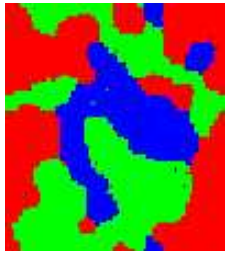


QUARK MATTER 2004

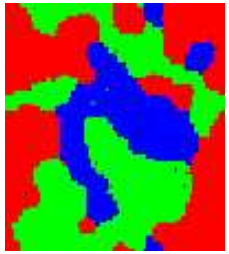
News from Lattice QCD

News from Lattice QCD

On

Heavy Quark Potentials and

Spectral Functions of Heavy Quark States



OUTLINE

1) From color averaged heavy quark free energies to color singlet potentials

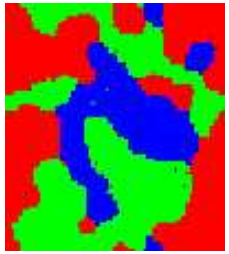
   A new look at the basic input for the discussion of
   charmonium suppression in the framework of potential models;
   short vs. long distance physics in the QGP (running coupling and screening)

2) Spectral analysis of hadronic correlators at high temperature

   ab initio calculation of the charmonium spectrum at finite temperature
   based on the maximum entropy method;
   heavy quark bound states at high temperature

3) ... topics that cannot be discussed in 30 min.

   QCD phase diagram and chiral critical point; equation of state and baryon
   number fluctuations; chiral symmetry and light quark bound states;
   LGT and resonance gas, LGT and HTL;



Deconfinement, screening and heavy quark bound states

Deconfinement $\sim$ screening of the static potential between heavy quarks

- $T = 0$: heavy quark bound states well described by a confining potential

$$V_{\bar{q}q}(r) = -\frac{4\alpha}{3r} + \sigma\, r\ , \quad \alpha \equiv g^2(r)/4\pi$$

- $T > T_c$: no bound state in a Debye screened potential:

$$V_{\bar{q}q}(r,T) \sim -\frac{\alpha}{r}\, \mathrm{e}^{-\mu\, r}\ , \quad \alpha \equiv g^2(T)/4\pi$$

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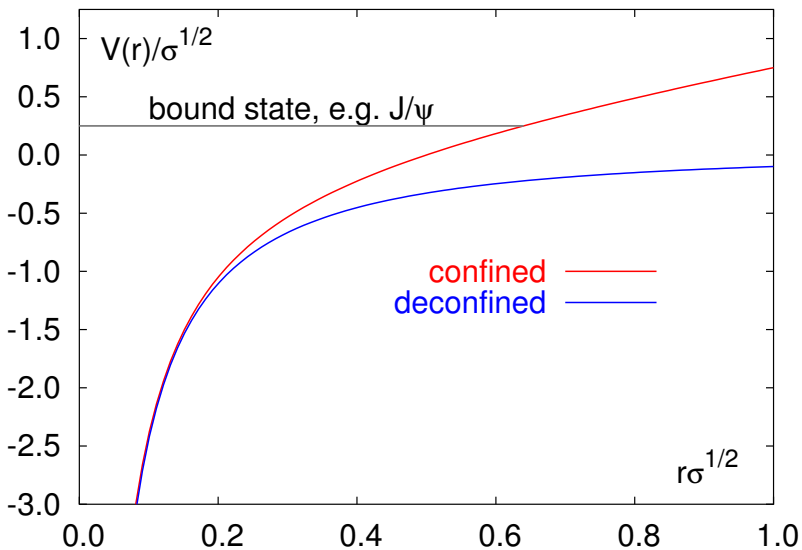


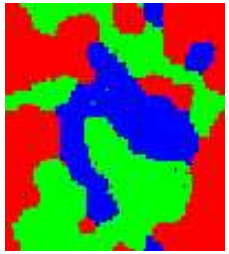
$V_{\bar{q}q}(r,T) \to \infty$ confinement

$V_{\bar{q}q}(r,T) < \infty$ deconfinement

J/ψ suppression

Polyakov loop: $L_{\vec{x}} \sim \mathrm{e}^{i \int_0^{1/T} \mathrm{d}x_0\, \mathcal{A}_0(x_0,\vec{x})}$ (operator for a static quark source)

order parameter for deconfinement ($m_q = \infty$); however, string breaking for $m_q < \infty$



Color averaged heavy quark free energies

$$e^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{9}\langle \mathrm{Tr} L_{\vec{x}} \mathrm{Tr} L^{\dagger}_{\vec{0}} \rangle$$

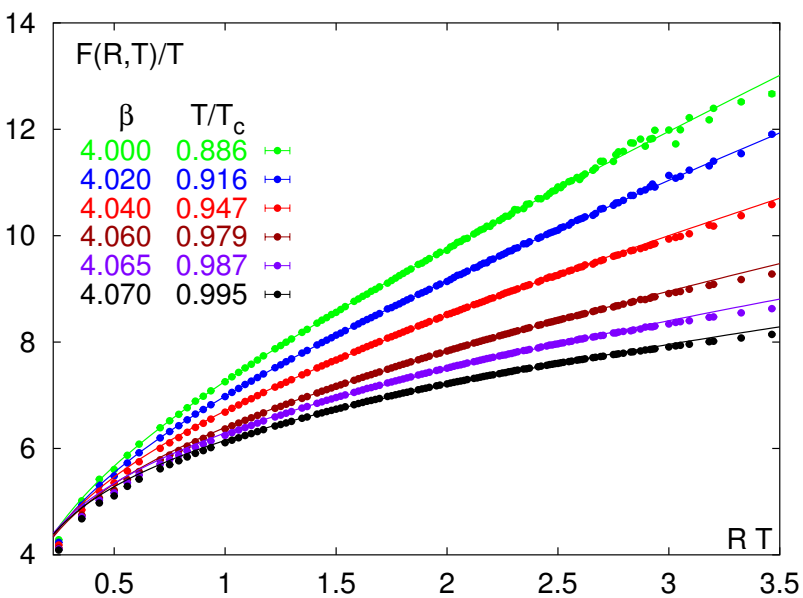
L.G. McLerran, B. Svetitsky, Phys. Rev. D24 (1981) 450

confined phase $(T < T_c)$:

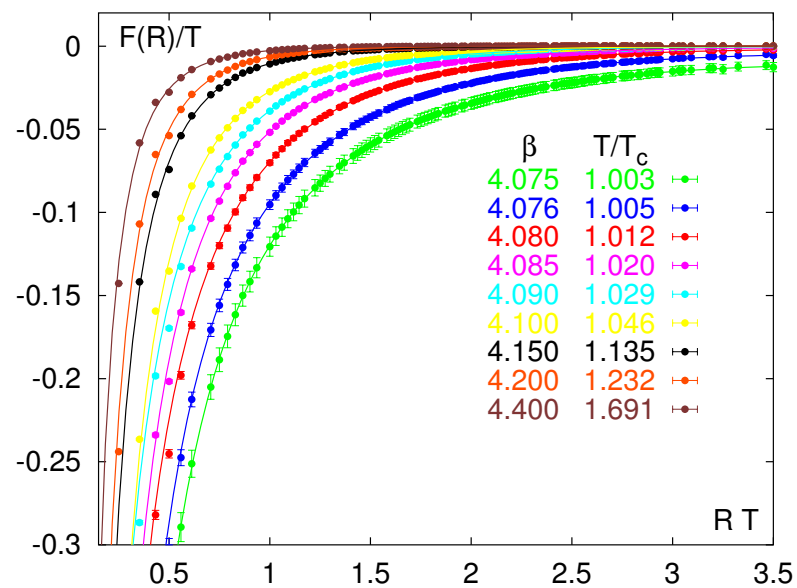
$$\frac{F(r,T)}{T} \simeq -\frac{\tilde{\alpha}}{rT} + \frac{\sigma(T)}{T^2} rT + \ln(rT)$$

deconfined phase $(T > T_c)$:

$$\frac{F(r,T)}{T} = \frac{\tilde{\alpha}}{(rT)^n} e^{-\mu r} + c(T)$$



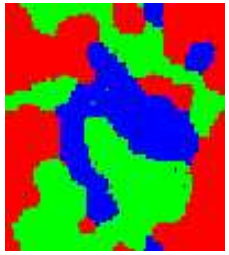
T-dependent string tension



SU(3);

$m_q = \infty$

screening in the plasma phase



Color averaged heavy quark free energies

$$e^{-V_{\bar{q}q}(r,T)/T} = \frac{1}{9}\langle \mathrm{Tr} L_{\vec{x}} \mathrm{Tr} L^{\dagger}_{\vec{0}} \rangle$$

so far used as input in finite-T spectroscopy!!

confined phase $(T < T_c)$:

$$\frac{V(r,T)}{T} \simeq -\frac{\tilde{\alpha}}{rT} + \frac{\sigma(T)}{T^2} rT + \ln(rT)$$

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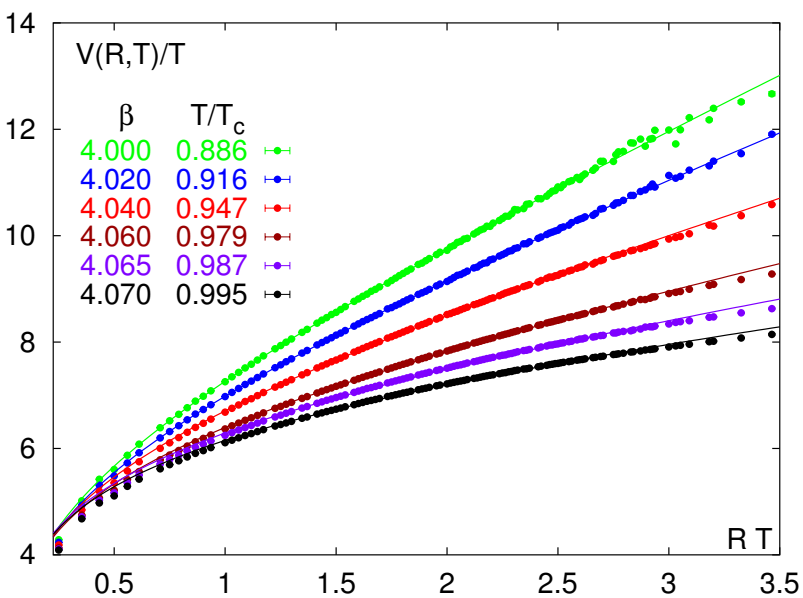

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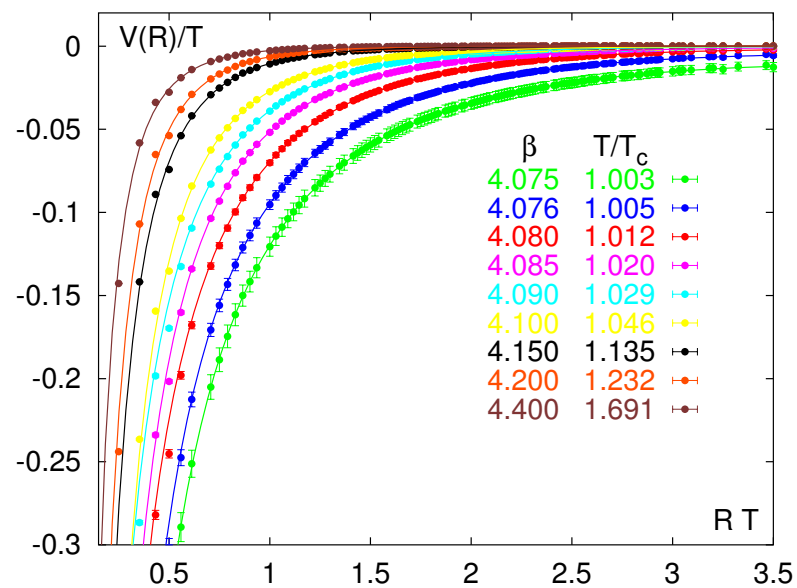

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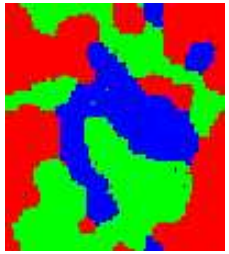

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confined phase $(T < T_c)$:

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- perturbation theory: $g^2 \equiv g^2(r)$ for $r \ll 1$
- matching at short distances

deconfined phase $(T > T_c)$:

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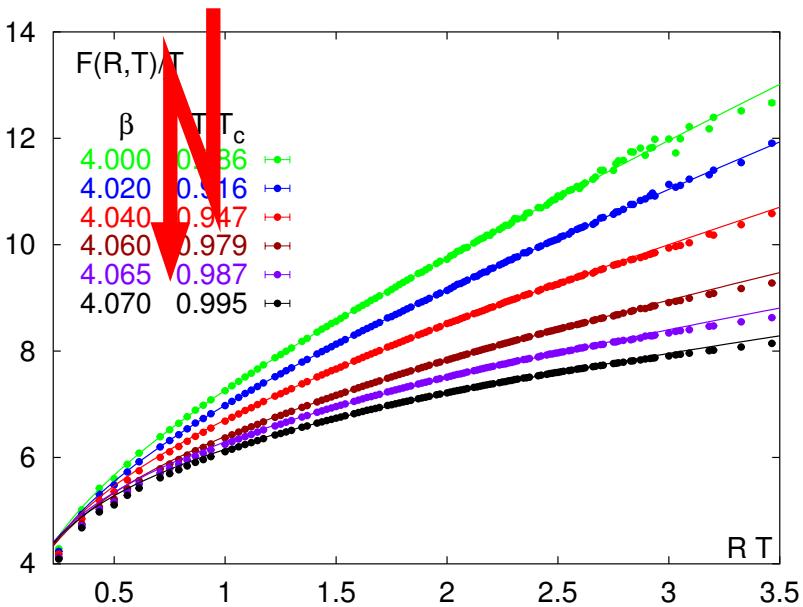
- perturbation theory: $g^2 \equiv g^2(T)$ for $rT \gg 1$
- matching at $r = \infty$: const = 0



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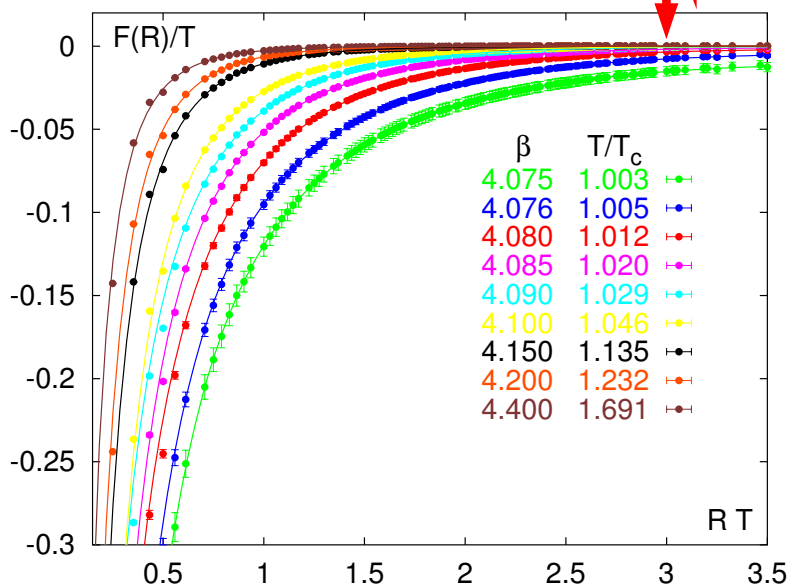

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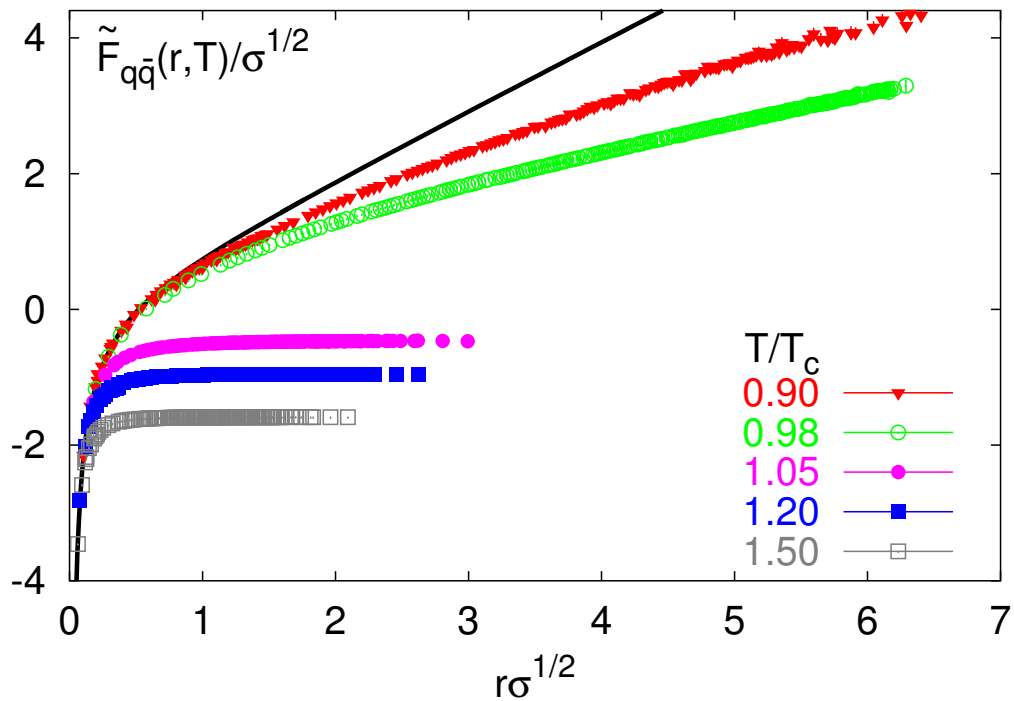
Color averaged heavy quark free energies

O. Kaczmarek, FK, P. Petreczky, F.Zantow, PLB 543(2002)41

confined phase $(T < T_c)$: deconfined phase $(T > T_c)$:

matching ($\equiv$ renormalization) at short distances to $T = 0$ potential for all T



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$$F = E - TS$$

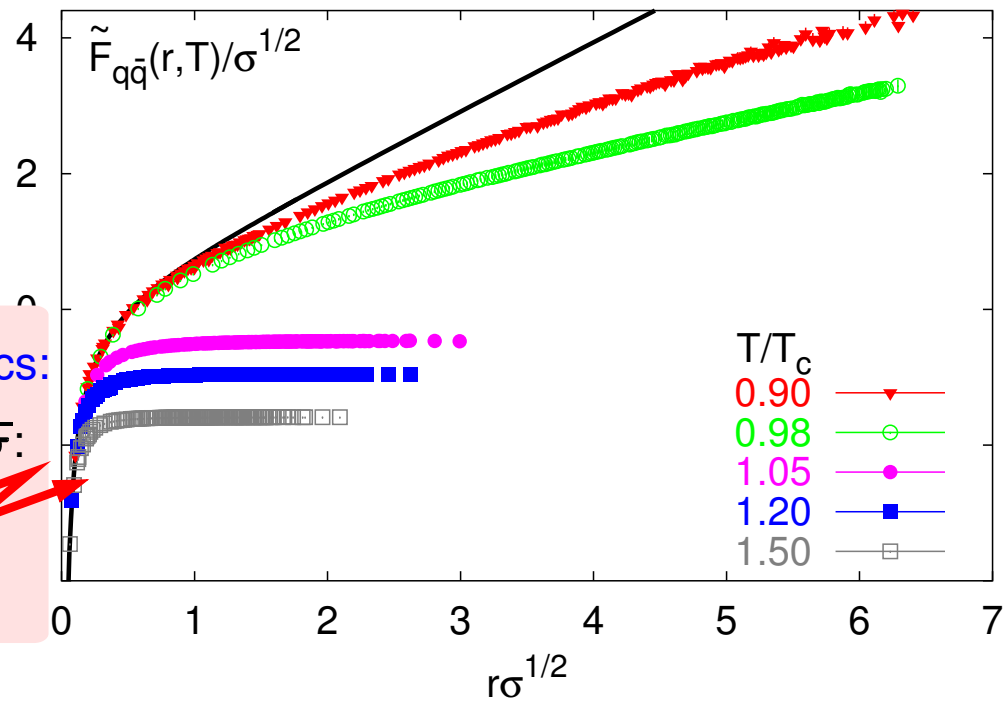
$$E = -T^2 \frac{\partial F/T}{\partial T}$$

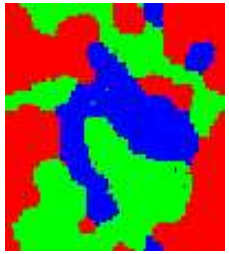
$T = 0$, vacuum physics:

$rT \ll 1,\ r \ll 1/\sqrt{\sigma}$:

$F(r,T) \sim g^2(r)/r$

(energy dominated!!)





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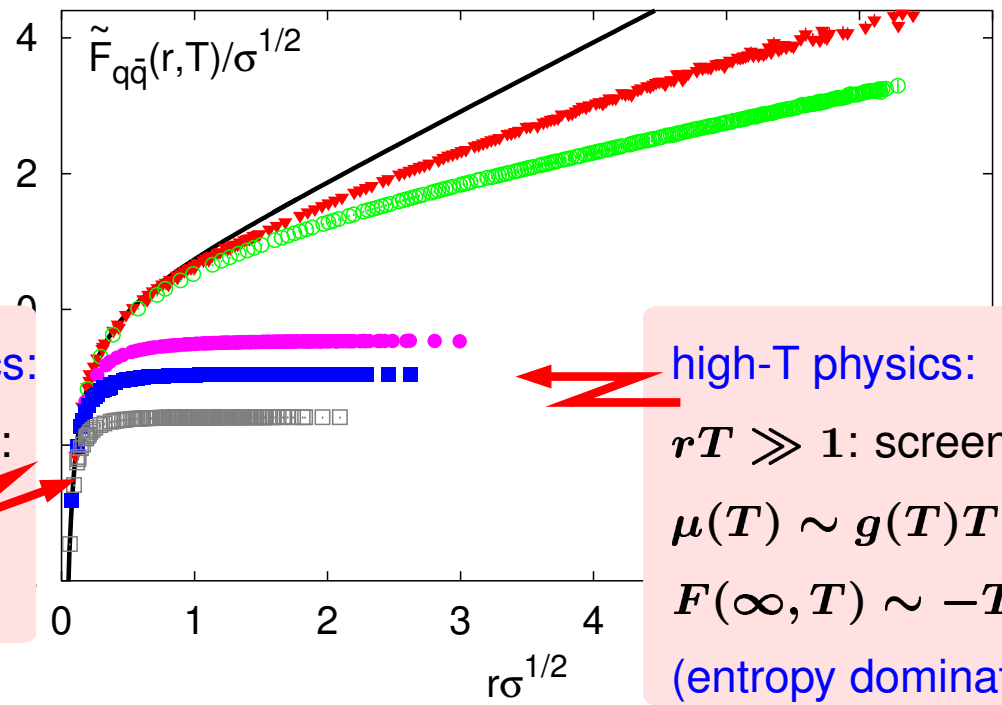


$T = 0$, vacuum physics:

$rT \ll 1,\ r \ll 1/\sqrt{\sigma}$:

$F(r,T) \sim g^2(r)/r$

(energy dominated!!)

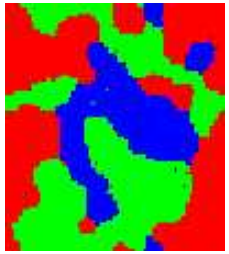
high-T physics:

$rT \gg 1$: screening

$\mu(T) \sim g(T)T$;

$F(\infty,T) \sim -T$

(entropy dominated!!)



Heavy quark free energies

- Static quark and anti-quark sources in a thermal heat bath

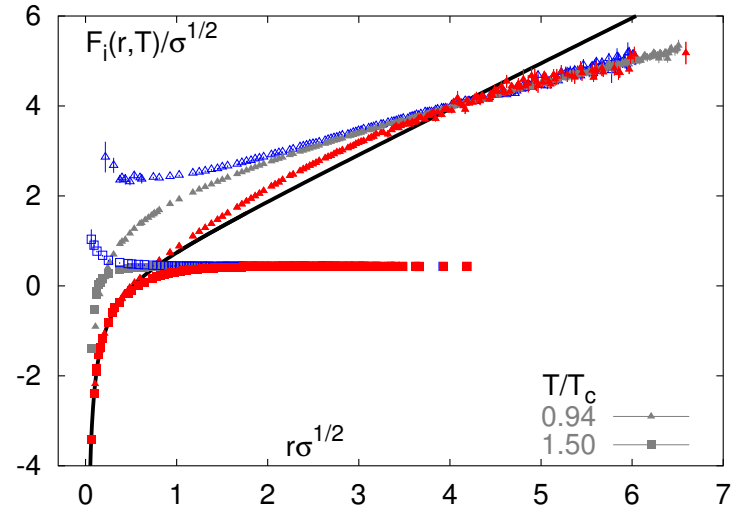
  → change in free energy due to presence of external sources

$$e^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{9}\langle \mathrm{Tr} L_{\vec{x}} \mathrm{Tr} L^{\dagger}_{\vec{0}} \rangle$$

- $3 \times \bar{3} = 1 + 8$; $(q\bar{q})$-pair can be in a singlet or octet state

$$e^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{9}\, e^{-F_1(r,T)/T} + \frac{8}{9}\, e^{-F_8(r,T)/T}$$

$$e^{-F_1(r,T)/T} = \frac{1}{3}\langle \mathrm{Tr} L_{\vec{x}} L^{\dagger}_{\vec{0}} \rangle$$

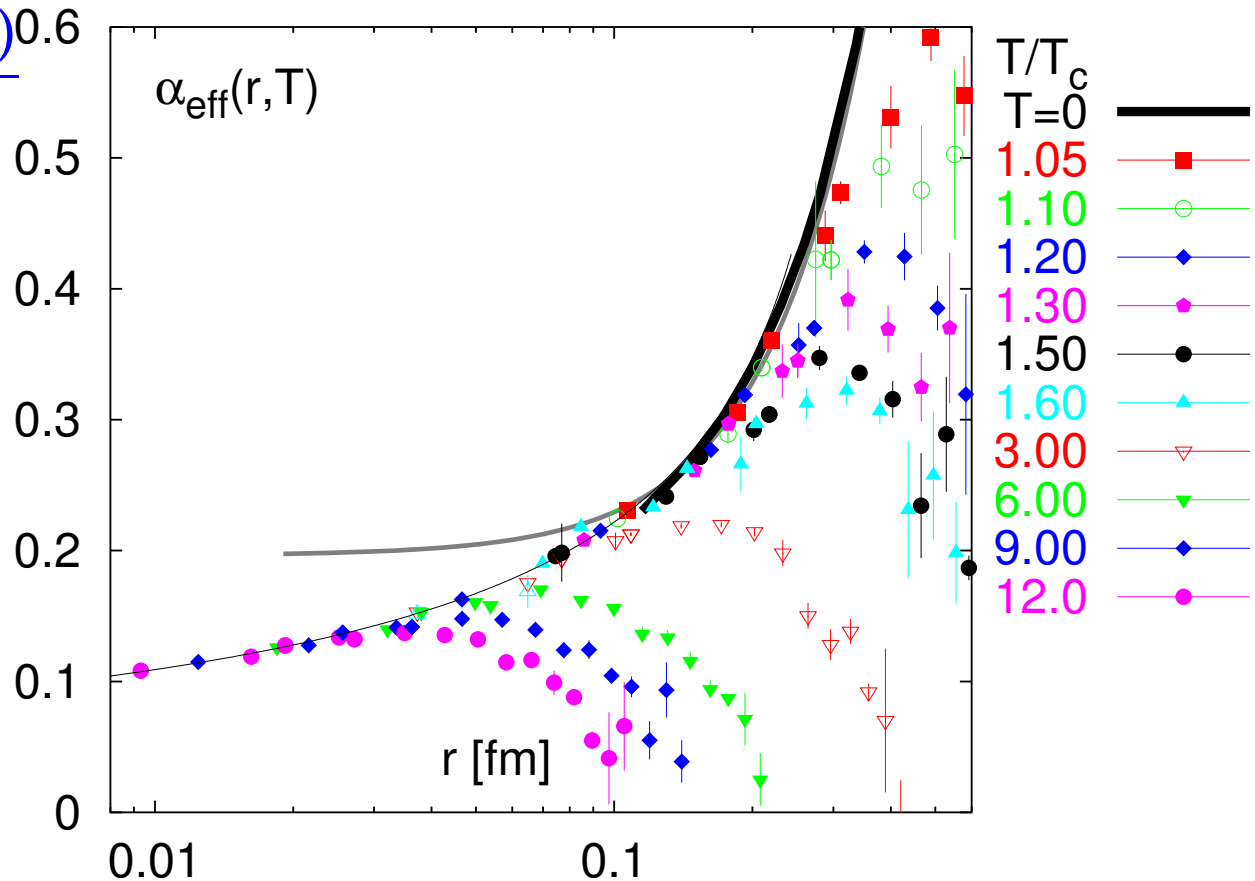


- F_1, F_8 require gauge fixing: Coulomb gauge;
- gauge invariant interpretation: O. Philipsen, PLB 535 (2002) 138)

Running coupling from singlet free energy

- singlet free energy defines a running coupling:

$$\alpha_{\mathrm{eff}} = \frac{3r^2}{4}\,\frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}$$



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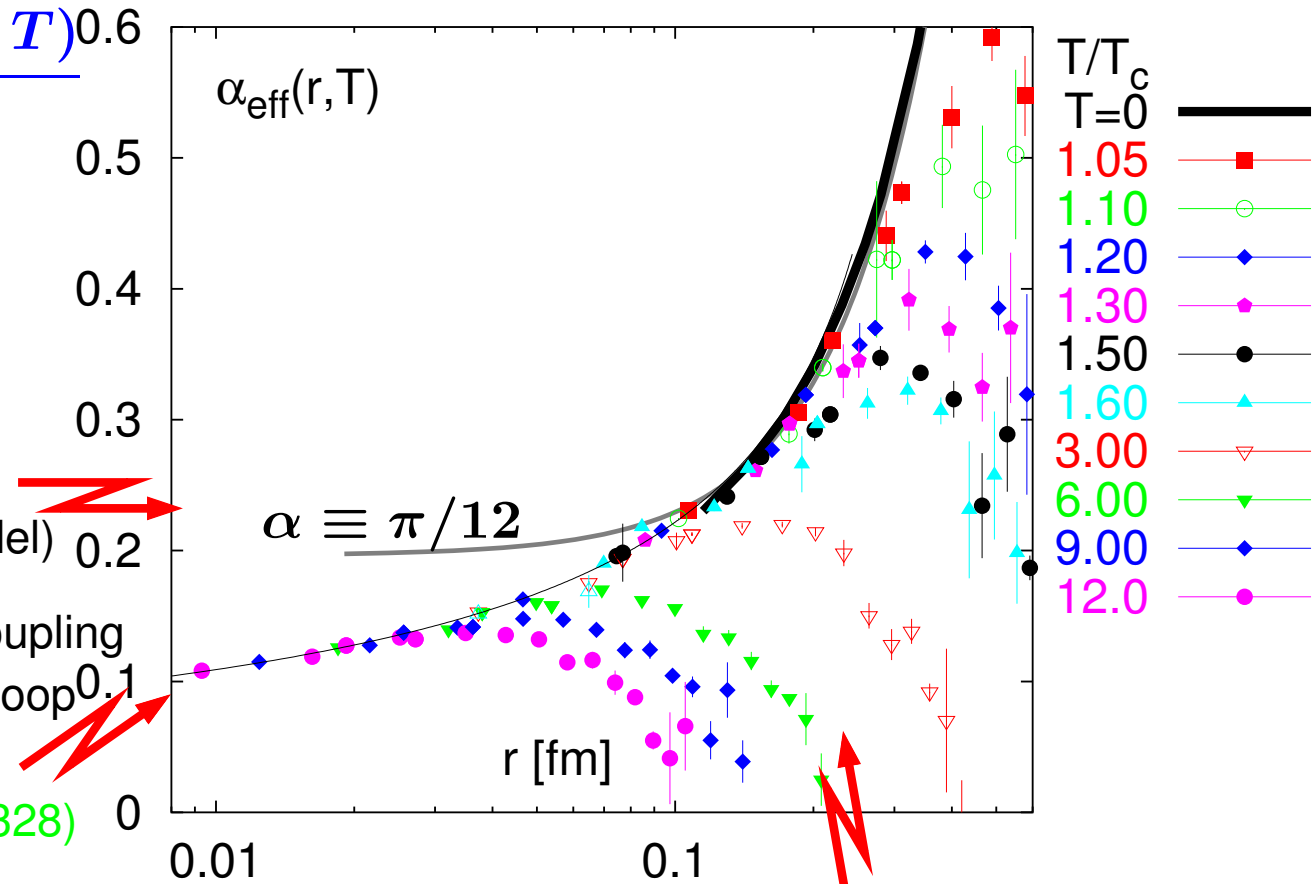


large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop (S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative regime for $T \lesssim 3\,T_c$

- short distance physics $\Leftrightarrow$ vacuum physics

From heavy quark free energies to heavy quark potentials

O. Kaczmarek, FK, P. Petreczky, F.Zantow, in preparation

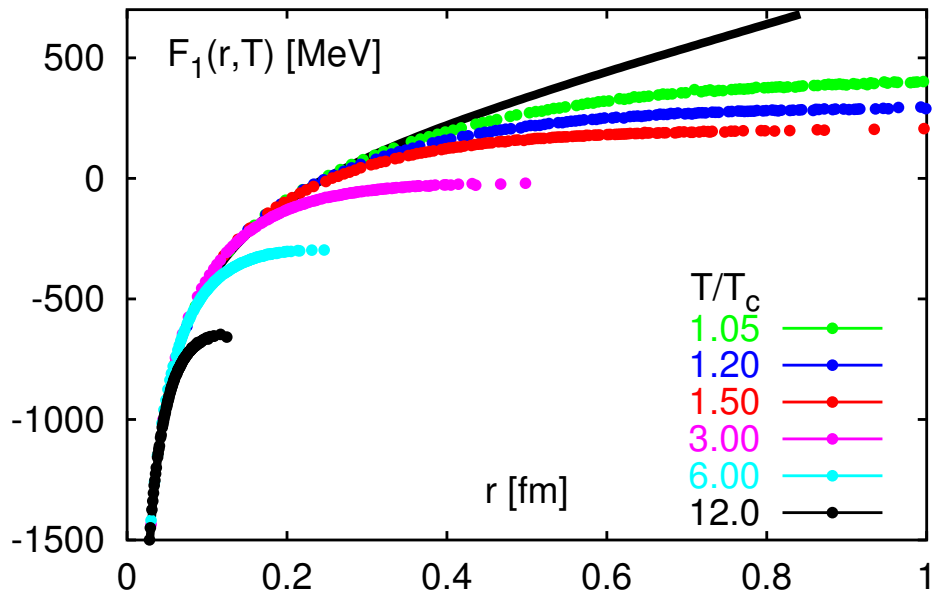
$$\lim_{T\to\infty} F(r,T) = -\infty \text{ !!}$$

$$\Downarrow$$

$$F = E - T \cdot S$$

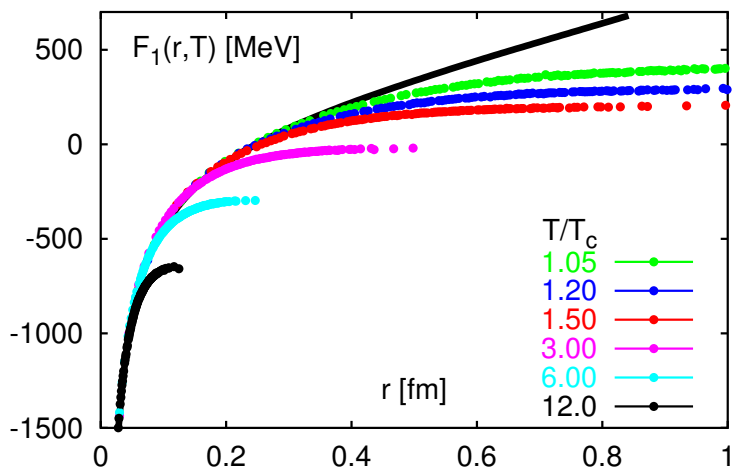
$$\Downarrow$$

$$E(r,T) = -T^2 \frac{\partial\, F(r,T)/T}{\partial\, T} \quad , \quad S(r,T) = -\frac{\partial\, F(r,T)}{\partial\, T}$$



- reconstruct energies from free energies;
- approximate derivatives through finite differences at T_1, T_2 and fixed r
- requires good control over scaling behaviour of the cut-off "a" (complicated!)

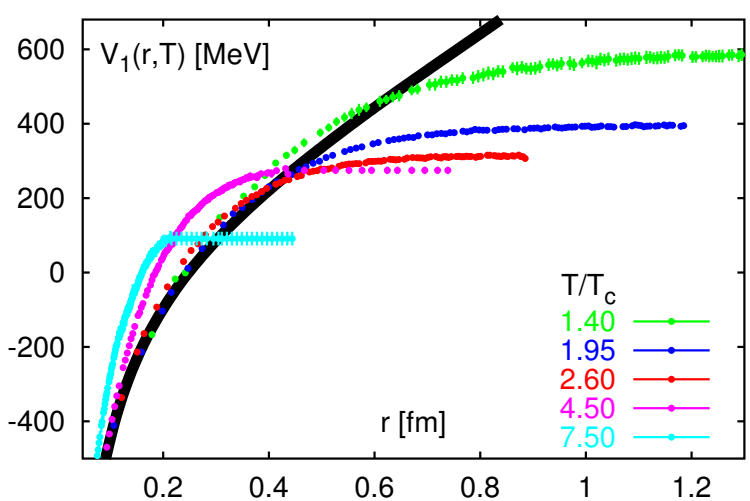
From heavy quark free energies to heavy quark potentials



i) singlet free energy

$$\exp(-F_1(r,T)/T) = \frac{1}{3}\langle \mathrm{Tr} L_{\vec{x}} L^\dagger_{\vec{0}} \rangle$$

(Coulomb gauge)

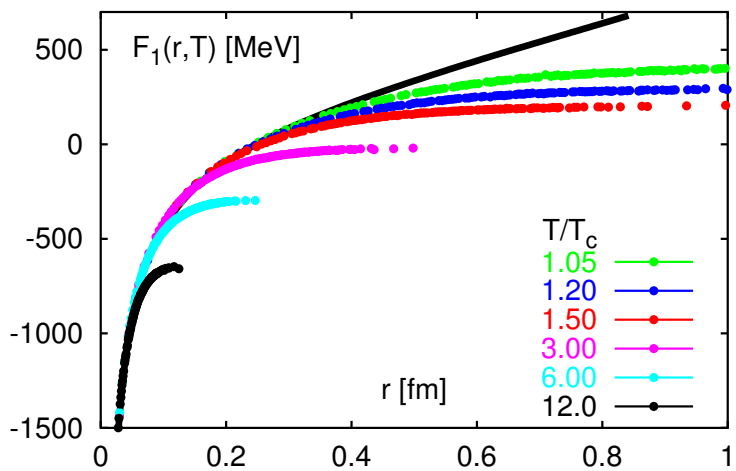


ii) singlet energy $\Leftrightarrow$ ”potential” energy

$$V_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

- potential is ”deeper”: $V(r,T) > F(r,T)$
- potential ”barrier” high also well above T_c
- ”potential” screened at short distances

From heavy quark free energies to heavy quark potentials

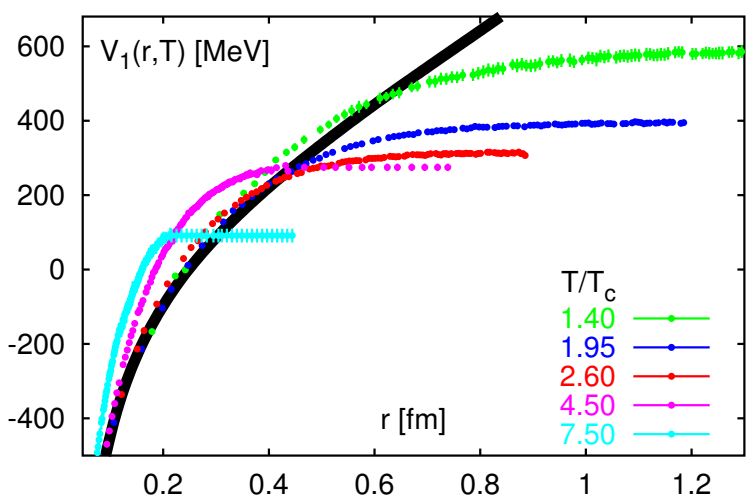


i) singlet free energy

NOTE:

$F_{\bar{q}q}(r,T)$ decreases with increasing T and fixed r $\Rightarrow$ positive entropy

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \geq 0$$



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$$V_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

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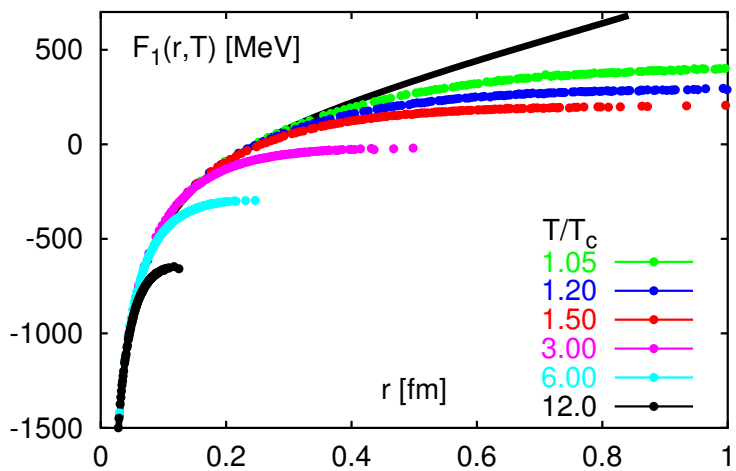
From heavy quark free energies to heavy quark potentials



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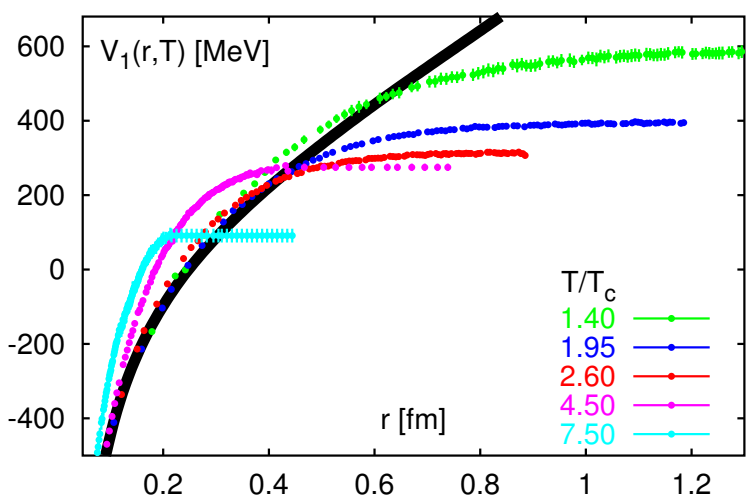

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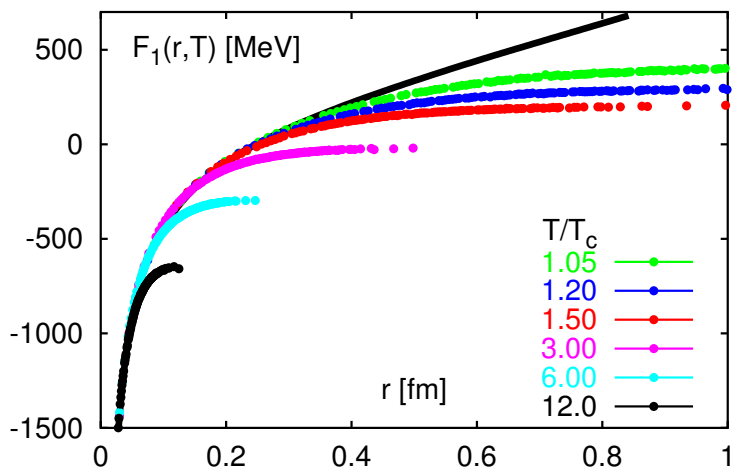

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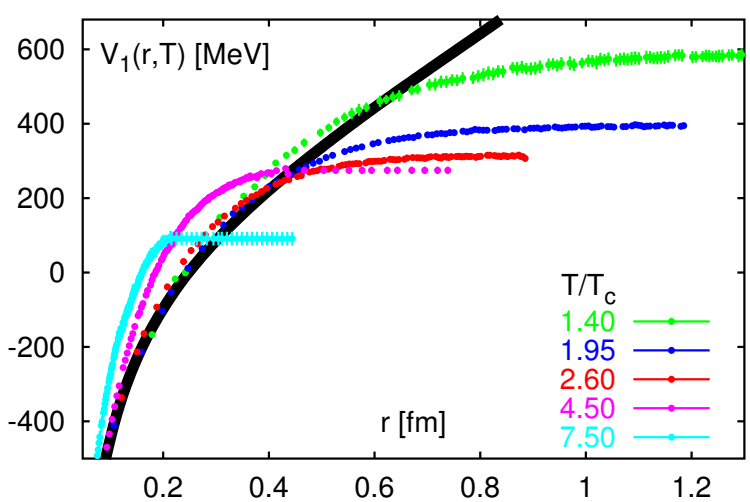
$$F_1(\infty, 1.4T_c) \simeq 200 \text{ MeV}$$

$$V_1(\infty, 1.4T_c) \simeq 600 \text{ MeV}$$



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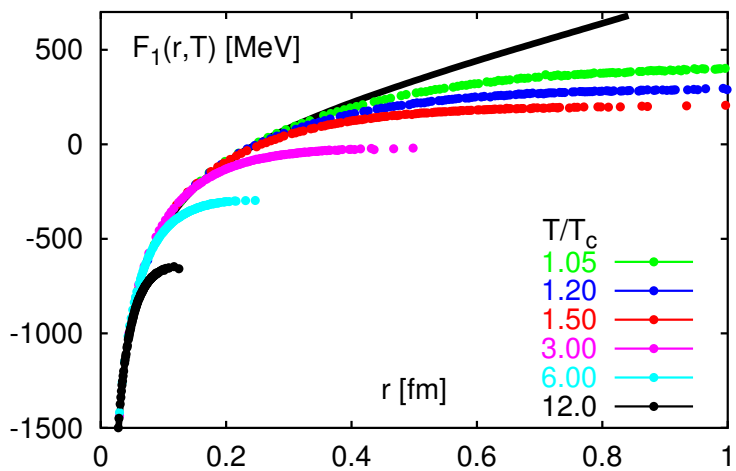

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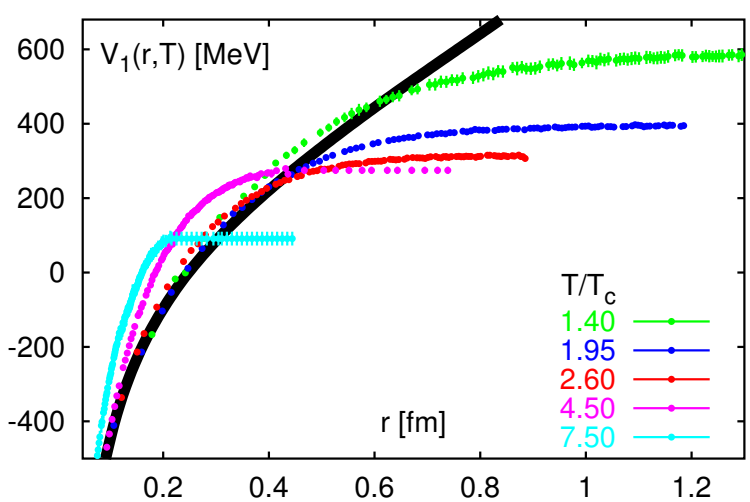
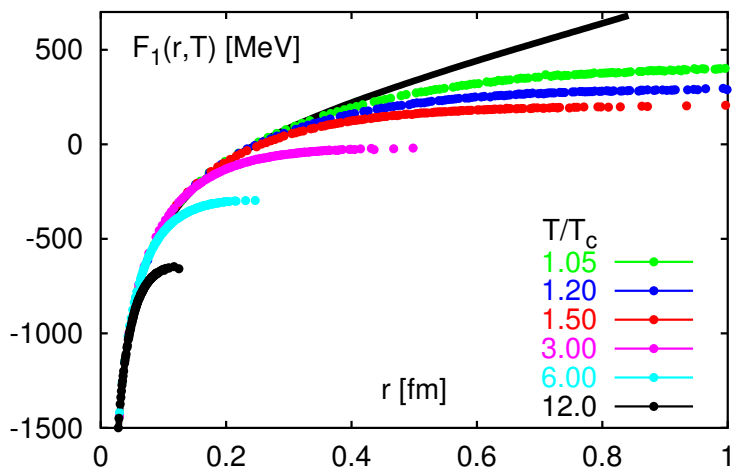


ii) singlet energy $\Leftrightarrow$ "potential" energy

When do heavy quark bound states really disappear?

From heavy quark free energies to heavy quark potentials



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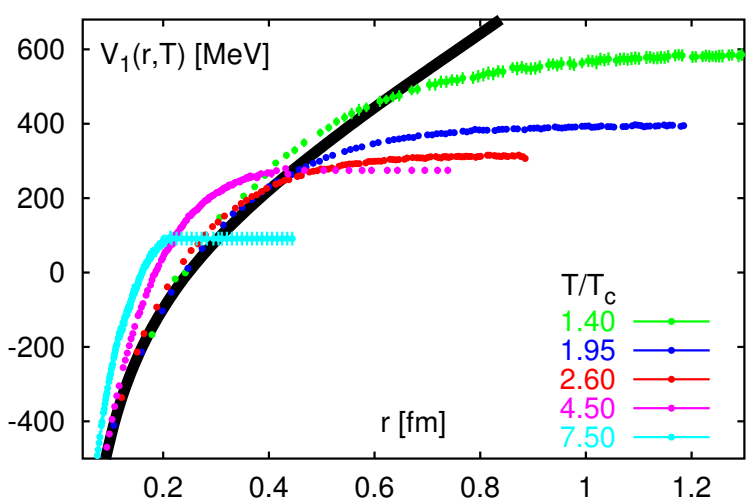
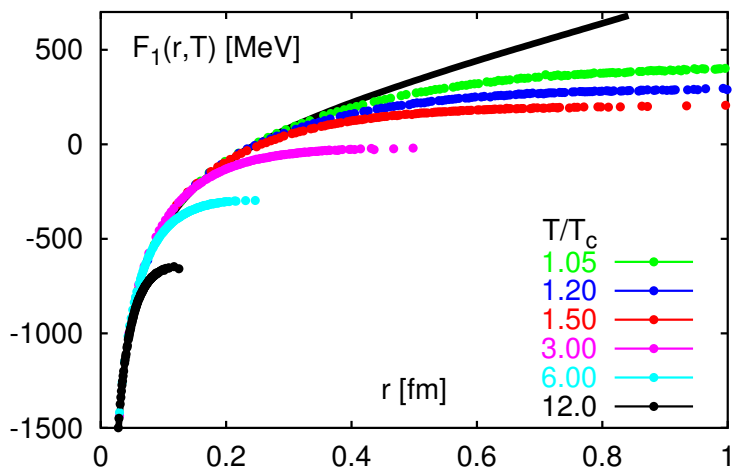


ii) singlet energy $\Leftrightarrow$ "potential" energy

When do heavy quark bound states really disappear?

i) redo calculations for QCD (string breaking!)
F. Zantow, poster
K. Petrov, poster

From heavy quark free energies to heavy quark potentials



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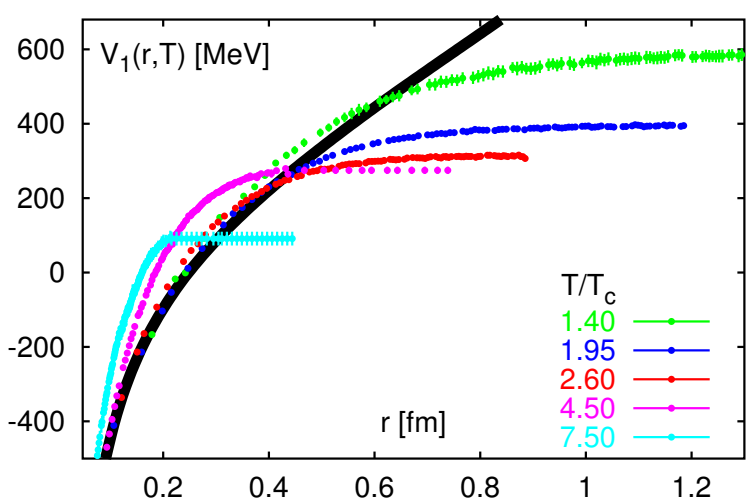
$$V_1(\infty, 1.4T_c) \simeq 600 \text{ MeV}$$



ii) singlet energy $\Leftrightarrow$ "potential" energy

When do heavy quark bound states really disappear?

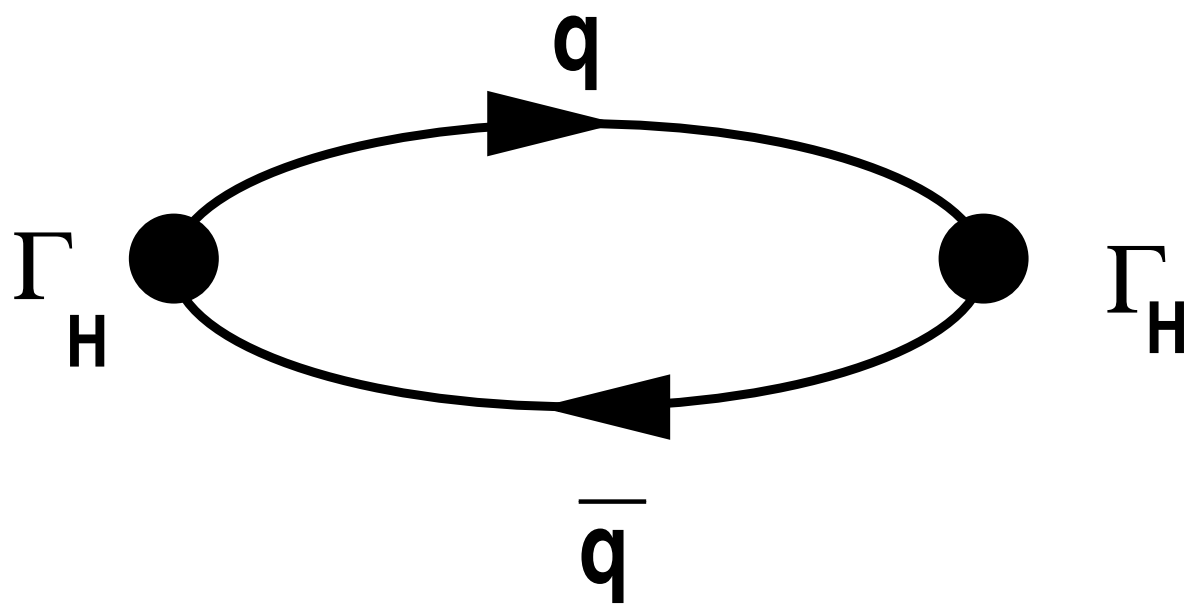
ii) redo potential model calculations

(IN PROGRESS) or... $\Rightarrow$

Thermal meson correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$-pair

spectral representation of correlator $\Rightarrow$ in-medium properties of hadrons;
thermal dilepton (photon) rates



$$G_H^\beta(\tau,\vec{r}) \;=\; \langle J_H(\tau,\vec{r}) J_H^\dagger(0,\vec{0})\rangle\,; \quad J_H(\tau,\vec{r}) = \bar{q}(\tau,\vec{r})\Gamma_H q(\tau,\vec{r})$$

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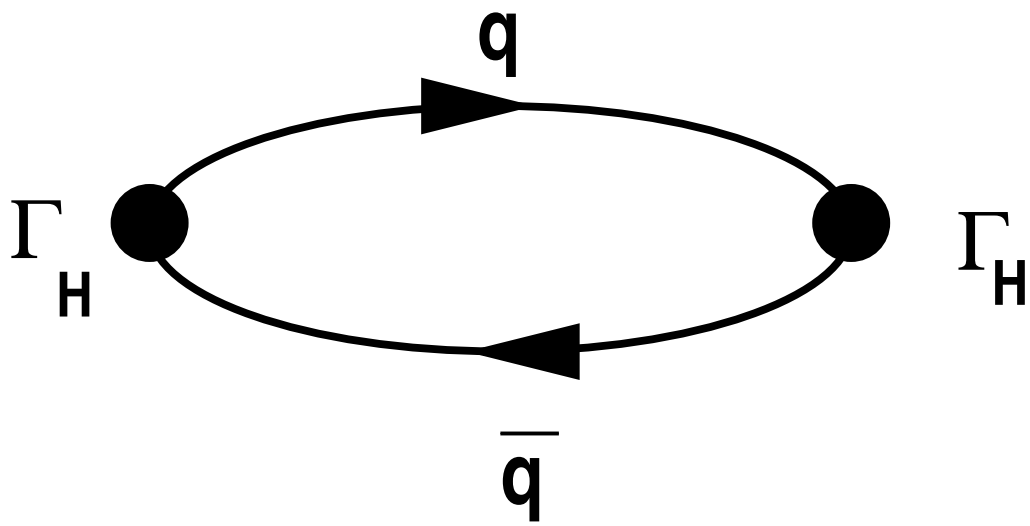

spectral representation of

Euclidean correlation functions

$$G_H^\beta(\tau,\vec{r}) = \int_0^\infty \mathrm{d}\omega \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\, \sigma_H(\omega,\vec{p},T)\, \mathrm{e}^{i\vec{p}\vec{r}}\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

Thermal meson correlation functions and spectral functions

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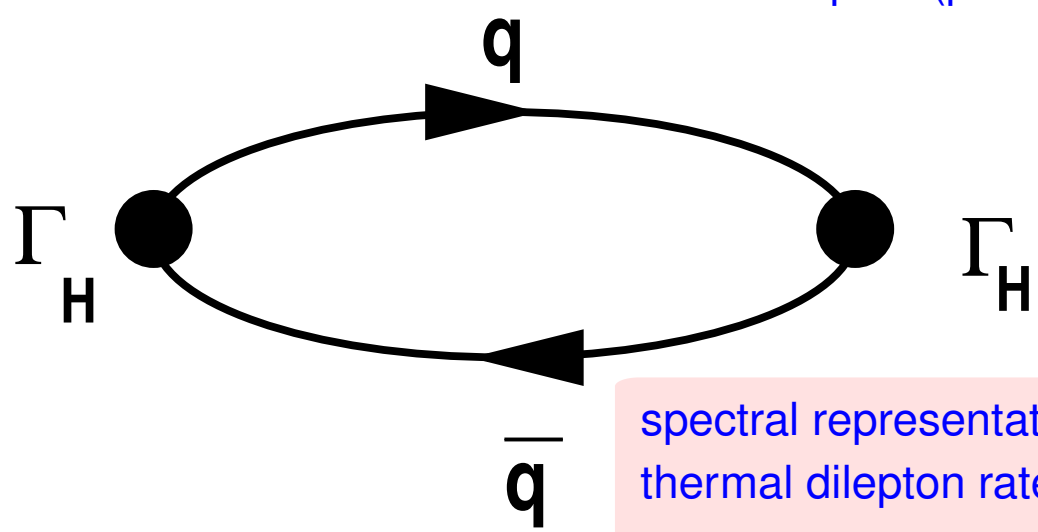


spectral representation of thermal dilepton rate

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2(e^{\omega/T} - 1)}$$

spectral representation of Euclidean correlation functions

$$G_H^\beta(\tau, \vec{r}) = \int_0^\infty d\omega \int \frac{d^3\vec{p}}{(2\pi)^3} \sigma_H(\omega, \vec{p}, T)\, e^{i\vec{p}\vec{r}} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

Reconstruction of the spectral function

- spectral functions can be reconstructed from meson correlation functions using the Maximum Entropy Method (MEM)

  Y. Nakahara, M. Asakawa, T. Hatsuda, PR D60 (1999) 091503

- MEM $\equiv$ statistical tool which needs input: entropy function, default model

$$G_H(\tau, T) = \int_0^\infty d\omega \, \sigma_H(\omega, T) \, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

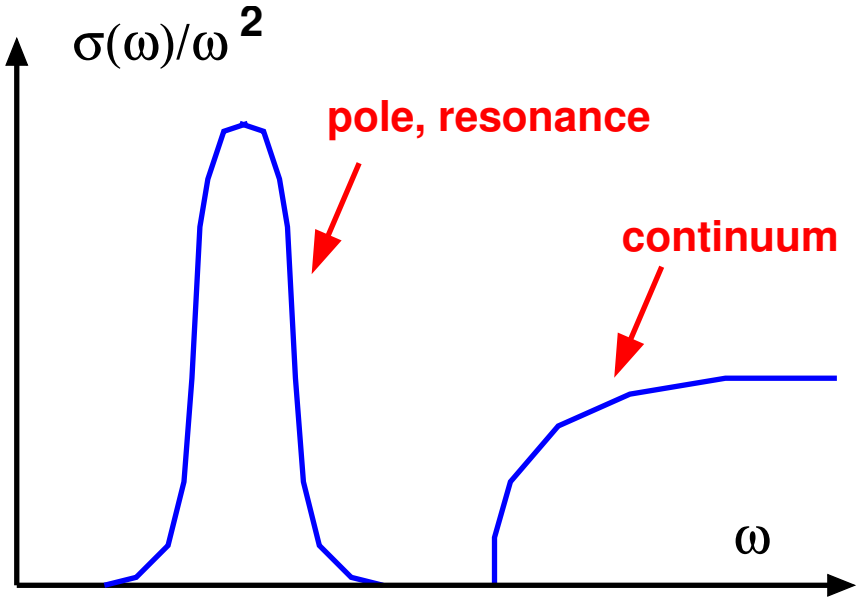
where G_H is known only for a discrete set of points, $\tau_k = k/N_\tau$, $k = 1, \ldots N_\tau$

maximize $Q = \alpha S - \chi^2/2$ with

entropy $S = \int d\omega [\sigma(\omega) - m(\omega) - \sigma(\omega) \ln(\sigma(\omega)/m(\omega))]$ and

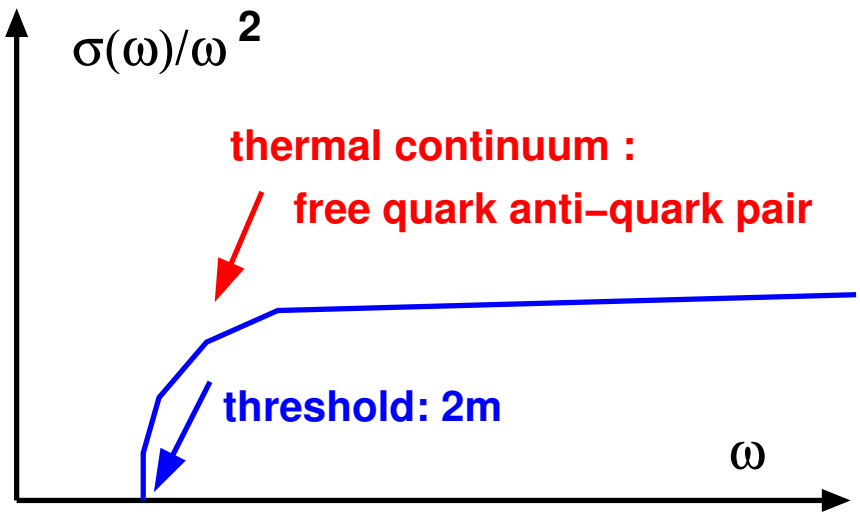
default model $m(\omega)$: incorporates known short distance properties, $m(\omega) \sim \omega^2$ (may include lattice artefacts)

Examples for spectral functions at $\mathbf{T = 0}$ and $\mathbf{T = \infty}$



vector meson spectral function ($T = 0$):

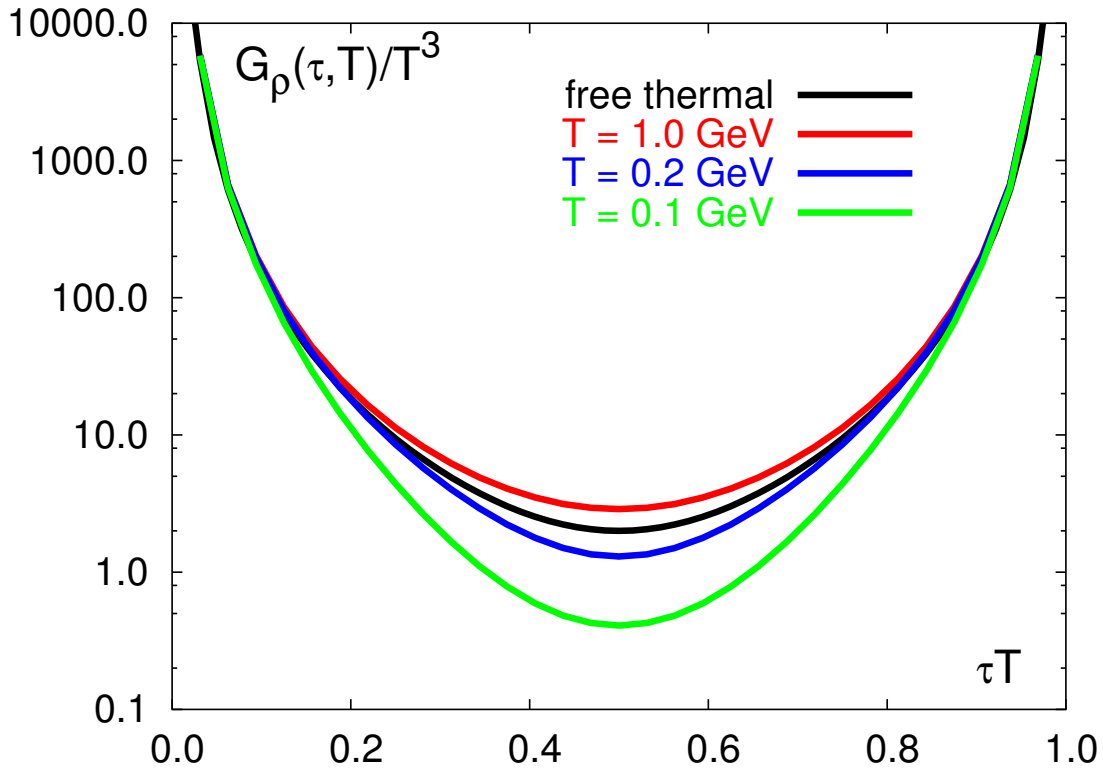
$$\sigma_\rho(\omega, \vec{0}) = \frac{2\omega^2}{\pi}\left[F_\rho^2 \frac{\Gamma_\rho m_\rho}{(\omega^2 - m_\rho^2)^2 + \Gamma_\rho^2 m_\rho^2} + \frac{1}{8\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\frac{1}{1 + \exp((\omega_0 - \omega)/\delta)}\right]$$



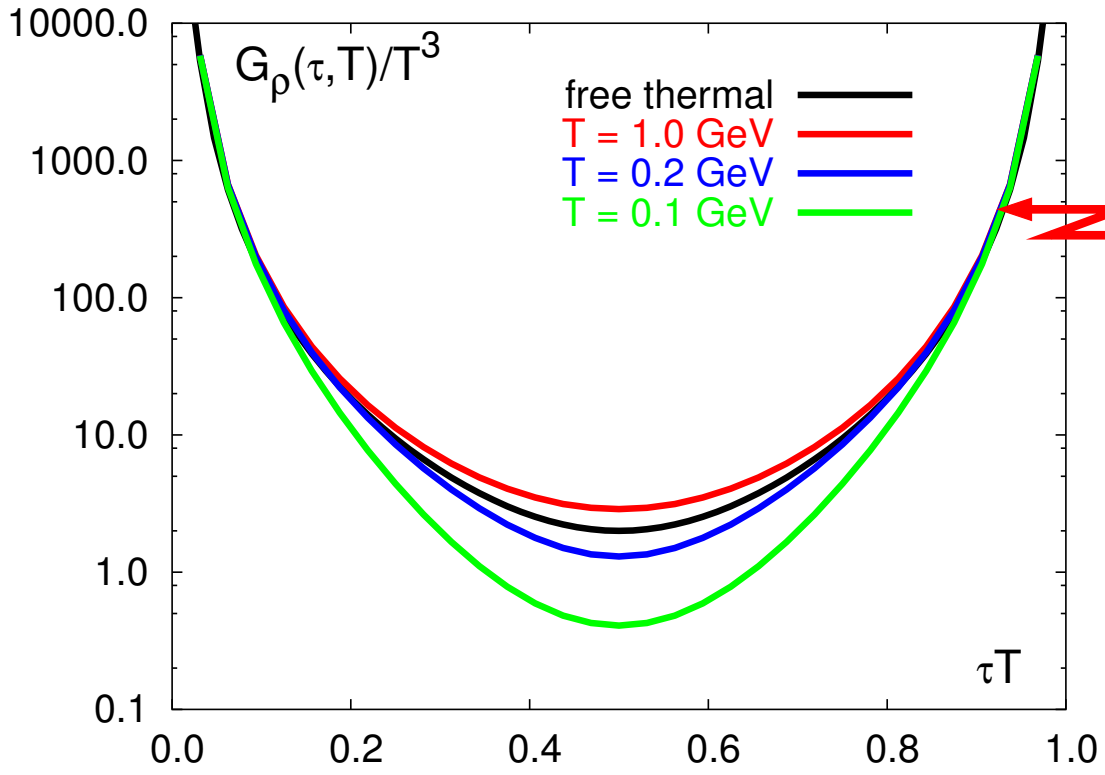
free vector meson spectral function ($T = \infty$):

$$\sigma_V(\omega, T) = \frac{N_c}{8\pi^2}\,\omega^2\,\Theta(\omega^2 - 4m^2)\tanh\left(\frac{\omega}{4T}\right) \cdot \sqrt{1 - \left(\frac{2m}{\omega}\right)^2}\left[2 + \left(\frac{2m}{\omega}\right)^2\right]$$

Thermal vector meson correlators

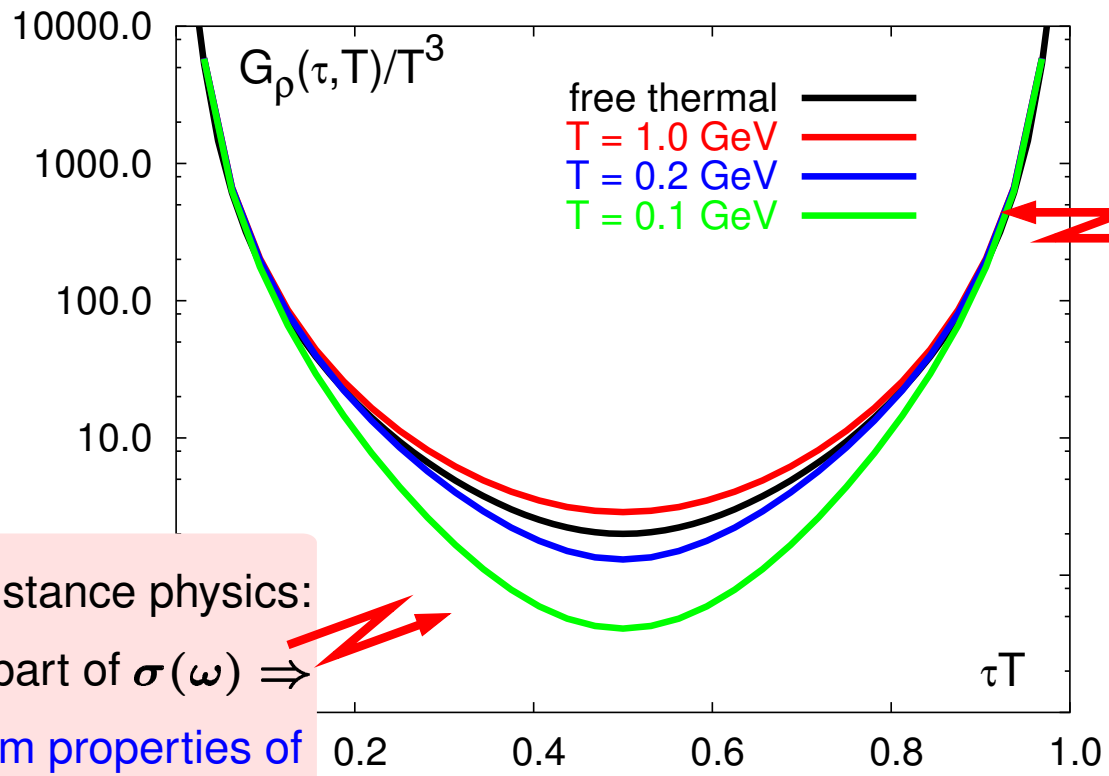


Thermal vector meson correlators



short distance physics: large ω, continuum part of $\sigma(\omega)$; uninteresting for "thermal masses"

Thermal vector meson correlators



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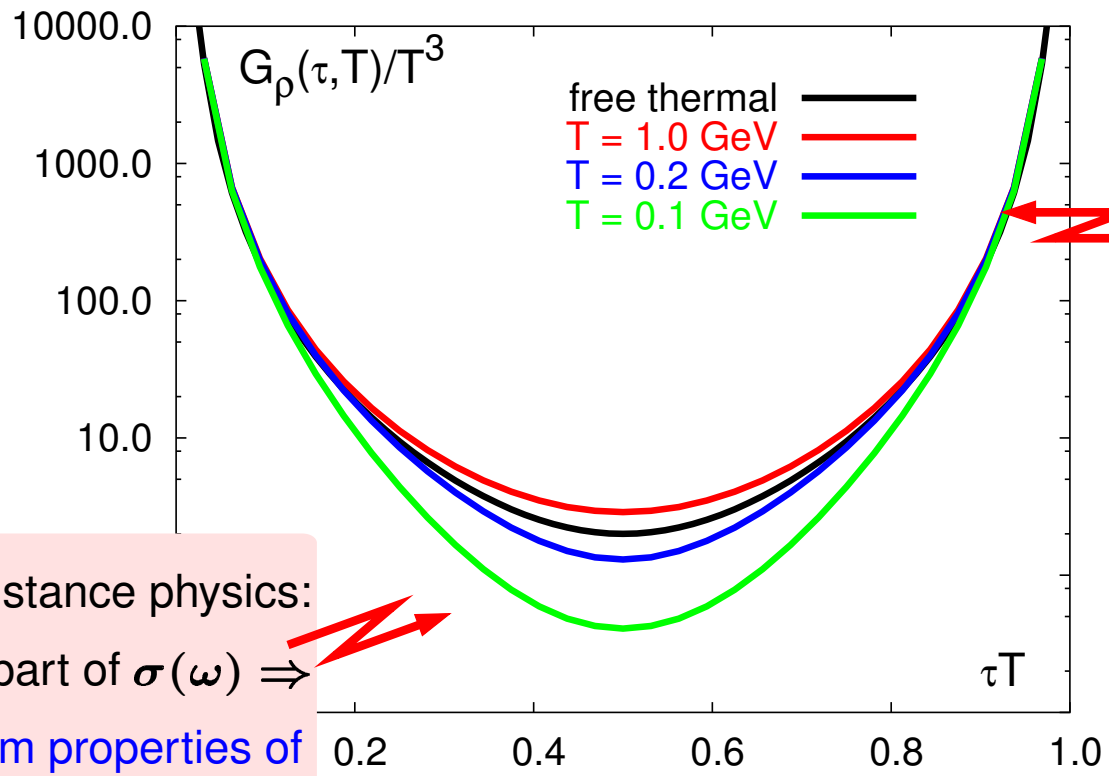
"large" distance physics: small ω part of $\sigma(\omega)$ $\Rightarrow$ in-medium properties of hadrons

Thermal vector meson correlators



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"large" distance physics: small ω part of $\sigma(\omega)$ $\Rightarrow$ in-medium properties of hadrons

goal: maximize information on small ω region; minimize influence of large ω region

Thermal vector meson correlators

goal: minimize influence of large ω region
maximize information on small ω region;

three approaches:

i) smeared operators: modify currents $J_H(\vec{x}) \to \tilde{J}_H(\vec{x}) \sim \sum_{\vec{y}} \exp\left(-A|\vec{x}-\vec{y}|^2\right)\ J_H(\vec{y})$

T. Umeda et al., hep-lat/0211003

ii) anisotropic lattices: increase number of data points, $k/N_\tau \ \to\ k/N_\tau\xi$
increases resolution around $\tau T = 1/2$

M. Asakawa and T. Hatsuda, hep-lat/0308034

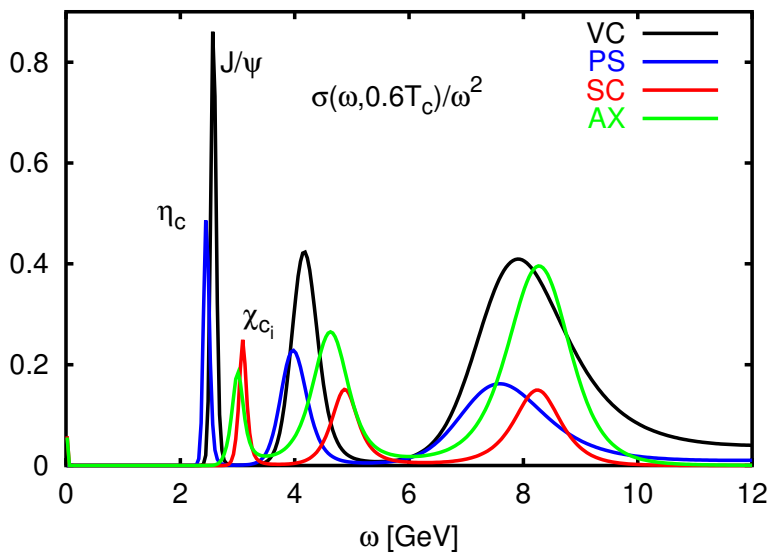
iii) low-T default model: model short distance part of correlation functions using information on σ at small T; $m(\omega) = \sigma(\omega, T \simeq 0)$ for $\omega > \omega_0$;
increases MEM sensitivity for $\omega < \omega_0$

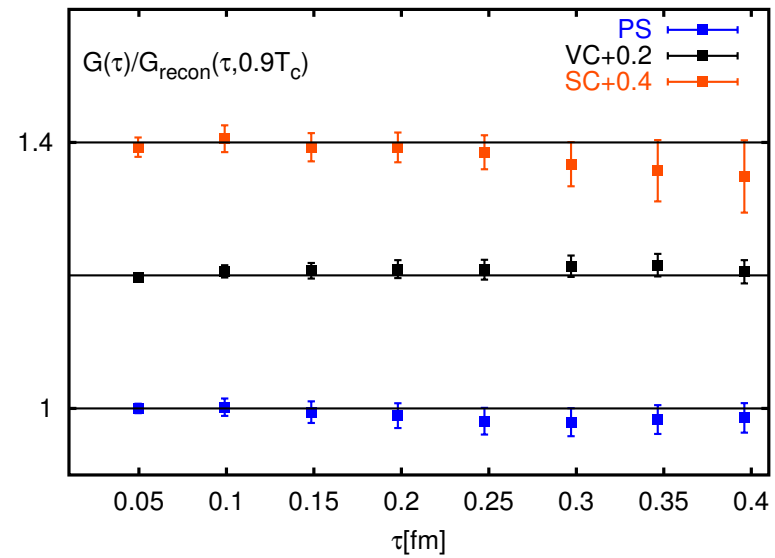
S. Datta et al., hep-lat/0312037

Heavy quark spectral functions and correlation functions

- left: charmonium spectral functions below T_c, *i.e.* at $T \simeq 0.6\ T_c$, lattice size $48^3 \times 24$
- right: correlation function at $T = 0.9T_c$ over reconstructed correlation function at $T \simeq 0.9\ T_c$ using the spectral function generated at $T \simeq 0.6\ T_c$, *i.e.*

$$G_{recon}(\tau, 0.9T_c) = \int d\omega\ \sigma(\omega, 0.6T_c) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$





→ no significant temperature dependence below T_c

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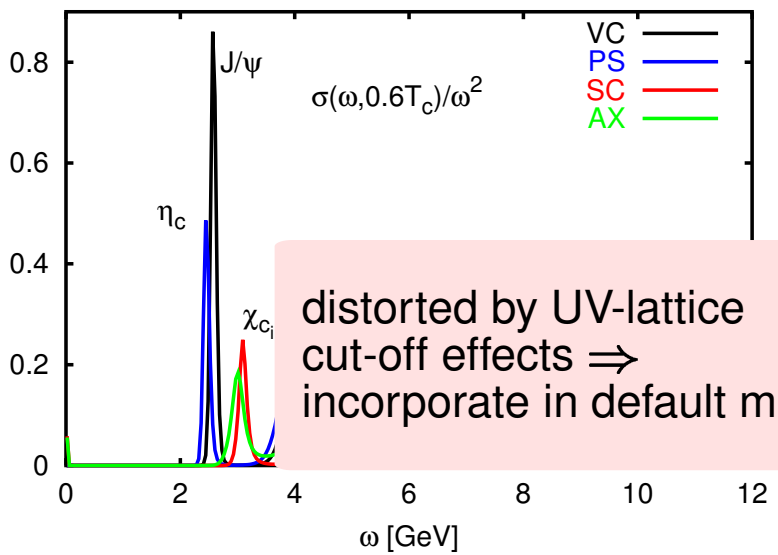
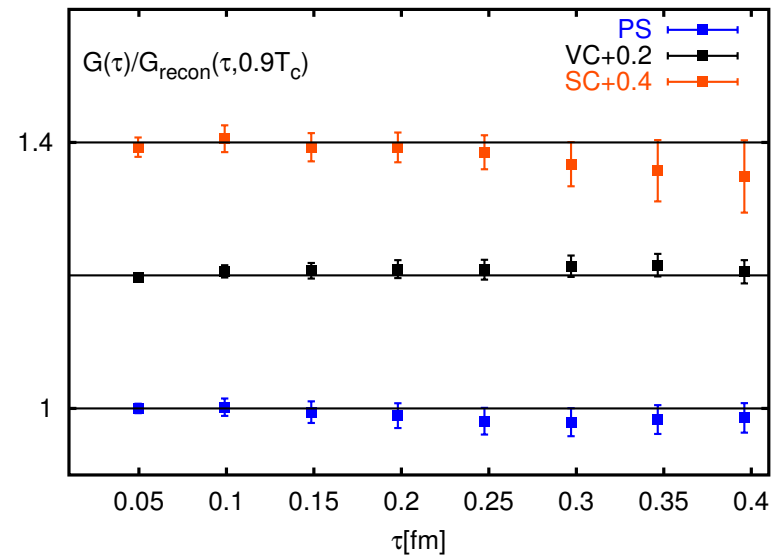


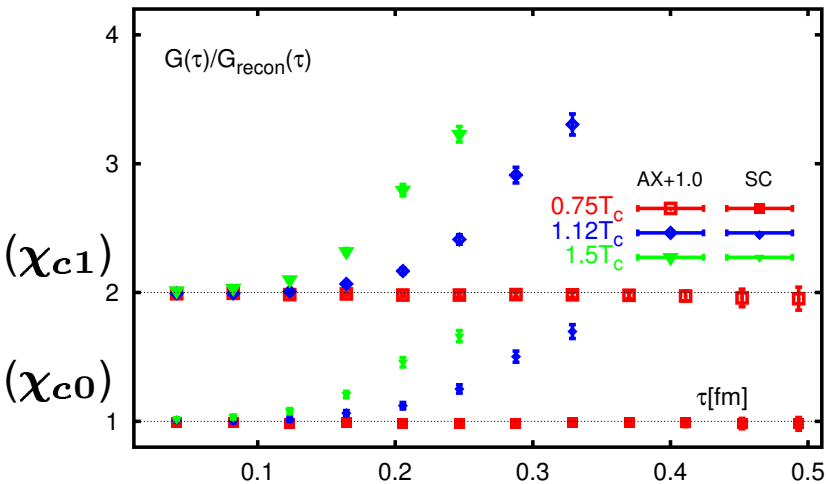
distorted by UV-lattice cut-off effects ⇒ incorporate in default model



⇒ no significant temperature dependence below T_c

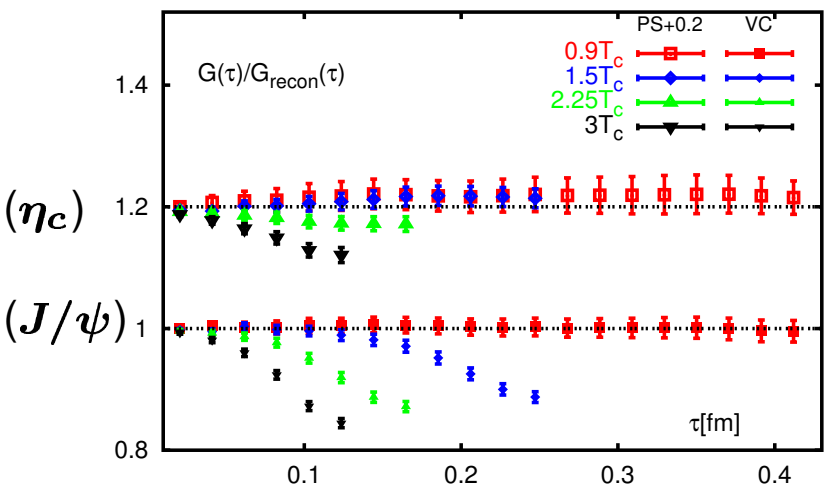
Heavy quark spectral functions and correlation functions

data for $G_H(\tau, T)$ over reconstructed correlation functions at T from data below T_c



scalar and axial-vector correlation functions:

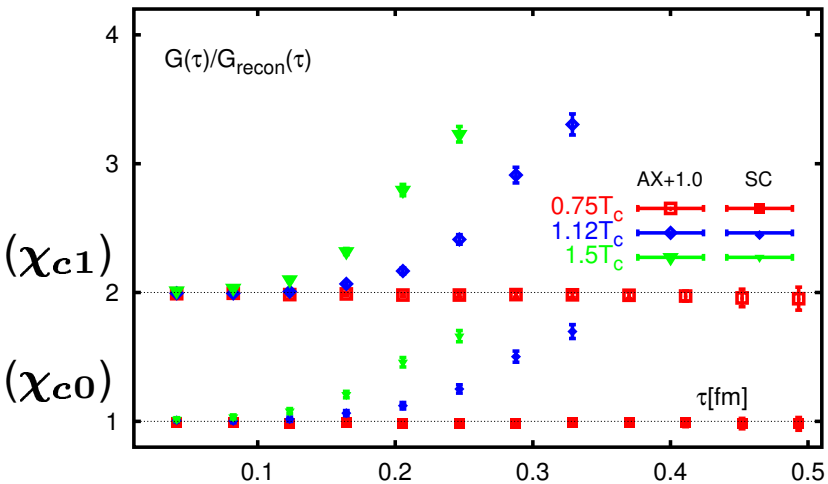
strong temperature dependence just above T_c for χ_c states
(normalized at $T < T_c$)
($48^3 \times N_\tau$, $N_\tau = 12,\ 16,\ 24$, $a = 0.04$ fm)

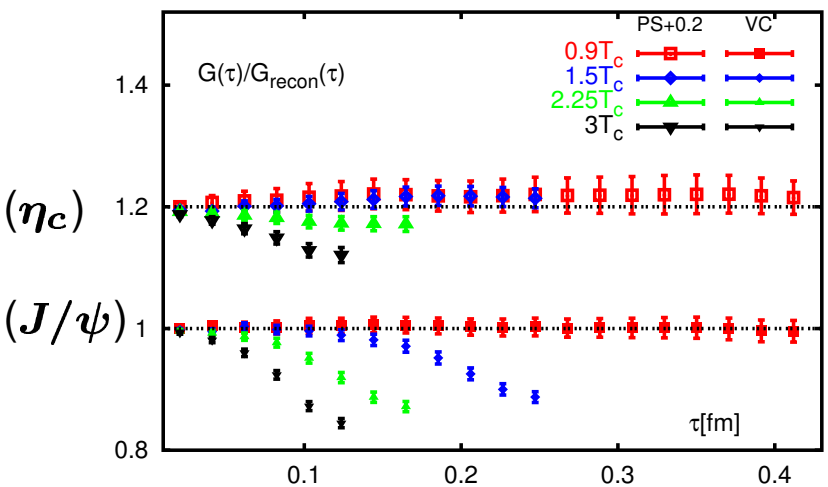


vector and pseudoscalar correlation functions:

no temperature dependence for η_c up to $1.5\ T_c$; only mild but systematic temperature dependence of J/ψ
(normalized at $T < T_c$)
($N_\sigma = 40,\ 48,\ 64$,
$N_\tau = 12,\ 16,\ 24,\ 40$, $a = 0.02$ fm)

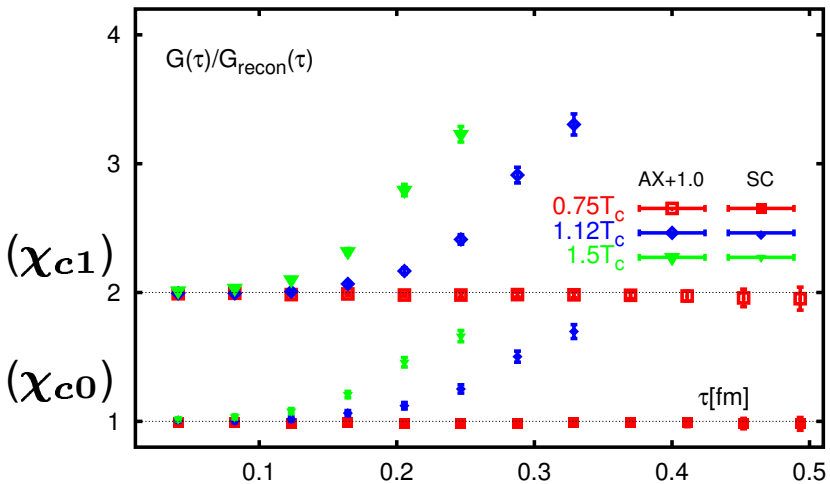
Heavy quark spectral functions and correlation functions



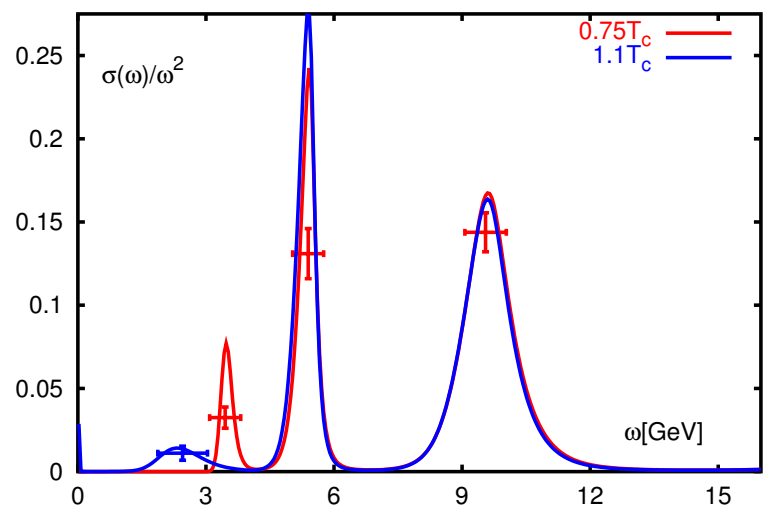


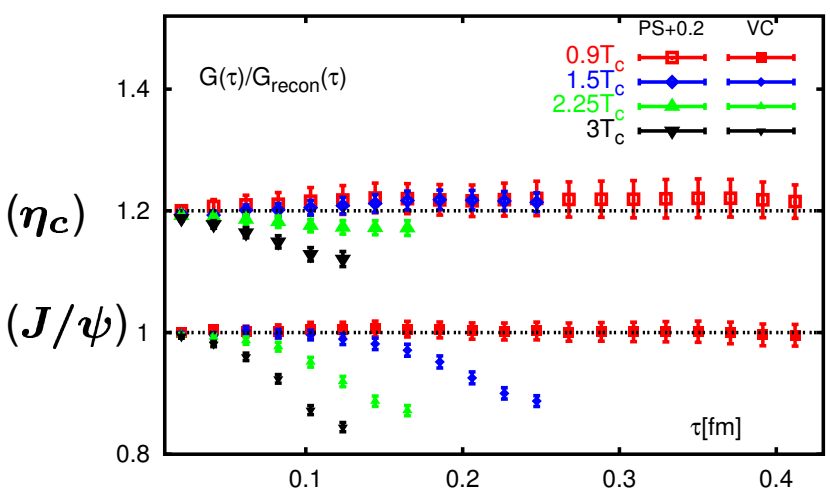
pattern seen in correlation functions also visible in spectral functions

Heavy quark spectral functions and correlation functions

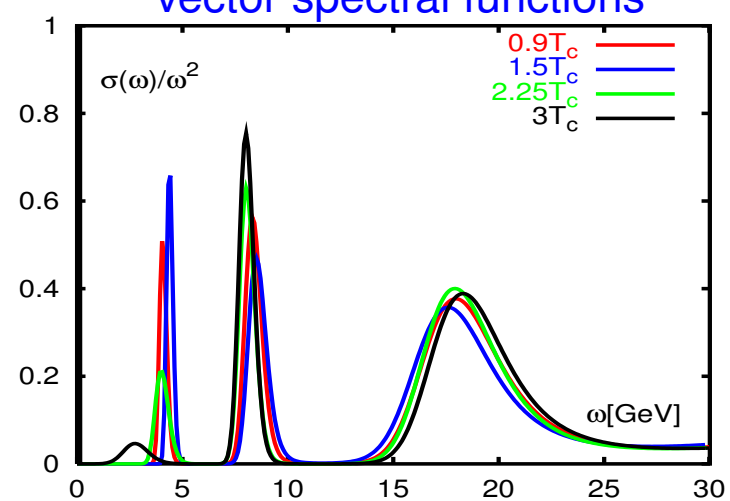


scalar spectral functions

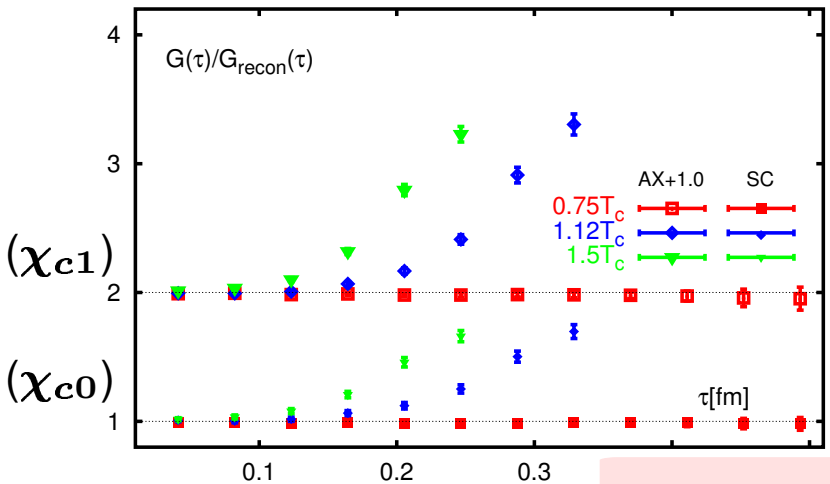




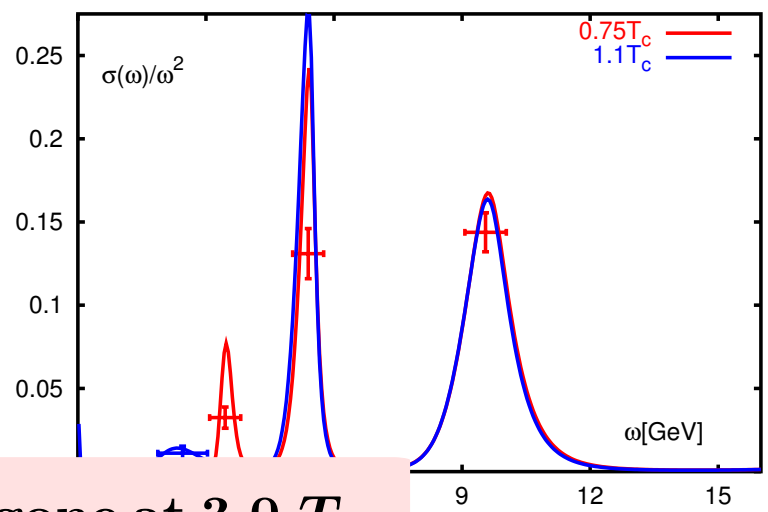
vector spectral functions



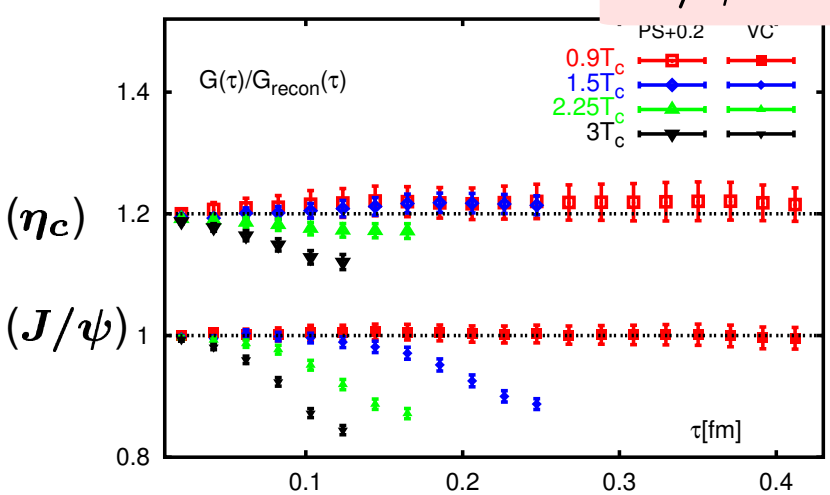
Heavy quark spectral functions and correlation functions



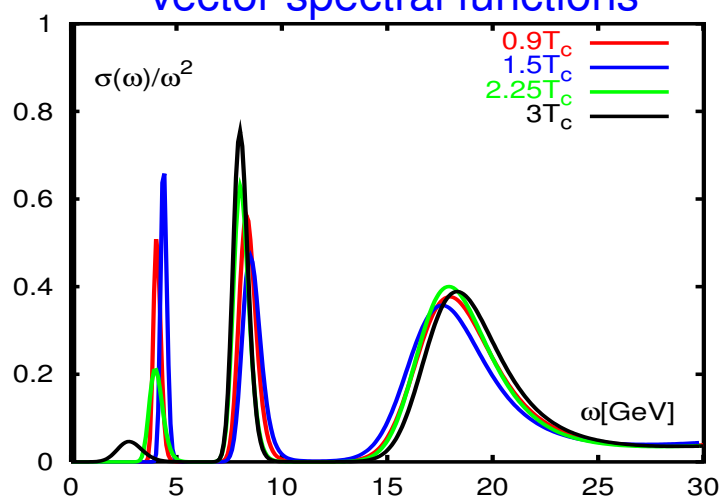
scalar spectral functions



J/ψ and η_c gone at 3.0 T_c



vector spectral functions

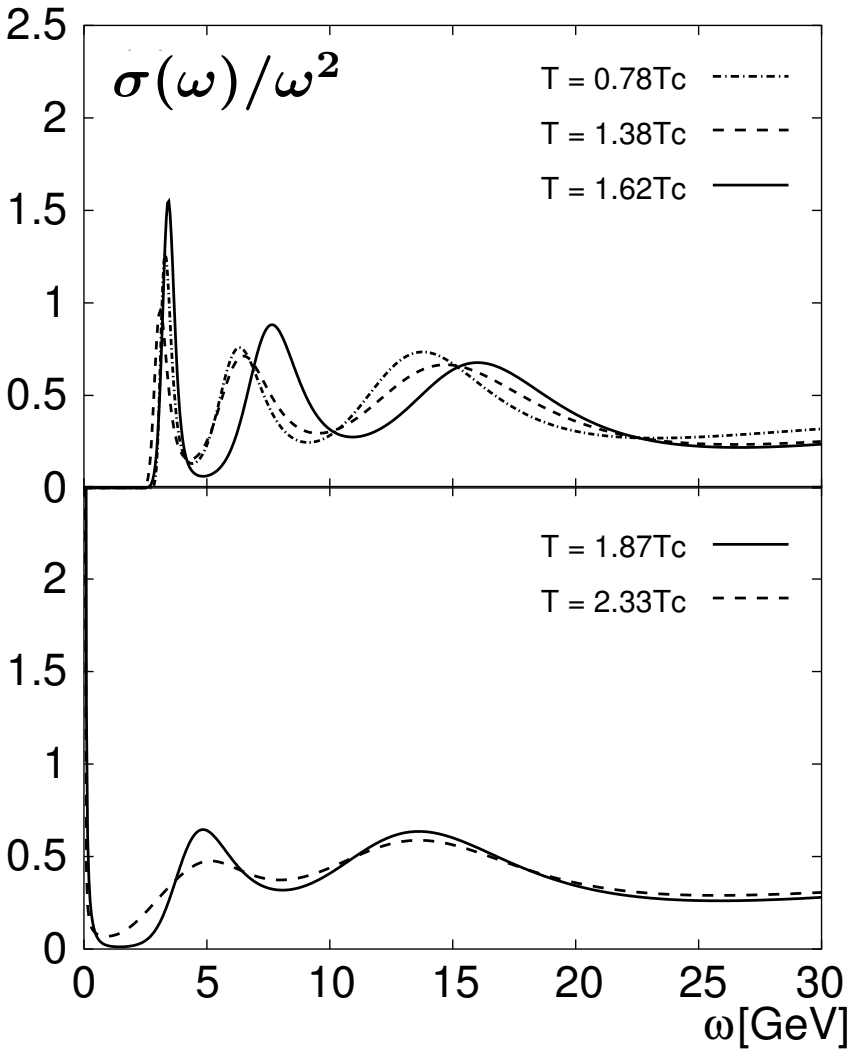


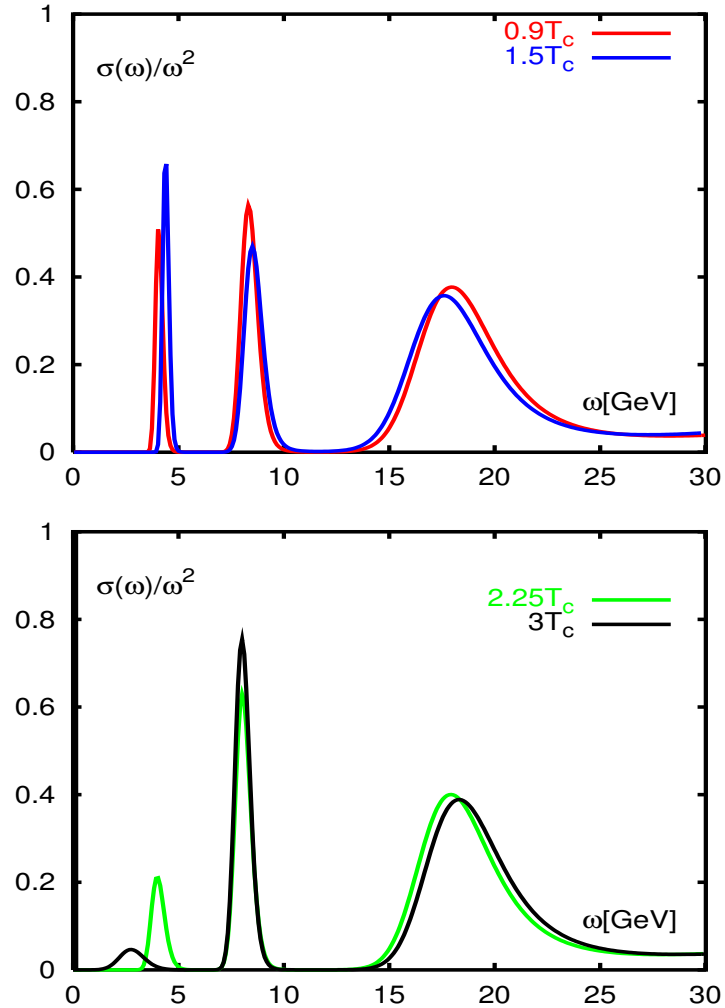
Heavy quark spectral functions
comparison of different approaches

M. Asakawa, T. Hatsuda, hep-lat/0308034

S. Datta et al., hep-lat/0312037

J/ψ spectral function





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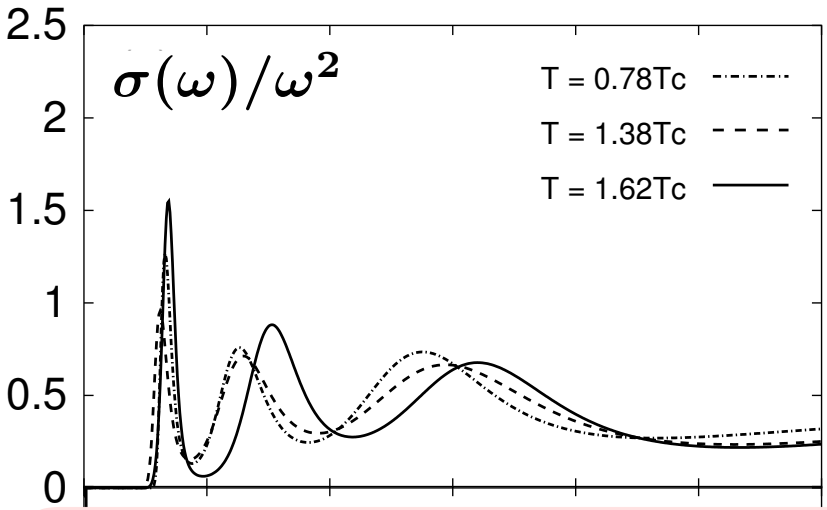
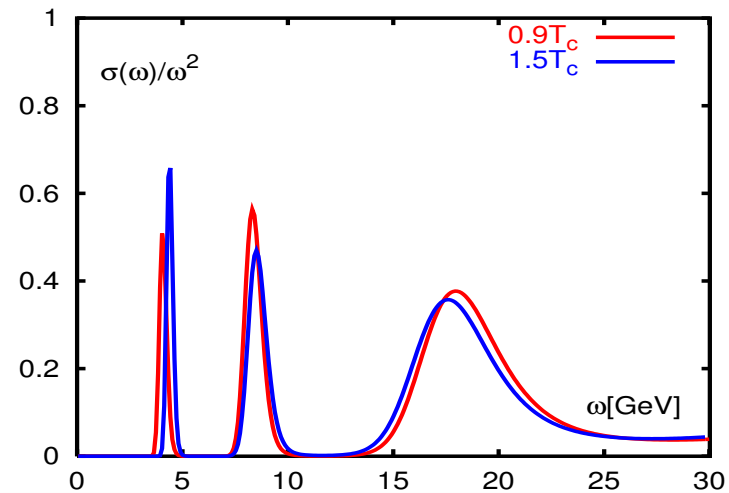
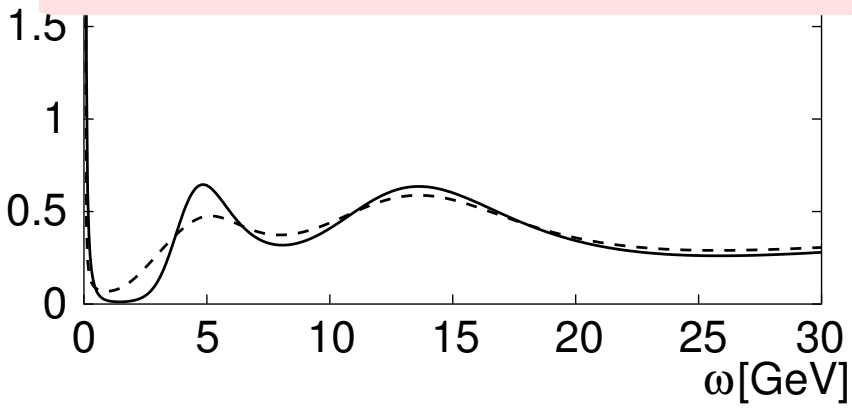
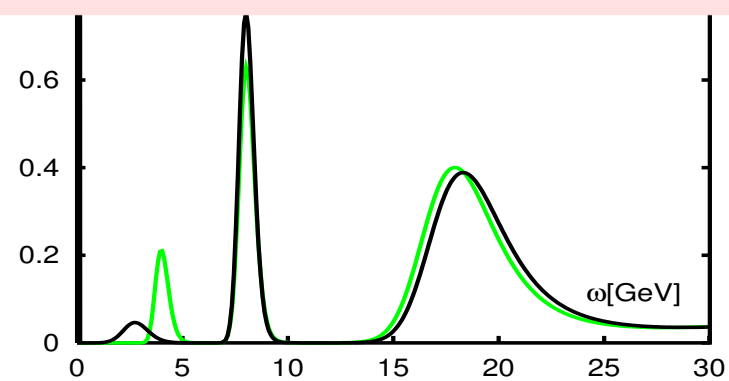




J/ψ dissociates for $1.6T_c \lesssim T \lesssim 1.9T_c$
rather abrupt disappearance of J/ψ

J/ψ gradually disappears for $T \gtrsim 1.5T_c$
J/ψ strength reduced by 25% at $T = 2.25T_c$



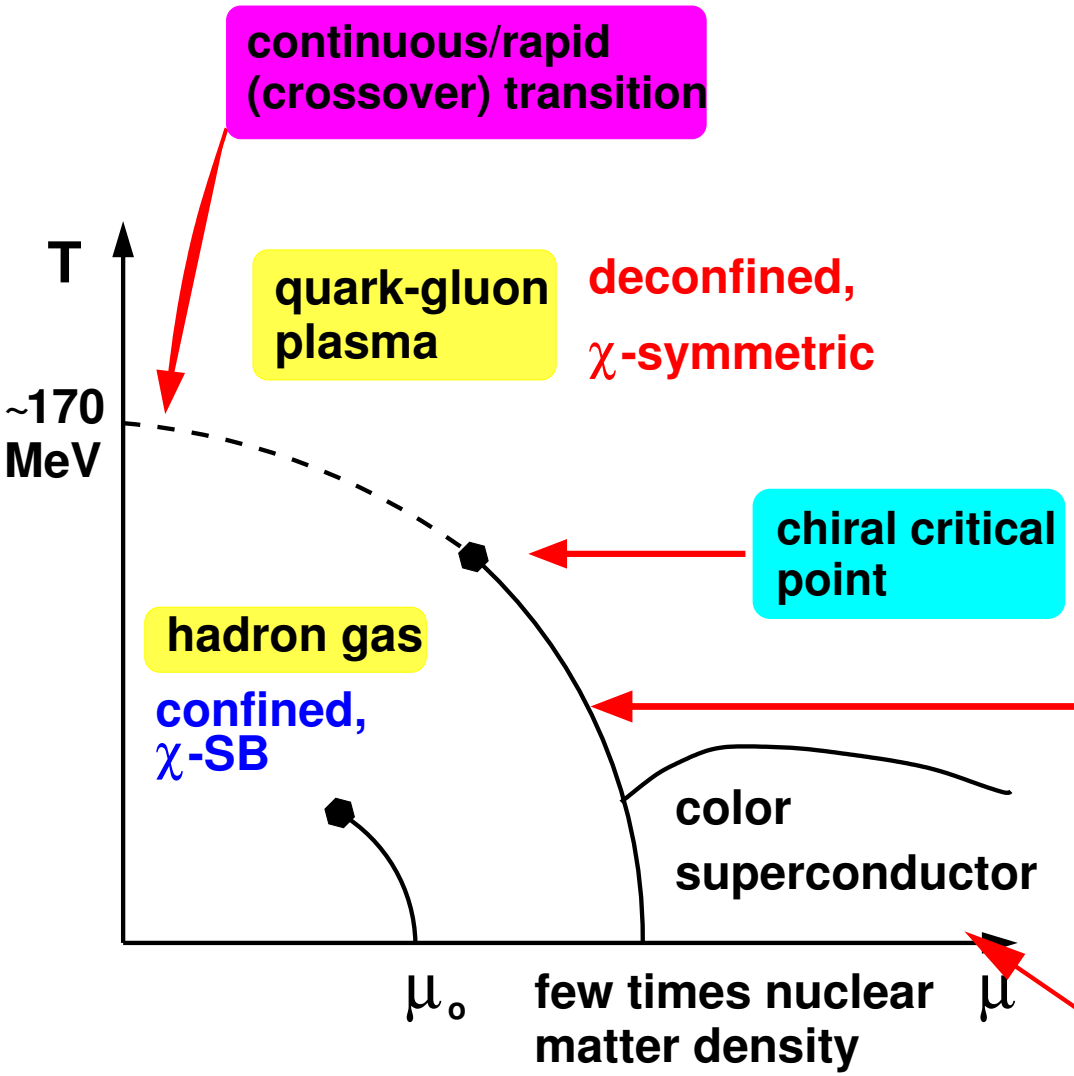


News from Lattice QCD on ...

...topics that cannot be discussed in detail:

1) Update on the QCD phase diagram
   - $T_c(\mu)$ and the chiral critical point: quark mass dependence, lattice artefacts
2) QCD equation of state for $\mu > 0$
   - baryon number fluctuations;
   - comparison with resonance gas
3) Chiral symmetry and in-medium properties of light quark mesons
   - thermal dilepton rates

Critical behavior in hot and dense matter: phase diagram



continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170\,MeV$$
$$\epsilon_c \simeq 0.7\,GeV/fm^3$$

2nd order phase transition; Ising universality class

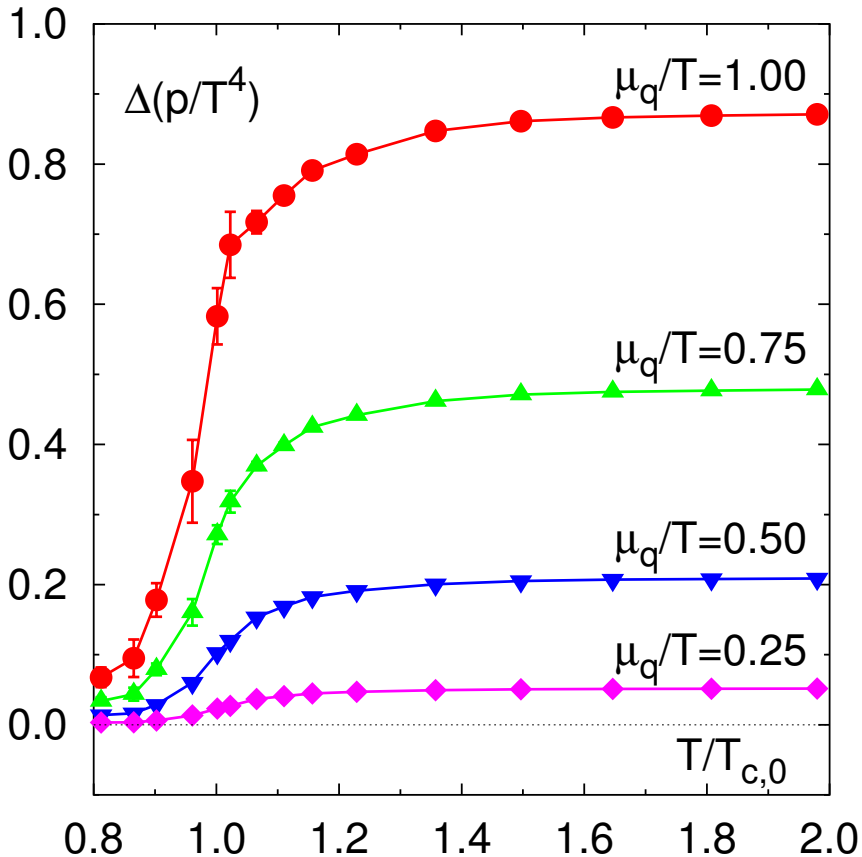
$T_c(\mu)$ under investigation (cut-off and m_q-dependence!!)

1st order phase transition ???

expected - however, so far no direct evidence from lattice QCD

attractive 1–gluon exchange => qq–condensates

Equation of State ($\mu > 0$)



new results from Taylor expansion up to $\mathcal{O}(\mu^6)$: poster by S. Ejiri

pressure for $\mu > 0$:

high-T, massless limit: polynomial in (μ/T)

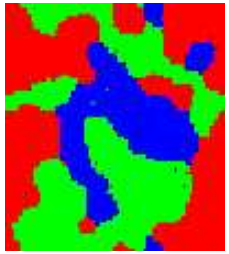
$$\left(\frac{p_{SB}}{T^4}\right) = \frac{\pi^2}{45}\left[8 + \frac{21}{4}n_f\right] + \frac{n_f}{2}\left(\frac{\mu}{T}\right)^2 + \frac{n_f}{4\pi^2}\left(\frac{\mu}{T}\right)^4$$

contribution for $(\mu/T) \leq 1$, $\frac{\Delta p}{p} \leq 30\%$;

Bielefeld-Swansea, hep-lat/0305007
(similar: Z. Fodor et al., hep-lat/0209114)

pattern similar to the $\mu = 0$ case

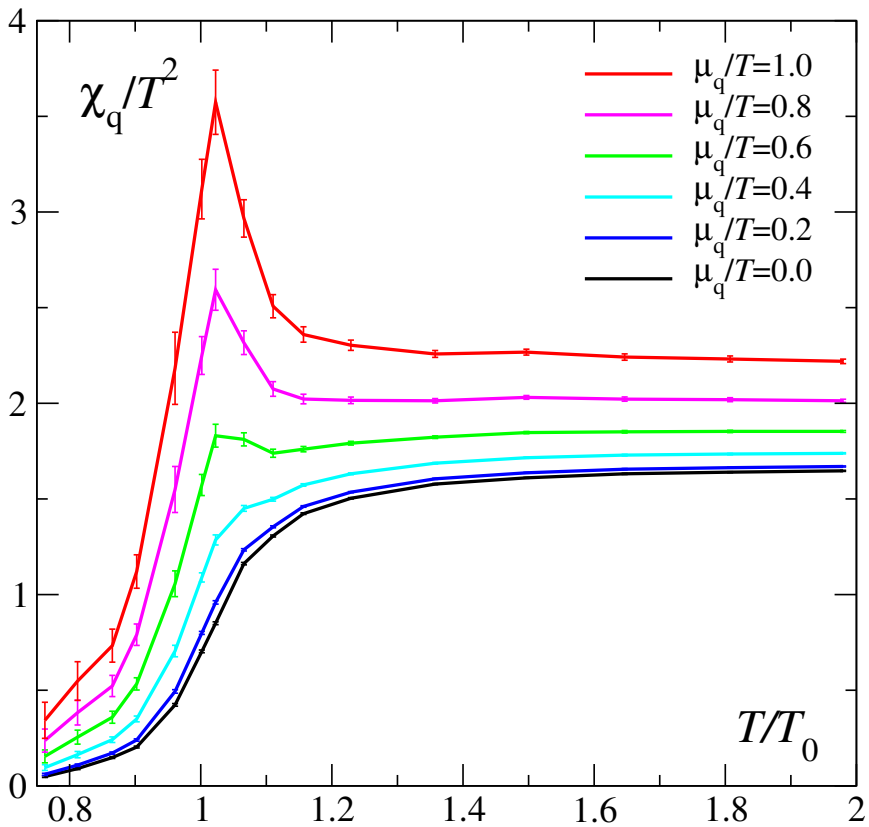
HOWEVER:
cut-off effects still large in these studies



Fluctuations of the baryon number density ($\mu > 0$)

baryon number density fluctuations:
(Bielefeld-Swansea, PRD68 (2003) 014507)

$$\frac{\chi_B}{T^2} = \left(\frac{\mathrm{d}^2}{\mathrm{d}(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$



high-T, massless limit: polynomial in (μ/T)

$$\frac{\chi_{B,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2} \left(\frac{\mu}{T} \right)^2$$

large density fluctuations closer to the chiral critical point

$$\frac{\chi_q}{T^2} \sim \frac{1}{V} \left(\langle N_q^2 \rangle - \langle N_q \rangle^2 \right)$$

Taylor expansion up to $\mathcal{O}(\mu^4)$
new: improved statistics, $\mathcal{O}(\mu^6)$
poster by S. Ejiri

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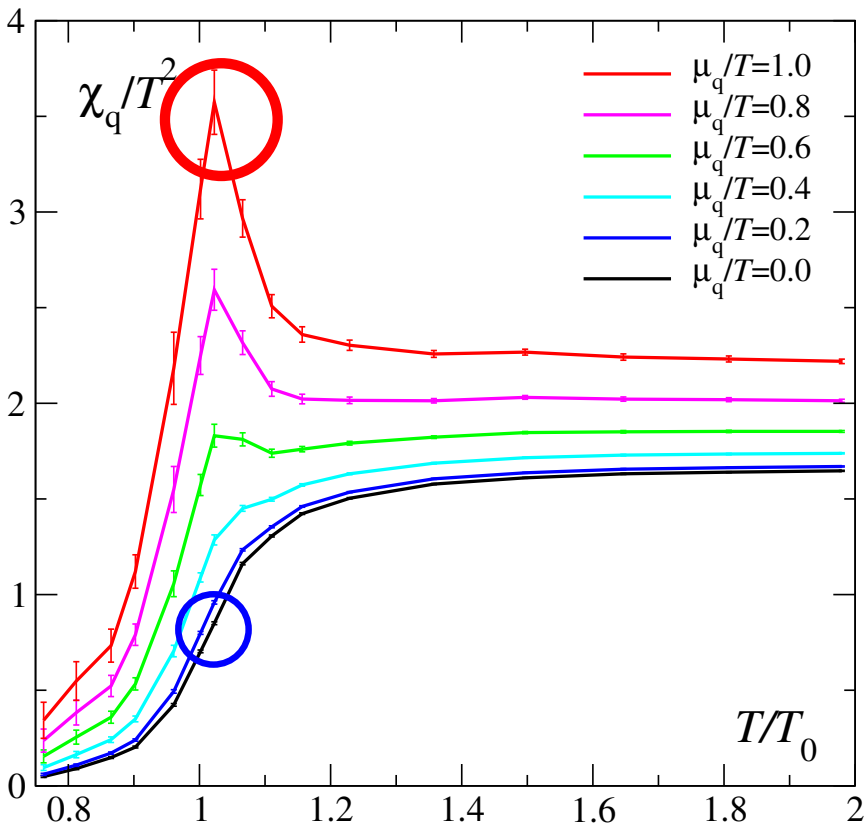

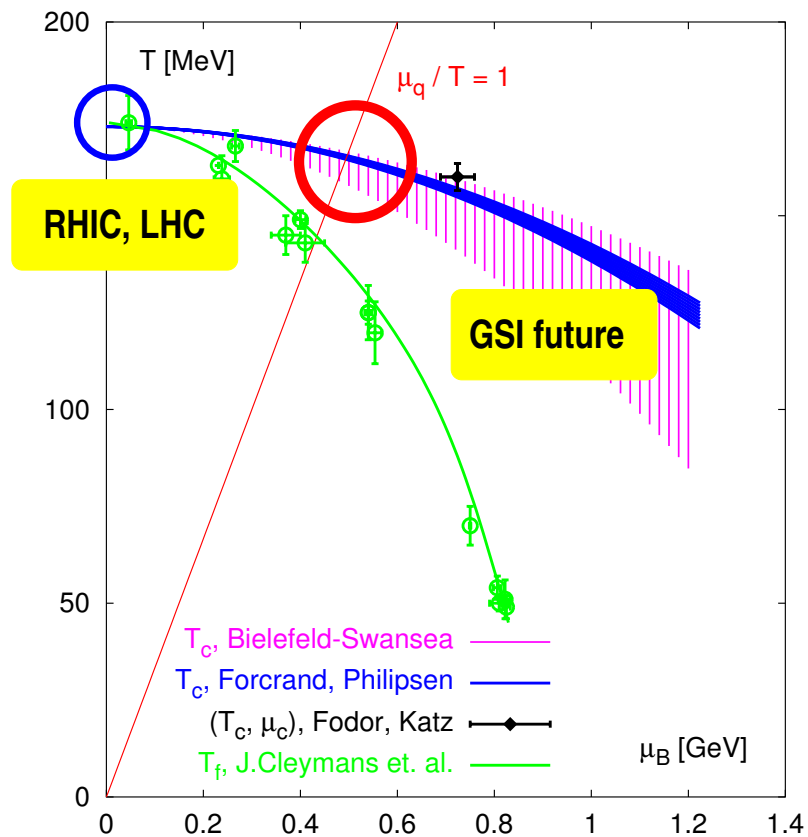

Resonance gas: spectrum independent consequences

K. Redlich, parallel session

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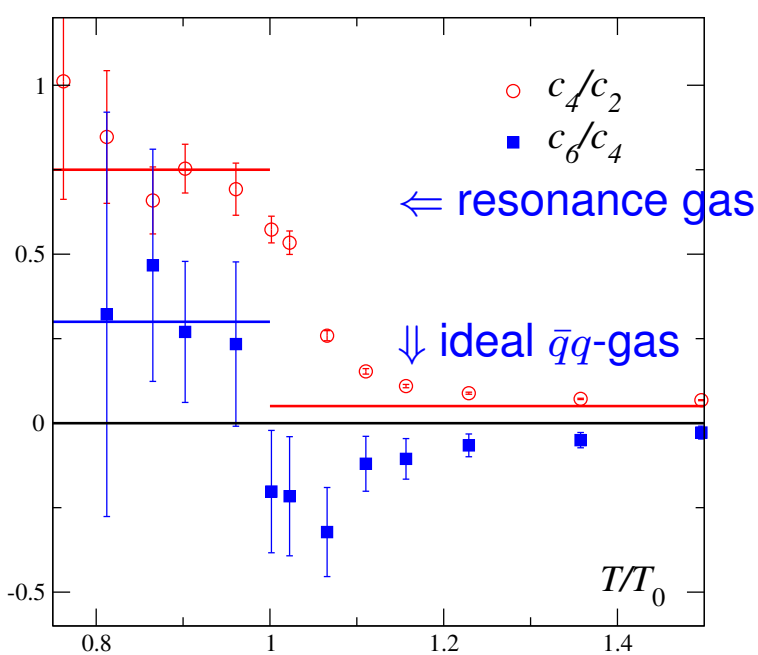
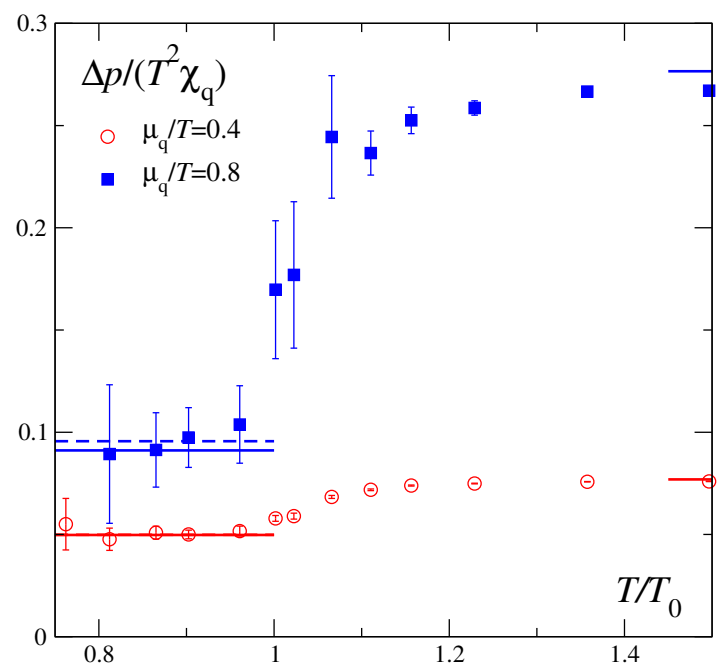
- Boltzmann approximation $\Rightarrow$ factorization $\Rightarrow$ temperature independent ratios; spectrum independent results

$$\frac{\Delta p}{T^2\chi_q} = \frac{1}{9}\left(1-\cosh^{-1}(3\mu_q/T)\right) \sim \frac{\left(\frac{\mu_q}{T}\right)^2 + \frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^4 + \frac{c_6}{c_2}\left(\frac{\mu_q}{T}\right)^6}{2 + 12\frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^2 + 30\frac{c_6}{c_2}\left(\frac{\mu_q}{T}\right)^4}$$

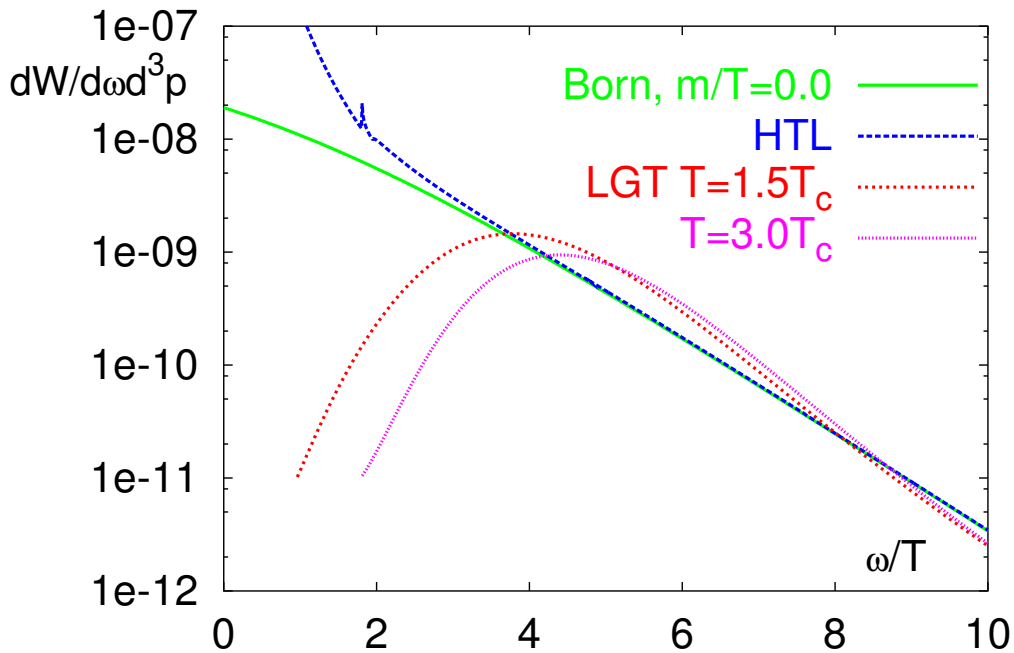




NEW: $\mathcal{O}(\mu^6)$

solid: exact; dashed $\mathcal{O}((\mu/T)^6)$

Dilepton rate: HTL and lattice calculations



thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3 p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2 (\mathrm{e}^{\omega/T} - 1)}$$

HTL and lattice disagree for: $\omega/T \lesssim (3-4)$

- infra-red sensitivity of HTL-calculations $\Leftrightarrow$ "massless gluon" cut in HTL-propagator
- infra-red sensitivity of lattice calculations $\Leftrightarrow$ thermodynamic limit, $V \to \infty$
- $VT^3 = (N_\sigma/N_\tau)^3 < \infty \quad \Rightarrow$ momentum cut-off: $p/T > 2\pi N_\tau/N_\sigma$

$\Rightarrow$ need large lattices to analyze infra-red regime

$\Rightarrow$ in future also thermal photon rates

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Disclaimer: The entire charmonium discussion was based on lattice calculations in quenched QCD! We need a much larger computer to do better!!