Future Prospects in QCD at High Energy
July 17-22 at BNL

Recent results from lattice QCD on nucleon structure

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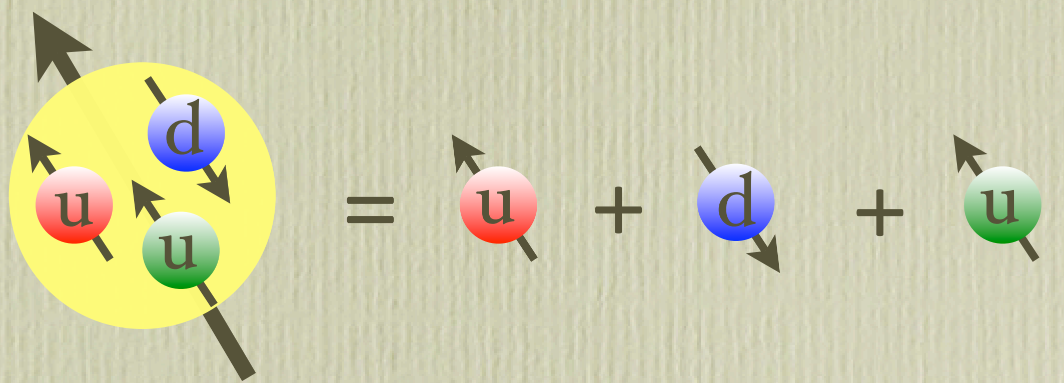
(RBRC / Univ. of Tokyo)

Outline

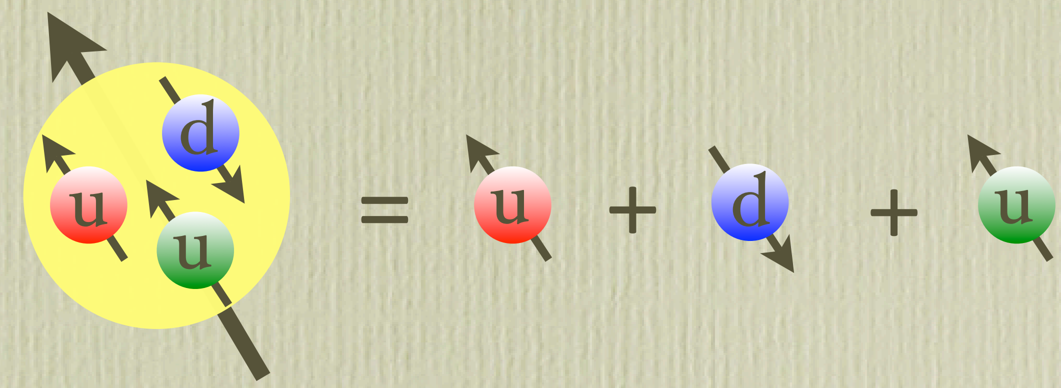
- Introduction:
- Review of lattice results for proton spin
  - quark spin fraction $\Delta\Sigma$ (Δu, Δd, Δs)
  - quark orbital angular momentum L_q
- Results from RBC (Riken-Bnl-Columbia) collab.
  - Old quench DWF results on iso-vector observables
  - New projects (hyeron beta decay)
  - Preliminary 2+1 flavors DWF results
- Summary

Proton spin



$$\frac{1}{2} \approx \frac{1}{2}\Delta\Sigma$$ (sum of quark spins)

Proton spin crisis?

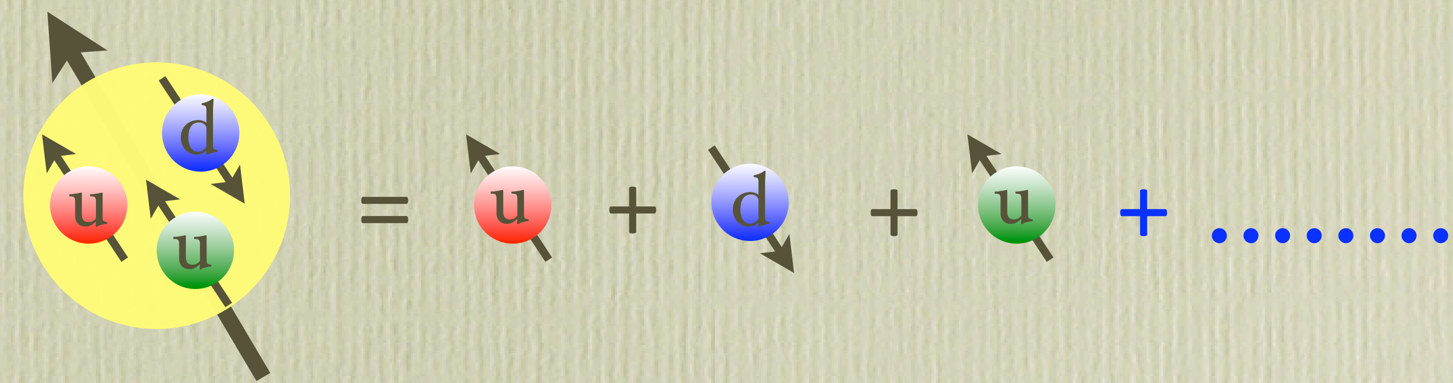


$$\frac{1}{2} \not\approx \frac{1}{2}\Delta\Sigma$$ (sum of quark spins)

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}(0.1 - 0.3) \ll \frac{1}{2}$$ **EMC (1988)**

$$\begin{aligned}\Delta\Sigma &= \Delta u(0.802) + \Delta d(-0.456) + \boxed{\Delta s(-0.124)} \\ &= 0.213 \pm 0.138\end{aligned}$$

Proton spin puzzle.....



$$\frac{1}{2} = (\frac{1}{2}\boxed{\Delta\Sigma} + L_q) + (\Delta g + L_g)$$

$$= \boxed{J_q(\mathrm{quark})} + J_g(\mathrm{gluon})$$

Lattice QCD

decomposition in a gauge invariant way

X. Ji, PRL 78 (97) 610

Methodology

- quark spin fraction $\Delta\Sigma$

$$\langle p, s|\bar{q}\gamma_5\gamma_\mu q|p, s\rangle = s_\mu \Delta q$$

- quark total angular momentum $T^q_{\mu\nu} = \frac{1}{2}\bar{q}\gamma_{(\mu}[\overrightarrow{D} - \overleftarrow{D}]_{\nu)}q$

$$\langle p', s'|T^q_{\mu\nu}|p, s\rangle = \bar{u}(p', s')\Gamma^q_{\mu\nu}u(p, s)$$

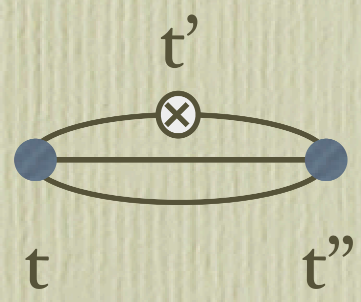
Generalized form factors

$$\Gamma^q_{\mu\nu} = A_q(Q^2)\gamma_{(\mu}\bar{P}_{\nu)} + B_q(Q^2)\frac{i\bar{P}_{(\mu}\sigma_{\nu)\alpha}Q^\alpha}{2M} + \ldots\ldots.$$

$$J_q = \frac{1}{2}[A_q(0) + B_q(0)] = \frac{1}{2}[\langle x\rangle_q + B_q(0)]$$ Ji's sum rule

Calculation of Matrix Elements (I)

$$\langle \psi_N(t) O(t') \overline{\psi}_N(t'') \rangle = \sum_{n,m} e^{-E_n(t-t')} \langle \psi_N | n \rangle \langle n | O | m \rangle \langle m | \overline{\psi}_N \rangle e^{-E_m(t'-t'')}$$



$$\rightarrow \langle \psi_N | N \rangle \langle N | O | N \rangle \langle N | \overline{\psi}_N \rangle e^{-E_N(t-t'')}$$

$t \gg t'$
and
$t' \gg t''$

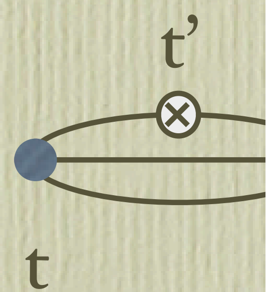
no dependence of t'

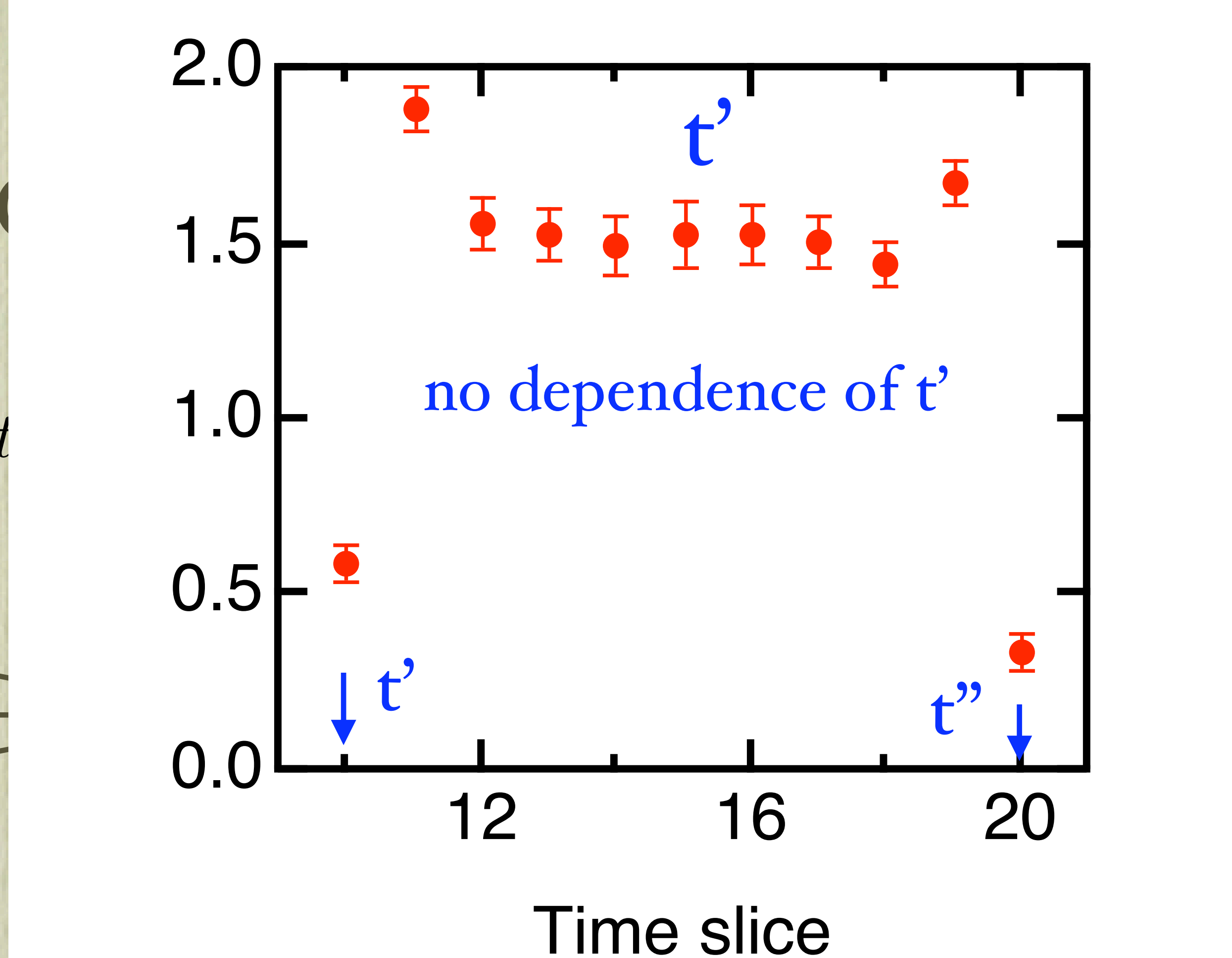
- Matrix element can be extracted from the following ratio

$$\frac{\langle \psi_N(t) O(t') \overline{\psi}_N(t'') \rangle}{\langle \psi_N(t) \overline{\psi}_N(t'') \rangle} \rightarrow \langle N | O | N \rangle$$

... (I)

$\langle\psi_N(t)O(t$ ... $|\overline{\psi}_N\rangle e^{-E_m(t'-t'')}$





... $_N(t-t'')$

ence of t'

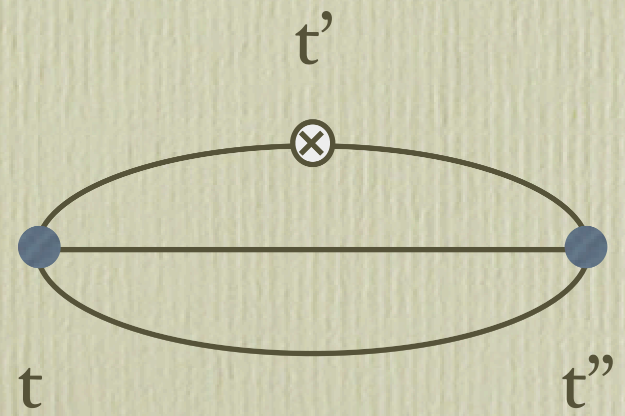
- Matrix element can be extracted from the following ratio

$$\frac{\langle\psi_N(t)O(t')\overline{\psi}_N(t'')\rangle}{\langle\psi_N(t)\overline{\psi}_N(t'')\rangle} \to \langle N|O|N\rangle$$

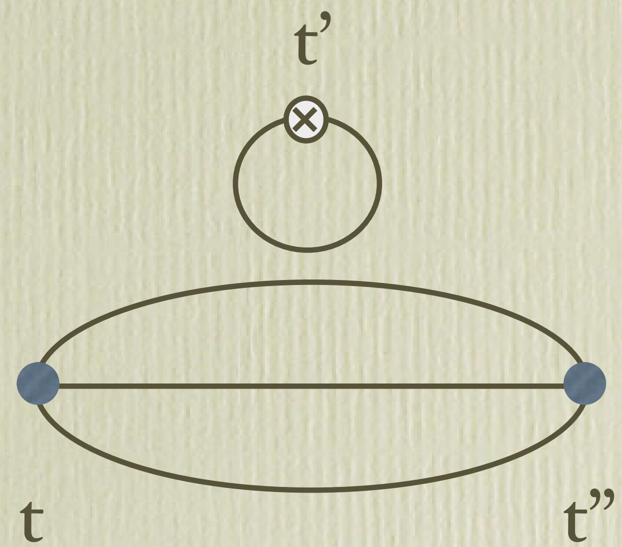
Calculation of Matrix Elements (II)

$$\langle \psi_N(t) O(t') \overline{\psi}_N(t'') \rangle$$

may possess two types of the Wick contraction



connected contribution



disconnected contribution

(~~$L^3 \times T$~~) times expensive calculation

T

typical orders of **L, T** are **O(10)**

Quench approximation

$$\langle O(U,\psi)\rangle = \frac{1}{Z}\int D\bar{\psi}D\psi DU\; O(U,\psi)e^{-S_G(U)-\bar{\psi}M(U)\psi}$$

$$= \frac{1}{Z}\int DU\; O(U,M^{-1}(U))(\cancel{\det\{M(U)\}})^{N_f}e^{-S_G(U)}$$

$$= \frac{1}{Z}\int DU\; O(U,M^{-1}(U))e^{-S_G(U)+\cancel{N_f \,\mathrm{TrLn} M(U)}}$$

$$\det\{M(U)\} = 1 \Longleftrightarrow N_f = 0$$

quark spin fraction $\Delta\Sigma$

	Kentucky	KEK-Tsukuba	SESAM
N_f	0	0	2
La	1.5 fm	2.2 fm	1.5 fm
Statistics	24	260	200
M_π(GeV)	0.75 -1.36	0.51 - 0.96	0.64 - 1.02
Renormalization	one-loop PT	one-loop PT	one-loop PT
Δu	0.79(11)	0.638(54)	0.62(7)
Δd	-0.42(11)	-0.347(46)	-0.29(6)
Δs	-0.12(10)	-0.109(30)	-0.12(7)
$\Delta\Sigma$	0.25(12)	0.18(10)	0.20(12)
$g_A=\Delta u-\Delta d$	1.20(10)	0.985(25)	0.907(20)

quark spin fraction $\Delta\Sigma$ (cont'd)

From old calculations (1995,1999)

- ✓ $\Delta\Sigma$(Lattice)~ + 0.20 (exp. ~ 0.20 ± 0.10)
- ✓ Δs(Lattice)~ - 0.10 (exp. ~ - 0.12 ± 0.05)

Surprisingly agrees with experimental values, but .....

- ✓ All calculations are performed
  - at single lattice spacing with Wilson fermions
  - in the heavy quark region (M_π > 0.5-0.6 GeV)
  - on the small physical volume

- Furthermore, g_A(Lattice)=Δu - Δd ~ 1.0 (exp. 1.2670)

quark angular momentum

	Kentucky	QCDSF	LHPC-SESAM
N_f	0	0	2
La	1.5 fm	1.5 fm	1.5 fm
Statistics	100	O(100)	200
M_π(GeV)	0.75 -1.36	0.64 - 1.07	0.90
Renormalization	one-loop PT	NPT	one-loop PT
at the scale	μ = 2 GeV	μ = 2 GeV	μ = 2 GeV
$J_u + J_d + J_s$	0.30(7)	–	–
$J_u + J_d$ (con)	0.44(7)	0.33(7)	0.34(4)
$L_u + L_d + L_s$	0.17(6)	–	–
$L_u + L_d$ (con)	0.13(7)	0.03(7)	0.01(4)

quark angular momentum (cont'd)

iso-scalar

Orbital angular momentum: $L_q = J_q - \frac{1}{2}\Delta\Sigma$

- Kentucky group, PRD 62 (2000) 114504

$$L_{u+d+s} = 0.17(6)$$

  - ➡ Full calculation (quench), but poor statistics for ΔΣ

- QCDSF and LHPC-SESAM

QCDSF: PRL 92 (04) 042002

LHPC-SESAM: PRL 93 (04) 112001

$$J_{u+d} \simeq \frac{1}{2}\langle x\rangle_{u+d} \approx \frac{1}{2}\Delta\Sigma(\mathrm{con}) \Longrightarrow L_{u+d} \approx 0$$

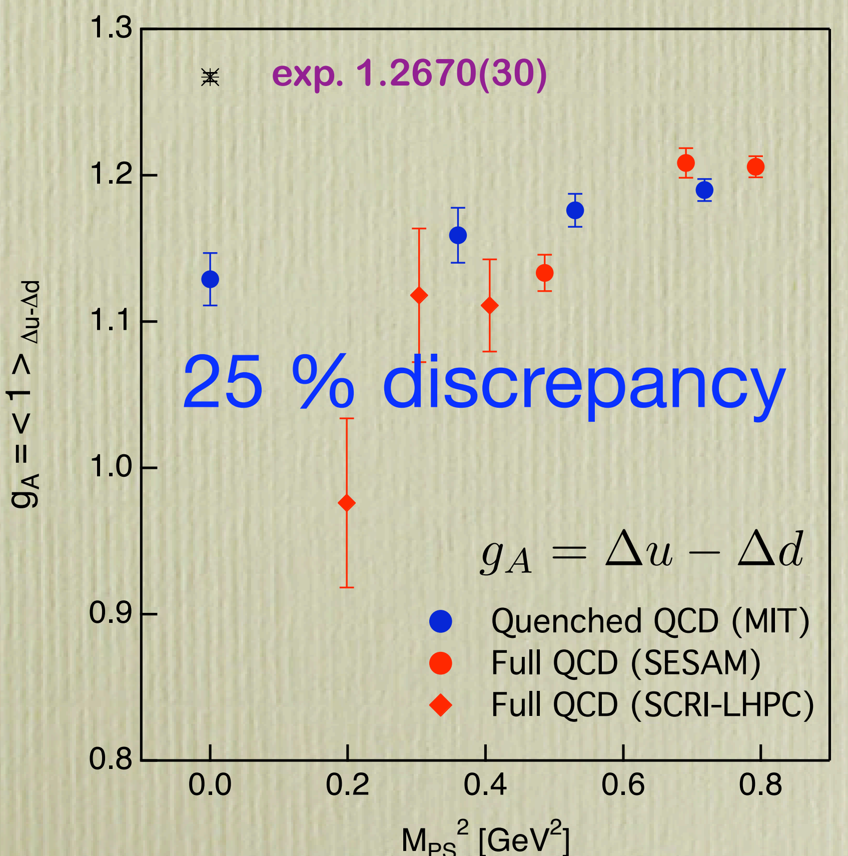
  - ➡ Not including disconnected contributions

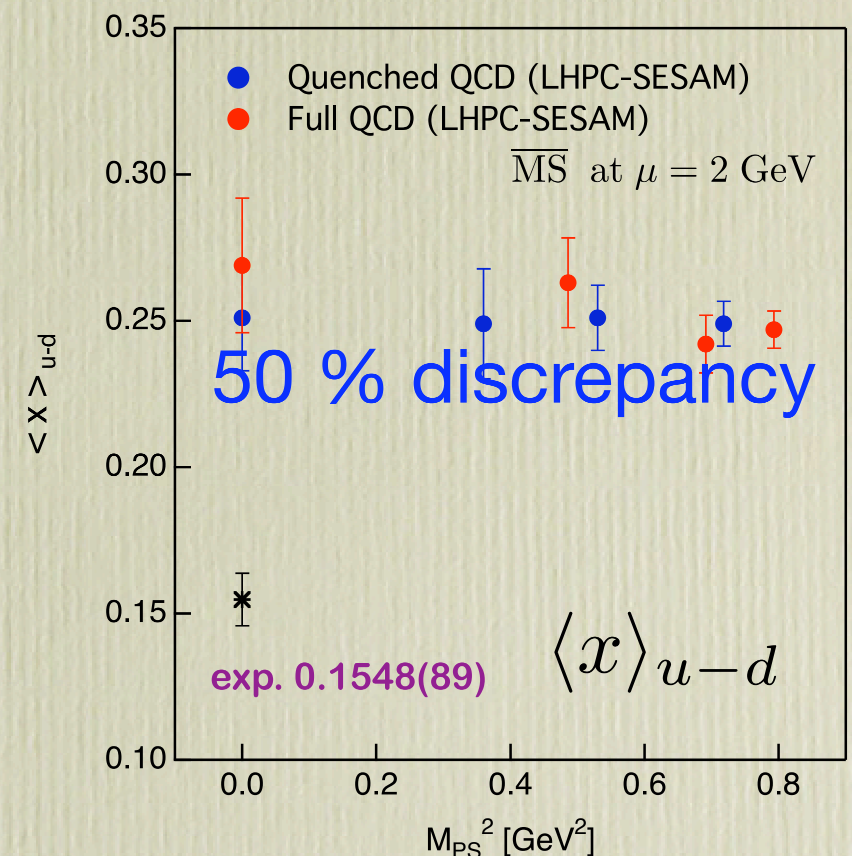
iso-vector

Unresolved puzzles

Before drawing definite conclusion from lattice QCD

➡We have to resolve serious discrepancies in $g_A, \langle x \rangle_{u-d}$





V=16^{3}x32, β=6.0 (quenched), β=5.6 (full) **LHPC-SESAM collaboration, Phys. Rev. D66 (02) 034506**

Messages from this talk

- Don't take too much seriously lattice numbers of $\Delta\Sigma$ and L_q at this moment for various reasons:
  - isovector quantities are not well reproduced
  - disconnected contributions are missing in some case

Activities related to Nucleon Structure in RIKEN-BNL-Columbia collaboration

- Nucleon excited states
  - S. Sasaki, T. Blum, S. Ohta, Phys. Rev. D65 (2002) 074503.
- Nucleon axial charge
  - S. Sasaki, K. Orginos, S. Ohta, T. Blum, Phys. Rev. D68 (2003) 054509.
- Nucleon structure functions
  - K. Orginos, T. Blum, S. Ohta, Phys. Rev. D73 (2006) 094503.
- Neutron electric dipole moment
  - F. Berruto, T. Blum, K. Orginos, A. Soni, Phys. Rev. D73 (2006) 054509.

Activities related to Nucleon Structure in RIKEN-BNL-Columbia collaboration

Domain wall fermions

- Nucleon excited states
  - S. Sasaki, T. Blum, S. Ohta, Phys. Rev. D65 (2002) 074503.
- Nucleon axial charge g_A iso-vector
  - S. Sasaki, K. Orginos, S. Ohta, T. Blum, Phys. Rev. D68 (2003) 054509.
- Nucleon structure functions $\langle x \rangle_{u-d}$ iso-vector
  - K. Orginos, T. Blum, S. Ohta, Phys. Rev. D73 (2006) 094503.
- Neutron electric dipole moment
  - F. Berruto, T. Blum, K. Orginos, A. Soni, Phys. Rev. D73 (2006) 054509.

Domain Wall Fermions

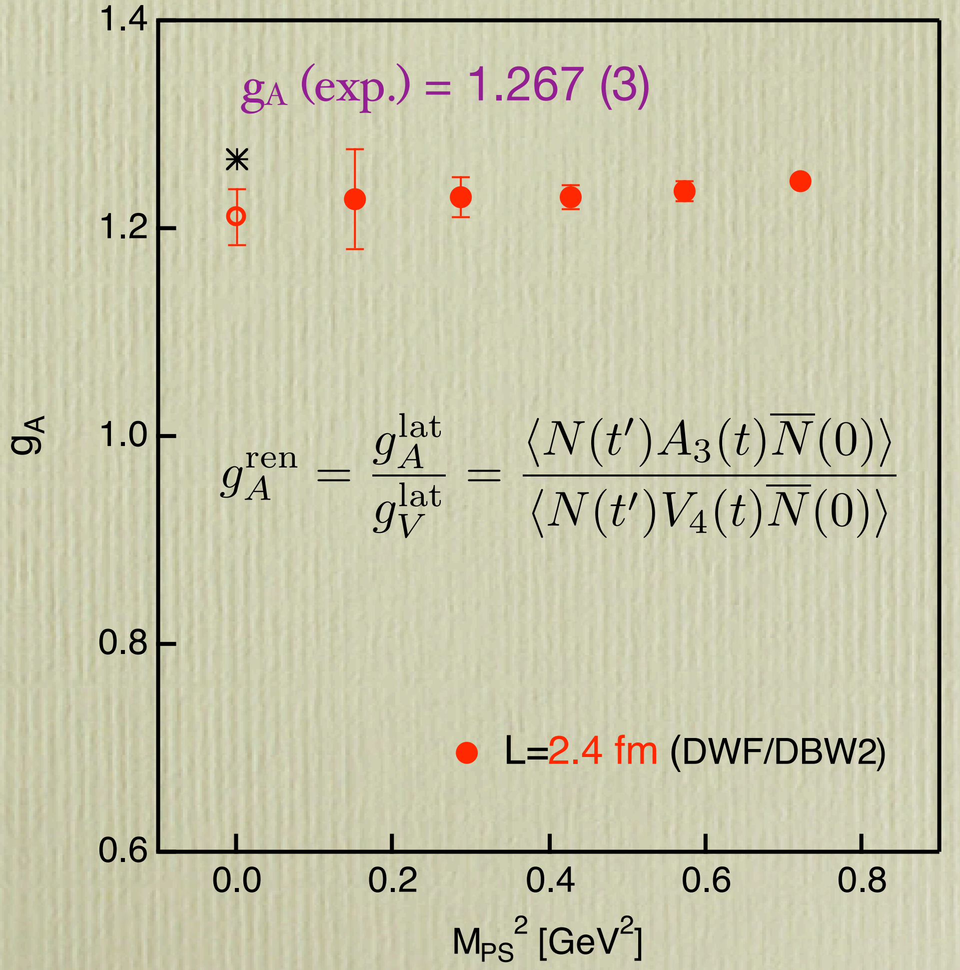
- New discretization scheme to preserve chiral symmery on the lattice
  - 5-dim extenstion of the Wilson fermion (L_s: the 5-D extent)
  - $L_s \to \infty$: Exact chiral symmetry even at finite lattice spacing
  - Finite L_s : Residual chiral breaking (m_{res} ~ 1 MeV for L_s ~O(10))
  - Leading discretization errors are expected to be O(a) exp(-L_s)
  - Simpler renormalization thanks to exact chiral symmetry
    - ✓ can eliminate mixing of lattice operators

Simulations (quench)

- Domain wall fermion + DBW2 gauge action
- Lattice cutoff: 1/a ~ 1.3 GeV ($\beta = 0.87$, $c_1 = -1.4069$)
- Small residual quark mass: 0.7 MeV ($L_5 = 16$, $M_5 = 1.8$)
- Lattice size: $V = 16^3 \times 32$, La ~ 2.4 fm
- Lightest pion mass: 390 MeV
- # of statistics: 416
- Non-perturbative renormalization

iso-vector

Quenched DWF calculation of g_A



Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

- ✓ the lightest pion mass, $M_\pi \sim 0.39$ GeV
- ✓ relatively large volume, $V \sim (2.4\ \text{fm})^3$
- ✓ large statistics, 416 configs
- ✳ mild quark mass dependence

➡ Linear extrapolation yields

g_A = 1.212 (27)

iso-vector

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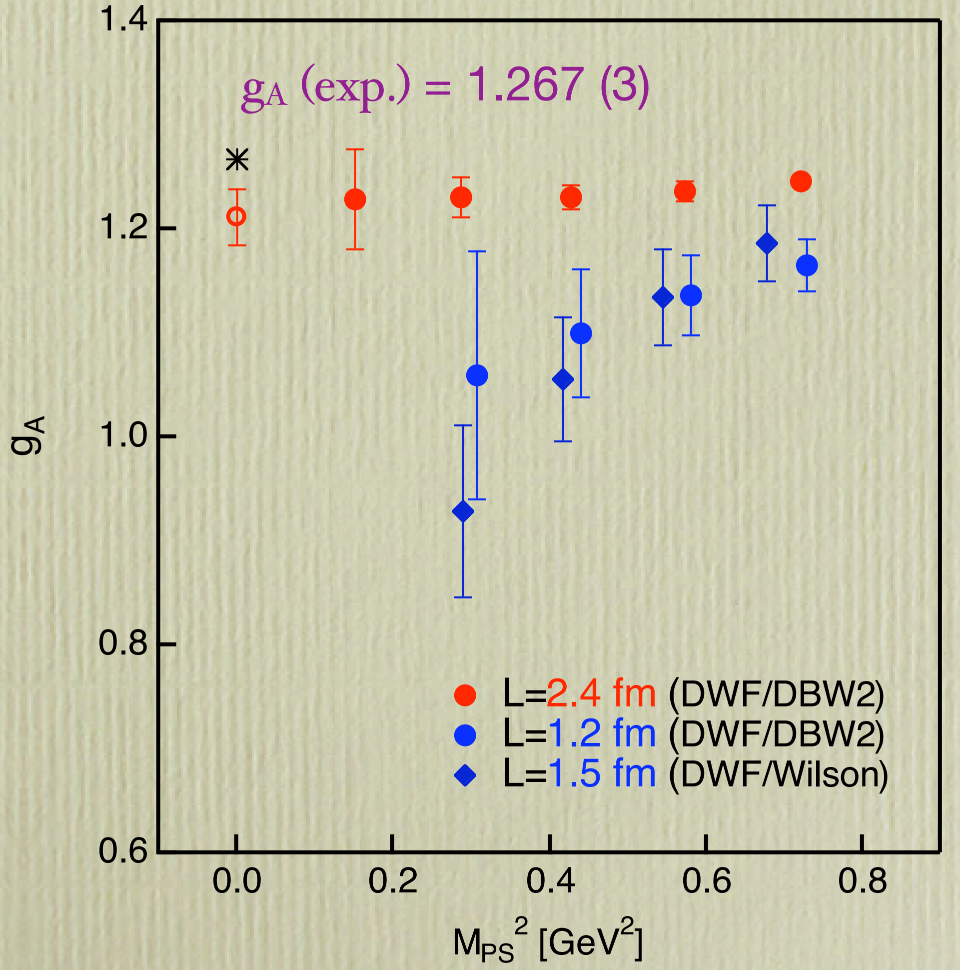

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✓ relatively large volume, $V \sim (2.4\ \text{fm})^3$

✓ large statistics, 416 configs

✳ mild quark mass dependence

✳ clear finite volume dependence

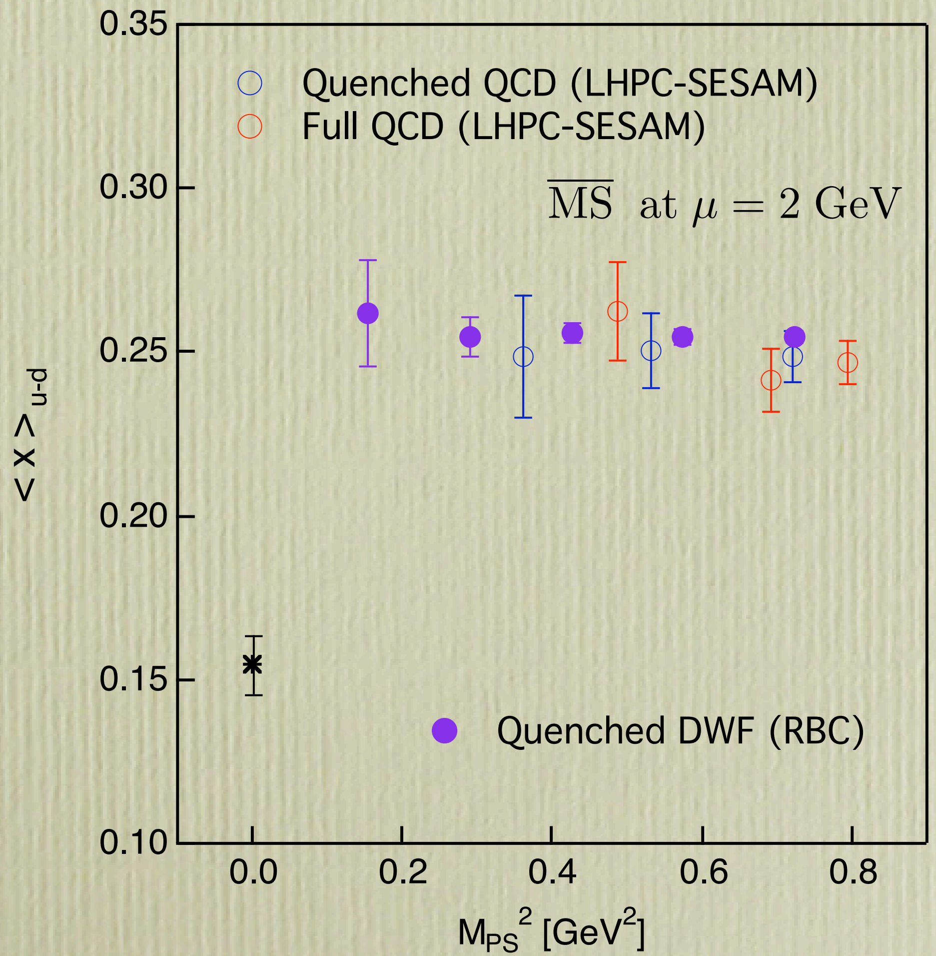
- a 20 % increase from 1.2 fm to 2.4 fm

resolve the long-standing problem!

iso-vector

Quenched DWF calculation of $\langle x \rangle_{u\text{-}d}$

K. Orginos, T. Blum, S. Ohta, hep-lat/050524.



very mild dependence of quark mass

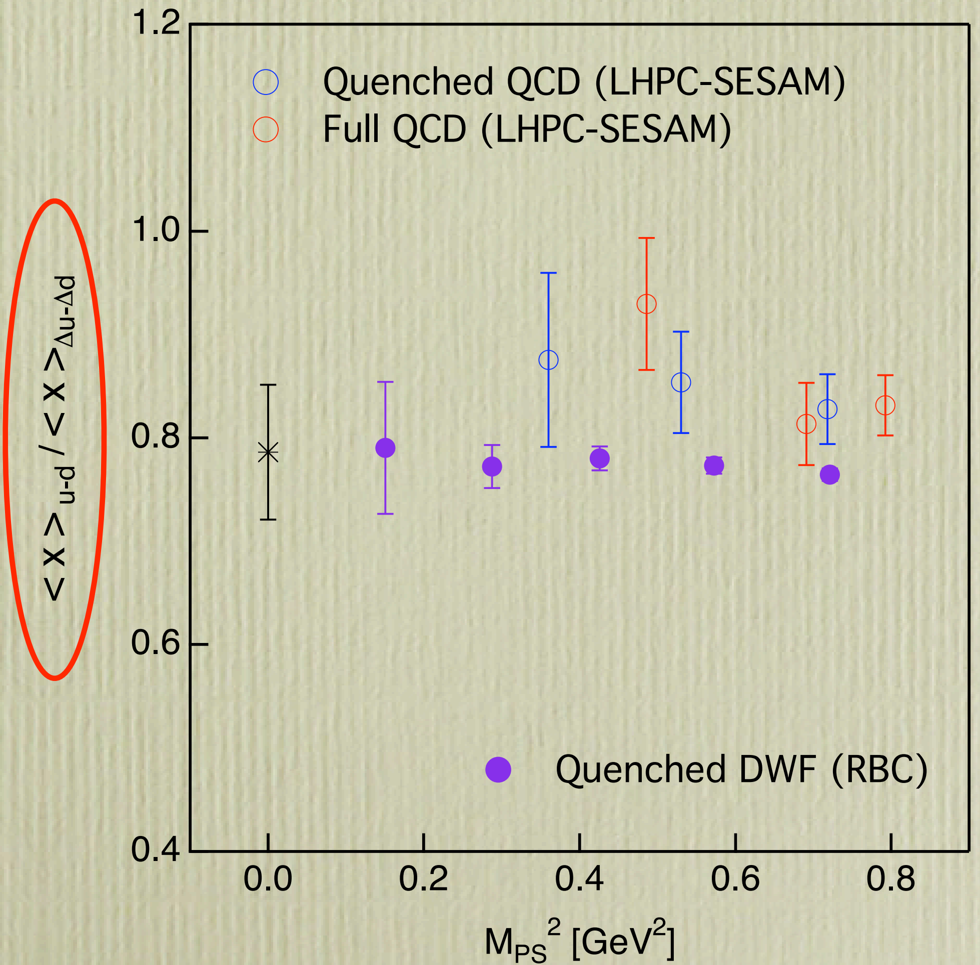
- sophisticated gauge/fermion action
  - ✓ No O(a) error
- large physical volume
  - ✓ La ~ 2.4 fm
- lighter quark mass region
  - ✓ M_π ~ 390 MeV
- non-perturbative renormalization
- high statistics

But, the puzzle still remains.

iso-vector

Quenched DWF calculation (cont'd)

K. Orginos, T. Blum, S. Ohta, hep-lat/050524.



very mild dependence of quark mass

Ratio $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$

✓ Renormalization invariant quantity

✤ Chiral symmetry is responsible for

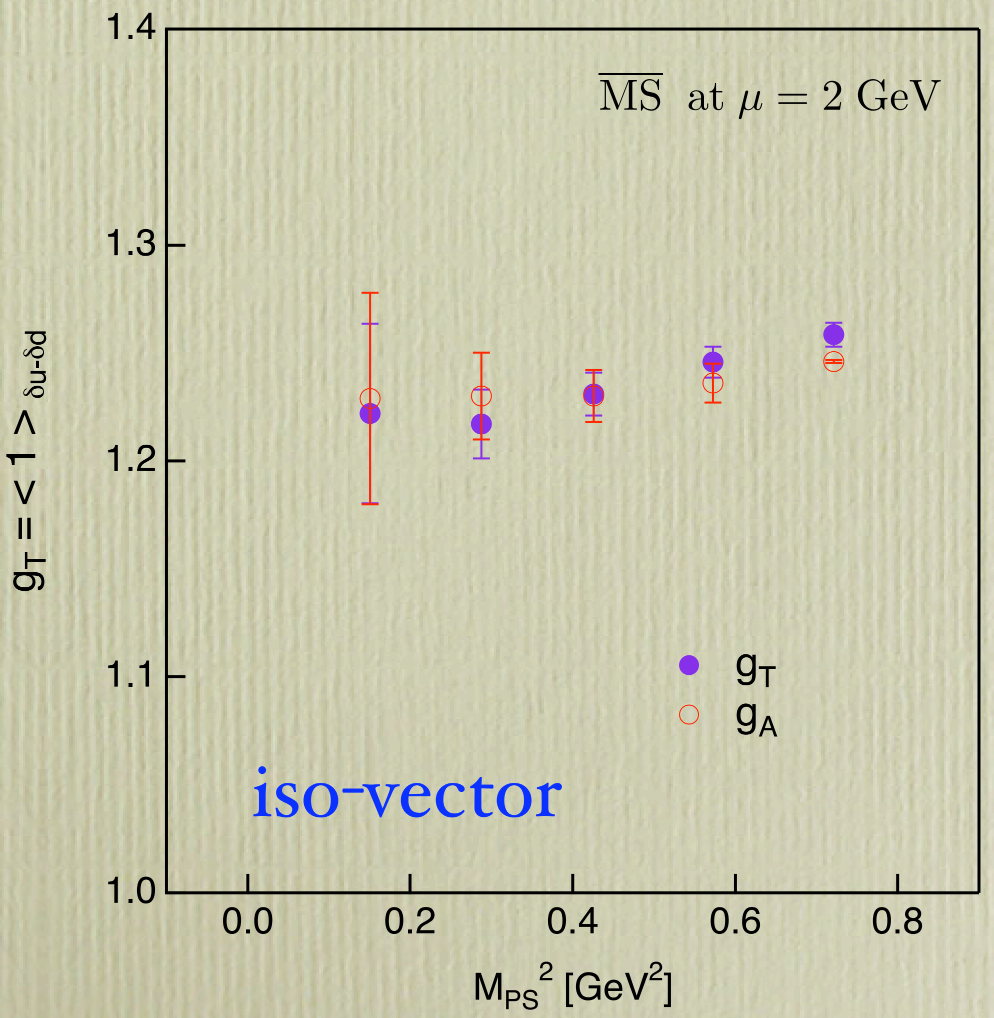
$$Z_{\langle x \rangle_q}(\mu) = Z_{\langle x \rangle_{\Delta q}}(\mu)$$

- DWF calculation has an advantage in this point

iso-vector

Quenched DWF calculation of g_T

K. Orginos, T. Blum, S. Ohta, hep-lat/050524.



- Iso-vector tensor charge

$$g_T = \delta u - \delta d = 1.193(30)$$

at μ= 2 GeV in the $\overline{\mathrm{MS}}$ scheme

Hyperon beta decay (New Project)

S. Sasaki and T. Yamazaki

- the baryonic-version of semileptonic decay
  - *Alternative way to determine $|V_{us}|$ other than K_{l3} decays*
- the SU(3)-extension of neutron beta decay
  - *Vital input to analysis of strange quark spin fraction*

$$\Delta\Sigma (= \Delta u + \Delta d + \Delta s)_{\mathrm{Expt.}} = 0.213 \pm 0.138$$

$$\left.\begin{array}{l} (g_A/g_V)_{np} = \Delta u - \Delta d \\ (g_A/g_V)_{\Lambda p} = (2\Delta u - \Delta d - \Delta s)/3 \\ (g_A/g_V)_{\Xi\Sigma} = (\Delta u + \Delta d - 2\Delta s)/3 \\ (g_A/g_V)_{\Sigma n} = \Delta d - \Delta s \end{array}\right\} \text{Assumption : SU(3) symmetry}$$

Hyperon beta decay (New Project)

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$$\Delta s = -0.124 \pm 0.046$$

Hyperon beta decay (New Project)

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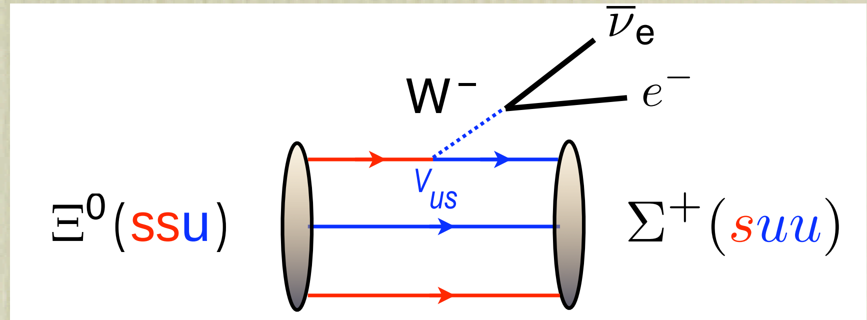
$$\Delta s = -0.124 \pm 0.046$$

  - *The hidden uncertainty of Δs coming from unknown SU(3) breaking in hyperon beta decays.*

Hyperon Beta Decay ($\Xi^0 \to \Sigma^+$)

	$B' \to Bl\nu$	$f_1^{SU(3)}$	g_1/f_1 (Exp.)	$(g_1/f_1)^{SU(3)}$
✔	$n \to p$	1	1.2670± 0.0030	$F+D$
	$\Lambda \to p$	$-\frac{\sqrt{6}}{2}$	0.718± 0.015	$F+\frac{1}{3}D$
	$\Xi^- \to \Lambda$	$\frac{\sqrt{6}}{2}$	0.25 ± 0.05	$F-\frac{1}{3}D$
	$\Sigma^- \to n$	-1	-0.340 ± 0.017	$F-D$
✔	$\Xi^0 \to \Sigma^+$	1	1.32± 0.21	$F+D$
	$\Xi^- \to \Xi^0$	-1	N/A	$F-D$



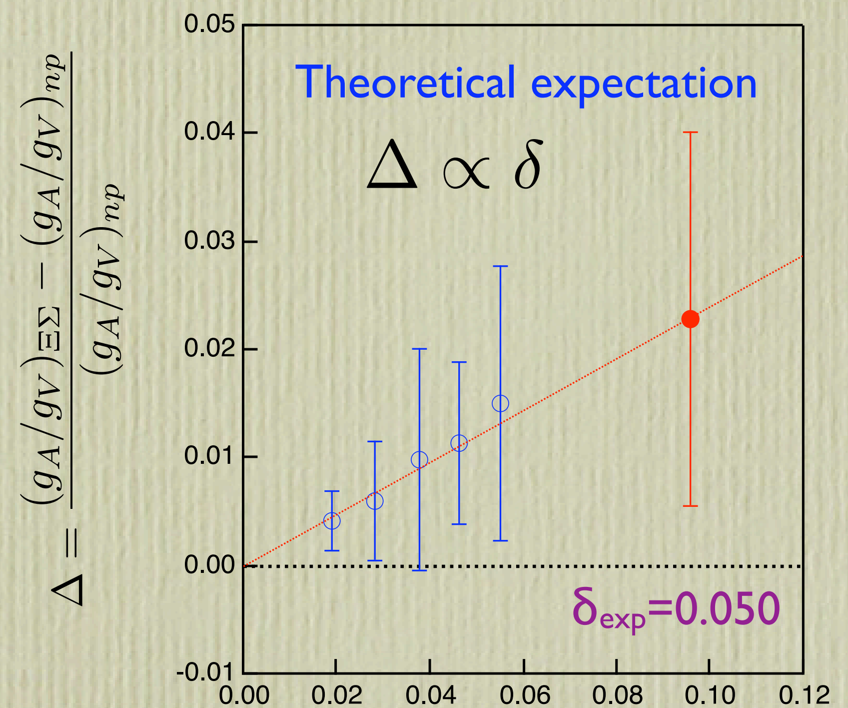
✴ $\Xi^0 \to \Sigma^+$ is the direct analogue of n→p under d ↔ s

✓ highly sensitive to SU(3) breaking

✴ So far, a single experiment and no lattice result

- DWF-DBW2 (Quench) at beta=0.87 (a^{-1}=1.3GeV)
  - 16^3x32x16 (L=2.4 fm): 377 statistics (QCDOC)
  - m_l=0.04, 0.05, 0.06 (M_π=0.53, 0.60, 0.65 GeV)
  - fixed "strange" quark masses at m_s=0.08 (0.10)

SU(3) breaking effect on g_A/g_V



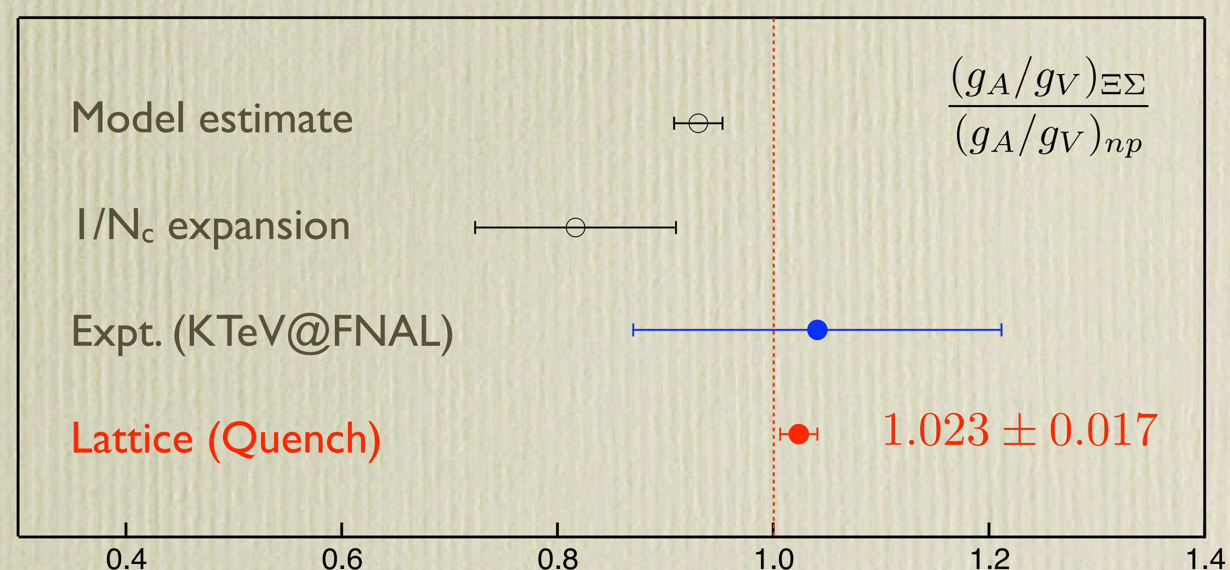
$$\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi + M_\Sigma}$$

$$\frac{\langle \Sigma(t') A_3(t) \overline{\Xi}(0)\rangle}{\langle \Sigma(t') V_4(t) \overline{\Xi}(0)\rangle} \frac{\langle N(t') V_4(t) \overline{N}(0)\rangle}{\langle N(t') A_3(t) \overline{N}(0)\rangle} = \frac{g_1(q^2_{\rm max})}{f_1(q^2_{\rm max}) - \delta f_3(q^2_{\rm max})} \left(\frac{f_1(0)}{g_1(0)}\right)_{\rm SU(3)}$$

$$= \frac{g_1(0)/f_1(0)}{(g_1(0)/f_1(0))_{\rm SU(3)}} + \mathcal{O}(\delta^2)$$

$$q^2_{\rm max} = -(M_\Xi - M_\Sigma)^2$$

Tiny symmetry breaking (~2%)!



- This doesn't conflict with Cabibbo-model fits for HBD
- However, this doesn't mean that an estimation of Δs is reliable.

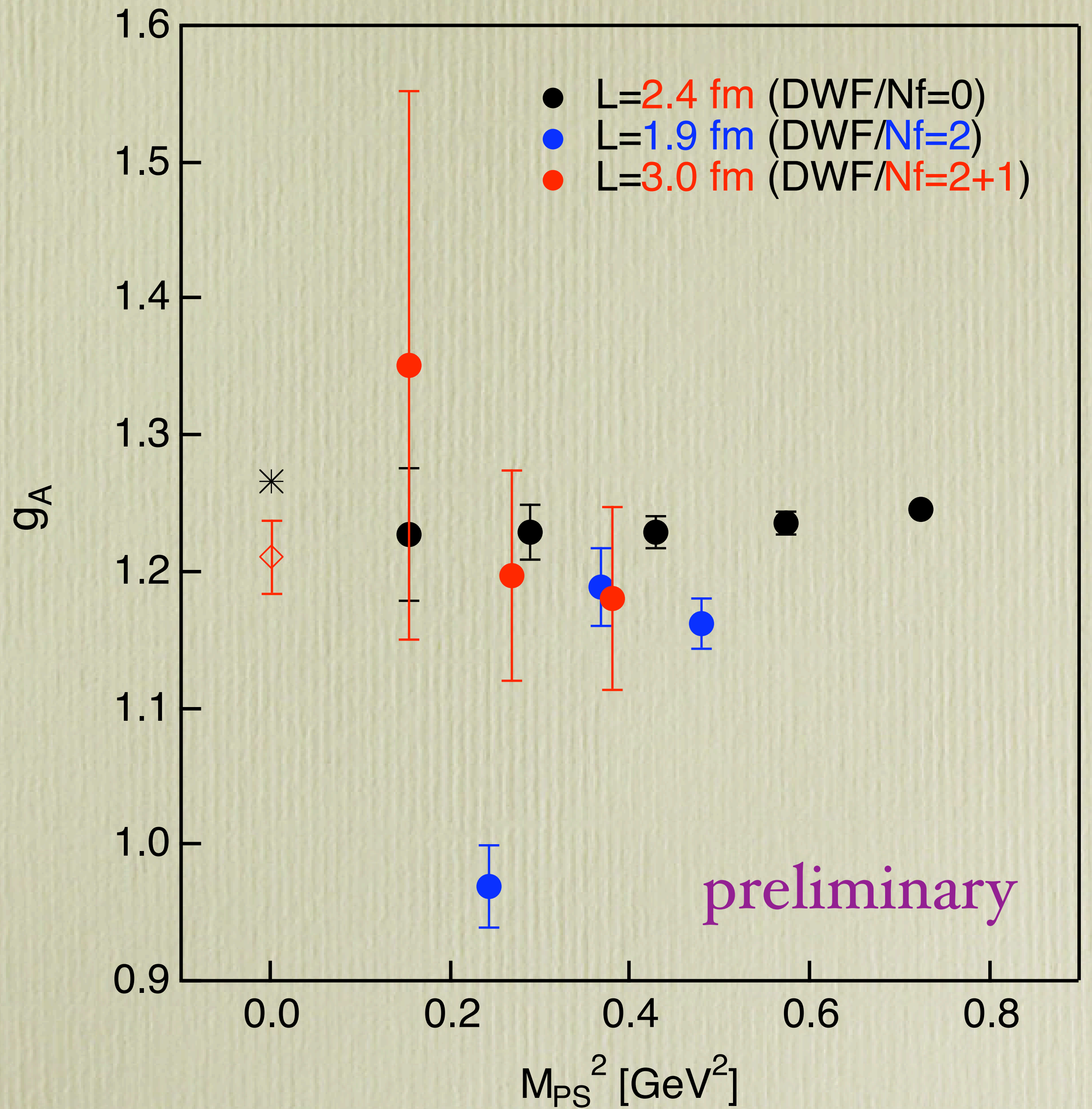
2+1 flavors DWF QCD on QCDOC

RIKEN BNL Research Center Dedicates
New Supercomputer (QCDOC) for Physics Research

- DWF + Iwasaki gauge action
- Lattice cutoff: $1/a \sim 1.6$ GeV
  ($\beta = 2.13$, $c_1 = -0.331$)
- Lattice size: $V = 24^3 \times 64 \times 16$
  $La \sim 3.0$ fm
- $m_{light} = 1/4, 1/2, 3/4$ of $m_{strange}$
  $M_\pi \sim 350, 500, 750$ MeV

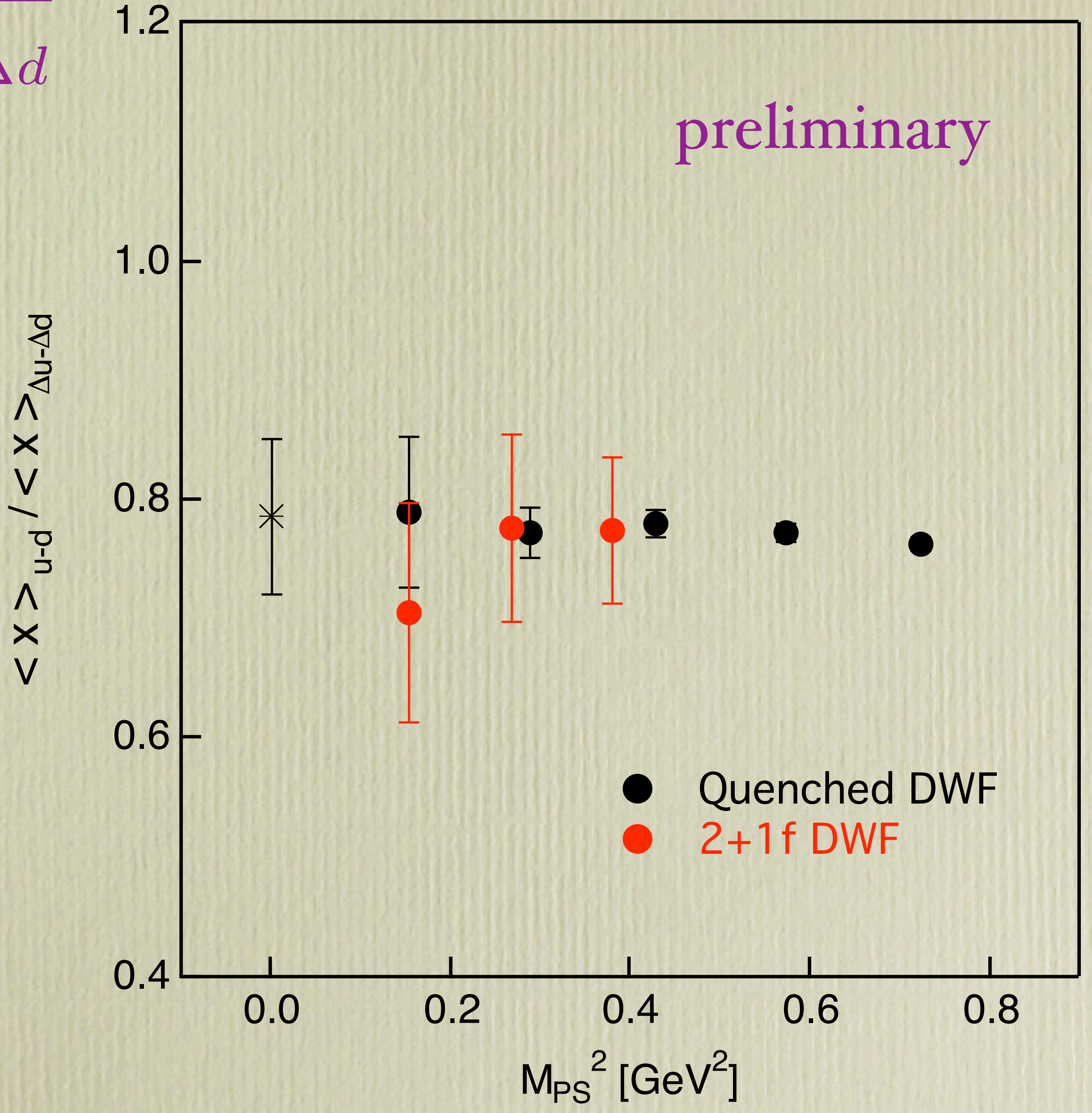
in collaboration with Columbia, UKQCD

g_A



Blum, Lin, Ohta, Orginos, Sasaki, Yamazaki

$$\frac{\langle x \rangle_{u-d}}{\langle x \rangle_{\Delta u - \Delta d}}$$



Summary

- The computation of nucleon properties in lattice QCD is now progressing with steadily increasing accuracy by using Dowain Wall Fermions
- Spin physics program in RBC collaboration will run on QCDOC to achieve high precision calculations using 2+1 flavors DWF QCD (underway)