27. PASSAGE OF PARTICLES THROUGH MATTER

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27.1. Notation

Table 27.1: Summary of variables used in this section. The kinematic variables β and γ have their usual meanings.

Symbol	Definition	Units or Value
α	Fine structure constant ($e^2/4\pi\epsilon_0\hbar c$)	1/137.035 999 11(46)
M	Incident particle mass	MeV/c^2
E	Incident particle energy γMc^2	MeV
T	Kinetic energy	MeV
$m_e c^2$	Electron mass $\times$ c^2	0.510 998 918(44) MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 325(28) fm
N_A	Avogadro's number	$6.022\,1415(10) \times 10^{23}$ mol^{-1}
ze	Charge of incident particle	
Z	Atomic number of absorber	
A	Atomic mass of absorber	g mol^{-1}
K/A	$4\pi N_A r_e^2 m_e c^2/A$	0.307 075 MeV g^{-1} cm^2 for A = 1 g mol^{-1}
I	Mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	Density effect correction to ionization energy loss	
$\hbar\omega_p$	Plasma energy ($\sqrt{4\pi N_e r_e^3}\ m_e c^2/\alpha$)	$28.816\sqrt{\rho\langle Z/A\rangle}$ eV$^{(a)}$
N_c	Electron density	(units of r_e)$^{-3}$
w_j	Weight fraction of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
—	$4\alpha r_e^2 N_A/A$	(716.408 g cm^{-2})$^{-1}$ for $A = 1$ g mol^{-1}
X_0	Radiation length	g cm^{-2}
E_c	Critical energy for electrons	MeV
$E_{\mu c}$	Critical energy for muons	GeV
E_s	Scale energy $\sqrt{4\pi/\alpha}\ m_e c^2$	21.2052 MeV
R_M	Molière radius	g cm^{-2}

$^{(a)}$ For ρ in g cm^{-3}.

$$= z_1\, x\, \theta_0/\sqrt{12} + z_2\, x\, \theta_0/2\ ; \tag{27.18}$$

$$\theta_{\text{plane}} = z_2\, \theta_0\ . \tag{27.19}$$

Note that the second term for y_{plane} equals $x\,\theta_{\text{plane}}/2$ and represents the displacement that would have occurred had the deflection θ_{plane} all occurred at the single point $x/2$.

For heavy ions the multiple Coulomb scattering has been measured and compared with various theoretical distributions [34].

27.4. Photon and electron interactions in matter

27.4.1. ***Radiation length*** : High-energy electrons predominantly lose energy in matter by bremsstrahlung, and high-energy photons by e^+e^- pair production. The characteristic amount of matter traversed for these related interactions is called the radiation length X_0, usually measured in g cm^{-2}. It is both (a) the mean distance over which a high-energy electron loses all but $1/e$ of its energy by bremsstrahlung, and (b) $\frac{7}{9}$ of the mean free path for pair production by a high-energy photon [35]. It is also the appropriate scale length for describing high-energy electromagnetic cascades. X_0 has been calculated and tabulated by Y.S. Tsai [36]:

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 \left[L_{\text{rad}} - f(Z) \right] + Z\, L'_{\text{rad}} \right\}\ . \tag{27.20}$$

For $A = 1$ g mol^{-1}, $4\alpha r_e^2 N_A/A = (716.408\ \text{g cm}^{-2})^{-1}$. L_{rad} and L'_{rad} are given in Table 27.2. The function $f(Z)$ is an infinite sum, but for elements up to uranium can be represented to 4-place accuracy by

$$\begin{aligned} f(Z) = a^2 \big[& (1+a^2)^{-1} + 0.20206 \\ & -0.0369\, a^2 + 0.0083\, a^4 - 0.002\, a^6 \big]\ , \end{aligned} \tag{27.21}$$

where $a = \alpha Z$ [37].

Table 27.2: Tsai's L_{rad} and L'_{rad}, for use in calculating the radiation length in an element using Eq. (27.20).

Element	Z	L_{rad}	L'_{rad}
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

Although it is easy to use Eq. (27.20) to calculate X_0, the functional dependence on Z is somewhat hidden. Dahl provides a compact fit to the data [38]:

$$X_0 = \frac{716.4\ \text{g cm}^{-2}\, A}{Z(Z+1)\ln(287/\sqrt{Z})}\ . \tag{27.22}$$

Results using this formula agree with Tsai's values to better than 2.5% for all elements except helium, where the result is about 5% low.

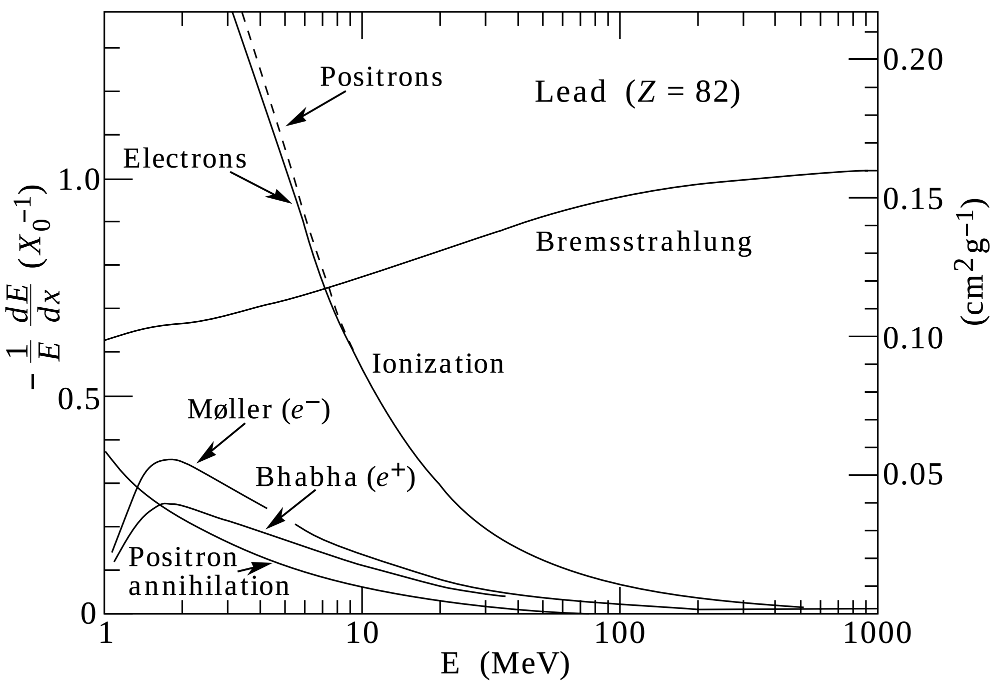

Figure 27.10: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Møller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\mathrm{Pb}) = 5.82$ g/cm^2, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0(\mathrm{Pb}) = 6.37$ g/cm^2).

The radiation length in a mixture or compound may be approximated by

$$1/X_0 = \sum w_j/X_j \ , \tag{27.23}$$

where w_j and X_j are the fraction by weight and the radiation length for the jth element.

27.4.2. ***Energy loss by electrons*** : At low energies electrons and positrons primarily lose energy by ionization, although other processes (Møller scattering, Bhabha scattering, e^+ annihilation) contribute, as shown in Fig. 27.10. While ionization loss rates rise logarithmically with energy, bremsstrahlung losses rise nearly linearly (fractional loss is nearly independent of energy), and dominates above a few tens of MeV in most materials

Ionization loss by electrons and positrons differs from loss by heavy particles because of the kinematics, spin, and the identity of the incident electron with the electrons which it ionizes. Complete discussions and tables can be found in Refs. 7, 8, and 27.