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Mechanisms for Single-Spin Asymmetries in SIDIS

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Based on works done with Sterman, and Ji, Vogelsang, and Yuan

Outline

- Single spin asymmetry – definition
- Single spin asymmetry within the collinear factorization – high twist matrix elements
- K_T- factorization – Sivers and Collins effects
- Connection between high twist matrix elements and Sivers and Collins functions
- Single spin asymmetry in SIDIS
- Summary and outlook

Single Spin Asymmetry – definition

- Spin-avg X-section: $\sigma(\ell) = \frac{1}{2}[\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})]$
- Spin-dep X-section: $\Delta\sigma(\ell,\vec{s}) = \frac{1}{2}[\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})]$
- Single spin asymmetry:

$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

- single longitudinal spin asymmetry: A_L

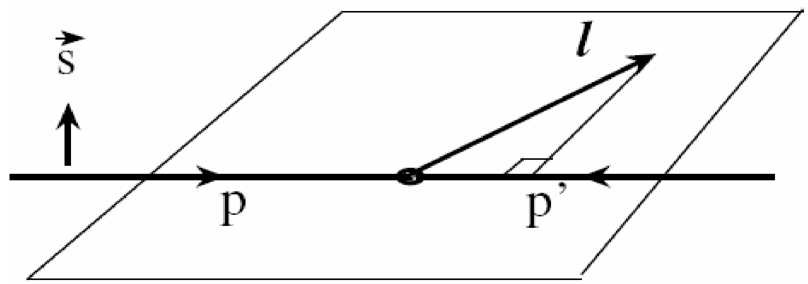
  particle spin $\vec{s}$ is parallel to its momentum $\vec{p}$

- single transverse spin asymmetry: A_N

  particle spin $\vec{s}$ is perpendicular to its momentum $\vec{p}$

Single spin asymmetry corresponds to a T-odd triple product

$$A_N \propto i\, \vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$

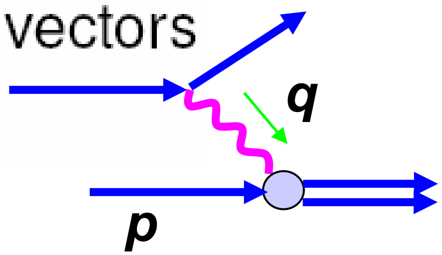


- the phase "i" is required by time-reversal invariance
- covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$

Nonvanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

- Inclusive DIS does not have enough vectors

  Note: q and p can only fix a line



$A_N = 0$ for inclusive DIS

- **DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$
- **Leptionic tensor is symmetric:** $L^{\mu\nu} = L^{\nu\mu}$
- **Hadronic tensor:** $W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$
- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right]$$

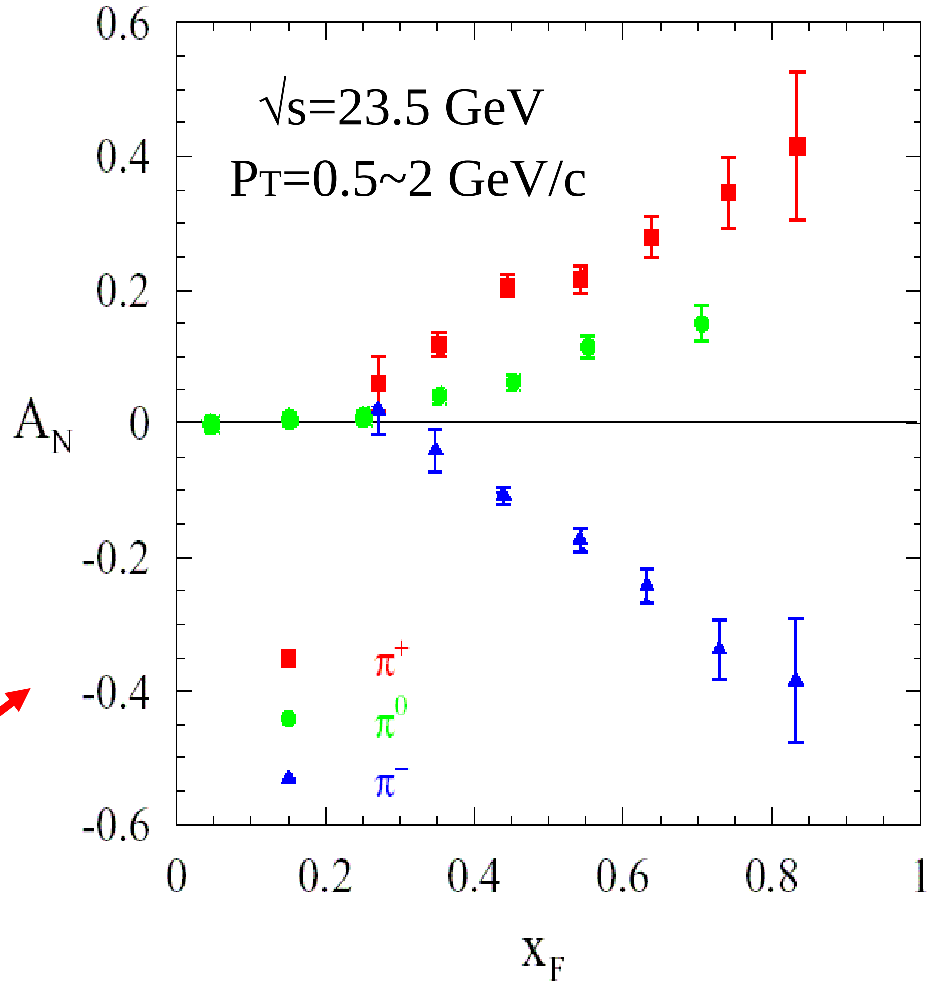
- **P and T invariance:**

$$A_N = 0 \quad \Leftrightarrow \quad \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle = \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle$$

Large A_N observed in hadronic collisions

- process: only one hadron is transversely polarized:

- Large asymmetries A_N observed in hadron collisions:
  - decay of Λ
  - production of π's



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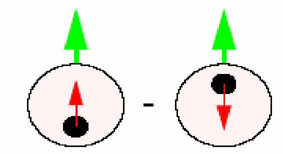
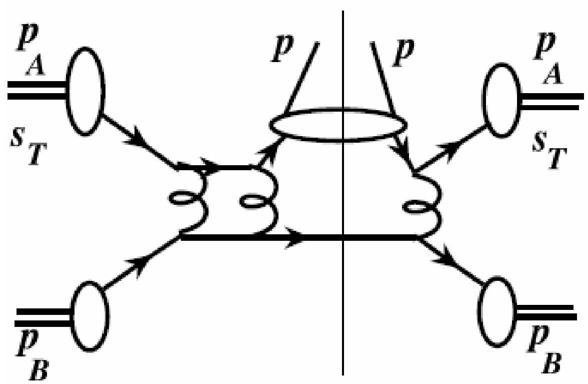
Single transverse spin asymmetry – A_N in the parton model

- transverse spin information at leading twist – transversity:

$\delta q(x) =$  = Chiral-odd helicity-flip density

- the operator for δq has even γ's ⟹ quark mass term
- the phase requires an imaginary part ⟹ loop diagram



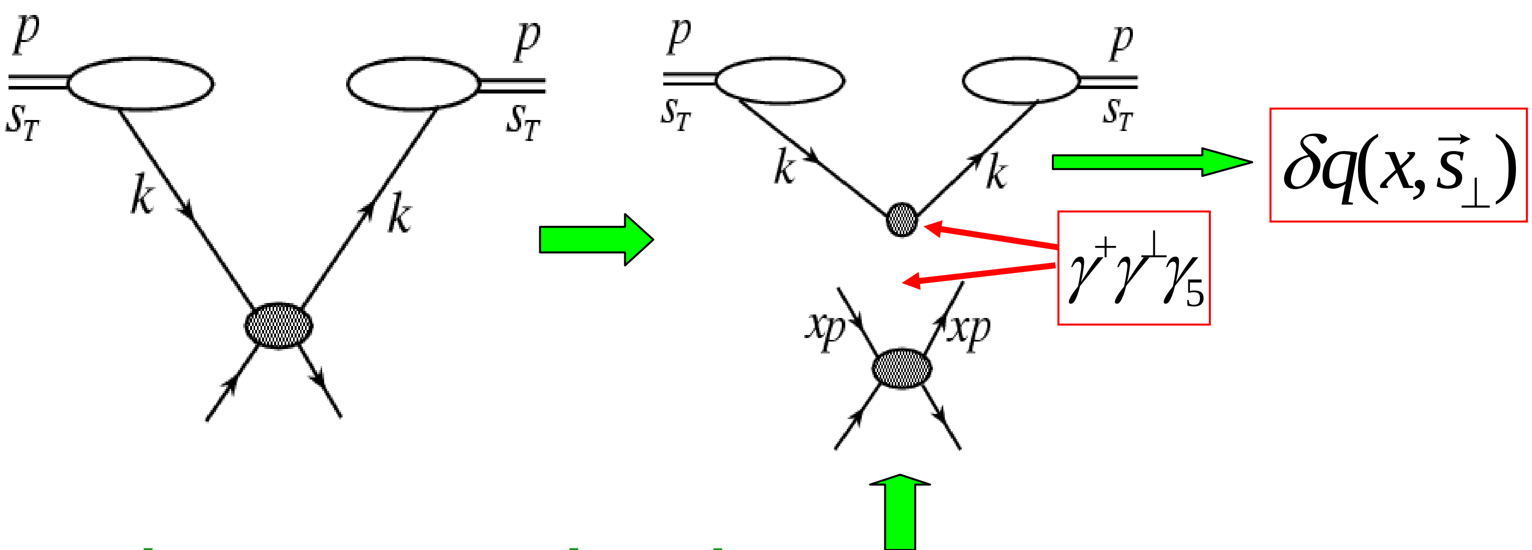
$\sim m_q \longrightarrow 0$

Asymmetry is expected to be small

- **Puzzle: the size of the observed single-spin asymmetries**

Single transverse spin asymmetry – A_N in collinear factorization approach

❖ **Leading twist PDF with transverse hadron spin:**



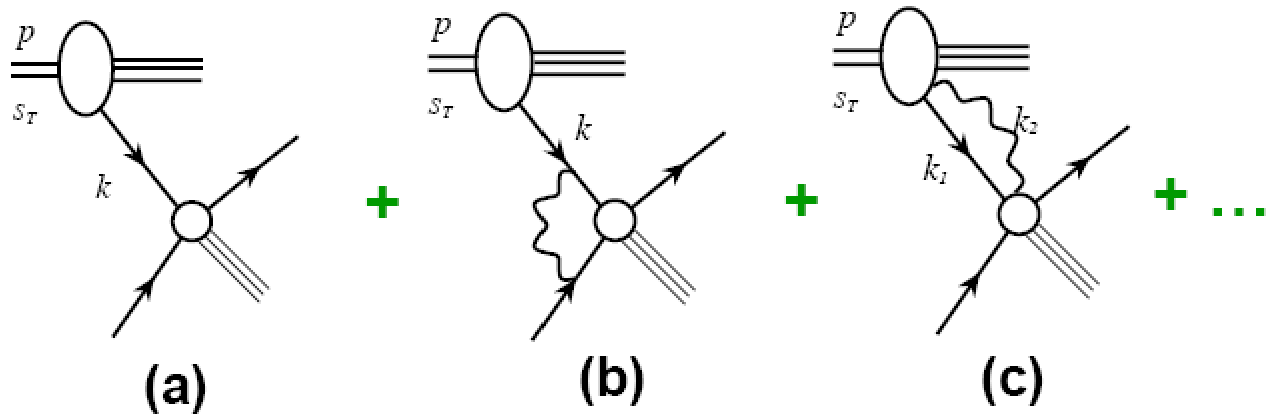
❑ **need an even number γ's:**

$\Gamma_i = I, \quad i\gamma_5$ ⟹ **Twist-3 matrix elements**

$\sigma^{\mu\nu}\,(i\gamma_5)$ ⟹ **An extra transverse index**

- **extra gluon (its polarization)**
- **extra vector direction**

High twist contribution to A_N

$\sigma(s_T)$ ~ $\Big|$  $\Big|^2$

- Leading spin dependent part of the cross section

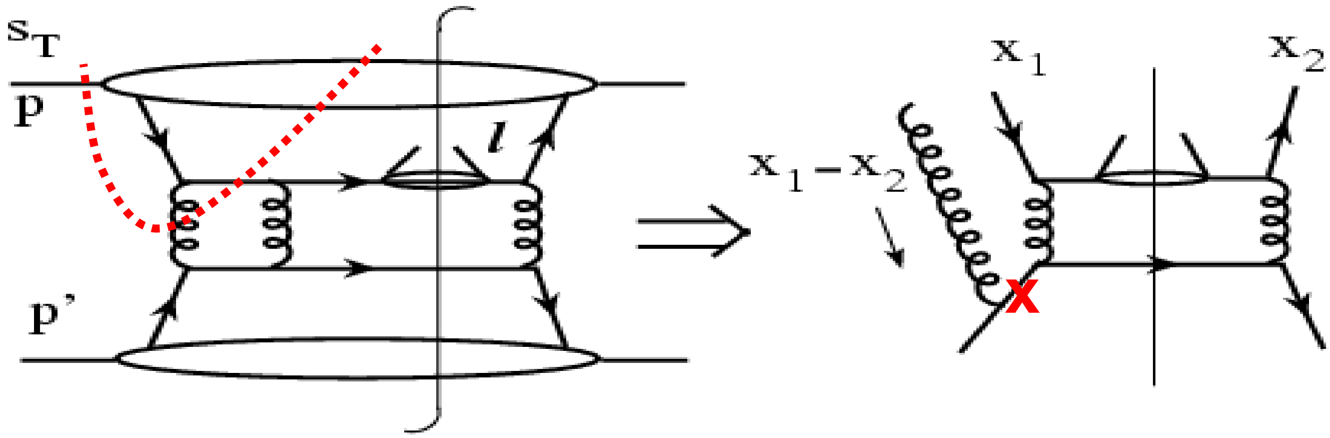
  ➡ Interference between amplitudes (a) and (b) or (c)

- The hadronic phase – the "i"

  ➡ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$

- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q\ \delta q(s_\perp)$

Leading contribution to A_N



- Unpinched pole to give the phase: $i\delta(x_1 - x_2)$

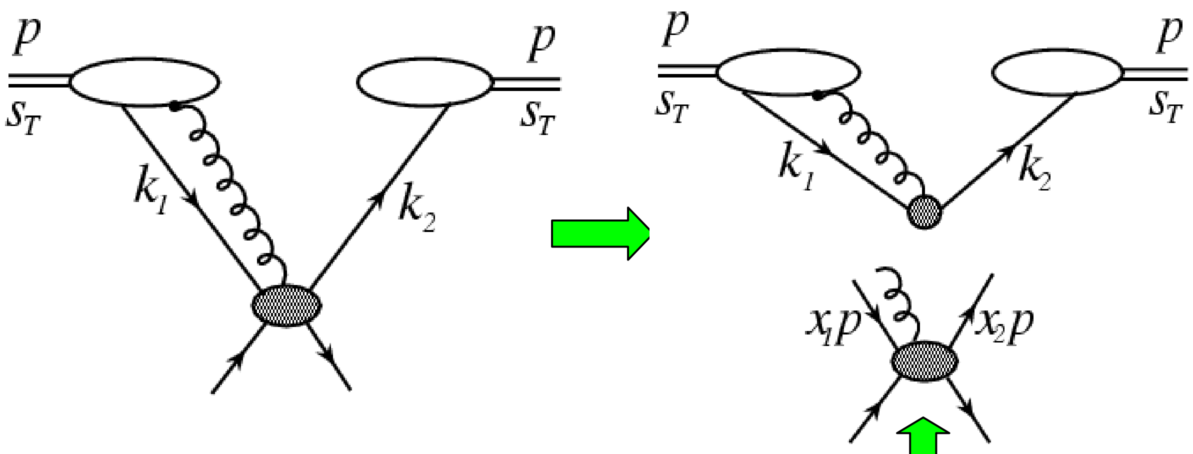
- **Spin flip** from interference between a quark state and a quark-gluon composite state

- Observed hadron momentum provides the 3rd vector

$$A_N \propto i\, \vec{s}_\perp \cdot (\vec{p} \times \vec{\ell})$$

A_N from polarized twist-3 correlations

❖ Factorization:



$$T_F(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

$$T_D(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp \psi \rangle$$

Normal twist-2 distributions

❖ Twist-3 correlation functions:

- $T_F(x_1, x_2)$ and $T_D(x_1, x_2)$ **have different properties under the P and T transformation**
- $T_D(x_1, x_2)$ **does not contribute to the A_N**
- $T_F(x_1, x_2)$ **is universal, $x_1 = x_2$ for A_N due to the pole**

Single spin asymmetry within the collinear factorization

❖ **Generic twist-3 factorized contributions**

$$\Delta\sigma_{AB\to h}(\vec{s}_T) = \sum_{abc} T^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes f_{b/B}(x') \otimes \hat{\sigma}_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z)$$
$$+ \sum_{abc} \delta q^{(2)}_{a/A}(x, \vec{s}_T) \otimes \left\{ f_{b/B}(x') \otimes \hat{\sigma}'_{ab\to c}(\vec{s}_T) \otimes D^{(3)}_{c\to h}(z_1, z_2) + f^{(3)}_{b/B}(x'_1, x'_2) \otimes \hat{\sigma}''_{ab\to c}(\vec{s}_T) \otimes D_{c\to h}(z) \right\}$$

Provides hadron spin dependence

transversity

(1) Calculated by Qiu and Sterman, Phys. Rev. D, 1999

(2) Calculated by Kanazawa and Koike, Phys. Lett. B, 2000

What is the $T^{(3)}(x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \times \langle P,\vec{s}_T|\bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \epsilon^{s_T\sigma n\bar{n}} F_\sigma^{\ +}(y_2^-)\right] \psi_a(y_1^-)|P,\vec{s}_T\rangle$$

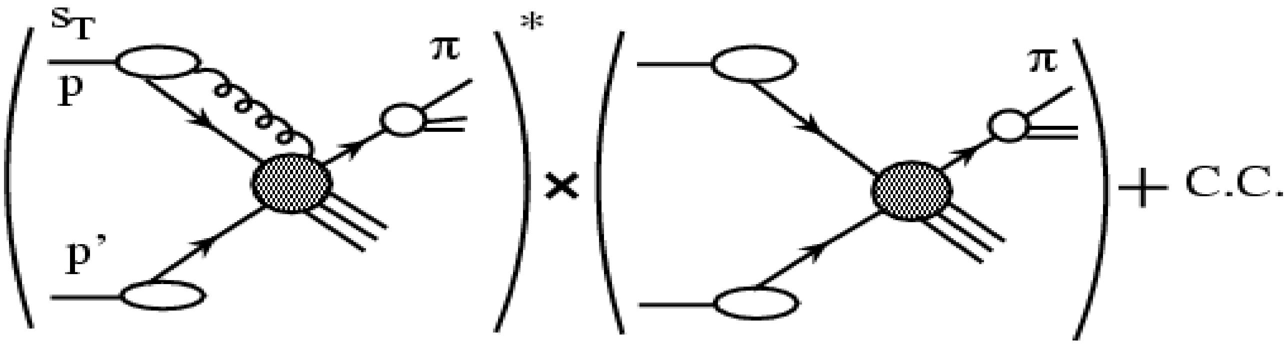
- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \langle P,\vec{s}_T|\bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-)|P,\vec{s}_T\rangle$$

T_F **Represents a fundamental quantum correlation between quark and gluon inside a hadron**

Leading twist-3 contribution to A_N

❖ **Minimal approach (within the collinear factorization):**



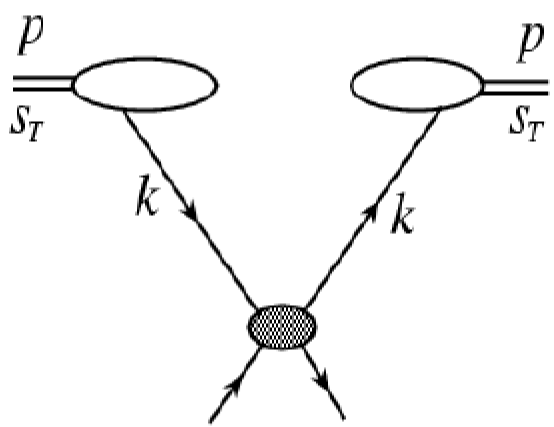
❖ Leading $(\partial/\partial x)T_F(x,x)$ contribution to the asymmetries

$$E\frac{d\Delta\sigma}{d^3\ell} \propto \epsilon^{\ell_T s_T n \bar{n}}\, D_{c\to\pi}(z) \otimes \left[-x\frac{\partial}{\partial x}T_F(x,x)\right]$$

$$\otimes \frac{1}{-\hat{u}}\left[G(x')\otimes\Delta\hat{\sigma}_{qg\to c} + \sum_{q'} q'(x')\otimes\Delta\hat{\sigma}_{qq'\to c}\right]$$

❖ $A_N \propto \left(\frac{\ell_\perp}{-\hat{u}}\right)\frac{n}{1-x}$ if $T_F(x,x)\propto q(x)\propto(1-x)^n$

Single transverse spin asymmetry – A_N
In k_T – factorization approach



$$k^\mu = xp^\mu + \frac{k^2 + k_\perp^2}{2xp\cdot n} n^\mu + k_\perp^\mu$$

If $|k^2| \ll |Q^2| \sim (xp)^2$, the parton state of momentum, *k*, lives much longer than the time scale of hard collision

$$\Rightarrow \int \frac{dx}{x} d^2k_\perp \; \mathrm{H}(Q, k^2 = 0) \int dk^2 \left(\frac{1}{k^2 + i\varepsilon}\right)\left(\frac{1}{k^2 - i\varepsilon}\right) \mathrm{T}(k, \frac{1}{r_0})$$

Transverse momentum dependent (TMD) PDFs

$$\varphi_q(x, k_\perp, \vec{s}_\perp)$$

A_N – generated from initial state

❖ Sivers' function: $q_T(x, k_\perp)$

$$\varphi_q(x, k_\perp, \vec{s}_\perp) - \varphi_q(x, k_\perp, -\vec{s}_\perp) = q_T(x, k_\perp)\ \varepsilon_{\mu\nu\rho\sigma} \frac{\gamma^\mu n^\nu k_\perp^\rho s_\perp^\sigma}{M}$$

Sivers's function is a unknown nonperturbative function

❖ Spin-dependence of the cross section:

$$\Delta\sigma(k_\perp, s_\perp) \propto [\varphi_q(x, k_\perp, \vec{s}_\perp) - \varphi_q(x, k_\perp, -\vec{s}_\perp)] \otimes D(z)$$

$$\propto \left(\varepsilon_{\mu\nu\rho\sigma} P^\mu n^\nu k_\perp^\rho s_\perp^\sigma\right)\ q_T(x, k_\perp) \otimes D(z)$$

Single transverse-spin asymmetry is generated or parameterized by the Sivers' function

A_N – generated from final state

❖ Collins' function: $D_T(z, k_\perp)$

$$D(z,k_\perp,\vec{s}_\perp) + D(x,k_\perp,-\vec{s}_\perp) = D_T(z,k_\perp)\ \sigma_{\mu\nu}\frac{k_\perp^\mu \overline{n}^\nu}{M}$$

Collins' function represents a fragmentation to a hadron from a sum of polarized partons

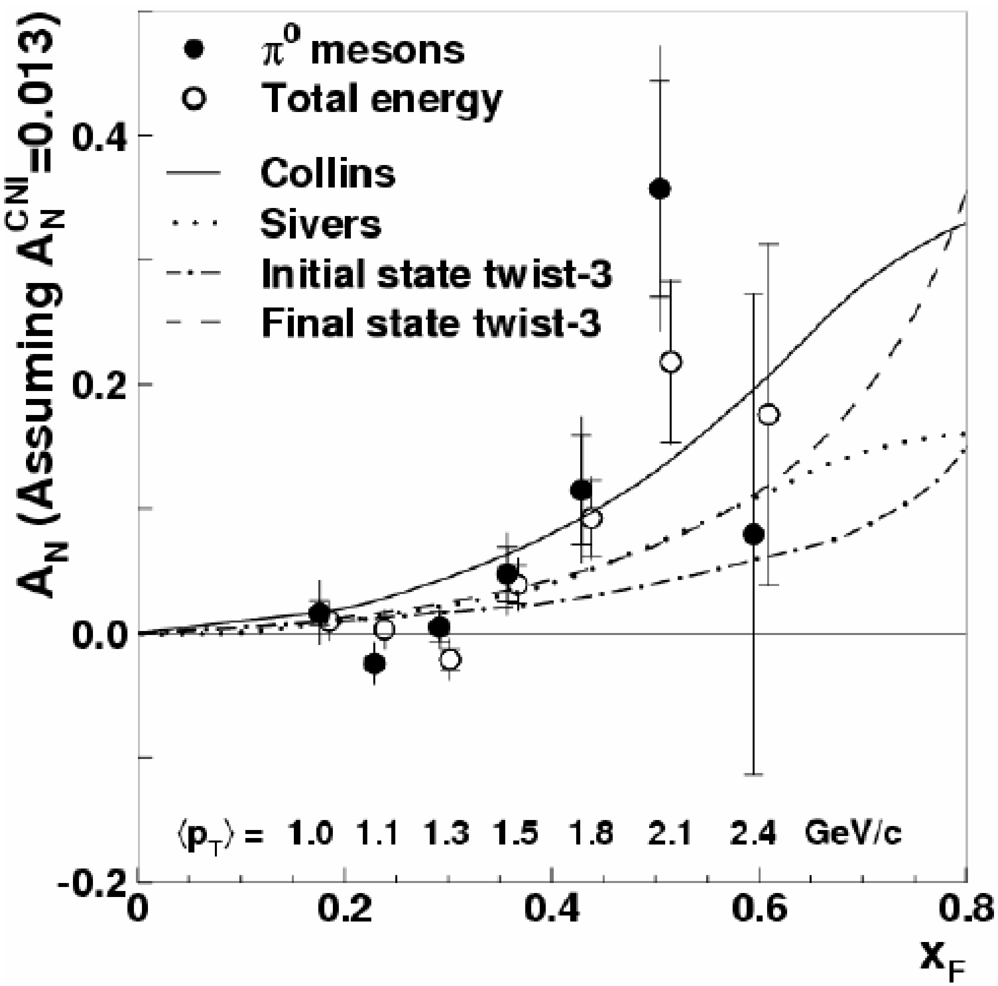
❖ Spin-dependence of the cross section:

$$\Delta\sigma(k_\perp,s_\perp) \propto \delta q(x,\vec{s}_\perp) \otimes [D(z,k_\perp,\vec{s}_\perp) + D(z,k_\perp,-\vec{s}_\perp)]$$

$$\propto \left(\sigma_{\mu\nu}k_\perp^\mu \overline{n}^\nu\right)\ \delta q(x,\vec{s}_\perp) \otimes D_T(z,k_\perp)$$

Single transverse-spin asymmetry is generated or parameterized by the Collins' function

Numerical results – “Predictions”

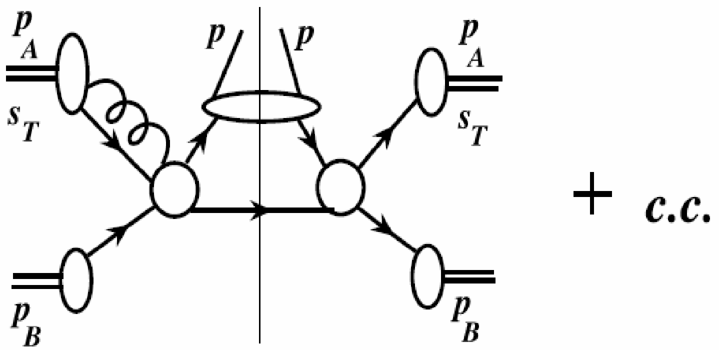


- Comparison with STAR data
- Too small P_T value to be comfortable for twist-3 calculation
- Is the k_T-factorization valid for this case
- Which mechanism is correct, any overlap?

Collinear twist-3 vs. k_T - approach

❑ Twist-3 contribution in collinear factorization

– leading corrections from parton correlation (a minimal approach)



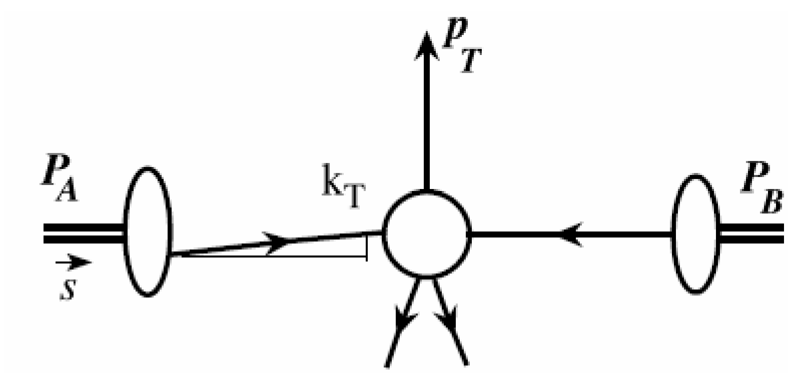
$+ \; c.c.$

$$A_N \propto \frac{1}{S} \varepsilon^{p_A p_B s_T p_T} \frac{1}{T} \frac{-x \frac{\partial}{\partial x} T^{(3)}(x)}{\phi(x)}$$

$$\Rightarrow \frac{1}{p_T} \left(-x \frac{\partial}{\partial x} T^{(3)}(x) \right)$$

$$\frac{d\Delta\sigma}{dy dp_T^2} = \sum_{abc} T^{(3)}_{a/A}(x_1) \otimes \phi_{b/B}(x_2) \otimes H_{abc} \otimes D_c(z) + \ldots$$

❑ Effect of non-vanish parton k_T (when $k_T \sim p_T$):



$$A_N \propto \frac{1}{S} \varepsilon^{p_A p_B s_T p_T} \frac{1}{M} f^{\perp}(x) \Rightarrow \frac{p_T}{M}$$

M = Non-perturbative scale, e.g., di-quark mass, …

Existing QCD based mechanisms

- In collinear factorization, SSAs are generated by
  - multi-parton correlation functions – initial-state
  - multi-parton fragmentation functions – final-state

- In k_T factorization, SSAs are generated by
  - Sivers function – initial state
  - Collins function – final state

- Any connections between these mechanisms?

Yes. They are expected to describe the same physics in any region where they are both applicable

A_N in Drell-Yan lepton-pair production

$$\frac{d\Delta\sigma(s_\perp)}{dQ^2 dy d^2 q_\perp} \quad \text{with} \quad \Delta\sigma(s_\perp) = \frac{1}{2}\left[\sigma(s_\perp) - \sigma(-s_\perp)\right]$$

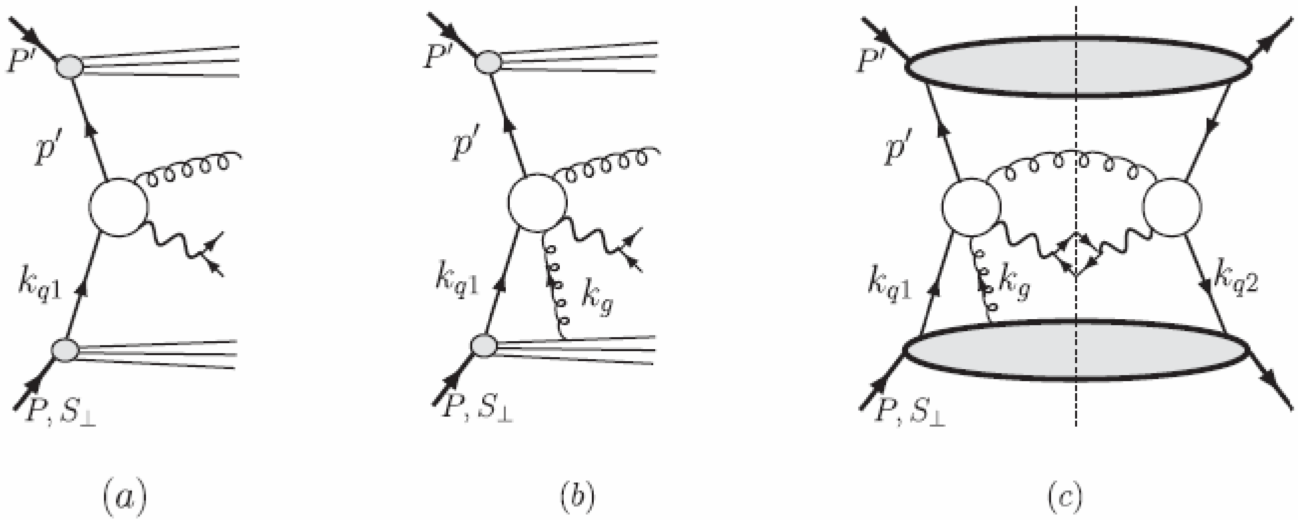
- **Collinear factorization is valid if** $Q^2,\ q_\perp^2 \gg \Lambda_{\rm QCD}$
- **The k_T - factorization is valid if** $Q^2 \gg q_\perp^2$

➡ **Both factorizations should work if**

$$Q^2 \gg q_\perp^2 \gg \Lambda_{\rm QCD}$$

➡ **Calculation A_N in both factorization schemes, And expect same results in the overlap region**

A_N in collinear factorization – (I)



- **A_N – interference of Re[(a)] and Im[(b)]:**

- **Imaginary part of amplitude (b):** $T_F(x_1, x_2, s_\perp)$

  k_g integration is fixed by an unpinched pole: soft, hard

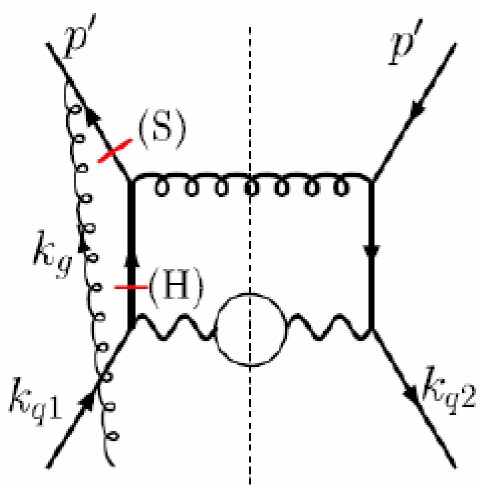
- **Extract twist-3 quark-gluon correlation:**

  convert gluon field to corresponding field strength

A_N in collinear factorization – (II)

$$\frac{d^4\Delta\sigma(S_\perp)}{dQ^2dyd^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_{\perp\alpha}q_{\perp\beta}\frac{\alpha_s}{2\pi^2}\int\frac{dx}{x}\frac{dx'}{x'}$$
$$\times\sum_q e_q^2[(H_q^s + H_q^h)\bar{q}(x') + (H_g^s + H_g^h)g(x')]$$
$$\times\,\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$$

- $H_{q,g}^s$ and $H_{q,g}^h$ **are soft and hard pole contributions**



$$H_q^s = \left[x\frac{\partial}{\partial x}T_F(x,x)\right]\frac{D_{q\bar{q}}^s}{-\hat{u}} + T_F(x,x)\frac{N_{q\bar{q}}^s}{-\hat{u}}$$

$$D_{q\bar{q}}^s = \frac{1}{2(N_C^2-1)}\hat{\sigma}_{q\bar{q}}(\hat{s},\hat{t},\hat{u}),$$

$$N_{q\bar{q}}^s = \frac{1}{2N_C}\frac{1}{\hat{t}^2\hat{u}}[Q^2(\hat{u}^2-\hat{t}^2) + 2Q^2\hat{s}(Q^2-2\hat{t}) - (\hat{u}^2+\hat{t}^2)\hat{t}],$$

Limit of $Q \gg Q_T \gg \Lambda_{QCD}$

❖ **Mandelstam variables:**

$$\hat{s} = \frac{q_\perp^2}{(1-\xi_1)(1-\xi_2)} \qquad \hat{t} = -\frac{q_\perp^2}{1-\xi_2} \qquad \hat{u} = -\frac{q_\perp^2}{1-\xi_1}$$

$$\xi_1 = z_1/x \qquad \xi_2 = z_2/x' \qquad z_1 = Q/\sqrt{s}e^{y} \qquad z_2 = Q/\sqrt{s}e^{-y}$$

❖ **On-shell delta-function:**

$$\begin{aligned}\delta(\hat{s}+\hat{t}+\hat{u}-Q^2) &= \delta(\hat{s}(1-\xi_1)(1-\xi_2)-q_\perp^2) \\ &= \frac{1}{\hat{s}}\left[\frac{\delta(\xi_2-1)}{(1-\xi_1)_+} + \frac{\delta(\xi_1-1)}{(1-\xi_2)_+}\right. \\ &\quad \left. + \delta(\xi_1-1)\delta(\xi_2-1)\ln\frac{Q^2}{q_\perp^2}\right]\end{aligned}$$

Asymptotic Results

- quark-antiquark annihilation:

$$\frac{d^4\Delta\sigma^{q\bar{q}\to\gamma^* g}(S_\perp)}{dQ^2\,dy\,d^2q_\perp} = \sigma_0 \epsilon^{\alpha\beta} S_{\perp\alpha} \frac{q_{\perp\beta}}{(q_\perp^2)^2} \frac{\alpha_s}{2\pi^2} \int \frac{dx}{x}\frac{dx'}{x'} \times \bar{q}(x')\{\delta(\xi_2 - 1)A + \delta(\xi_1 - 1)B\}$$

where

$$A = \frac{1}{2N_C}\left\{\left[x\frac{\partial}{\partial x}T_F(x,x)\right](1+\xi_1^2) + T_F(x, x-\hat{x}_g) \times \frac{1+\xi_1}{(1-\xi_1)_+} + T_F(x,x)\frac{(1-\xi_1)^2(2\xi_1+1)-2}{(1-\xi_1)_+}\right\} + C_F T_F(x, x-\hat{x}_g)\frac{1+\xi_1}{(1-\xi_1)_+},$$

$$B = C_F T_F(x,x)\left[\frac{1+\xi_2^2}{(1-\xi_2)_+} + 2\delta(\xi_2 - 1)\ln\frac{Q^2}{q_\perp^2}\right]$$

A_N in k_T – factorization – (I)

- Factorized formula in $v \cdot A = 0$ gauge:

$$\frac{d^4\Delta\sigma(S)}{dQ^2 dy d^2 q_\perp} = \sigma_0 \epsilon^{\alpha\beta} S_{\perp\alpha} q_{\perp\beta} \frac{1}{M_P} \int d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{\lambda}_\perp \times \frac{\vec{k}_{1\perp}\cdot\vec{q}_\perp}{q_\perp^2} \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_\perp - \vec{q}_\perp) \times q_T(z_1, k_{1\perp}, \zeta_1)\bar{q}(z_2, k_{2\perp}, \zeta_2) H(Q^2) \times (S(\lambda_\perp))^{-1}$$

$$\zeta^2 = (2v\cdot P)^2/v^2$$

- Need to calculate at $q_\perp \gg \Lambda_{\text{QCD}}$
  - Sivers's function $q_T(x, k_\perp)$
  - unpolarized k_T-dependent (anti)quark PDFs $\bar{q}(x, k_\perp)$
  - Soft factor $S(k_\perp)$

A_N in k_T – factorization – (II)

- Expand transverse momenta in $\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_\perp - \vec{q}_\perp)$ to the first power

- Use the one-loop moment relations:

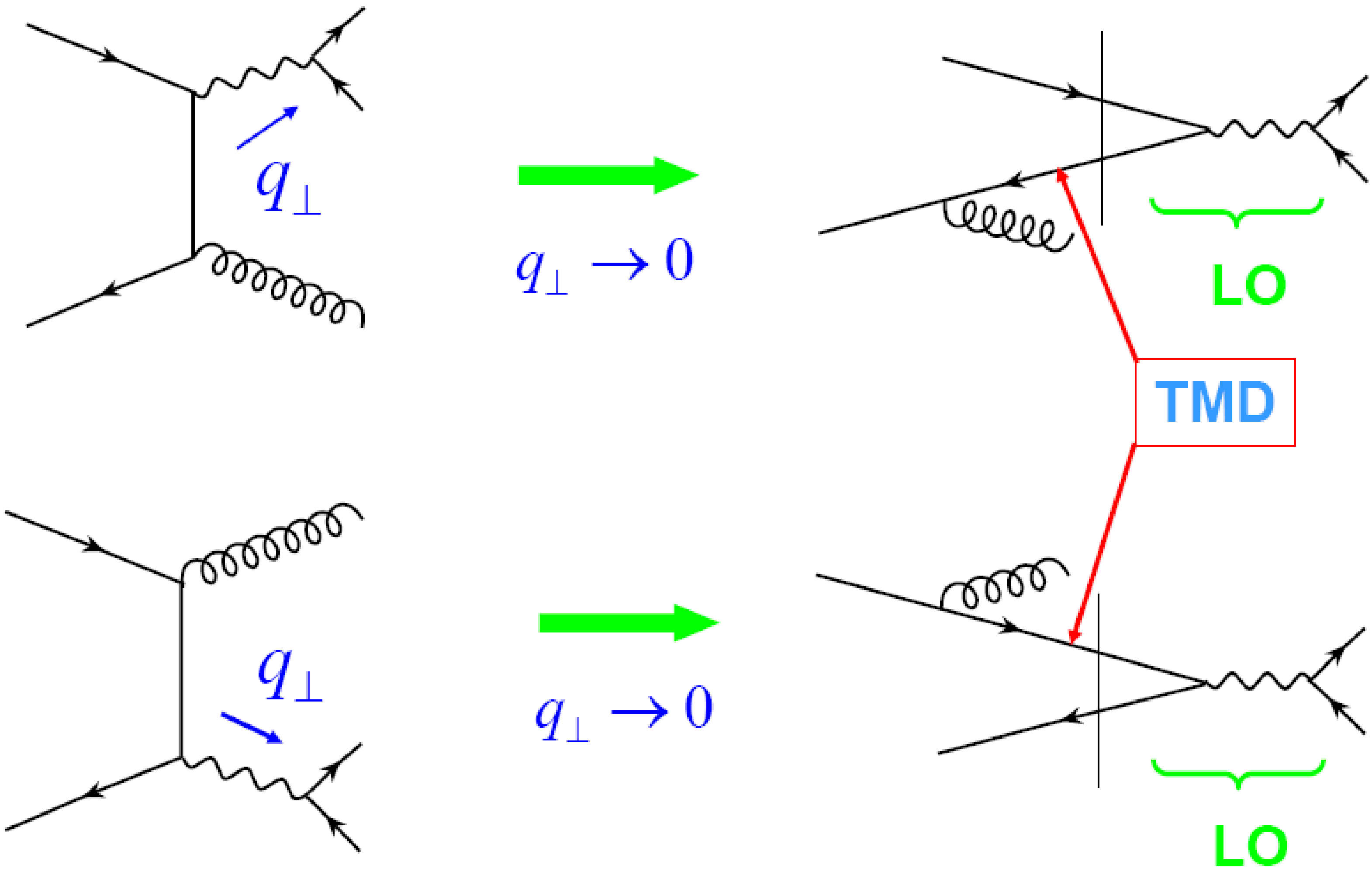
$$\int d^2\vec{k}_\perp q(z_1, k_\perp) = q(z_1), \qquad \int d^2\vec{k}_\perp \bar{q}(z_2, k_\perp) = \bar{q}(z_2)$$

$$\frac{1}{M_P}\int d^2\vec{k}_\perp \vec{k}_\perp^2 q_T(x, k_\perp) = T_F(x, x) \qquad \int d^2\vec{\lambda}_\perp S(\lambda_\perp) = 1$$

Spin-dependent cross section calculated in k_T-factorization approach is the same as the asymptotic limit of what calculated in collinear factorization

Is this matching obvious? No!

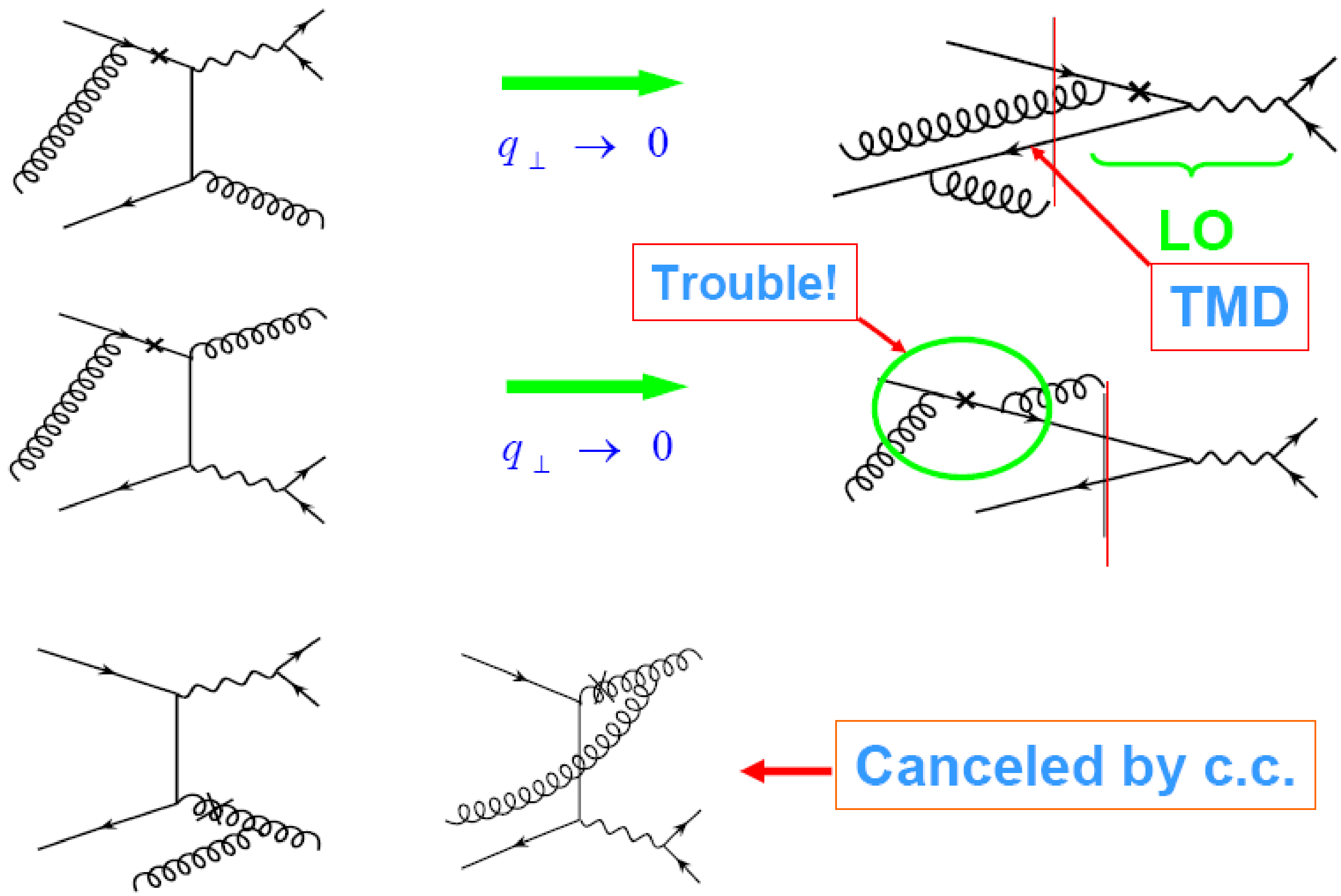
Spin-averaged Leading Twist



also need a soft part

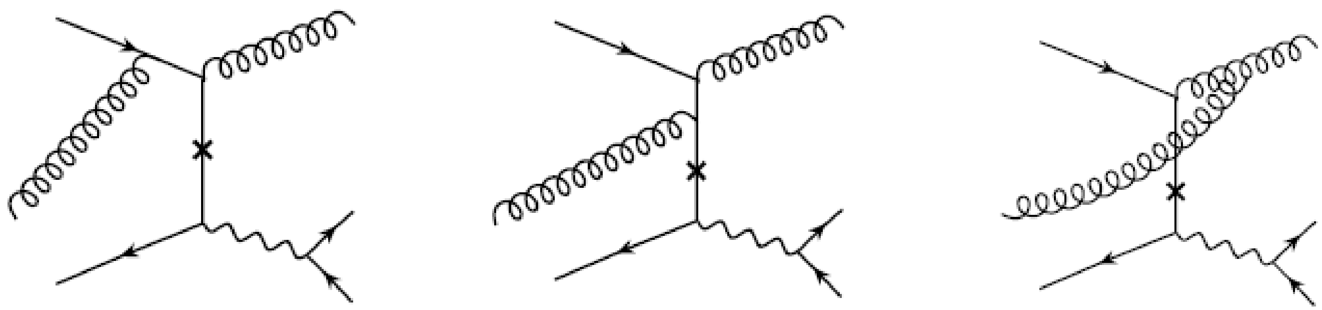
Spin-dependent – "Soft Pole"

$\int dk_g$ – fixed by a unpinched pole $\rightarrow k_g = 0$



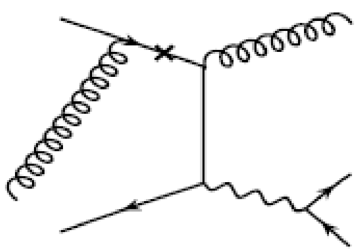
Spin-dependent – "Hard Pole"

$$\int dk_g \text{ – fixed by a unpinched pole } \rightarrow k_g \neq 0$$

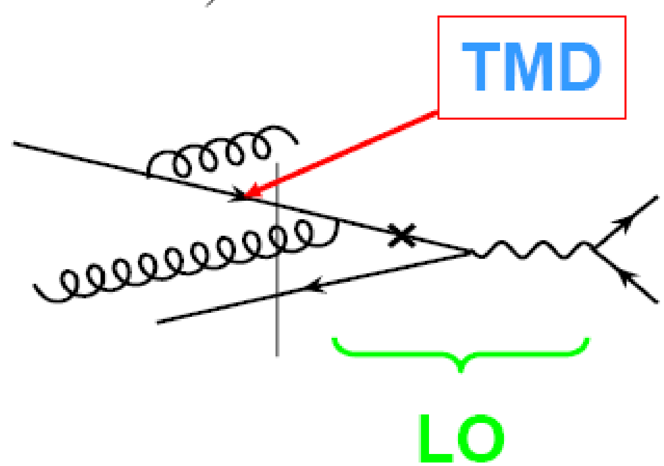


$$= \left(q+g \rightarrow q+g\right) \otimes \left(q+\bar{q} \rightarrow \gamma^*\right)$$

$+$

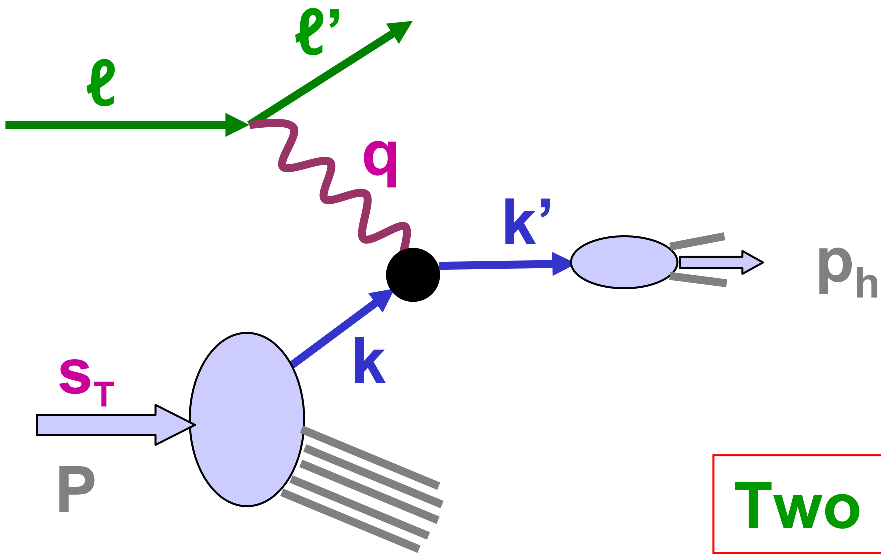


$q_\perp \rightarrow 0$

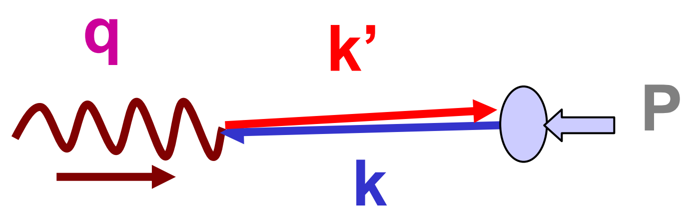


Semi-inclusive DIS (SIDIS)

❑ **Process:**



Breit frame:



Two scale problem: $Q^2=-q^2$, p_T

❖ **Fixed order pQCD:** $Q \sim p_T \gg \Lambda_{\rm QCD}$

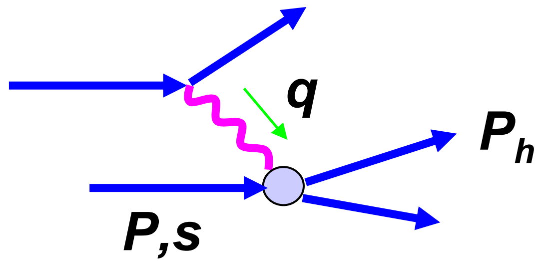
❖ **Single spin asym:** $A_N \propto \vec{s}_T \cdot (\vec{P} \times \vec{p}) \Rightarrow 0$

If P is anti-parallel to p_h

❖ **When** $Q \gg p_T$, **p_T sensitive to parton k_T**

A_N for Semi-inclusive DIS (SIDIS)

❑ **SIDIS cross section:**



$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \frac{2\pi\alpha_{\rm em}^2}{Q^4} y L_{\mu\nu}(\ell,q) W^{\mu\nu}(P,S_\perp,q,P_h)$$

$$z_h \equiv P\cdot P_h / P\cdot q \qquad y \equiv P\cdot q / P\cdot \ell$$

$$L^{\mu\nu}(\ell,q) \;=\; 2\left(\ell^\mu \ell'^\nu + \ell^\mu \ell'^\nu - g^{\mu\nu} Q^2/2\right)$$

$$W^{\mu\nu}(P,S_\perp,q,P_h) = \frac{1}{4z_h}\sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \langle PS|J_\mu(\xi)|XP_h\rangle\langle XP_h|J_\nu(0)|PS\rangle$$

❑ **Polarized SIDIS cross section:**

$$\Delta\sigma(S_\perp) = [\sigma(S_\perp) - \sigma(-S_\perp)]/2$$

Enough vectors to form the invariant

Explicit model calculation ⟹ $A_N^{SIDIS} \neq 0$ **Brodsky et al**

Hadronic tensor and structure functions

- Virtual photon momentum:

$$q^\mu = q_t^\mu + \frac{q\cdot P_h}{P\cdot P_h} P^\mu + \frac{q\cdot P}{P\cdot P_h} P_h^\mu$$

with $q_t^\mu P_\mu = q_t^\mu P_{h\mu} = 0$

- Independent structure functions:

$$W^{\mu\nu} = \sum_{i=1}^{5} \mathcal{V}_i^{\mu\nu} W_i \qquad W_i = W_{\alpha\beta} \tilde{\mathcal{V}}_i^{\alpha\beta}$$

With five parity and current conserving tensors $\mathcal{V}_i^{\mu\nu}$

and five corresponding inverse tensors $\tilde{\mathcal{V}}_i^{\alpha\beta}$

- Conserving tensors:

$$\mathcal{V}_1^{\mu\nu} = X^\mu X^\nu + Y^\mu Y^\nu$$

$$\tilde{\mathcal{V}}_1^{\mu\nu} = \frac{1}{2}\left(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu\right)$$

All other tensors give the Sub-leading term when q_T << Q

❑ Orthonormal basis vectors:

$$T^\mu = \frac{1}{Q}\left(q^\mu + 2x_B P^\mu\right) \qquad X^\mu = \frac{1}{q_\perp}\left[\frac{P_h^\mu}{z_h} - q^\mu - \left(1 + \frac{q_\perp^2}{Q^2}\right) x_B P^\mu\right]$$

$$Y^\mu = \epsilon^{\mu\nu\rho\sigma} Z_\nu X_\rho T_\sigma \qquad Z^\mu = -\frac{q^\mu}{Q} \qquad \vec{q}_\perp^{\,2} \equiv -q_t^2$$

With normalization: $T^\mu T_\mu = 1, X^\mu X_\mu = Y^\mu Y_\mu = Z^\mu Z_\mu = -1$

❑ Leading result when q_T << Q:

$$\frac{d\Delta\sigma(S_\perp)}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = -\frac{4\pi\alpha_{\rm em}^2 S_{ep}}{Q^4}\epsilon^{\alpha\beta} S_\perp^\alpha \frac{z_h P_{h\perp}^\beta}{(\vec{P}_{h\perp}^2)^2}\frac{\alpha_s}{2\pi^2}\int\frac{dxdz}{xz}\hat{q}(z)$$

$$\times\left\{\delta(\hat{\xi}-1)A + \delta(\xi-1)B\right\} ,$$

Where A and B are the same as those for Drell-Yan

But, the asymmetry has an opposite sign due to the expected sign difference between the Sivers functions

Summary and outlook

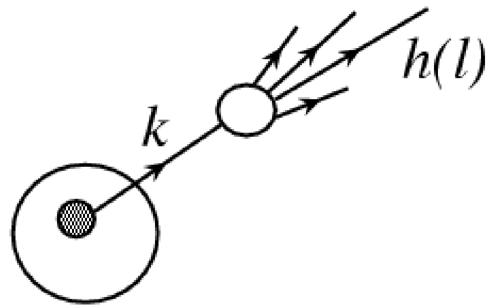
- Two mechanisms for generating single transverse-spin asymmetry are closely connected
- They describe the same physics in the region where they are both valid
  – an important constraint to phenomenological fits to data
- What should we use for hadronic processes where QCD factorization may not be valid
- "First" test of QCD beyond the leading twist level
- …

Backup transparencies

When does the factorization lose its predictive power?

At the time when the nonperturbative functions lose their universality

❖ For final-state fragmentation:



Factorization breaks if the fragmentation took place inside the hadronic medium

⟺ **If the lifetime of the parton state of momentun *k* is shorter than the medium size**

Lifetime: $$\Delta y_\perp \sim \Delta t \sim \frac{1}{\Delta E} \sim \frac{2\ell_\perp}{m_{jet}^2} \gg L_{medium} \sim 2r_0 \sim 2\ \text{fm}$$

⟹ $$\ell_\perp \gg 5\ \text{GeV}\left(\frac{m_{jet}^2}{\text{GeV}^2}\right)$$

Open questions – (I)

❖ How much overlap between k_T-approach at low p_T and twist expansion at high p_T?

	k_T approach	Twist-3
Sivers:	$f_{1T}^{\perp}(x)$	$T_F(x,x)$
Collins:	$H_1^{\perp}(z)$	$D^{(3)}(z,z)$

❖ What these nonperturbative functions try to tell us?

"direct k_T" vs "k_T – moments"

Seperate Sivers' and Collins' functions

Transversely polarized target

Sivers	Collins
$\langle \sin(\phi - \phi_s) \rangle$ moment	$\langle \sin(\phi + \phi_s) \rangle$ moment
$\downarrow$	$\downarrow$
$f_{1T}^{\perp}(x)$	$h_1(x)$, $H_1^{\perp}(z)$

ϕ_S : Angle between $\vec{S}_\perp$ and L.P.

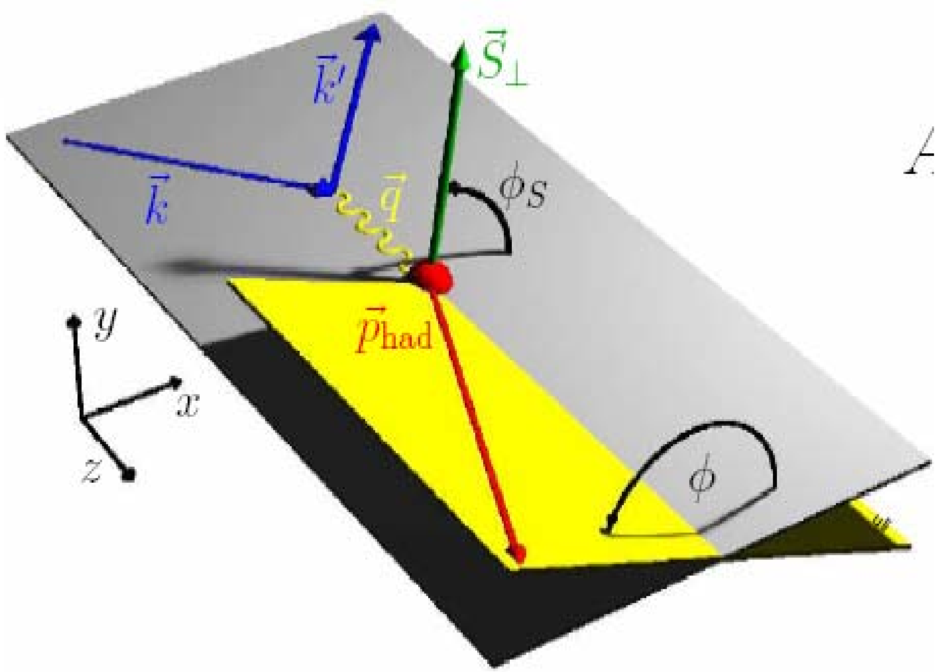
ϕ : Angle between L.P. and H.P.

Measure Sivers' and Collins' functions

- **SIDIS:** $ep^{\uparrow} \longrightarrow e'\pi X$

study azimuthal distribution of π's:

with transversely polarized target:
(unpolarized beam)



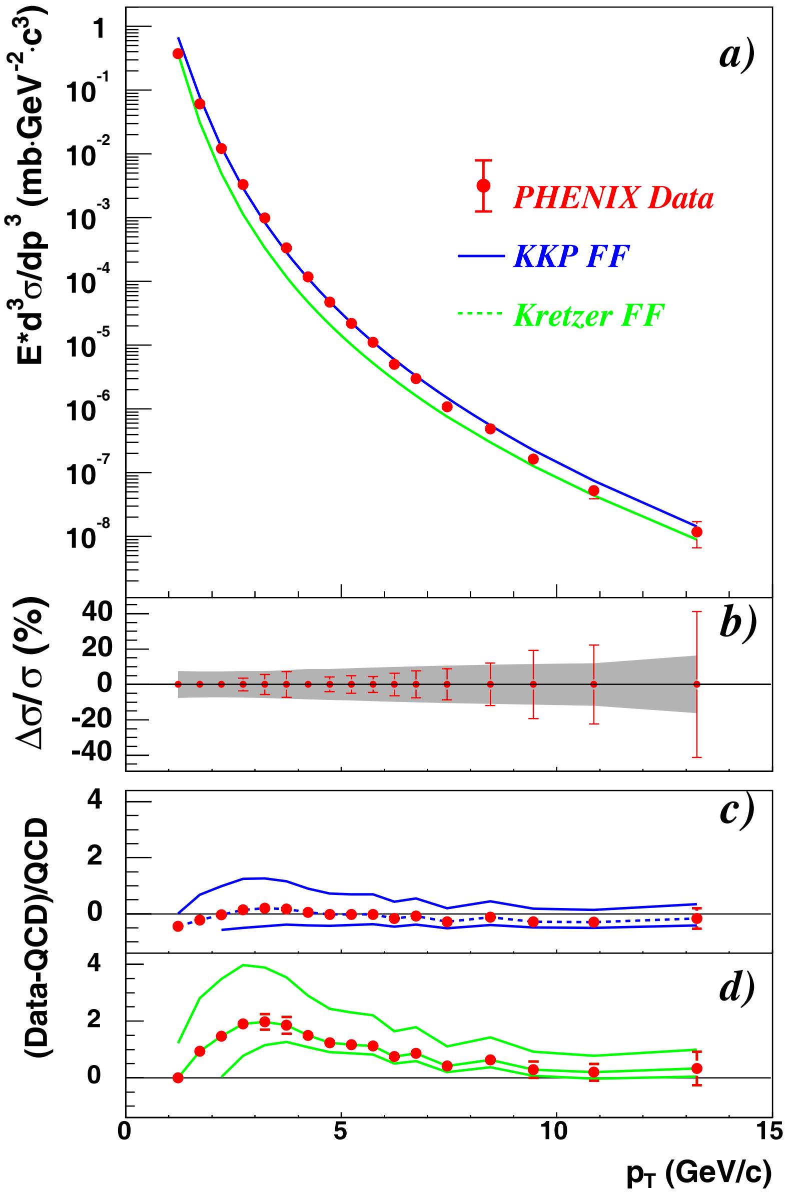
- **Collins functions:**

$$A_{UT}^{\mathrm{Sin}\Phi} = \sum_q \frac{e_q^2 \delta q(x) H_1^{\perp}(z)}{e_q^2 q(x) D(z)}$$

$\Phi = \phi + \phi_S$ Collins angle

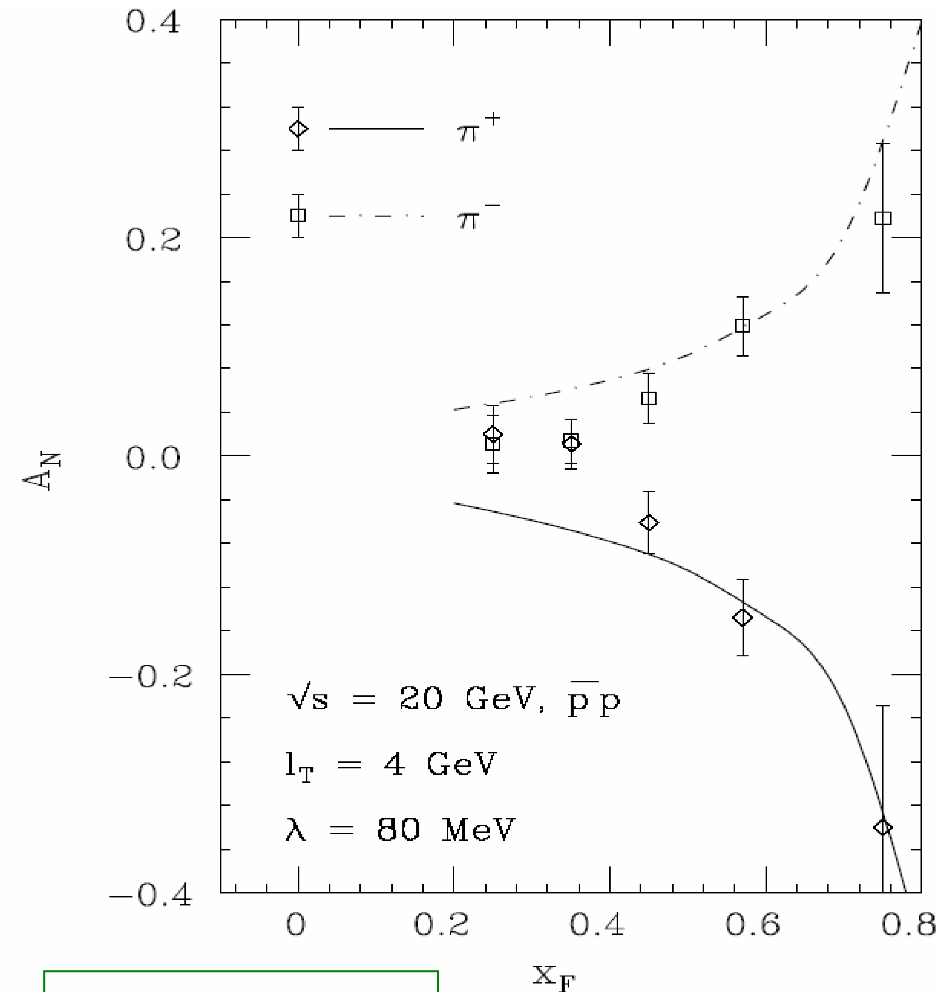
Initial success of RHIC pp runs

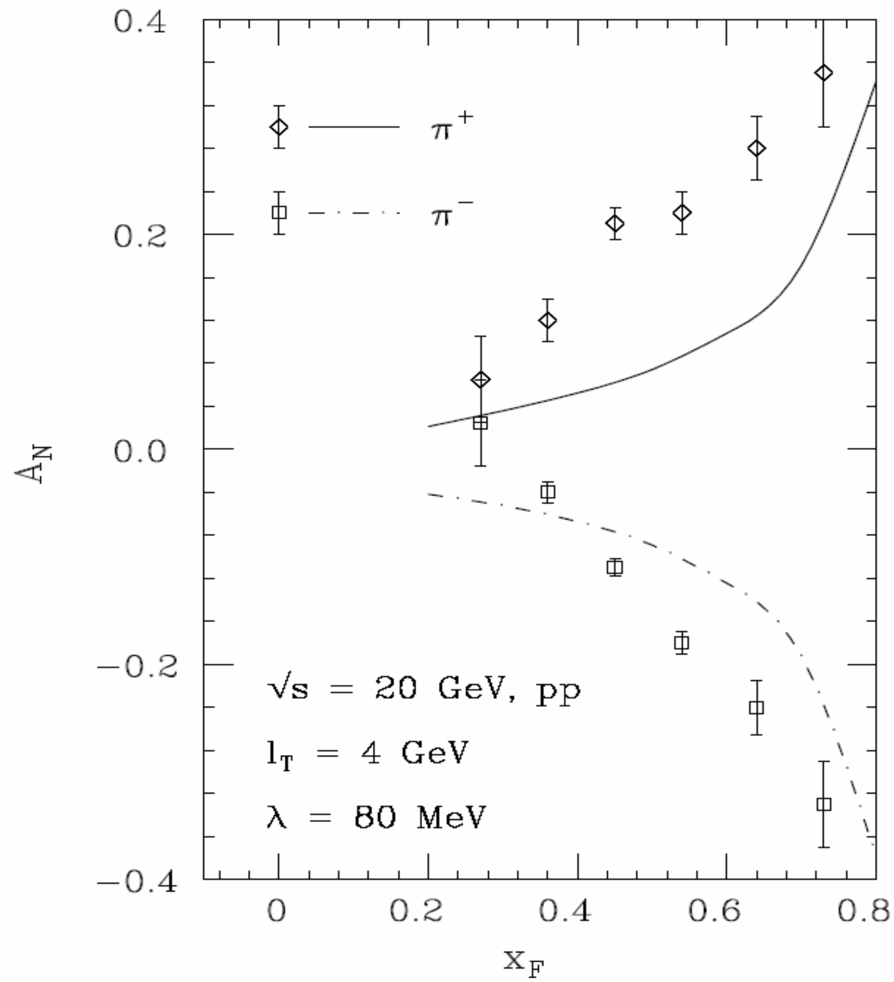
- π^0 cross section measured over 8 order of magnitude [PRL 91, 241803 (2003)]
- Good agreement with NLO pQCD calculation at low p_T
- Can be used in interpretation of spin-dependent results



9.6% normalization error not shown

Numerical results – (I)
(compare apples with oranges)

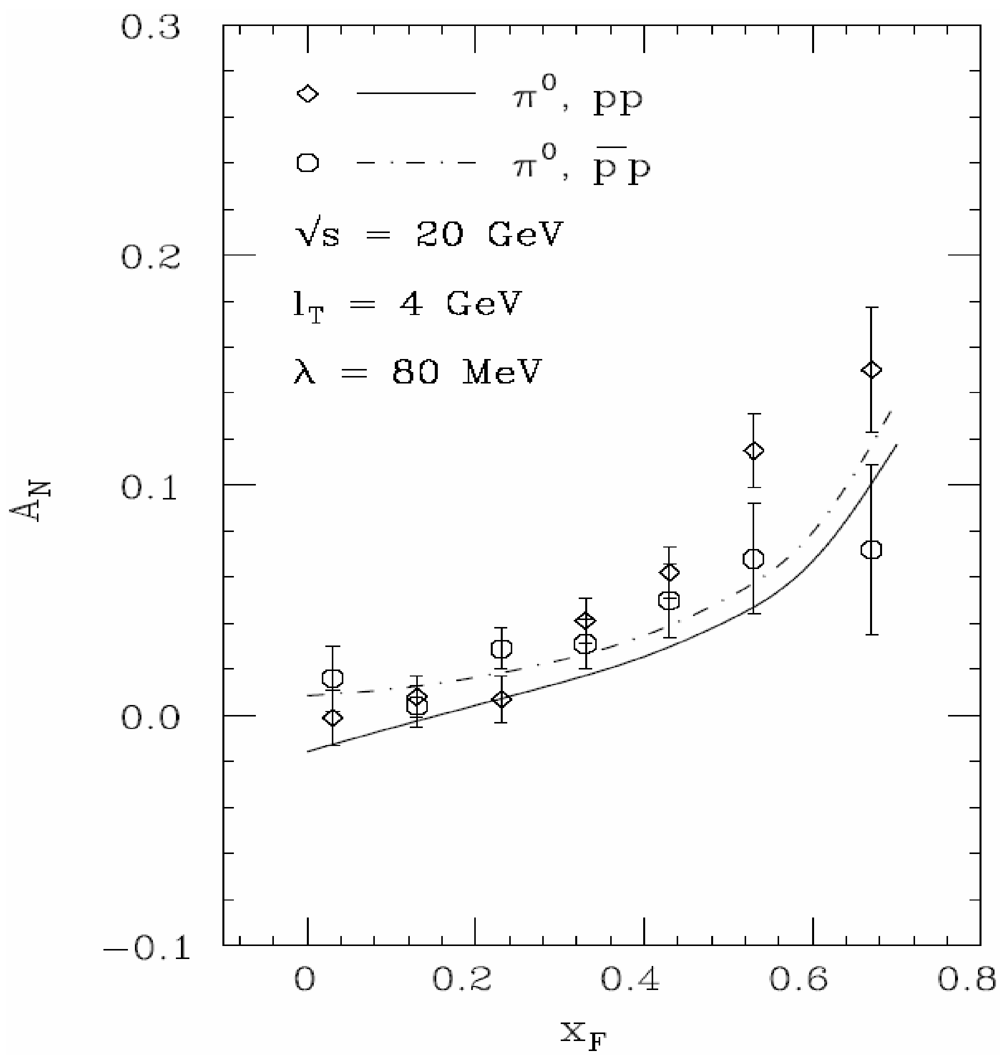




Qiu and Sterman
Phy. Rev. D, 1999

Fermilab data with ℓ_T up to 1.5 GeV

Numerical results – (II)
(compare apples with oranges)



Qiu and Sterman
Phy. Rev. D, 1999

Model for $T_F(x,x)$

- $T_F\,(x,x)$ **tells us something about quark's transverse motion in a transversely polarized hadron**

- **It is non-perturbative, has unknown x-dependence**

$$T_F(x,x) \propto \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Model for $T_F(x,x)$ of quark flavor a:

$$T_{F_a}(x,x) \equiv \kappa_a \, \lambda \, q_a(x)$$

with $\kappa_u = +1$ and $k_d = -1$ for proton

Fitting parameter $\lambda \sim O(\Lambda_{\rm QCD})$

$\Rightarrow$

$$A_N \propto \left(\frac{\ell_\perp}{-\hat{u}} \right) \frac{n}{1-x}$$

if $T_F(x,x) \propto q(x) \propto (1-x)^n$

One parameter and one sign!

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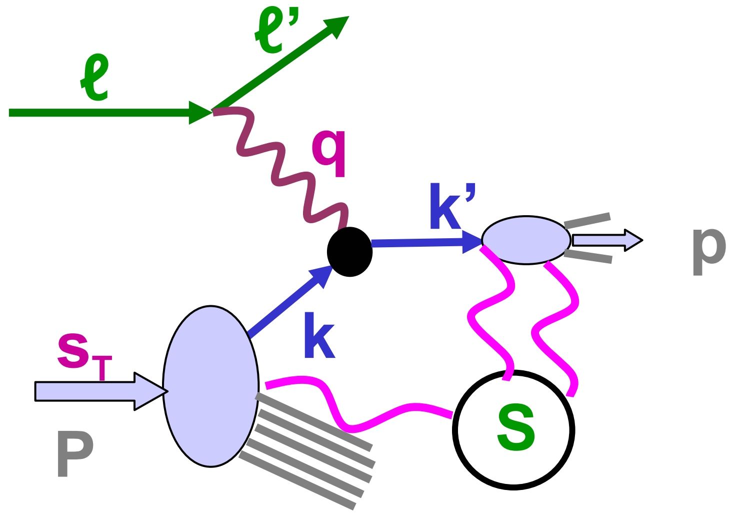
Intrinsic vs dynamical k_T



In q-P frame, if $k_T \sim p_T \ll Q$

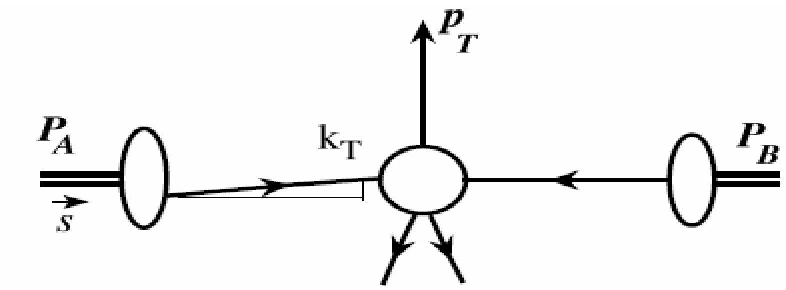
- we can neglect k^2 in partonic part
- But, we cannot neglect k_T in partonic part

$$k^\mu = xP^\mu + k_\perp^\mu + \frac{k_T^2}{2k\cdot n} n^\mu$$

- One can define k_T-dependent and gauge invariant parton distributions
- Soft interaction between the hadrons can spoil factorization
- Sudakov resummation (done in b- or k_T-space) resums dynamical k_T from gluon shower
- Parton orbital motion is more relevant to the intrinsic k_T

K_T - Factorization

- k_T-factorization measures parton k_T directly, while twist-expansion gives integrated k_T information
- No formal proof of k_T-factorization for hadronic collisions at $k_T \sim p_T$



$Q \sim p_T >> k_T$

- Factorization requires a separation of perturbative hard scale from nonperturbative hadronic scale
  ⟹ a physical hard scale, Q, much larger than the k_T
- k_T-factorization works for semi-inclusive DIS and Drell-Yan, or others with a large scale Q