$

Walk a positive ion through the field with a fixed step size and record position and time of progress. Terminate the walk if the ion reaches a high field indicating that it has arrived at a wire or terminate if has walked out of designated region.

$\delta L := .1$ step size

$K_I := 7$ ion mobility in $\frac{cm^2}{volt\cdot sec}$

$$\delta t = \frac{\delta L \cdot 10^5}{K_I \cdot E}$$

time to take the step, in μs

$EML := 5000$

Magnitude of the field where the walk will be stopped. Units V/cm

stop if mag(E)>5000V/cm

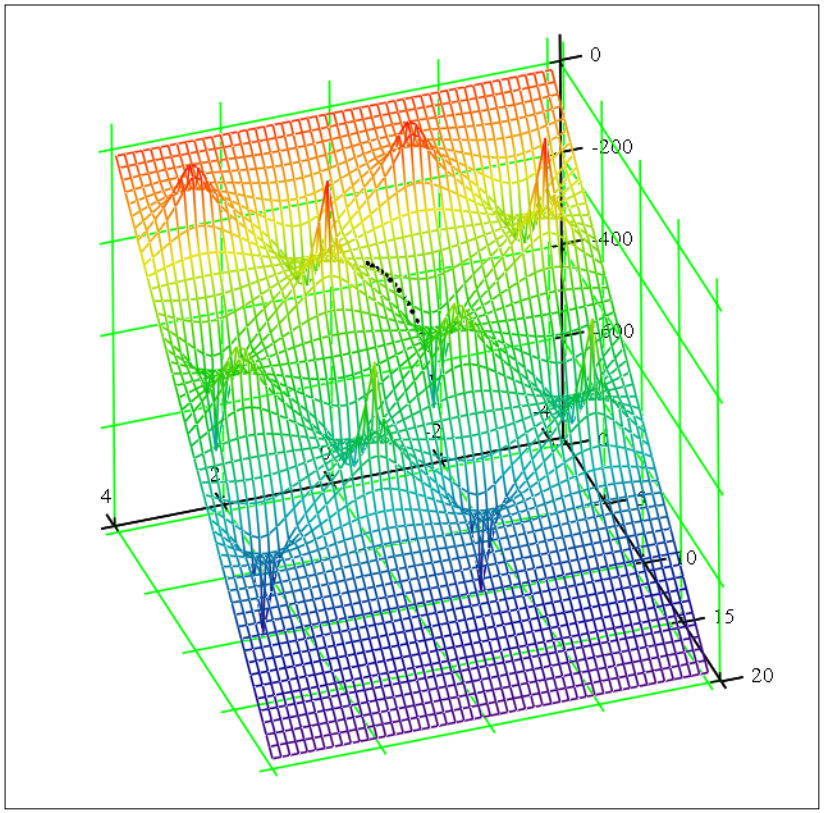
$$
\mathrm{Wk}(x,z) := \left[\begin{array}{l}
p \leftarrow (x \quad z \quad V(x,z) \quad 0) \\
tw \leftarrow 0 \\
xw \leftarrow x \\
zw \leftarrow z \\
Ex \leftarrow E_x(x,z) \\
Ez \leftarrow E_z(x,z) \\
E \leftarrow \sqrt{Ex^2 + Ez^2} \\
\text{while } E < EML \wedge xw > -5 \wedge xw < 5 \wedge zw > 0 \wedge zw < 20 \\
\quad xw \leftarrow xw + \dfrac{Ex}{E} \cdot \delta L \\
\quad zw \leftarrow zw + \dfrac{Ez}{E} \cdot \delta L \\
\quad tw \leftarrow tw + \dfrac{\delta L \cdot 10^5}{K_I \cdot E} \\
\quad Ex \leftarrow E_x(xw,zw) \\
\quad Ez \leftarrow E_z(xw,zw) \\
\quad E \leftarrow \sqrt{Ex^2 + Ez^2} \\
\quad pw \leftarrow (xw \quad zw \quad V(xw,zw) \quad tw) \\
\quad p \leftarrow \mathrm{stack}(p,pw) \\
p
\end{array}\right]
$$

ion walk program

returns a 4 column matrix of step points where the columns are

x, z, Potential at the point, elapsed time at the point

$try := Wk(.1, 5)$ test with ion starting at x = .1, z = 5



$V, \left(try^{\langle 0 \rangle}, try^{\langle 1 \rangle}, try^{\langle 2 \rangle}\right)$

Example track of an ion starting at (0.1,5) as it falls into the potential well of a grid wire.

Program to launch multiple ions and track there demise.

Nx by Nz patch of start locations with limits
xs - xe
zs - ze

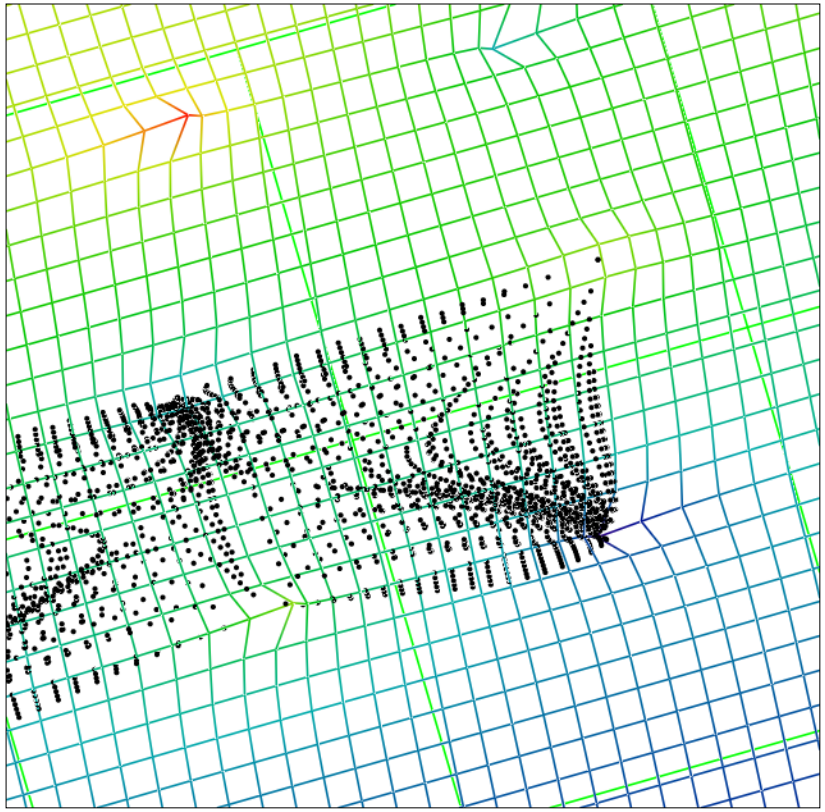
$Nx := 20$

$Nz := 10$

$$MWk(xs, xe, zs, ze) := \left| \begin{array}{l} p \leftarrow Wk(xs, zs) \\ \text{for } kx \in 0..Nx \\ \quad \text{for } kz \in 0..Nz \\ \quad\quad \left| \begin{array}{l} x \leftarrow xs + \dfrac{(xe - xs)}{Nx} \cdot kx \\ z \leftarrow zs + \dfrac{(ze - zs)}{Nz} \cdot kz \\ p \leftarrow stack(p, Wk(x, z)) \end{array} \right. \\ p \end{array} \right.$$

Returns rows of consecutive steps for each start ion stacked in a single matrix. The 4 columns are as in the walk program.

$$p := MWk(-.9, 2.95, 9, 12)$$



$V, \left(p^{\langle 0 \rangle}, p^{\langle 1 \rangle}, p^{\langle 2 \rangle}\right)$

Multiple ion paths from the defined region. In this case they are all captured.

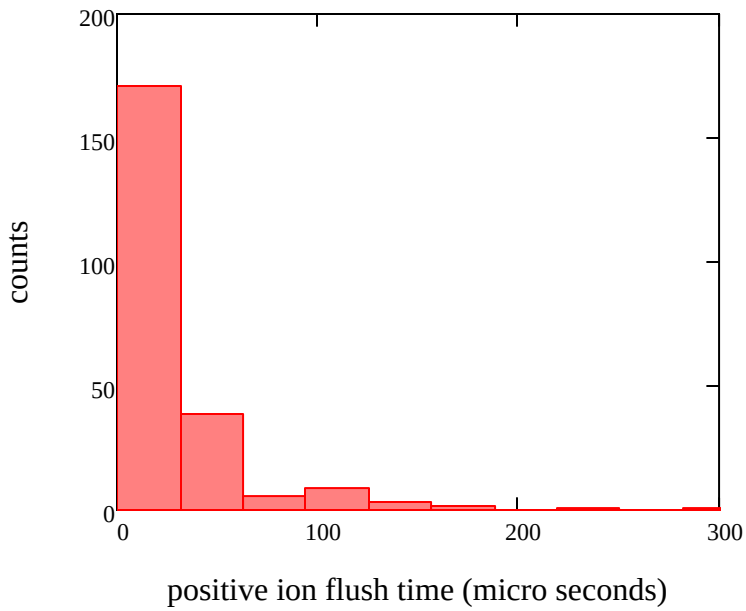
$$tt(p) := \left| \begin{array}{l} N \leftarrow \mathrm{last}\left(p^{\langle 0 \rangle}\right) \\ t_0 \leftarrow 0 \\ \text{for } k \in 1..N \\ \quad t \leftarrow \mathrm{stack}\left[t, \left(p^{\langle 3 \rangle}\right)_{k-1}\right] \text{ if } \left(p^{\langle 3 \rangle}\right)_k = 0 \\ t \end{array} \right.$$

program to extract all the final elapsed times for the different ion paths.

$t := tt(p)$

$h := \mathrm{histogram}(10, t)$ $\mathrm{mean}(t) = 28$

mean ion capture time



$$
\begin{pmatrix}
\Delta z & \Delta z & \Delta z \\
\Delta z & \Delta z & \Delta z \\
2\cdot\Delta z & 2\cdot\Delta z & 2\cdot\Delta z \\
2\cdot\Delta z & 2\cdot\Delta z & 2\cdot\Delta z \\
3\cdot\Delta z & 3\cdot\Delta z & 3\cdot\Delta z \\
3\cdot\Delta z & 3\cdot\Delta z & 3\cdot\Delta z \\
4\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & 4\Delta z - \frac{s}{2\pi}\cdot\ln(2) & 4\cdot\Delta z \\
4\Delta z - \frac{s}{2\pi}\cdot\ln(2) & 4\Delta z - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right) & 4\cdot\Delta z \\
4\cdot\Delta z & 4\cdot\Delta z & zc - \frac{s}{2\pi}\cdot\ln\left(\frac{2\pi\cdot r}{s}\right)
\end{pmatrix}
$$

$$
\begin{pmatrix}
)^{-6} \\
)^{-6} \\
0^{-5} \\
0^{-5} \\
0^{-4} \\
0^{-4} \\
)^{-3} \\
)^{-3}
\end{pmatrix}
$$